MA1200 Hand-in Assignment #3 due Dec. 1

Instructions to students:

1. *Please submit it via Canvas in a PDF file (you can handwrite the answers and take photos by your phone, then make it into a PDF file, see, for example, https://www.wikihow.com/Convert-JPG-to-PDF for how to combine JPG files to a PDF; you can also do it by note-taking apps on an iPad or a Surface)*
2. *The assignment is due on* **23:59 of December 1 (Friday)**. *Late submissions will* ***NOT*** *be marked.*
3. *Please write down your name and student ID.*

Questions:

1. Compute the following limits:

$$\text{(a)} \lim_{x\to 3}\frac{x^2-5x+6}{x^2-8x+15}, \quad \text{(b)} \lim_{x\to 0+}\left(\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}-\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}\right),$$

$$\text{(c)} \lim_{x\to\infty}\left(\frac{x^2-1}{x^2+1}\right)^{\frac{x-1}{x+1}}, \quad \text{(d)} \lim_{x\to 0}\frac{\sin(a+2x)-2\sin(a+x)+\sin a}{x^2}.$$

2. Let

$$f(x)=\begin{cases}3x^2+1, & \text{if } x<0,\\ cx+d, & \text{if } 0\le x\le 1,\\ \sqrt{x+3}, & \text{if } x>1.\end{cases}$$

Determine the values of c and d, such that $f(x)$ is continuous everywhere.

3. Which of the following functions is differentiable at $x=0$?

$$f(x)=x\,|x|, \quad g(x)=\ln|x|, \quad h(x)=x-|x|, \quad j(x)=\begin{cases}x & \text{if } x<0,\\ \ln(1+x) & \text{if } x\ge 0.\end{cases}$$

4. Find derivatives of the following functions $y=f(x)$:

$$\text{(a)} f(x)=x[\sin(\ln x)-\cos(\ln x)], \quad \text{(b)} f(x)=\frac{x}{\sqrt{a^2-x^2}}, \quad \text{(c)} f(x)=\tan^{-1}(x+\sqrt{1+x^2}),$$

$$\text{(d)} f(x)=(\sin x)^{\cos x}+(\cos x)^{\sin x}, \quad \text{(e)} \sqrt{x}+\sqrt{y}=\sqrt{a}, \quad \text{(f)} \begin{cases}x=a\cos^3 t\\ y=a\sin^3 t\end{cases}.$$

5. Find the tangent line of the curve $\frac{x^2}{100}+\frac{y^2}{64}=1$ at the point $(6,\frac{32}{5})$.

6. Find the tangent line of the curve $\begin{cases}x=2t-t^2,\\ y=3t-t^3,\end{cases}$ at the point when $t=1$.

7. Find two nonnegative numbers whose sum is 9 and so that the product of one number and the square of the other number is a maximum.

8. Let $f(x) = (1 + x^2)^{-1}$.

   (a) Show that
   $$(1 + x^2)f'(x) + 2xf(x) = 0.$$

   (b) Let n be a positive integer, show that
   $$(1 + x^2)f^{(n+1)}(x) + (2n + 2)xf^{(n)}(x) + n(n + 1)f^{(n-1)}(x) = 0.$$

   (c) Hence, or otherwise, find the Maclaurin series of $(1 + x^2)^{-1}$ up to the term x^4.

End