MHF4U Exam Review Outline

Chapter 1 – Functions: Characteristics and Properties
- Determine whether a relation is a function and state domain and range
- Absolute values: evaluate, graph on axis and number line
- Properties of graphs of functions: domain/range, intervals of increasing/decreasing, x/y intercepts, even/odd function, continuity/discontinuity
- Inverse relations
- Piecewise functions

Practice questions- 1 ace, 2 ace, 4 ace, 5, 6, 7ace

Chapter 2- Functions: Understanding Rates of Change
- Calculate average rate of change
- Instantaneous rates of change from table of values, equations and graphs
- Preceding interval, following interval and centred interval, and difference quotient
- Verifying maximum or minimum points using tangent lines

Practice questions- 1, 2, 4

Chapter 3 – Polynomial Functions
- Characteristics of polynomial functions, end behaviors, turning points, number of zeros and symmetry
- Transformations of cubic functions
- Dividing polynomials, long division, synthetic division
- Factoring polynomials, remainder theorem, and factor theorem.
- Factoring sum or difference of cubes

Practice questions- 1, 2ac, 4, 6ac, 7ac, 8ac, 9ac, 10ac, 11ac

Chapter 4 – Polynomial Equations and Inequalities
- Solving polynomial equations, finding zeros using a variety of factoring techniques
- Solving linear inequalities
- Solving polynomial inequalities
- Calculate rates of change of polynomials for a given interval

Practice questions – 1ace, 3ace, 4ace, 5ace, 7ace, 8ace

Chapter 5- Rational Functions, Equations, and Inequalities
- Graph reciprocal functions using key characteristics from the related function
- Properties of rational functions including horizontal, vertical, and oblique asymptotes and holes.
- Graphing rational functions
- Solving rational equations by finding lowest common denominators
- Solving rational inequalities

Practice questions- 1, 2, 3,4,5,7

Chapter 6 – Trigonometric Functions
- Converting radian measure to degrees and vice versa
- Use special triangles to determine related angles
- Transformations of trigonometric functions
- Determine periods, amplitude, horizontal translations and equation of the axis for trigonometric functions

Practice questions – 1, 2, 3

Chapter 7- Trigonometric Identities and Equations
- Use compound angle, double angle and addition/subtraction formulas to solve
- Prove trigonometric identities (know all identities)
- Solve linear trigonometric equations
- Solve quadratic trigonometric equations

Practice questions – 1ace, 2ace, 3, 4ace, 6, 7

Chapter 8 – Exponential and Logarithmic Functions
- Transformations of Logarithmic functions
- Evaluate simple logarithms using properties
- Evaluate logarithms using laws of logarithms
- Application problems with exponential and logarithmic functions

Practice questions- 2, 3, 4, 5, 6ac, 7ac, 9abc

Chapter 1 Review Extra Practice

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1. State the domain and range of the relation and determine whether the relation is a function.

$=\{0,1,2,3\}$
$R=\{-9,-8,-5,-4,0\}$

a)

x	0	1	1	2	3
y	0	−4	−5	−8	−9

b)

x	−2	−1	0	1	2
y	2	1	0	1	2

c)

x	2	4	6	8	10
y	−3	−6	0	−6	−3

d)

x	5	15	25	35	45
y	10	10	10	10	10

e)

x	12	12	12	12	12
y	−10	−5	0	−5	−10

f)

x	−0.2	−0.1	0	0.1	0.2
y	3	2	1	2	3

2. State the domain and range of each function.
   a) $y = |x - 1|$
   b) $y = -2|x + 3| - 4$
   c) $y = \left|\frac{x - 3}{6}\right|$
   d) $y = -\frac{1}{4}|3x| + 7$
   e) $y = 8 + \left|\frac{1}{2}x - 1\right|$
   f) $y = -\frac{1}{2} + |x| - 3$

3. Determine the parent functions that match each of the following characteristics.
   a) As $x \to \infty$, $y \to 0$
   b) Symmetric about y-axis
   c) $D = \{x \in \mathbf{R}\}$
      $R = \{y \in \mathbf{R} | y \geq 0\}$
   d) As $x \to 0$, $y \to 1$
   e) passes through $(0, 0)$
   f) $D = \{x \in \mathbf{R} | x \geq 0\}$
      $R = \{y \in \mathbf{R} | y \geq 0\}$

4. Determine the transformations of $f(x) = x^2 + 3$ described by the following.
   a) $f(2(x - 4)) + 2$
   b) $-3f(-(x + 1))$
   c) $f(5(x - 3)) + 5$
   d) $-f(x) - 12$
   e) $-4f(6(x)) - 11$
   f) $-f(-(x + 2)) - 1$

5. Determine the inverse of each function.
   a) $y = 2x - 7$
   b) $y = \frac{3}{x + 4}$
   c) $y = \sqrt{x^2 + 3}$
   d) $y = x^3 + 1$
   e) $y = \frac{1}{5}x^2 - 3$

6. Determine each value for the given function.

$$f(x) = \begin{cases} 3^x, \text{ if } x < 0 \\ 5, \text{ if } x = 0 \\ -2x^2 + 5, \text{ if } x > 0 \end{cases}$$

   a) $f(-2)$
   b) $f(0)$
   c) $f(-5)$
   d) $f(1)$
   e) $f(5)$
   f) $f(-3)$

7. Given $f = \{(0, -4), (1, 2), (3, 0)(5, 5)\}$, $g = \{(-1, 4), (0, \frac{1}{2}), (3, 12), (4, -1)\}$, and $h = \{(-1, \frac{1}{4}), (0, 3), (4, -1), (5, 6)\}$, determine the following.
   a) $f(x) + g(x)$
   b) $g(x) + h(x)$
   c) $f(x) - h(x)$
   d) $[f(x)][h(x)]$
   e) $[h(x)][g(x)]$
   f) $[f(x) + h(x)][f(x)]$

Chapter 2 Review Extra Practice

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1. A golfer hits a golf ball from the fairway. The function $h(t) = -5t^2 + 27t$ models the height of the golf ball, where $h(t)$ is the height in metres and t is the time it is in the air in seconds.
   a) Determine the average rate of change in the golf ball's height over the time interval $0 \le t \le 1$.
   b) Determine the average rate of change in the golf ball's height over the time interval $0 \le t \le 2$.
   c) Determine the average rate of change in the golf ball's height over the time interval $0 \le t \le 3$.
   d) Is the golf ball's average rate of change in height increasing, decreasing, or does it stay the same? Explain how you made your decision.

2. For the function $g(x) = -3x^2 + 4x - 6$, estimate the instantaneous rate of change for the given values of x.
   a) $x = -2$
   b) $x = 0$
   c) $x = 3$
   d) $x = 5$

3. Sketch the graph of each of the following functions. Draw a secant line on each sketch that you could use to estimate the slope of the tangent when $x = -2$. Use the results to estimate the slope of the line tangent to each of the functions when $x = -2$.
   a) $f(x) = 4x^2 + 8x - 1$
   b) $f(x) = 5^x - 1$
   c) $f(x) = -2x - 5$

4. Matthew leaves his house to go for a run. He ran at a constant speed of 10 km/h for 30 minutes. The path sloped upward, so he ran at a rate of 6 km/h for 15 minutes. At the top of the hill, he increased his speed to 8 km/h for 30 minutes. He stopped for 15 minutes to rest. Finally, he ran at a constant speed of 10 km/h for 30 minutes.
   a) Sketch a graph of Matthew's distance versus time while running.
   b) What is Matthew's average speed during the entire time that he ran?
   c) What is Matthew's average speed between 15 and 45 minutes?
   d) What is Matthew's instantaneous speed at 60 minutes?

5. Susan is a track-and-field athlete who is practising with the javelin. The table shows the height of the javelin and the time that it is in the air during one of her throws.

Time (s)	Height (m)
0	1.80
0.3	8.25
0.6	13.80
0.9	18.45
1.2	22.20
1.5	25.05
1.8	27.00
2.1	28.05
2.4	28.20
2.7	27.45
3.0	25.80
3.3	23.25

   a) Plot the points on a graph and draw the curve of best fit.
   b) Estimate at the speed at which the javelin is travelling at 2.3 s.

Chapter 3 Review Extra Practice

STUDENT BOOK PAGES 183–185

1. Draw the graph of a polynomial function that has all of the following characteristics:
   $f(1) = 8, f(-1) = 0, f(5) = 0$
   The y-intercept is 10.
   $f(x) > 0$ when $-1 < x < 2$
   $f(x) < 0$ when $x < -1$
   The domain is the set of real numbers.

2. Describe the end behaviour of each polynomial function using the degree and the leading coefficient.
   a) $f(x) = x^3 + 4x^2 + 5x - 2$
   b) $f(x) = -2x^3 + 3x + 1$
   c) $f(x) = 5x^4 - 2x^2 + 1$
   d) $f(x) = -x^4 + 3x^3 - 2x^2 + x + 7$

3. For each of the following, write the equations of three quartic functions that have the given zeros and belong to the same family of functions.
   a) 2, 1, −4, −1
   b) 5, 6, −2, 3
   c) −2, −3, 4, 1
   d) 8, −6, −4, −3

4. Sketch the graph of $f(x) = (x - 1)(x + 5)(x + 6)$ using the zeros and end behaviours.

5. Describe the transformations that were applied to $y = x^2$ to obtain each of the following functions.
   a) $y = 3(x + 2)^2 - 8$
   b) $y = -\left(\frac{4}{3}(x - 4)\right)^2 + 6$
   c) $y = 5(2(x - 7))^2 - 9$
   d) $y = -\frac{1}{4}(x + 5)^2 + 12$

6. Calculate each of the following using long division.
   a) $(3x^3 - 4x + 5) \div (x - 2)$
   b) $(x^4 - 5x^3 + 6x^2 - 4x + 8) \div (x^2 - 4)$
   c) $(5x^4 - 6x^3 - x^2 + 4x + 7) \div (x^3 + 3x^2 - 4x + 9)$
   d) $(x^5 - 7x^4 + 3x^3 - 2x^2 - 4x + 8) \div (x^4 + 5x^3 - 4x^2 - 8x + 1)$

7. Divide each polynomial by $x - 3$ using synthetic division.
   a) $3x^3 - 2x^2 + 4x - 6$
   b) $4x^3 + 5x^2 - 2x - 4$
   c) $6\ ^4 - 4x^3 - x^2 + 5x + 8$
   d) $5x^4 - 4x^2 + 3x - 9$

8. Factor each polynomial using the factor theorem.
   a) $x^3 + 2x^2 - 5x - 6$
   b) $2x^3 + 3x^2 - 59x - 30$
   c) $4x^4 - 23x^3 + 38x^2 - 13x - 6$
   d) $x^4 + 8x^3 + 11x^2 - 8x - 12$

9. Factor fully.
   a) $x^3 + 9x^2 + 15x - 25$
   b) $x^3 + 2x^2 - 16x - 32$
   c) $4x^4 + 11x^3 - 30x^2 - 99x - 54$
   d) $2x^4 + 7x^3 - 5x^2 - 28x - 12$

10. Factor each difference of cubes.
    a) $125x^3 - 64$
    b) $1000x^3 - 27$
    c) $729x^3 - 8$
    d) $27x^3 - 1$

11. Factor each sum of cubes.
    a) $2744x^3 + 729$
    b) $1331x^3 + 343$
    c) $1728x^3 + 216$
    d) $3375x^3 + 512$

Chapter 4 Review Extra Practice

STUDENT BOOK PAGES 238–241

1. Determine the zeros of the following functions algebraically.
   a) $x^3 = 7x + 6$
   b) $16 - 12x = 8x^2 - 3x^3 - x^4$
   c) $2x^3 - 8x^2 + 2x = -12$
   d) $2x^4 = -6x^3 + 6x^2 + 14x - 12$
   e) $-20 = -35x + 10x^2 + 5x^3$
   f) $7x^4 - 14x^2 - 56 = 21x^3 - 84x$

2. Determine the zeros of the following functions with graphing technology. Round to the nearest hundredth, if necessary.
   a) $x^3 + 6x^2 - 5x + 12 = 0$
   b) $x^4 - 5x^2 + 9x = 6$
   c) $3x^3 + x^2 = 12x$
   d) $2x^4 = x^3 + 1$
   e) $18 = -7x^3 + x^2$
   f) $-\frac{1}{2}x^4 + \frac{7}{2}x^3 = 9$

3. Solve the following inequalities algebraically. Express your solution in set notation.
   a) $4x - 7 > 3x - 5$
   b) $\frac{13}{2}x < -2 + 6x$
   c) $3 - 4x \geq -2 - 3x$
   d) $\frac{2x + 1}{2} \geq \frac{5 + 3x}{4}$
   e) $\frac{x + 6}{3} + 2x < \frac{7}{3} + 2x$
   f) $\frac{6x}{5} \leq \frac{10 + 7x}{5}$

4. Solve the following double inequalities algebraically and determine if $x = 2$ is a solution. Express your solution in interval notation.
   a) $2x + 1 < 3x + 2 < 9 + 2x$
   b) $-\frac{3}{2} < \frac{x}{4} \leq \frac{1}{2}$
   c) $-2 + 5x > 4x > -8 + 5x$
   d) $3 \leq 3x - 6 \leq 39$
   e) $x + 5 < 2x + 4 < x + 6$
   f) $0 \leq (3x + 6) + (x - 13) \leq 3$

5. Solve the following inequalities.
   a) $(x + 2)(x - 4) < 0$
   b) $(3x - 6)(x - 5) \geq 0$
   c) $(2x - 2)(x + 6)(x - 1) > 0$
   d) $-2x(x + 4)(x - 6)(x + 1) \leq 0$
   e) $(-3x + 9)(x + 1)(x - 4) > 0$
   f) $x^2(x - 4)(-5x + 10) \geq 0$

6. Use graphing technology and state the intervals when $f(x) < 0$. Express your solution in interval notation.
   a) $f(x) = x^3 + x^2 - 44x - 84$
   b) $f(x) = x^4 + 12x^3 + 52x^2 + 96x + 64$
   c) $f(x) = 8x^2 + 8x - 48$
   d) $f(x) = 8x^4 - 56x^2 + 48x$
   e) $f(x) = 12x^4 - 84x^2 + 72x$
   f) $f(x) = 14x^5 + 14x^4 - 56x^3 - 56x^2$

7. Determine the average rate of change of the following functions on the interval $-2 \leq x \leq 2$.
   a) $f(x) = -3x - 7$
   b) $g(x) = x^2 - x + 9$
   c) $h(x) = x^3 - x^2 + x - 1$
   d) $j(x) = -2x^3 + 5x^2 - 5$
   e) $k(x) = 5$
   f) $l(x) = -6x^2 + 7$

8. Estimate the instantaneous rate of change at $x = 1$ and let $h = 0.001$ for the following functions.
   a) $f(x) = -3x + 2$
   b) $g(x) = -x^2 - 8$
   c) $h(x) = 2^x + 1$
   d) $j(x) = \sqrt{x + 7} - 5$
   e) $k(x) = 2|x + 4| - 1$
   f) $l(x) = 3x^4 - x^2 + 8x - 1$

Chapter 5 Review Extra Practice

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1. For each function:
   i) Determine the domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals.
   ii) Determine the reciprocal of the original function.
   iii) Sketch both graphs.
   a) $f(x) = 2x - 1$
   b) $f(x) = 5 - 2x$

2. For each pair of functions determine where the zeros of the original function occur and state the equations of the vertical asymptotes of the reciprocal function. If no zeros exist, state so.
   a) $f(x) = 3x - 6,\ g(x) = \dfrac{1}{3x - 6}$
   b) $f(x) = 8x + 10,\ g(x) = \dfrac{1}{8x + 10}$
   c) $f(x) = 5x^2 - 15,\ g(x) = \dfrac{1}{5x^2 - 15}$
   d) $f(x) = x^2 + 3,\ g(x) = \dfrac{1}{x^2 + 3}$
   e) $f(x) = 3x^2 + 14x - 24,$
   $g(x) = \dfrac{1}{3x^2 + 14x - 24}$

3. For each function, state the equation of the vertical asymptote, if it exists.
   a) $f(x) = \dfrac{3}{3x + 8}$
   b) $f(x) = \dfrac{1 - 4x}{2}$
   c) $f(x) = \dfrac{7 + 5x}{2 - x}$
   d) $f(x) = \dfrac{9x + 3}{2x - 9}$

4. State the horizontal asymptote for each function in Question 3, if it exists.

5. Solve each equation algebraically. Check your answers.
   a) $0 = \dfrac{x - 3}{500}$
   b) $\dfrac{x + 3}{7} = 10$
   c) $\dfrac{5x + 2}{3} = 6 - x$
   d) $\dfrac{4}{2x + 3} = 1(-2 + 3x)$
   e) $\dfrac{4}{5x} = \dfrac{6}{2x - 1}$

6. Does the equation $\dfrac{x - 1}{x - 5} = \dfrac{x}{x - 4}$ have any solutions? If it does, list them. If it does not, explain why.

7. Use an algebraic process to find the solution set of each inequality. Verify your answers using graphing technology.
   a) $\dfrac{1}{x^2} > 0$
   b) $x - 2 < \dfrac{1}{x + 2}$
   c) $\dfrac{100}{x + 20} > -x$

8. Estimate the slope of the line tangent to the function at the given value of x. At what point(s) is it not possible to do so?
   a) $f(x) = \dfrac{x + 10}{x - 20},\ x = 30$
   b) $f(x) = \dfrac{1}{x + 6},\ x = -4$
   c) $f(x) = \dfrac{2x}{3x + 4},\ x = -2$

9. a) Does the instantaneous rate of change exist for the function $f(x) = \dfrac{x + 4}{x^2 + 2x - 24}$ at $x = -4$? Does it exist at $x = 4$? Explain.
   b) Does the instantaneous rate of change exist for the same function at $x = 4.000\,001$? Explain.

Chapter 6 Review Extra Practice

STUDENT BOOK PAGES 374–377

1. Convert each of the following radian measurements to degrees. Give your answers to two decimal places, if necessary.
   a) $\frac{11\pi}{12}$
   b) 74
   c) 314π
   d) $\frac{4\pi}{7}$
   e) $\frac{21\pi}{20}$
   f) $\frac{\pi}{22}$

2. For each of the following expressions, state an equivalent expression based on a related angle.
   a) $\cot \frac{7\pi}{6}$
   b) $\cos \left(-\frac{\pi}{6}\right)$
   c) $\tan \frac{\pi}{2}$
   d) $\sin \frac{\pi}{4}$
   e) $\sec \frac{5\pi}{3}$
   f) $\csc \frac{7\pi}{4}$

3. The function $y = \cos x$ is the parent function of each of the following trigonometric functions. State the transformations that have been applied to each.
   a) $y = -\frac{8}{21} \cos \left(\frac{3}{5}(x - 9)\right) + 14$
   b) $y = 77 \cos \left(-\left(x + \frac{1}{8}\right)\right) - 22$
   c) $y = 16 \cos \left(\frac{7}{15}(x - 5)\right) + 3$
   d) $y = \frac{2}{13} \cos (8(x + 7)) - 17$

4. A clock is hanging on a wall, with the centre of the clock 4.5 metres above the ground. Both the minute hand and the second hand are 13 cm long, while the hour hand is 6 cm long. Determine the equations of the sine function that describe the distance of the tip of each hand above the ground as a function of time. Assume that the time t is in hours and that the distance $D(t)$ is in cm. Also assume that at $t = 0$ it is 3 AM.

5. State two points where each of the following functions has an instantaneous rate of change that is a positive value.
   a) $y = -\frac{19}{20} \sin (24\pi x) + \frac{1}{40}$
   b) $y = 35 \cos \left(\frac{x}{12}\right) - 31$
   c) $y = \frac{1}{36} \sin (20x) + \frac{1}{18}$
   d) $y = 58 \cos \left(\frac{9\pi x}{10}\right) - 62$
   e) $y = -\cos \left(\frac{x}{100}\right) - 49$
   f) $y = \frac{3}{8} \sin (60\pi x) + \frac{1}{8}$

6. State the average rate of change of each of the following functions over the interval $\frac{\pi}{3} \leq x \leq \pi$ to two decimal places, if necessary.
   a) $y = \frac{5}{9} \cos (16x) - \frac{1}{9}$
   b) $y = 27 \sin \left(\frac{2x}{3}\right) + 28$
   c) $y = -4 \cos \left(\frac{3x}{4}\right) - 1$
   d) $y = \frac{17}{20} \sin (4x) + \frac{1}{20}$
   e) $y = 33 \cos \left(\frac{x}{6}\right) - 31$
   f) $y = 5 \sin (101x) + 4$

Chapter 7 Review Extra Practice

STUDENT BOOK PAGES 438–440

1. Write each of the following expressions as a single trigonometric function.
   a) $\sin 16° \cos 99° - \cos 16° \sin 99°$
   b) $\dfrac{\tan\frac{\pi}{18} + \tan\frac{2\pi}{9}}{1 - \tan\frac{\pi}{18}\tan\frac{2\pi}{9}}$
   c) $\sin\frac{13\pi}{20}\cos\frac{\pi}{5} + \cos\frac{13\pi}{20}\sin\frac{\pi}{5}$
   d) $\cos 88° \cos 9° - \sin 88° \sin 9°$
   e) $\dfrac{\tan 61° - \tan 49°}{1 + \tan 61° \tan 49°}$
   f) $\cos\frac{4\pi}{11}\cos\frac{4\pi}{5} + \sin\frac{4\pi}{11}\sin\frac{4\pi}{5}$

2. Write each of the following expressions as a single trigonometric function.
   a) $\dfrac{2\tan 114°}{1 - \tan^2 114°}$
   b) $1 - 2\sin^2\frac{3\pi}{22}$
   c) $2\cos^2\frac{19\pi}{25} - 1$
   d) $\cos^2 77° - \sin^2 77°$
   e) $2 \sin 159° \cos 159°$
   f) $\dfrac{2\tan\frac{3\pi}{41}}{1 - \tan^2\frac{3\pi}{41}}$

3. Give a counterexample to show that each of the following statements is not an identity.
   a) $\cos 2x = \dfrac{2\cos x}{1 - \cos^2 x}$
   b) $\sin^3 x + \cos^3 x = 1$
   c) $\cot x = \dfrac{\sec x}{\csc x}$
   d) $\cot(x - y) = \cot x - \cot y$

4. Determine the solutions for each equation where $0 \le x \le 2\pi$.
   a) $1 + \frac{17}{3}\tan x = -\frac{14}{3}$
   b) $\sin x + 2 + \frac{\csc x}{10} = \frac{9}{5} + \sin x$
   c) $5 + \pi + 2\cos x = \pi + 4$
   d) $\sqrt{3} + 4\cot x - 1 = 2\sqrt{3} - 1 + \cot x$
   e) $\dfrac{\sin x + \cos x}{15} = 0$
   f) $\dfrac{1 + 9\sec x + \pi}{\sec x} = \dfrac{17}{2} + \dfrac{\pi}{\sec x}$

5. With the help of a graphing calculator, sketch a graph that could be used to help solve each of the following equations. Also state the solution(s) to the equations, where $0 \le x \le 2\pi$.
   a) $\sin x = \frac{1}{2}$
   b) $\dfrac{\cot x}{8} = -\dfrac{\sqrt{3}}{24}$
   c) $20\csc x + 40 = 0$

6. Use factoring to solve each of the following equations where $0 \le x \le 2\pi$. Round your answers to two decimal places, if necessary.
   a) $36\sin^2 x + 5\sin x - 1 = 0$
   b) $\sec^2 x + \frac{17}{7}\sec x + \frac{10}{7} = 0$

7. Use the quadratic formula to solve each of the following equations where $0 \le x \le 2\pi$. Round your answers to two decimal places, if necessary.
   a) $0.3\csc^2 x + 0.1\csc x - 0.9 = 0$
   b) $0.9\tan^2 x + 0.6\tan x - 0.1 = 0$

Chapter 8 Review Extra Practice

STUDENT BOOK PAGES 509–511

1. Write the equation of the inverse of each exponential function in exponential form and logarithm form.
   a) $y = 4^x$
   b) $y = \left(\frac{2}{5}\right)^x$
   c) $y = (1.2)^x$
   d) $y = 0.2^x$

2. Let $f(x) = \log_{10}(x)$. For each of the following functions, state the transformations that must be applied to $f(x)$.
   a) $g(x) = 4\log_{10}\left(\frac{1}{2}x\right) + 3$
   b) $g(x) = \frac{2}{5}\log_{10}(-x)$
   c) $g(x) = \log_{10}(3(x+2)) - \frac{7}{8}$
   d) $g(x) = 3\log_{10}(6 - x)$

3. Solve for x.
   a) $\log_2 \frac{1}{256} = x$
   b) $\log_x 32 = -5$
   c) $\log_5 \sqrt[3]{5} = x$
   d) $\log_4 512 = x$
   e) $\log_x 36 = -2$
   f) $\log_x \sqrt{\frac{4}{9}} = \frac{1}{2}$

4. Express the following logarithms in terms of $\log_3 64$, $\log_3 100$, and $\log_3 25$.
   a) $\log_3(16)$
   b) $\log_3\left(\frac{25}{16}\right)^3$
   c) $\log_3\left(\frac{2500}{16^2}\right)^{\frac{1}{3}}$
   d) $\log_3 \frac{\sqrt{64}\sqrt{2500}}{625}$

5. Solve for x.
   a) $\left(\frac{2}{3}\right)^{x-1} + \left(\frac{2}{3}\right)^x = \frac{20}{27}$
   b) $2^{x+2} - 2^x = 96$
   c) $4^{x-1} + 4^{x-3} = 272$
   d) $\left(\frac{1}{2}\right)^{x-2} - \left(\frac{1}{2}\right)^x = \frac{3}{32}$

6. Solve each equation for x.
   a) $\log_2(4x - 3) = \log_2\left(\frac{1}{2}x + 2\right)$
   b) $\log_3(3x) = \log_3(8x - 10)$
   c) $\log_4\left(-\frac{4}{5}x\right) = \log_4\left(-x - \frac{9}{5}\right)$
   d) $\log_{10}(11 - x) = \log_{10}(1 - 2x)$

7. Solve each equation for x.
   a) $\log x + \log x^2 = 3$
   b) $\log_4 x^2 - \log_4 x = 2$
   c) $\log_{12} x^3 - \log_{12} x^5 = -2$
   d) $\log_3 x^4 + \log_3 x = 5$

8. Solve the following equations for n using logarithms. Round your answers to 3 decimal places.
   a) $0.32 = (0.78)^n$
   b) $1000 = (1.9)^n$
   c) $300 = \left(\frac{10}{9}\right)^n$
   d) $0.20 = (0.0945)^n$
   e) $20\,000 = (1.42)^n$
   f) $0.12 = (0.976)^n$

9. A town's population grows 2.3% each year. In 2005, its population was 1040.
   a) Give an equation that models the town's population for a given year n.
   b) About how many years will it take for the population to reach 20 000 people?
   c) In about what year will the population reach 30 000 people?
   d) Estimate the rate of change of the population in 2015.
   e) Estimate the rate of change of the population in 2025.
   f) What is the average rate of change per year of the population from 2005 to 2010?
   g) What is the average rate of change per year of the population from 2025 to 2030?

1. Sketch each polynomial function:

a) $y=\frac{1}{2}(x-2)^3-3$ b) $y=-2x^2(x+3)$ c) $y=(x+1)(x-2)^3$ d) $y=x^4-13x^2+36$

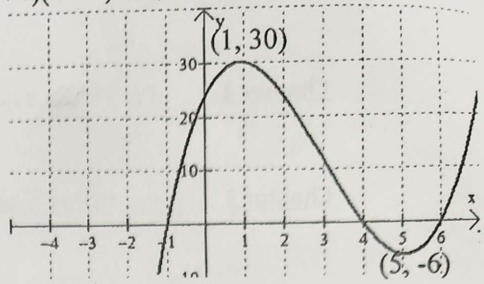
2. Given $y=f(x)$

a) State the degree of the function.

b) State the coordinates of the zeros, local minimums and local maximums.

c) State the intervals of increasing and decreasing.

d) Find the equation of the function



3. a) Find the quotient and the remainder when $3x^3-4x+3$ is divided by $x-2$.

b) Find the value of k in x^3-2x^2+kx-3 if $(x+3)$ is a factor.

4. Factor a) $x^3-4x^2-4x+16$ b) $8x^3+27$ c) $5x^3+3x^2-12x+4$ d) x^4-11x^2+18

5. Solve each: a) $2x^3+x^2-18x-9=0$ b) $x(x^2+9x+3)=-5(2x+1)$

c) $-2x(x+3)(x-2)(x-5)<0$ d) $-x^3-5x^2+6x\geq 0$

6. Find the cubic polynomial function with two of its zeros 2 and $-3+\sqrt{2}$ and a y-intercept of 7.

7. a) Find the value of k if $2x^3-4x^2-3x+k$ is divisible by $2x-3$.

b) Given ax^3+x^2+x+b, find the value of a and b if the remainder when divided by $(x-1)$ and $(x+1)$ are 6 and 2 respectively.

8. Given $y=x^3-2x+3$

a) Find the average rate of change of y with respect to x in the interval [-2, 3]

b) Estimate the slope of the tangent to the curve at $x=3$

c) Find the equation of the tangent at at $x=2$.

9. State the asymptotes to each curve:

a) $y=\dfrac{3x}{x^2-4}$ b) $y=\dfrac{2x^3+x^2+5}{x^2}$ c) $y=\dfrac{2x^2-18}{x^2+x-12}$ d) $y=\dfrac{x^2+5x-1}{x+2}$ e) $y=2^{x+1}-3$

10. Analysis each rational function the sketch the graph. Consider the domain, x and y-intercepts, asymptotes and symmetry.

a) $y=\dfrac{2}{x+3}-2$ b) $y=\dfrac{2x+6}{x-3}$ c) $y=\dfrac{x}{x^2-4}$

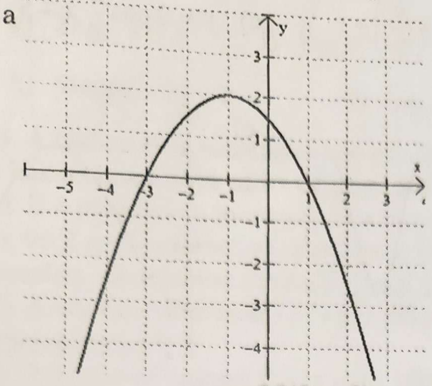
11. Solve the following rational equations and inequalities.

a) $\dfrac{2x^2+5x-12}{x+1}=0$ b) $\dfrac{2x}{6x+5}=\dfrac{x+3}{3x-1}$ c) $\dfrac{3x+2}{2x+1}=\dfrac{3x+1}{x-1}-\dfrac{1}{3}$

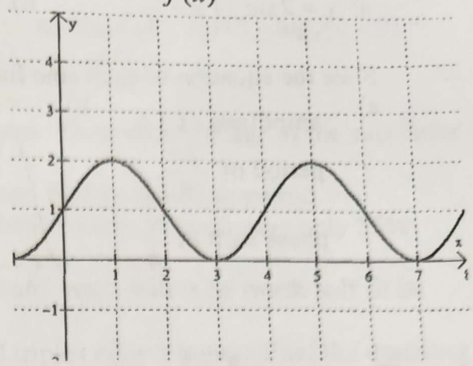
d) $\dfrac{-x+5}{2x+3}\geq 2$ e) $\dfrac{1}{x-1}<\dfrac{-1}{x+2}$ f) $\dfrac{x}{x-2}\leq\dfrac{x-1}{x+1}$

12. Given $y = f(x)$ sketch the graph of the reciprocal function $y = \dfrac{1}{f(x)}$

a



b



13. The function $P(t) = \dfrac{25(5t+8)}{2t+1}$ models the population, in thousands, of a town t years since 1990. Find: a) the initial population.

b) the average rate of change of the population from 1995 to 2005.

c) the rate at which the population is changing in the year 2000.

14. a) Convert to radians i) 75° ii) 330° b) Convert to degrees: i) $\frac{3\pi}{5}$ ii) 4.5^r

15. Find the exact value of: a) $\sin\dfrac{4\pi}{3}$ b) $\cot\dfrac{5\pi}{6}$ c) $\sec\dfrac{7\pi}{4}$

16. Solve for all θ, $0 \le x \le 2\pi$. Find exact values where possible.

a) $\cot\theta = \sqrt{3}$ b) $\sqrt{2}\cos\theta + 1 = 0$ c) $\sin\theta = 0.325$ d) $\sin(2\theta) = \dfrac{\sqrt{3}}{2}$

e) $3\sin^2\theta - 5\sin\theta - 2 = 0$ f) $2\cos^2\theta - \cos\theta = 0$ g) $2\sin^2\theta = \cos\theta$

h) $\sin(2\theta) - \cos\theta = 0$ i) $2\cos\frac{\pi}{3}(\theta - 1) + 5 = 4$

17. Simplify $\cos\left(\dfrac{3\pi}{4} + x\right) + \sin\left(\dfrac{3\pi}{4} - x\right)$

18. Find the exact value of: a) $\sin\dfrac{\pi}{12}$ b) $\cos\dfrac{13\pi}{12}$.

19. If $\cos\theta = \dfrac{-2}{\sqrt{7}}$, $0 \le \theta \le \pi$ find the exact value of $\cos 2\theta$ and $\sin 2\theta$.

20. Prove: a) $\tan\theta + 1 = \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta - \sin\theta\cos\theta}$ b) $\sec^2\theta + \sin^2\theta - 2 = \tan^2\theta - \cos^2\theta$

c) $(\sec x - \cos x)(\csc x - \sin x) = \dfrac{\tan x}{1 + \tan^2 x}$ d) $\sin 2A = \dfrac{2\tan A}{1 + \tan^2 A}$

e) $\cot\theta + \tan\theta = 2\csc 2\theta$ f) $\dfrac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$

g) $\sin(x+y)\sin(x-y) = \cos^2 y - \cos^2 x$

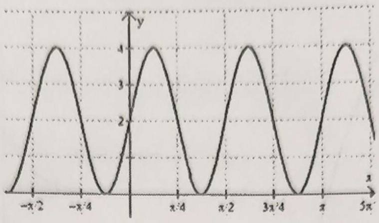
21. For each, state the amplitude, period, phase shift and sketch the graph.

a) $y = 2\sin\frac{1}{3}\left(x - \frac{\pi}{3}\right)$ b) $y = -\cos\left(2x + \frac{\pi}{2}\right) + 1$ c) $y = 2\cos\frac{\pi}{6}x + 5$

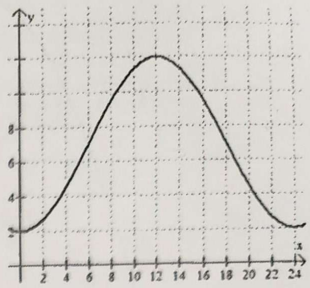
22. State the equation of each sine function.

a) amplitude of 3.5, period of $\frac{2\pi}{5}$, phase shift of $\frac{\pi}{4}$

b)



c)



23. The points $\left(\frac{\pi}{2}, 0\right)$ and $(\pi, -1)$ are on the curve $y = \cos\theta$. State the new coordinates of these points under the transformations given by $y = 3\cos\frac{2}{5}\left(\theta - \frac{\pi}{4}\right) - 1$.

24. The average monthly temperature, T, in degrees Celsius, for any month, t, for the town of Someplace, is modelled by the function $T(t) = 19.1\sin\left(\frac{\pi t}{6} + 4.2\right) + 7.5$. For $t = 0$, the month is January.

a) What is the maximum average monthly temperature?
b) What is the period of the function and what does it mean?
c) Determine the month with minimum average monthly temperature.
d) When is the average monthly temperature about $24°C$.

25. A pedal on a bicycle has an arm length of 20 cm and rotates about an axle 32 cm above the ground. If the pedal states at its lowest point and rotates at 20 revolutions every minute, find a sinusoidal function that will model the height h, in centimetres, of the pedal after t seconds. At what time during the first 5 seconds will the pedal be 40 cm above the ground?

$y = -20\cos(\frac{2\pi}{3}x) + 32$, $t = 0.946s, 2.043s, 3.924s$

26. Evaluate without a calculator:

a) $\log_3 \frac{1}{27}$ b) $\log_5 \sqrt[3]{625}$ c) $\log_8 32$ d) $\log 0.001$ e) $\log 25 - \log\frac{5}{2}$

27. Simplify: a) $\log_a \sqrt{a^5}$ b) $\log_a \frac{1}{\sqrt[5]{a^2}}$ c) $4^{3\log_4 x}$ d) $3\log 2x - 5\log x + 2\log 2$

e) $(\log 10^x)(\log a + 2\log b)$

28. For each function:

i) State the domain and range ii) sketch the graph iii) state the equation of its inverse

a) $y = -3^{x-2} - 1$ b) $y = -2\log_3(x+1)$ c) $y = 1 + \log_2(x-4)$

29. Solve for x: a) $7^x = 21$ b) $4(3^{2x}) - 5 = 29$ c) $3^{x-1} = 5^{1-2x}$ d) $5\log_3 x - 8 = 22$

e) $2\log_x 3 = 3$ f) $\log(x^2+4) = 1 + 2\log x$ g) $\log_2(3x+2) = 5 - \log_2(x-9)$

h) $5\log(x-3) = 2$ i) $\log_2(\log x) = 3$

30. a) A radioactive substance has a half-life of 250 years. Determine the age of the substance if it has retained 74% of its original amount.

b) If a population increases at a rate of 1.5% per year, find its doubling period.

c) 96.2 g of radioactive material is placed in a nuclear reactor. After 8 days only 35% remains. Determine the half-life of the substance.

d) A car depreciates at 15% per year. After how many years will it be worth half of its original value?

e) A bacterial culture starts with 1000 bacteria and triples after 5 hours. Find the doubling period and the amount of bacteria after 8 hours.

31. The half-life of a radioactive substance is 56 years. If there is 250 mg of the substance now:

a) Find the average rate of decay per year over the first 30 years

b) Estimate the rate of decay of the substance after 100 years.

32. Given $f(x) = 2x^2 - x$, $g(x) = \dfrac{x}{2-x}$, $h(x) = x + \cos x$ and $k(x) = \log(x-1)$,

i) find each function and state its domain:

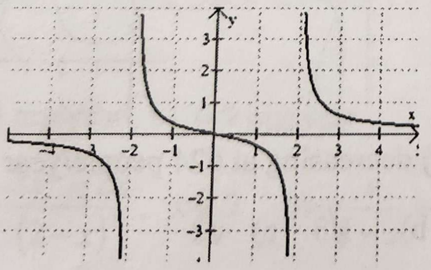
a) $f(g(x))$ b) $h \circ g(x)$ c) $g \circ k(x)$ d) $(k \circ f)(x)$ e) $(fg)(x)$ f) $\left(\frac{g}{f}\right)(x)$

ii) find x if $g(f(x)) = 1$

33. Determine if the given functions are even, odd or neither. Support your answer.

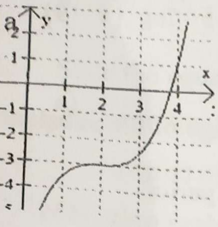
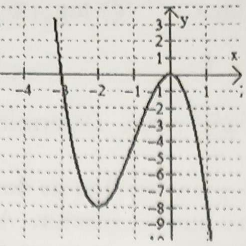
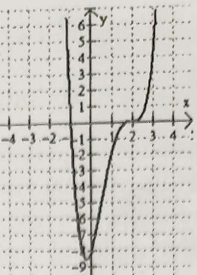
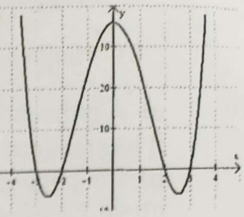
a) $y = x^3 + 4x$ b) $y = 3x^4 + 5x^2 - 1$ c) $y = \dfrac{x}{x-4}$ d) $y = 3\sin x$ e) $y = \cos 2x$

f)



g) $y = 2^{x+4}$ h) $y = \dfrac{\sin x}{x}$ i) $y = x^2 + \sin x$

Answers:

1a 
b 
c 
d 

2. a) 3^{rd} degree b) zeros (-1,0), (4,0) and (6,0) local max (1, 30), local minimum (5, -6) c) increasing $x<1$ or $x>5$, decreaing $1<x<5$ d) $y=k(x+1)(x-4)(x-6), k=1$

3.a) $3x^2+6x+8,\ R:19$ b) $k=-16$

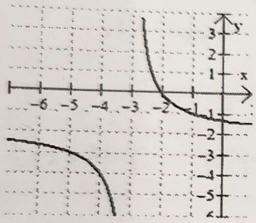
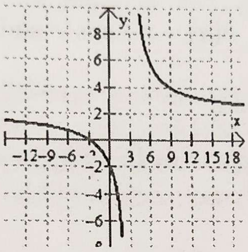
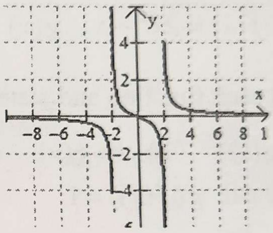
4. a) $(x-2)(x+2)(x-4)$ b) $(2x+3)(4x^2-6x+9)$ c) $(x+2)(x-1)(5x-2)$ d) $(x-3)(x+3)(x^2-2)$

5. a) $\pm 3, -\frac{1}{2}$ b) $-1,\ -4\pm\sqrt{11}$ c) $x<-3,\ 0<x<2,\ x>5$ d) $x\le -6,\ 0\le x\le 1$

6. $y=-0.5(x-2)(x^2+6x+7)$ **7.** a) $\frac{27}{4}$ b) $a=1,\ b=3$

8. a) $\frac{\Delta y}{\Delta x}=5$ b) $\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=25$ c) $y=10x-13$

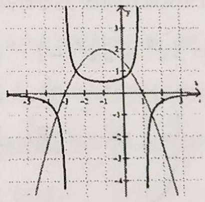
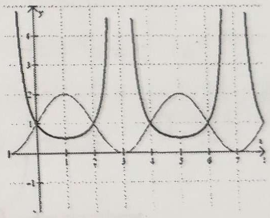
9. a) $x=\pm 2, y=0$ b) $x=0, y=2x+1$ c) $x=-4, y=2$ d) $x=-2, y=x+3$ e) $y=-3$

10.a) 
b) 
c) 

11. a) -4, 3/2 b) -0.6 c) -5/7, -2

d) $-\frac{3}{2}\le x\le -\frac{1}{5}$ e) $x<-2$ or $-\frac{1}{2}<x<1$

f) $x<-1$ or $\frac{1}{2}\le x<2$

12. a 
b 

13. a) 200 000 people b) decreasing 806 people/year c) decreasing at 624 people/year

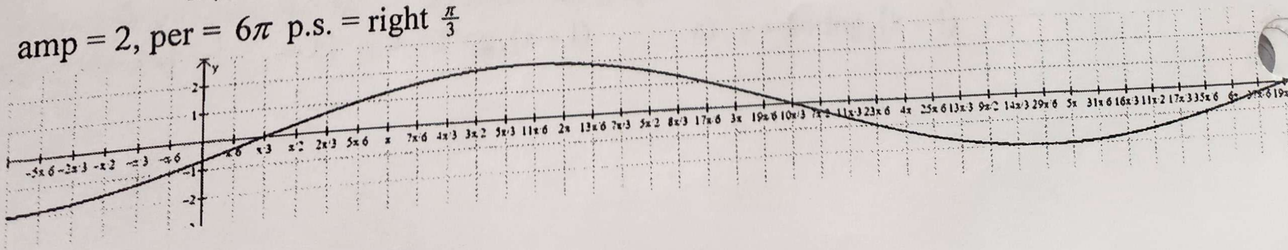
14. a) i) $\frac{5\pi}{12}$ ii) $\frac{11\pi}{6}$ b) i) 108° ii) 258° **15.** a) $-\frac{\sqrt{3}}{2}$ b) $-\sqrt{3}$ c) $\sqrt{2}$

16. a) $\frac{\pi}{6}, \frac{7\pi}{6}$ b) $\frac{3\pi}{4}, \frac{5\pi}{4}$ c) 0.331, 2.811 d) $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$ e) 3.48, 5.94 f) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$

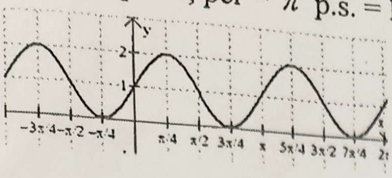
g) 0.675, 5.60 h) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ i) 3, 5

17. 0 **18.** a) $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$ b) $\dfrac{-1-\sqrt{3}}{2\sqrt{2}}$ **19.** $\frac{1}{7}$, $\frac{-4\sqrt{3}}{7}$

21 a) amp = 2, per = 6π p.s. = right $\frac{\pi}{3}$

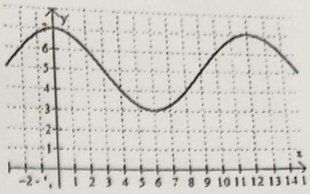


21 b) amp = 1, per = π p.s. = left $\frac{\pi}{4}$



c)

amp = 2
Per = 12
p.s. none



22. a) $y = 3.5\sin 5(x - \frac{\pi}{4})$ b) $y = 2\sin(4x) + 2$ c) $y = 5\sin\frac{\pi}{12}(x-6)+7$

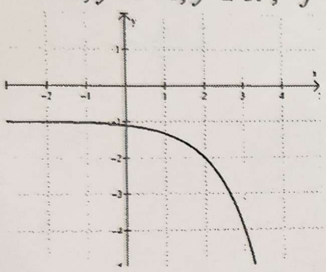
23. $(\frac{3\pi}{2}, -1)$, $(\frac{11\pi}{4} - 4)$ **24.** a) 26.6° b) 12, annual cycle c) $t = 0.9786$ ∴ Feb d) June, Aug

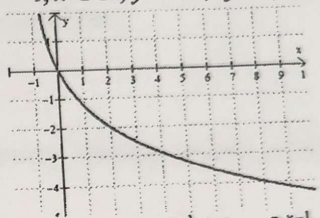
25. $y = -20\cos(\frac{2\pi}{3}t) + 32$, $t = 0.946s, 2.053s, 3.924s$

26. a) -3 b) $\frac{4}{3}$ c) $\frac{5}{3}$ d) -3 e) 1

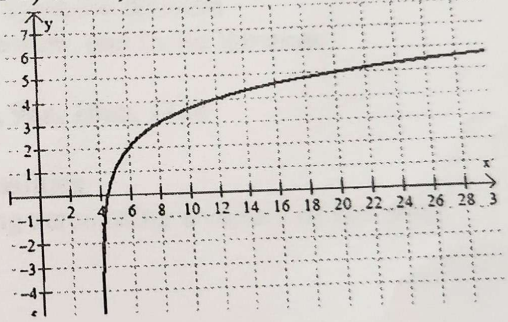
27. a) $\frac{5}{2}$ b) $-\frac{2}{5}$ c) x^3 d) $\log\frac{32}{x^2}$ e) $\log a^x b^{2x}$

28 a) $x \in R, y < -1, y \in R$, $f^{-1}(x) = \log_3(-x-1)+2$ b) $x > -1, x \in R, y \in R$, $f^{-1}(x) = 3^{-\frac{x}{2}} - 1$





28 c) $x > 4, x \in R, y \in R$, $f^{-1}(x) = 2^{x-1} + 4$



29. a) 1.565 b) 0.974 c) 0.627 d) 729
e) 2.08 f) $\frac{2}{3}$ g) 10 h) 5.512 i) 10^8

30 a) 108.6 yrs b) 46.6 yrs c) 5.28 d
d) 4.27 yrs e) 3.15 hrs, 5800 bacteria

31. a) 2.58 mg/year b) 0.9 mg/year

32. i) a) $\frac{3x^2-2x}{(2-x)^2}$, $x \neq 2$ b) $\frac{x}{2-x} + \cos\left(\frac{x}{2-x}\right)$, $x \neq 2$ c) $\frac{\log(x-1)}{2-\log(x-1)}$, $x > 1$

d) $\log(2x^2 - x - 1)$ $x < -0.5$ or $x > 1$ e) $\frac{x^2(2x-1)}{2-x}, x \neq 2$ f) $\frac{1}{(2x-1)(2-x)}, x \neq 0, \frac{1}{2}, 2$

ii) $-\frac{1}{2}, 1$

33. a) odd b) even c) neither d) odd e) even f) odd g) neither h) even i) neither