Clue to Resolving Forward Guidance Puzzle *

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Abstract

This paper aims to find a clue to resolving the forward guidance puzzle which central banks in the developed countries have struggled with recently. The heterogeneous agent New Keynesian (HANK) model is one of the powerful candidates to give a solution to the forward guidance puzzle. I estimate the HANK model along with the standard Representative New Keynesian (RANK) model by Bayesian approach and calculate the impulse response function. The result suggests the HANK mutes the effect of the forward guidance and it could be a hint for resolving forward guidance puzzle.

keywords: Forward guidance puzzle, Heterogeneous agent New Keynesian, Bayesian method

JEL classification: E52, E58, C11, C32

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1 Introduction

Forward guidance is an innovative policy tool that is defined as a central bank's announcement about the future path of monetary policy to the public. A dynamic macroeconomics theory strongly justifies the use of this tool which could be effective especially under the zero lower bound and the liquidity trap. However, Del Negro et al. (2015) points out that evidence on the quantitative effects of this policy tool on the macroeconomy is still limited. This is known as the *"forward guidance puzzle"*. In Del Negro et al. (2015), their medium-scale DSGE model (the FRBNY DSGE) implies an unrealistically large response to the forward guidance compared to the empirical results. Similarly, Gertler (2017) provides insight into the power of the forward guidance using the example of Japan's large quantitative easing policy. He explains the disconnect between the Japanese experience and the theory and suggests relaxing the part of the rational expectations assumptions to mute the power of the forward guidance. He adds that the heart of the forward guidance puzzle in standard New Keynesian model (representative agent New Keynesian model, or RANK) is a substantial role for intertemporal substitution, which is derived from the consumption Euler equation.

This paper aims to seek a clue to filling the gap between the model prediction and empirical observations about the effect of the forward guidance. As Gertler (2017) points out, the key modification for resolving the forward guidance puzzle is how a model relaxes the assumptions of rational expectations. One of the powerful candidates to modify the assumption is a heterogeneous agent New Keynesian (HANK) model. HANK model allows households to be heterogeneous by incorporating idiosyncratic income risk and incomplete market.

As the application of HANK for monetary policy analysis, Kaplan et al. (2018) studies the transmission mechanism from monetary policy to household consumption with the HANK model, which yields a realistic distribution of wealth and marginal propensities to consume due to idiosyncratic income shocks and borrowing constraint. This assumption could attenuate intertemporal substitution that RANK relies on the main driver of the transmission from interest rates to consumption. Instead, the HANK model reveals that the indirect effect of monetary policy shocks is substantial.

McKay et al. (2016) contributes to the development of the HANK model as the means to understand and solve the forward guidance puzzle. Like Kaplan et al. (2018), they argue that people are likely to face some risk of hitting a borrowing constraint or idiosyncratic income risk. These risks are supposed to shorten households' planning horizon and increase precautionary savings, which both could reduce households' ability to smooth consumption. In the paper, they illustrate the power of the forward guidance in the RANK model and show that the effect is cumulatively large as the guidance gets further out in the future. Then, they estimate the impulse responses to the output and inflation from the monetary policy shock caused by the forward guidance in both the RANK and HANK model with calibrated parameters. The result shows the power of the forward guidance that the HANK model predicts is weakened compared to the prediction from the RANK model, which indicates that the HANK model can mute the effect.

Farhi and Werning (2019) compares the HANK model involving bounded rationality (level-k thinking), which means they combine incomplete markets, idiosyncratic risk, borrowing constraint, and bounded rationality, to the RANK model. They estimate the model with calibrated parameters and find that the interaction of these frictions leads to the mitigation of the power of monetary policy.

From the point of the central bank's view, Acharya and Dogra (2018) provides analytical tractability for a good understanding of the difference between the HANK and RANK model. They emphasize the importance of the cyclicality of income risk to resolve the forward guidance puzzle while other HANK literature tends to stress the role of marginal propensity to consume heterogeneity and precautionary savings.

The existing literature that has been seen so far shows the HANK model is a good candidate to resolve the forward guidance puzzle. However, they have mainly focused on the theoretical aspect and just simulate the model to measure the magnitude of using calibrated parameters. Therefore, this paper attempts to empirically estimate both HANK and RANK models from the macro data and measure the magnitude of the forward guidance by simulating impulse response functions with empirically estimated structure parameters. This approach can provide the empirical evaluation of the validity of the HANK model for resolving forward guidance puzzle as well as hands-on analytical tools to see the fitness of the HANK model to empirical observations.

This paper is structured as follows. Section 2 provides setups for the RANK and the HANK model. Section 3 describes Bayesian approach with Markov Chain Monte Carlo algorithm as estimation methods. Section 4 shows the data and the prior distributions of parameters I use. In section 5, I present estimated parameters and impulse response functions. Section 6 concludes the paper.

2 Model

In this section, I set up a RANK model following An and Schorfheide (2007) and a one-asset HANK model in continuous time following Ahn et al. (2017) and Kaplan et al. (2018), which are used to estimate structural parameters with Bayesian method and to calculate impulse responses from aggregate monetary policy shock.

2.1 Representative Agent New Keynesian model

The RANK model supposes that economy consists of a representative household, a final good producing firm, a continuum of intermediate goods producing firms, a government (fiscal authority) and a central bank.

2.1.1 Household

The representative household considers utility from consumption goods C_t with a habit shock which is given by the level of technology A_t, real money balance M_t/P_t, and disutility from hours worked H_t. Then the household maximizes

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i}/A_{t+i})^{1-\tau} - 1}{1-\tau} + \chi_M \left(\frac{M_{t+i}}{P_{t+i}} \right) - \chi_H H_{t+i} \right] \tag{1}$$

where β is the discount factor, $1/\tau$ is the intertemporal elasticity of substitution and χ_M and χ_H are scale factors that decide steady state real money balances and hours worked, respectively (assume $\chi_H = 1$). The household provides labor services to the firms getting the real wage W_t and has access to a bond market where government bonds are traded with paying nominal interest R_t. Also, the household receives residual real profits D_t from firms and pays lump-sum taxes T_t. Hence, the household's budget constraint is

$$P_tC_t + B_t + M_t - M_{t-1} + T_t = P_tW_tH_t + R_{t-1}B_{t-1} + P_tD_t \tag{2}$$

2.1.2 Firm

The representative final good producing firm under perfect competitive environment uses a continuum of intermediate goods $Y_t(j),\ j \in [0,1]$ expressed as

$$Y_t = \left[\int_0^1 Y_t(j)^{1-\nu} dj\right]^{\frac{1}{1-\nu}}, \quad 1/\nu > 1 \tag{3}$$

where $1/\nu$ is the elasticity of demand for each intermediate good. The firm takes input prices $P_t(j)$ and output prices P_t as given and maximizes its profit. Then, the demand for intermediate goods are

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-1/\nu} Y_t \tag{4}$$

Also, using (3) and (4), the final goods price in terms of intermediate good is defined as

$$P_t = \left[\int_0^1 P_t(j)^{\nu-1}\nu dj\right]^{\frac{\nu}{\nu-1}} \tag{5}$$

Intermediate goods firms produce intermediate goods under monopolistically competitive market and their production function is

$$Y_t(j) = A_t N_t(j) \tag{6}$$

where A_t is an exogenous productivity process and $N_t(j)$ is the labor input of firm j. Labor is employed at the real wage W_t in a perfectly competitive market. As a result, firm j faces marginal cost MC_t can be calculated as the minimization problem of cost. That is,

$$\begin{aligned} &W_t N_t(j) + MC_t(Y_t(j) - A_t N_t(j)) \\ \Leftrightarrow &MC_t = \frac{W_t}{A_t} \end{aligned}$$

Firm j also faces nominal rigidities and chooses its labor input $N_t(j)$ and the price $P_t(j)$ to maximize the present value of future profits

$$E_t\left[\sum_{i=0}^{\infty} Q_{t+i}\left(\frac{P_{t+i}(j)}{P_{t+i}}Y_{t+i}(j) - W_{t+i}N_{t+i}(j) - \frac{\phi}{2}\left(\frac{P_{t+i}(j)}{P_{t+i-1}(j)-\pi}\right)^2 Y_{t+s}(j)\right)\right] \tag{7}$$

where ϕ is the price stickiness and π is the steady state inflation rate associated with the final good. Q_{t+i} is the stochastic discount factor (i.e., the marginal value of a unit of the consumption good to the household. This is exogenous variable for the firm).

2.1.3 Central bank and Government

Suppose monetary policy is decided by following the Taylor rule.

$$R_t = \left[\frac{\pi^*}{\beta}\left(\frac{\pi_t}{\pi^*}\right)^{\psi_1} \frac{Y_t}{Y_t^*}^{\psi_2}\right]^{1-\rho_R} R_{t-1}^{\rho_R} e^{\epsilon_{R,t}} \tag{8}$$

Here, $\epsilon_{R,t}$ is monetary policy shock, π_t is the gross inflation rate ($\pi_t = P_t/P_{t-1}$), π^* is the target inflation rate, and Y_t^* is the level of output without nominal rigidities.

The fiscal authority spends a fraction ζ_t of aggregate output Y_t, where $\zeta_t \in [0,1]$ which follows an exogenous process ($G_t = \zeta_t Y_t$). The government imposes a lump-sum tax to finance any deficit in government revenue. The government's budget constraint is

$$P_tG_t + R_{t-1}B_{t-1} = T_t + B_t + M_t - M_{t-1} \tag{9}$$

2.1.4 Exogenous Processes

Suppose three exogenous processes disturb the model economy. Aggregated productivity evolves according to random walk. That is,

$$\ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t \tag{10}$$

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t} \tag{11}$$

Government expenditures ($g_t = 1/(1-\zeta_t)$) fluctuate as the following

$$\ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-1} + \epsilon_{g,t} \tag{12}$$

Finally, the monetary policy shock $\epsilon_{R,t}$ is assumed to be serially uncorrelated. Three shocks $\epsilon_{z,t}, \epsilon_{g,t}, \epsilon_{R,t}$ are independent of each other and are normally distributed with mean zero and standard deviations σ_z,σ_g and σ_R, respectively.

2.1.5 Equilibrium Relationship

Here we assume that the symmetric equilibrium in which all intermediate good producers make the same choice so that the j subscript can be removed. The market clearing conditions are

$$Y_t = C_t + G_t + \frac{\phi}{2}\left(\frac{P_{t+i}(j)}{P_{t+i-1}(j) - \pi}\right)^2 Y_t \tag{13}$$

From the household's optimal solution based on (1) and (2), we have

$$\frac{1}{R_t} = \beta E_t \frac{1}{\pi_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\tau}\left(\frac{A_t}{A_{t+1}}\right)^{1-\tau} \tag{14}$$

Similarly, intermediate firms maximizes equation (7) subject to the demand for their goods as shown in equation (4)

$$1 = \beta E_t \left[\left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{A_t}{A_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \tag{15}$$

$$\begin{aligned} 1 =& \frac{1}{\nu} \left[1 - \left(\frac{C_t}{A_t} \right)^{\tau} \right] + \phi(\pi_t - \pi) \left[\left(1 - \frac{1}{2\nu} \right) \pi_t + \frac{\pi}{2\nu} \right] \\ & - \phi\beta E_t \left[\left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{Y_{t+1}/A_{t+1}}{Y_t/A_t} (\pi_{t+1} - \pi)\pi_{t+1} \right] \end{aligned} \tag{16}$$

This equation is known as the New Keynesian Phillips Curve. In the absence of nominal rigidities ($\phi = 0$) aggregate output is

$$Y_t^* = (1 - \nu)^{1/\tau} A_t g_t \tag{17}$$

As a result, we can describe the model which determines endogenous variables $(Y_t, C_t, R_t, \pi_t, Y_t^*)$ under exogenous variables (A_t, z_t, g_t).

Since the productivity A_t is non-stationary process, it induces stochastic trend in the output and consumption. Thus, we should introduce detrended variables $c_t = C_t/A_t, y_t = Y_t/A_t$. The model economy can be described with a unique steady state with detrended variables. The steady state inflation π equals the target rate π^* and

$$r = \frac{\gamma}{\beta}, R = r\pi^*, c = (1 - \nu)^{1/\tau}, \text{ and } y = g(1 - \nu)^{1/\tau} \tag{18}$$

Let $\hat{x}_t = \ln(x_t/x)$ denote the percentage deviation of a variable x_t from its steady state x. Then, we can have the model with detrended variables as the following.

$$1 = E_t[e^{-\tau\hat{c}_{t+1}+\tau\hat{c}_t+\hat{R}_t-\hat{z}_{t+1}-\hat{\pi_{t+1}}}] \tag{19}$$

$$\frac{1-\nu}{\nu\phi\pi^2}(e^{\tau\hat{c}_t}-1) = (e^{\hat{\pi}_t}-1)\left[\left(1-\frac{1}{2\nu}\right)e^{\hat{\pi}_t}+\frac{1}{2\nu}\right] \tag{20}$$
$$-\beta E_t[e^{-\tau\hat{c}_{t+1}+\tau\hat{c}_t+\hat{R}_t-\hat{z}_{t+1}-\hat{\pi_{t+1}}}]$$

$$e^{\hat{c}_t-\hat{y}_t} = e^{-\hat{g}_t} - \frac{\phi\pi^2 g}{2}(e^{\hat{\pi}_t}-1)^2 \tag{21}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\psi_1\hat{\pi}_t + (1-\rho_R)\psi_2(\Delta\hat{y}_t - \hat{z}_t) + \epsilon_{R,t} \tag{22}$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + \epsilon_{g,t} \tag{23}$$

$$\hat{z}_t = \rho_z\hat{z}_{t-1} + \epsilon_{z,t} \tag{24}$$

Since the equation (19)-(24) are a nonlinear rational expectations system, it is necessary to linearlize them to solve the model. Applying log linearization, the model can be written as

$$\hat{y}_t = E_t[\hat{y}_{t+1}] + \hat{g}_t - E_t[\hat{g}_{t+1}] - \frac{1}{\gamma}\left(\hat{R}_t - \hat{E}_t[\pi_{t+1}] - E_[\hat{z}_{t+1}]\right) \tag{25}$$

$$\hat{\pi}_t = \beta\hat{E}_t[\hat{\pi}_{t+1}] + \kappa(\hat{y}_t - \hat{g}_t) \tag{26}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\psi_1\hat{\pi}_t + (1-\rho_R)\psi_2(\Delta\hat{y}_t - \hat{z}_t) + \epsilon_{R,t} \tag{27}$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + \epsilon_{g,t} \tag{28}$$

$$\hat{z}_t = \rho_z\hat{z}_{t-1} + \epsilon_{z,t} \tag{29}$$

where $\kappa = \tau(1-\nu)/(\nu\pi^2\phi)$. To solve the model described as equation (25)-(29), I follow the solution algorithm provided by Sims (2002). The linearized expressions and rational expectations solution of the model via solution algorithm lead to transition equations as a part of a state-space representation of the DSGE model which can be analyzed with the Kalman filter. The transition equations and measurement equations from observations allow us to conduct estimation of DSGE model with Bayesian method by likelihood approach.

2.2 One-asset Heterogeneous Agent New Keynesian model

Like the RANK model, one-asset HANK model also assumes that the model economy consists of a representative final good producing firm, a continuum of intermediate goods producing firms, government and a central bank. However, HANK model considers heterogeneous households who face uninsurable, idiosyncratic income risk and borrowing constraints. This realistic modification in the model would allow us to capture a feasible monetary policy effect.

Also, following the model specification by Ahn et al. (2017), the HANK model in this paper is presented in continuous time rather than discrete time. This is because continuous time is easier to capture occasionally binding constraints and inactions, which the HANK model cares about.

2.2.1 Household

In the model economy, there is a continuum of households with indexed by $j \in [0, 1]$ who have preferences represented by the expected utility function. Households receive utility from a consumption good $c_{j,t}$ and disutility from hours worked $l_{j,t}$. Then the household maximizes

$$E_0 \int_0^\infty e^{-\rho t} \left(\frac{c_{j,t}^{1-\gamma}}{1-\gamma} - \frac{l_{j,t}^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}} \right) dt \tag{30}$$

where ρ is the discount factor, γ is the coefficient of relative risk aversion and ϕ is Frisch elasticity of labor supply. At the each time t, a household face idiosyncratic productivity shock $z_{j,t} \in \{z_L, z_H\}$ with $z_L < z_H$ (high and low productivity), which the expectation is taken over. Households transit between this two values for labor productivity following a Poisson process. Households can only trade in productive asset a_t at the real interest rate r_t. Therefore, households face two different heterogeneous property: labor productivity z_t and their asset position a_t. A household optimizes the decision problem subject to the following equations

$$a_{t+1} = (1 - \tau_t) w_t z_t l_t + r_t a_t + T_t + D_t - c_t \tag{31}$$

$$a_{t+1} \geq 0 \tag{32}$$

where τ is tax rate of income, w_t is wage, T_t is lump-sum transfer and D_t is residual profit from firms. The household budget constraint contains stochastic idiosyncratic productivity shock (z_t) and borrowing constraing (equation (32)). Those contraints and uncertainty may cause the decline in households' ability to smooth consumption (intertemporal substitution) which the forward guidance mainly rely on its effect.

2.2.2 Firm

The model setup for the firms is basically the same as RANK model I elaborate so far. That is, HANK model also assumes there are final good producing firm which use intermediate goods from monopolistically competitive firms.

2.2.3 Central bank and Government

A Central bank decides the nominal interest rate according to Taylor rule.

$$R_t = r_t^* + \psi_0 \pi_t + \psi_1 (y_t - y^*) + \epsilon_{R,t} \tag{33}$$

r_t^* is the steady state of real interest rate, y^* denotes steady state of output and $\epsilon_{R,t}$ represents the monetary policy shock, which follows

$$d\epsilon_{R,t} = -\theta_R \epsilon_{R,t} + \sigma_{R,t} dW_t \tag{34}$$

Government faces exogenous expenditures G_t and raises revenue via tax on household labor income $w_t z_t l_t$ that consists of lump-sum transfer T_t. Only government can issue liquid assets in the economy, which are real bonds of infinitesimal maturity B_t^g.

Government's an intertemporal budget constraint is

$$B_t^g + G_t + T_t = \pi_t \int w_t z_t l_t(a,z) g_t(a,z) dadz + r_t B_t^g \tag{35}$$

2.2.4 Equilibrium Relationship

For households' equilibrium, we make use of the Hamilton-Jacobi-Bellman (HJB) equation from the model set up as the following.

$$\begin{aligned}\rho V(a,z) = \max_{c,l} \; & u(c,l) + ((1-\tau)wzl + ra + T + D - c)\partial_a V(a,z) \\ & + \lambda(z)(V(a,z') - V(a,z))\end{aligned} \tag{36}$$

where λ is transition probabilities of idiosyncratic labor productivity z (follow a Poisson process with arrival rates λ_L and λ_H).

Although firm's equilibrium can be obtained to solve a maximization problem of profit like RANK model, one needs to adjust a continuous-time form. I follow Kaplan et al. (2018) which presents the combination of a continuous-time formulation of the problem and quadratic price adjustment costs. The identification yields a simple equation characterizing a New Keynesian Phillips Curve without the need for log linearization.

$$\left(r_t - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\nu}{\theta}(m_t - m^*) + \dot{\pi}_t, \quad m^* = \frac{\nu - 1}{\nu} \tag{37}$$

where θ is price stickiness, m_t is marginal cost which is assumed to be common across all intermediate good producers.

The bond market clearing condition is given by

$$B_t^g = \int a g_t(a,z) dadz \tag{38}$$

Also, the labor market clearing condition can be written

$$\int z l_t(a,z) g_t(a,z) da dg = L_t \tag{39}$$

where L_t is aggregate level of labor demand.

2.2.5 Linearlization

In order to solve HANK model, we need to linearlize the model. I employ the three step linearization procedure provided by Ahn et al. (2017).

- **Step 1: Approximate Steady State**

  Solve for the steady state of the model without aggregate shocks but with idiosyncratic shocks.

- **Step 2: Linearize Equilibrium Conditions**

  Take a first-order Taylor expansion of the discretized equilibrium conditions around the steady state. Note that heterogeneous agent models have too large size of variables to estimate it. Thus, in addition to linearization, we need to conduct model reduction procedure.

- **Step 3: Solve Linear System**

  Like RANK model, solve the model which can be obtain from the step 2 with standard solution algorithm. I utilize Sims (2002) method as the same as RANK model for soling the model.

3 Estimation Method

To estimate the structure parameters in both RANK and HANK model, I use Bayesian approach based on likelihood evaluation. Through linearization procedures, both models can be expressed in the state space model, which allows us to enjoy the Kalman filter to calculate likelihood.

Although Bayesian approach has several merits for estimating dynamic Macroeconomics models, the most crucial point is that the approach is more likely to be able to find the optimal solutions. When one implements ordinary maximum likelihood estimation for DSGE model, since the log-likelihood function would be very flat around the optimum, and the estimates are very sensitive to their initial values, the solutions easily fall into local maxima. On the other hand, by combining estimation process with sampling algorithms, Bayesian approach can provide estimations which can be robust for local maxima.

Suppose data as $Y = (y_1, \cdots, y_r)$, the Bayesian estimation procedure is the following.

i. Set the posterior distributions of parameters $f(\theta)$.

ii. Based on the Bayesian Theorem,

$$f(\theta|Y) = \frac{f(Y|\theta)f(\theta)}{\int f(Y|\theta)f(\theta)d\theta} \tag{40}$$

calculate posterior distributions $f(\theta|Y)$.

iii. Using posterior distributions, estimate the structure parameters.

The denominator of the Bayesian Theorem is independent from the parameters and we can't calculate it although the likelihood $f(Y|\theta)$ can be available via Kalman filter. In this case, we should make use of Markov chain Monte Carlo (MCMC) methods to sample parameters from the posterior distribution.

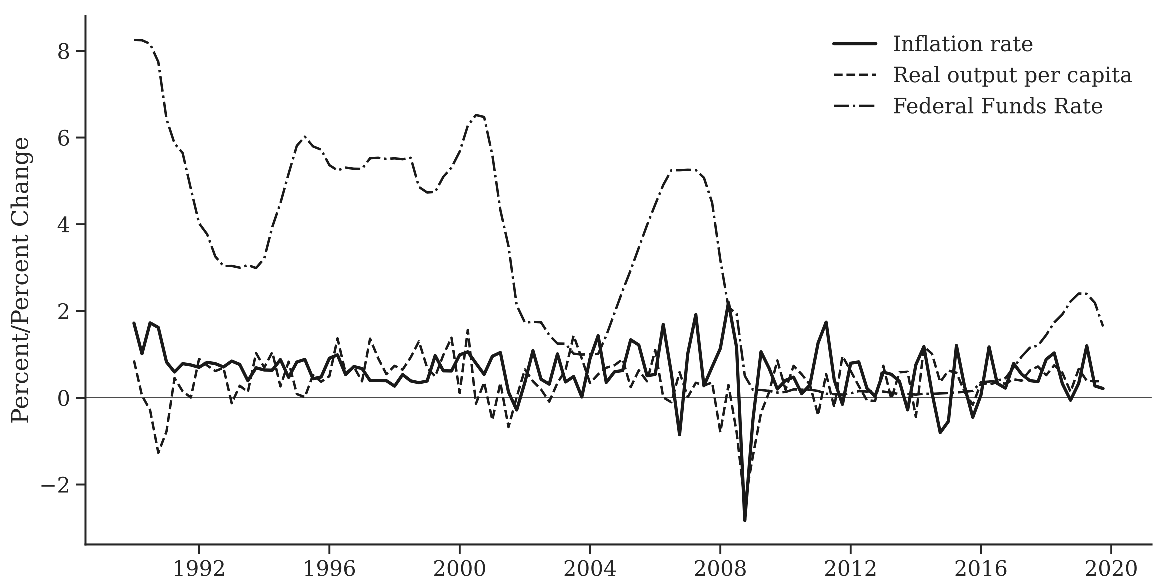
Among several methods of MCMC family, I adopt the Metropolis–Hastings (MH) algorithm for RANK model estimation and the Sequential Monte Carlo (SMC) algorithm for HANK model estimation. The MH algorithm is much simpler than SMC but MH sampler can't be parallelized for the computation. Meanwhile, since HANK model needs to take additional procedure and manipulate large size of variables as I described in the model section, it takes much longer time to draw proposed parameters than RANK model. Therefore, I use SMC for the HANK model estimation.

In the estimation, I choose 300,000 sampling for the MH algorithm (but discard 200,000 drawings as the burn-in and use the rest for evaluating posterior distribution). Also, SMC algorithm takes 3,000 particles for the estimation.

4 Data and Prior distribution

The data used for estimation are the real output per capita, the inflation rate and the nominal interest rate in the United States. The real output per capita is calculated as quarter-to-quarter percent change of real GDP per capita. Annualized quarter-to-quarter percent change of CPI urban consumers index is used as the inflation. The nominal interest rate is the Federal Funds Rate at a quarterly rate annualized. The data period is from 1990:Q1 to 2019:Q4. All data is obtained from Federal Reserve Bank of St. Louis' economic database (FRED). Figure 1 shows the time-series of those data.

Figure 1: Time series Macro data of U.S.



The Bayesian estimation needs the prior distributions for parameters. Table 1 provides the prior distributions for parameters in RANK model, which are set as following An and Schorfheide (2007) and New York Fed's DSGE model (FEDNY-DSGE) specifications. For HANK model, I calibrate six parameters as shown in table 2 and table 3 shows prior settings of estimated parameters. Both calibration and prior settings for HANK model are based on FEDNY-DSGE specification. The descriptions of parameters are available in Appendix.

Table 1: Prior Distribution for RANK model

Parameter	Domain	Distribution	Mean	Std.Dev
τ	$(0, \infty)$	Gamma	2.00	0.50
κ	$(0, \infty)$	Gamma	0.20	0.10
ψ_1	$(0, \infty)$	Gamma	1.50	0.25
ψ_2	$(0, \infty)$	Gamma	0.50	0.25
r_A	$(0, \infty)$	Gamma	0.50	0.50
π^*	$(0, \infty)$	Gamma	7.00	2.00
γ_Q	$(-\infty, \infty)$	Normal	0.40	0.20
ρ_R	(0,1)	Beta	0.50	0.20
ρ_g	(0,1)	Beta	0.80	0.10
ρ_z	(0,1)	Beta	0.66	0.15
σ_R	$(0, \infty)$	InvGamma	0.40	4.00
σ_g	$(0, \infty)$	InvGamma	1.00	4.00
σ_z	$(0, \infty)$	InvGamma	0.50	4.00

Table 2: Calibrated parameters for HANK model

Parameter	Description	Value
h	Mean working hours	0.33
γ	Relative risk aversion	1.00
ν	Elasticity of demand	10.00
τ	Marginal tax rate on labor income	0.20
B_t^g	Government bond target	6.00
T	Lump-sum transfer per GDP	0.06

Table 3: Prior Distribution for HANK model

Parameter	Domain	Distribution	Mean	Std.Dev
ϕ	$(0, \infty)$	Normal	0.50	0.10
θ	$(0, \infty)$	Normal	100.00	10.00
ψ_1	$(0, \infty)$	Normal	1.25	0.20
ψ_2	$(0, \infty)$	Normal	0.10	0.05
σ_R	$(0, \infty)$	InvGamma	0.40	4.00
θ_R	(0,1)	Beta	0.25	0.10

5 Result

Table 4 and 5 show the structure parameters estimated from the posterior distributions. Every estimator is statistically significant and seems to be converged as shown in the shape of posterior distribution for each parameter that is available in the Appendix.

Using those estimated parameters, I calculate impulse response function of aggregate variables to the monetary policy shock which is -5% shock (i.e., monetary easing) at $t = 0$ for 20 quarters horizon in RANK and HANK model as shown in Figure 2. The black dash line represents the impulse response from RANK model and the red line is that from HANK model. As standard Macroeconomics theory indicates, the monetary easing policy increases output and raises inflation, and then the effect is gradually diminishing. However, the magnitude of the effect in HANK model is much smaller than that in RANK model for both observations. This difference would indicate that the HANK model somewhat mutes the effect of monetary policy impact which RANK model might overestimate in its specification.

Figure 2: Impulse Response Function from Monetary Policy shock at $t = 0$

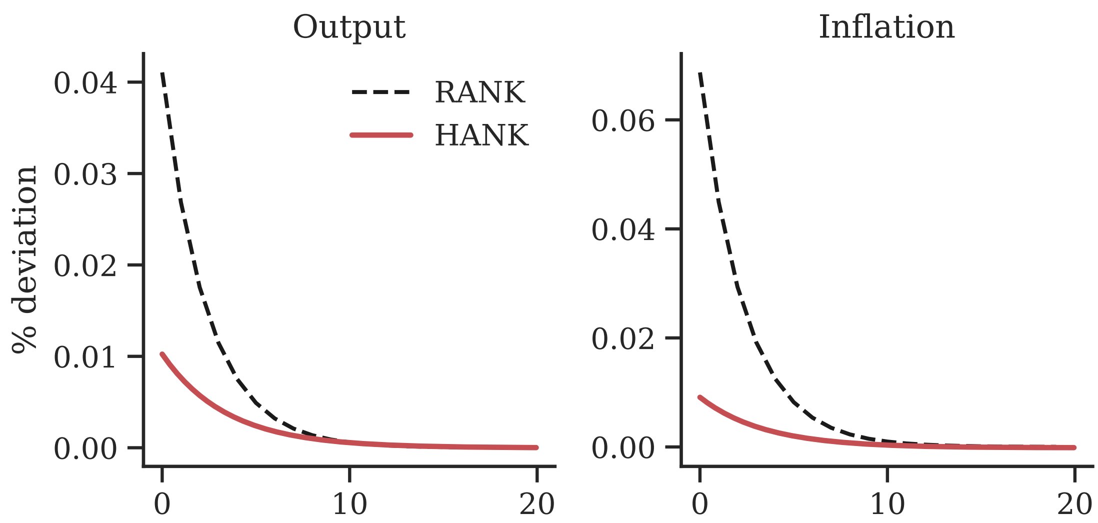


Figure 3: Impulse Response Function from Forward guidance

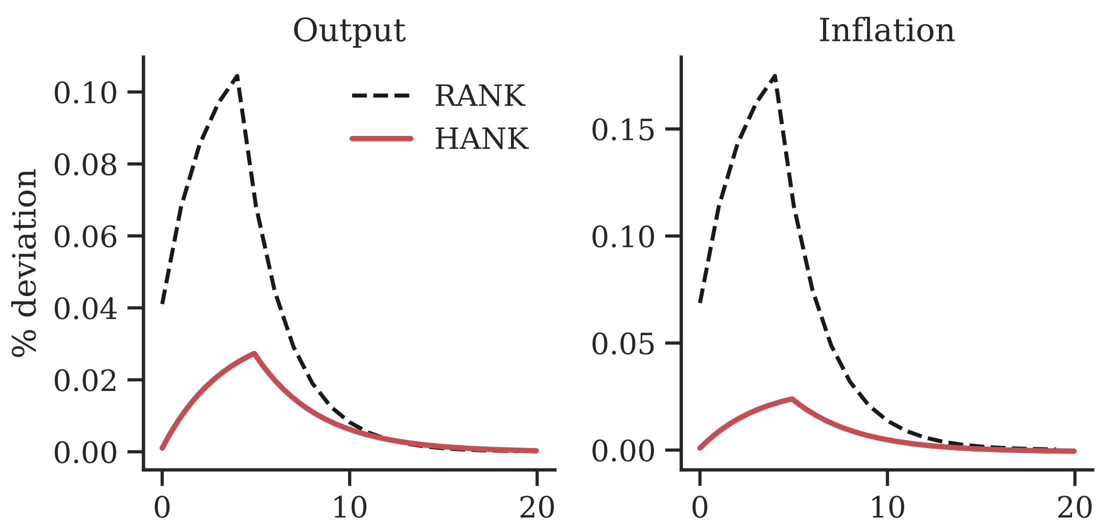


Figure 3 shows the impulse response function to the forward guidance which a central bank makes a commitment to continue monetary easing policy until $t = 5$. The impulse response function from HANK model implies the magnitude of the easing policy is smaller and the the momentum of the output and inflation increasing is slower than RANK model's prediction. This result is consistent with the discussion in McKay et al. (2016). HANK model constrains households to bind the borrowing behavior and make them face idiosyncratic income risk and such uncertainty reduces the effect of intertemporal smooth consumption. This mechanism in HANK model leads to the reduction of the effect of forward guidance as those impulse response functions suggest.

Table 4: Estimated Posterior Distribution for RANK model

Parameter	Mean	Std.Dev	90% Confidence Interval
τ	3.5294	0.6210	(2.588, 4.6116)
κ	0.1449	0.0415	(0.0814, 0.2192)
ψ_1	1.9615	0.2979	(1.4817, 2.4729)
ψ_2	0.8651	0.3583	(0.3531, 1.5183)
r_A	0.2419	0.1785	(0.0201, 0.5795)
π^*	1.4112	0.2683	(1.0086, 1.8799)
γ_Q	0.7548	0.1237	(0.5594, 0.9611)
ρ_R	0.8400	0.0235	(0.7994, 0.8768)
ρ_g	0.9760	0.0087	(0.9623, 0.9905)
ρ_z	0.9494	0.0122	(0.9288, 0.969)
σ_R	0.1793	0.0148	(0.1587, 0.2076)
σ_g	0.6877	0.0506	(0.6146, 0.7779)
σ_z	0.1965	0.0199	(0.1652, 0.2312)

Table 5: Estimated Posterior Distribution for HANK model

Parameter	Mean	Std.Dev	90% Confidence Interval
ϕ	0.5305	0.0922	(0.374, 0.6851)
θ	100.6775	10.2741	(83.7623, 117.4115)
ψ_1	1.2692	0.2100	(0.8844, 1.5926)
ψ_2	0.1051	0.0465	(0.031, 0.1843)
σ_R	0.4535	0.1847	(0.2567, 0.7995)
θ_R	0.2935	0.0978	(0.1427, 0.4613)

6 Conclusion

This paper estimates the representative agent New Keynesian (RANK) model and one-asset heterogeneous agent New Keynesian (HANK) model for the United States using three observations (real output per capita, inflation, and nominal interest rate). Given the structural parameters estimated using Bayesian methods with Markov Chain Monte Carlo algorithm (the Metropolis-Hastings (MH) algorithm for RANK model estimation and the Sequential Monte Carlo (SMC) algorithm for HANK model estimation), I simulate the impulse response functions with monetary easing policy shock and the forward guidance policy shock.

I find that the estimated HANK model substantially reduces the power of the forward guidance which RANK model predicts tremendous effects on the output and inflation but the magnitude is inconsistent with empirical observations. In other words, the HANK model could be a solution to the inconsistency between the model prediction and observations. The mechanism behind the reduction of the power of the forward guidance is, as McKay et al. (2016) argues, that the HANK model which incorporates idiosyncratic income risk and borrowing constraints with household's decision problem can weaken the household's ability to intertemporal substitution on which standard New Keynesian model and forward guidance rely.

This paper adopts the one-asset HANK model which assumes households can hold only liquid assets. However, households are also able to choose to save in illiquid assets in the real world. Thus, for evaluating forward guidance effect with empirically more realistic heterogeneity, it is worth considering the two-asset HANK model as the extension.

Moreover, I simply calibrate some parameters from specifications of the New York Fed and Ahn et al. (2017) but to reflect empirical settings to the model more, micro household data can be used for calibrations of parameters. For example, using the flow of funds, we can see the households' balance sheets to examine asset holding situations. Or, Social Security Administration data allows us to obtain earning processes for knowing parameters of the distribution of idiosyncratic income shocks. Incorporating micro household data with the model also enables the study to compare the magnitude of the forward guidance with the HANK model internationally because we can reflect the heterogeneous circumstance of each country's household to each HANK model, not just using each macro data. This is another way for the future extension of this paper.

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APPENDIX

A-1 Posterior Distributions

Figure 4: Posterior Distributions for RANK

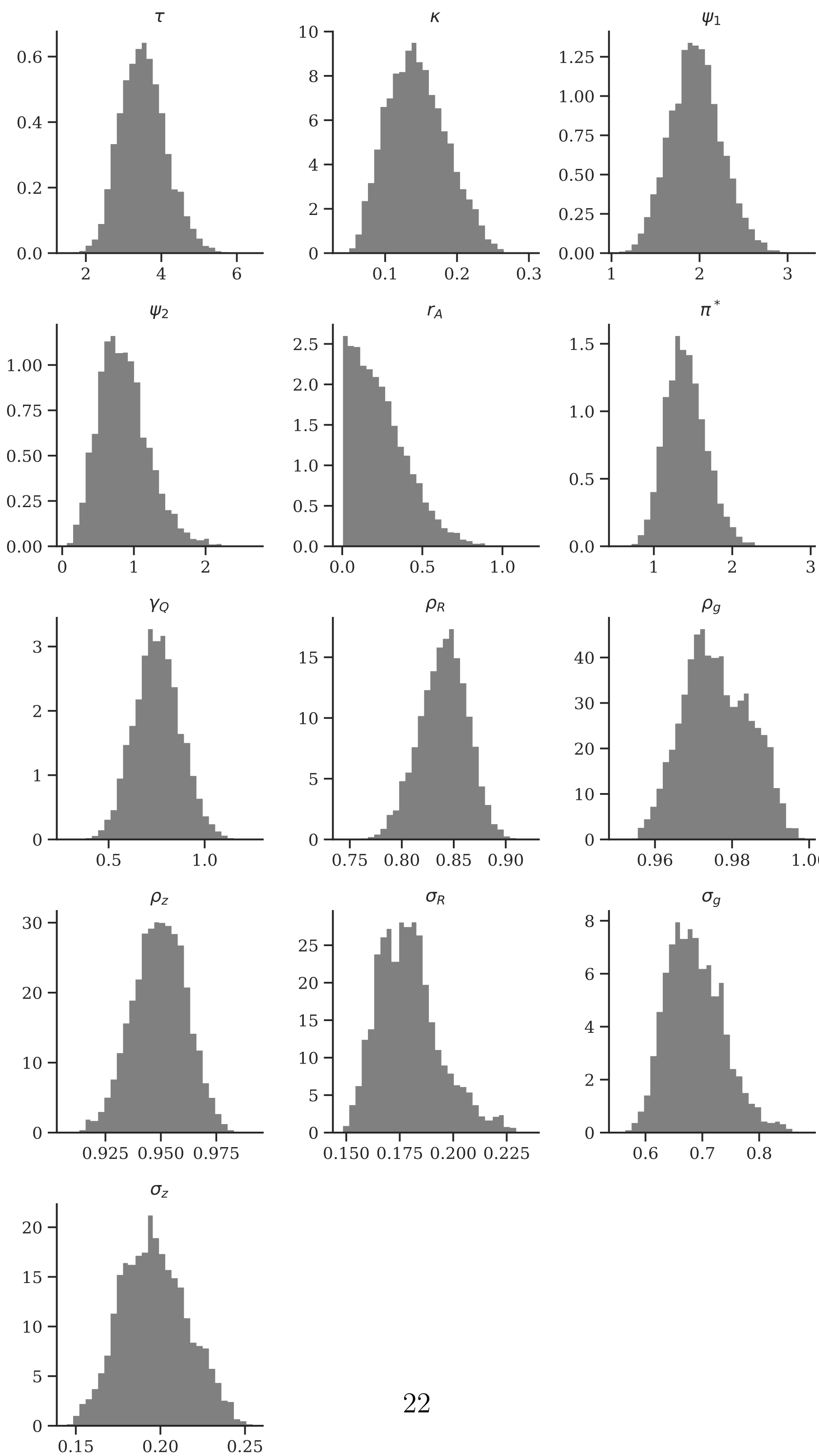
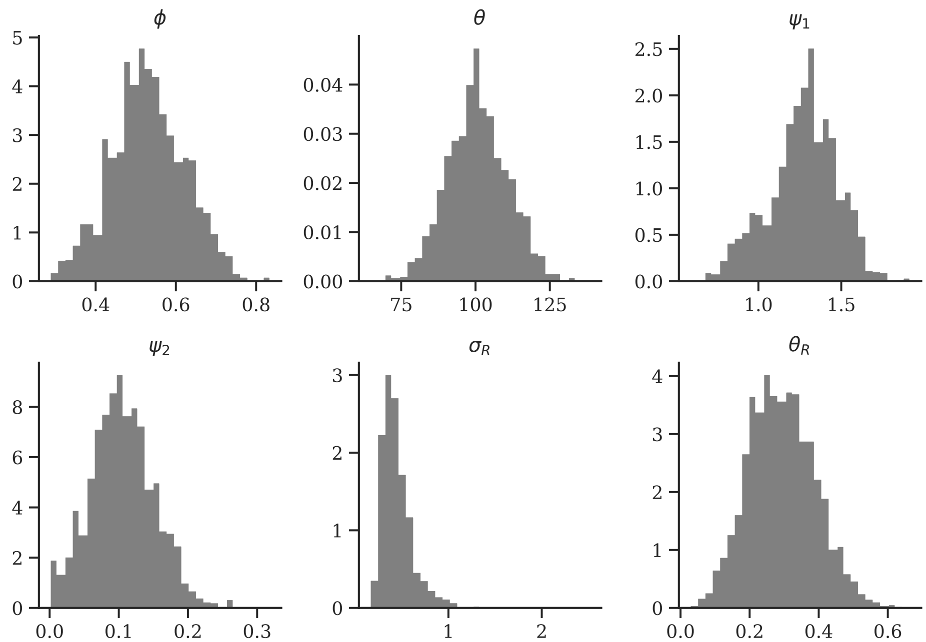


Figure 5: Posterior Distributions for HANK



A-2 Description of Parameters

Table 6: Description of Structure parameters in RANK model

Parameter	Description
τ	The intemporal elasticity of substitution
κ	Composite parameter in NKPC
ψ_1	Weight on inflation in monetary policy rule
ψ_2	Weight on the output gap in monetary policy rule
r_A	Discount factor
π^*	Target inflation rate
γ_Q	Steady state growth rate of technology
ρ_R	AR(1) coefficient on interest rate
ρ_g	AR(1) coefficient on government spending
ρ_z	AR(1) coefficient on shocks to the technology
σ_R	Std.dev of shocks to the nominal interest rate.
σ_g	Std.dev of shocks to the gov. spending process.
σ_z	Std.dev of shocks to the technology process.

Table 7: Description of Structure parameters in HANK model

Parameter	Description
ϕ	Frisch elasticity of labor supply
θ	Price stickiness
ψ_1	Weight on inflation in monetary policy rule
ψ_2	Weight on the output gap in monetary policy rule
σ_R	Volatility of monetary policy shocks
θ_R	Rate of mean reversion in monetary policy shocks