MHF4U

Advanced Functions
Grade 12 University

Unit 1 Polynomials

7 Video Lessons

Allow **no more** than **15** class days for this unit!
This includes time for review and to write the test.

Lesson #	Lesson Title	Practice Questions	Date Completed
1	Power Functions	Page 11 #1-4, 7-11, 13, 14, 16	
2	Characteristics of Polynomial Functions	Page 26 #1-7, 12, 13, 15, 16	
3	Graphs of Polynomial Functions	Page 39 #1, 2, 3, 6, 8, 13	
4	Symmetry in Polynomial Functions	Page 39 3, 4, 8	
5	Transformations of Power Functions	Page 49 #1-5, 7, 8, 10, 14	
6	Slopes of Secants	Page 62 #1-5, 7, 8, 9, 12	
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Test Written on : ______________________________________

Topic: Power Functions

Goal : I know what a polynomial function is and can describe the key features and end behaviours of power functions.

Power Functions

Linear and Quadratic functions are the two most encountered polynomial functions. Polynomials are defined as follows… (from you text)

A polynomial expression is an expression of the form

$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_3x^3 + a_2x^2 + a_1x + a_0$,

where

- n is a whole number
- x is a variable
- the **coefficients** $a_0, a_1, \ldots, a_n$ are real numbers
- the **degree** of the function is n, the exponent of the greatest power of x
- a_n, the coefficient of the greatest power of x, is the **leading coefficient**
- a_0, the term without a variable, is the **constant term**

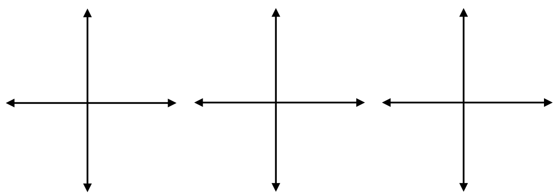
A **polynomial function** has the form

$f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_3x^3 + a_2x^2 + a_1x + a_0$

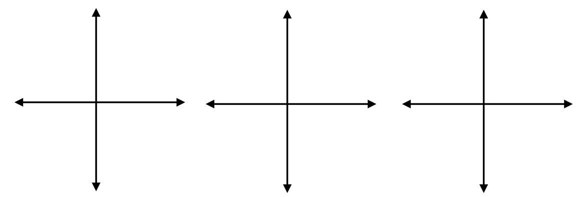
Power functions are the simplest of the polynomial functions. They only have one term, such as f(x) = x (the linear power function) or f(x) = x^2 (the quadratic power function) and so on. When you transform a power function, it is then a general polynomial function.

Can we tell by looking if a function has an even or odd degree?

<u>Even</u>



<u>Odd</u>



What is meant by end behaviour? How do we describe the end behaviours?

What can be said about domain, range and symmetry of power functions?

Domain

Range

Symmetry

Example 1. Give an example of each and explain how you know?

a) a function that goes from Quad 2 to Quad 4

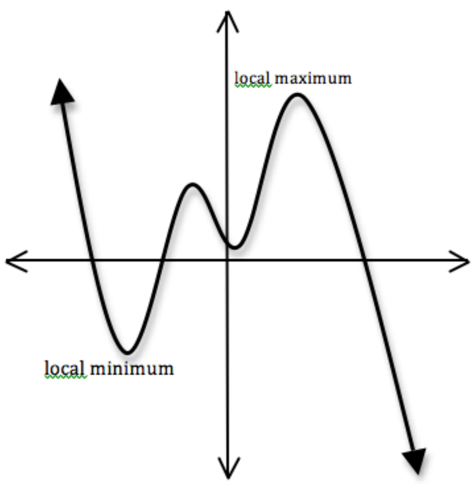
b) a function that starts at -infinity and ends at + infinity

Practice Questions - Page 11 #1-4, 7-11, 13, 14, 16

Topic : Polynomial Functions

Goal : I can describe the characteristics of a polynomial function, including degree, end behaviour, and local extremes.

Characteristics of Polynomial Functions



A polynomial function of degree n will have ***at most*** _______ local extreme (max/min) points.

A polynomial function that has r local max/min points will have a degree of ***at least*** _______________

A polynomial function of degree n will have at most ___________ x-intercepts (places where it crosses the x-axis)

For the graph given, there are 4 local extremes which means it must ***at least*** be degree _________________.

Finite Differences: The nth differences for a polynomial of degree n will all be the same, and this common difference will be equal to the product of n! and the leading coefficient (a_n)

Common Difference $= a_n n!$

$= a_n[n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1]$

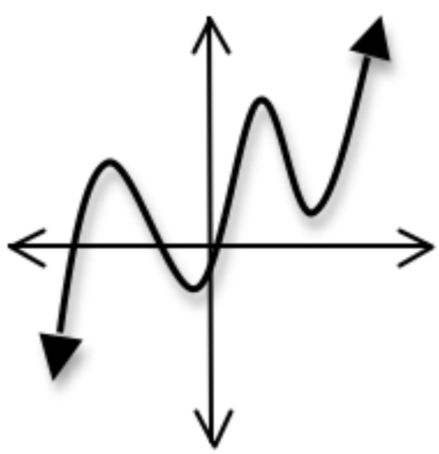
Example 1. The finite differences are taken for a polynomial function, and the 6^{th} differences are found to all be -2880. What was the leading coefficient of the polynomial function.

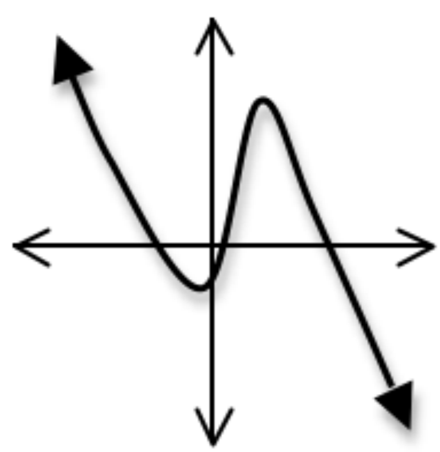
MHF 4U U1L2 Characteristics of Polynomial Functions

For a polynomial of degree *n*, where *n* is odd…

- The end behaviours are opposite.
  - if the leading coefficient is positive, it will go from $-\infty$ to $+\infty$ (from 3rd to 1st quadrant). The overall "slope" of the graph will be _____________.
  - if the leading coefficient is negative, it will go from $+\infty$ to $-\infty$ (from 2nd to 4th quadrant. The overall "slope" of the graph will be ______________.

- There will be at least one x-intercept and no more than ____ x-intercepts.
- There are no ________________ extremes (min/max points)
- Domain : _______
- Range : ________
- An odd degree polynomial may show _____________ symmetry.





For a polynomial of degree *n*, where *n* is even…

The end behaviours are the same.

* if the leading coefficient is positive, it will go from $+\infty$ to $+\infty$ (from 2nd to 1st quadrant). The overall "opening" of the graph will be _____________.
* if the leading coefficient is negative, it will go from $-\infty$ to $-\infty$ (from 3rd to 4th quadrant. The overall "opening" of the graph will be ______________.

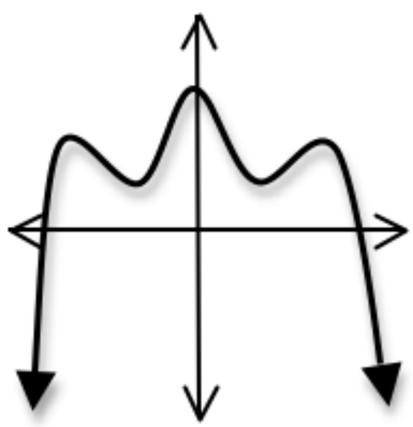
There may be anywhere from zero to ____ x-intercepts.

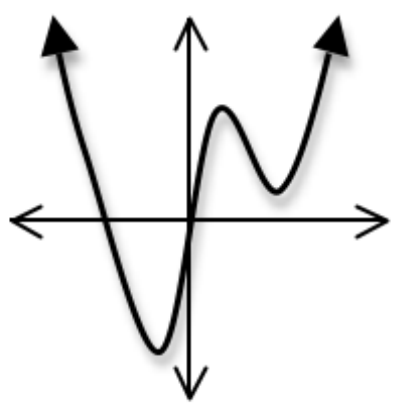
There will be at least one absolutes extreme (__________ or _________)

Domain : _______

Range : ___________________________________ if $a_n > 0$
___________________________________ if $a_n < 0$

An even degree polynomial may have a _____________ of symmetry.





Practice Questions - Page 26 #1-7, 12, 13, 15, 16

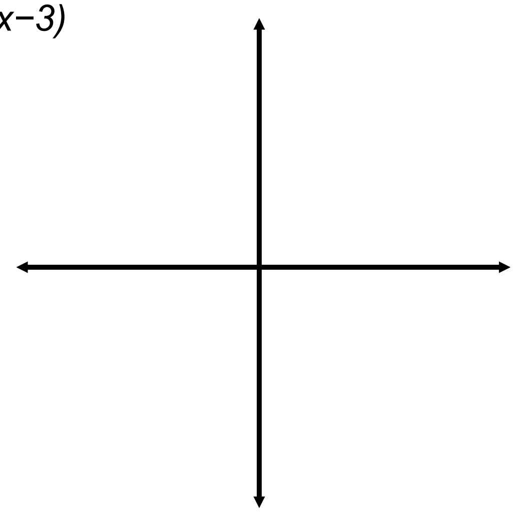
Topic : Graphs of Polynomial Functions

Goal : I can sketch a function based on the factored form of its equation and the order of its roots.

Equations and Graphs of Polynomial Functions

Think back to our work on parabolas. The function $f(x) = 2x^2 + 2x - 24$ can be written in factored form $f(x)=2(x+4)(x-3)$

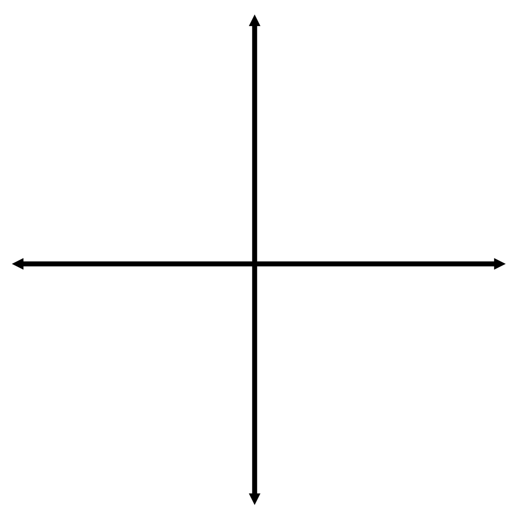
What do we know about this function?



We can graph higher degree polynomials with similar methods.

Example 1. Sketch the function $f(x) = (x + 5)(x + 3)(x - 1)(x - 3)(x - 4)$, and determine the sign of the function in the intervals between x-intercepts.

Interval	$(x+5)(x+3)(x-1)(x-3)(x+4)$					f(x)



Where does the function cross the y-axis?

In the example we just completed, there were 5 distinct factors in factored form. This tells us that there are 5 x-intercepts (or zeros) and the polynomial must have been of degree 5. Not all degree 5 polynomials will have 5 distinct factors. Take the following example.

$$f(x) = (x-3)(x-3)(x+4)(x+5)(x-2)$$

The (x-3) factor is repeated. There are two of them, so the zero x=3 is said to have order 2. In general, if a factor (x-a) is repeated n times for a polynomial function, the zero x=a will be of order n.

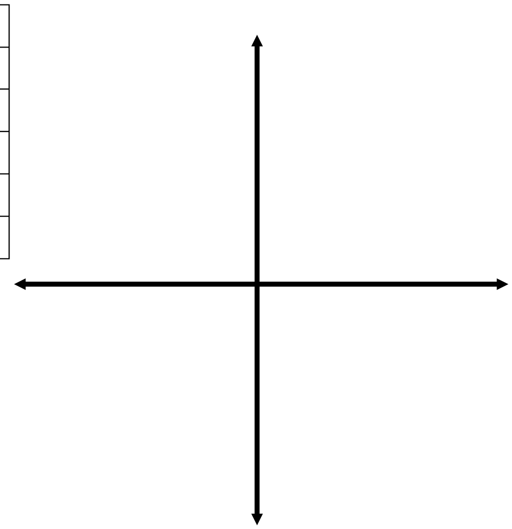
How does the repeated factor affect the graph?

Example 2. Sketch the function

$$f(x)=(x-3)^2(x+4)(x+5)(x-2),$$

and determine the sign of the function in the intervals between x-intercepts.

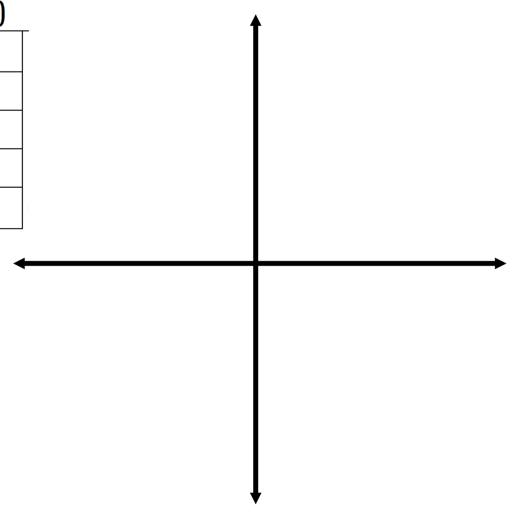
Interval	$(x-3)(x-3)(x+4)(x+5)(x-2)$					f(x)



Notice, that at the repeated zero, the function does not change signs. This means the graph doesn't cross the x-axis at this point, it only touches it.

Example 3. Sketch the function $f(x) = (x-3)^3(x+4)(x+5)(x-2)$, and determine the sign of the function in the intervals between x-intercepts.

Interval	$(x-3)(x-3)(x-3)(x+4)(x+5)(x-2)$						f(x)
$(-\infty,-5)$							
$(-5,-4)$							
$(-4,2)$							
$(2,3)$							
$(3,\infty)$							



This time the sign does change at the repeated root. If the repeated root has an even order, the function will not cross the axis at that point. If the zero has an odd order, the function will cross the axis at the zero. But, like our power functions, the higher the ORDER of a root, the flatter it will appear at the intercept.

Practice Questions - Page 39 #1, 2, 3, 6, 9, 13

eop is fine if you don't have any problems with the concepts.

Today's Topic : Polynomial Functions

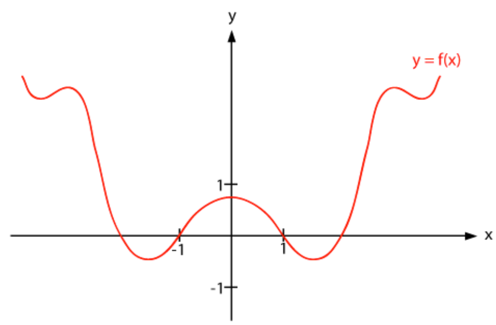
Today's Goal : I know the difference between and odd and even function and what that means about its symmetry.

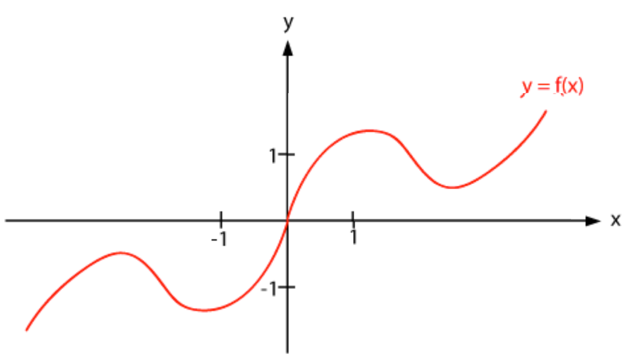
Symmetry in Polynomial Functions

★ A polynomial function is called an even function if the **exponent of each term** of the equation is **even**. The value of the function would be the same if you subbed in a positive value or its opposite negative value.

f(x) = f(-x)

Because of this, the function will be symmetric about the y-axis.





★ A polynomial function is called an odd function if the **exponent of each term** of the equation is **odd**. The value of the function would have the opposite sign if you subbed in a positive value of its opposite negative value

f(-x) = -f(x)

Because of this, the function will be symmetric about the origin.

Example 4. Determine if the following functions are even, odd or neither.

a) $f(x)=(x-4)(x+3)(2x-1)$

b) $f(x)=-2(x+2)(x-2)(1+x)(x-1)$

Practice Questions - Page 39 #3, 4, 8

Topic : Transformations of Power Functions

Goal : I know how changes in a power function's equation affects its position on the Cartesian plane, and I know how to use a mapping to apply the transformation.

Transforming the Power Function

With your graphing calculator or desmos.com, graph the following functions. Keep Y_1 the same and compare each new function to the original. (Try this before watching the video).

1. $Y_1 = x^3$

   $Y_2 = x^3 + 4$
   $Y_2 = (x - 4)^3$
   $Y_2 = 3x^3$
   $Y_2 = (3x)^3$

2. $Y_1 = x^4$

   $Y_2 = x^4 + 5$
   $Y_2 = (x - 5)^4$
   $Y_2 = 3x^4$
   $Y_2 = (3x)^4$

The general form of a transformed power function is...

$$y = a\,[\,k\,(x - d)]^n + c$$

Example 1. Describe the transformation of $y = x^4$ into $y = 3[2(x+5)]^4 + 7$

Example 2. Describe the transformation of $y = x^3$ into $y = 4f(3(x-3))+2$

Example 3. Describe the transformation of $y = x^8$ into $y = -3f(2x+5)$

To generally describe what the transformations do...

- ★ The k and d transform the x-coordinate of the function. Since k is a horizontal compression, it divides the x-coordinate. Then the d slides it around.

$$\frac{x}{k} + d$$

- ★ The a and c transform the y-coordinate of the function. Since a is a vertical stretch it multiplies the y-coordinate.

$$ay + c$$

What we now have is a formula for each coordinate that we can use to transform the table of values for a function.

Example 4. Use the transformation of $y = x^3$ into $y = 2[5(x-4)^3] + 6$ to come up with a table of values for the latter function.

x	y
-2	-8
-1	-1
0	0
1	1
2	8

In general, to represent a transformation, we use a mapping....

$$(x, y) \longrightarrow \left(\frac{x}{k} + d \ , \ ay + c \right)$$

Example 5. Give the mapping of $y = x^3$ into $y = 2[5(x-4)]^3 + 6$

Practice Questions - Page 49 #1-5, 7, 8, 10, 14

Topic : Average rate of change

Goal : I can use the slope of a secant to find the average rate of change if we are given a graph, a table or a function's equation.

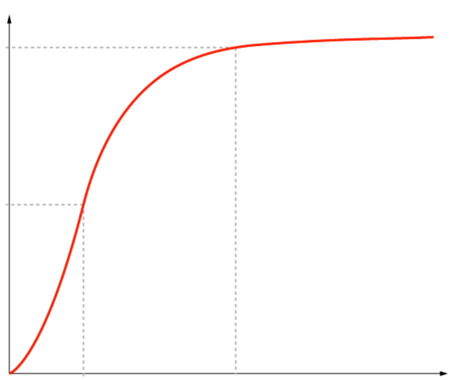
Slopes of Secants : Average Rate of Change (AROC)

Rate of Change is how one quantity compares to another. The most notable rate of change in life is speed or velocity.

$$\text{Average Speed} = \frac{\text{the change in distance}}{\text{the change in time}}$$

Example 1. A trip to Windsor took 3.5 hours to drive the 250 km. What was the average speed driven during the trip?

In general, if we have two quantities changing, say x and y on a grid, the average rate of change will be...



$$\text{AROC} = \frac{\Delta y}{\Delta x}$$

=

=

MHF 4U 1L6 Slopes of Secants

Example 1. Finding A.R.O.C using a graph.

What is the average rate of change during the first 12 seconds of this graph?

Example 2. Finding A.R.O.C using a table.

a) What is the rate of change of money within this account for the whole 6 months?

b) What is the rate at which money is changing in the account during the first 3 months?

Month	Bank Balance
0	2500
1	2700
2	2900
3	3100
4	3100
5	2800
6	2700

Example 3. A projectile follows a parabolic path according to the function...

$$h(d) = -0.09d^2 + 0.9d + 2$$

where h(d) is the height in metres, after a horizontal distance of "d" metres is travelled.

a) What is the average rate of change in height during the first 11 minutes of the flight?

b) What is the average rate of change in height from 5 - 8 m of horizontal distance?

Practice Questions - Page 62 #1-5, 7, 8, 9, 12

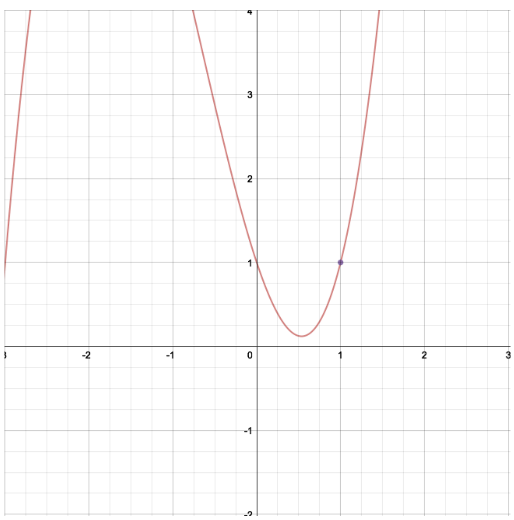
Topic : Rate of Change

Goal : I can use the slope of a secant to approximate the instantaneous rate of change of an object at any given point.

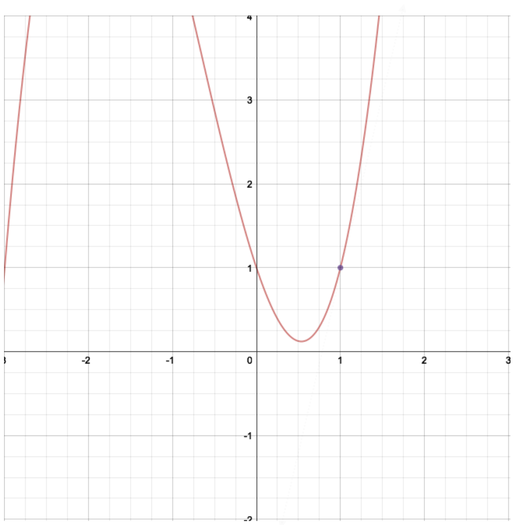
Slopes of Tangents and Instantaneous Rate of Change (IROC)

If we want to find out how fast something is changing at any given moment, we are looking for the slope of a tangent line, in stead of a secant.

Example 1. Estimate the instantaneous rate of change at (1, 1)?



Method 1: pick another point on the function and take the slope of the secant line between them



Method 2: Draw on a tangent line. Find another point on the tangent and use it to find the slope.

Slope of the Tangent Applet
http://www.calvin.edu/~rpruim/courses/m161/F01/java/SecantTangent.shtml

Notice the closer the points are together, the closer closer the secant gets to BECOMING a tangent line. So if we pick a point VERY close to the one we are interested in, and find the slope of the secant, it will be a good estimate for the tangent.

Example 2. Find the slope of the tangent line of $f(x) = 2x^2 - 3x + 2$, when x is 2.

To find the slope of any line we need two points. Since we want the tangent when x is 2, we need find another point very close to where x is 2. Just slightly bigger than 2 would be sufficient, say 2.001.

$$\text{IROC} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Practice Questions - Page 71 #1, 2, 4, 6, 7, 9, 11, 12

MHF 4U Chapter 1 Review : Polynomial Functions

A polynomial function has the form

- {$n \in$________} and n is the _________________ of the function since it is the largest exponent of the variable *x*.
- a_n is called the _______________________ coefficient
- a_0 has no variable attached to it and so is called the ____________________ term

A polynomial function of degree *n* will have

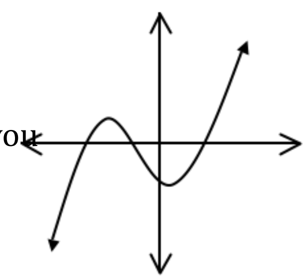
- the ______________ finite difference with common values
- a common difference = an! , where *a* is the leading coefficient and *n* is the _________________
- at most _________ x-intercepts
- at most _________ local extreme points (maximums or minimums)

Polynomial Functions of Odd Degree

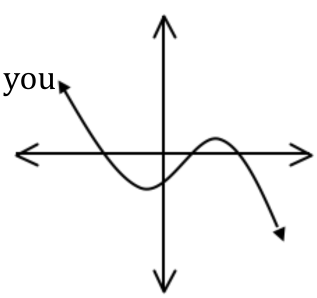
- the largest exponent on all the variables is an ________ number
- End Behaviour : what the function looks like as __ goes off to $\pm\infty$

> Will always begin and end in ______________ directions

> **If the leading coefficient is positive -** will start at negative infinity (quad) and stop at positive infinity (quad) (if you look at the "big picture" the overall "slope" of the graph is ________________________)
as $x \to -\infty$, $y \to -\infty$ and as $x \to +\infty$, $y \to +\infty$)



> **If the leading coefficient is negative -** will start at positive infinity (quad) and stop at negative infinity (quad) (if you look at the "big picture" the overall "slope" of the graph is _______________________)
as $x \to -\infty$, $y \to +\infty$ and as $x \to +\infty$, $y \to -\infty$)



- A polynomial function of odd degree is said to be an ____________ function if ALL the variables have odd exponents.
  > If a polynomial function is odd, it has the property that f(-x) = -f(x) for all values of x

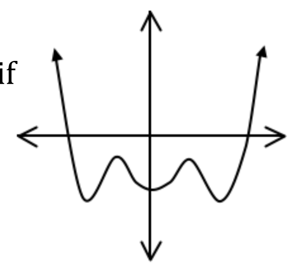
  > If a polynomial function is odd, it will have a point of _________________________ symmetry at the origin.

Polynomial Functions of Even Degree

- the largest exponent on all the variables is an ____________ number
- End Behaviour : what the function looks like as ___ goes off to ±∞

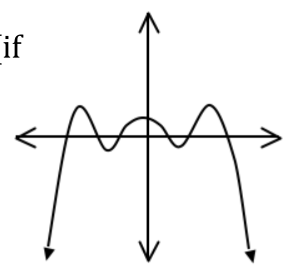
  > Will always begin and end in the _____________ direction

  > **If the leading coefficient is positive** - will start at positive infinity (quad) and stop at positive infinity (quad) (if you look at the "big picture" the overall "shape" of the graph opens __________________)



  as $x \to -\infty$, $y \to +\infty$ and as $x \to +\infty$, $y \to +\infty$)

  > **If the leading coefficient is negative** - will start at negative infinity (quad) and stop at negative infinity (quad) (if you look at the "big picture" the overall "shape" of the graph opens ____________________)



  as $x \to -\infty$, $y \to -\infty$ and as $x \to +\infty$, $y \to -\infty$)

- A polynomial function of even degree is said to be an _____________ function if ALL the variables have even exponents.
  > If a polynomial function is even, it has the property that f(-x) = f(x) for all values of x

  > If a polynomial function is even, it will have a ______________ of symmetry in the y-axis

Graphing Polynomial Functions using the X-intercepts

- The function must be in _______________________ form to find the x-intercepts.
- Plot the x-intercepts, and the y-intercept (get the y-intercept by subbing _____________ in for x)
- Use an _________________ test to determine the sign of the polynomial in the intervals divided by the x-intercepts.
- The function $f(x) = (x-3)(x-1)(x+2)^2(x+5)^3$ is of degree 7. It has _____ x-intercepts that are _________________________. The zero from the factor$(x+2)^2$ is repeated and so is said to have an ____________ of 2. The zero from the factor $(x+5)^3$ is repeated and so is said to have order ________________.
- The function will pass through the axis at any zero with an ____________ order, and just skim the x-axis for zeros with an ____________________ order.
- For a zero of order 1, the function will pass through the x-axis looking ______________. For a zero of order 2 the function will pass through the x-axis looking _______________________. For a zero of order 3 the function will pass through the x-axis looking __________________ … and so on…

Transformations of Power Functions

$$y = a[k(x-d)]^n + c$$

We can use the mapping $(x,y) \rightarrow \left(\frac{x}{k}+d, ay+c\right)$ to transform every point on the original

power function into the new power function.

Average and Instantaneous Rates of Change

- The average rate of change of a function in an interval is given by the **slope** of the ________________ line that passes through the interval's end points

$$\text{average rate of change} = \frac{\text{change in y}}{\text{change in x}}$$

- The smaller the interval the closer the secant line is to being a ______________________ line.
- Instantaneous rates of change are given by the slope of the ________________________.

Page 74-77
All questions are fair and you should know how to do them. Be sure that you understand applications such as #8, 16, 17, & 16