ANSWERS

1. Answers

1: Assume that $y = kx + b$, then one has $k = \frac{2}{3}$. In addition, $\frac{2}{3} \cdot 3 + b = -8$, then $b = -10$, so

$$y = \frac{2}{3}x - 10. \tag{1.1}$$

2: Assume $y = k(x-4)+3$. Let $k(x_0 - 4) + 3 = 0$, then $x_0 = 4 - \frac{3}{k}$. Let $y_0 = -4k + 3$, then

$$\frac{1}{2}(4 - \frac{3}{k})(-4k+3) = 27. \tag{1.2}$$

Then

$$k = -\frac{3}{2}, \quad k = -\frac{3}{8}. \tag{1.3}$$

Thus,

$$y = -\frac{3}{2}x + 9, \quad y = -\frac{3}{8}x + \frac{9}{2}. \tag{1.4}$$

3: Assume $(x-a)^2 + (y-b)^2 = c^2$, then one has

$$(2-a)^2 + (7-b)^2 = c^2, \tag{1.5}$$

$$(1-a)^2 + b^2 = c^2, \tag{1.6}$$

$$a^2 + (3-b)^2 = c^2. \tag{1.7}$$

(1.5) minus (1.7) yields that

$$-a - 2b + 11 = 0. \tag{1.8}$$

(1.6) minus (1.7) yields that

$$-a + 3b - 4 = 0. \tag{1.9}$$

Then $a = 5, b = 3, c^2 = 25$. So

$$(x-5)^2 + (y-3)^2 = 25. \tag{1.10}$$

4: (a):

$$9(x-1)^2 - 9 + 4(y+1)^2 - 4 - 23 = 0, \tag{1.11}$$

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that is

$$9(x-1)^2 + 4(y+1)^2 = 36, \tag{1.12}$$

that is

$$\frac{(x-1)^2}{4} + \frac{(y+1)^2}{9} = 1. \tag{1.13}$$

Elliptic. Center: $(1, -1)$.

(b):

$$(y+1)^2 + 8 - 8x = 0. \tag{1.14}$$

That is

$$x - \frac{1}{8}(y+1)^2 = 1. \tag{1.15}$$

Parabolic. Vertices: $(1, -1)$.

(c):

$$9(x^2 - 4x) - 16(y^2 - 2y) - 124 = 0, \tag{1.16}$$

that is

$$9(x-2)^2 - 16(y-1)^2 = 144, \tag{1.17}$$

that is

$$\frac{(x-2)^2}{16} - \frac{(y-1)^2}{9} = 1. \tag{1.18}$$

Hyperbolic. Vertices: $(6, 1)$, $(-2, 1)$.

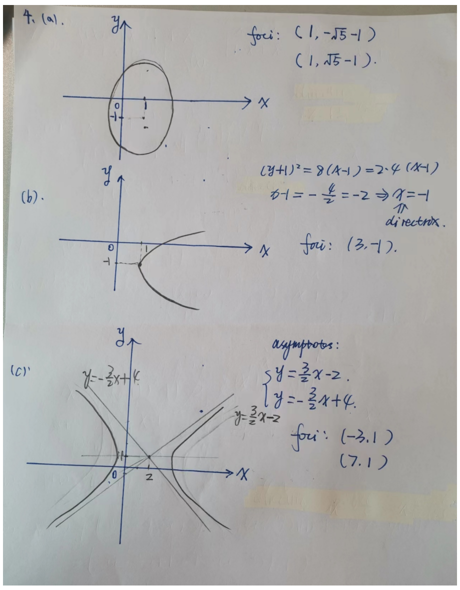

FIGURE 1.1.

5:

(i):

$$x \in (-\infty, -3) \cup (-3, 3) \cup (3, +\infty); \quad f(x) \in (-\infty, -\frac{5}{9}) \cup (0, +\infty). \tag{1.19}$$

(ii):

$$g(x) = \sqrt{(x-1)(x-3)} \in [0, +\infty); \quad x \in (-\infty, 1) \cup (3, +\infty). \tag{1.20}$$

6: (a):

$$F(x) \in (-\infty, 0) \cup (0, +\infty), \quad x \in (-\infty, 2) \cup (2, +\infty), \tag{1.21}$$

$$G(x) \in (-\infty, 1) \cup (1, +\infty) \quad x \in (-\infty, 0) \cup (0, +\infty). \tag{1.22}$$

(b):

$$(G \circ F)(x) = 1 - \frac{2}{F(x)} = 1 - 2(2-x) = -3 + 2x. \tag{1.23}$$

7: see figure 1.2

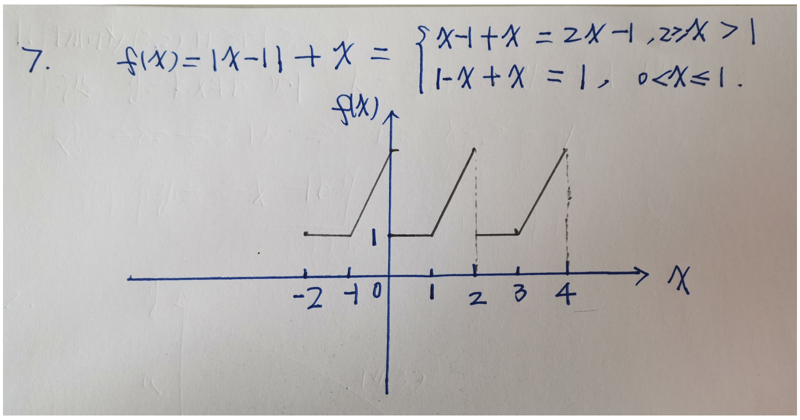

FIGURE 1.2.

8:

$$5 - 2x = 3(x+1), \quad 5 - 2x = -3(x+1), \tag{1.24}$$

then

$$x = \frac{2}{5}, \quad x = -8. \tag{1.25}$$

9:(see figure 1.3)

10: (a): $f(x) = -f(-x)$, odd function.

(b): $f(x) \neq f(-x)$ and $f(x) \neq -f(-x)$, neither even or odd.

11: Let $y = (x-2)^2 + 3$, then $x = 2 \pm \sqrt{y-3}$. Since $x \in (2, +\infty)$, then $x = 2 + \sqrt{y-3}$. Then

$$x = F^{-1}(y) = 2 + \sqrt{y-3}. \tag{1.26}$$

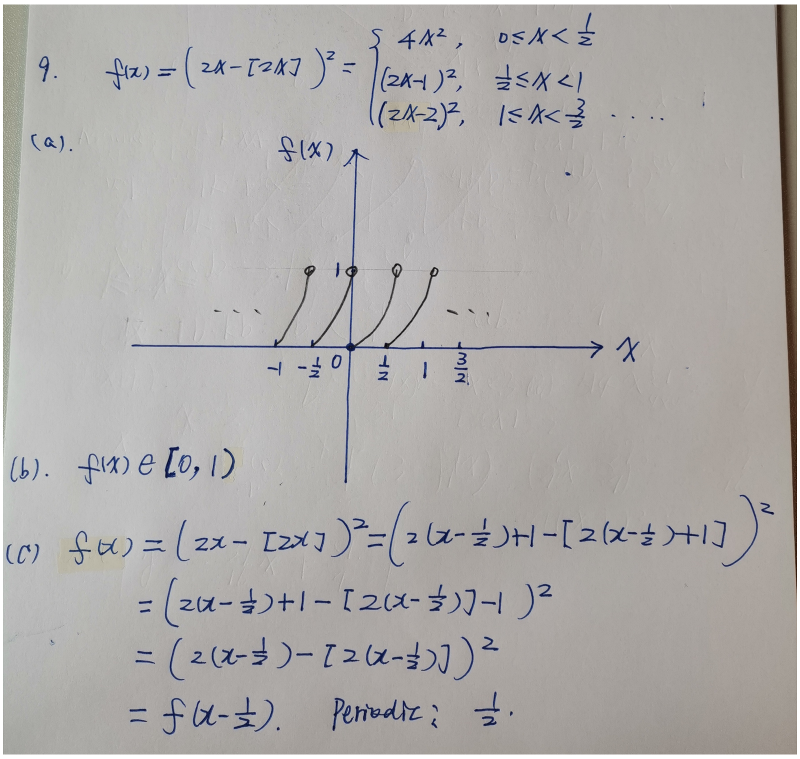

FIGURE 1.3.

Domain: $y \in [3, +\infty)$; Range: $x \in [2, +\infty)$.

one to one: for any $x_1, x_2 \in [2, +\infty)$ and $x_1 \neq x_2$, if

$$0 = F(x_1) - F(x_2) = (x_1 - 2)^2 - (x_2 - 2)^2 = (x_1 - x_2)(x_1 + x_2 - 4), \qquad (1.27)$$

which contradicts to $x_1 \neq x_2$ and $x_1 + x_2 - 4 > 0$. Thus, $F(x_1) \neq F(x_2)$.