MHF4U

Advanced Functions
Grade 12 University

Unit 3 Rational Functions & Equations

6 Video Lessons

Allow **no more** than **15** class days for this unit!
This includes time for review and to write the test.

Lesson #	Lesson Title	Practice Questions	Date Completed
1	Reciprocal of Linear Functions	Page 153 #1, 2, 6, 7, 9, 10, 13	
2	Reciprocal of a Quadratic Function	Page 166 #1-3, 5, 7, 8, 9, 11, 13	
3	Rational Functions (Linear over Linear)	Page 174 #1-4, 7, 8, 9, 11, 12, 13	
4	Solving Rational Equations ***NOTE - Leave more time for this lesson and the next one - the questions are harder!***	Page 183 #1, 2, 3, 8, 18	
5	Solving Rational Inequalities	Page 184 #4, 5, 12, 19	
6	Making Connections	Page 189 #1-4, 6, 8, 9, 11, 12	

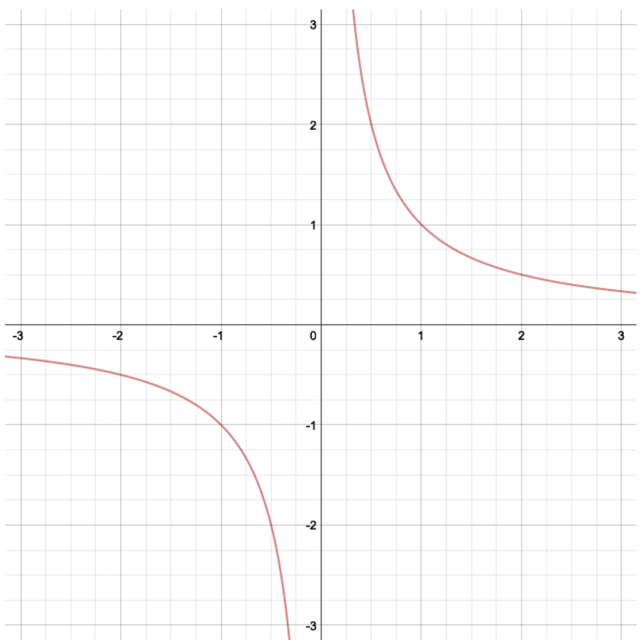
Test Written on : ____

Topic : reciprocal functions

Goal : I am able to determine key aspects of the reciprocal of a linear function, especially asymptotes.

Reciprocal of a Linear Function

If you have not already done so, take 15 minutes to complete the investigation on page 148/49 of your text.



If $f(x) = \dfrac{1}{x}$

End Behaviour

* as x → -∞, y → 0^-
* as x → +∞, y → 0+

Behaviour at Asymptote

* as x → , y →
* as x → , y →

Domain : { }

Range : { }

Now what do we know about the following function?

If $f(x) = \dfrac{1}{-3x+5}$

End Behaviour

* as x → -∞, y →
* as x → +∞, y →

If x is a really big number, what does f(x) approach?

Range : { } *Are there any y-values that aren't possible?*

Domain : { } *Are there any values of x that won't work?*

MHF4U U3L1 Reciprocal of a Linear Function

Behaviour at Asymptote
* as y → , y →
* as y → , y →

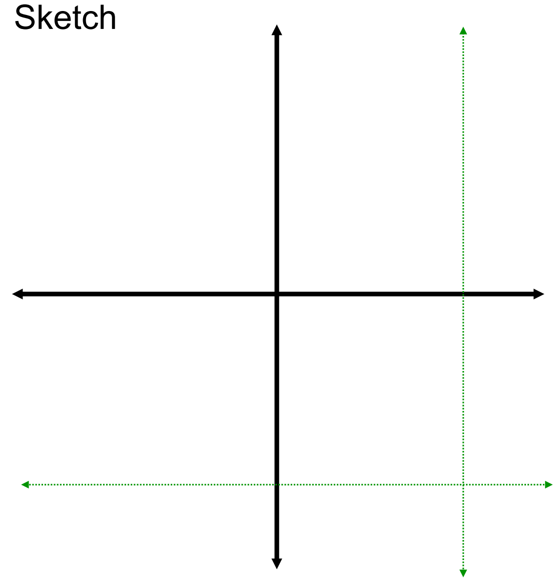
Let x get really close the asymptote from both sides.
It will go to ±∞

x	f(x)
1.5	
1.6	
1.66	
1.666	

x	f(x)
1.7	
1.67	
1.667	
1.6667	

Find the x and y intercepts...

Sketch



Practice Questions - Page 153 #1, 2, 6, 7, 9, 10, 13

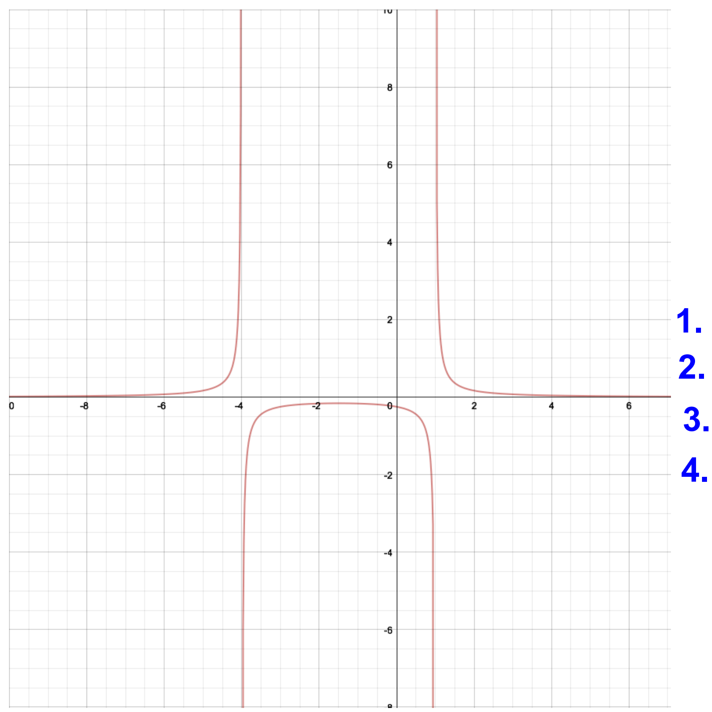
Topic : **Reciprocal Functions**

Goal : **I know the key properties of a reciprocal quadratic function.**

Reciprocal of a Quadratic Function

This is a fairly simply reciprocal quadratic function.

For most reciprocal quadratics, there are 4 key intervals of interest.



Each of these intervals split the graph into section where it is either increasing (y-values get bigger) or decreasing (y-values get smaller) through the entire interval.

Within each interval, the slope of a tangent line (or any secant line) will either be positive or negative.

The reciprocal of any quadratic function that has two zeros will look like the graph above.

The two zeros of the denominator are responsible for the asymptotes.

The behaviour near the asymptotes is similar to that of reciprocal linear functions.

The local max (or min if the function is inverted), lies directly between the two vertical asymptotes.

MHF4U U3L2 Reciprocal of a Quadratic Function

Example: For the function, $f(x) = \dfrac{1}{-x^2 - 3x + 10}$ determine

A. the asymptotes

B. the intercepts

C. domain and range

D. intervals of increase or decrease

E. sketch the function

Practice Questions - Page 166 #1-3, 5, 7, 8, 9, 11, 13

Topic : Rational Functions

Goal : I understand how to find the asymptotes and end behaviour of a function that is the quotient of two linear expressions.

Rational Functions of the Form $f(x) = \dfrac{ax + b}{cx + d}$

Let's look at $f(x) = \dfrac{2x - 3}{5x + 6}$ find....

A. the vertical asymptote (and behaviour at the asymptote)

A vertical asymptote occurs when the denominator is zero.

B. the horizontal asymptote (end behaviour)

Imagine x being a very large postive (+∞) number or very large negative (-∞) number. What happens to f(x)?

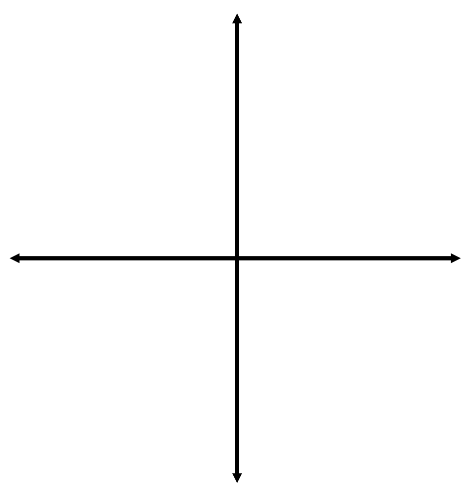
MHF4U U3L3 Rational Functions (Linear over Linear)

C. the intercepts

Let's look at $f(x) = \frac{2x - 3}{5x + 6}$ find....

D. the domain and range

E. sketch the function



Summary

For a rational function of the form $f(x) = \dfrac{ax + b}{cx + d}$

Vertical Asymptote is $-\dfrac{d}{c}$

Horizontal Asymptote is $\dfrac{a}{c}$

x - intercept is $-\dfrac{b}{a}$

y - intercept is $\dfrac{b}{d}$

Practice Questions - Page 174 #1-4, 7, 8, 9, 11, 12, 13

Topic : Solving Rational Equations

Goal : I can solve equations that involve rational expressions

Solving Rational Equations

We'll tackle these kinds of problems by working through some typical examples.

Example 1. A SIMPLE ratio

With all instances where a variable appears in the DENOMINATOR of a rational expression, you need to state the restrictions on the variable.

If you multiply BOTH expressions by the denominators, that will in fact clear the denominators and you no longer have a rational expression to solve.

a) $\frac{x + 3}{x - 1} = 2$

b) $\frac{x - 1}{x} = \frac{x + 1}{x + 3}$

Example 2. Getting a Common Denominator

If you have a sum or a difference of two rational expressions, it will be easiest to get a common denominator and simplify it into a single rational expression before you solve.

$$\frac{3}{x} + \frac{4}{x + 1} = 2$$

Example 3. You may want to factor first!

Sometimes we need to factor our expression before we try to solve it. This will allow us to state variable restrictions and may even let us simplify the equations before we start.

$$\frac{x^2 - 6x + 5}{x^2 + 2x - 3} = \frac{2 - 3x}{-3x^2 - x + 2}$$

Practice Questions - Page 183 #1, 2, 3, 9, 18

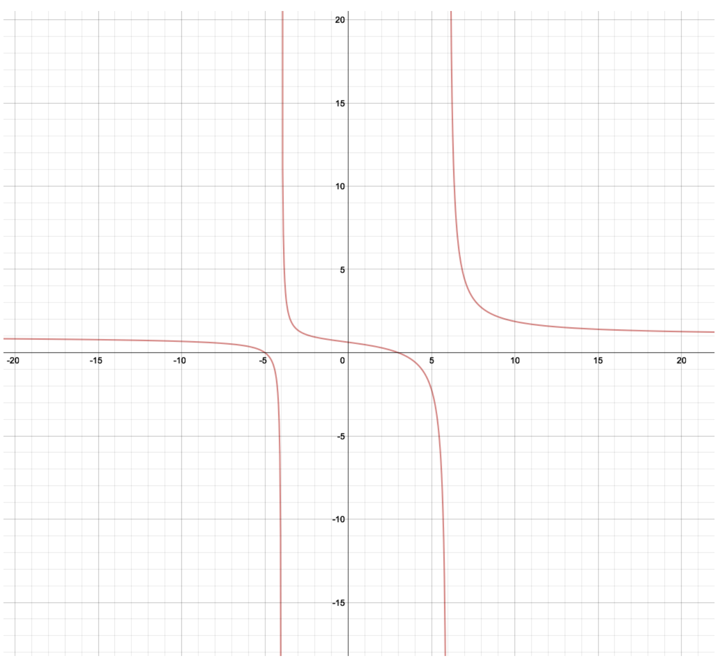
Topic : Rational Inequalities

Goal : I can solve rational inequalities and state restrictions on the variables.

Solving Rational Inequalities

You can NOT use cross multiplying to solve rational inequalities. When you multiply an inequality by a negative number you must flip the sign. Expressions that contain an unknown will be both positive and negative depending on the value of x - so it's impossible to know which way the sign will be.

To solve, we are simply going to get everything to one side and put it all together as a single rational expression (i.e. get a common denominator). Then we will do an interval check for each factor in the expression.



Notice, that the function switches from positive to negative and vice versa at x-intercepts, but also at asymptotes.

For a rational expression, our interval boundaries will be both zeros and vertical asymptotes.

Zeros - numerator is zero

Vertical Asymptotes - denominator is zero.

MHF4U U3L5 Solving Rational Inequalities

Example 1. Solve by sketching a graph.

$$\frac{2}{3x + 7} - 4 < 0$$

Example 2. Solve using an interval test.

$$\frac{(x + 3)(x - 4)}{(x - 7)(x - 2)} \leqq 0$$

Zeros :

V. Asymptotes :

Example 3. Solve by first simplifying.

$$\frac{x}{x+1} - \frac{2x}{x-2} < 0$$

Practice Questions - Page 184 #4, 5, 12, 19

Topic : making connections

Goal : I know about special cases of rational functions that have slant asymptotes, and ones that have “holes”.

Making Connections With Rational Functions and Equations

Dealing with Special Cases

State the restrictions for the following two functions and then graph on the graphing calculator...

$$f(x)=\frac{(x+3)(x-4)}{(x-2)}$$

$$g(x)=\frac{(x+3)(x-4)}{(x-4)}$$

Both functions are quadratic over linear - what accounts for the difference in the graphs. What effect does the variable restriction have on the graph?

Notice in the first graph, there is no horizontal asymptote. If the numerator is exactly one degree higher than the denominator, the graph tends to an oblique (or slant) asymptote. To find it, you must perform a division.

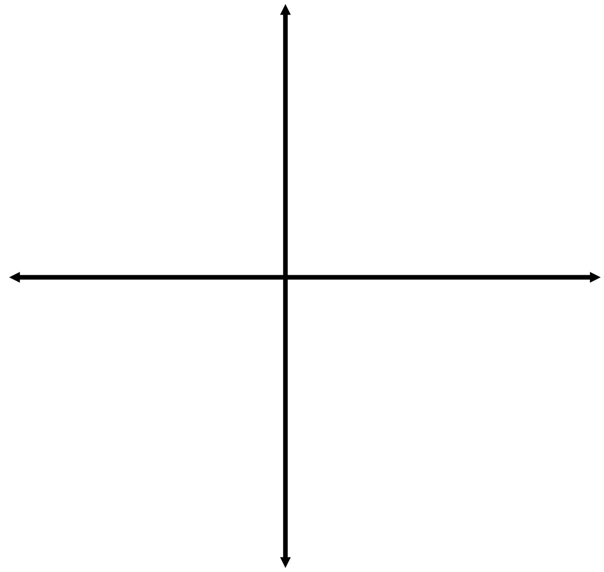
$$f(x)=\frac{(x+3)(x-4)}{(x-2)}$$

Q(x)

MHF4U U3L6 Making Connections

Example 1. Graph the following functions by examining asymptotes and intercepts.

$$f(x) = \frac{2x^2 + 13x + 15}{6x^2 + 29x - 5}$$



Example 2. A Greek mathematician, Pythagorus, is credited with the discovery of the Golden Rectangle. This is considered to be the rectangle with the dimensions that are the most visually appealing. In a Golden Rectangle, the length and width are related by the proportion

$$\frac{l}{w} = \frac{w}{l-w}$$

A billboard length of 15 m is going to be built. What must its width be if it is to form a Golden Rectangle?

Practice Questions - Page 189 #1-4, 6, 8, 9, 11, 12