MHF4U

Advanced Functions
Grade 12 University

Unit 5 Trig Functions & Equations

5 Video Lessons

Allow **no more** than **12** class days for this unit!
This includes time for review and to write the test.

Lesson #	Title	Practice Questions	Date Completed
1	Graphs of Sine, Cosine and Tangent	Page 258 #1-11(pick & choose), 13, 14, 17, 19, 20	
1.5	Changing the Period of a Sinusoidal Function		
2	Reciprocal Trig Functions	Page 268 #10, 13, 15	
3	Sinusoidal Functions	Page 275 #1, 2, 5b, 6b, 8, 10a, 11a, 18, 20	
4	Solving Trigonometric Equations - Part 1 additional example	Page 287 #(1, 3, 5, 7) pick & choose 9-11, 16, 18, 19, 23	
5	Making Connections to the "Real World"	Page 296 #1, 2, 4, 5, 10, 11, 15	

Test Written on : ________________________________

MHF4U U5L1 Graphs of Sine, Cosine and Tangent

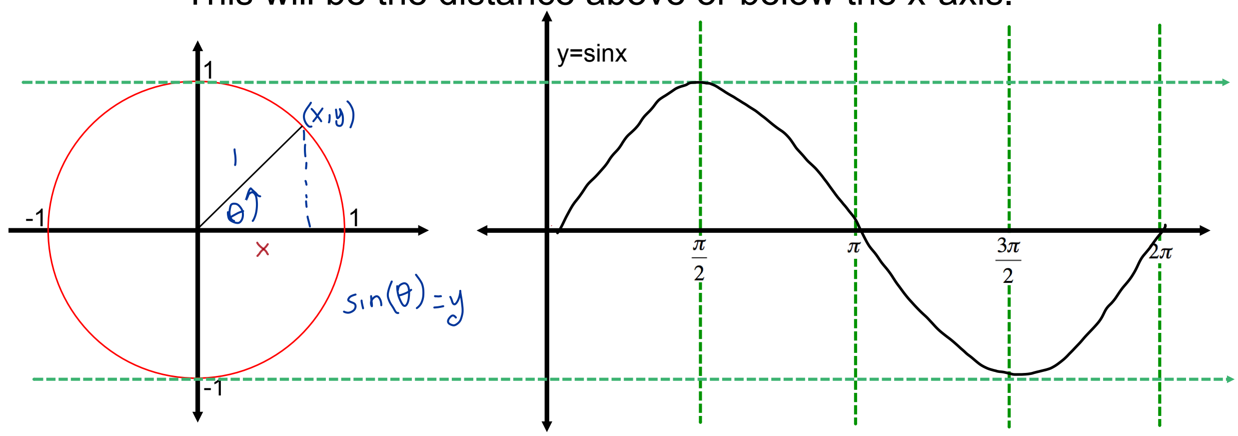
Topic : Graphing the primary trig functions

Goal : I can transfer my knowledge of graphing the primary trig functions from working in degrees to working in radians.

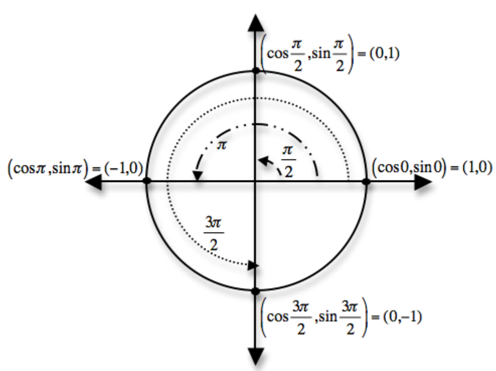
Graphs of Sine, Cosine and Tangent

http://www.absorblearning.com/media/item.action;jsessionid=61E51F2E41731631956F10F1D1F4797D?quick=je

Remember that sinx is defined as the y-coordinate of the unit circle. This will be the distance above or below the x-axis.



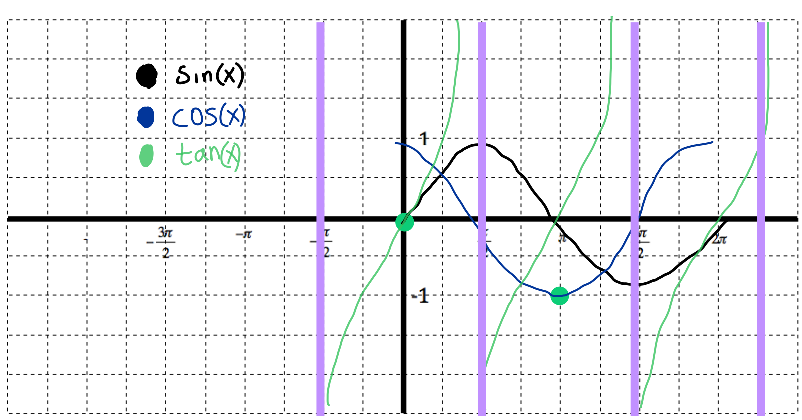
So having now looked at exactly where the sine function comes from in relation to the unit circle, let's think about the table of values for all three primary trig ratios, as they relate to the unit circle.



x	y=sinx	y=cosx
0	0	1
$\frac{\pi}{2}$	1	0
π	0	−1
$\frac{3\pi}{2}$	−1	0
2π	0	1

x	y=tanx
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	UND
$\frac{3\pi}{4}$	−1
π	0
$\frac{5\pi}{4}$	1
$\frac{3\pi}{2}$	UND

$\frac{\sin(x)}{\cos(x)}$



Note that every $\frac{\pi}{2}$ there is a "special" point

MHF4U U5L1 Graphs of Sine, Cosine and Tangent

Some Definitions :

the length along the x-axis before the function repeats Period :

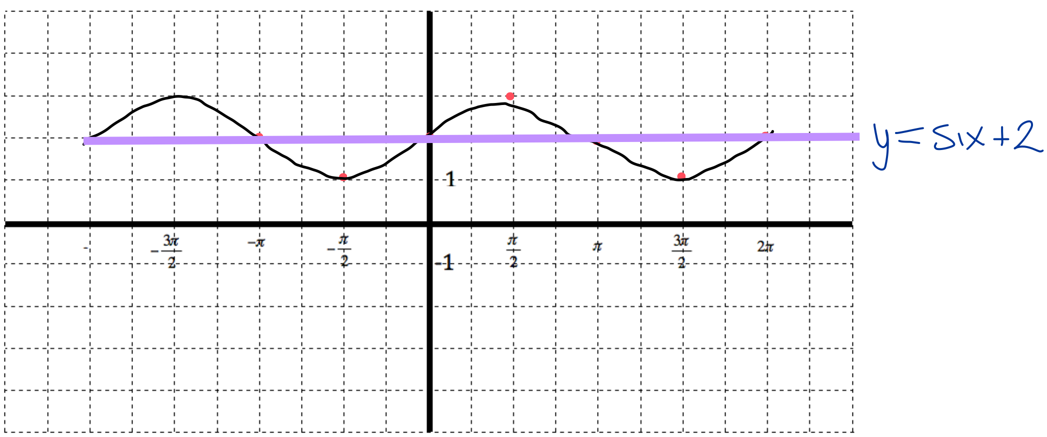
half the distance between the function's maximum and minimum points Amplitude :

the distance from midline to an extreme value

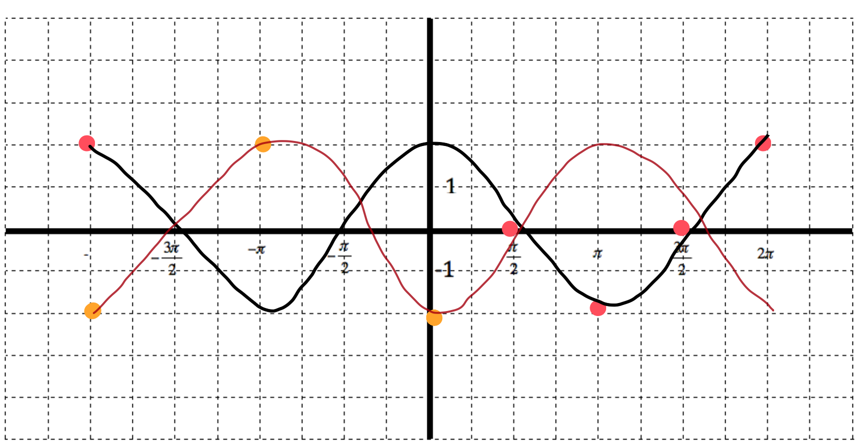
For untransformed sinusoidal functions we have the following properties...

	Sine	Cosine	Tangent
period	2π	2π	π
amplitude	1	1	UND
asymptote	UND	UND	$\frac{\pi}{2}+n(\pi)$

Example 1. Graph $y = \sin x + 2$

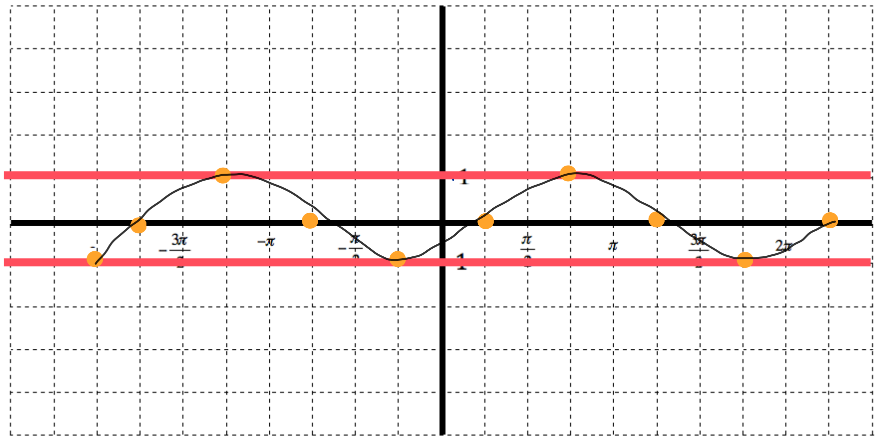


Example 2. Graph $y = 2\cos x$ and $y = -2\cos x$



Example 3. Graph $y = \sin(x - \frac{\pi}{4})$

phase shift (horizontal translation) moves the function left or right remember like all other function the shift is the opposite direction than your intuition would tell you.



Practice Questions
Page 258 #1-11 (pick & choose), 13, 14, 17, 19, 20

Topic : Changing the Period Length

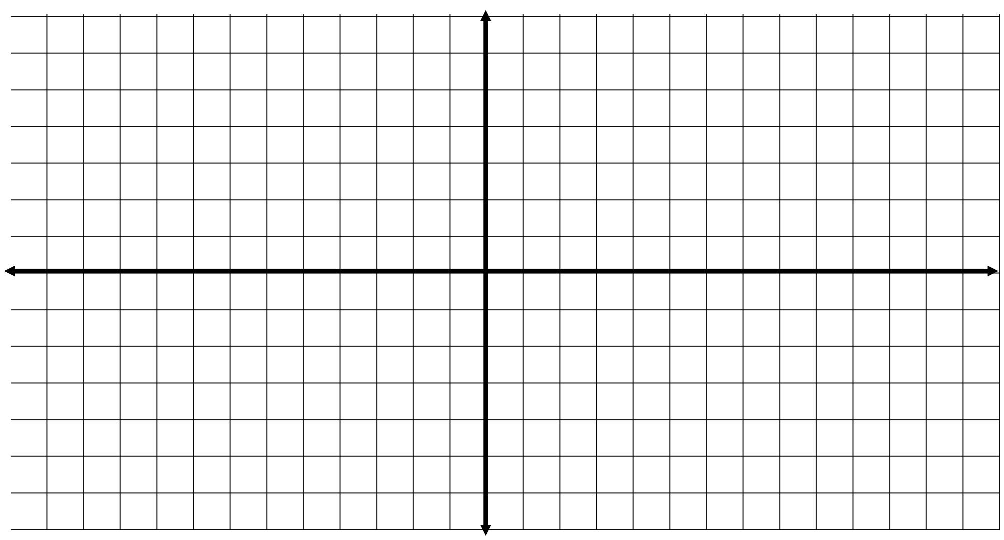
Goal : I can graph a sinusoidal function that has had its period length changed.

<u>Example 4</u> Graph the function y=sin3x

The 3 corresponds to a horizontal **compression** - in other words it divides by three. We can calculate the period length as follows.

Period = $\frac{2\pi}{k}$

NOTE



Example 5. What is the value of k in y=coskx, if the period is $\frac{3\pi}{4}$

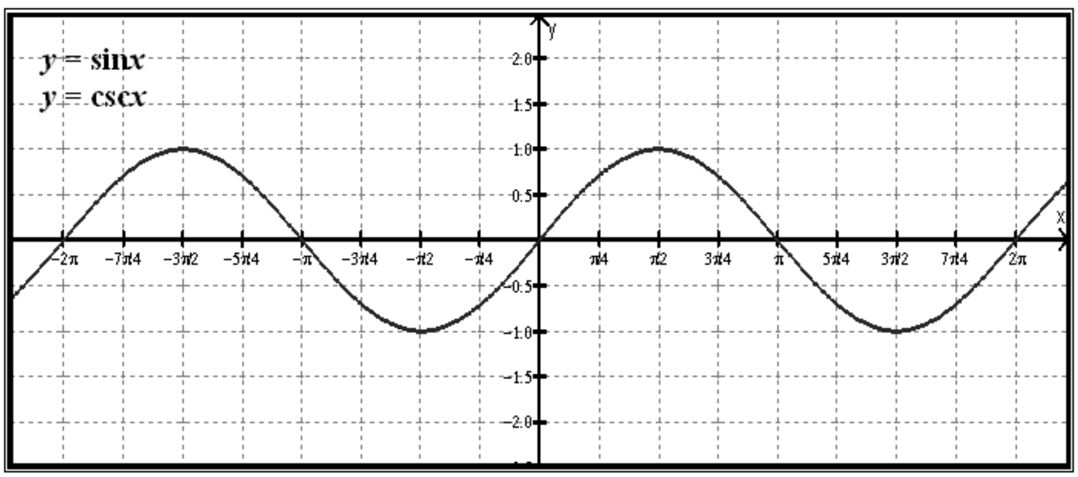
Topic : Graphing the Secondary Trig Ratios

Goal : I can use the graph of the primary trig ratios to graph the secondary trig ratios.

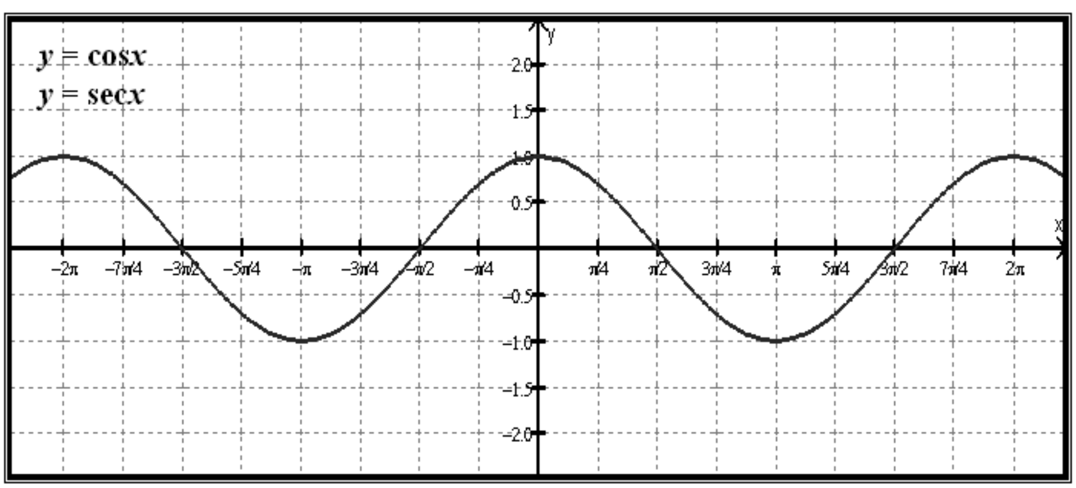
Reciprocal Trig Functions

We are not going to dwell much on the reciprocal trig functions. We will complete one exercise here on how the reciprocal functions can be graphed from their primary counter parts.

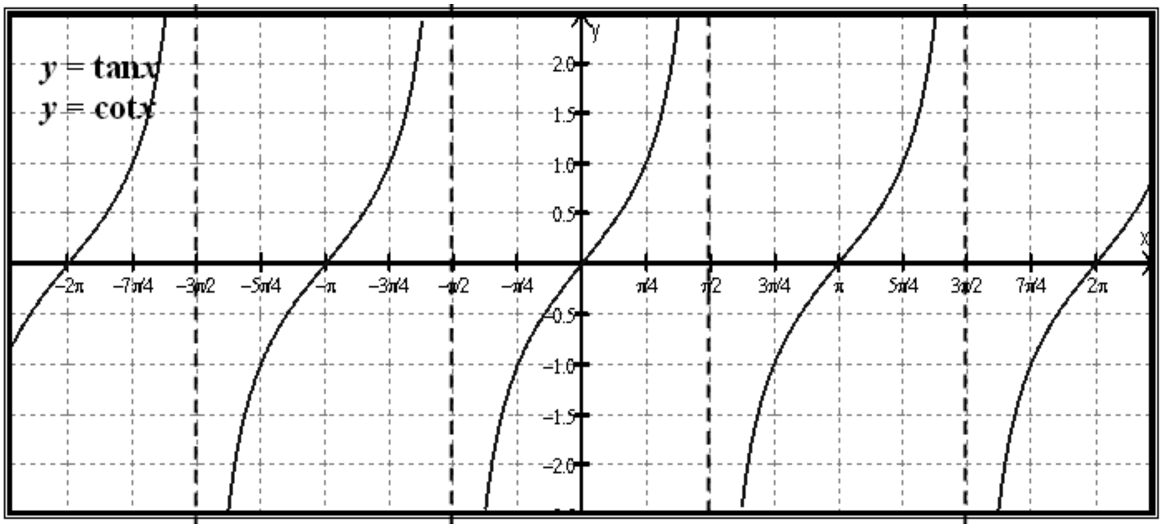
$$y = \csc x \text{ is equivalent to } y = \frac{1}{\sin x}$$



Complete the following graphs for y=secx and y=cotx



MHF 4U U5L2 Reciprocal Trig Functions



Practice Questions - Page 268 10, 13, 15

Topic : Graphing Sinusoidal Functions

Goal : I can use the general form of a sinusoidal function to be able to graph and create equations for specified functions.

Sinusoidal Functions

We've dealt with each transformation individually, now we put them all together to get the general transformation.

Phase Shift

Horizontal translation left d<0
Horizontal translation right d>0
(but remember the sign of d is the opposite as it appears)

$$f(x) = a \sin [k (x - d)] + c$$

Vertical Stretch if |a|>1
Vertical Compression if |a|<1

Reflection in x-axis a<0

Horizontal Stretch if |k|<1
Horizontal Compression if |k|>1

Reflection in y-axis k<0

Vertical translation down c<0
Vertical translation up c>0

Suggested Steps to Follow when Graphing

Step 1

Determine the period of the function using the formula...

Step 2

Transform the midline of the curve (formerly the x-axis) by moving it up/down c space

Step 3

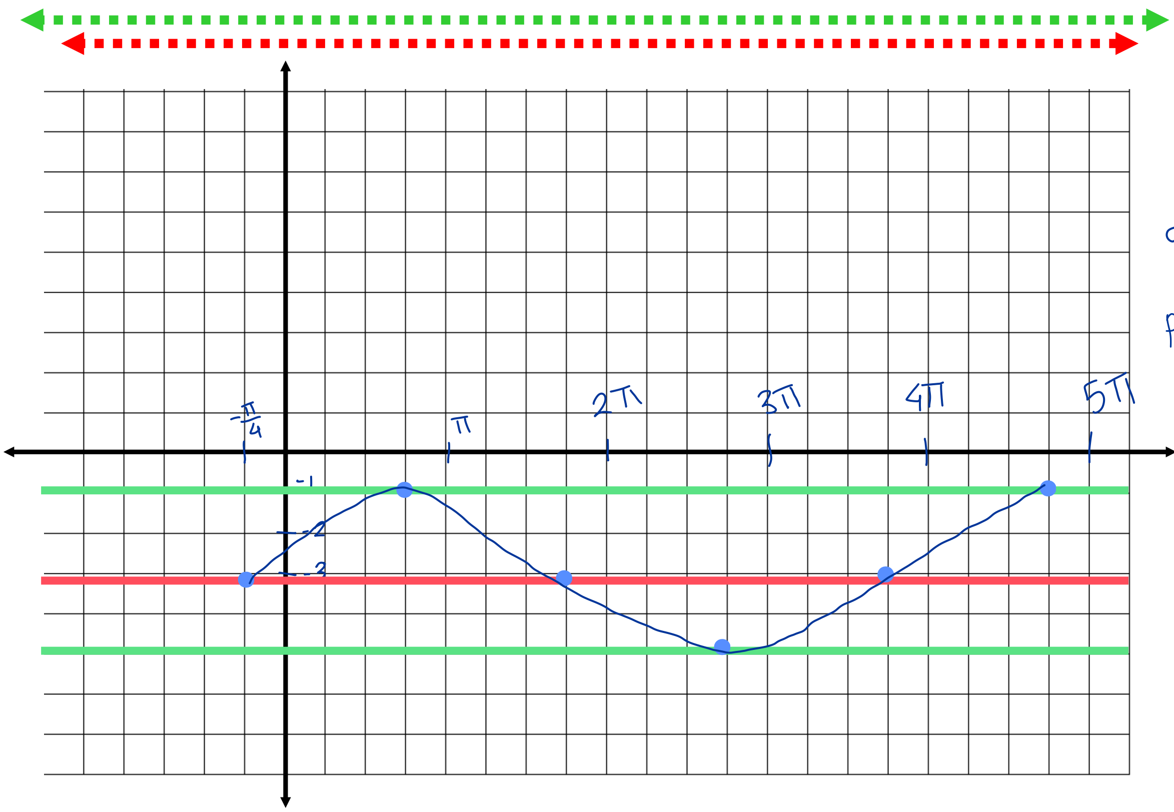
For sine curve plot your beginning point on the midline at phase shift (d)

For cosine curve plot your beginning point one amplitude (a) above the midline at the phase shift (d)

Step 4

Plot your end point one period length from you beginning point and divide up the space between into 4 equal sections accordingly. Remember what regular sine and cosine graphs look like and pay attention to reflections.

Example 1. Graph $y = 2\sin(\frac{1}{2}(x + \frac{\pi}{4})) - 3$

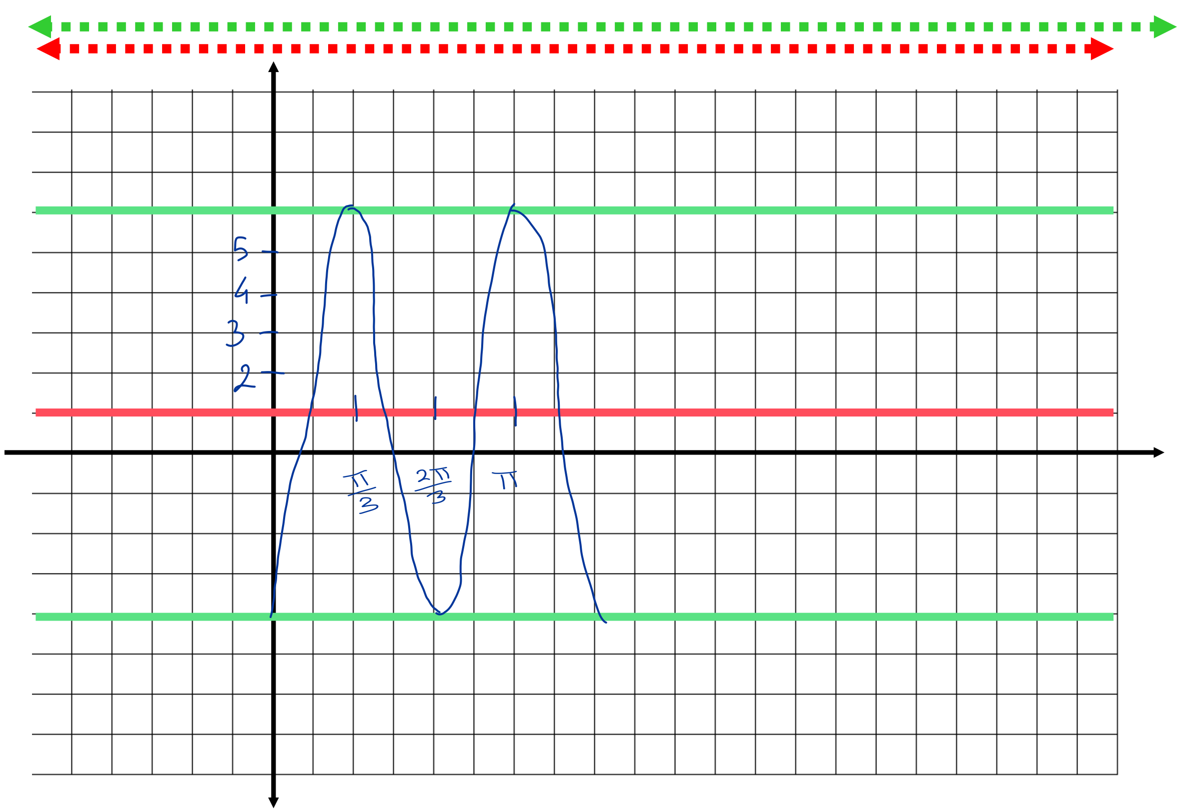


$k = \frac{1}{2}$

amp = 2

$p = \frac{2\pi}{k} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

Example 2. Graph y = -5cos(3x) +1



Example 3. Find the form of the cosine curve that has an amplitude of 4, a period of $\frac{\pi}{2}$ a left phase shift of $\frac{\pi}{3}$ and a vertical translation of 7.

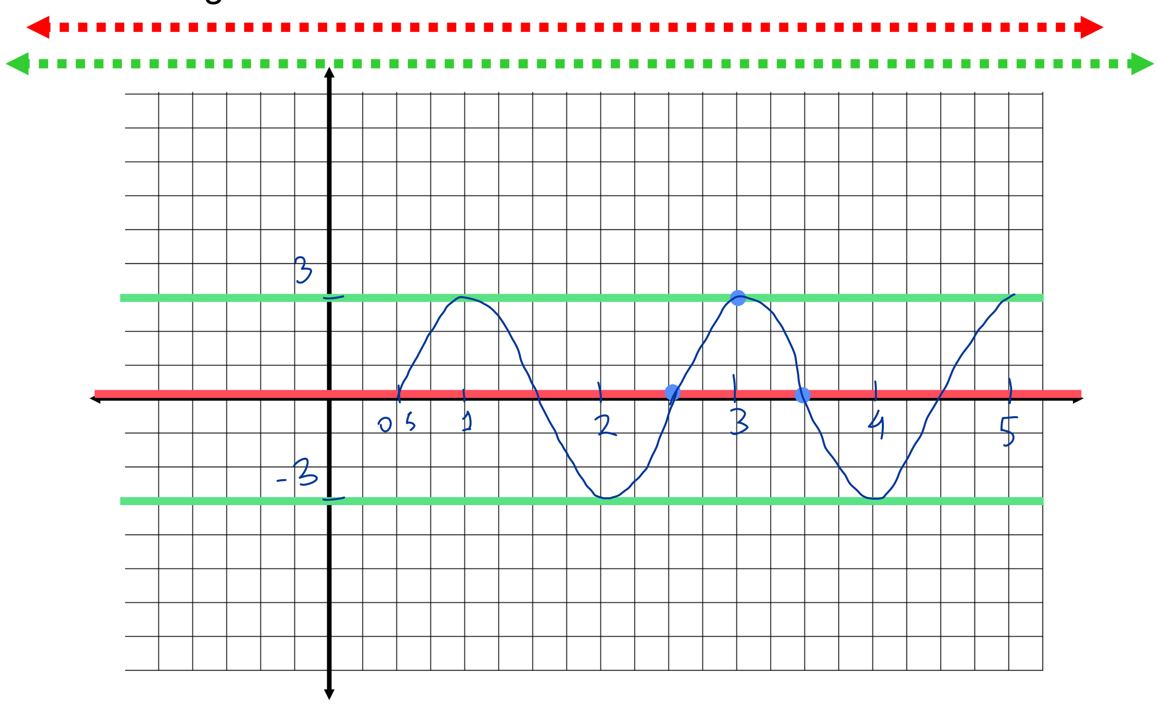
$p = \frac{2\pi}{k}$

$\frac{\pi}{2} = \frac{2\pi}{k}$

$k = 4$

$y = 4\cos\left(4\left(x + \frac{\pi}{3}\right)\right) + 7$

Example 4. Sketch the transformation of f(x) = sinx with an amplitude of 3, period 2 and a phase shift of 0.5 rads to the right.



Practice Questions - Page 275 #1, 2, 5b, 6b, 8, 10a, 11a, 18, 20

Topic : Solving Trig Equations

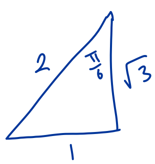
Goal : I can solve equations that involve trigonometric ratios.

Solve Trigonometric Equations

Solving a trig equation is the same as solving any other equation. We have to find a value for x that will make the equation true. Just remember that as we move through one complete rotation, **every value of sine will have two angles to match it.**

Example 1. Determine the solution to the equation from [0, 2π]

$$2\sin x + 1 = 0$$



$2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x_1 = \pi + \frac{\pi}{6}$

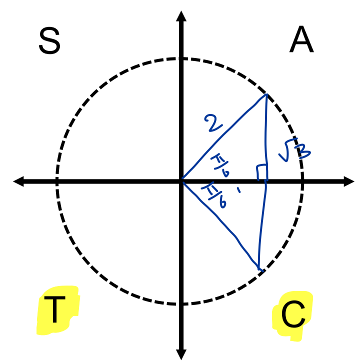
$= \frac{6\pi}{6} + \frac{\pi}{6}$

$= \frac{7\pi}{6}$

$x_2 = 2\pi - \frac{\pi}{6}$

$= \frac{12\pi}{6} - \frac{\pi}{6}$

$= \frac{11\pi}{6}$



Example 2. Determine the solution to the equation from [0, 2π]

$$3(\tan x + 1) = 2$$

$\tan x + 1 = \frac{2}{3}$

$\tan x = -\frac{1}{3}$

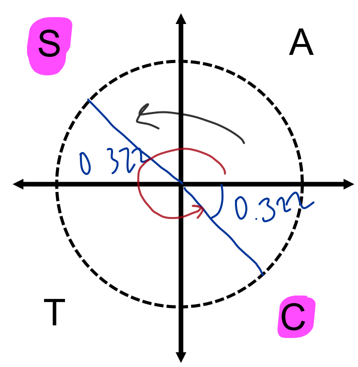
$x = -0.322$

$x_1 = 2\pi - 0.322$

$= 5.961$ rad

$x_2 = \pi - 0.322$

$= 2.820$ rad



You may have to apply some identities before you can solve. Try to get your equation into just one primary trig ratio in some way.

Example 3. Determine the solution to the equation from $[0, 2\pi]$

$$2\sin^2 x - 3\sin x + 1 = 0$$

let $u = \sin x$

$2u^2 - 3u + 1 = 0$

$(2u-1)(n-1) = 0$

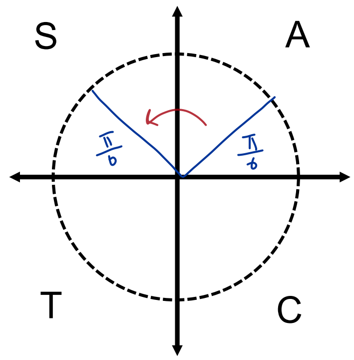
$n = \frac{1}{2}$, $n = 1$

$\sin x = \frac{1}{2}$, $\sin x = 1$

$x_1 = \frac{\pi}{6}$ $\quad$ $x_2 = \frac{\pi}{2}$

$x_3 = \pi - \frac{\pi}{6}$

$x_3 = \frac{5\pi}{6}$



Example 4. Determine the solution to the equation from $[0, 2\pi]$

$$2\sec^2 x - 3 + \tan x = 0$$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x + \cos^2 x = \frac{1}{\cos^2 x}$

$2(1+\tan^2 x) - 3 + \tan x = 0$

$2 + 2\tan^2 x - 3 + \tan x = 0$

$2\tan^2 x + \tan x - 1 = 0$

$2u + u - 1 = 0$

$(2u-1)(n+1) = 0$

$u = \frac{1}{2}$, $u = -1$

$\tan x = \frac{1}{2}$ $\quad$ $\tan x = -1$

$x_1 = 0.464$

$x_2 = \pi + 0.464$

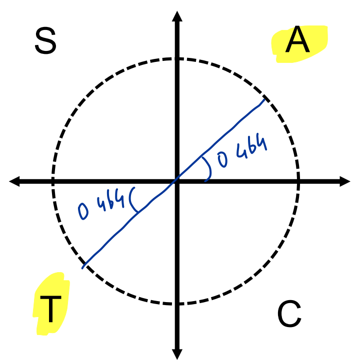
$= 3.606$

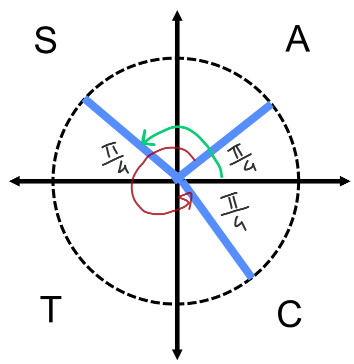
$x_1 = \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

$x_2 = 2\pi - \frac{\pi}{4}$

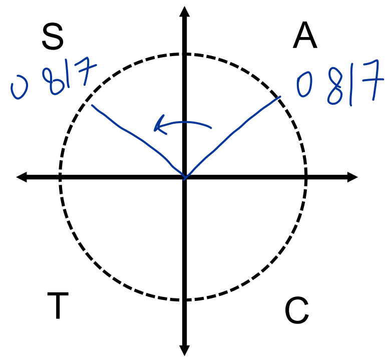
$= \frac{7\pi}{4}$





Example 5. Determine the solution to the equation from [0, 2π]

$$3\sin x + 3\cos 2x = 2$$



$$3\sin x + 3(1 - 2\sin^2 x) = 2$$

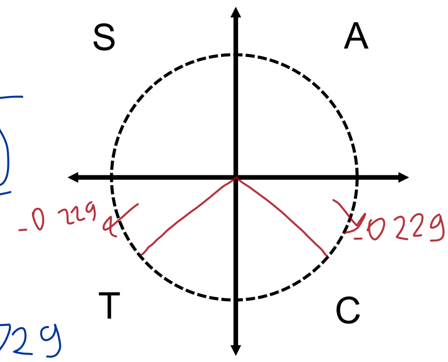
$$3\sin x + 3 - 6\sin^2 x = 2$$

$$0 = 6\sin^2 x - 3\sin x - 1$$

$$u = \sin x$$

$$6u^2 - 3u - 1 = 0$$

$$t_{1,2} = \frac{3 \pm \sqrt{(-3)^2 - 4(6)(-3)}}{2(6)}$$



$$t = \frac{3 \pm \sqrt{33}}{12} \qquad t_1 = 0.729 \qquad t_2 = -0.229$$

$\sin x = 0.729$

$\boxed{x_1 = 0.817 \text{ rad}}$

$\sin x = -0.229$

$x_2 = \pi - 0.817$

$\boxed{= 2.325 \text{ rad}}$

$\sin x = -0.229$

$\boxed{x_3 = -0.231 \text{ rad}}$

$x_4 = \pi + 0.234$

$\boxed{= 3.373 \text{ rad}}$

Practice Questions -
Page 287 #(1, 3, 5, 7) pick and choose 9, 10, 11, 16, 18, 19, 23

Topic : making connections

Goal : I can make "real world" connections with the sinusoidal curves and determine the rate of change.

Making Connections

We are going to use the graphing calculators for instantaneous rate of change.

Remember that the instantaneous rate of change at any point is given by the slope of the tangent line to that point.

On the graphing calculator...

* Graph the curve y = sinx (adjust your window so you see the curve better)

* Use the DRAW button [2nd PRGM] to draw on a tangent line

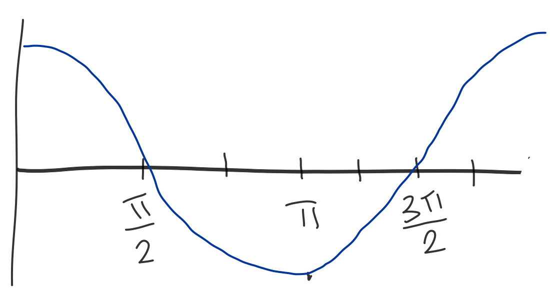
  Press 2nd PRGM, choose 5: Tangent

* It will take you back to the graph, key in the x-value where you wan the tanget line and press enter. Fill in the following table.

x	y=sinx	IROC
0	0	1
$\frac{\pi}{4}$	0.707	0.707
$\frac{\pi}{2}$	1	0
$\frac{3\pi}{4}$	0.707	-0.707
π	0	-1
$\frac{5\pi}{4}$	-0.707	-0 707
$\frac{3\pi}{2}$	-1	0
$\frac{7\pi}{4}$	-0.707	0 707
2π	0	1

What do you notice about your points?

The IRCO of the normal sine curve is the cosine curve



Many "real world" application follow a cyclical or sinusoidal pattern. We can use a sinusoidal curve to model them. Usually we will choose the sine curve and apply a transformation to suit.

Example 1. The number of employees at a City Bicycle Company for each of the last 11 years is shown below. Find a sine curve that will model this data. Use technology to help you.

Year	Employees
1	228
2	241
3	259
4	233
5	226
6	209
7	212
8	225
9	240
10	251
11	261

Amplitude (half the distance from max to min)

$261 - 209 = 52$

$\frac{52}{2} = 26$

Midline (the horizontal line right between max and min)

$$\frac{261 + 206}{2} = 235$$

Period (for k-value)

$11 - 3 = 8$ $\quad p = \frac{2\pi}{k}$ $\quad 8 = \frac{2\pi}{k}$ $\quad 8k = 2\pi$ $\quad k = \frac{2\pi}{8}$ $\quad k = \frac{\pi}{4}$

Phase Shift (the x-value when the y is first on the midline)

1 3

$y = 26\sin\frac{\pi}{4}(x - 13) + 235$

Practice Questions - Page 296 #1, 2, 4, 5, 10, 11, 15