EE 2000 Logic Circuit Design
Semester A 2024/25

Tutorial 3

1. Design a combinational circuit for a 4-bit BCD-2421-to-Gray code converter.

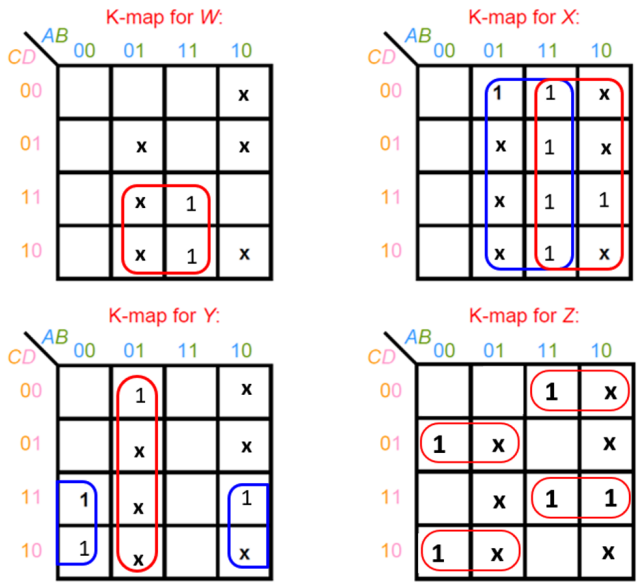
Decimal numbers	Minterm	Inputs (2421)				Outputs (Gray)			
		A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	0	0	1
2	2	0	0	1	0	0	0	1	1
3	3	0	0	1	1	0	0	1	0
4	4	0	1	0	0	0	1	1	0
5	11	1	0	1	1	0	1	1	1
6	12	1	1	0	0	0	1	0	1
7	13	1	1	0	1	0	1	0	0
8	14	1	1	1	0	1	1	0	0
9	15	1	1	1	1	1	1	0	1
Unused	5	x	x	x	x	x	x	x	x
Unused	6	x	x	x	x	x	x	x	x
Unused	7	x	x	x	x	x	x	x	x
Unused	8	x	x	x	x	x	x	x	x
Unused	9	x	x	x	x	x	x	x	x
Unused	10	x	x	x	x	x	x	x	x

$$W(A,B,C,D) = \sum m(14,15) + \sum d(5,6,7,8,9,10)$$

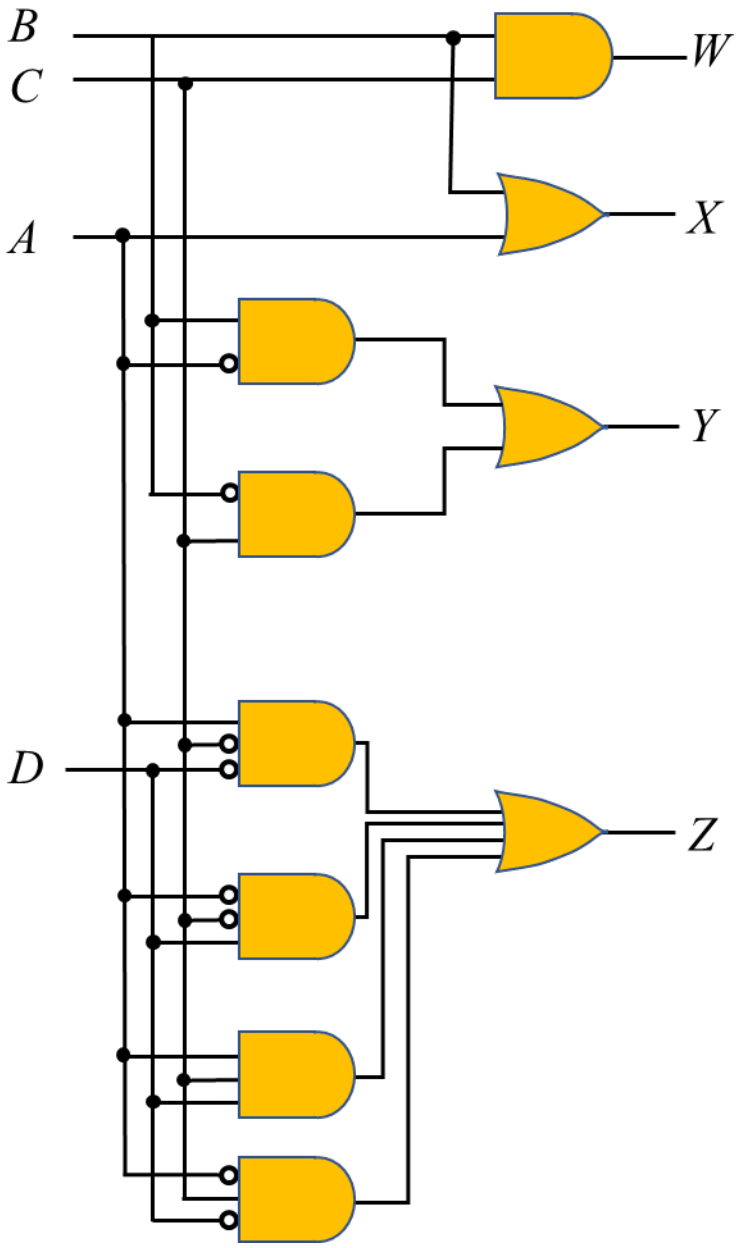
$$X(A,B,C,D) = \sum m(4,11,12,13,14,15) + \sum d(5,6,7,8,9,10)$$

$$Y(A,B,C,D) = \sum m(2,3,4,11) + \sum d(5,6,7,8,9,10)$$

$$Z(A,B,C,D) = \sum m(1,2,11,12,15) + \sum d(5,6,7,8,9,10)$$

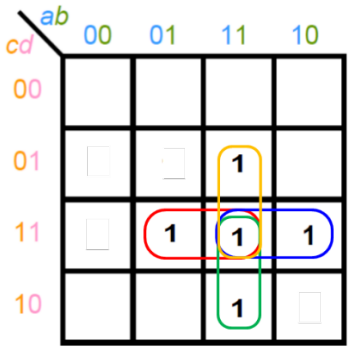


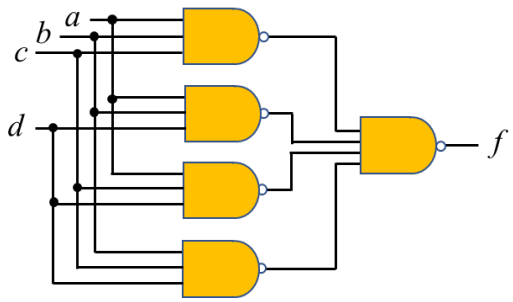
$W(A,B,C,D) = BC$
$X(A,B,C,D) = A + B$
$Y(A,B,C,D) = A'B + B'C$
$Z(A,B,C,D) = AC'D' + A'C'D + ACD + A'CD'$


B
C
A
D
W
X
Y
Z

2. Jack, Jen, Joe, and Jim get together once a week to either go to a movie or go bowling. To decide what to do, they vote and a simple majority wins. They will go to a movie if a TIE (2 votes each) occurs. Assuming a vote for the movie is represented as a 0, design a NAND gate circuit that automatically computes the decision.

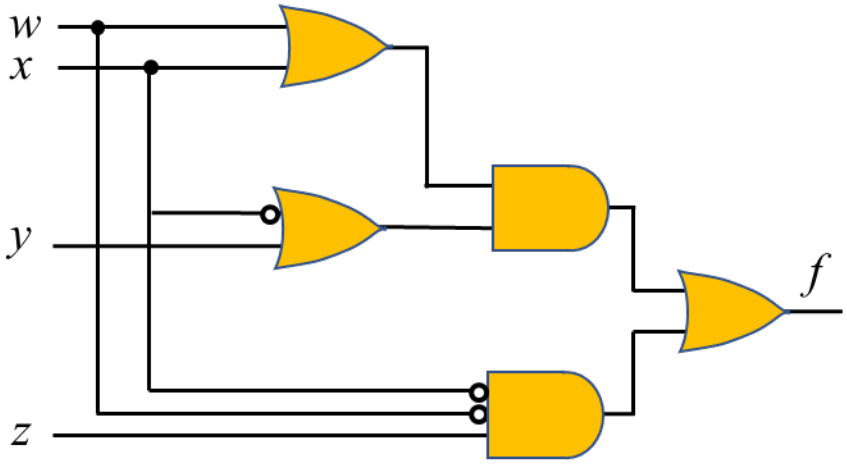
Jack/a	Jen/b	Joe/c	Jim/d	Decision/f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

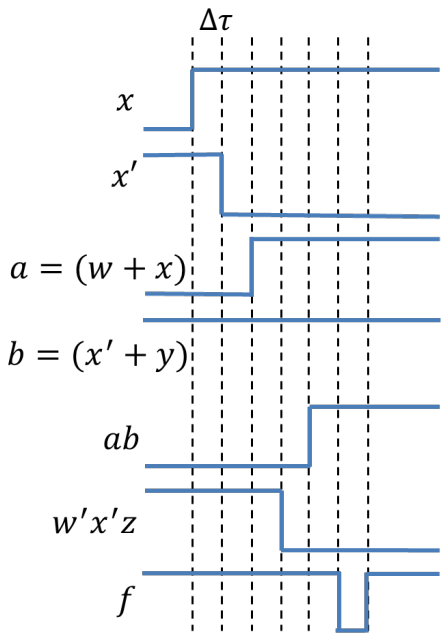




$f(a,b,c,d) = abc + abd + acd + bcd$

3. (a) Given the following combinational circuit, work out the timing diagram to identify the presence of any timing hazard when the input condition changes from $(w, x, y, z) = (0,0,1,1)$ to $(0,1,1,1)$. Assume that the propagation delay for NOT gate is $\Delta\tau$, and other gates is $2\Delta\tau$.

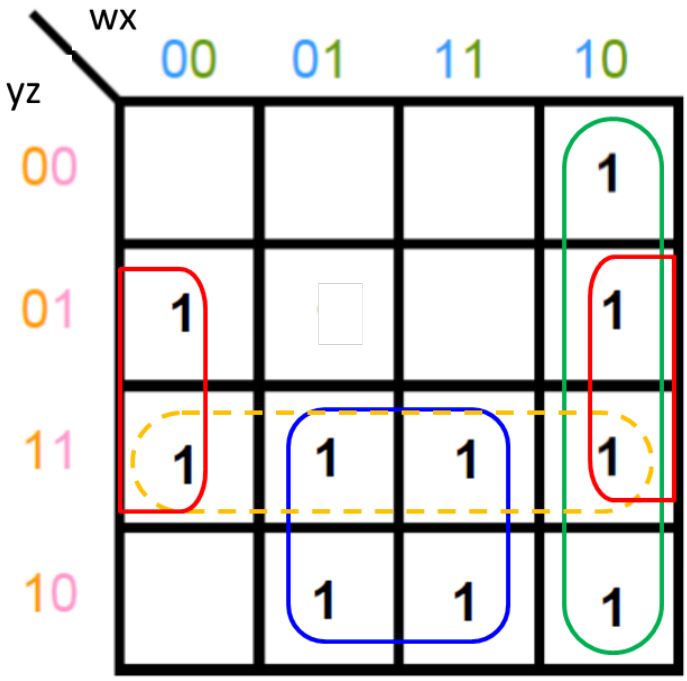




Static-1 hazard!!!

(b) Redesign the circuit to eliminate the hazard.

w	x	y	z	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1



$$f(w,x,y,z) = x'z + xy + wx' + yz$$

4. (a) Determine the Hamming code using both odd and even parity bit for a data code of 11001.

Calculate extra bit (***k***) needed for a ***n*** bit of code.
$2^k \geq n + k + 1$
For a 5-bit data $d_5d_4d_3d_2d_1$, $n = 5$
$2^k \geq 6 + k$
Therefore, $k = 4$ (4 parity bits).

Hamming Code	H_9 d_5	H_8 p_4	H_7 d_4	H_6 d_3	H_5 d_2	H_4 p_3	H_3 d_1	H_2 p_2	H_1 p_1
Bit	9	8	7	6	5	4	3	2	1
Binary Code	1001	1000	0111	0110	0101	0100	0011	0010	0001
p_1	1		1		0		1		
p_2			1	0			1		
p_3			1	0	0				
p_4	1								
Even parity	1	1	1	0	0	1	1	0	1
Odd Parity	1	0	1	0	0	0	1	1	0

(b) Check for error should we receive the following codes, consider odd parity, and determine the original data code.
(i) 100000110
(ii) 101100110

(i)

$$c_1 = (H_9 \oplus H_7 \oplus H_5 \oplus H_3 \oplus H_1)' = (1 \oplus 0 \oplus 0 \oplus 1 \oplus 0)' = 1$$
$$c_2 = (H_7 \oplus H_6 \oplus H_3 \oplus H_2)' = (0 \oplus 0 \oplus 1 \oplus 1)' = 1$$
$$c_3 = (H_7 \oplus H_6 \oplus H_5 \oplus H_4)' = (0 \oplus 0 \oplus 0 \oplus 0)' = 1$$
$$c_4 = (H_9 \oplus H_8)' = (1 \oplus 0)' = 0$$
$$c_4c_3c_2c_1 = (0111)_2 = 7$$

Error on bit 7
Correct hamming code: 101000110
Original data code: 11001

(ii)

$$c_1 = (H_9 \oplus H_7 \oplus H_5 \oplus H_3 \oplus H_1)' = (1 \oplus 1 \oplus 0 \oplus 1 \oplus 0)' = 0$$

$$c_2 = (H_7 \oplus H_6 \oplus H_3 \oplus H_2)' = (1 \oplus 1 \oplus 1 \oplus 1)' = 1$$

$$c_3 = (H_7 \oplus H_6 \oplus H_5 \oplus H_4)' = (1 \oplus 1 \oplus 0 \oplus 0)' = 1$$

$$c_4 = (H_9 \oplus H_8)' = (1 \oplus 0)' = 0$$

$$c_4 c_3 c_2 c_1 = (0110)_2 = 6$$

Error on bit 6
Correct hamming code: 101000110
Original data code: 11001