MCR3U - Test #2

Name: SOLNS Per: ____

Date: November 10th, 2023

54 marks

- Approved calculators are permitted
- Answers should belong to the real number set
- Answers should be exact unless otherwise stated

PART A – Write your answer in the box to the right (rough work will not be marked)

QUESTION	ANSWER
1. The Canada Calendar Company is a small business that sells calendars and the function below models the company's profits ($), where x is the price of a calendar in dollars. $P(x) = -25x^2 + 400x - 1825$ At what calendar price will the company receive maximum profits? $x = \frac{-400}{2(-25)}$ $x = 8$	$8 ✓
2. A parabola has an axis of symmetry of $x = 2$ and a *y-intercept* of $(0, 8)$, list another point that is on this parabola.	$(4, 8)$ ✓
3. What is the minimum value of $g(x) = (x + 3)^2$?	min. of 0 ✓
4. What is the equation of the inverse of $h(x) = x^2, x \leq 0$? $x = y^2$ $y = \pm\sqrt{x}$ $\int$ $h^{-1}(x) = -\sqrt{x}$	$h^{-1}(x) = -\sqrt{x}$ ✓
5. Write the 'vertex-form' of a quadratic function for parabola with a vertex of (2,6) and passes through the point (3, 5) $5 = a(3-2)^2 + 6$ $-1 = a$	$f(x) = -(x-2)^2 + 6$ ✓
6. Determine the value(s) of b that will make $h(x) = 2x^2 + bx + 20$ have an axis of symmetry of $x = 7$. $x = \frac{-b}{2a}$ $7 = \frac{-b}{4}$ $\int$ $b = -28$	$b = -28$ ✓
7. What is the axis of symmetry of the function $h(x) = -2x^2 + 10$?	$x = 0$ ✓
8. Express the following in standard form $g(x) = -2(x - 1)^2 + 2$ $= -2(x^2 - 2x + 1) + 2$ $= -2x^2 + 4x - 2 + 2$ $= -2x^2 + 4x$	$g(x) = -2x^2 + 4x$ ✓
9. What is the *y-intercept* of the function $g(x) = 3(x - 2)^2 + 1$? $g(0) = 3(4) + 1$	$(0, 13)$ ✓

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PART B – Full Solutions

- Approved calculators are permitted
- Answers should belong to the real number set
- Answers should be exact unless otherwise stated

1. Determine an equation for a quadratic function with *x-intercepts* of $(-5,0)$ and $(15,0)$ and a *y-intercept* of $(0$

$f(x) = a(x+5)(x-15)$

sub in $(0,-15)$

$-15 = a(5)(-15)$ ✓

$-15 = -75a$

$\frac{1}{5} = a$

$\therefore f(x) = \frac{1}{5}(x+5)(x-15)$

3

2. Fill in the chart for the function: $h(x) = -x^2 + 20x - 36$

y-intercept	$(0,-36)$ ✓
x-intercept(s)	$(2,0)$ $(18,0)$ ✓
Axis of symmetry	$x=10$ ✓
Direction of Opening	DOWN ✓
Optimal Value (min / max)	MAX OF 64 ✓
Vertex	$(10,64)$ ✓
Domain	$\{x \in \mathbb{R}\}$ ✓
Range	$\{y \in \mathbb{R} \mid y \leq 64\}$ ✓

$h(x) = -(x-2)(x-18)$

8

3. Given $f(x) = 2(x+8)^2 - 4$, where $x \leq -8$.

a) Find $f^{-1}(x)$

$x = 2(y+8)^2 - 4$

$x+4 = 2(y+8)^2$

$\frac{1}{2}(x+4) = (y+8)^2$

SRBS

$\pm\sqrt{\frac{1}{2}(x+4)} = y+8$

$y = \pm\sqrt{\frac{1}{2}(x+4)} - 8$

$f^{-1}(x) = -\sqrt{\frac{1}{2}(x+4)} - 8$ ✓✓

4

b) State the domain and range of $f^{-1}(x)$

Domain	$\{x \in \mathbb{R} \mid x \geq -4\}$ ✓	Range	$\{y \in \mathbb{R} \mid y \leq -8\}$ ✓

15

4. Determine the point(s) of intersection of the functions $f(x) = 2x^2 - 9x$ and $g(x) = 4x - 6$ algebraically.

set $f(x) = g(x)$

$2x^2 - 9x = 4x - 6$

$2x^2 - 13x + 6 = 0$

$(2x - 1)(x - 6) = 0$

$x = \frac{1}{2}$ $\quad$ $x = 6$

$g(\frac{1}{2}) = 4(\frac{1}{2}) - 6$

$= -4$

$g(6) = 4(6) - 6$

$= 18$

4

$\therefore$ POIs ARE $(6, 18)$ AND $(\frac{1}{2}, -4)$

5. Determine the nature of roots and complete the chart by using the following:

If there are...	Write in the column
Zero real roots	0
Two equal real roots	2E
Two distinct real roots	2D

Question	Function	How many real roots?
a)	$d(x) = 25x^2 - 1$	2D
b)	$k(x) = x^2 - 6x + 5$	2D
c)	$h(x) = 9x^2 - 12x + 4$	2E
d)	$m(x) = -4x^2 - 5x$	2D
e)	$p(x) = (x - 9)^2 - 5$	2D
f)	$f(x) = -\frac{1}{2}(x + 3)^2 - 10$	0
g)	$q(x) = 3 - x^2$	2D
h)	$g(x) = -(x + 3)^2$	2E

4

6. Determine the value of k such that the linear function $g(x) = -10x + k$ intersects $f(x) = 3x^2 - 40x + 67$ at exactly one point.

$D = 0$

set $f(x) = g(x)$ ✓

$3x^2 - 40x + 67 = -10x + K$

$3x^2 - 30x + 67 - K = 0$

$D = 0$

$b^2 - 4ac = 0$ ✓

$(-30)^2 - 4(3)(67 - K) = 0$

$900 - 12(67 - K) = 0$ ✓

$900 - 804 + 12K = 0$

$12K = -96$

$K = -8$

3

11

7. Travis and Taylor have 640m of fencing and would like to build a rectangular pen. They are going to divide up th into two parts for their horses and pigs.

a) Develop a function to express the total area enclosed as a function of the width.
b) Determine the dimensions that give the maximum area.

$P = 3w + 2l$

$640 = 3w + 2l$

(solve for l)

$l = \frac{640 - 3w}{2}$

$A = lw$

$A = \left(\frac{640-3w}{2}\right)(w)$

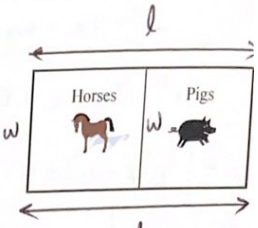
$A(w) = \left(\frac{640-3w}{2}\right)(w)$

$w = \frac{640}{3}$ $\quad w = 0$

AOS

$w = \frac{640}{6}$

4



(b) $w = \frac{640}{6} \approx 106.6\text{m}$

$l = \frac{640 - 3\left(\frac{640}{6}\right)}{2}$

$l = 160\text{m}$

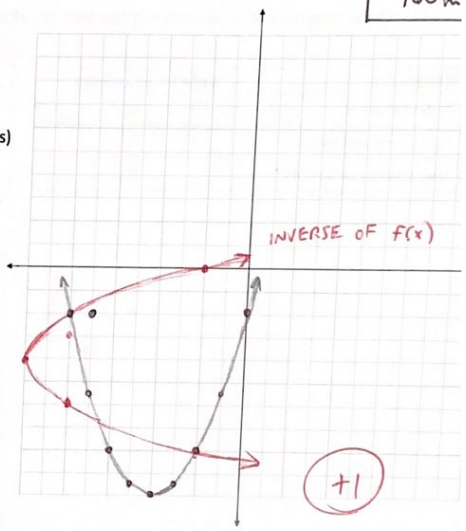
$\therefore$ 106.6 x 160m

8. Graph the following using any method:

$f(x) = \frac{1}{2}(x+4)^2 - 10$

(graph 5 lattice points or 7 total points to receive full marks)

3marks



9. You have a ferry boat business on the Detroit river. You currently charge $18 per ticket and sell an average 372 tickets per day. An industry journal says that, for every one dollar increase in price, you can expect to lose six ticket sales per day. It costs $2 per person to operate the ferry plus a $500 fixed cost for fuel and wages. How much should you charge for a ticket to maximize your profit?

Let x represent the # of $1 price increases.

$P(x) = (372 - 6x)(18 + x) - 2(372 - 6x) - 500$

$P(x) = 6696 + 372x - 108x - 6x^2 - 744 + 12x - 500$

$P(x) = -6x^2 + 276x + 5452$

4

AOS

$x = \frac{-276}{2(-6)}$

$x = 23$

$\therefore$ Increase 23 times for a ticket price of $41/ticket

11

10. Solve the following when $x \in \mathbb{R}$ (answers must be exact).

a) $0 = -\frac{1}{2}(x-1)^2 + 50$

$-50 = -\frac{1}{2}(x-1)^2$

$100 = (x-1)^2$

$\pm 10 = x - 1$

$1 \pm 10 = x$

$x = -9$

$x = 11$

$\{-9, 11\}$

b) $x^2 - 1 = -8x$

$x^2 + 8x - 1 = 0$

$x = \frac{-8 \pm \sqrt{(8)^2 - 4(-1)(1)}}{2(1)}$

$x = \frac{-8 \pm \sqrt{64+4}}{2}$

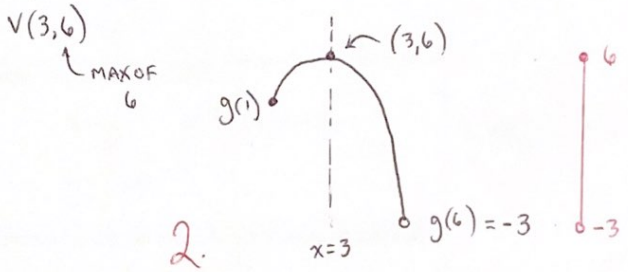
$x = \frac{-8 \pm \sqrt{68}}{2}$ ← $\sqrt{4}\sqrt{17}$

$x = \frac{-8 \pm 2\sqrt{17}}{2}$

$\boxed{x = -4 \pm \sqrt{17}}$

4.

11. Find the range of $g(x) = -(x-3)^2 + 6$ if the domain is restricted to $1 \leq x < 6$.



2.

RANGE: $\{y \in \mathbb{R} \mid -3 < y \leq 6\}$

12. Determine the equation of the parabola that has:

- an axis of symmetry of $x = 8$
- *y-intercept* of $(0,9)$ and
- passes through the point $(4,6)$

$x = \frac{-b}{2a}$

$8 = \frac{-b}{2a}$

$16a = -b$

$-16a = b$ ①

$y = ax^2 + bx + 9$ ②

sub ① into ②

$y = ax^2 - 16ax + 9$

sub $(4,6)$ (x y)

$6 = a(4)^2 - 16a(4) + 9$

$6 = 16a - 64a + 9$

$-3 = -48a$

$\frac{1}{16} = a$

$\therefore f(x) = \frac{1}{16}x^2 - x + 9$

sub $a = \frac{1}{16}$ into ①

$b = -16\left(\frac{1}{16}\right)$

$b = -1$

3

IN VERTEX FORM:

$f(x) = \frac{1}{16}(x-8)^2 + 5$

* For 1 bonus mark – graph the inverse of the parabola from question 8 (use the same provided grid from question 8)

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