Name : ________________________

MHF4U1
Unit 5: Trigonometry Part 2

K/U	APP	COM	TH
/13	/24	/4	/14

TOTAL **/49**

KNOWLEDGE/UNDERSTANDING

______1. For the function $y = A\sin\left(B(x - C)\right) - D$, which variable determines amplitude? **[1K]**

a. *A*
b. *B*
c. *C*
d. *D*

______2. What is the period of the function $y = 10\sin\left(\frac{6\pi}{4}\left(x - \frac{\pi}{2}\right)\right) + 25$? **[1K]**

a. $\frac{4}{3}$
b. $\frac{4}{6}$
c. $\frac{6}{3}$
d. $\frac{3}{4}$

______3. If $k = \frac{2\pi}{45}$, what is the period? **[1K]**

a. $\frac{45}{2}$
b. 45
c. $\frac{45}{2\pi}$
d. $\frac{2\pi}{45}$

____ 4. The temperature of a swimming pool is cyclic and modelled by a trigonometric function. If its highest temperature is 82 °F and its lowest temperature is 76 °F, and it takes 12 hours for the temperature to change between its extremes, what equation models the temperature of the pool as a function of time in hours? **[1K]**

a. $y = 3\cos\left(\frac{2\pi}{24}t\right) + 79$
b. $y = 3\cos\left(\frac{2\pi}{12}t\right) + 79$
c. $y = 3\sin\left(\frac{2\pi}{24}t\right) + 79$
d. $y = 6\cos\left(\frac{2\pi}{24}t\right) + 79$

5. Use transformations to sketch the graph of the following function: $y = 5\cos(x - \pi/4) - 2$. $(-2\pi, 2\pi)$ **[6 K]**

Find the following:

(a) Amplitude?

(b) Period?

(c) Phase shift (left or right? And by how much?)

(d) Describe the vertical translation

(e) Graph the function

Application

1. Mike Tyson was training for his fight and he was jabbing his punching bag. With Tyson's raw poser, the punching bag reached a maximum and minimum horizontal distance of 75cm. One full cycle of the swinging punching bag had occurred every 2.5 seconds. Assume that the bag is at the equilibrium position at t = 0 sec. **[7 A]**

   a.) What is the equation of a **cosine** function?

   b) Graph the function. Fully label the axes. (3 cycles)

   c) Where would the bag be at 6.2 seconds?

← ↕ →

-75cm 0cm 75cm

2. The TREC Windshare turbine at the Essex, Ontario, has the following specifications:

[8 A]

- 90 metres high to the tip of the blade
- Rotor diameter 52 metres
- Normal rotation speed 30 rpm

(a) Using the information provided above, draw a graph which represents 2 complete cycles for a point at the tip of a blade on the wind turbine. Remember to fully label your axes. Assume that the first blade is at the equilibrium position at t = 0 sec

(b) Write a "Sine" function that models the rotating motion of the blades.

COMMUNICATION

1. What adjustment is needed to change the cosine function to the sine function? **[2C]**

2. Explain graphically why sin x = 0.75 has 2 solutions for $0 \le \theta \le 2\pi$. **[2C]**

THINKING

1. A cosine function has a maximum value of 1, a minimum value of -5, a phase shift of $\pi/4$ to the right, and a period of 2. Write an equation for the function. **[4 T]**

2. Determine all the solutions for the following trigonometric equations: $0 \le \theta \le 2\pi$.

(a) $\text{Cos}^2 x - \cos x - 2 = 0$ **[4T]**

(b) $2 \sin^2 x - 1 = 0$ **[4T]**

(c) Sin $(\Theta + \frac{\pi}{4}) = \sqrt{2} \cos \Theta$ **[4T]**

3. Determine all the solutions of the equation: [**4K]**

$$3 \sin^2 x + \sin x - 1 = 0$$

Basic Trigonometric Identities

You will be using these basic trig identities to prove more obscure trig identities.

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identitie

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta = 1 + \tan^2\theta$$
$$\csc^2\theta = 1 + \cot^2\theta$$

Related Angles Identities

$$\sin(\pi - \theta) = \sin\theta \qquad \sin(\pi + \theta) = -\sin\theta \qquad \sin(2\pi - \theta) = -\sin\theta \qquad \sin(-\theta) = -\sin\theta$$
$$\cos(\pi - \theta) = -\cos\theta \qquad \cos(\pi + \theta) = -\cos\theta \qquad \cos(2\pi - \theta) = \cos\theta \qquad \cos(-\theta) = \cos\theta$$
$$\tan(\pi - \theta) = -\tan\theta \qquad \tan(\pi + \theta) = \tan\theta \qquad \tan(2\pi - \theta) = -\tan\theta \qquad \tan(-\theta) = -\tan\theta$$

Co-related Angle Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta \qquad \sin\left(\frac{3\pi}{2} - \theta\right) = -\cos\theta \qquad \sin\left(\frac{3\pi}{2} + \theta\right) = -\cos\theta$$
$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta \qquad \cos\left(\frac{3\pi}{2} - \theta\right) = -\sin\theta \qquad \cos\left(\frac{3\pi}{2} + \theta\right) = \sin\theta$$
$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \tan\left(\frac{\pi}{2} + \theta\right) = -\cot\theta \qquad \tan\left(\frac{3\pi}{2} - \theta\right) = \cot\theta \qquad \tan\left(\frac{3\pi}{2} + \theta\right) = -\cot\theta$$

Compound Angle Formulas

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Special Triangles and the Unit Circle