Solutions
Mon/Wed..

Name : ______________________

MHF4U1
Unit 1: Polynomial Functions

K/U /20	APP /10	COM -------------	TH /10

LIFE LINES ☐ Phone ☐ Notebook ☐ 50/50

KNOWLEDGE/UNDERSTANDING

Multiple Choice
Identify the choice that best completes the statement or answers the question.

_d_ 1. The restriction on the degree 'n' for all polynomial functions of the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x^1 + a_0x^0$ is: [1K]
a. 'n' must be non-negative
b. $n \in \{0, 1, 2, 3, 4, \ldots\}$
c. 'n' must be a whole number
d. All of the above

_b_ 2. Which function does not classify as a polynomial function? [1K]
a. $y = -3$
b. $y = 2\sqrt{x}$
c. $y = 2x^2-1$
d. $y = -2x^7$

_c_ 3. If the leading coefficient of an odd-degree polynomial function is negative, then the function extends from: [1K]
a. 1st quadrant → 3rd quadrant
b. 1st quadrant → 2nd quadrant
c. 2nd quadrant → 4th quadrant
d. 2nd quadrant → 3rd quadrant

_a_ 4. The degree 'n' of a polynomial provides information about all of the following except: [1K]
a. The shape of the graph
b. The end behaviours of the graph
c. The roots of the graph
d. All of the above

_b_ 5. Which polynomial function has its end behaviour extending from quadrants 1→2? [1K]
a. $f(x) = 7x^5 - 8x^4 - 2x^3 + x^2 + 3x - 2$
b. $f(x) = -4x^2 + 3x^4 - 6x^3 + 2x + 8$
c. $f(x) = -6x^3 + 3x^2 + x - 11$
d. $f(x) = x^3 - 9$

_d_ 6. The graph of an odd-degree polynomial function has at least __?__ root(s) and up to a maximum of __?__roots. [1K]
a. 3, n
b. 0, n-1
c. 1, n-1
d. 1, n

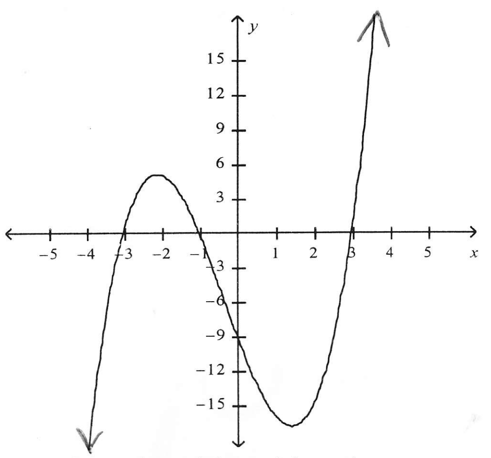
b 7. The graph of an even degree polynomial function can have at least__?__ root(s) and a maximum of up to__?__ roots. [1K]

a. 2, n
b. 0, n
c. 0, n-1
d. n, n-1

b 8. At “first glance”, the polynomial function f(x) = $3x^4-2x^3+7x^2-1$ provides information about all of the following except: [1K]

a. End behaviours
b. Exact shape of the graph
c. Degree of the function
d. Sign of leading coefficient

a 9. What is the equation of the graph shown below? [1K]



a. $f(x) = (x-3)(x+1)(x+3)$
b. $f(x) = (x-3)(x-1)(x+3)$
c. $f(x) = (x-3)(x+3)$
d. $f(x) = (x-3)^2(x+1)$

d 10. The degree and x-intercepts of the polynomial function f(x) = $x(x+3)^3(x+1)(x-5)$ are: [1K]

a. n=6, x=0, 3 (order 3), -1, -5
b. n=5, x=0, -3 (order 2), -1, -5/2
c. n=5, x=0, -3 (order 3), -1, 5/2
d. n=6, x=0, -3 (order 3), -1, 5

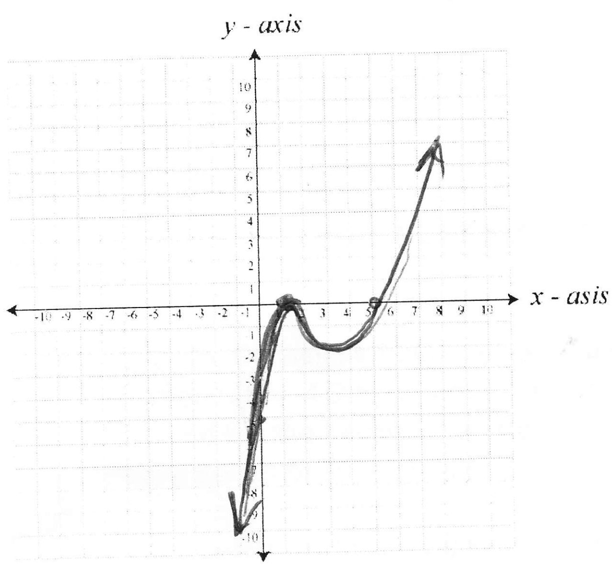
d 11. The function $h(t) = -4.9t^2 + 150$, where h is in metres and t in seconds, models the following scenario: [1K]

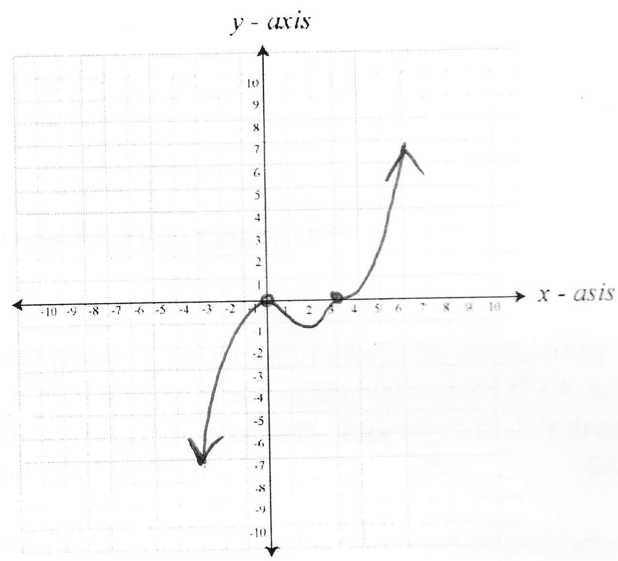
a. A football punt on goal
b. A ball being thrown downward from the roof of a building 120 metres high
c. A cannon being shot upward from the edge of a cliff
d. Releasing a ball from a bridge 120 metres above ground

12. Sketch a possible graph of the functions

a) y= (x-1)(x-1)(x-5) $=(x-1)^2(x-5)$

b) $y = x^2(x-3)^3$ [6K]





13. Write an equation of a cubic function that has zeros of -1, 2, and 3. The function has a y-intercept of 6. [3K]

(0,6)

$f(x) = k(x+1)(x-2)(x-3)$

$6 = k(0+1)(0-2)(0-3)$

$6 = k(1)(-2)(-3)$

$\frac{6}{6} = \frac{k(6)}{6}$

$k = 1$

$\therefore f(x) = (x+1)(x-2)(x-3)$

PLICATION

1. This table of values represents a polynomial function f(x). Use **finite differences** to determine the following: [4A]

(a) the degree 'n' of the polynomial function f(x)
(b) the sign of the leading coefficient
(c) the value of the leading coefficient, a_n

X	Y
-3	0
-2	-4
-1	0
0	6
1	8
2	0
3	-24

3rd
-
-6
-6
-6
-6.

c) $-6 = a_n(3!)$
$-6 = a_n(3 \times 2 \times 1)$
$\frac{-6}{6} = \frac{a_n(\cancel{6})}{\cancel{6}}$
$-1 = a_n$.

a) $\therefore n = 3$

b) negative.

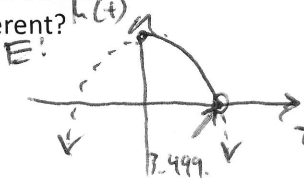
CHOOSE ONE FROM THE FOLLOWING TWO QUESTIONS…

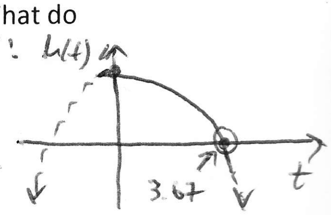
2. On Earth, the height, h, in metres, of a free falling object after t seconds can be modeled by the function $h(t) = -4.9t^2 + k$, while on Venus, the height can be modeled by $h(t) = -4.45\,t^2 + k$, where $t \geq 0$ and k is the height, in metres, from which the object is dropped. Suppose a rock is dropped from a height of 60 m on each planet. For each planet, [6A]

E: Earth
V: Venus.

(a) Determine the **average rate of change** of the height of the rock after the first 3 seconds after it is dropped (i.e. $0 \leq t \leq 3$)
(b) Estimate the **instantaneous rate of change** of the rock 3 seconds after it is dropped (i.e. @ t= 3 sec). Hint: using average rate of change, choose two to three intervals of time that get closer and closer to 3 sec.
(c) Compare the average rates of change of the falling rock on Earth and on Venus. What do these rates represent, and why are these values different?

Earth: $h(t) = -4.9t^2 + 60$
Venus: $h(t) = -4.45t^2 + 60$.





(a) $ARoC_E = M_{sec} = \frac{\Delta h}{\Delta t}$ (for $0 < t < 3$)
$= \frac{h_2 - h_1}{t_2 - t_1} = \frac{h(3) - h(0)}{3 - 0}$.
$= \boxed{-14.7\,m/s}$

$ARoC_V = \frac{\Delta h}{\Delta t} = \frac{h(3) - h(0)}{3 - 0}$
$= \boxed{-13.35\,m/s}$

(b) $IROC = \lim_{t_2 \to t_1} \frac{\Delta h}{\Delta t}$ (Concept)

Earth

(i) $3 < t < 3.01$

$AROC = \frac{\Delta h}{\Delta t} = \frac{h(3.01) - h(3)}{3.01 - 3} = \boxed{-29.444\ m/s}$

$3 < t < 3.001$

(ii) $AROC = \frac{\Delta h}{\Delta t} = \frac{h(3.001) - h(3)}{3.001 - 3} = \boxed{-29.4049\ m/s}$

$\therefore IROC_E \approx -29.4\ m/s.$

Venus

(i) $3 < t < 3.01$

$AROC = \frac{\Delta h}{\Delta t} = \frac{h(3.01) - h(3)}{3.01 - 3} = \boxed{-26.7945\ m/s}$

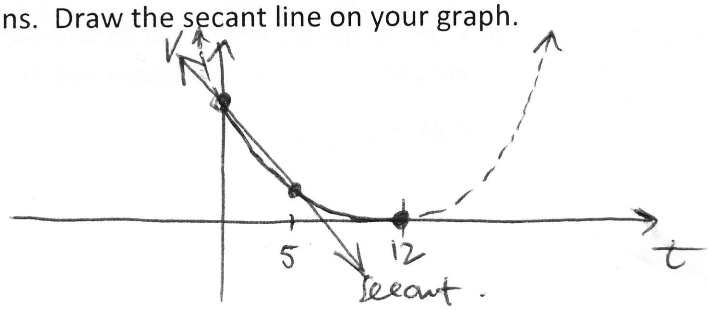
(ii) $3 < t < 3.001$

$AROC = \frac{\Delta h}{\Delta t} = \frac{h(3.001) - h(3)}{3.001 - 3} = \boxed{-26.70495\ m/s}$

$\therefore IROC_V \approx -26.7\ m/s$

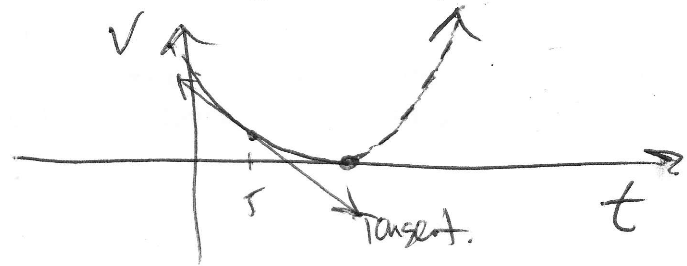
3. Zohan drains the water from a hot tub. The tub holds 144 L of water. It takes 12 min for the water to drain completely. The volume of water in the tub is modeled by the function $V(t) = (12 - t)^2$, where V is the volume on litres at t mins between $0 \le t \le 12$. [6A]

a) Draw the graph and fully label the axes. Determine the average rate of change of volume from 5 mins to 12 mins. Draw the secant line on your graph.



$AROC = \frac{\Delta V}{\Delta t} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{V(5) - V(0)}{5 - 0} =$

b) Determine the instantaneous rate of change after exactly 5 mins. Draw the tangent line on your graph.



$5 < t < 5.01$

$AROC = \frac{\Delta V}{\Delta t} = \frac{V(5.01) - V(5)}{5.01 - 5} =$

$> IROC \approx$

$5 < t < 5.001$

$AROC = \frac{\Delta V}{\Delta t} = \frac{V(5.001) - V(5)}{5.001 - 5} =$

(a) $AROC_E = \frac{\Delta h}{\Delta t} = \frac{h(2)-h(0)}{2-0} = \frac{25.4 - 45}{2-0} = -9.8 \text{ m/s}$
$0 \le t \le 2$

$AROC_V = \frac{\Delta h}{\Delta t} = \frac{h(2)-h(0)}{2-0} = \frac{27.36 - 45}{2-0} = -8.82 \text{ m/s}$

$IROC_E \approx -19.6 \text{ m/s}$
$IROC_V \approx -17.6 \text{ m/s}$

(b) $AROC_E = \frac{25.20351 - 25.4}{0.01} = -19.649 \text{ m/s}$
$2 < t < 2.01$

$AROC_V = \frac{27.183 - 27.36}{0.01} = -17.68 \text{ m/s}$
$2 < t < 2.01$

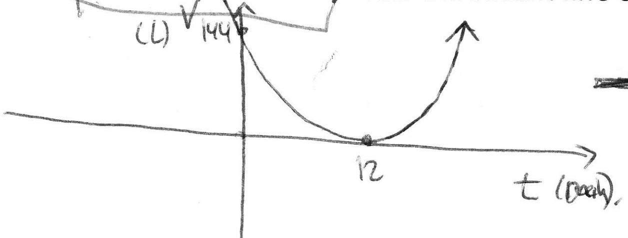
$AROC = \frac{25.3804 - 25.4}{0.001} = -19.6 \text{ m/s}$
$2 < t < 2.001$

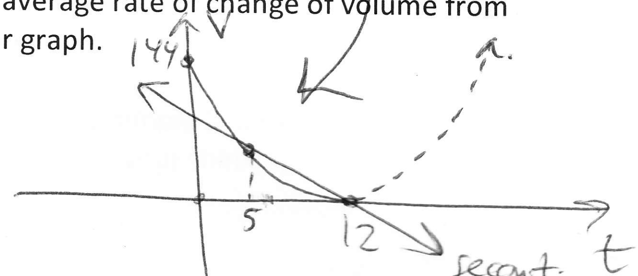
$AROC_V = \frac{27.343 - 27.36}{0.001} = -17.64 \text{ m/s}$
$2 < t < 2.001$

3. Zohan drains the water from a hot tub. The tub holds 144 L of water. It takes 12 min for the water to drain completely. The volume of water in the tub is modeled by the function $V(t) = (12 - t)^2$, where V is the volume (in litres) at time (in mins) between $\boxed{0 \le t \le 12}$. [6A]

Restriction.

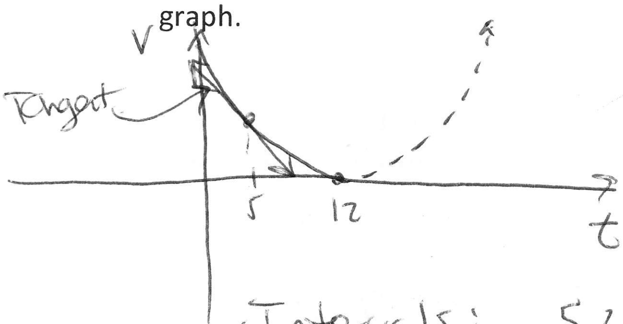
a) Draw the graph and fully label the axes. Determine the average rate of change of volume from 5 mins to 12 mins. Draw the secant line on your graph.





AROC for $0 < t < 12 \Rightarrow AROC = \frac{\Delta V}{\Delta t} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{V(12) - V(5)}{12 - 5} = \boxed{-7 \frac{L}{min}}$

b) Determine the instantaneous rate of change (IROC) @ exactly t= 5 min. Draw the tangent line on your graph.

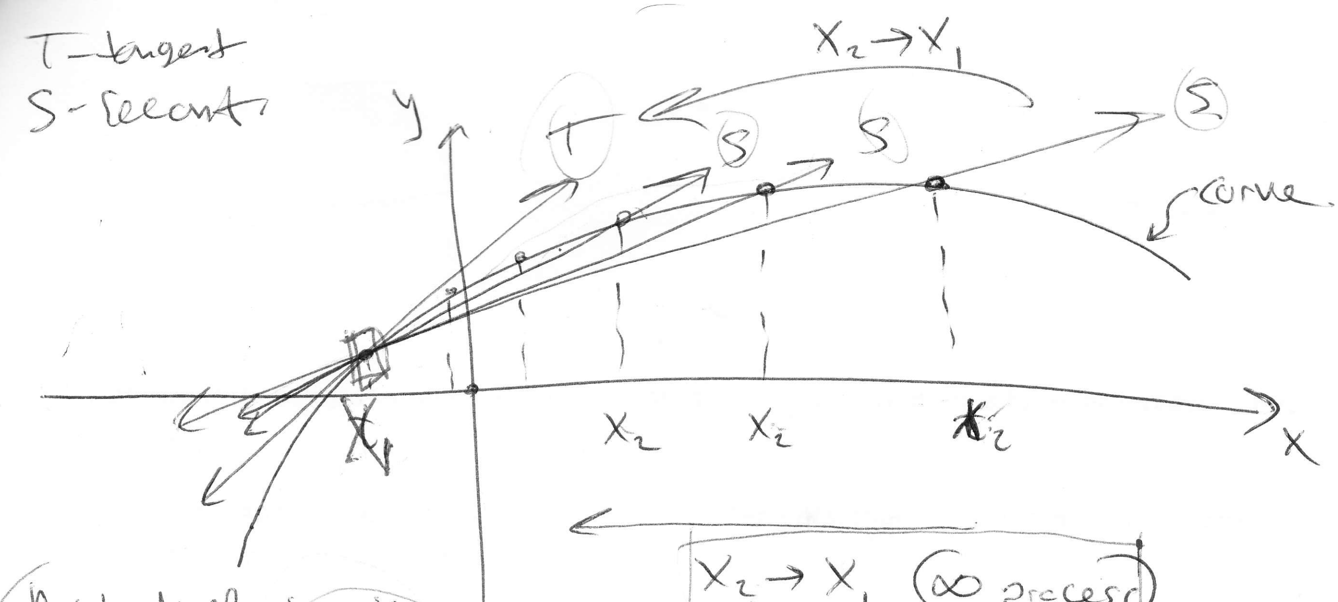


$IROC = \lim_{t_2 \to t_1} \frac{\Delta V}{\Delta t}$ Concept

Intervals: $5 \le t \le 5.01$ $AROC = \frac{\Delta V}{\Delta t} = \frac{V(5.01) - V(5)}{5.01 - 5} = \boxed{-13.99 \frac{L}{min}}$

$5 \le t \le 5.001$ $AROC = \frac{\Delta V}{\Delta t} = \frac{V(5.001) - V(5)}{5.001 - 5} = \boxed{-13.999 \frac{L}{min}}$

$IROC \approx -14 \text{ L/min}$



Driving Mechanism is lim $X_2 \to X_1$ b/c $\Delta y / \Delta x$ is a static slope

THINKING

lim $X_2 \to X_1$ rotates the secant towards the tangent.

1. Explain the meaning of the following equation, using simple language. What is the driving mechanism of this equation? Use a diagram to illustrate your thinking. [4T]

$$M_{Tangent} = IROC = \lim_{x_2 \to x_1} \frac{\Delta y}{\Delta x}$$

The slopes of the secant as $X_2 \to X_1$ approximates the slope of the tangent (M_{Tan}). It approximates b/c $X_2 \to X_1$ is an infinite process, however, there is a limit to the slopes of the secants as you approach the slope of the tangent!

2. Explain why odd degree polynomial functions can have only local maximums and minimums, but even – degree polynomial functions can have maximum and minimums. [2T]

odd degree polynomials have their end behaviours going in opposite directions. Therefore odd degree polynomials must cross the X-axis at least once!

3. Determine if each statement is either true or false? If the statement is false, give reasons for your answer.

a) If the graph of a quartic function has two x – intercepts, then the corresponding quartic equation has four real roots.

[3] False. It would only have 2 roots (each having degree 2, since it bounces).

e.g. $f(x) = (x+2)^2 (x-4)^2$.

b) A polynomial equation of degree three must have at least one real root.

[1] True