

Name : Solutions

**MHF4U1**  
**Unit 3: Rational Functions**

K/U /25	APP /16	COM	TH /14
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**KNOWLEDGE/UNDERSTANDING**

**Multiple Choice**

*Identify the choice that best completes the statement or answers the question.*

D

1. Which of the following is the reciprocal of a linear function?

a.  $f(x) = \frac{3}{x^2 + 1}$

c.  $f(x) = \frac{x}{x + 3}$

b.  $f(x) = \frac{1}{x^2 + 1}$

d.  $f(x) = \frac{1}{x + 3}$

D

2. Which of the following has a horizontal asymptote at  $y = 0$ ?

a.  $f(x) = \frac{1}{3 - x}$

c.  $f(x) = -\frac{1}{x + 2}$

b.  $f(x) = \frac{1}{17x + 4}$

d. all of the above

B

3. What is true about the function  $f(x) = \frac{1}{3x + 5}$  as  $x \rightarrow -\frac{5}{3}^+$ ?

a.  $f(x) \rightarrow 0$

c.  $f(x) \rightarrow -\infty$

b.  $f(x) \rightarrow \infty$

d.  $f(x)$  is undefined

C 4. What is true about the function  $f(x) = \frac{2x+5}{x+3}$  as  $x \rightarrow \infty$ ?

a.  $f(x) \rightarrow \frac{5}{3}$  from above

c.  $f(x) \rightarrow 2$  from above

b.  $f(x) \rightarrow \frac{5}{3}$  from below

d.  $f(x) \rightarrow 2$  from below

D 5. What is the x-intercept of  $f(x) = \frac{1}{3x-4}$ ?

a.  $-\frac{1}{4}$

c.  $\frac{1}{4}$

b.  $\frac{4}{3}$

d. There is no x-intercept.

D 6. What is the y-intercept of the function  $f(x) = -\frac{3}{x-3} + 1$ ?

a. 2

c. 1

b. -3

d. 0

A 7. What is the equation of the horizontal asymptote of  $f(x) = -\frac{1}{2x+10}$ ?

a.  $y = 0$

c.  $x = 0$

b.  $y = 5$

d.  $x = 5$

A 8. What is the value of  $k$  in the function  $f(x) = \frac{3-k}{2x+k}$  if its graph passes through the point  $(5, -0.35)$ ?

a. 10

c.  $\frac{13}{4}$

b.  $-\frac{47}{6}$

d. No such  $k$  exists

D

9. Which function has positive y-values on the entire domain?

a.  $f(x) = \frac{3}{2x+4}$

c.  $f(x) = \frac{1}{x^2+4}$

b.  $f(x) = \frac{1}{(x-4)^2}$

d. B and C

C

10. Which function has a y-intercept of  $\frac{1}{2}$ ?

a.  $f(x) = \frac{2}{(2x-1)(x+1)}$

c.  $f(x) = -\frac{4}{x^2-7x-8}$

b.  $f(x) = \frac{2}{2x^2+5x-3}$

d. all of the above

B

11. Solve the equation  $\frac{1}{x-4} = \frac{5}{x}$ .

a.  $x = -1$

c.  $x = -5$

b.  $x = 5$

d. no solution

A

12. Solve the equation  $\frac{3}{x-2} = \frac{7}{4x-8}$ .

a.  $x = 2$

c.  $x = -2$

b.  $x = \frac{4}{5}$

d. no solution

$$7x \cdot 14 = 12x - 24$$

$$\frac{10}{7} = 5x$$

$$x = 2$$

B

13. What are the x-intercepts of the graph of  $f(x) = \frac{x^2 + 4x - 21}{x^2 - 8x + 15}$ ?

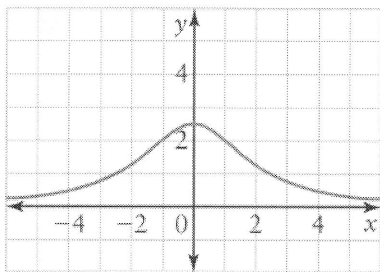
a. -4, 5

c. 4, -5

b.  $-7, 3$

d.  $7, -3$

14. Use the graph of  $f(x) = \frac{10}{x^2 + 4}$  to solve the equation  $2 = \frac{10}{x^2 + 4}$



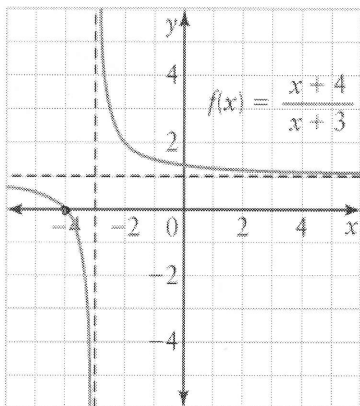
a.  $x = -1.5, +1.5$

c.  $x = 0$

b.  $x = -1, +1$

d. no solution

15. Use the graph of  $f(x) = \frac{x+4}{x+3}$  to solve the inequality  $\frac{x+4}{x+3} \leq 0$ .



a.  $x = -4$

c.  $-4 \leq x < -3$

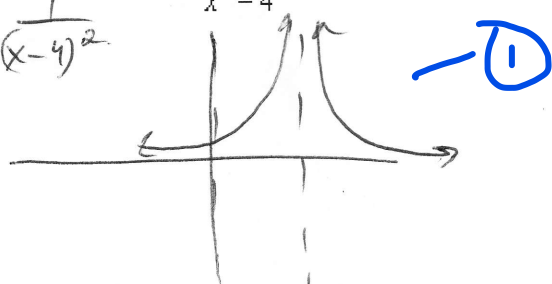
b.  $-4 < x < -3$

d. no solution

2. In point-form, list as many similarities/differences between the graphs of  $f(x) = \frac{1}{(x-4)^2}$  and

[3K]

$f(x) = \frac{1}{(x-4)^2}$

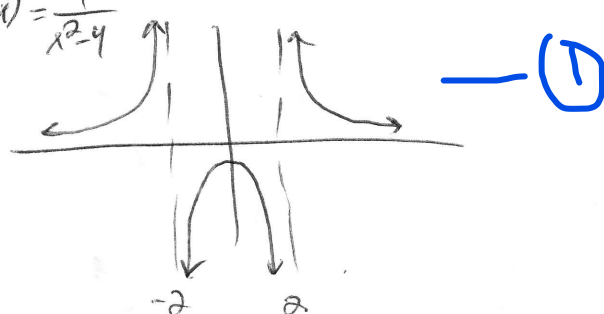


positive for all  $x$   
One VA.

①

Both have  
HA at  $y=0$ .

$f(x) = \frac{1}{x^2-4}$



two VAs

3. Graph the rational function  $f(x) = -\frac{x+3}{x^2-9}$

State the Domain & Range

[3K]

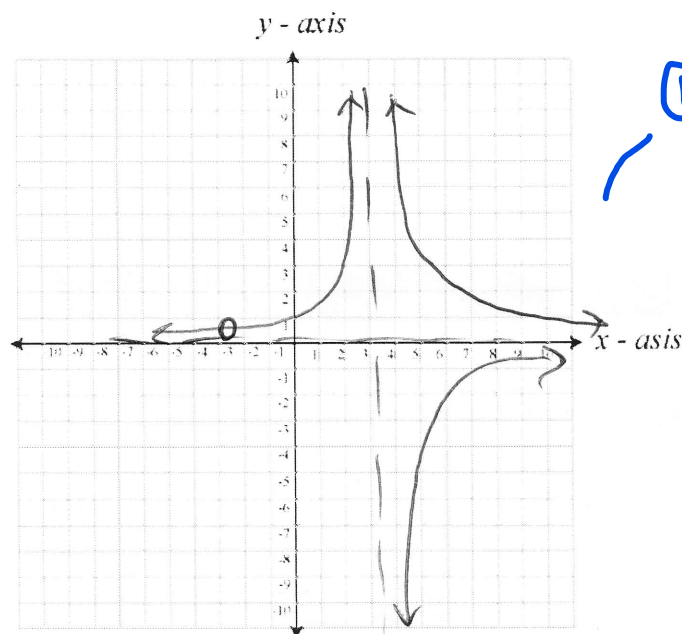
$$f(x) = \frac{-(x+3)}{(x-3)(x+3)}$$

$$= \frac{-1}{x-3}$$

VA @  $x=3$

HA @  $y=0$

Hole @  $x=-3$



Domain:  $\{x \in \mathbb{R} \mid x \neq 3\}$   
Range:  $\{y \in \mathbb{R} \mid y \neq 0\}$

4. Solve the following equality  $\frac{1}{x^2} = 4 = f(x)$ . Show both a geometric and algebraic representation of your solution. State the domain & range. [4T]

Algebraic:

$$\frac{1}{x^2} = 4$$

$$\frac{4x^2}{4} = \frac{1}{4}$$

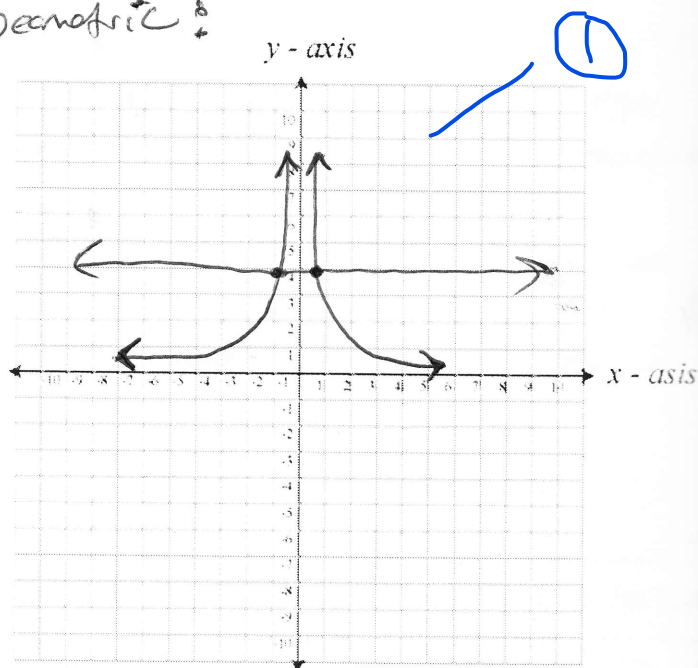
$$\sqrt{x^2} = \pm \sqrt{\frac{1}{4}}$$

$$x = \pm \frac{1}{2} \quad \text{--- (2)}$$

Domain:  $\{x \in \mathbb{R} \mid x \neq 0\}$   
Range:  $\{y \in \mathbb{R} \mid y > 0\}$

--- (1)

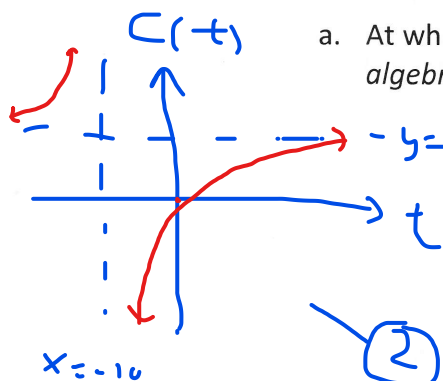
Geometric:



### APPLICATION

1. Salt water flows into a large tank of pure water. The concentration of the salt in the tank at  $t$  minutes is given by the function  $c(t) = \frac{6t}{10+t}$ , where  $c$  is measured in grams/liter. Graph the function and properly label the axes.

- a. At what time does the concentration in the tank reach 5 grams/liter? Provide both an algebraic and geometrical representation of your solution.



$$c(t) = \frac{6t}{t+10} = 5$$

$$5t + 50 = 6t$$

$$50 = t$$

reaches 5 grams/liter

--- (2)

- b. At what time interval(s) is the concentration of salt less than 3 grams/litre? Provide both an algebraic and geometrical representation of your solution. [8A]

$$C(t) = \frac{6t}{t+10}$$

$$\frac{6t}{t+10} < 3$$

$$\frac{6t}{t+10} - 3 < 0$$

$$\frac{6t - 3t - 30}{t+10} < 0$$

$$\frac{3t - 30}{t+10} < 0$$

one:  $t = 10$   $t = -10$

2. The intensity of illumination is modeled by the following function  $I(d) = \frac{k}{d^2}$ , where  $I$  is intensity, in lux;  $d$  is the distance, in metres, from the light source; and  $k$  is the constant. When the distance from a certain light source is 50m, the intensity is 6 lux.

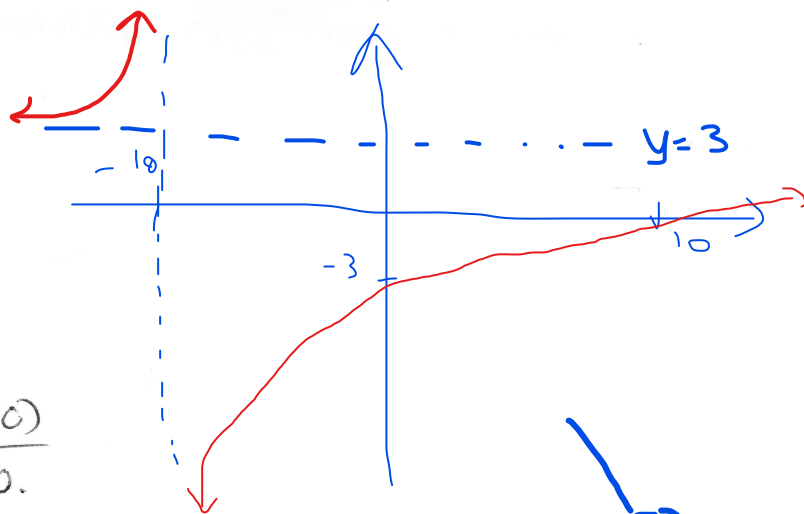
- (a) Sketch a graph of this relation and properly label the axes.  
 (b) Describe what happens to the light intensity as the distance away from the light source becomes greater  
 (c) Comment on the level of intensity for values of  $d$  close to 0

(b) As distance becomes greater, the intensity drops and approaches 0 lux. (2)

(c) As  $d \rightarrow 0$  the intensity approaches  $\infty$  lux. (2)

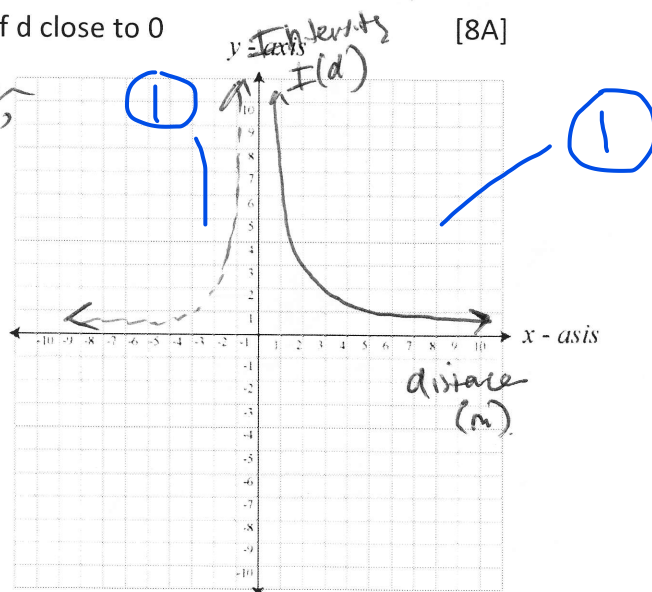
a)  $6 = \frac{k}{50^2}$ ,  $k = 15000$  (1)

$$\Rightarrow I(d) = \frac{15000}{d^2}$$
 (1)



(2)

Solution of  $-10 < t < 10$  m/h



[8A]



# THINKING

1. A For what values of  $k$  does the graph of  $f(x) = \frac{x^2 + 16x + k}{x^2 + 7x}$  have no  $x$ -intercepts? [1T]

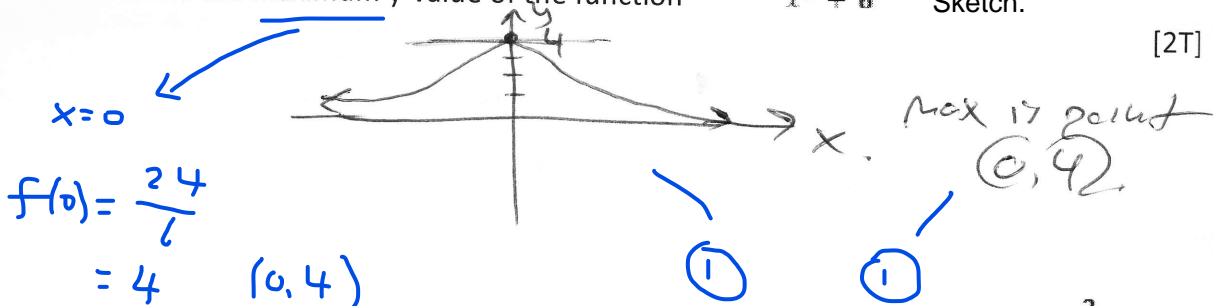
a.  $k > 64$

c.  $k > 4$

b.  $k < 64$

d.  $k < 4$

2. Indicate the maximum  $y$ -value of the function  $f(x) = \frac{24}{x^2 + 6}$  Sketch. [2T]



3. Determine the equation of a rational function with a vertical asymptote  $x = \frac{2}{3}$ , a hole at  $x = 4$  and a horizontal asymptote at  $y = 1$  [4T]

Handwritten solution for question 3:

$$f(x) = \frac{3x(x-4)}{(3x-2)(x-4)} = \frac{3x^2 - 12x}{3x^2 - 14x + 8}$$

4. Solve the following inequality (Hint: use "holla at yo boy" test #2). Show both a geometric and algebraic representation of your solution. [7K]

(a)  $\frac{x^2 - x - 6}{x - 2} \geq 3$

B: Geometric

A: Algebraic

Algebraic steps:

$$\frac{x^2 - x - 6}{x - 2} - 3 \geq 0$$

$$\frac{x^2 - x - 6 - 3x + 6}{x - 2} \geq 0$$

$$\frac{x^2 - 4x}{x - 2} \geq 0$$

$$\frac{x(x-4)}{x-2} \geq 0$$

