MA1200 Mid-term Test for CA1, CB1, CC1 and CD1
Nov. 10 (Friday)

Instructions to candidates:

1. This paper has **six** questions. Answer **all** of them.
2. Show all steps.
3. Please write down your **name** and **student ID**.

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*This is a **closed-book** examination.*

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1. [16 marks] Classify the type of conic section described by the equation

$$3x^2 + 2y^2 + 6x - 8y + 5 = 0.$$

Sketch its graph with the coordinates of **vertices and foci** clearly shown.

(Hint: You may use the method of completing the squares.)

2. [17 marks] Express $\dfrac{6x^3 + 5x^2 + 2x - 10}{6x^2 - x - 2}$ in partial fractions.

3. [17 marks] Solve the equation: $\tan x - \sec x = 1$.

4. [15 marks] Solve the equation: $2^x - 2^{-x} = 1$.

5. [21 marks] Find the following limits

$$[a]\ \lim_{x\to 16} \frac{\sqrt{x} - 4}{x - 16} \qquad [b]\ \lim_{x\to 0} \frac{\sin 5x}{\sin 3x} \qquad [c]\ \lim_{x\to\infty} \frac{x^2 + 2x - 1}{2x^2 - 3x - 2}$$

6. [14 marks] Let $f(x) = \begin{cases} x - 3 & \text{if } x < 2 \\ -1 & \text{if } x = 2 \\ 6x & \text{if } x > 2 \end{cases}$.

Determine whether the function is continuous at $x = 2$. Justify your answer.

End

1. $3x^2+2y^2+6x-8y+5=0.$

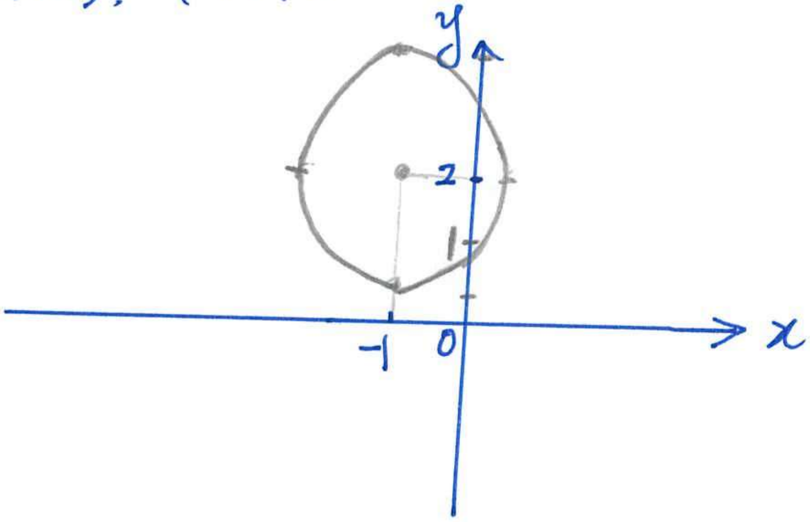
$\Rightarrow 3(x^2+2x)+2(y^2-4y)+5=0$

$\Rightarrow 3(x+1)^2+2(y-2)^2=6$

$\Rightarrow \dfrac{(x+1)^2}{2}+\dfrac{(y-2)^2}{3}=1$. Elliptic.

vertices: $(-1,2-\sqrt{3})$. $(-1,2+\sqrt{3})$. $(-1-\sqrt{2},2)$. $(-1+\sqrt{2},2)$.

foci: $(-1,3)$. $(-1,1)$



2.

$I:=\dfrac{6x^3+5x^2+2x-10}{6x^2-x-2}$

$=\dfrac{6x^3-x^2-2x}{6x^2-x-2}+\dfrac{6x^2+4x-10}{6x^2-x-2}$

$=x+\dfrac{6x^2-x-2+5x-8}{6x^2-x-2}$

$=x+1+\dfrac{5x-8}{6x^2-x-2}$

Note $\dfrac{5x-8}{6x^2-x-2}=\dfrac{5x-8}{(2x+1)(3x-2)}=\dfrac{A}{2x+1}+\dfrac{B}{3x-2}.$

$=\dfrac{(3A+2B)x-2A+B}{(2x+1)(3x-2)}.$

$\therefore \begin{cases}3A+2B=5\\ B-2A=-8\end{cases} \Rightarrow \begin{cases}A=3\\ B=-2\end{cases}$

$\therefore \boxed{I=x+1+\dfrac{3}{2x+1}-\dfrac{2}{3x-2}.}$

3. $\tan x - \sec x = 1 \Leftrightarrow \sin x - 1 = \cos x$ & $\cos x \neq 0$.

Note: $\cos x = \sin x - 1 < 0$. $\therefore \sin x - 1 = -\sqrt{1-\sin^2 x}$.

$\therefore 1 - \sin x = \sqrt{(1-\sin x)(1+\sin x)} \Rightarrow \sqrt{1-\sin x} = \sqrt{1+\sin x}$.

$\therefore \sin x = 0 \Rightarrow x = k\pi,\ k \in \mathbb{Z}$

By $\cos x < 0 \Rightarrow \boxed{x = \pi + 2k\pi,\ k \in \mathbb{Z}}$

4. $2^x - 2^{-x} = 1$.

Let $t = 2^x$, then $t - \frac{1}{t} = 1 \Rightarrow t^2 - t - 1 = 0$

$\Rightarrow (t - \frac{1}{2})^2 = \frac{5}{4} \Rightarrow t - \frac{1}{2} = \pm \frac{\sqrt{5}}{2} \Rightarrow t = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$.

By $t = 2^x > 0$ $\therefore t = \frac{1+\sqrt{5}}{2}$ i.e. $2^x = \frac{1+\sqrt{5}}{2}$.

$\therefore x = \log_2 \frac{1+\sqrt{5}}{2}$.

5. [a]. $\lim\limits_{x\to 16} \frac{\sqrt{x}-4}{x-16} = \lim\limits_{x\to 16} \frac{\sqrt{x}-4}{(\sqrt{x}-4)(\sqrt{x}+4)} = \lim\limits_{x\to 16} \frac{1}{\sqrt{x}+4} = \frac{1}{8}$.

[b]. $\lim\limits_{x\to 0} \frac{\sin 5x}{\sin 3x} = \frac{5}{3}$.

[c] $\lim\limits_{x\to \infty} \frac{x^2+2x-1}{2x^2-3x-2} = \lim\limits_{x\to \infty} \frac{1+\frac{2}{x}-\frac{1}{x^2}}{2-\frac{3}{x}-\frac{2}{x^2}} = \frac{1}{2}$.

6. $f(x) = \begin{cases} x-3, & \text{if } x<2 \\ -1 & \text{if } x=2 \\ 6x & \text{if } x>2. \end{cases}$

Since $\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} 6x = 12 \neq f(2) = -1$.

$\therefore f(x)$ is not continuous at $x=2$.