

Power finding

$$2^m = 2^{m/2} \cdot 2^{m/2}$$

$a = 2, b = 64$

$$2^{64} = 2^{32} \cdot 2^{32}$$

$$\begin{array}{c} 2^{64} \\ \diagdown \quad \diagup \\ 2^{32} \quad 2^{32} \end{array}$$

$$\begin{array}{c} 2^{32} \\ \diagdown \quad \diagup \\ 2^{16} \quad 2^{16} \\ a = 2, \underline{b = 65} \end{array}$$

$$\begin{array}{c} 2^{16} \\ \diagdown \quad \diagup \\ 2^8 \times 2^8 = 2^{16} \\ 2^8 \\ 2^4 \end{array}$$

$$\begin{array}{c} 2^4 \\ \diagdown \quad \diagup \\ 2^2 \times 2^2 = 2^4 \end{array}$$

$$\begin{array}{c} 2^2 \\ \diagdown \quad \diagup \\ 2^1 \times 2^1 = 2^2 \end{array}$$

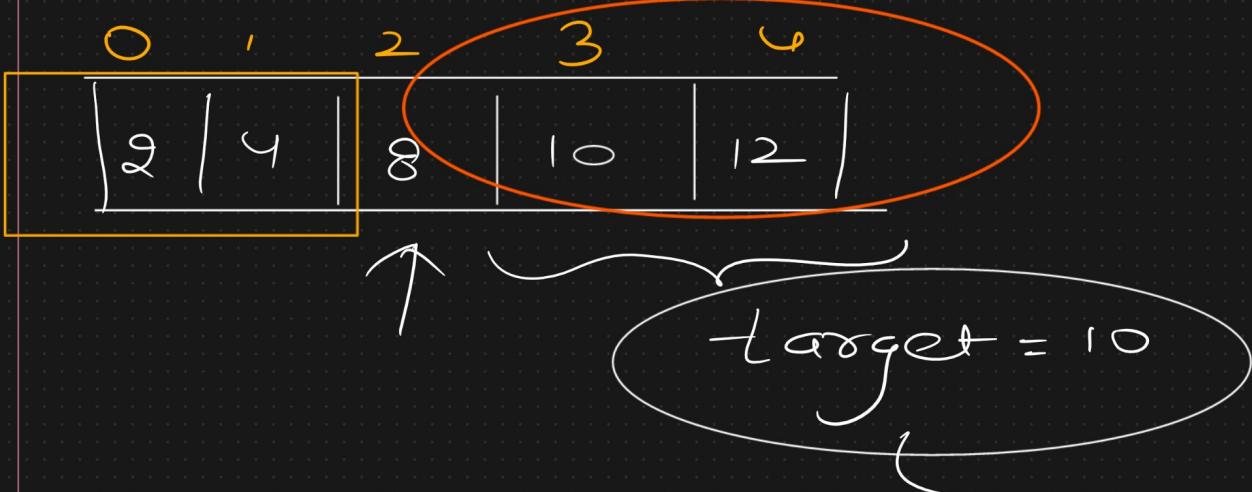
$$\begin{array}{c} 2^{65} = \textcircled{2} \times \textcircled{2^{64}} \\ \textcircled{2} \times \underline{\textcircled{2^{64}}} \\ a \times \underline{= \text{Recursion}} \end{array}$$

$$2^{67} = 2^* 2^{66}$$

$$\begin{array}{c}
 2^{66} \\
 \diagdown \qquad \diagup \\
 2^{33} \qquad 2^{32} \qquad 2^{33}
 \end{array}$$

Big integers

Binary Search



Output = 3

Recursion

$$\text{low} = 0, \text{high} = n - 1 \\ = 4$$

function name

$$\text{mid} = 2$$

binarySearch (arr, low, high, target)

While ($\text{low} <= \text{high}$) \wedge

base case condition

if ($\text{arr}(\text{mid}) == \text{target}$)

return mid;



else if ($\text{arr}(\text{mid}) < \text{target}$)

Right

low

binarySearch (arr, mid + 1,

high, target)

$T(n/2)$

\nearrow

else

Left

$T(n/2)$

binarySearch (arr, low,
mid - 1, target - 1)

3

0	1	2	3	4	5
2	4	6	8	10	12

↑ ↑

low = 0

target = 10

high = 5

mid = 2

Example 1

binarySearch (arr, 3,
5, 10)

|

mid = 4

↓, ↓

0

1

2

3

result = 4

4

5

2	4	6	8	10	12
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low = 0

target = 2

high = 5

mid = 2

Example 2

binarySearch (arr, 0, 1, 2)

↓,

$\text{mid} = \emptyset$

binarySearch($\alpha\infty, 0, 0, 2$)

→ $\text{mid} = 0$

→ return mid

output = 0

Binary Search

$$T(n) = T\left(\frac{n}{2}\right) + c \quad \text{--- 1}$$
$$T\left(\frac{n}{2}\right) = \overline{T\left(\frac{n}{2^2}\right)} + c$$

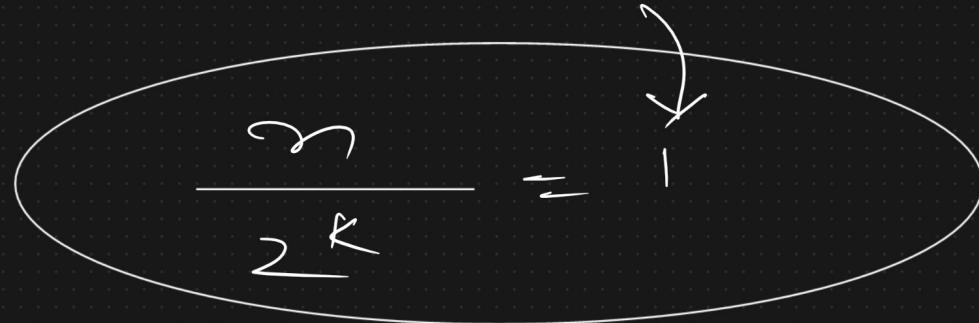
$$T(n) = T\left(\frac{n}{2^2}\right) + 2c$$

$$T(n) = T\left(\frac{n}{2^3}\right) + 3c$$

↓ K times

$$T(n) = T\left(\frac{n}{2}\right) + 3k$$

$T(1) = r$



$$n = \underline{2^k}$$

$$\log_2 n = k \cancel{\log_2 2}$$

$$k = \log_2 n$$

$$T(n) = T\left(\frac{n}{2^{\log_2 n}}\right) + (\log_2 n) \cdot c$$

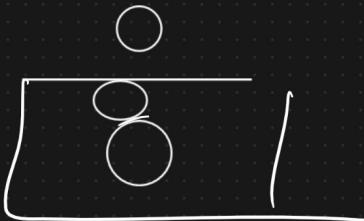
$$a^{\log_2 b} = b^{\log_2 a} = T\left(\frac{n^{\frac{1}{\log_2 \frac{b}{a}}}}{2^{\log_2 \frac{b}{a}}}\right) + c \cdot \log_2 n$$

$$\log_a a = 1 = \frac{1}{\log_2 \frac{b}{a}} + c \cdot \log_2 n$$

$$= \mathcal{O}(\log_2 n)$$

Searching

$$m = r$$



if ($arr(0) == target \rightarrow target = 9$

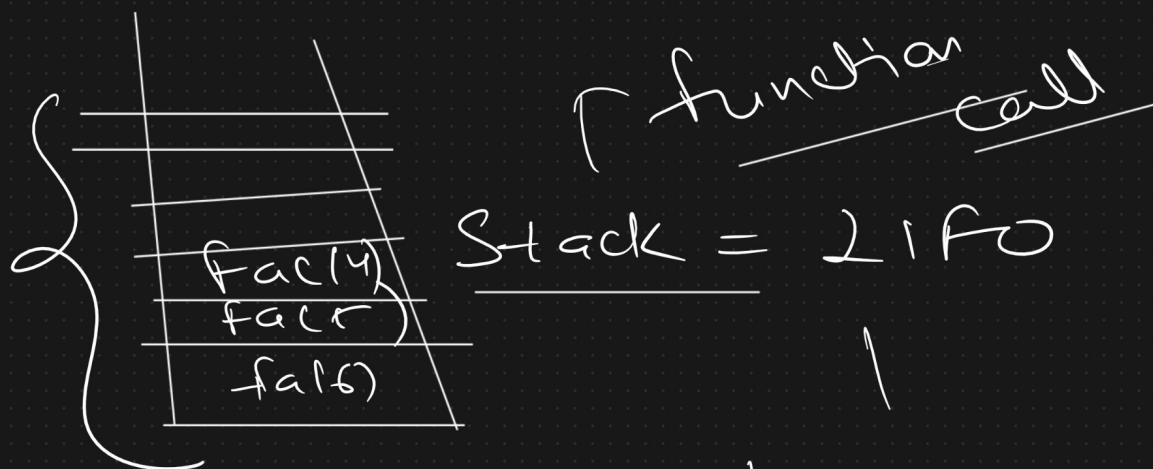
return 0;

y

c

return -1)

Space Complexity



Last In
first Out

in time

long time

~ 10

$\text{Num} = 12345$

Sum of digit =

$$1 + 2 + 3 + 4 + 5 = 15$$

$$\begin{array}{r} 972 \\ \hline \end{array}$$

$$9 + 7 + 2$$

18

Sum digits(num) &
Base case condition

$$\left\{ \begin{array}{l} \text{num} = 0 \\ \rightarrow 0 \end{array} \right.$$

$\text{num} = 12345$

else

$\left(\text{num} / 10 \right) + \underline{\text{SumDigits}} \left(\underline{\text{num} / 10} \right)$

x

$$\begin{array}{r} & \swarrow \text{remain } \text{ed} \\ 12345 & \overline{) 10 = 5} \end{array}$$

$\text{fun}(1234)$

\downarrow

$4 + \text{fun}(123)$

fun(12345) 15

10

5 + fun(1234)

6

4 + fun(123)

3

3 + fun(12)

1

2 + fun(1)

0

1 + fun(0)

Assignment

Problem

$$num = 4$$

$$k = 5$$

Statement

4, 8, 12, 16, 20

$$num = 12$$

$$k = 4$$

12, 24, 36, 48

$$\cancel{result} = 12$$

$$num = 12$$

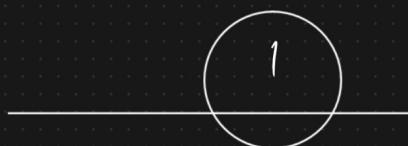
$$\cancel{k} = 1$$

$$k = 1 \quad \alpha$$

return num

8

$\text{PointMult}(n, k)$



$\text{if } (k = 1)$

print n
Y



$\text{PointMult}(n, k-1)$



$\hookrightarrow \text{Point}(n \times k)$

Y

60

$\text{PointMult}(1, 2, 5)$

1 48

$\text{PointMult}(1, 2, 4)$

1

36

$\hookrightarrow \text{PointMult}(1, 2, 3)$

1

24

Raintnuff (12, 12)

12

RainwMelt (12, 1)