Linear Algebra Course Notes

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1 Introduction

1.1 Definitions and Notations

In linear algebra, we deal with scalars, vectors, and matrices. Scalars are single numbers, vectors are ordered lists of numbers, and matrices are rectangular arrays of numbers.

1.2 Scalars, Vectors, and Matrices

A scalar is a single number, usually denoted by a lowercase letter (e.g., a, b, c). A vector is an ordered list of numbers, denoted by a lowercase bold letter (e.g., $\mathbf{v}$). A matrix is a rectangular array of numbers, denoted by an uppercase letter (e.g., A, B, C).

2 Systems of Linear Equations

2.1 Row Reduction and Echelon Forms

To solve systems of linear equations, we use row reduction to transform the augmented matrix into row echelon form.

$$\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 2 & 6 \end{array}$$

2.2 Solutions to Linear Systems

A system of linear equations can have no solution, exactly one solution, or infinitely many solutions. This can be determined by the row echelon form of the augmented matrix.

3 Matrix Algebra

3.1 Matrix Operations

Matrices can be added, subtracted, and multiplied, and they can also be multiplied by scalars.

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$$

3.2 Inverses and Transposes

The transpose of a matrix A is denoted A^T. A matrix A is invertible if there exists a matrix B such that $AB = BA = I$.

4 Determinants

4.1 Properties of Determinants

The determinant is a scalar value that can be computed from a square matrix.

$$\det(A) = a_{11}a_{22} - a_{12}a_{21}$$

4.2 Cofactor Expansion

The determinant can be computed using cofactor expansion.

5 Vector Spaces

5.1 Definitions and Examples

A vector space is a collection of vectors that can be added together and multiplied by scalars.

5.2 Subspaces, Basis, and Dimension

A subspace is a subset of a vector space that is also a vector space. A basis is a set of vectors that span the vector space, and the dimension is the number of vectors in the basis.

6 Eigenvalues and Eigenvectors

6.1 Characteristic Equation

The eigenvalues of a matrix A are found by solving the characteristic equation $\det(A - \lambda I) = 0$.

6.2 Diagonalization

A matrix is diagonalizable if it has enough eigenvectors to form a basis.

7 Orthogonality

7.1 Inner Product, Norm, and Orthogonality

The inner product of two vectors is a scalar. The norm of a vector is its length, and two vectors are orthogonal if their inner product is zero.

7.2 Gram-Schmidt Process

The Gram-Schmidt process is a method for orthogonalizing a set of vectors in an inner product space.

8 Applications

8.1 Linear Transformations

A linear transformation is a mapping between vector spaces that preserves addition and scalar multiplication.

8.2 Applications in Differential Equations

Linear algebra techniques are used to solve systems of differential equations.