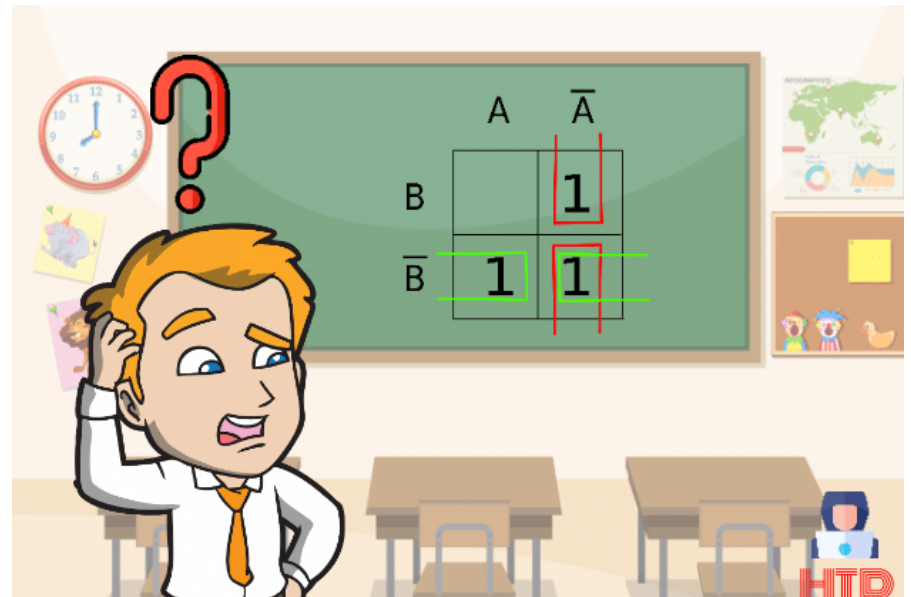


EE2000 Logic Circuit Design

Lecture 2 – Karnaugh Map and Quine-McCluskey (QM) Method



Exercise

Find all minimum sum of products and all minimum product of sums expressions for the following Boolean Function.

$$f(a, b, c, d) = \sum m(1, 3, 4, 6, 11) + \sum d(0, 8, 10, 12, 13)$$

cd \ ab	ab			
	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

cd \ ab	ab			
	00	01	11	10
00	X	1	X	X
01	1		X	
11	1			1
10		1*		X

Exercise

	ab			
	00	01	11	10
cd				
00	x	1	x	x
01	1		x	
11	1			1
10		1*		x

	ab			
	00	01	11	10
cd				
00	x	1	x	x
01	1		x	
11	1			1
10		1*		x

	ab			
	00	01	11	10
cd				
00	x	1	x	x
01	1		x	
11	1			1
10		1*		x

$$f = a'bd' + a'b'd + b'cd$$

$$f = a'bd' + a'b'c' + b'cd$$

$$f = a'bd' + a'b'd + ab'c$$

Exercise

Find all minimum sum of products and all minimum product of sums expressions for the following Boolean Function.

		<i>ab</i>			
<i>cd</i>		00	01	11	10
	00	X	1	X	X
	01	1	0	X	0
	11	1	0	0	1
	10	0	1	0	X

		<i>ab</i>			
<i>cd</i>		00	01	11	10
	00	X	1	X	X
	01	1	0	X	0
	11	1	0	0	1
	10	0	1	0	X

Exercise

Find all minimum sum of products and all minimum product of sums expressions for the following Boolean Function.

<i>cd</i> \ <i>ab</i>	00	01	11	10
00	X	1	X	X
01	1	0*	X	0*
11	1	0*	0	1
10	0*	1	0	X

Exercise

		ab			
		00	01	11	10
cd	00	X	1	X	X
	01	1	0*	X	0*
	11	1	0*	0	1
	10	0*	1	0	X

$$f = (b' + d')(b + d) \\ (a' + c)(a' + b')$$

		ab			
		00	01	11	10
cd	00	X	1	X	X
	01	1	0*	X	0*
	11	1	0*	0	1
	10	0*	1	0	X

$$f = (b' + d')(b + d) \\ (a' + c)(a' + d)$$

Exercise (Don't Care Case)

Step 5-6: Reduce PI chart & express the Boolean Function

PI	Minterms	<i>abcd</i>	4	8	9	10	12	15
PI₁	m_8, m_9, m_{12}, m_{13}	1-0-		x	x		x	
PI ₂	m_2, m_6	0-10						
PI ₃	m_2, m_{10}	-010				x		
PI ₄	m_4, m_6	01-0	x					
PI ₅	m_4, m_{12}	-100	x				x	
PI ₆	m_8, m_{10}	10-0		x		x		
PI₇	m_{13}, m_{15}	11-1						x

PI	Minterms	<i>abcd</i>	4	10
PI ₃	m_2, m_{10}	-010		x
PI ₄	m_4, m_6	01-0	x	
PI ₅	m_4, m_{12}	-100	x	
PI ₆	m_8, m_{10}	10-0		x

$$\begin{aligned}
 f(a, b, c, d) &= \text{PI}_1 + \text{PI}_3 + \text{PI}_4 + \text{PI}_7 = ac' + b'cd' + a'bd' + abd \\
 &= \text{PI}_1 + \text{PI}_3 + \text{PI}_5 + \text{PI}_7 = ac' + b'cd' + bc'd' + abd \\
 &= \text{PI}_1 + \text{PI}_4 + \text{PI}_6 + \text{PI}_7 = ac' + a'bd' + ab'd' + abd \\
 &= \text{PI}_1 + \text{PI}_5 + \text{PI}_6 + \text{PI}_7 = ac' + bc'd' + ab'd' + abd
 \end{aligned}$$