

- Unless otherwise stated, all variables belong to the real number set
- All restrictions must be stated
- Approved calculators are permitted

Part A – Full Solutions

1. State the domain in set-builder notation: $\frac{5x-1}{x(x-2)(2x+3)(x^2+1)}$

$$\text{Domain} = \{x \in \mathbb{R} \mid x \neq -\frac{3}{2}, 0, 2\} \quad \checkmark$$

2. Simplify and state restrictions:

a) $\frac{m^2-6m+8}{20-5m}$

$$= \frac{(m-4)(m-2)}{-5(m-4)}, m \neq 4$$

$$= -\frac{m-2}{5}, m \neq 4$$

✓✓

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b) $\frac{30x-15y}{20x-10y}$

$$= \frac{15(2x-y)}{10(2x-y)}, x \neq \frac{1}{2}y$$

$$= \frac{3}{2}, x \neq \frac{1}{2}y$$

✓✓

2

3. Simplify and state restrictions:

a) $\frac{48x(7x-1)^2}{x^3} \cdot \frac{(x-2)(x-3)}{(7x-1)(3x-9)}$

$$= \frac{16 \cancel{48}x(7x-1)^2}{x^3 \cancel{x^2}} \cdot \frac{(x-2)(x-3)}{3(7x-1)(x-3)}, x \neq 0, \frac{1}{7}, 3$$

$$= \frac{16(x-2)(7x-1)}{x^2}, x \neq 0, \frac{1}{7}, 3$$

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b) $\frac{2x^2+11x-21}{4x^2-10x+6} \div \frac{2x^2-98}{x^2-x+ax-a}$

$$= \frac{(2x-3)(x+7)}{2(2x^2-5x+3)} \div \frac{2(x^2-49)}{x(x-1)+a(x-1)}$$

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$$= \frac{(2x-3)(x+7)}{2(2x-3)(x-1)} \div \frac{2(x-7)(x+7)}{(x-1)(x+a)}, x \neq \frac{3}{2}, 1, -a, \pm 7$$

$$= \frac{(2x-3)(x+7)}{2(2x-3)(x-1)} \cdot \frac{(x-1)(x+a)}{2(x-7)(x+7)}$$

$$= \frac{x+a}{4(x-7)}, x \neq 1, \frac{3}{2}, -a, \pm 7$$

/ 12

$$\begin{aligned}
 \text{c) } & \frac{9-x^2}{2x^2+x-15} \div \frac{1}{4x^2-20x+25} \\
 &= \frac{-(x^2-9)}{(2x-5)(x+3)} \cdot (2x-5)^2 \\
 &= \frac{-(x-3)\cancel{(x+3)}}{\cancel{(2x-5)}(x+3)} \cdot (2x-5)^2, x \neq -3, \frac{5}{2} \\
 &= -(x-3)(2x-5), x \neq -3, \frac{5}{2}
 \end{aligned}$$

3

$$\begin{aligned}
 \text{d) } & \frac{1-\frac{49}{c^2}}{\frac{7}{c}+1} \cdot \frac{c^2}{c^2} \\
 &= \frac{c^2-49}{7c+c^2} \\
 &= \frac{(c-7)\cancel{(c+7)}}{c\cancel{(c+7)}}, c \neq -7, 0 \\
 &= \frac{c-7}{c}, c \neq -7, 0
 \end{aligned}$$

2

4. Simplify and state restrictions:

$$\begin{aligned}
 \text{a) } & \frac{3x-2}{x^2+4x-12} - \frac{5}{2x+12} \\
 &= \frac{3x-2}{(x+6)(x-2)} - \frac{5}{2(x+6)}, x \neq 2, -6 \\
 &= \frac{2(3x-2)}{2(x+6)(x-2)} - \frac{5(x-2)}{2(x+6)(x-2)} \\
 &= \frac{6x-4-5x+10}{2(x+6)(x-2)} \\
 &= \frac{\cancel{x+6}}{2\cancel{(x+6)}(x-2)} \\
 &= \frac{1}{2(x-2)}, x \neq -6, 2
 \end{aligned}$$

4

$$\begin{aligned}
 \text{b) } & \frac{x}{4(x+4)} + \frac{5}{6(x+4)} + 1, x \neq -4 \\
 &= \frac{3x}{12(x+4)} + \frac{10}{12(x+4)} + \frac{12(x+4)}{12(x+4)} \\
 &= \frac{3x+10+12x+48}{12(x+4)} \\
 &= \frac{15x+58}{12(x+4)}, x \neq -4
 \end{aligned}$$

3

12

$$c) \frac{x}{x^2+5x-50} - \frac{1}{x^2-7x+10} + \frac{1}{x^2+8x-20}$$

$$= \frac{x}{(x+10)(x-5)} - \frac{1}{(x-2)(x-5)} + \frac{1}{(x+10)(x-2)}, x \neq -10, 2, 5$$

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$$= \frac{x(x-2) - (x+10) + (x-5)}{(x+10)(x-2)(x-5)}$$

$$= \frac{x^2 - 2x - x - 10 + x - 5}{(x+10)(x-2)(x-5)}$$

$$= \frac{x^2 - 2x - 15}{(x+10)(x-2)(x-5)}$$

$$= \frac{(x-5)(x+3)}{(x+10)(x-2)(x-5)}$$

$$= \frac{x+3}{(x+10)(x-2)}, x \neq -10, 2, 5$$

5. Solve the following rational equation:

$$\frac{4y}{y+2} - \frac{y}{y-1} = \frac{9}{y^2+y-2}, y \neq 1, -2$$

$$\frac{4y}{y+2} - \frac{y}{y-1} = \frac{9}{(y+2)(y-1)}$$

[mult BS by $(y+2)(y-1)$]

$$4y(y-1) - y(y+2) = 9$$

$$4y^2 - 4y - y^2 - 2y = 9$$

$$3y^2 - 6y = 9$$

$$3y^2 - 6y - 9 = 0$$

$$3(y^2 - 2y - 3) = 0$$

$$3(y-3)(y+1) = 0$$

$$\downarrow \quad \downarrow$$

$$y=3 \quad y=-1$$

$$\{-1, 3\}$$

4

6. The Massey Girls basketball team went out for a team dinner to celebrate winning the WECSSAA Championship. The total bill was \$288. The players were supposed to split the bill evenly but 4 of the players forgot their money, so each person that paid had to pay \$12 more dollars than they expected.

Solve a rational equation to determine how many basketball players went out to dinner?

Let x rep. the # of players that went out to dinner.

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$$\frac{288}{x-4} - \frac{288}{x} = 12$$

[mult BS by $x(x-4)$]

$$288x - 288(x-4) = 12(x)(x-4)$$

$$288x - 288x + 1152 = 12x^2 - 48x$$

$$0 = 12x^2 - 48x - 1152$$

$$0 = 12(x^2 - 4x - 96)$$

$$0 = 12(x-12)(x+8)$$

$$\downarrow \quad \downarrow$$

$$x=12 \quad x=-8$$

$\therefore 12$ players went out to dinner.

12

7. Determine A and B if $\frac{A}{4x-1} + \frac{B}{x+6} = \frac{19x+14}{4x^2+23x-6}$ where $A, B \in \mathbb{Z}$.

$$\begin{aligned} & A(x+6) + B(4x-1) \\ &= Ax + 6A + 4Bx - B \\ &= \underbrace{(A+4B)}_{19}x + \underbrace{(6A-B)}_{14} \end{aligned}$$

$$(x+6)(4x-1)$$

$$\textcircled{1} A + 4B = 19 \longrightarrow A + 4B = 19 \textcircled{1}$$

$$\textcircled{2} 6A - B = 14 \xrightarrow{\times 4} 24A - 4B = 56 \textcircled{2}$$

$$25A = 75 \quad \textcircled{1} + \textcircled{2}$$

$$\boxed{A = 3} \textcircled{4}$$

sub $\textcircled{4}$ into $\textcircled{1}$

$$3 + 4B = 19$$

$$\boxed{B = 4}$$

Part B - Multiple Choice (1 mark each)

ANSWER:

C

1. Which is the LCD (lowest common denominator) for $\frac{3}{2x+10} + \frac{5x}{3x^2-75}$

A) $(2x+10)(3x^2-75)$

B) $(2x+10)(x-5)$

C) $6(x+5)(x-5)$

D) $2(x+5)(x-5)$

D) $(x+5)(x-5)$

F) $3(x+5)(x-5)$

ANSWER:

B

2. How many of the following rational expressions do not have any restrictions on the variable?

$\frac{x^2-1}{x} \quad x \neq 0$

$\frac{x^2+1}{x^2} \quad x \neq 0$

$\frac{x^2+4x+3}{6}$

$\frac{x^2-25}{x-5} \quad x \neq 5$

$\frac{x^2+1}{x^3+1} \quad x \neq -1$

A) 0

B) 1

C) 2

D) 3

E) 4

F) 5

G) 6

ANSWER:

C

3. Simplify completely $\frac{x+\frac{x}{3}}{x+\frac{x}{4}}$ where $x \neq 0$

A) $\frac{3}{4}$

B) $\frac{4}{3}$

C) $\frac{16}{15}$

D) $\frac{15}{16}$

E) $\frac{5}{4}$

ANSWER:

E

4. Which of the following is equivalent to $(x+2)^3$?

A) $x^3 + 8$

B) $x^3 + 6$

C) $x^3 + 8x$

D) $x^3 - 6x^2 + 12x - 8$

E) $x^3 + 6x^2 + 12x + 8$

ANSWER:

D

5. Which of the following is equivalent to $\frac{4x^2-28x+49-y^2}{2x+y-7}$?

A) $2x - y + 7$

B) $2x + y + 7$

C) $2x + y - 7$

D) $2x - y - 7$

E) None of the above

ANSWER:

E

6. Which of the following is equivalent to $\frac{8x^2-20xy+2x}{2x}$?

A) $4x - 10y$

B) $4x + 10y + 1$

C) $4x - 10xy + 2x$

D) $4x - 1$

E) None of the above

ANSWER:

A

7. When simplified, $\frac{6x^2+11x+5}{x+1} = Ax + B$. What is the value of $A + B$?

A) 11

B) 6

C) -6

D) 16

E) 2

F) None of the above

ANSWER:

C

8. For what values of x is the following expression equal to zero?

$$\frac{x^2+2x+1}{x^2-4x+4}$$

A) 2

B) 2, -1

C) -1

D) -2

E) 1

F) None of the above