MA1200 Hand-in Assignment #3 due at 3:00PM on November 27, 2024

Instructions to students:

1. *Please submit it via Canvas in a PDF file (you can handwrite the answers and take photos by your phone, then make it into a PDF file, see, for example, https://www.wikihow.com/Convert-JPG-to-PDF for how to combine JPG files to a PDF; you can also do it by note-taking apps on an iPad or a Surface)*

2. *The assignment is due on* **3:00PM of November 27, 2024**. *Your score of this assignment is only based on what appears on Canvas as a successful submission. Any unsuccessful submissions will* ***NOT*** *be marked, which results in your getting zero point.*

3. *Please write down your name and student ID.*

10 points for every question below. There are totally ten questions. **Questions**:

1. Compute the following limits:

$$\text{(a)} \lim_{x\to -1} \frac{x^4+2x+1}{x^3+5x^2+7x+3}, \quad \text{(b)} \lim_{x\to+\infty} \left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x-\sqrt{x+\sqrt{x}}}\right),$$

$$\text{(c)} \lim_{x\to 0} \frac{\cos(3x)-\cos(2x)}{\sin(3x)-\sin(2x)}, \quad \text{(d)} \lim_{x\to 0} \frac{\tan(x)}{2\sin(x+\frac{\pi}{6})-1}.$$

2. Compute the following limits:

$$\text{(a)} \lim_{x\to+\infty} \frac{1+a^x+(2a)^x}{1-a^x-(3a)^x} \text{ where (i) } a=0.4 \text{ (ii) } a=0.8, \quad \text{(b)} \lim_{x\to+\infty} \left(1-\frac{1}{2x}\right)^{3x}.$$

3. Compute the following limits:

$$\text{(a)} \lim_{n\to+\infty} \left(\frac{1}{n^2+1}+\frac{2}{n^2+1}+\frac{3}{n^2+1}+\cdots+\frac{n}{n^2+1}\right),$$

$$\text{(b)} \lim_{n\to+\infty} \left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+3}}+\cdots+\frac{1}{\sqrt{n^2+n}}\right).$$

4. Let

$$f(x)=\begin{cases} x^2+1, & \text{if } x<0,\\ c(e^{2x})+d, & \text{if } 0\le x\le 1,\\ (x+7)^{1/3}, & \text{if } x>1. \end{cases}$$

Determine the values of c and d, such that $f(x)$ is continuous everywhere.

5. Which of the following functions are differentiable at $x = 0$?

$$f(x) = |x|\sin(x), \quad g(x) = \ln(x^2), \quad h(x) = x+|x|, \quad j(x) = \begin{cases} x & \text{if } x < 0, \\ \ln(1+x+3x^2) & \text{if } x \geq 0. \end{cases}$$

6. Find derivatives of the following functions $y = f(x)$:

(a) $f(x) = x[\sin(\ln(2x))-\cos(\ln(3x))]$, (b) $f(x) = \dfrac{x}{\sqrt{1+x^2}}$, (c) $f(x) = \tan^{-1}(x+\sqrt{1+x^2})$,

(d) $f(x) = (\sin x)^{\tan x}$, (e) $x^{1/3} + y^{1/3} = a^{1/3}$ $(a \neq 0)$, (f) $\begin{cases} x = a\cos^3 t \\ y = a\tan^3 t \end{cases}$.

7. Find the tangent line of the curve $y^2 - 3x^2 + 6x + 2y = 0$ at the point $(2, 0)$.

8. Find the tangent line of the curve $\begin{cases} x = 2t - \sqrt{3t}, \\ y = 3t^2 - t^3, \end{cases}$ at the point when $t = 1$.

9. Find two nonnegative numbers whose sum is 10 and so that the product of one number and the square of the other number is a minimum.

10. Let $f(x) = \dfrac{e^{-2x}}{(1-x)^2}$.

(a) Show that

$$(1-x)f'(x) - 2xf(x) = 0.$$

(b) Let n be a positive integer, show that

$$(1-x)f^{(n+1)}(x) - (n+2x)f^{(n)}(x) - 2nf^{(n-1)}(x) = 0.$$

(c) Hence, or otherwise, find the Taylor series of $f(x)$ at $a = 3$ up to the term $(x-3)^3$.

End