

Why we need differentiation ?

We consider the following problem:

In 1931, the HK population is about 864,117

In 1999, the HK population is about 6,840,000

Question:

What is the population growth per year?

Simple Answer:

$$\text{The population growth} = \frac{\overbrace{6,840,000 - 864,117}^{\text{change in population}}}{\underbrace{1999 - 1931}_{\text{duration (time horizon)}}} = 87,881/\text{year}$$

Is it a correct estimation?

The number given above is just the *average population growth*.

In fact, the population growth is different in different time due to various conditions (economic conditions, wars, culture, personality etc.)

<u>Year</u>	<u>Population</u>
1931	864,117
1941	1,600,000
1945	750,000
1947	1,750,000
1951	2,013,000
1961	3,133,131
1971	3,950,000
1981	4,986,560
1991	5,647,114

In order to have a better estimation on the population growth at certain time, one needs to shorten the time horizon (say 10 years, 5 years, 1 year, 3 month ...) so that the occasion is small. Then we can obtain a better estimation on the population growth

$$\underbrace{\frac{P(t+10) - P(t)}{10}}_{\text{less accurate}} \Rightarrow \frac{P(t+5) - P(t)}{5} \Rightarrow \frac{P(t+1) - P(t)}{1} \Rightarrow \underbrace{\frac{P\left(t + \frac{1}{4}\right) - P(t)}{\frac{1}{4}}}_{\text{more accurate}}$$

(*Note: $P(t)$ is the population at year t .)

In the limiting case when time horizon Δt is close to 0, the corresponding estimation should be the most accurate one since every condition between t and $t + \Delta t$ is essentially identical and the population growth should be uniform (grows at same “speed”), i.e.

$$\lim_{\Delta t \rightarrow 0} \frac{P(t + \Delta t) - P(t)}{\Delta t}$$

This number is commonly used to describe the “rate of change” of population.