Clue to Resolve Forward Guidance Puzzle: (Work in Progress/Initial Result) *

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May 2020

Abstract

This paper aims to find a clue to resolve forward guidance puzzle which central banks in the developed countries have struggled recently. The heterogeneous agent New Keynesian (HANK) model is one of the powerful candidates to give a solution to the forward guidance puzzle. I estimate HANK model along with standard Representative New Keynesian (RANK) model by Bayesian approach and calculate the impulse response function. The result suggests the HANK mutes the effect of forward guidance and it could be a hint for resolving forward guidance puzzle

keywords: Forward guidance puzzle, Heterogeneous agent New Keynesian, Bayesian method

JEL classification: E52, E58, C30

*This paper is a final project for MACS 30250 Perspectives on Computational Research in Economics. I would like to thank Dr. Richard W. Evans for his helpful support and advice. The views expressed in the paper are solely the responsibility of the author.

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1 Introduction

2 Model

In this section, I set up a Representative Agent New Keynesian (RANK) model following An and Schorfheide (2007) and a one-asset Heterogeneous Agent New Keynesian (HANK) model in continuous time following Ahn et al. (2017) and Kaplan et al. (2018), which are used to estimate structural parameters with Bayesian method and to calculate impulse responses from aggregate monetary policy shock.

2.1 Representative Agent New Keynesian model

The RANK model supposes that economy consists of a representative household, a final good producing firm, a continuum of intermediate goods producing firms, a government (fiscal authority) and a central bank.

2.1.1 Household

The representative household considers utility from a composite consumption good C_t with a habit shock which is given by the level of technology A_t, real money balance M_t/P_t, and disutility from hours worked H_t. Then the household maximizes

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i}/A_{t+i})^{1-\tau} - 1}{1-\tau} + \chi_M \left(\frac{M_{t+i}}{P_{t+i}} \right) - \chi_H H_{t+i} \right] \tag{1}$$

where β is the discount factor, $1/\tau$ is the intertemporal elasticity of substitution and χ_M and χ_H are scale factors that decide steady state real money balances and hours worked, respectively (assume $\chi_H = 1$). The household provides labor services to the firms getting the real wage W_t and has access to a bond market where government bonds are traded with paying nominal interest R_t. Also, the household receives residual real profits D_t from firms and pays lump-sum taxes T_t. Hence, the household's budget constraint is

$$P_tC_t + B_t + M_t - M_{t-1} + T_t = P_tW_tH_t + R_{t-1}B_{t-1} + P_tD_t \tag{2}$$

2.1.2 Firm

The representative final good producing firm under perfect competitive environment uses a continuum of intermediate goods $Y_t(j),\ j \in [0,1]$ expressed as

$$Y_t = \left[\int_0^1 Y_t(j)^{1-\nu} dj\right]^{\frac{1}{1-\nu}}, \quad 1/\nu > 1 \tag{3}$$

where $1/\nu$ is the elasticity of demand for each intermediate good. The firm takes input prices $P_t(j)$ and output prices P_t as given and maximizes its profit. Then, the demand for intermediate goods are

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-1/\nu} Y_t \tag{4}$$

Also, using (3) and (4), the final goods price in terms of intermediate good is defined as

$$P_t = \left[\int_0^1 P_t(j)^{\nu-1} \nu dj\right]^{\frac{\nu}{\nu-1}} \tag{5}$$

Intermediate goods firms produce intermediate goods under monopolistically competitive market and their production function is

$$Y_t(j) = A_tN_t(j) \tag{6}$$

where A_t is an exogenous productivity process and $N_t(j)$ is the labor input of firm j. Labor is employed at the real wage W_t in a perfectly competitive market. As a result, firm j faces marginal cost MC_t can be calculated as the minimization problem of cost. That is,

$$
\begin{aligned}
&W_t N_t(j) + MC_t(Y_t(j) - A_t N_t(j)) \\
\Leftrightarrow &MC_t = \frac{W_t}{A_t}
\end{aligned}
$$

Firm j also faces nominal rigidities and chooses its labor input $N_t(j)$ and the price $P_t(j)$ to maximize the present value of future profits

$$
E_t \left[\sum_{i=0}^{\infty} Q_{t+i} \left(\frac{P_{t+i}(j)}{P_{t+i}} Y_{t+i}(j) - W_{t+i} N_{t+i}(j) - \frac{\phi}{2} \left(\frac{P_{t+i}(j)}{P_{t+i-1}(j) - \pi} \right)^2 Y_{t+s}(j) \right) \right] \quad (7)
$$

where ϕ is the price stickiness and π is the steady state inflation rate associated with the final good.Q_{t+i} is the stochastic discount factor (i.e., the marginal value of a unit of the consumption good to the household. This is exogenous variable for the firm).

2.1.3 Central bank and Government

Suppose monetary policy is decided by following the Taylor rule.

$$
R_t = \left[\frac{\pi^*}{\beta} \left(\frac{\pi_t}{\pi^*} \right)^{\psi_1} \frac{Y_t}{Y_t^*}^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R} e^{\epsilon_{R,t}} \quad (8)
$$

Here, $\epsilon_{R,t}$ is monetary policy shock, π_t is the gross inflation rate ($\pi_t = P_t/P_{t-1}$), π^* is the target inflation rate, and Y_t^* is the level of output without nominal rigidities.

The fiscal authority spends a fraction ζ_t of aggregate output Y_t, where $\zeta_t \in [0, 1]$ which follows an exogenous process ($G_t = \zeta_t Y_t$). The government imposes a lump-sum tax to finance any deficit in government revenue. The government's budget constraint is

$$
P_t G_t + R_{t-1} B_{t-1} = T_t + B_t + M_t - M_{t-1} \quad (9)
$$

2.1.4 Exogenous Processes

Suppose three exogenous processes disturb the model economy. Aggregated productivity evolves according to random walk. That is,

$$\ln A_t = \ln\gamma + \ln A_{t-1} + \ln z_t \tag{10}$$

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t} \tag{11}$$

Government expenditures ($g_t = 1/(1-\zeta_t)$) fluctuate as the following

$$\ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-1} + \epsilon_{g,t} \tag{12}$$

Finally, the monetary policy shock $\epsilon_{R,t}$ is assumed to be serially uncorrelated. Three shocks $\epsilon_{z,t}, \epsilon_{g,t}, \epsilon_{R,t}$ are independent of each other and are normally distributed with mean zero and standard deviations σ_z,σ_g and σ_R, respectively.

2.1.5 Equilibrium Relationship

Here we assume that the symmetric equilibrium in which all in which all intermediate good producers make the same choice so that the j subscript can be removed. The market clearing conditions are

$$Y_t = C_t + G_t + \frac{\phi}{2}\left(\frac{P_{t+i}(j)}{P_{t+i-1}(j)-\pi}\right)^2 Y_t \tag{13}$$

From the household's optimal solution based on (1) and (2), we have

$$\frac{1}{R_t} = \beta E_t \frac{1}{\pi_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\tau}\left(\frac{A_t}{A_{t+1}}\right)^{1-\tau} \tag{14}$$

Similarly, intermediate firms maximizes equation (7) subject to the demand for their goods as shown in equation (4)

$$1 = \beta E_t \left[\left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{A_t}{A_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \tag{15}$$

$$\begin{aligned} 1 =& \frac{1}{\nu} \left[1 - \left(\frac{C_t}{A_t} \right)^{\tau} \right] + \phi(\pi_t - \pi) \left[\left(1 - \frac{1}{2\nu} \right) \pi_t + \frac{\pi}{2\nu} \right] \\ & - \phi \beta E_t \left[\left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{Y_{t+1}/A_{t+1}}{Y_t/A_t} (\pi_{t+1} - \pi) \pi_{t+1} \right] \end{aligned} \tag{16}$$

This equation is known as the New Keynesian Phillips Curve. In the absence of nominal rigidities ($\phi = 0$) aggregate output is

$$Y_t^* = (1 - \nu)^{1/\tau} A_t g_t \tag{17}$$

As a result, we can describe the model which determines endogenous variables $(Y_t, C_t, R_t, \pi_t, Y_t^*)$ under exogenous variables (A_t, z_t, g_t).

Since the productivity A_t is non-stationary process, it induces stochastic trend in the output and consumption. Thus, we should introduce detrended variables $c_t = C_t/A_t, y_t = Y_t/A_t$. The model economy can be described with a unique steady state with detrended variables. The steady state inflation π equals the target rate π^* and

$$r = \frac{\gamma}{\beta}, R = r\pi^*, c = (1 - \nu)^{1/\tau}, \text{ and } y = g(1 - \nu)^{1/\tau} \tag{18}$$

Let $\hat{x}_t = \ln(x_t/x)$ denote the percentage deviation of a variable x_t from its steady state x. Then, we can have the model with detrended variables as the following.

$$1 = E_t[e^{-\tau\hat{c}_{t+1}+\tau\hat{c}_t+\hat{R}_t-\hat{z}_{t+1}-\pi_{\hat{t+1}}}] \tag{19}$$

$$\frac{1-\nu}{\nu\phi\pi^2}(e^{\tau\hat{c}_t}-1) = (e^{\hat{\pi}_t}-1)\left[\left(1-\frac{1}{2\nu}\right)e^{\hat{\pi}_t}+\frac{1}{2\nu}\right] \tag{20}$$

$$-\beta E_t[e^{-\tau\hat{c}_{t+1}+\tau\hat{c}_t+\hat{R}_t-\hat{z}_{t+1}-\pi_{\hat{t+1}}}]$$

$$e^{\hat{c}_t-\hat{y}_t} = e^{-\hat{g}_t}-\frac{\phi\pi^2 g}{2}(e^{\hat{\pi}_t}-1)^2 \tag{21}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1}+(1-\rho_R)\psi_1\hat{\pi}_t+(1-\rho_R)\psi_2(\Delta\hat{y}_t-\hat{z}_t)+\epsilon_{R,t} \tag{22}$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1}+\epsilon_{g,t} \tag{23}$$

$$\hat{z}_t = \rho_z\hat{z}_{t-1}+\epsilon_{z,t} \tag{24}$$

Since the equation (19)-(24) are a nonlinear rational expectations system, it is necessary to linearlize them to solve the model. Applying log linearization, the model can be written as

$$\hat{y}_t = E_t[\hat{y}_{t+1}]+\hat{g}_t-E_t[\hat{g}_{t+1}]-\frac{1}{\gamma}\left(\hat{R}_t-\hat{E}_t[\pi_{t+1}]-E_[\hat{z}_{t+1}]\right) \tag{25}$$

$$\hat{\pi}_t = \beta\hat{E}_t[\hat{\pi}_{t+1}]+\kappa(\hat{y}_t-\hat{g}_t) \tag{26}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1}+(1-\rho_R)\psi_1\hat{\pi}_t+(1-\rho_R)\psi_2(\Delta\hat{y}_t-\hat{z}_t)+\epsilon_{R,t} \tag{27}$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1}+\epsilon_{g,t} \tag{28}$$

$$\hat{z}_t = \rho_z\hat{z}_{t-1}+\epsilon_{z,t} \tag{29}$$

where $\kappa = \tau(1-\nu)/(\nu\pi^2\phi)$. To solve the model described as equation (25)-(29), I follow the solution algorithm provided by Sims (2002). The linearlized expressions and rational expectations solution of the model via solution algorithm lead to a state-space representation of the DSGE model which can be analyzed with the Kalman filter. Those allow us to conduct estimation of DSGE model with Bayesian method by likelihood approach.

2.2 One-asset Heterogeneous Agent New Keynesian model

Like RANK model, one-asset HANK model also assumes that the model economy consists of a representative final good producing firm, a continuum of intermediate goods producing firms, government and a central banks. However, HANK model considers heterogeneous households who face uninsurable, idiosyncratic income risk and borrowing constraints. This realistic modification in the model would allow us to capture a robust monetary policy effect.

Also, following the model specification by Ahn et al. (2017), HANK model is presented in continuous time rather than discrete time. This is because continuous time is easier to capture occasionally binding constraints and inactions, which HANK model cares about.

2.2.1 Household

In the model economy, there is a continuum of households with indexed by $j \in [0, 1]$ who have preferences represented by the expected utility function. Households receive utility from a consumption good $c_{j,t}$ and disutility from hours worked $l_{j,t}$. Then the household maximizes

$$E_0 \int_0^\infty e^{-\rho t} \left(\frac{c_{j,t}^{1-\theta}}{1-\theta} - \phi_0 \frac{l_{j,t}^{1+\frac{1}{\phi_1}}}{1+\frac{1}{\phi_1}} \right) dt \tag{30}$$

where ρ is the discount factor, $1/\theta$ is the coefficient of relative risk aversion and ϕ_1 is Frisch elasticity of labor supply. At the each time t, a household face idiosyncratic productivity shock $z_{j,t} \in \{z_L, z_H\}$ with $z_L < z_H$ (high and low productivity), which the expectation is taken over. Households transit between this two values for labor productivity following a Poisson process. Households can only trade in productive asset a_t at the real interest rate r_t. Therefore, households face two different heterogeneous property: labor productivity z_t and their asset position a_t. A household optimizes the decision problem subject to the following equations

$$a_{t+1} = (1 - \tau_t) w_t z_t l_t + r_t a_t + T_t + D_t - c_t \tag{31}$$

$$a_{t+1} \geq 0 \tag{32}$$

where τ is tax rate of income, w_t is wage, T_t is lump-sum transfer and D_t is residual profit from firms. The household budget constraint contains stochastic idiosyncratic productivity shock (z_t) and borrowing constraing (equation (32)). Those contraints and uncertainty may cause the decline in households' ability to smooth consumption (intertemporal substitution) which the forward guidance mainly rely on its effect.

2.2.2 Firm

The model setup for the firms is basically the same as RANK model I elaborate so far. That is, HANK model also assumes there are final good producing firm which use intermediate goods from monopolistically competitive firms.

2.2.3 Central bank and Government

A Central bank decides the nominal interest rate according to Taylor rule.

$$R_t = r_t^* + \phi_\pi \pi_t + \phi_y (y_t - y^*) + \epsilon_{R,t} \tag{33}$$

r_t^* is the steady state of real interest rate, y^* denotes steady state of output and ϵ_R, t represents the monetary policy shock, which follows

$$d\epsilon_{R,t} = -\theta_R \epsilon_{R,t} + \sigma_{R,t} dW_t \tag{34}$$

Government faces exogenous expenditures G_t and raise revenue via tax on household labor income $w_t z_t l_t$ that consists of lump-sum transfer T_t. Only government can issue liquid assets in the economy, which are real bonds of infinitesimal maturity B_t^g.

Government's an intertemporal budget constraint is

$$B_t^g + G_t + T_t = \pi_t \int w_t z_t l_t(a,z) g_t(a,z) da dz + r_t B_t^g \tag{35}$$

2.2.4 Equilibrium Relationship

For households' equilibrium, we make use of the Hamilton-Jacobi-Bellman (HJB) equation from the model set up as the following.

$$\begin{aligned} \rho V(a,z) = \max_{c,l} u(c,l) + ((1-\tau)wzl + ra + T + D - c)\partial_a V(a,z) \\ + \lambda(z)(V(a,z') - V(a,z)) \end{aligned} \tag{36}$$

where λ is transition probabilities of idiosyncratic labor productivity z (follow a Poisson process with arrival rates λ_L and λ_H).

Although firm's equilibrium can be obtained to solve a maximization problem of profit like RANK model, one needs to adjust a continuous-time form. I follow Kaplan et al. (2018) which presents the combination of a continuous-time formulation of the problem and quadratic price adjustment costs. The identification yields a simple equation characterizing a New Keynesian Phillips Curve without the need for log linearization.

$$\left(r_t - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\nu}{\theta}(m_t - m^*) + \dot{\pi}_t, \quad m^* = \frac{\nu - 1}{\nu} \tag{37}$$

where θ is price stickiness, m_t is marginal cost which is assumed to be common across all intermediate good producers.

The bond market clearing condition is given by

$$B_t^g = \int a g_t(a,z) da dz \tag{38}$$

Also, the labor market clearing condition can be written

$$\int z l_t(a,z) g_t(a,z) da dg = L_t \tag{39}$$

where L_t is aggregate level of labor demand.

2.2.5 Linearlization

In order to solve HANK model, we need to linearlize the model. I employ the three step linearization procedure provided by Ahn et al. (2017).

- **Step 1: Approximate Steady State**

  Solve for the steady state of the model without aggregate shocks but with idiosyncratic shocks.

- **Step 2: Linearize Equilibrium Conditions**

  Take a first-order Taylor expansion of the discretized equilibrium conditions around the steady state. Note that heterogeneous agent models have too large size of variables to estimate it. Thus, in addition to linearization, we need to conduct model reduction procedure.

- **Step 3: Solve Linear System**

  Like RANK model, solve the model which can be obtain from the step 2 with standard solution algorithm. I utilize Sims (2002) method as the same as RANK model in the estimation.

3 Estimation Method

To estimate the structure parameters in both RANK and HANK model, I use Bayesian approach based on likelihood evaluation. Through linearization procedure, the both models can be expressed in the state space model, which allows us to enjoy the Kalman filter to calculate likelihood.

Although Bayesian approach has several merits for estimating dynamic Macroeconomics models, the most crucial point is that the approach is more likely to be able to find the optimal solutions. When one implements ordinary maximum likelihood estimation for DSGE model, since the log-likelihood function would be very flat around the optimum, and the estimates are very sensitive to their initial values, the solutions easily fall into local maxima. On the other hand, by combining estimation process with sampling algorithms, Bayesian approach can provide estimations which are robust for local maxima.

Suppose data as $Y = (y_1, \cdots, y_r)$, the Bayesian estimation procedure is the following.

i. Set the posterior distributions of parameters $f(\theta)$.

ii. Based on the Bayesian Theorem,

$$f(\theta|Y) = \frac{f(Y|\theta)f(\theta)}{\int f(Y|\theta)f(\theta)d\theta} \tag{40}$$

calculate posterior distributions $f(\theta|Y)$.

iii. Using posterior distributions, estimate the structure parameters. The denominator of the Bayesian Theorem is independent from the parameters and we can't calculate it although the likelihood $f(Y|\theta)$ can be available via Kalman filter. In this case, we should make use of Markov chain Monte Carlo (MCMC) methods to sample parameters from the posterior distribution.

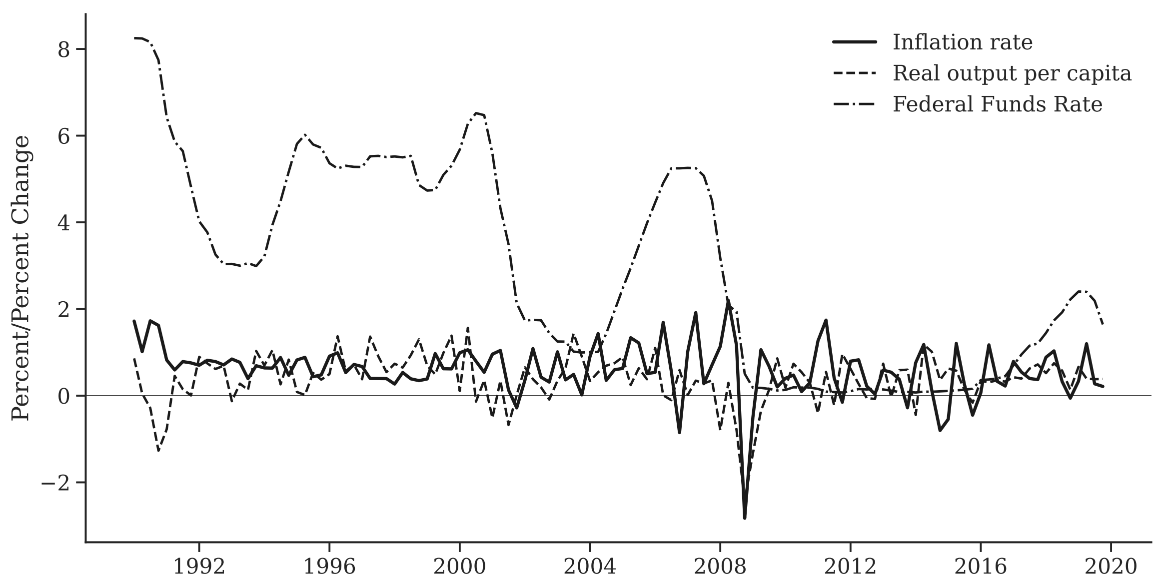
Among several methods of MCMC family, I adopt the Metropolis–Hastings (MH) algorithm for RANK model estimation and the Sequential Monte Carlo (SMC) algorithm for HANK model estimation. The MH algorithm is much simpler than SMC but MH sampler can't be parallelized for the computation. Meanwhile, since HANK model needs to take additional procedure and manipulate large size of variables as I described in the model section, it takes much longer time to draw proposed parameters than RANK model. Therefore, I use SMC for the HANK model estimation.

In the estimation, I choose 300,000 sampling for the MH algorithm (but discard 200,000 drawings as the burn-in and use the rest for evaluating posterior distribution). Also, SMC algorithm takes 3,000 particles for the estimation.

4 Data and Prior distribution

The data used for estimation are the real output per capita, the inflation rate and the nominal interest rate in the U.S.. The real output per capita is calculated as quarter-to-quarter percent change of real GDP per capita. Annualized quarter-to-quarter percent change of CPI urban consumers index is used as the inflation. The nominal interest rate is the Federal Funds Rate at a quarterly rate annualized. The data period is from 1990:Q1 to 2019:Q4. All data is obtained from Federal Reserve Bank of St. Louis' economic database (FRED). Figure 1 shows the time-series of those data.

Figure 1: Time series Macro data of U.S.



The Bayesian estimation needs the prior distributions for parameters. Table 1 provides the prior distributions for parameters in RANK model, which are set as following An and Schorfheide (2007) and New York Fed's DSGE model (FEDNY-DSGE) specifications. For HANK model, I calibrate six parameters as shown in table

2 and table 3 shows prior settings of estimated parameters. Both calibration and prior settings for HANK model are based on FEDNY-DSGE specification.

Table 1: Prior Distribution for RANK model

Parameter	Domain	Distribution	Mean	Std.Dev
τ	$(0, \infty)$	Gamma	2.00	0.50
κ	(0,1)	Uniform	-	-
ψ_1	$(0, \infty)$	Gamma	1.50	0.25
ψ_2	$(0, \infty)$	Gamma	0.50	0.25
r_A	$(0, \infty)$	Gamma	0.50	0.50
π^*	$(0, \infty)$	Gamma	7.00	2.00
γ_Q	$(-\infty, \infty)$	Normal	0.40	0.20
ρ_R	(0,1)	Uniform	-	-
ρ_g	(0,1)	Uniform	-	-
ρ_z	(0,1)	Uniform	-	-
σ_R	$(0, \infty)$	InvGamma	0.40	4.00
σ_g	$(0, \infty)$	InvGamma	1.00	4.00
σ_z	$(0, \infty)$	InvGamma	0.50	4.00

Table 2: Calibrated parameters for HANK model

Parameter	Description	Value
h	Mean working hours	0.33
γ	Relative risk aversion	1.00
ν	Elasticity of demand	10.00
τ	Marginal tax rate on labor income	0.20
B_t^g	Government bond target	6.00
T	Lump-sum transfer per GDP	0.06

Table 3: Prior Distribution for HANK model

Parameter	Domain	Distribution	Mean	Std.Dev
ϕ_1	$(0, \infty)$	Normal	0.50	0.10
θ	$(0, \infty)$	Normal	100.00	10.00
ψ_1	$(0, \infty)$	Normal	1.25	0.20
ψ_2	$(0, \infty)$	Normal	0.10	0.05
σ_R	$(0, \infty)$	InvGamma	0.40	4.00
θ_R	(0,1)	Beta	0.25	0.10

5 Result

Table 4 and 5 show the structure parameters estimated from the posterior distributions. Every estimator is statistically significant.

Using those estimated parameters, I calculate impulse response function of aggregate variables to the monetary policy shock which is -5% shock (i.e., monetary easing) at $t = 0$ for 20 quarters horizon in RANK and HANK model as shown in Figure 2. The black dash line represents the impulse response from RANK model and the red line is that from HANK model. As standard Macroeconomics theory indicates, the monetary easing policy increases output and raises inflation, and then the effect is gradually diminishing. However, the magnitude of the effect in HANK model is much smaller than that in RANK model for every observation. This difference would indicate that the HANK model somewhat mutes the effect of monetary policy impact which RANK model might overestimate in its specification.

Figure 2: Impulse Response Function from Monetary Policy shock at $t = 0$

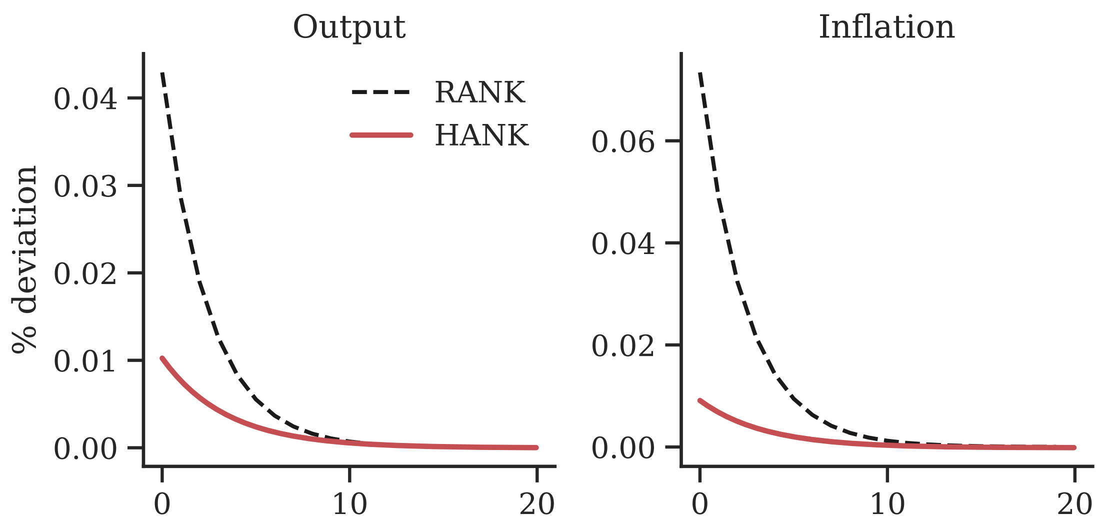


Figure 3: Impulse Response Function from Forward guidance

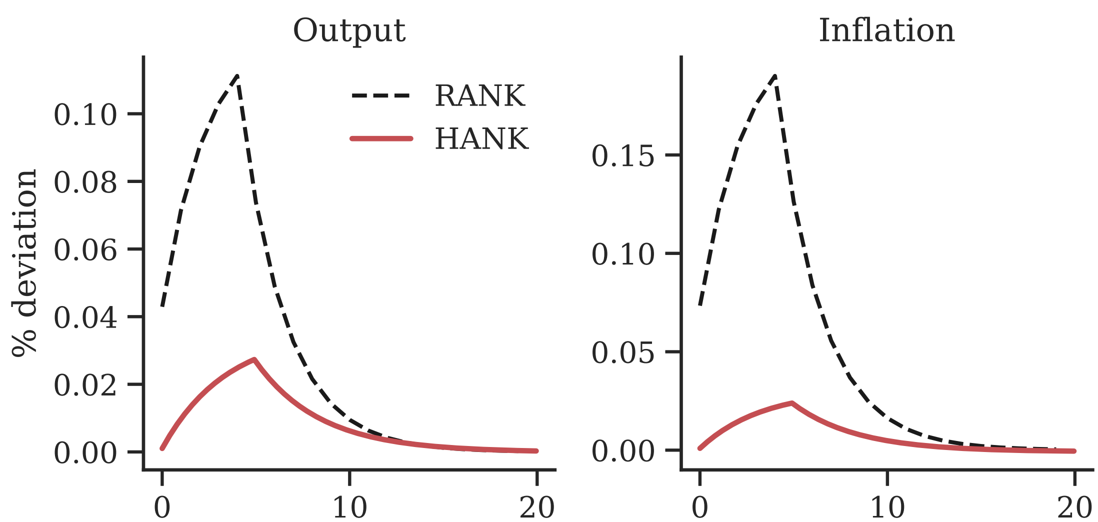


Figure 3 shows the impulse response function to the forward guidance which a central bank makes a commitment to continue monetary easing policy until $t = 5$. The impulse response function from HANK model implies the magnitude of the easing policy is smaller and the the momentum of the output and inflation increasing is slower than RANK model's prediction. This result is consistent with the discussion in McKay et al. (2016). HANK model constrains households to bind the borrowing behavior and make them face idiosyncratic income risk and such uncertainty reduces the effect of intertemporal smooth consumption. This mechanism in HANK model leads to the reduction of the effect of forward guidance as this impulse response function suggests.

Table 4: Estimated Posterior Distribution for RANK model

Parameter	Mean	Std.Dev	90% Confidence Interval
τ	3.5349	0.6377	(2.5482, 4.6415)
κ	0.1440	0.0429	(0.0811, 0.2178)
ψ_1	2.0274	0.3091	(1.5282, 2.5346)
ψ_2	0.9192	0.3555	(0.3976, 1.5794)
r_A	0.2703	0.1990	(0.0193, 0.6441)
π^*	1.4275	0.2922	(0.9913, 1.9448)
γ_Q	0.7650	0.1321	(0.5532, 0.9833)
ρ_R	0.8544	0.0255	(0.8114, 0.8946)
ρ_g	0.9891	0.0073	(0.9763, 0.999)
ρ_z	0.9529	0.0148	(0.9268, 0.9752)
σ_R	0.1685	0.0148	(0.1454, 0.1928)
σ_g	0.6757	0.0493	(0.5944, 0.7612)
σ_z	0.1896	0.0177	(0.1625, 0.2203)

Table 5: Estimated Posterior Distribution for HANK model

Parameter	Mean	Std.Dev	90% Confidence Interval
ϕ_1	0.5305	0.0922	(0.374, 0.6851)
θ	100.6775	10.2741	(83.7623, 117.4115)
ψ_1	1.2692	0.2100	(0.8844, 1.5926)
ψ_2	0.1051	0.0465	(0.031, 0.1843)
σ_R	0.4535	0.1847	(0.2567, 0.7995)
θ_R	0.2935	0.0978	(0.1427, 0.4613)

6 Conclusion

References

Ahn, SeHyoun, Greg Kaplan, Benjamin Moll, Thomas Winberry, and Christian Wolf, "When Inequality Matters for Macro and Macro Matters for Inequality," *National Bureau of Economic Research Working Paper*, 2017, (No.23494).

An, Sungbae and Frank Schorfheide, "Bayesian analysis of DSGE models," *Econometric reviews*, 2007, *26* (2-4), 113–172.

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante, "Monetary policy according to HANK," *American Economic Review*, 2018, *108* (3), 697–743.

McKay, Alisdair, Emi Nakamura, and Jón Steinsson, "The power of forward guidance revisited," *American Economic Review*, 2016, *106* (10), 3133–3158.

Sims, Christopher A., "Solving Linear Rational Expectations Models," *Computational Economics*, oct 2002, *20* (1-2), 1–20.

APPENDIX

A-1 Description of Parameters

Table 6: Description of Structure parameters in RANK model

Parameter	Description
τ	The intemporal elasticity of substitution
κ	Composite parameter in NKPC
ψ_1	Weight on inflation in monetary policy rule
ψ_2	Weight on the output gap in monetary policy rule
r_A	Discount factor
π^*	Target inflation rate
γ_Q	Steady state growth rate of technology
ρ_R	AR(1) coefficient on interest rate
ρ_g	AR(1) coefficient on government spending
ρ_z	AR(1) coefficient on shocks to the technology
σ_R	Std.dev of shocks to the nominal interest rate.
σ_g	Std.dev of shocks to the gov. spending process.
σ_z	Std.dev of shocks to the technology process.

Table 7: Description of Structure parameters in HANK model

Parameter	Description
ϕ_1	Frisch elasticity of labor supply
θ	Price stickiness
ψ_1	Weight on inflation in monetary policy rule
ψ_2	Weight on the output gap in monetary policy rule
σ_R	Volatility of monetary policy shocks
θ_R	Rate of mean reversion in monetary policy shocks

A-2 Markov chain Monte Carlo Algorithm

TODO: add details

Metropolis–Hastings algorithm (MH)

- Set prior distributions and start initial values of parameters. Then, draw proposed parameters from normal distribution with previous parameters as mean and a certain variance (random work sampling).

- If the proposed parameters are satisfied with the model conditions (e.g., parameter boundary and rational expectations solutions condition), calculate the acceptance probability defined as the following equation

$$q = \min\left[\frac{f(\theta_n^{\text{proposal}}|Y)}{g(\theta_{n-1}|Y)}, 1\right] \tag{A.2.1}$$

- The algorithm accept the proposed parameters $\theta_n^{\text{proposal}}$ in the probability q, if the parameters are accepted, update it. Recursive this procedure until reaching maximized iterations.

Sequential Monte Carlo algorithm (SMC)

- SMC, or also known Particle filters, is an alternative to Metropolis Hastings. First, create a sequence of intermediate distributions $\{\pi_n(\theta)\}_{n=0}^{N_\phi}$ that converge to the target posterior distribution, $\pi(\theta)$, where we call a sequence.

- Suppose ϕ_n for $n = 0, \cdots, N_\phi$ is a sequence that slowly increases from zero to one. Define a sequence of bridge distributions, $\{\pi_n(\theta)\}_{n=0}^{N_\phi}$ for $n = N_\phi$ and $\phi_n = 1$ as

$$\pi_n(\theta) = \frac{[f(Y|\theta)]^{\phi_n} f(\theta)}{\int [f(Y|\theta)]^{\phi_n} f(\theta) d\theta} \text{ for } n = 0, \cdots, N_\phi \tag{A.2.2}$$

- The rest of procedure is similar to MH algorithm.

TODO?: elaborate HANK's linearization procedure?