MHF4U

Advanced Functions
Grade 12 University

Unit 2 Polynomial Functions

9 Video Lessons

Allow **no more** than **15** class days for this unit!
This includes time for review and to write the test.

Lesson #	Title	Practice Questions	Date Completed
1	The Remainder Theorem - Part 1	Page 91 #1-6	
2	The Remainder Theorem - Part 2	Page 91 #(7-9)eop, 10, 13, 15, 16, 17, 19, 20, 21	
3	The Factor Theorem	Page 102 #1-11 (eop when appropriate)	
4	Sum and Difference of Cubes	Page 102 #4, 5, (6, 7)eop, 9, 10, 12, 13, 18, 20	
5	Solving Polynomial Equations	Page 110 #1, 2, 5, 7, 8, 9, 10, 14, 17, 19 (eop when appropriate)	
6	Families of Polynomial Functions	Page 119 #(1-5)eop, 8, 10, 19, 20	
7	Solving Inequalities Using Technology	Page 129 #3, 5, 6, 9, 10, 12, 16	
8	Solving Polynomial Inequalities	Page 138 #3, 4, 6, 8, 9, 12, 13	
9	Solving Polynomial Word Problems	Page 11 #10-16, 19, 21 Page 131 #10-12 Page 139 #8, 9, 13	

Test Written on : ________________________________

Topic : Dividing Polynomials

Goal : I can divide polynomials using long division and/or synthetic division and I can state my answer in quotient form or as a division statement.

The Remainder Theorem - Part 1

Long Division

Let's see if we remember how to do long division with constants...

$$15\overline{)62345}$$

The process of long division is similar with polynomials.

Example 1. Divide $3x^4 - 12x^3 - 20x^2 - 30x + 2$ by $x - 5$.

$$x-5\overline{)3x^4 - 12x^3 - 20x^2 - 30x + 2}$$

You know you are finished when the degree of the remainder is less than the degree of the divisor. For example – we have a linear divisor so our remainder will be constant.

If we state our result in quotient form it would look like this…

$$\frac{3x^4 - 12x^3 - 20x^2 - 30x + 2}{x - 5} = 3x^3 + 3x^2 - 5x - 55 - \frac{273}{(x-5)}$$

In general that would be...

$$\frac{P(x)}{(x-b)} = Q(x) + \frac{R}{(x-b)}$$

Often I'll want you to conclude a division question with a division statement in the proper form.

Original Polynomial = Divisor x Quotient + Remainder

In function notation we write:

$$P(x) = (x-b)Q(x) + R$$

For the example above, the division statement will be

$$3x^4 - 12x^3 - 20x^2 - 30x + 2 = (x-5)(3x^3 + 3x^2 - 5x - 55) - 273$$

$$P(x) = (x-b)Q(x) + R$$

Note: if you expand the right side of the division statement and simplify, you should get what's on the left side.

Example 2. Divide $6z^3 + 13z^2 - 9$ by $2z + 3$.

You must always place the polynomial in descending powers of the variable. If one power of the variable is missing, it means its coefficient was zero, and you need to put it in as a placeholder.

$$2z + 3 \overline{\smash{\big)}\, 6z^3 + 13z^2 + 0z - 9}$$

Since the remainder is zero, we know that the divisor went evenly into the polynomial. That makes it a factor of the original polynomial.

Example 3. One factor of $x^3 - 4x^2 - 11x + 30$ is $x - 5$. Factor the polynomial completely.

$$x - 5 \overline{\smash{\big)}\, x^3 - 4x^2 - 11x + 30}$$

This is in factored form, but it's not completely factored. You always need to check to see if the new quadratic factors as well.

Synthetic Division – used when you have a linear divisor

To use synthetic division you must have

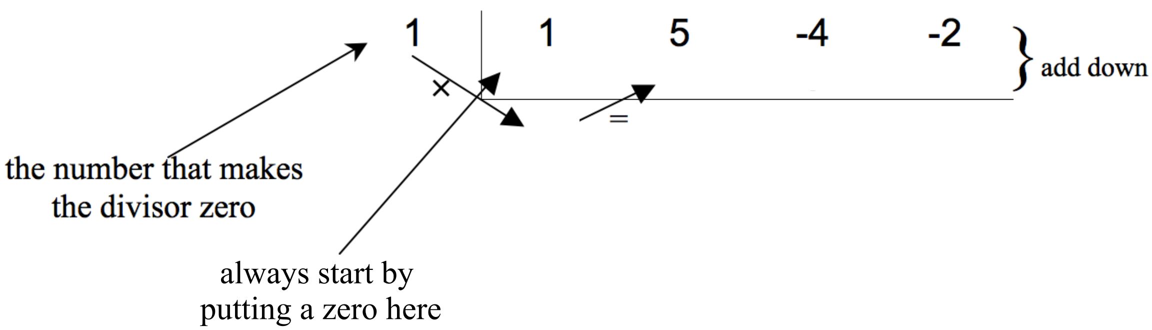
> a linear divisor where the coefficient of the variable is ONE.

> a polynomial written in descending powers of the variable

> if you are missing a power of the variable, you must fill in a zero for its coefficient.

Now, we only write the coefficients and fill in the variable when we are finished.

Example 4. Divide $5w^2 - 4w - 2 + w^3$ by $w - 1$



The final numbers (1,6,2,0) represent the coefficients of the quotient, starting with the power that is one less than the original polynomial. In this case, our answer will be $w^2 + 6w + 2$, the last number is the remainder (zero).

So our division statement is

Example 5. Divide $6w^4 - 3w^3 + 2w - 5$ by $w + 3$

Example 6. Divide $6z^3 + 13z^2 - 9$ by $2z + 3$.

Since the divisor doesn't have a coefficient of one, we must force out a factor of 2, to make the coefficient one. This will leave us with a fraction.

Now we are going to do this division in two steps. First we use synthetic division to divide out the linear factor, then we'll divide our answer by two.

Practice Questions - Page 91 #1-6

Topic : The Remainder Theorem

Goal : I know what the remainder theorem is and can apply it in appropriate situations.

The Remainder Theorem - Part 2

Example 1. Divide the polynomial $P(x) = x^3 - 2x^2 - 4$ by $x - 2$.

We'll use synthetic division…

The Remainder Theorem

If $P(x)$ is divided by $(x-b)$ and the remainder is constant, then the remainder will be $P(b)$

Proof: $P(x)$ is a polynomial
$(x-b)$ is the divisor
$Q(x)$ is the quotient
R is the remainder

The General Remainder Theorem

If $P(x)$ is divided by $(ax-b)$ and the remainder is constant, then the remainder will be $P\left(\frac{b}{a}\right)$ where a, b$\in$ I and a$\neq$0.

<u>Example 2.</u> Find the remainder for each division.

a) $\left(2x^2-3x+7\right)\div(x+4)$

b) $\left(4x^3-2x^2+6x-1\right)\div(2x-1)$

<u>Example 3.</u> When the polynomial $x^3 - 3x^2 + kx - 7$ is divided by $(x-4)$ the remainder is 29. What is the value of k?

Always start by filling in what you know...

Example 4. For what value of b will the polynomial $P(x) = -2x^3 + bx^2 - 5x + 2$ have the same remainder when divided by $(x - 2)$ and $(x + 1)$

If the remainders are the same, we know that $P(2) = P(-1)$ so we sub in and set them equal.

Practice Questions - Page 91 #(7-9)eop, 10, 13, 15, 16, 17, 19, 20, 21

Topic : Factoring Polynomials

Goal : I can factor polynomials of a higher degree than quadratic using the Factor Theorem

The Factor Theorem - Part 1

-
-
-
-

The Factor Theorem

(x - b) is a factor of P(x) if and only if P(b)=0

(ax - b) is a factor of P(x) if and only if $P(\frac{b}{a})$

Proof : Given (x-b) is a factor of P(x) then

That means that the quotient $\frac{P(x)}{(x-b)}$ will have a remainder of zero.

Example 1. Is $(x - 2)$ a factor of the following polynomials?

a) $x^3 - 7x^2 + 9x + 2$

b) $x^3 - 3x^2 + 2x - 5$

Integral Zero Theorem

When we have a polynomial in factored form say

$$P(x) = (2x - 3)(x + 4)(x - 5),$$

we know that the constant term in the expanded polynomial comes from multiplying all the constant terms of the factors.

So if this polynomial were in expanded form, we would know that any zeros would have to be a factor of _____.

In General...

For (x-b) to be a factor of the polynomial P(x), b must be a factor of the constant term of P(x).

Example 2. Factor the following...

$$x^3 + 2x^2 - 5x - 6$$

We need to divide out a linear factor, then we can employ our methods of factoring quadratics.

By the integral zero theorem, any zeros we find (that are integers) will be factors of 6.

When you check, always start with ±1.

The Rational Zero Theorem

Factor $3x^3 - 17x^2 + 31x - 14$

Now since there is a number other than 1 as the leading coefficient, we could have some zeros that are fractions.

To get $3x$ at the front we could have

To get 14 at the back we could have

Basically our factor would look like this.

so all our possible values that might give us a zero are...

Checking all these possibilities is a daunting task. Most calculators work with fractions, and most allow you to go back and change things, so you don't have to type the whole the polynomial in again, just change what is in the bracket.

We find that $P\left(\frac{2}{3}\right) = 0$ so using synthetic division, we divide it out.

So we get the division statement...

$3x^3 - 17x^2 + 31x - 14 =$

The quadratic factor may be able to factor further. In this case it doesn't. A quick check of the descriminant of the quadratic formula can tell you if a quadratic factors or not...

Practice Questions - Page 102 (#1-11)eop (when appropriate)

Topic : Factoring cubic expressions

Goal : I know how to factor a sum or difference of cubes without using the factor theorem.

Sum and Difference of Cubes

Example 1. Use factor theorem to factor $P(x) = x^3 + 8$

Example 2. Use factor theorem to factor $P(x) = x^3 - 8$

MHF 4U U2L4 Sum and Difference of Cubes

Look for patterns in a sum or difference of cubes...

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

Example 3. Use the patterns you found in sum and difference of cubes to factor the following...

a) $8t^3 - 64 =$

b) $y^3 + 27z^3 =$

c) $125 + 64x^3 =$

Factoring by Grouping

You may be able to group terms with common factors to find a binomial factor.

Example 4. Factor $x^3 + 2x^2 - 9x - 18$ by grouping terms.

Practice Questions - Page 102 #4, 5, (6, 7)eop, 9, 10, 12, 13, 18, 20

Topic : Polynomial Equations

Goal : I can use the factor theorem to solve polynomial equations of degrees higher than 2.

Solving Polynomial Equations

Solving a polynomial equation (or finding the roots of a polynomial function) is easy when things are factored.

Example 1. What are the roots of $2(x + 3)(3x - 4)(x-7) = 0$

What about equations that aren't in factored form?

We have a few tools at our disposal.

1.

2.

3.

Example 2. Solve $2x^3 - 3x^2 - 5x + 6 = 0$

MHFU U2L5 Solving Polynomial Equations

Example 3. Solve $x^4 - x^3 + 41x = 21x^2 + 20$

Example 4. Write equations, in standard form, that have the following roots.

a) 5, 3, -6

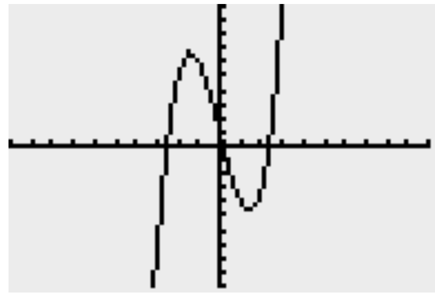
b) $1 \pm \sqrt{3}$

Example 5. Solve $4x^4 - 25x^2 + 36 = 0$

Notice - this kind of looks quadratic! Let's make a substitution. Let's let $n = x^2$

Example 6. Solve $x^3 + 27 = 0$

Example 7. Use technology to solve $x^3 - 6x + 1 = 0$



Practice Questions - Page 110 #1, 2, 5, 7, 8, 9 , 10, 14, 17, 19
(do eop when appropriate)

Topic : Families of Polynomial Functions

Goal : I know what is meant by a polynomial family, and I can find specific members of a polynomial family when given some extra information.

Families of Polynomial Functions

Example 1. Determine the family of cubic functions that have the intercepts -3, $3 \pm \sqrt{2}$

Example 2. A family of cubic functions have x-intercepts -1, 1 and 2. Find the member that passes through the point (3, 40)

Practice Questions - Page 119 #(1-5)eop, 8, 10, 19, 20

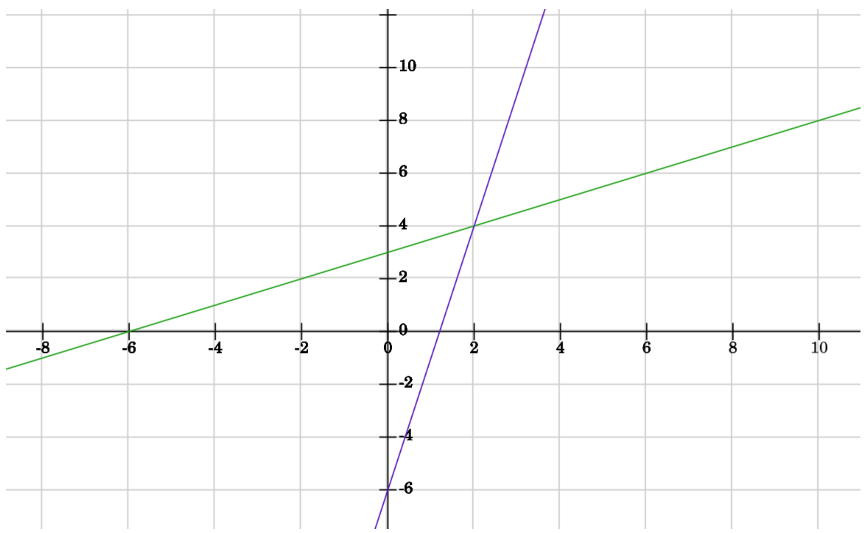
Topic : Solving Inequalities

Goal : I know what an inequality is, and I can solve it using graphing technology.

Solving Inequalities Using Technology

Example 1. For what values of x will $0.5x + 3 \leqq 5x - 6$

Graph



What we are really asking ourselves from this graph is for what values of x, does the green line lie below the purple line.

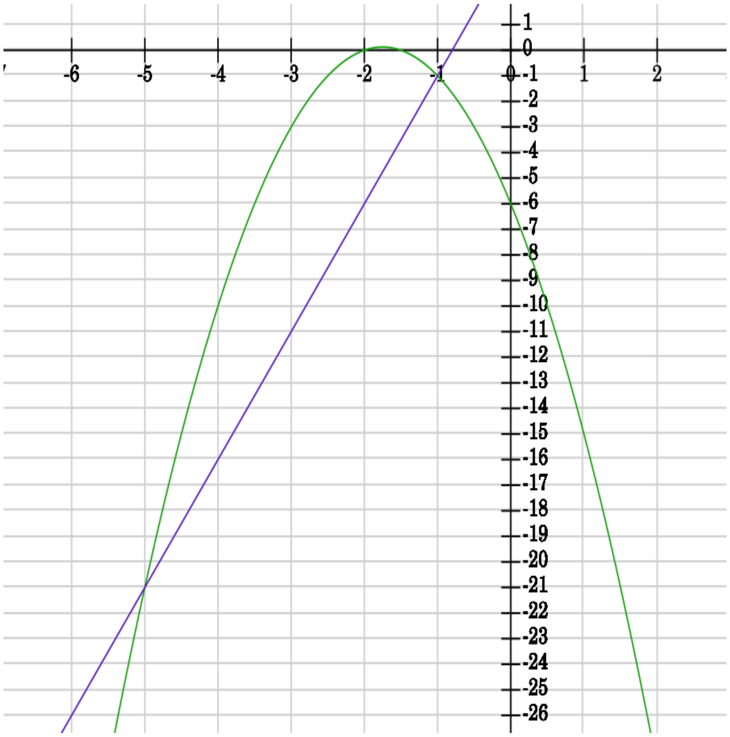
We can solve for this algebraically...

Note :

MHF4U U2L7 Solving Inequalities Using Technology

Example 2. For what values of x, is $-2x^2 - 7x - 6 \leqq 5x + 4$

Graph

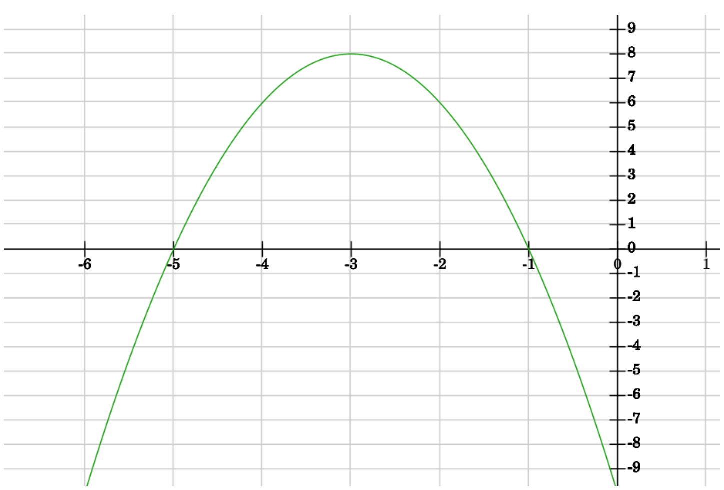


What we are really asking ourselves from this graph is for what values of x, does the green parabola lie BELOW the purple line.

This question would be easier if we were just dealing with one function, and trying to decide when it was above or below the x-axis. It would be easier to graph and easier to see. We can change it into that kind of question, by rearranging the equation to get one side equal to zero.

Rearranging $-2x^2 - 7x - 6 \leqq 5x + 4$

Graph



Now our question becomes, where is the parabola a BELOW ZERO (the x-axis)

The following homework questions you will do with the graphing calculator. You need to rearrange each inequality so that all you are really asking is where is the function Above or Below Zero. That way the x-intercepts are the boundaries between intervals. If you have difficulty remembering how to find the x-intercepts I can help you, but here are some quick reminders.

Press 2nd TRACE and choose 2 : Zero

Right now the calculator is asking you for a **left bound?** Ignore it.

Move the curser to the intercept you want to find. Do NOT press ENTER

Now pay attention to the **left bound?** question. Click the left cursor key twice. And press ENTER

The calculator is now asking for a **right bound?**. Click the right cursor key 4 times. This should take you past your intercept. Press ENTER

It is now asking you for a Guess? Move your cursor back as close as you can to the intercept and press ENTER. The intercept should be displayed.

Repeat this process for all your x-intercepts.

Now you just have to decide which intervals satisfy the given inequality.

Practice Questions - Page 129 #3, 5, 6, 9, 10, 12, 16

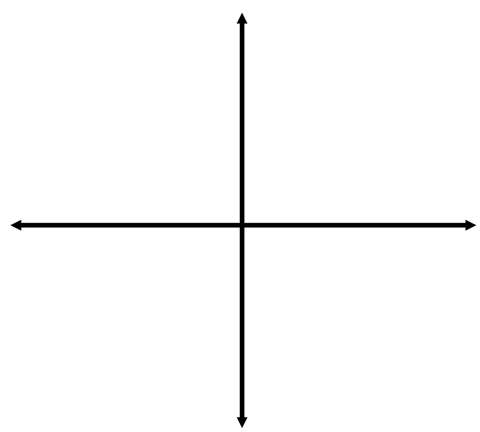
Topic : Solving polynomial inequalities

Goal : I can solve polynomial inequalities without technology, using curve sketching or interval tests.

Solving Polynomial Inequalities

Example 1. Solve $x^2 - x > 12$

Method 1 : Draw a rough sketch of the function, by first factoring and finding the zeros.



Method 2 : Make an interval table and determine when the polynomial is above and below zero from there.

MHF4U U2L8 Solving Polynomial Inequalities

Example 2. Solve $x^3 + x \leqq 4x^2 - 6$

interval				

Example 3. An object moves along the horizontal in a straight line according to the function

$$d = -t^2 + 16t$$

where d is the distance in metres and t is the time in seconds. When will the object be more than 50m away?

Practice Questions - Page 138 #3, 4, 6, 8, 9, 12, 13

Topic : **Polynomial Equations and Inequations**

Goal : **I can use technology to assist us in solving polynomial word problems**

Using Technology for Word Problems

Example 1. The volume, V, in cubic centimetres, of a block of ice that a sculptor uses to carve the wings of a dragon can be modelled by $V(x) = 9x^3 + 60x^2 + 249x$, where x represents the thickness of the block, in centimetres. What maximum thickness of wings can be carved from a block of ice with volume 2532 cm^3?

Example 2. An open-topped box can be made from a sheet of aluminium measuring 50 cm by 30 cm by cutting congruent squares from the four corners and folding up the sides. Write a polynomial function to represent the volume of the box, and then use it to determine the dimensions that result in a volume greater than 4000 cm^3.

Practice Questions

Page 11 #10 - 16, 19, 21 **Page 131** #10-12 **Page 139** #8, 9, 13