

MHF 4U Chapter 3 Rational Functions

Key Concepts

Key Features

- By analyzing the key _____ of a rational function we are able to get a picture of what the function will look like.
- The key features of rational functions are – Asymptotes and the _____ near them, _____ (both x and y), intervals of increase and _____, domain, range, and positive and _____ intervals.

Behaviour Tendencies at Asymptotes

- The value of the function will get closer and closer to the _____ asymptote when $x \rightarrow \pm\infty$.
- To determine if the function approaches the horizontal asymptote from _____ or below it, sub sufficiently large positive and negative values in for x and see if the value is _____ or _____ than the horizontal asymptote.
- A vertical asymptote will occur when the denominator of the function is _____ (unless the function can be _____)
- As the function gets close to the _____ asymptote, the value of the function will tend to $\pm\infty$. To determine which direction it will go, sub in a value of x that is sufficiently close to the vertical asymptote BOTH to the _____ and _____ of the asymptote and determine the function's sign.

Reciprocal of a Linear Function

- The reciprocal of a linear function is $f(x) = \frac{1}{kx - c}$
- The domain of a reciprocal linear function cannot include where the denominator is zero (_____). At this x value we will find a vertical asymptote.
- The reciprocal of a linear function will have a horizontal asymptote along the x-axis (_____).
- To find the slope of any function at a given value of x, we must approximate the slope of the _____ to the function by finding the slope of a _____ for two x-values that are very close together.

Reciprocal of a Quadratic Function

- The reciprocal of a quadratic function that has two distinct zeros, will have two _____, dividing the function into three intervals.
- The middle interval of a reciprocal quadratic function is somewhat similar to a parabola in that it will have a _____ or _____ value and be symmetric about a vertical line through that point. This _____ or _____ value lies directly in between the asymptotes. The x value of this min/max point can be found by _____ the asymptotes. Then sub that value in for x in the function to find the y-value at that point.
- The intervals of increase and decrease can be found by dividing the domain into 4 sections : $-\infty$ to left asymptote, left _____ to local max/min value, local max/min value to right _____, right asymptote to $+\infty$.
- To check whether the function is increasing or decreasing in those intervals, pick two points within the interval and find the _____. If the _____ is negative, the function is _____, if the _____ is positive, the function is _____.

Rational Functions of the Form $f(x) = \frac{ax + b}{cx + d}$

- Functions of this type have _____ asymptotes, $x = -\frac{d}{c}$
- Functions of this type will have _____ asymptotes of $y = \frac{a}{c}$
- Functions of this type will have an _____-intercept of $x = -\frac{b}{a}$
- Functions of this type will have a _____-intercept of $y = \frac{b}{d}$

Rational Equations

- To solve a rational equation, get a common _____ on each side of the equal side and simplify. Then _____, expand, simplify, get one side equal to zero and solve as any normal polynomial equations (factoring, factor theorem or the quadratic formula). Be sure to state the restrictions on the _____.

Rational Inequations

- You cannot use cross multiplication.
- Get one side equal to zero, get a common denominator and _____ both numerator and denominator.
- Use an interval test to determine the _____ of each factor in intervals between _____ and _____.
- Determine the sign of the expression in each _____. Choose the appropriate interval for the inequality (<0 will be negative intervals, >0 will be positive intervals)

Special Case Functions

- sometimes when you factor an expression, a factor will _____ in the numerator and denominator.
- The function will be _____ at the x value that makes the cancelled factor zero (i.e. there will be a small hole in the function at that point. To find the y-value at this point, sub the x-value into the simplified function.

Suggested questions

Page 192 #1a), 2b), 3b)d), 5b)d), 6, 7, 9b)d), 10, 12a)c), 13 c)d), 15, 16