## City University of Hong Kong Department of Electrical Engineering

## **EE3009 Data Communications and Networking**

## **Solution to Tutorial 5**

1.

$$E[t_{SW}] = t_0 + \sum_{i=1}^{\infty} (i-1)t_{out}P_f^{i-1}(1-P_f)$$

$$= t_0 + t_{out}(1-P_f)\sum_{i=1}^{\infty} (i-1)P_f^{i-1}$$

Using the identity  $\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$ 

Differentiate both side with respect to x, we have  $\sum_{i=0}^{\infty} ix^{i-1} = \frac{1}{(1-x)^2}$ 

Hence, 
$$x \sum_{i=0}^{\infty} i x^{i-1} = \sum_{i=0}^{\infty} i x^{i} = \frac{x}{(1-x)^{2}}$$

Therefore, 
$$E[t_{SW}] = t_0 + t_{out} (1 - P_f) \frac{P_f}{(1 - P_f)^2} = t_0 + \frac{t_{out} P_f}{1 - P_f}$$

Alternatively,

$$\begin{split} E[t_{SW}] &= t_0 + \sum_{i=1}^{\infty} (i-1)t_{out}P_f^{i-1}(1-P_f) \\ &= t_0 + t_{out}\sum_{i=1}^{\infty} (i-1)P_f^{i-1}(1-P_f) \\ &= t_0 + t_{out}[P_f(1-P_f) + 2P_f^2(1-P_f) + 3P_f^3(1-P_f) + \dots ] \\ &= t_0 + t_{out}[P_f - P_f^2 + 2P_f^2 - 2P_f^3 + 3P_f^3 + \dots ] \\ &= t_0 + t_{out}[P_f + P_f^2 + P_f^3 + \dots ] \\ &= t_0 + \frac{t_{out}P_f}{1-P_f} \end{split}$$

2.

i) First, we have the following:

$$n_f = 256 \times 8 = 2048$$

$$P_f = 1 - (1 - 10^{-4})^{n_f} = 0.1852$$

$$t_{prop} = 100 \,\mathrm{ms}$$

Using the results for Stop-and-Wait,

$$\eta = (1 - P_f) \frac{1 - \frac{n_0}{n_f}}{1 + \frac{n_a}{n_f} + \frac{2(t_{prop} + t_{proc})}{n_f}} R$$

$$= 0.126.$$

ii) The same parameters as in (i) are used.

For the window size,  $W_s \le 2^3 - 1 = 7$ , and at the same time,

$$W_s = \frac{2 \times R \times d}{n_f} + 1$$
, where d is the one one-way propagation delay

$$W_s = \frac{56 \times 10^3 \times 2 \times 100 \times 10^{-3}}{2048} + 1 = 6.468$$

So, the maximum number of complete frames allowed by  $W_s$  is 6 Considering the two constraints,  $W_s = 6$  is suitable.

Using 
$$\eta = (1 - P_f) \frac{1 - \frac{n_0}{n_f}}{1 + (W_s - 1)P_f}$$

$$\eta = \frac{1 - 0.1852}{1 + 5 \times 0.1852} = \frac{0.8148}{1.926} = 0.423$$