Name : ____________________

MHF4U1

**Unit 1: Polynomial Functions**

| K/U | APP | COM | TH | Total |
|---|---|---|---|---|
| /26 | /16 | ------------ | /10 | /52 |

## KNOWLEDGE/UNDERSTANDING

**Multiple Choice**

*Identify the choice that best completes the statement or answers the question.*

D 1. The restriction on the degree 'n' for all polynomial functions of the form $f(x) = a_nx^n + a_{n-1}x^{n-1}+\cdots+a_1x^1+a_0x^0$ is: [1K]

a. 'n' must be non-negative
b. $n \in \{0, 1, 2, 3, 4,\ldots\}$
c. 'n' must be a whole number
d. All of the above

B 2. Which function does not classify as a polynomial function? [1K]

a. $y = -3$
b. $y = 2\sqrt{x}$
c. $y = 2x^2-1$
d. $y = -2x^7$

C 3. If the leading coefficient of an odd-degree polynomial function is negative, then the function extends from: [1K]

a. $1^{st}$ quadrant → $3^{rd}$ quadrant
b. $1^{st}$ quadrant → $2^{nd}$ quadrant
c. $2^{nd}$ quadrant → $4^{th}$ quadrant
d. $2^{nd}$ quadrant → $3^{rd}$ quadrant

A 4. The degree 'n' of a polynomial provides information about all of the following except: [1K]

a. The shape of the graph
b. The end behaviours of the graph
c. The roots of the graph
d. All of the above

B 5. Which polynomial function has its end behaviour extending from quadrants 1→2? [1K]

a. $f(x) = 7x^5 - 8x^4 - 2x^3 + x^2 + 3x - 2$
b. $f(x) = -4x^2 + 3x^4 - 6x^3 + 2x + 8$
c. $f(x) = -6x^3 + 3x^2 + x - 11$
d. $f(x) = x^3 - 9$

D 6. The graph of an odd-degree polynomial function has at least __?__ root(s) and up to a maximum of __?__ roots. [1K]

a. 3, n
b. 0, n-1
c. 1, n-1
d. 1, n

B 7. The graph of an even degree polynomial function can have at least__?__ root(s) and a maximum of up to__?__ roots.

[1K]

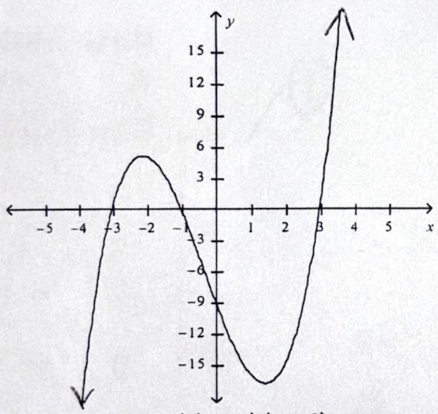
a. 2, n
b. 0, n
c. 0, n-1
d. n, n-1

B 8. At "first glance", the polynomial function $f(x) = 3x^4-2x^3+7x^2-1$ provides information about all of the following except: [1K]

a. End behaviours
b. Exact shape of the graph
c. Degree of the function
d. Sign of leading coefficient

A 9. What is the equation of the graph shown below?

[1K]




a. $f(x) = (x-3)(x+1)(x+3)$
b. $f(x) = (x-3)(x-1)(x+3)$
c. $f(x) = (x-3)(x+3)$
d. $f(x) = (x-3)^2(x+1)$

D 10. The degree and x-intercepts of the polynomial function $f(x) = x(x+3)^3(x+1)(x-5)$ are:

[1K]

a. n=6, x=0, 3 (order 3), -1, -5
b. n=5, x=0, -3 (order 2), -1, -5/2
c. n=5, x=0, -3 (order 3), -1, 5/2
d. n=6, x=0, -3 (order 3), -1, 5

D 11. The function $h(t) = -4.9t^2 + 120$, where h is in metres and t in seconds, models the following scenario: [1K]

a. A football punt on goal
b. A ball being thrown downward from the roof of a building 120 metres high
c. A cannon being shot upward from the edge of a cliff
d. Releasing a ball from a bridge 120 metres above ground

12. Sketch a possible graph of the functions [6K]

a) y= (x+1)(x-1)(x-5)

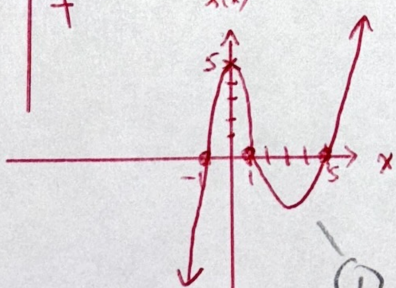
$x = -1, 1, 5$

(1)

| x | (x+1) | (x-1) | (x-5) | y |
|---|---|---|---|---|
| $[-\infty, -1]$ | − | − | − | − |
| $[-1, 1]$ | + | − | − | + |
| $[1, 5]$ | + | + | − | − |
| $[5, \infty]$ | + | + | + | + |

$f(0) = (0+1)(0-1)(0-5)$
$= 5$

(1)




(1)

b) $y = x^2(x-3)^3$

$x = 0, 3$

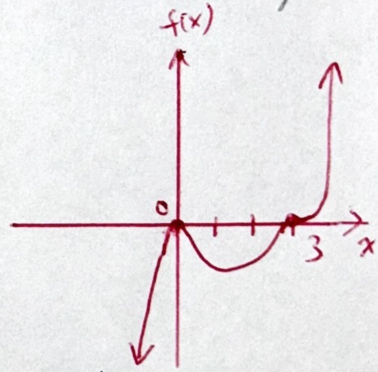
(1)

| | $x^2$ | $(x-3)^3$ | y |
|---|---|---|---|
| $[-\infty, 0]$ | + | − | − |
| $[0, 3]$ | + | − | − |
| $[3, \infty]$ | + | + | + |

(1)

$f(0) = 0^2(0-3)^3$
$= 0$

(1)




13. Write an equation of a cubic function that has zeros of -1, 2, and 3. The function has a y-intercept of 6. [3K]

$\therefore \begin{cases} f(x) = a(x+1)(x-2)(x-3) \\ y\text{-int} = (0,6) \end{cases}$ —— (1)

$6 = a(0+1)(0-2)(0-3)$
$6 = a(6)$
$a = 1$ —— (1)

$\therefore f(x) = (x+1)(x-2)(x-3)$ —— (1)

## CATION

This table of values represents a polynomial function f(x). Use **finite differences** to determine the following: [4A]

(a) the degree 'n' of the polynomial function f(x)
(b) the sign of the leading coefficient
(c) the value of the leading coefficient, $a_n$

| X | Y |
|---|---|
| -3 | 0 |
| -2 | -4 |
| -1 | 0 |
| 0 | 6 |
| 1 | 8 |
| 2 | 0 |
| 3 | -24 |

1st: -4, 4, 6, 2, -8, -24
2nd: 8, 2, -4, -10, -16
3rd: -6, -6, -6, -6

a) $n = 3$ — (1)

c) $c.d. = a_n(n!)$
$-6 = a_n(3!)$
$-6 = a_n(3 \times 2 \times 1)$
$a_n = -1$ — (2)

b) leading coefficient is negative — (1)

2. On Earth, the height, h, in metres, of a free falling object after t seconds can be modeled by the function $h(t) = -4.9t^2 + k$, while on Venus, the height can be modeled by $h(t) = -4.45t^2 + k$, where $t \geq 0$ and k is the height, in metres, from which the object is dropped. Suppose a rock is dropped from a height of 60 m on each planet. For each planet, [6A]

(a) Determine the **average rate of change** of the height of the rock after the first 3 seconds after it is dropped (i.e. $0 \leq t \leq 3$)
(b) Estimate the **instantaneous rate of change** of the rock 3 seconds after it is dropped (i.e. @ t= 3 sec). Hint: using average rate of change, choose two to three intervals of time that get closer and closer to 3 sec.
(c) Compare the average rates of change of the falling rock on Earth and on Venus. What do these rates represent, and why are these values different?

Earth: $h_E(t) = -4.9t^2 + k$, $k = 60$
Venus: $h_V(t) = -4.45t^2 + k$, $k = 60$
$\Rightarrow$ $h_E(t) = -4.9t^2 + 60$, $h_V(t) = -4.45t^2 + 60$

a) $AROC_E = \dfrac{\Delta h}{\Delta t}$
$= \dfrac{h(3) - h(0)}{3 - 0}$
$= \dfrac{-4.9(3)^2 + 60 - [-4.9(0)^2 + 60]}{3}$
$= -14.7\ m/s$ — (1)

$AROC_V = \dfrac{\Delta h}{\Delta t}$
$= \dfrac{h(3) - h(0)}{3 - 0}$
$= \dfrac{-4.45(3)^2 + 60 - [-4.45(0)^2 + 60]}{3}$
$= -13.35\ m/s$ — (1)

b) $IROC_E = \lim_{h \to 0} \frac{h(t+h) - h(t)}{h}$

$= \lim_{h \to 0} \frac{-4.9(t+h)^2 + 60 - [-4.9t^2 + 60]}{h}$

$= \lim_{h \to 0} \frac{\cancel{-4.9t^2} - 4.9h^2 - 9.8ht \cancel{+60} \cancel{+4.9t^2} \cancel{-60}}{h}$

$= \lim_{h \to 0} \frac{-4.9h^2 - 9.8ht}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(-4.9h - 9.8t)}{\cancel{h}}$

$= -9.8t$

$= -9.8(3)$

$IROC_E = -29.4 \text{ m/s}$ — 2

$IROC_V = \lim_{h \to 0} \frac{h(t+h) - h(t)}{h}$

$= \lim_{h \to 0} \frac{-4.5(t+h)^2 + 60 - [-4.5t^2 + 60]}{h}$

$= \lim_{h \to 0} \frac{\cancel{-4.5t^2} - 4.5h^2 - 9th \cancel{+60} \cancel{+4.5t^2} \cancel{-60}}{h}$

$= \lim_{h \to 0} \frac{-4.5h^2 - 9th}{h}$

$= \lim_{h \to 0} \frac{h(-4.5h - 9t)}{h}$

$= -9t$

$= -9(3)$

$IROC_E = -27 \text{ m/s}$ — 2

c) - AROC represents the speed between two points. — 1

- The rates are different between Earth & Venus are because of different gravity force. — 1

4. Without graphing, determine if each polynomial function has line symmetry about the y-axis, point symmetry about the origin, or neither. [2K]
State the type of odd function or even function or neither.

$f(x) = -x^5 + 7x^3 + 2x$

sub $x$ with $-x$ — $\frac{1}{2}$

$f(-x) = -(-x)^5 + 7(-x)^3 + 2(-x)$

$= x^5 - 7x^3 - 2x$

$= (-1)(-x^5 + 7x^3 + 2x)$

$f(-x) = -f(x)$ — $1\frac{1}{2}$

= odd function with a symmetry point — 1

5.
i) Describe the transformations that must be applied to the graph of each power function, f(x), to obtain the transformed function.
Then, write the corresponding equation.
ii) State the domain and range of the transformed function. For even functions, state the vertex and the equation of the axis of symmetry.
iii) Find the new transformed coordinate with $f(-2)$

$f(x) = x^4$, $y = -5f[2x - 6] + 1$ [4k]

iv) Graph

i) $f(x) = -5(2x-6)^4 + 1$ — $\frac{1}{2}$

$f(x) = -5[(2)(x-3)]^4 + 1$ — 1

a = 5, Vertical stretch ×5
k = 2, Horizontal Compression ÷ 2
d = 3, Horizontal translation → 3
c = 1, Vertical translation ↑ 1
a = -ve, Vertical Reflection

$2\frac{1}{2}$

ii) $D = \{x \in \mathbb{R}\}$ — $\frac{1}{2}$

$R = \{y \in \mathbb{R} \mid y \leq 1\}$ — $\frac{1}{2}$

Axis of Symmetry:
$x = 3$ — $\frac{1}{2}$

$V_{(0,0)} = V_{(x,y)} = (\frac{x}{k} + d, ay + c)$

$= (3, 1)$ — $\frac{1}{2}$

iii) $V_{(x,y)} = \left( \frac{x}{k} + d \ , \ ay + c \right)$

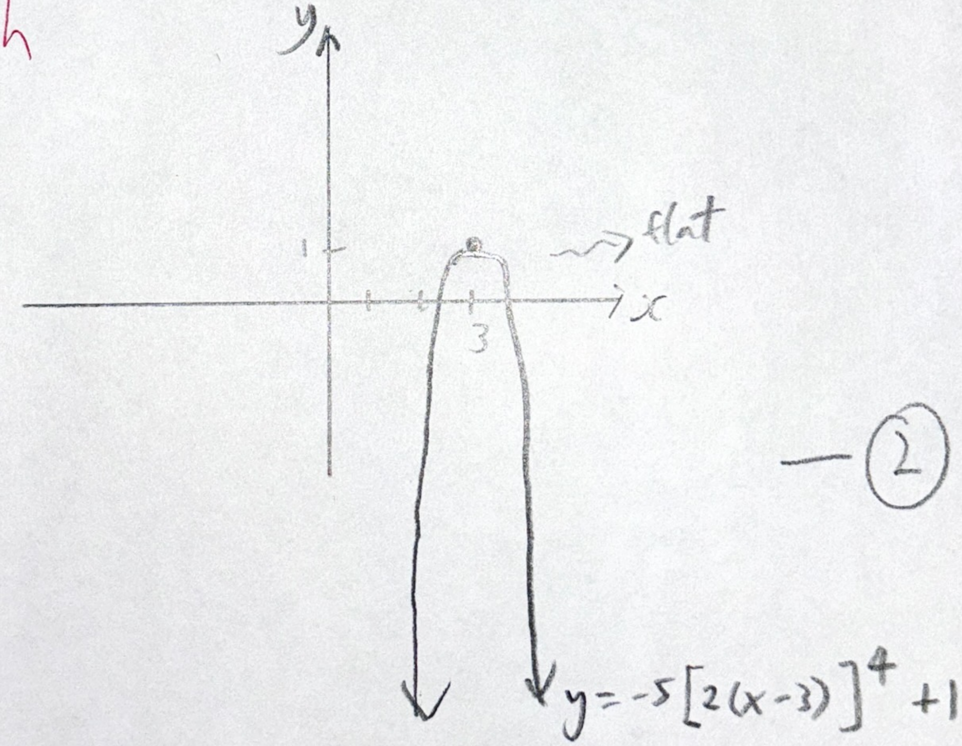
$= \left( \frac{(-2)}{2} + 3 \ , \ -5(16) + 1 \right)$

$= \left( 2 \ , \ -79 \right)$ — 2

$f(-2) = 16$

$(-2, 16)$

iv) Graph

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