MCR3U Exam Prep Package – Basics – Unit 1

This package includes **basic** questions. If you can't do these types of questions:
- You should not attempt the questions in the next booklet.
- Your highest priority should be to learn this content. You may be in danger of failing.

Covers:
- Function notation (using $f(x)$)
- Domain and Range
- Sketching Parabolas
- Factoring
- Solving Quadratic Equations
- Linear-Quadratic Systems (Intersection of Line and Parabola)
- Families of Quadratic Functions (Finding a quadratic function given x-intercepts and a point)
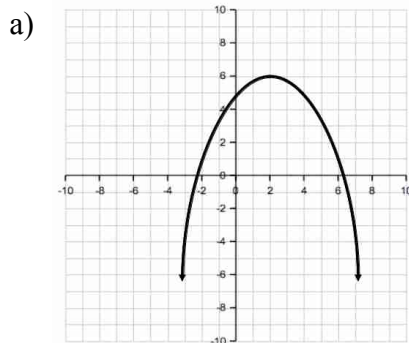
Function Notation

1. Given that $f(x)=2x^2-6x+7$ and $h(x)=mx^2+2$, evaluate:
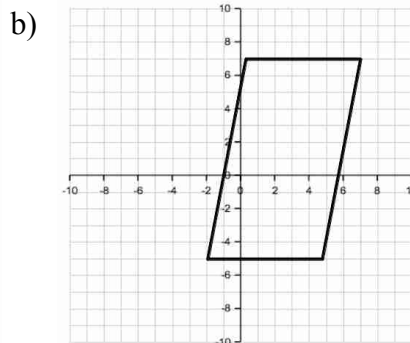
a) $f(-2)$ $= 2(-2) - 6(-2) + 7 = 27$

b) $f(\frac{1}{2})$ $= 2(\frac{1}{2}) - 6(\frac{1}{2}) + 7 =$
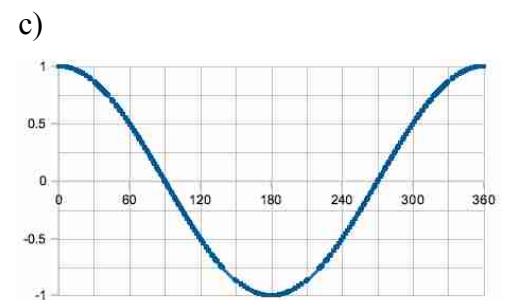
c) $h(0)$ $m(0)^2 + 2 = 2$

2. Are each of the following relationships "**functions**"?

a)

yes

b)

NO

c)
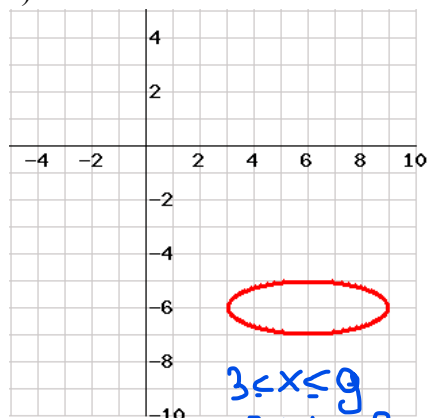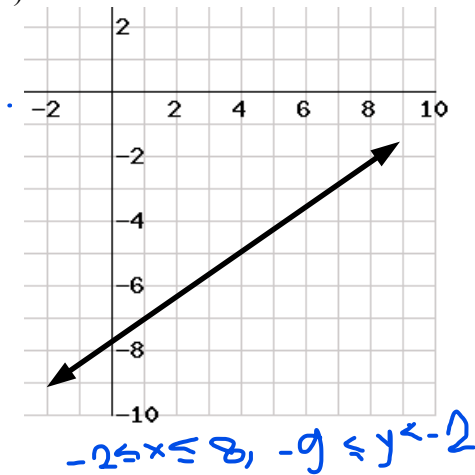

yes

Domain and Range

3.   Find the domain and range of each of the following functions:

a)

$3 \le x \le 9$
$-7 \le y \le -3$

b)

$-2 \le x \le 8, \ -9 \le y < -2$

c)

$x \in \mathbb{R}, \ y \ge 4$
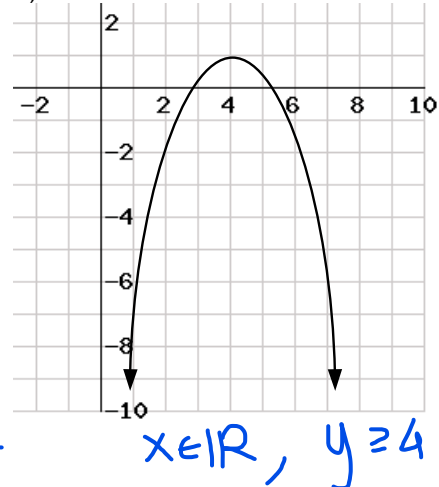
4.   What is the domain and range for each of the following functions?

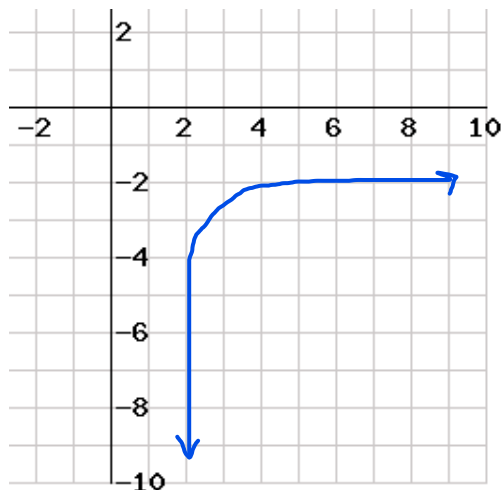a) $g(x)=4(x-7)^2+9$

$x \in \mathbb{R}, \ y \ge 9$

b) $y=-\dfrac{1}{4}(x+9)^2$

$x \in \mathbb{R}, \ y < 0$

5.   Draw a curve with the following domain and range. It does not have to be a "**function**".

$D:\{2 \le x \le 9, x \in \mathbb{R}\}$

$R:\{-9 \le y \le -2, y \in \mathbb{R}\}$

Sketching Parabolas

$$\frac{1}{2} x \left(x - \frac{1}{12}\right) + 13$$

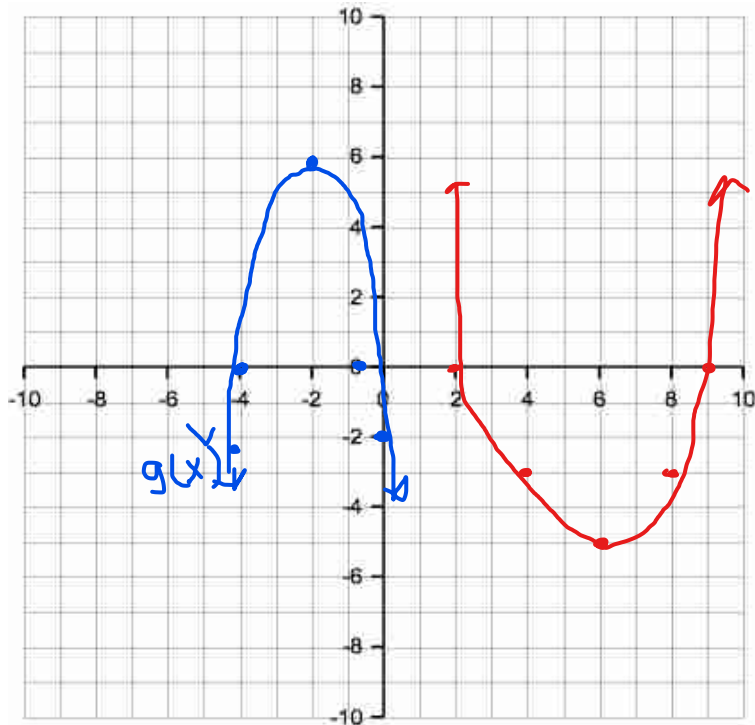6. On the grid provided, sketch the parabolas:

a) $g(x) = -2(x+2)^2 + 6$ b) $q(x) = \frac{1}{2}x^2 - 6x + 13$

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7. Sketch the parabola $h(x) = -2(x-4)(x+2)$

$h(-1) = -2(-1-4)(-1+2)$

$-2x - 5 \quad x + L$

$= 10$
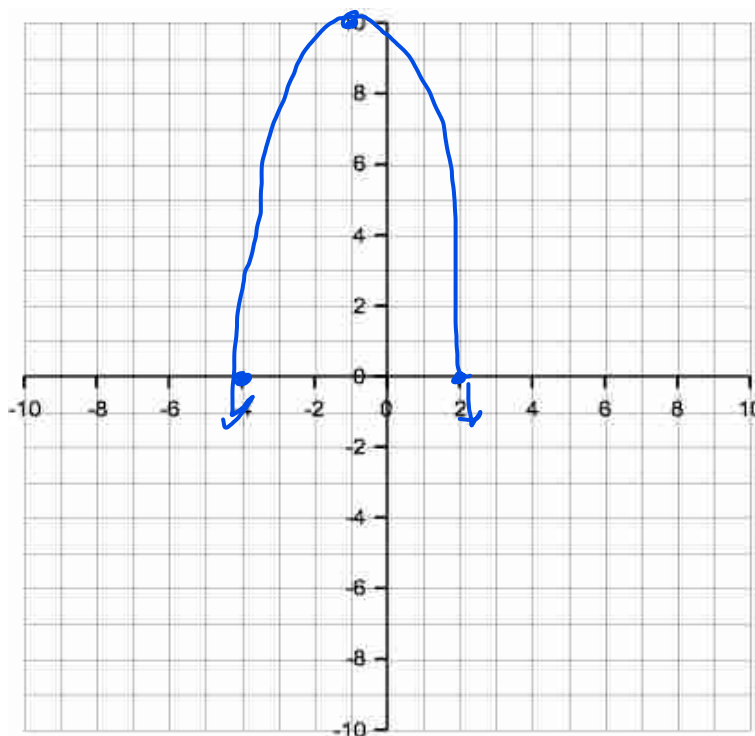
Factoring

8.   Factor each of the following equations:

a) $y = 2x^2 - 4x - 30$

$$2(x^2 - 2x - 15)$$

$$2(x - 5)(x + 3)$$

b) $y = 8x^2 - 10x - 3$

$$8x^2 - 12x + 2x - 3$$

$$4x(2x - 3) -$$

$$(2x - 3)(4x + 1)$$

-24

2   -12

-10

c) $y = -x^2 + 12x - 36$

d) $y = -3x^2 + x + 10$

e) $y = 5x^3 - 80x$

$$5x(x^2 - 16)$$

$$5x(x - 4)(x + 4)$$

f) $y = 36x^2 - 1$

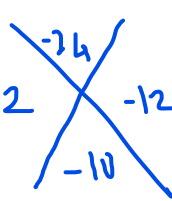**Solving Quadratic Equations**

9.   Solve for all values of x that satisfy the following equations:

a) $x^2+6x+9=0$ b) $x^2=-9(x+2)$

Linear-Quadratic Systems

10.   Find the point(s) of intersection between the line $w(x)=10x-60$ and the parabola $q(x)=2(x-6)(x+4)$.

Families of Quadratic Functions

1.   Find the equation of a parabola with x-intercepts of 7 and -9, if the parabola goes through the point $(-5,16)$.

$$f(x) = a(x-r)(x-s) \qquad x = \frac{-b}{2a}$$

$$16 = a(-5-7)(-5+9)$$

$$-12 \times 4$$

$$16 = \frac{-48a}{-48} = -\frac{1}{3} \qquad f(x) = -\frac{1}{3}(x-7)(x+9)$$

$$V(\,,\,)$$

2.   Find the equation of a parabola with a single x-intercept at 11, if the parabola goes through the point $(16,-50)$

Covers:

- Finding/Drawing the inverse of a function
- Sketching square root functions
- Sketching reciprocal functions
- Simplifying rational expressions

**Finding the Inverse of a Function**

1. Determine the equation of the inverse ($f^{-1}(x)$) of the function $f(x)=\frac{9}{5}(x+32)$

2. Determine the equation of the inverse, $g^{-1}(x)$ of the function $g(x)=-2(x+2)^2+6$

3. Draw the curve for the inverse ($q^{-1}(x)$) for the function $q(x)$ drawn below.
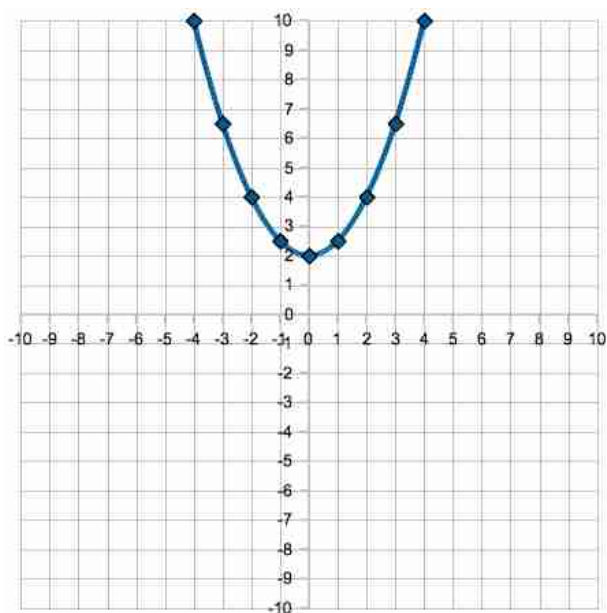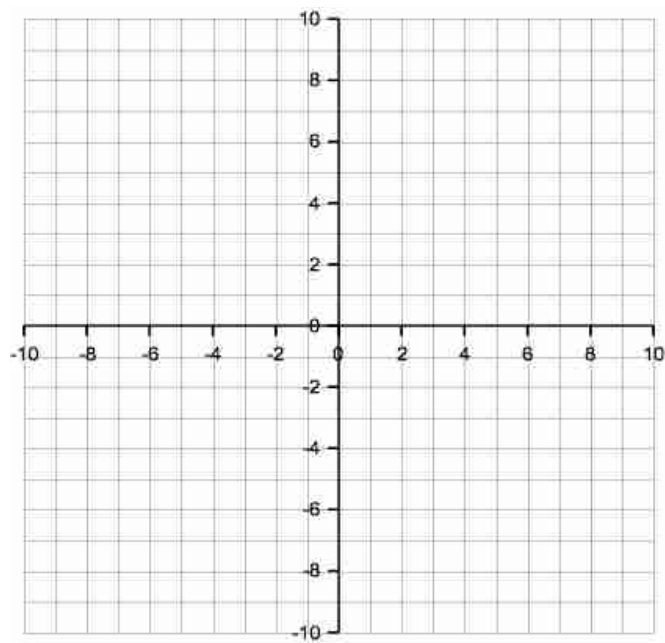
Graphing Square Root Functions

2.   Graph each of the following functions. Mark all points in the domain and range of the grids provided.

$$f(x)=-\sqrt{\frac{1}{2}(x+5)}$$
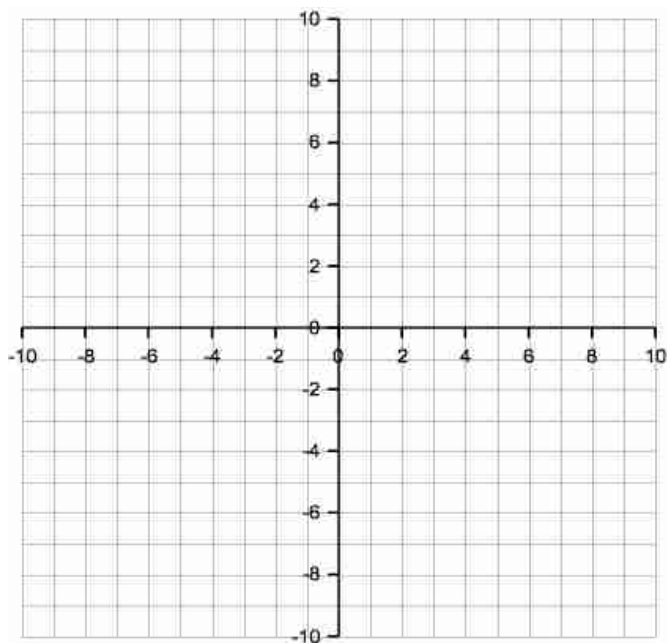
$$f(x)=3\sqrt{x+6}-6$$

Graphing Reciprocal Functions

3.   Graph each of the following functions accurately on the grids provided.

$$f(x)=\frac{-3}{x+1}$$
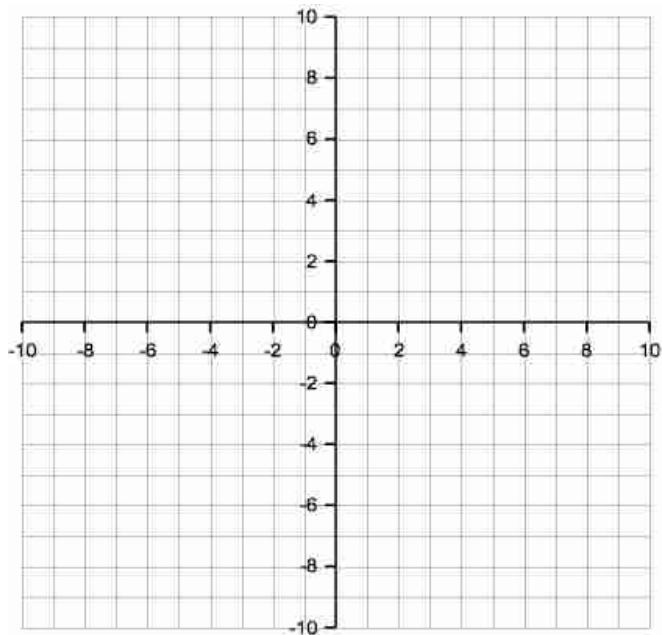
$$f(x)=\frac{1}{x-2}+2$$
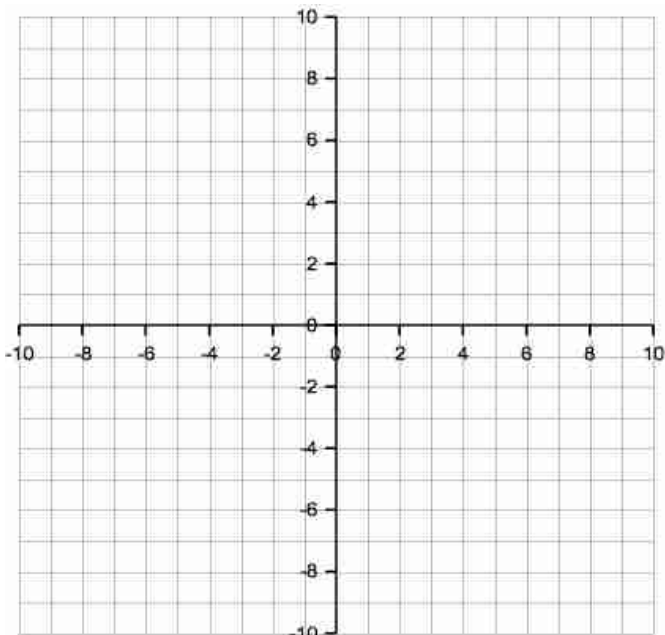
Simplifying Rational Expressions

4.   Simplify each of the following functions and state restrictions.

a) $f(x)=\dfrac{8(x-3)(x+4)(x-5)}{4(x-5)(x-4)(x-3)}$

b) $f(x)=\dfrac{2x-6}{x^2-6x+9}$

c) $f(x)=\dfrac{x+3}{x^2-4x-21}\div\dfrac{x}{x-7}$

d) $f(x)=\dfrac{x}{x+2}-\dfrac{2}{x}$

Covers:

- Simple Exponent Rules
- Rational Exponents
- Half-life questions
- Appreciation/Depreciation questions
- Sketching exponential functions

1. Simplify each as much as possible, and give your answer with only positive exponents.

a) $(r^4 r^{-2})^{-\frac{1}{2}} =$

b) $\left(\frac{2x}{3}\right)^{-2} =$

2. Evaluate the following, and give your answer as an **integer or fraction** (do not have exponents in your answer!)

a) $\left(\frac{1000}{64}\right)^{-\frac{2}{3}} =$

b) $\left(\frac{27}{-64}\right)^{-\frac{4}{3}} =$

2. A condo is bought for \$310,000. Assuming that the condo increases in value by 8% each year, how much will it be worth in four years?

4.   The half-life of a particular radioactive isotope is 1 day. $A = A_0 \left(\dfrac{1}{2}\right)^{\frac{t}{h}}$
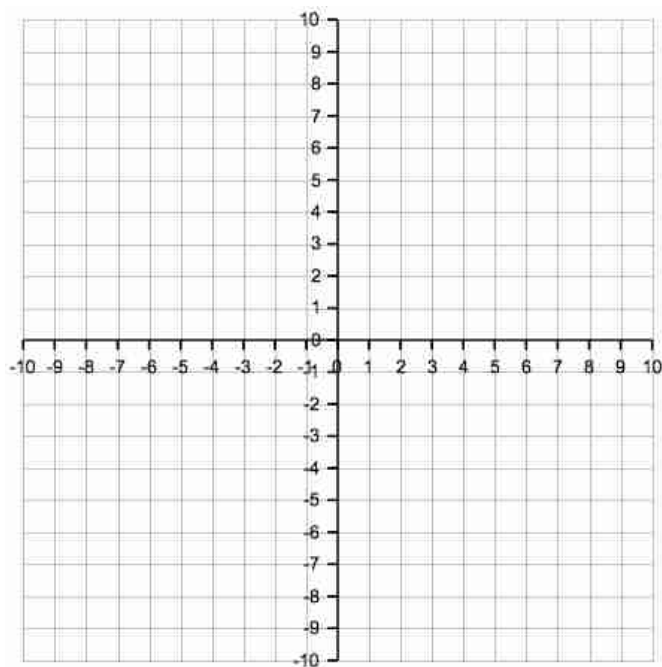
a)   Write an equation to relate the mass of radioactive material remaining to time (in hours).
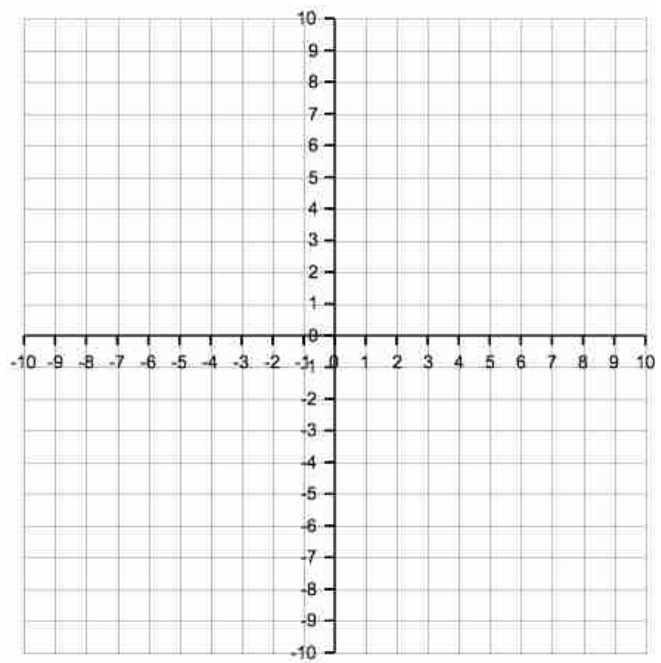
b)   Starting with 170 g, what mass will remain in 1 week?

5.   Graph the following functions on the grids provided:

a) $y = (2)^{x-5} - 7$ b) $h(x) = (3)^{\frac{1}{3}(x-2)} - 5$

Covers:

- Exact trigonometric ratios for special angles
- Sine law and cosine law
- Exact trigonometric ratios for angles given a point on the terminal arm
- Ambiguous case of sine law
- Trigonometric identities

Exact Trigonometric Ratios for Special Angles

1.   Please give the **exact** trigonometric ratios that correspond to the following angles. Include a sketch of the angle!

a) $\tan 210° \ =$

b) $\cos 315° \ =$

c) $\sin 60° \ =$

d) $\tan 120° \ =$

e) $\cot 135° \ =$

d) $\sec 300° \ =$

Sine Law and Cosine Law

2.   Please solve the following triangles FULLY. That is, give values for each of the following angles and sides. Please show ALL of your work, and **don't forget UNITS**:

a)
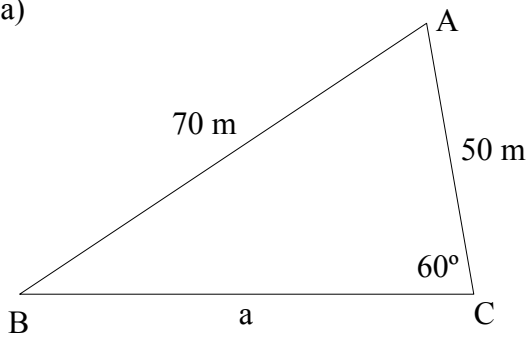
Angle A: [   ]

Angle B: [   ]

Side Length a: [   ]

b) $\triangle ABC$ where a = 15 ft, b = 7 ft and c = 10 ft.
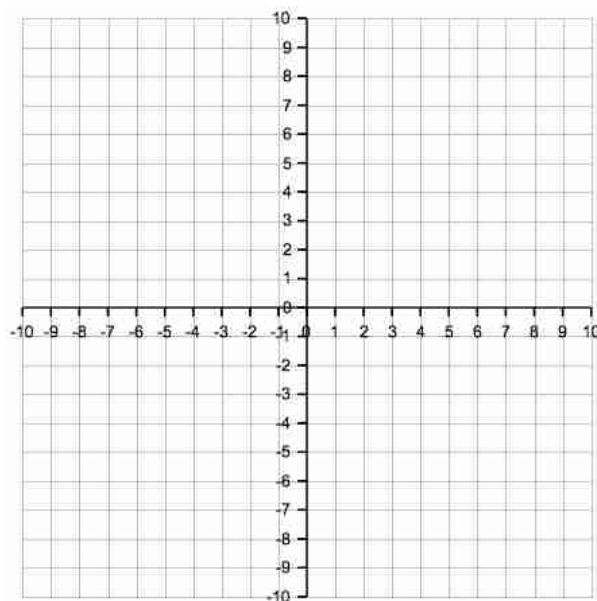
Angle A: [   ]

Angle B: [   ]

Angle C: [   ]

Exact Trigonometric Ratios for Angles given a Point on its Terminal Arm

3.   What are the exact trigonometric ratios for the angle (θ) represented by a terminal arm that goes through (-9 , -6)? [Please draw the terminal arm on the grid provided, and label the side lengths. Show any other work in the space below.]

sin θ = ☐   cos θ = ☐   csc θ = ☐   sec θ = ☐

tan θ = ☐   cot θ = ☐

5.   Solve **all** triangles that satisfy the given information. Be sure to show **how many** triangles can be formed.

a)   b = 52 m, a = 26 m, and angle A = 30°

b)   g = 74 m, f = 80 m, and angle G = 70°

c) $a = 35$ m, $b = 50$ m, and angle $A = 40°$

Trigonometric Identities

6.   Prove each of the following identities.

a) $\csc x \sec x \cos x \sin x = \sin^2 x + \cos^2 x$

b) $\sin^2 x + \cos^2 x = \sec^2 x - \tan^2 x$
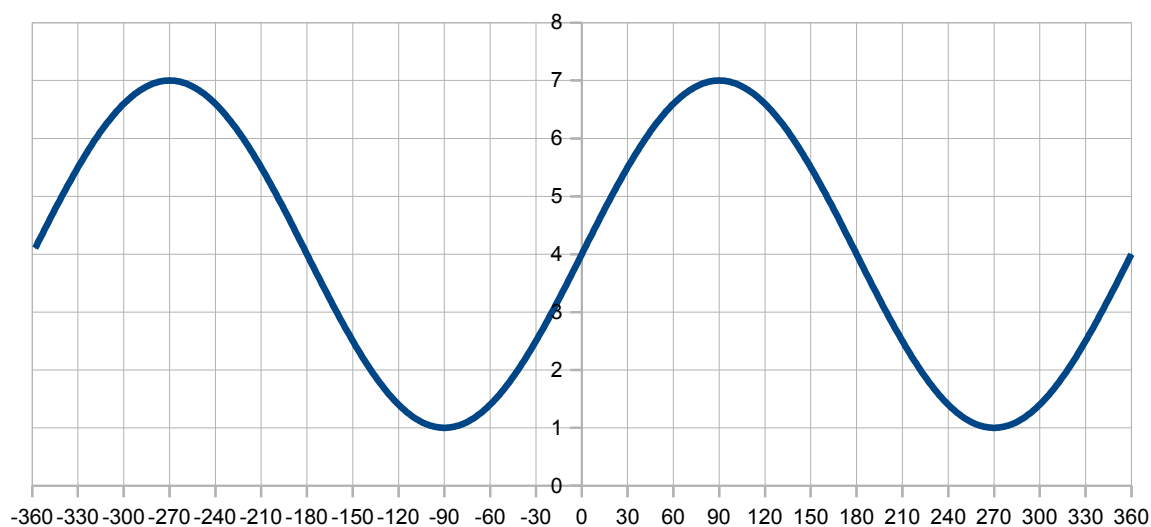
c) $\csc^2 x - \cot^2 x = 1$

Covers:

- Basics of amplitude, phase shift, vertical shift, period, k
- The relationship between period and k
- Sketching sinusoidal functions

1. For the function $y = 8\sin\left[\frac{1}{4}(x + 34°)\right] + 6$, what is the ...

a) Phase Shift: b) Amplitude:

c) Vertical Shift: d) k:

e) Period: f) Range:

g) Domain: h) Minimum Value:

2. For the function pictured, what is the ...
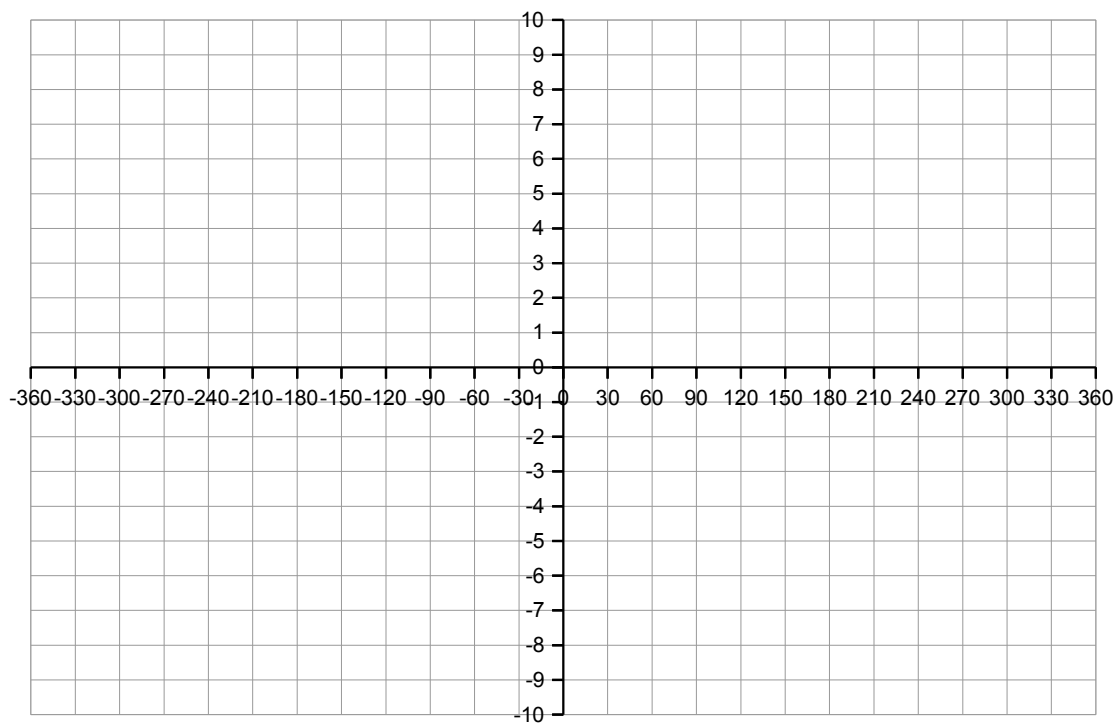

a) Phase Shift: b) Amplitude:

c) Vertical Shift: d) k:

e) Period: f) Range:

g) Domain: h) Minimum Value:

3.   Using the information you found collected in the previous question, determine the equation for the function.

4.   A sinusoidal function has an amplitude of 800, a period of 180°, and a maximum at (0, 100). Write a sinusoidal equation to represent this wave (can be sine or cosine, although you'll find one is easier than the other...)

5.   What is the period of the function $E=500\sin(0.02x)$?

6.   Draw the curve $y=4\cos\left[\frac{1}{3}(x-30°)\right]-6$.

Covers:

- Determining if a series is arithmetic or geometric
- Finding an arbitrary term in a sequence
- Finding the value of a series of an arbitrary length

1.   Please identify each of the following sequences as arithmetic, geometric or neither.

| Sequence	Arithmetic, Geometric, or Neither?
{ 3, -6, 12, -24, ... }	
{ 1, 1, 2, 3, 5, 8, ... }	
{ -15, -9, -3, 3, ... }	
{ 400, 200, 100, 50, ... }	

2.   What is the 12th term of the sequence{ 5, 10, 20, 40, ... }.

3.   Determine the 45th term of the sequence { -90, -83, -76, -69, ... }

4.   Calculate the sum of -64 – 57 – 50 – 43 – ... + 90 + 97

5.   Calculate the sum of $729 + 243 + 81 + \ldots + \dfrac{1}{9} + \dfrac{1}{27}$ to four decimal places.

6.   You have $4,000 and invest it in a savings account that pays 4%/year, simple interest. How much money will be in the account after 10 years?

7.   How much would $4,000 grow to in 10 years, if an investment pays 4%/year, compounded monthly?

8.   How much would you have if you had deposited $400 every month, for 10 years, and the investment earned 6%/year, compounded monthly?

Reference Material

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$
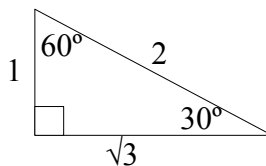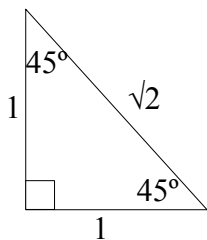
$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\csc q = \frac{1}{\sin q} \qquad \sec q = \frac{1}{\cos q} \qquad \cot q = \frac{1}{\tan q}$$

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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$$t_n = a + (n-1)d \qquad t_n = ar^{n-1} \qquad A = P(1+i)^n \qquad A = \frac{R[(1+i)^n - 1]}{i}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_n = \frac{n}{2}[a + t_n] \qquad I = Prt$$