MHF4U

Advanced Functions
Grade 12 University

Unit 4 Radian Measure

5 Video Lessons

Allow **no more** than **13** class days for this unit!
This includes time for review and to write the test.

NOTE : there are two additional pages with the notes on this chapter
Basic Trig Identities
The Trig Identity Checklist of Possibilities
Both pages may be used for all tests and assignments.

Complete Lessons 1-4 as quickly as possible – you will need additional days for the Trig Identity Lesson and Practice.

Lesson #	Lesson Title	Practice Questions	Date Completed
1	Radian Measure More Examples	Page 208 #(1-8)eop, 9, 10, 11, 15, 17, 19	
2	Trig Ratios and Special Angles	Page 216 #1-6, 7, 8, 11, 12, 13	
3	Equivalent Trigonometric Expressions ***NOTE - Don't waste too much time on this lesson, the next two lessons are more important.***	Pae 225 #1-10, 12, 14, 15-19	
4	Compound Angle Formulas	Page 232 #1c, 2b, 3ac, (4-7)a, 10, 11, 12, 15	
5	Prove Trig Identities Part 1 Part 2 ***NOTE - Leave more time for these lessons - the questions are harder!***	Page 240 #1-13, 16, 19, 21 **Extra Questions** Trig Identity Practice.pdf	

Test Written on : ______________________________________

MHF4U U4L1 Radian Measure

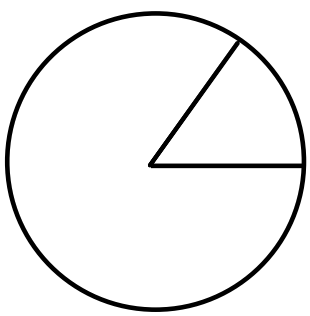
Topic : radian measure

Goal : I know the difference between degrees and radians and how to switch between them.

Radian Measure

Degrees divide one full rotation into 360 parts.
Radium measure divides up a circle into a different number of parts.

If we draw a circle, a radian is the angle at the centre, where two radii subtend and arc of the same length as the radius.



How many radians are in 360°?

Well one full circle has a circumference of 2πr, so...

$$\frac{2\pi r}{r} = 2\pi$$
$$\approx 6\ 28$$

Converting between radians and degrees is simply a ratio and proportion question. If you can remember one of these two ratios, you can set it up easily. $\frac{rad}{deg}$ $\frac{2\pi}{360}$ rad $\frac{\pi}{180}$

Example 1. Convert the following to radian measure...

a) 45°

$\frac{\pi}{180} = \frac{x}{45}$

$180x = \frac{45\pi}{180}$

$x = \frac{\pi}{4}$

b) 90°

$\frac{\pi}{180} = \frac{x}{90}$

$x = \frac{\pi}{2}$

c) 30°

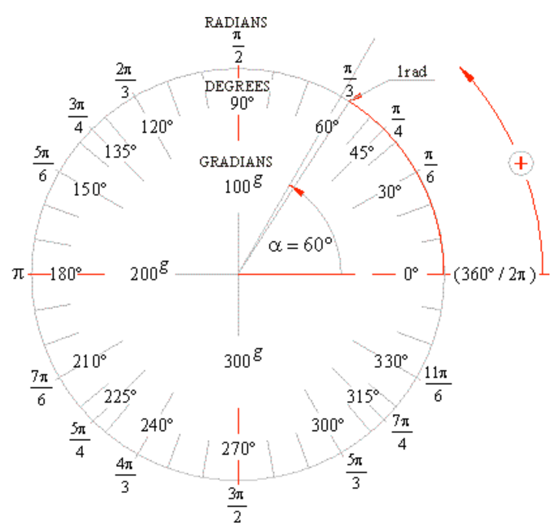
$\frac{\pi}{180} = \frac{x}{30}$

$x = \frac{\pi}{6}$

d)60°

$\frac{\pi}{180} = \frac{x}{60}$

$x = \frac{\pi}{3}$



Example 3. Angular Velocity

A figure skater spinsso that they rotate 15 times in 5 seconds. Calculate the angular velocity in a) degrees/second and b) radians per second.

angle velocity = angle / unit time

a) angle $= 15 \times 360$

$= 5400$

b) angle $= 15 \times 2\pi$

$= 30\pi$

$v = \frac{a}{t}$

$= \frac{5400}{5}$ s $\qquad = \frac{30\pi}{5s}$

$1080^{\circ}/s \qquad 6\pi$ rad/s

Practice Questions - Page 208 #(1-8)eop, 9, 10, 11, 15, 17, 19

MHF4U U4L2 Trig Ratios and Special Angles

Topic : Trig Ratios and Special Angles

Goal : I remember what the "special angles" are and can adapt them to adapt it to work with radian measure.

Trig Ratios and Special Angles

Evaluate each trig ratio for the given radian measure using your calculator. Be sure it is in radian mode.

use x^{-1} of the primary ratios

$\sin(\frac{2\pi}{3})=$ 0 866

$\cos(\frac{2\pi}{3})=$ -0 5

$\tan(\frac{2\pi}{3})=$ 1.732

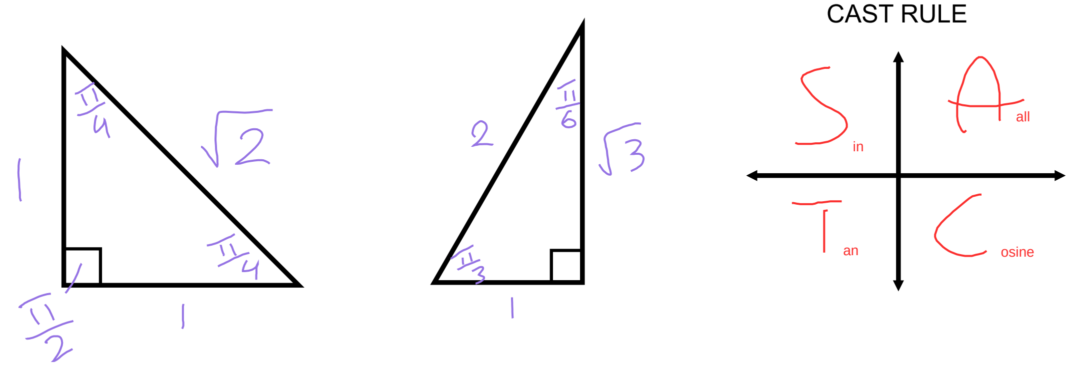
$\csc(\frac{2\pi}{3})=$ 1.155

$\sec(\frac{2\pi}{3})=$ -2

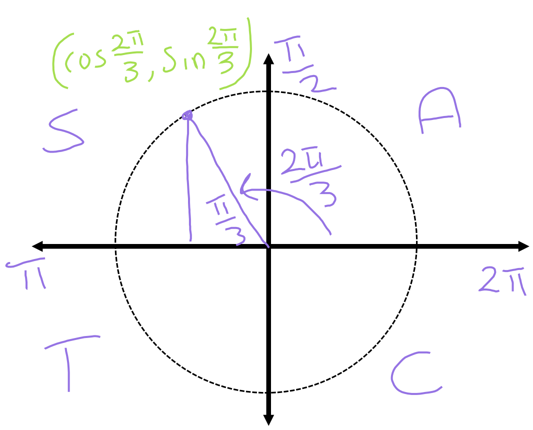
$\cot(\frac{2\pi}{3})=$ -0.577

The angle $\frac{2\pi}{3}$ is actually a multiple of a special angle. We can get an exact answer using the special triangles, the unit circle and CAST Rule.

Where are the trig ratios positive?
CAST RULE



Example 1. Find the 6 exact trig ratios for $\frac{2\pi}{3}$



$\sin(\frac{2\pi}{3})=\frac{\sqrt{3}}{2}$

$\cos(\frac{2\pi}{3})=\frac{1}{2}$

$\tan(\frac{2\pi}{3})=\sqrt{3}$

$\csc(\frac{2\pi}{3})=\frac{2}{\sqrt{3}}=\frac{\sqrt{3}}{2}$

$\sec(\frac{2\pi}{3})=2$

$\cot(\frac{2\pi}{3})=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{2}$

Example 2. Convert the following radian measure to degrees.

a) $\frac{3\pi}{4}$ $\pi = 180$

$$\frac{3(180)}{4} = 135^\circ$$

b) 7.5 radians

$$\frac{\pi}{180} = \frac{7.5}{x}$$

$$x = \frac{180(7.5)}{\pi}$$

$$\approx 429.7^\circ$$

Example 3. Arc Length

A compass rotates through an angle of $\frac{3\pi}{5}$ radians. How long is the arc drawn if the compass is set for a 12cm radius?

Let's set up a ratio of Arc Length to radian angle...

$$\frac{\text{Arc Length} \to a}{\text{Angle} \to \theta} \qquad \propto$$

$$\frac{a}{\theta} = r \quad \text{OR} \quad a = r\theta$$

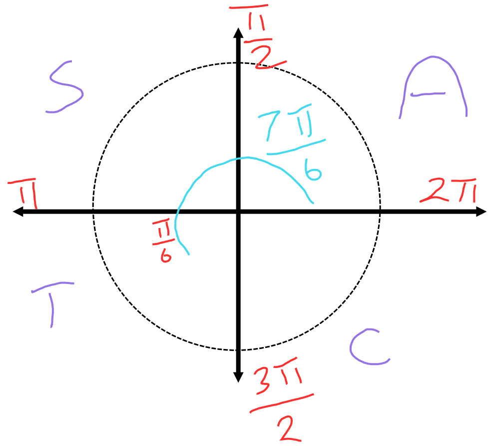
$$a = 12\left(\frac{13\pi}{5}\right)$$

$$= \frac{36\pi}{5}$$

$$\approx 22\ 6m$$

MHF4U U4L2 Trig Ratios and Special Angles

Example 2. Find the 6 exact trig ratios for $\frac{7\pi}{6}$



$\sin(\frac{7\pi}{6}) = -\frac{1}{2}$

$\cos(\frac{7\pi}{6}) = -\frac{\sqrt{3}}{2}$

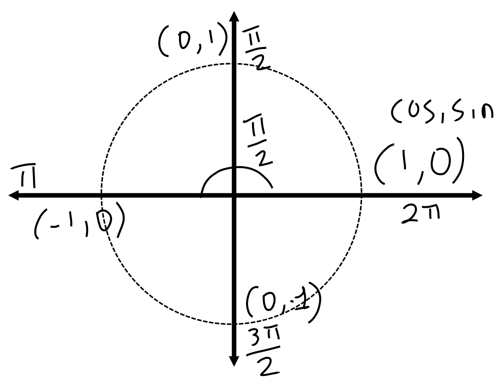
$\tan(\frac{7\pi}{6}) = \frac{1}{\sqrt{3}}$

$\csc(\frac{7\pi}{6}) = -2$

$\sec(\frac{7\pi}{6}) = -\frac{2}{\sqrt{3}}$

$\cot(\frac{7\pi}{6}) = \sqrt{3}$

For the quadrant boundaries $\frac{\pi}{2}$, π , $\frac{3\pi}{2}$, 2π and it's multiples we must think of the unit circle. Remember that the x-coordinate is cosine and the y-coordinate is sine. Also remember the Quotient Identity tan = $\frac{\sin}{\cos}$



$\cos 0 = 1$ $\quad \sin 0 = 0$

$\cos 2\pi = 1$ $\quad \sin 2\pi = 0$

$\cos \frac{\pi}{2} = 0$ $\quad \sin \frac{\pi}{2} = 1$

$\cos \pi = -1$ $\quad \sin \pi = 0$

$\cos \frac{3\pi}{2} = 0$ $\quad \sin \frac{3\pi}{2} = -1$

Example 3. Find the EXACT answer in simplest radical form.

$\sin(\frac{\pi}{6}) + \cos(\frac{\pi}{4})$

$\frac{1}{2}\cdot\frac{\sqrt{2}}{\sqrt{2}} \qquad \frac{1}{\sqrt{2}}\cdot\frac{2}{2}$

$= \frac{(\sqrt{2}+\sqrt{2})}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{2+2\sqrt{2}}{4}$

Practice Questions - Page 216 #1-6, 7, 8, 11, 12, 13

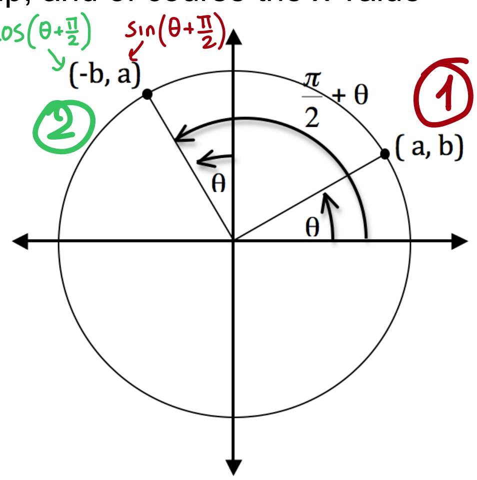
Topic : Equivalent Trig Expressions

Goal : I understand how the unit circle works and can determine the relationship between trig ratios for various angles.

Equivalent Trigonometric Expressions

When we add $\frac{\pi}{2}$ radians (90^{o}) onto an angle in the first quadrant, the coordinates at the end of the terminal arm flip, and of course the x-value will now be negative.(see diagram)

You can check the slopes of the two radii in the diagram to make sure they are in fact perpendicular.



$m_1 = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{b-0}{a-0}$

$= b/a$

$m_2 = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{a-0}{-b-0}$

$= -a/b$

Now from the diagram we know that

$\sin(\theta+\frac{\pi}{2}) = a$
$\cos\theta$

$\sin(\theta+\frac{\pi}{2}) = \cos\theta$

$\cos(\theta+\frac{\pi}{2}) = a$
$-\sin\theta$

$\cos(\theta+\frac{\pi}{2}) = -\sin\theta$

$$\tan(\theta+\frac{\pi}{2}) = \frac{\sin(\theta+\frac{\pi}{2})}{\cos(\theta+\frac{\pi}{2})}$$

$$= \frac{\cos\theta}{-\sin\theta}$$

$$= -\cot\theta$$

$\tan(\theta+\frac{\pi}{2}) = -\cot\theta$

MHF4U U4L3 Equivalent Trigonometric Expressions

We can do the same thing with the unit circle when we subtract an angle from $\frac{\pi}{2}$

$\sin\left(\frac{\pi}{2}-\theta\right) = \underset{\cos\theta}{a}$ } $\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

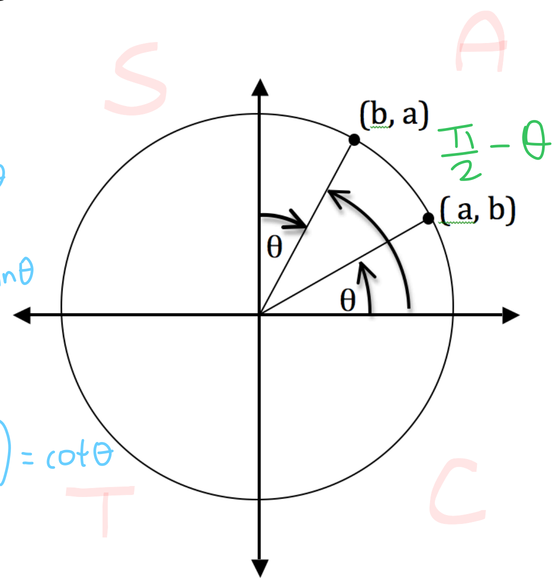
$\cos\left(\frac{\pi}{2}-\theta\right) = \underset{\sin\theta}{b}$ } $\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$

$\tan\left(\frac{\pi}{2}-\theta\right) = \dfrac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos\left(\frac{\pi}{2}-\theta\right)}$

$= \dfrac{\cos\theta}{\sin\theta}$

$= \cot\theta$

} $\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$



So to summarize...

The trig identities using $\frac{\pi}{2}$

a.k.a. cofunction identities

$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$	$\csc\left(\frac{\pi}{2}+\theta\right)=\sec\theta$	$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$	$\csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta$
$\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$	$\sec\left(\frac{\pi}{2}+\theta\right)=-\csc\theta$	$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$	$\sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$
$\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$	$\cot\left(\frac{\pi}{2}+\theta\right)=-\tan\theta$	$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$	$\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$

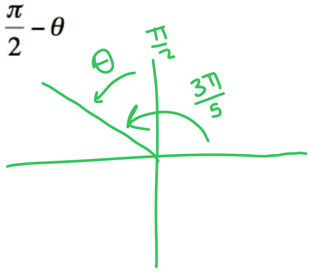
Example 1. If $\sin\frac{\pi}{10}$ = -0.30901 use this information to find $\cos\frac{3\pi}{5}$.

Solution:

We need to determine and equivalent expression for $\cos\frac{3\pi}{5}$

We first determine what quadrant it is in, then try to write it as either $\frac{\pi}{2}+\theta$

or $\frac{\pi}{2}-\theta$



$\frac{3\pi}{5} > \frac{\pi}{2}$ this is Q2 angle

$\frac{3\pi}{5} = \frac{\pi}{2}+\theta$

$\times 2\ \frac{3\pi}{5} - \frac{\pi}{2}\times 5 = \theta$

$\frac{6\pi - 5\pi}{10} = \theta$

$\frac{3\pi}{5} = \frac{\pi}{2}+\frac{\pi}{10}$

Use confun id

$\cos\frac{3\pi}{5} = \cos\left(\frac{\pi}{2}+\frac{\pi}{10}\right)$

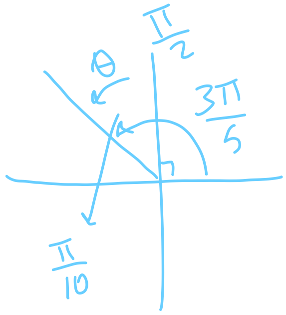
$= -\sin\frac{\pi}{10}$

$= 0.30901$

Example 2. If θ lies in the first quadrant, and cscθ = sec 1.45, determine the measure of θ using a cofunction identity.

Solution :

If the angle we want lies in the first quadrant we know...



$\therefore \frac{3\pi}{5} > \frac{\pi}{2}$ Q2 angle

$\frac{3\pi}{5} = \frac{\pi}{2} + \theta$

$\frac{6\pi - 5\pi}{10} = \theta$

$\frac{\pi}{10} = \theta$

$\frac{3\pi}{5} = \frac{\pi}{2} + \frac{\pi}{10}$

$\cos\frac{3\pi}{5} = \cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)$

$= -\sin\frac{\pi}{10}$

$\boxed{= 0.30901}$

Practice Questions - Page 225 #1-10, 12, 14, 15-19

Topic : Trig identities

Goal : I know what the compound angle formulas are, and can use them to find exact answers for trig problems.

Compound Angle Formulas

The compound angle formulas are as follows.

$$\sin(x+y)=\sin x\cos y+\cos x\sin y$$
$$\sin(x-y)=\sin x\cos y-\cos x\sin y$$
$$\cos(x+y)=\cos x\cos y-\sin x\sin y$$
$$\cos(x-y)=\cos x\cos y+\sin x\sin y$$

$\sin(x+y)$

$\cancel{\sin x+\sin y}$

Proof is on Page 228-230 of your text. We aren't going to go over the development of them at this time.

I do want you to notice some patterns that will make these easier to recognize.

1. The cosine formulas alway have products of cos and products of sine, where the sine formulas mix the two products.

2. The cosine formulas have the opposite sign in the brackets as in expanded form, where sine has the same.

Example 1. Use an appropriate compound angle formula to express the following as a single trig function.

$$\cos\frac{\pi}{5}\cos\frac{\pi}{3}-\sin\frac{\pi}{5}\sin\frac{\pi}{3}$$

Which double angle formula is involved? Notice the from (cos cos) - (sin sin)

Solution:

$\cos(x+y)=\cos\frac{\pi}{5}\cos\frac{\pi}{3}-\sin\frac{\pi}{5}\sin\frac{\pi}{3}$

$=\cos\left(\frac{\pi}{5}+\frac{\pi}{3}\right)$

$=\cos\left(\frac{8\pi}{15}\right)$

MHF4U U4L4 Compound Angle Formulas

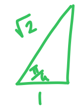
Example 2. Apply the compound angle formula for $\sin\left(\frac{5\pi}{6}-\frac{\pi}{4}\right)$ to get an exact answer.

Think which formula this is and expand it. The special angles or multiples of special angles.

Solution :

$$\sin(x+y) = \sin\frac{5\pi}{6}\cos\frac{\pi}{4} + \cos\frac{5\pi}{6}\sin\frac{\pi}{4}$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$



$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

Example 3. Use an appropriate compound angle formula to find the exact value of $\sin\frac{11\pi}{12}$.

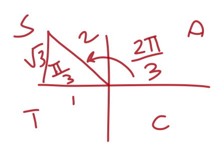
If you can express the given angles as sum or difference of two special angles we can use the formulas to expand and then evaluate

Solution :

$$\frac{11\pi}{12} = \frac{\cancel{8}^{2}\pi}{\cancel{12}_{3}} + \frac{\cancel{3}^{1}\pi}{\cancel{12}_{4}}$$

$$= \frac{2\pi}{3} + \frac{\pi}{4}$$

$$\sin\left(\frac{2\pi}{3}+\frac{\pi}{4}\right) = \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$$



$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}\,\frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

All are +

MHF4U U4L4 Compound Angle Formulas

Example 4. The angles α and θ are located in the FIRST quadrant.

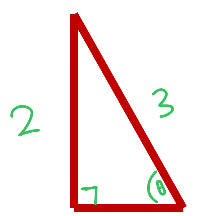
If $\sin\theta = \frac{2}{3}$ and $\sin\alpha = \frac{1}{2}$, find $\cos(\theta - \alpha)$

What is the expansion for the compound angle?

Solution : $\cos(\theta - a) = \cos\theta\cos a + \sin\theta\sin a$

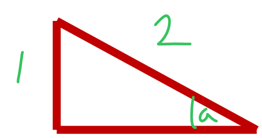
$$= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$$

We are told that these are first quarter so that we know the signs of all the trig ratios using CAST rule. Other than that we can use right angle to find all other trig ratios for the two angles



$\sqrt{3^2 - 2^2}$

$= \sqrt{5}$



$= \sqrt{2^2 - 1^2}$

$= \sqrt{3}$

$$= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{15}}{6} + \frac{2}{6}$$

$$= \frac{\sqrt{15} + 2}{6}$$

Practice Questions Page 232 #1c 2b 3ac (4-7)a, 10, 11 12 15

Topic : Proving Trig Identities

Goal : I understand the basic trig identities and I can use them to prove more obscure trig identities.

Prove Trigonometric Identities

The story so far...

Quotient Identity : $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Pythagorean Identity : $\sin^2\theta + \cos^2\theta = 1$ (from the unit circle)

Reciprocal Identities :

$$\csc\theta = \frac{1}{\sin\theta};\quad \sin\theta = \frac{1}{\csc\theta}$$
$$\sec\theta = \frac{1}{\cos\theta};\quad \cos\theta = \frac{1}{\sec\theta}$$
$$\tan\theta = \frac{1}{\cot\theta};\quad \cot\theta = \frac{1}{\tan\theta}$$

Compound Angle Formulas:

$$\sin(x+y) = \sin x\cos y + \cos x\sin y$$
$$\sin(x-y) = \sin x\cos y - \cos x\sin y$$
$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$
$$\cos(x-y) = \cos x\cos y + \sin x\sin y$$

Now let's develop a few more...

The Compound Angle Formula for Tangent

Quotient id = $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

with the compound angle formulas

$$\tan(x+y) = \frac{\sin(x+y)}{\cos(x+y)}$$

$$= \frac{\sin x\cos y + \cos x\sin y}{\cos x\cos y - \sin x\sin y}$$

The Double Angle Formulas

The double angle formulas are a special case of the compound angle formulas, except that the two angles are exactly the same.

For Sine :

$$\sin 2\theta = \sin(\theta + \theta)$$

$$= \sin\theta\cos\theta + \cos\theta\sin\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

For Cosine :

$$\cos 2\theta = \cos(\theta + \theta)$$

$$= \cos\theta\cos\theta - \sin\theta\sin\theta$$

$$\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$$

$$= (1 - \sin^2\theta) - \sin\theta$$

$$\boxed{\cos 2\theta = 1 - 2\sin^2\theta}$$

$$= 1 - 2(1 - \cos^2\theta)$$

For Tangent :

$$\tan 2\theta = \tan(\theta + \theta)$$

$$= \frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\tan^2\theta = \tan\theta\tan\theta$$

$$\neq \tan^2\theta$$

Applying the Basic Identities

When you prove trig identities, you are trying to do anything you can with the known trig identities in order to transform the two sides and make them look like each other.

You may need to work on BOTH sides of the equal sign, but start first with the side that looks the most complicated. Generally you want to take complicated and make it look more simple.

It is important the a for each step along the way, you state what identity or math operation you have used.

Example 1. Prove $\dfrac{1+\tan x}{1+\cot x} = \dfrac{1-\tan x}{\cot x - 1}$

Left Side	Right Side
$\dfrac{1+\tan x}{1+\cot x}$	$\dfrac{1+\tan x}{1/\tan x - 1}$ (recip id)
$\dfrac{1+\tan x}{1+\frac{1}{\tan x}}$ (Recip id)	$= \dfrac{1-\tan x}{\frac{1-\tan x}{\tan x}}$ (com dem)
$= \dfrac{1+\tan x}{\frac{\tan x+1}{\tan x}}$ (com dem)	$= (\cancel{1-\tan x})\left(\dfrac{\tan x}{\cancel{1-\tan x}}\right)$ (invert and multi)
$= (\cancel{1+\tan x}) \times \left(\dfrac{\tan x}{\cancel{\tan x+1}}\right)$ (invert and multi)	$= \tan x$ (cancelling)
$= \tan x$ (cancel)	

LS $\longleftrightarrow$ RS

QED!

Example 2. Prove $\csc 2x + \cot 2x = \cot x$

Left Side	Right Side
$= \csc 2x + \cot 2x$	$\cot x$
$= \dfrac{1}{\sin 2x} + \dfrac{1}{\tan 2x}$ (Recip id)	
$= \dfrac{1}{\sin 2x} + \dfrac{\sin 2x}{\cos 2x}$ (Quot id)	
$= \dfrac{1 + 2\cos^2 x - 1}{2\sin x \cos x}$ (double id)	
$= \dfrac{2\cos^2 x}{2\sin x \cos x}$ (Simplify)	
$= \dfrac{\cancel{2}\cos x \cancel{\cos x}}{\cancel{2}\sin x \cancel{\cos x}}$ (def^n of squ)	
$= \dfrac{\cos x}{\sin x}$ (cancelling)	
$= \cot x$ (Quot id)	

LS = RS

QED!

Practice Questions - Page 240 #1-13, 16, 19, 21

Trig Identity Practice

Part A – *Identities that involve Reciprocal, Quotient and Pythagorean Relationships*

1. $\sin x \tan x = \sec x - \cos x$
2. $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$
3. $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$
4. $(\cos^2 x)(\cos^2 y) + (\sin^2 x)(\sin^2 y) + (\sin^2 x)(\cos^2 y) + (\sin^2 y)(\cos^2 x) = 1$
5. $\sec^2 x - \sec^2 y = \tan^2 x - \tan^2 y$
6. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = (\tan x)(\tan y)$
7. $(\sec x - \cos x)(\csc x - \sin x) = \dfrac{\tan x}{1 + \tan^2 x}$
8. $\cos^6 x + \sin^6 x = 1 - 3\sin^2 x + 3\sin^4 x$
9. $\sec^6 x - \tan^6 x = 1 + 3(\tan^2 x)(\sec^2 x)$

Part B – *Identities that Involve Compound Angle Formulas*

10. $1 + \cot x \tan y = \dfrac{\sin(x+y)}{\sin x \cos y}$
11. $\cos(x+y)\cos y + \sin(x+y)\sin y = \cos x$
12. $\sin x - \tan y \cos x = \dfrac{\sin(x-y)}{\cos y}$
13. $\cos\left(\dfrac{3\pi}{4} + x\right) + \sin\left(\dfrac{3\pi}{4} - x\right) = 0$
14. $\dfrac{\tan\left(\dfrac{\pi}{4} + x\right) - \tan\left(\dfrac{\pi}{4} - x\right)}{\tan\left(\dfrac{\pi}{4} + x\right) + \tan\left(\dfrac{\pi}{4} - x\right)} = 2\sin x \cos x$
15. $\sin(x+y)\sin(x-y) = \cos^2 y - \cos^2 x$
16. $\tan(x+y)\tan(x-y) = \dfrac{\sin^2 x - \sin^2 y}{\cos^2 x - \sin^2 y}$
17. $\dfrac{\tan(x-y) + \tan y}{1 - \tan(x-y)\tan y} = \tan x$
18. $\sin 5x = \sin x(\cos^2 2x - \sin^2 2x) + 2(\cos x)(\cos 2x)(\sin 2x)$

Part C – *Identities Involving Related and Co-Related Angles*

19. $\sin\left(\dfrac{\pi}{2} - x\right)\cot\left(\dfrac{\pi}{2} + x\right) = -\sin x$
20. $\cos(-x) + \cos(\pi - x) = \cos(\pi + x) + \cos x$
21. $\dfrac{\sin(\pi - x)\cot\left(\dfrac{\pi}{2} - x\right)\cos(2\pi - x)}{\tan(\pi + x)\tan\left(\dfrac{\pi}{2} + x\right)\sin(-x)} = \sin x$
22. $\dfrac{\sin(-x)}{\sin(\pi + x)} - \dfrac{\tan\left(\dfrac{\pi}{2} + x\right)}{\cot x} + \dfrac{\cos x}{\sin\left(\dfrac{\pi}{2} + x\right)} = 3$
23. $\dfrac{\csc(\pi - x)}{\sec(\pi + x)}\dfrac{\cos(-x)}{\cos\left(\dfrac{\pi}{2} + x\right)} = \cot^2 x$
24. $\dfrac{\cos\left(\dfrac{\pi}{2} + x\right)\sec(-x)\tan(\pi - x)}{\sec(2\pi + x)\sin(\pi + x)\cot\left(\dfrac{\pi}{2} - x\right)} = -1$

BASIC TRIGONOMETRIC IDENTITIES

You will be using these basic trig identities to prove more obscure trig identities.

RECIPROCAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$

QUOTIENT IDENTITIES

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

PYTHAGOREAN IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta = 1 + \tan^2\theta$$
$$\csc^2\theta = 1 + \cot^2\theta$$

RELATED ANGLES IDENTITIES

$$\sin(\pi-\theta) = \sin\theta \qquad \sin(\pi+\theta) = -\sin\theta \qquad \sin(2\pi-\theta) = -\sin\theta \qquad \sin(-\theta) = -\sin\theta$$
$$\cos(\pi-\theta) = -\cos\theta \qquad \cos(\pi+\theta) = -\cos\theta \qquad \cos(2\pi-\theta) = \cos\theta \qquad \cos(-\theta) = \cos\theta$$
$$\tan(\pi-\theta) = -\tan\theta \qquad \tan(\pi+\theta) = \tan\theta \qquad \tan(2\pi-\theta) = -\tan\theta \qquad \tan(-\theta) = -\tan\theta$$

CO-RELATED ANGLE IDENTITIES

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta \qquad \sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta \qquad \sin\left(\frac{3\pi}{2}-\theta\right) = -\cos\theta \qquad \sin\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta$$
$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta \qquad \cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta \qquad \cos\left(\frac{3\pi}{2}-\theta\right) = -\sin\theta \qquad \cos\left(\frac{3\pi}{2}+\theta\right) = \sin\theta$$
$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta \qquad \tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta \qquad \tan\left(\frac{3\pi}{2}-\theta\right) = \cot\theta \qquad \tan\left(\frac{3\pi}{2}+\theta\right) = -\cot\theta$$

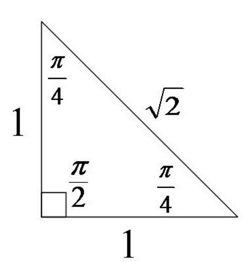
COMPOUND ANGLE FORMULAS

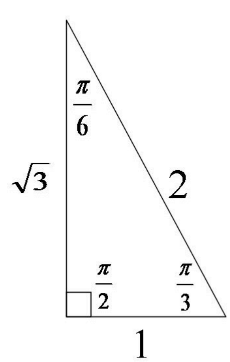
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

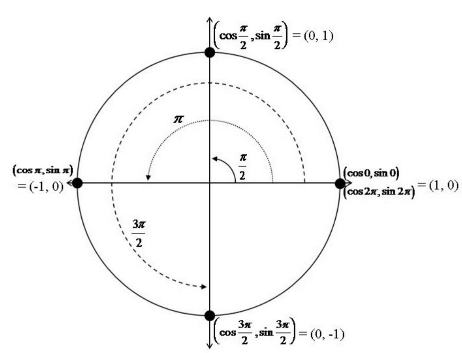
DOUBLE ANGLE FORMULAS

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

SPECIAL TRIANGLES AND THE UNIT CIRCLE







The Trig Identity Checklist of Possibilities

Start with the side of the identity that looks the most complicated and try to make it simpler.
Keep in mind that you can work on BOTH sides of the trig ratio

Making things simpler involves

- changing secondary ratios to primary
- changing TAN into SIN and COS
- changing double angles into single angles
- changing addition of angles into single angles

As you look at the following, always keep an eye on the other side. Make sure that if you have a choice in what identity to use, that it will match up with the other side.

Here are some questions that should be running through your head

- [ ] Can I use the reciprocal identities to turn secondary ratios into primary ratios
- [ ] Can I use a double angle formula to change a double angle into single angles
- [ ] Can I use a co-function identity to turn a “sum/difference” into a single angle
- [ ] Can I use a compound angle formula to turn a “sum/difference” into a single angle
- [ ] Can I use the quotient identity to turn Tan into Sin and Cos
- [ ] Can I get a common denominator
- [ ] Can I break apart a fraction into the sum/difference of two fractions (with the same denominator)
- [ ] Can I take out a common factor
- [ ] Can I factor as a difference of squares
- [ ] Can I factor it like a quadratic
- [ ] Can I group the terms to factor them
- [ ] Can I cancel any factors from the numerator and denominator
- [ ] Can I put any terms together (simplify)
- [ ] Can I put but a $\sin^2 x$ next to a $\cos^2 x$ and replace them with a “1” (Pythagorean identity)
- [ ] Can I expand and simplify
- [ ] Can I replace $\sin^2 x$ with a $(1-\cos^2 x)$ or vice versa (Pythagorean Identity)
- [ ] CAN I DO ANYTHING ABOVE ON THE OTHER SIDE OF THE IDENTITY

Please note the following :

From the quotient identity $\tan x = \dfrac{\sin x}{\cos x}$

this also means that $\tan^2 x = \dfrac{\sin^2 x}{\cos^2 x}$ or $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$ or $\tan(x+y) = \dfrac{\sin(x+y)}{\cos(x+y)}$

this applies to other identities as well.

To try and clear up some common misconceptions…

$\sin(x+y) \neq \sin x + \sin y$ (*see the compound angle identities*)

You can NOT make this cancellation

$$\frac{\cancel{\sin x} + \cos y}{\cancel{\sin x}} \neq \cos y \text{ or } 1+\cos y$$

(you may only cancel ***FACTORS*** of products. The numerator is a sum, not a product!)