MA1200 Mid-term Test for CE1, CF1, CG1 and CH1
Nov. 10 (Friday)

Instructions to candidates:

1. This paper has **six** questions. Answer **all** of them.
2. Show all steps.
3. Please write down your **name** and **student ID**.

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*This is a **closed-book** examination.*

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1. [16 marks] Classify the type of conic section described by the equation

$$3x^2 - 2y^2 + 6x + 8y - 11 = 0.$$

Sketch its graph with the coordinates of **vertices and foci** clearly shown.

(Hint: You may use the method of completing the squares.)

2. [17 marks] Express $\dfrac{3x^2 - 6x + 6}{(x+1)(x^2 - x + 1)}$ in partial fractions.

3. [17 marks] Solve the equation: $2\cos^2 x + 3\sin x = 3$.

4. [15 marks] Solve the equation: $5^{4-x} = 7^{3x+1}$.

5. [21 marks] Find the following limits

$$[a]\ \lim_{x\to 5} \frac{x-5}{x^2-25} \qquad [b]\ \lim_{x\to 0} \frac{\sin 5x}{x} \qquad [c]\ \lim_{x\to\infty} \frac{x^2-1}{2x^2+1}$$

6. [14 marks] Let $f(x) = \begin{cases} x^2 - 3 & \text{if } x < 2 \\ 1 & \text{if } x = 2 \\ \frac{1}{2}x & \text{if } x > 2 \end{cases}$.

Determine whether the function is continuous at $x = 2$. Justify your answer.

End

1. $3x^2-2y^2+6x+8y-11=0$.

$\Rightarrow 3(x^2+2x)-2(y^2-4y)-11=0$.

$\Rightarrow 3(x+1)^2-3-2(y-2)^2+8-11=0$.

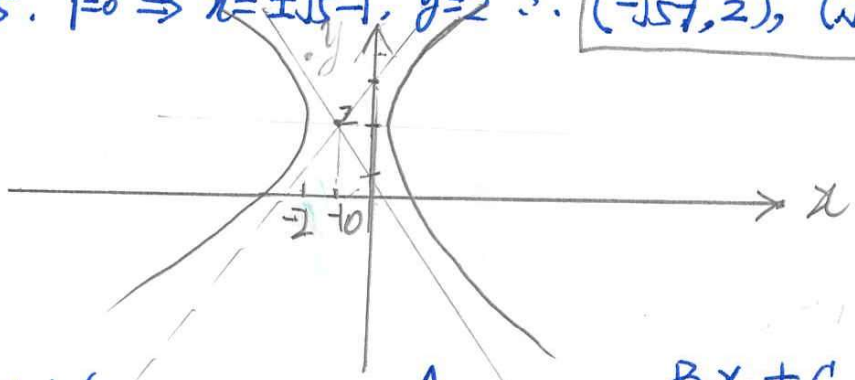
$\Rightarrow 3(x+1)^2-2(y-2)^2=6$

$\Rightarrow \dfrac{(x+1)^2}{2}-\dfrac{(y-2)^2}{3}=1$. Hyperbolle.

let $X=x+1$. $Y=y-2$. then $\dfrac{X^2}{(\sqrt{2})^2}-\dfrac{Y^2}{(\sqrt{3})^2}=1$, $a=\sqrt{2}$. $b=\sqrt{3}$. $c=\sqrt{5}$.

Vertices: $X=\pm\sqrt{2}$, $Y=0 \Rightarrow x=\pm\sqrt{2}-1$, $y=2$ $\therefore$ $\boxed{(-\sqrt{2}-1,2),(\sqrt{2}-1,2)}$

foci: $X=\pm\sqrt{5}$. $Y=0 \Rightarrow x=\pm\sqrt{5}-1$. $y=2$ $\therefore$ $\boxed{(-\sqrt{5}-1,2),(\sqrt{5}-1,2)}$



2. $I:=\dfrac{3x^2-6x+6}{(x+1)(x^2-x+1)}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2-x+1}$.

$$=\frac{A(x^2-x+1)+(Bx+C)(x+1)}{(x+1)(x^2-x+1)}$$

$$=\frac{(A+B)x^2+(B+C-A)x+A+C}{(x+1)(x^2-x+1)}$$

$\therefore \begin{cases} A+B=3, \\ B+C-A=-6, \\ A+C=6. \end{cases} \Rightarrow \begin{cases} A=5 \\ B=-2 \\ C=1 \end{cases}$

$\therefore I=\dfrac{5}{x+1}+\dfrac{1-2x}{x^2-x+1}$.

3. $2\cos^2 x + 3\sin x = 3$

$\Rightarrow 2(1-\sin^2 x) + 3\sin x = 3$

$\Rightarrow 2\sin^2 x - 3\sin x + 1 = 0$

$\Rightarrow (\sin x - 1)\cdot(2\sin x - 1) = 0$

$\therefore \sin x = 1$ or $\sin x = \frac{1}{2}$

$\therefore x = \frac{\pi}{2} + 2k\pi$. or $x = \frac{\pi}{6} + 2k\pi$. or $x = \frac{5\pi}{6} + 2k\pi$. $k \in \mathbb{Z}$.

4. $5^{4-x} = 7^{3x+1}$

$\Rightarrow (4-x)\ln 5 = (3x+1)\ln 7$.

$\Rightarrow (3\ln 7 + \ln 5)\cdot x = 4\ln 5 - \ln 7$

$\Rightarrow x = \dfrac{4\ln 5 - \ln 7}{3\ln 7 + \ln 5}$.

5. [a]. $\lim\limits_{x\to 5} \dfrac{x-5}{x^2-25} = \lim\limits_{x\to 5} \dfrac{1}{x+5} = \dfrac{1}{10}$.

[b]. $\lim\limits_{x\to 0} \dfrac{\sin 5x}{x} = 5$

[c]. $\lim\limits_{x\to \infty} \dfrac{x^2-1}{2x^2+1} = \lim\limits_{x\to \infty} \dfrac{1-\frac{1}{x^2}}{2+\frac{1}{x^2}} = \dfrac{1}{2}$.

6. $f(x) = \begin{cases} x^2-3, & \text{if } x<2. \\ 1, & \text{if } x=2. \\ \frac{1}{2}x, & \text{if } x>2. \end{cases}$

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} (x^2-3) = 1 = f(2)$

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} \frac{1}{2}x = 1 = f(2)$

$\therefore f(x)$ is continuous at $x=2$.