# Clue to Resolve Forward Guidance Puzzle: (Work in Progress/Initial Result) *

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### Abstract

This paper aims to find a clue to resolve forward guidance puzzle which central banks in the developed countries have struggled recently. The heterogeneous agent New Keynesian (HANK) model is one of the powerful candidates to give a solution to the forward guidance puzzle. I estimate HANK model along with standard Representative New Keynesian (RANK) model by Bayesian approach and calculate the impulse response function. The result suggests the HANK mutes the effect of forward guidance and it could be a hint for resolving forward guidance puzzle

*keywords:* Forward guidance puzzle, Heterogeneous agent New Keynesian, Bayesian method

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# 1 Model

In this section, I set up a Representative Agent New Keynesian (RANK) model following An and Schorfheide (2007) and a one-asset Heterogeneous Agent New Keynesian (HANK) model in continuous time following Ahn et al. (2017) and Kaplan et al. (2018), which are used to estimate structural parameters with Bayesian method and to calculate impulse responses from aggregate monetary policy shock.

## 1.1 Representative Agent New Keynesian model

The RANK model supposes that economy consists of a representative household, a final good producing firm, a continuum of intermediate goods producing firms, a government (fiscal authority) and a central bank.

Let $\hat{x}_t = \ln(x_t/x)$ denote the percentage deviation of a variable $x_t$ from its steady state $x$. Then, we can have the model with detrended variables as the following.

$$1 = E_t[e^{-\tau\hat{c}_{t+1}+\tau\hat{c}_t+\hat{R}_t-\hat{z}_{t+1}-\hat{\pi}_{t+1}}] \tag{1}$$

$$\frac{1-\nu}{\nu\phi\pi^2}(e^{\tau\hat{c}_t}-1) = (e^{\hat{\pi}_t}-1)\left[\left(1-\frac{1}{2\nu}\right)e^{\hat{\pi}_t}+\frac{1}{2\nu}\right] \tag{2}$$

$$-\beta E_t[e^{-\tau\hat{c}_{t+1}+\tau\hat{c}_t+\hat{R}_t-\hat{z}_{t+1}-\hat{\pi}_{t+1}}]$$

$$e^{\hat{c}_t-\hat{y}_t} = e^{-\hat{g}_t}-\frac{\phi\pi^2 g}{2}(e^{\hat{\pi}_t}-1)^2 \tag{3}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1}+(1-\rho_R)\psi_1\hat{\pi}_t+(1-\rho_R)\psi_2(\Delta\hat{y}_t-\hat{z}_t)+\epsilon_{R,t} \tag{4}$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1}+\epsilon_{g,t} \tag{5}$$

$$\hat{z}_t = \rho_z\hat{z}_{t-1}+\epsilon_{z,t} \tag{6}$$

Since the equation (1)-(6) are a nonlinear rational expectations system, it is necessary to linearlize them to solve the model. Applying log linearization, the model can be

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written as

$$\hat{y}_t = E_t[\hat{y}_{t+1}] + \hat{g}_t - E_t[\hat{g}_{t+1}] - \frac{1}{\gamma}\left(\hat{R}_t - \hat{E}_t[\pi_{t+1}] - E_[\hat{z}_{t+1}]\right) \tag{7}$$

$$\hat{\pi}_t = \beta\hat{E}_t[\hat{\pi}_{t+1}] + \kappa(\hat{y}_t - \hat{g}_t) \tag{8}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\psi_1\hat{\pi}_t + (1-\rho_R)\psi_2(\Delta\hat{y}_t - \hat{z}_t) + \epsilon_{R,t} \tag{9}$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + \epsilon_{g,t} \tag{10}$$

$$\hat{z}_t = \rho_z\hat{z}_{t-1} + \epsilon_{z,t} \tag{11}$$

where $\kappa = \tau(1-\nu)/(\nu\pi^2\phi)$. To solve the model described as equation (7)-(11), I follow the solution algorithm provided by Sims (2002). The linearlized expressions and rational expectations solution of the model via solution algorithm lead to a state-space representation of the DSGE model which can be analyzed with the Kalman filter. Those allow us to conduct estimation of DSGE model with Bayesian method by likelihood approach.

## 1.2   One-asset Heterogeneous Agent New Keynesian model

Like RANK model, one-asset HANK model also assumes that the model economy consists of a representative final good producing firm, a continuum of intermediate goods producing firms, government and a central banks. However, HANK model considers heterogeneous households who face uninsurable, idiosyncratic income risk and borrowing constraints. This realistic modification in the model would allow us to capture a robust monetary policy effect.

Also, following the model specification by Ahn et al. (2017), HANK model is presented in continuous time rather than discrete time. This is because continuous time is easier to capture occasionally binding constraints and inactions, which HANK model cares about.

### 1.2.1 Household

In the model economy, there is a continuum of households with indexed by $j \in [0,1]$ who have preferences represented by the expected utility function. Households receive utility from a consumption good $c_{j,t}$ and disutility from hours worked $l_{j,t}$. Then the household maximizes

$$E_0 \int_0^\infty e^{-\rho t} \left( \frac{c_{j,t}^{1-\theta}}{1-\theta} - \phi_0 \frac{l_{j,t}^{1+\frac{1}{\phi_1}}}{1+\frac{1}{\phi_1}} \right) dt \tag{12}$$

where $\rho$ is the discount factor, $1/\theta$ is the coefficient of relative risk aversion and $\phi_1$ is Frisch elasticity of labor supply. At the each time $t$, a household face idiosyncratic productivity shock $z_{j,t} \in \{z_L, z_H\}$ with $z_L < z_H$ (high and low productivity), which the expectation is taken over. Households transit between this two values for labor productivity following a Poisson process. Households can only trade in productive asset $a_t$ at the real interest rate $r_t$. Therefore, households face two different heterogeneous property: labor productivity $z_t$ and their asset position $a_t$. A household optimizes the decision problem subject to the following equations

$$a_{t+1} = (1 - \tau_t)w_t z_t l_t + r_t a_t + T_t + D_t - c_t \tag{13}$$

$$a_{t+1} \geq 0 \tag{14}$$

where $\tau$ is tax rate of income, $w_t$ is wage, $T_t$ is lump-sum transfer and $D_t$ is residual profit from firms. The household budget constraint contains stochastic idiosyncratic productivity shock ($z_t$) and borrowing constraing (equation (14)). Those contraints and uncertainty may cause the decline in households' ability to smooth consumption (intertemporal substitution) which the forward guidance mainly rely on its effect.

### 1.2.2 Firm

The model setup for the firms is basically the same as RANK model I elaborate so far. That is, HANK model also assumes there are final good producing firm which

use intermediate goods from monopolistically competitive firms.

### 1.2.3 Central bank and Government

A Central bank decides the nominal interest rate according to Taylor rule.

$$R_t = r_t^* + \phi_\pi \pi_t + \phi_y(y_t - y^*) + \epsilon_{R,t} \tag{15}$$

$r_t^*$ is the steady state of real interest rate, $y^*$ denotes steady state of output and $\epsilon_R, t$ represents the monetary policy shock, which follows

$$d\epsilon_{R,t} = -\theta_R \epsilon_{R,t} + \sigma_{R,t} dW_t \tag{16}$$

Government faces exogenous expenditures $G_t$ and raise revenue via tax on household labor income $w_t z_t l_t$ that consists of lump-sum transfer $T_t$. Only government can issue liquid assets in the economy, which are real bonds of infinitesimal maturity $B_t^g$. Government's an intertemporal budget constraint is

$$B_t^g + G_t + T_t = \pi_t \int w_t z_t l_t(a, z) g_t(a, z) da dz + r_t B_t^g \tag{17}$$

### 1.2.4 Equilibrium Relationship

For households' equilibrium, we make use of the Hamilton-Jacobi-Bellman (HJB) equation from the model set up as the following.

$$\rho V(a, z) = \max_{c,l} u(c, l) + ((1 - \tau)wzl + ra + T + D - c)\partial_a V(a, z) \tag{18}$$
$$+ \lambda(z)(V(a, z') - V(a, z))$$

where $\lambda$ is transition probabilities of idiosyncratic labor productivity $z$ (follow a Poisson process with arrival rates $\lambda_L$ and $\lambda_H$).

Although firm's equilibrium can be obtained to solve a maximization problem of profit like RANK model, one needs to adjust a continuous-time form. I follow Kaplan

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et al. (2018) which presents the combination of a continuous-time formulation of the problem and quadratic price adjustment costs. The identification yields a simple equation characterizing a New Keynesian Phillips Curve without the need for log linearization.

$$\left(r_t - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\nu}{\theta}(m_t - m^*) + \dot{\pi}_t, \quad m^* = \frac{\nu - 1}{\nu} \tag{19}$$

where $\theta$ is price stickiness, $m_t$ is marginal cost which is assumed to be common across all intermediate good producers.

The bond market clearing condition is given by

$$B_t^g = \int a g_t(a, z) da dz \tag{20}$$

Also, the labor market clearing condition can be written

$$\int z l_t(a, z) g_t(a, z) da dg = L_t \tag{21}$$

where $L_t$ is aggregate level of labor demand.

### 1.2.5 Linearlization

In order to solve HANK model, we need to linearlize the model. I employ the three step linearization procedure provided by Ahn et al. (2017).

- **Step 1: Approximate Steady State**

  Solve for the steady state of the model without aggregate shocks but with idiosyncratic shocks.

- **Step 2: Linearize Equilibrium Conditions**

  Take a first-order Taylor expansion of the discretized equilibrium conditions around the steady state. Note that heterogeneous agent models have too large size of variables to estimate it. Thus, in addition to linearization, we need to conduct model reduction procedure.

- **Step 3: Solve Linear System**

  Like RANK model, solve the model which can be obtain from the step 2 with standard solution algorithm. I utilize Sims (2002) method as the same as RANK model in the estimation.

# 2 Estimation Method

To estimate the structure parameters in both RANK and HANK model, I use Bayesian approach based on likelihood evaluation. Through linearization procedure, the both models can be expressed in the state space model, which allows us to enjoy the Kalman filter to calculate likelihood.

Although Bayesian approach has several merits for estimating dynamic Macroeconomics models, the most crucial point is that the approach is more likely to be able to find the optimal solutions. When one implements ordinary maximum likelihood estimation for DSGE model, since the log-likelihood function would be very flat around the optimum, and the estimates are very sensitive to their initial values, the solutions easily fall into local maxima. On the other hand, by combining estimation process with sampling algorithms, Bayesian approach can provide estimations which are robust for local maxima.

Suppose data as $Y = (y_1, \cdots, y_r)$, the Bayesian estimation procedure is the following.

i. Set the posterior distributions of parameters $f(\theta)$.

ii. Based on the Bayesian Theorem,

$$f(\theta|Y) = \frac{f(Y|\theta)f(\theta)}{\int f(Y|\theta)f(\theta)d\theta} \tag{22}$$

   calculate posterior distributions $f(\theta|Y)$.

iii. Using posterior distributions, estimate the structure parameters. The denominator of the Bayesian Theorem is independent from the parameters and we

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can't calculate it although the likelihood $f(Y|\theta)$ can be available via Kalman filter. In this case, we should make use of Markov chain Monte Carlo (MCMC) methods to sample parameters from the posterior distribution.

Among several methods of MCMC family, I adopt the Metropolis–Hastings (MH) algorithm for RANK model estimation and the Sequential Monte Carlo (SMC) algorithm for HANK model estimation. The MH algorithm is much simpler than SMC but MH sampler can't be parallelized for the computation. Meanwhile, since HANK model needs to take additional procedure and manipulate large size of variables as I described in the model section, it takes much longer time to draw proposed parameters than RANK model. Therefore, I use SMC for the HANK model estimation.

In the estimation, I choose 300,000 sampling for the MH algorithm (but discard 200,000 drawings as the burn-in and use the rest for evaluating posterior distribution). Also, SMC algorithm takes 3,000 particles for the estimation.

# 3   Data and Prior distribution

The data used for estimation are the real output per capita, the inflation rate and the nominal interest rate in the U.S.. The real output per capita is calculated as quarter-to-quarter percent change of real GDP per capita. Annualized quarter-to-quarter percent change of CPI urban consumers index is used as the inflation. The nominal interest rate is the Federal Funds Rate at a quarterly rate annualized. The data period is from 1990:Q1 to 2019:Q4. All data is obtained from Federal Reserve Bank of St. Louis' economic database (FRED). Figure 1 shows the time-series of those data.

**Figure 1: Time series Macro data of U.S.**
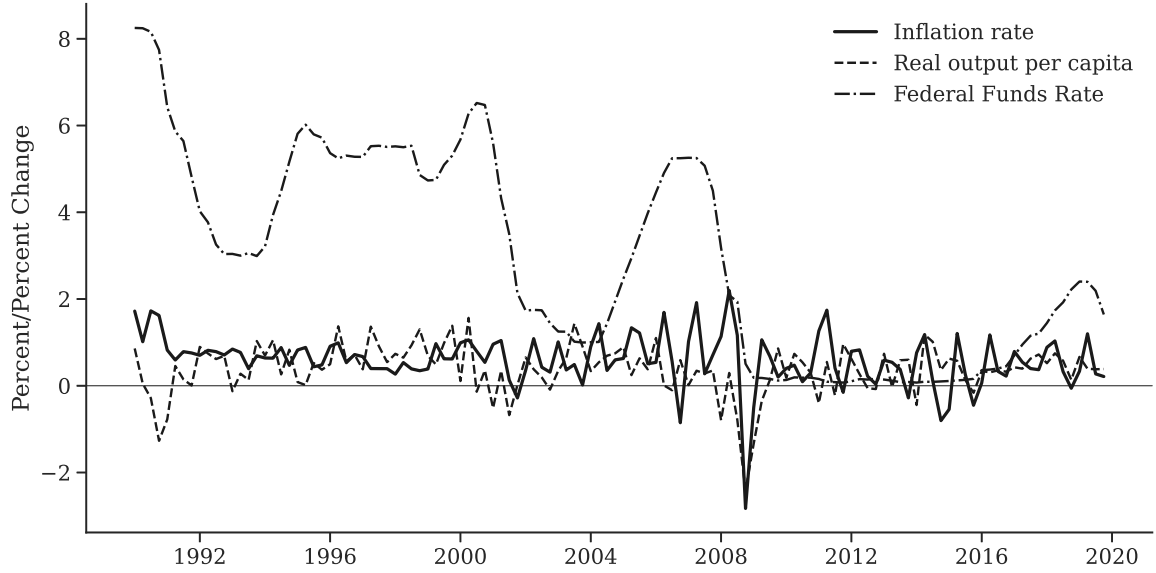


The Bayesian estimation needs the prior distributions for parameters. Table 1 provides the prior distributions for parameters in RANK model, which are set as following An and Schorfheide (2007) and New York Fed's DSGE model (FEDNY-DSGE) specifications. For HANK model, I calibrate six parameters as shown in table 2 and table 3 shows prior settings of estimated parameters. Both calibration and prior settings for HANK model are based on FEDNY-DSGE specification.

**Table 1:** Prior Distribution for RANK model

| Parameter | Domain | Distribution | Mean | Std.Dev |
|:---:|:---:|:---:|:---:|:---:|
| $\tau$ | $(0, \infty)$ | Gamma | 2.00 | 0.50 |
| $\kappa$ | $(0,1)$ | Uniform | - | - |
| $\psi_1$ | $(0, \infty)$ | Gamma | 1.50 | 0.25 |
| $\psi_2$ | $(0, \infty)$ | Gamma | 0.50 | 0.25 |
| $r_A$ | $(0, \infty)$ | Gamma | 0.50 | 0.50 |
| $\pi^*$ | $(0, \infty)$ | Gamma | 7.00 | 2.00 |
| $\gamma_Q$ | $(-\infty, \infty)$ | Normal | 0.40 | 0.20 |
| $\rho_R$ | $(0,1)$ | Uniform | - | - |
| $\rho_g$ | $(0,1)$ | Uniform | - | - |
| $\rho_z$ | $(0,1)$ | Uniform | - | - |
| $\sigma_R$ | $(0, \infty)$ | InvGamma | 0.40 | 4.00 |
| $\sigma_g$ | $(0, \infty)$ | InvGamma | 1.00 | 4.00 |
| $\sigma_z$ | $(0, \infty)$ | InvGamma | 0.50 | 4.00 |

**Table 2:** Calibrated parameters for HANK model

| Parameter | Description | Value |
|:---:|:---|:---:|
| $h$ | Mean working hours | 0.33 |
| $\gamma$ | Relative risk aversion | 1.00 |
| $\nu$ | Elasticity of demand | 10.00 |
| $\tau$ | Marginal tax rate on labor income | 0.20 |
| $B_t^g$ | Government bond target | 6.00 |
| $T$ | Lump-sum transfer per GDP | 0.06 |

**Table 3:** Prior Distribution for HANK model

| Parameter | Domain | Distribution | Mean | Std.Dev |
|:---:|:---:|:---:|:---:|:---:|
| $\phi_1$ | $(0, \infty)$ | Normal | 0.50 | 0.10 |
| $\theta$ | $(0, \infty)$ | Normal | 100.00 | 10.00 |
| $\psi_1$ | $(0, \infty)$ | Normal | 1.25 | 0.20 |
| $\psi_2$ | $(0, \infty)$ | Normal | 0.10 | 0.05 |
| $\sigma_R$ | $(0, \infty)$ | InvGamma | 0.40 | 4.00 |
| $\theta_R$ | $(0,1)$ | Beta | 0.25 | 0.10 |

# 4 Result

Table 4 and 5 show the structure parameters estimated from the posterior distributions. Every estimator is statistically significant.

Using those estimated parameters, I calculate impulse response function of aggregate variables to the monetary policy shock which is -5% shock (i.e., monetary easing) at $t = 0$ for 20 quarters horizon in RANK and HANK model as shown in Figure 2. The black dash line represents the impulse response from RANK model and the red line is that from HANK model. As standard Macroeconomics theory indicates, the monetary easing policy increases output and raises inflation, and then the effect is gradually diminishing. However, the magnitude of the effect in HANK model is much smaller than that in RANK model for every observation. This difference would indicate that the HANK model somewhat mutes the effect of monetary policy impact which RANK model might overestimate in its specification.

**Figure 2: Impulse Response Function from Monetary Policy shock at $t = 0$**
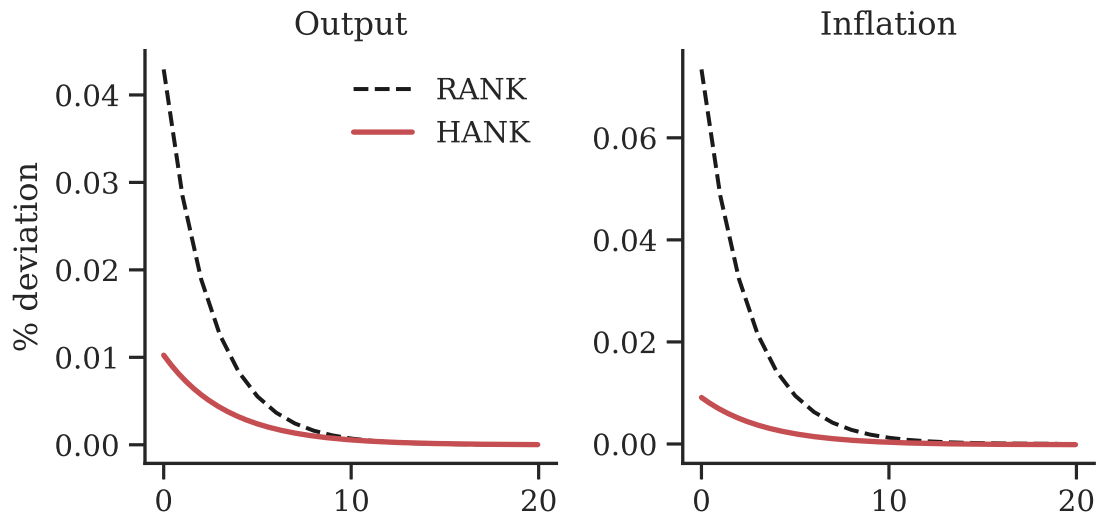


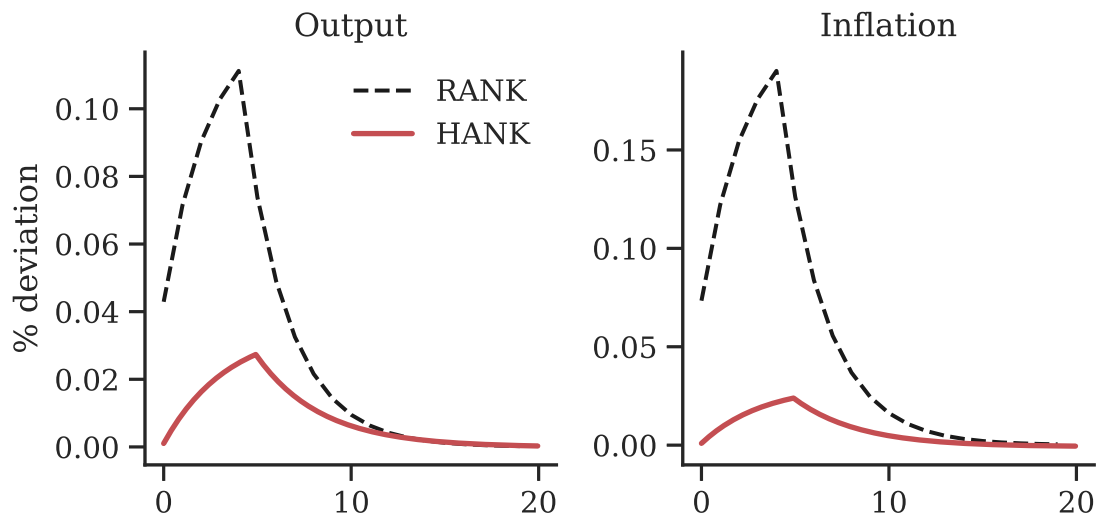**Figure 3: Impulse Response Function from Forward guidance**



Figure 3 shows the impulse response function to the forward guidance which a central bank makes a commitment to continue monetary easing policy until $t = 5$. The impulse response function from HANK model implies the magnitude of the easing policy is smaller and the the momentum of the output and inflation increasing is slower than RANK model's prediction. This result is consistent with the discussion in McKay et al. (2016). HANK model constrains households to bind the borrowing behavior and make them face idiosyncratic income risk and such uncertainty reduces

the effect of intertemporal smooth consumption. This mechanism in HANK model leads to the reduction of the effect of forward guidance as this impulse response function suggests.

**Table 4:** Estimated Posterior Distribution for RANK model

| Parameter | Mean | Std.Dev | 90% Confidence Interval |
|:---:|:---:|:---:|:---:|
| $\tau$ | 3.5349 | 0.6377 | (2.5482, 4.6415) |
| $\kappa$ | 0.1440 | 0.0429 | (0.0811, 0.2178) |
| $\psi_1$ | 2.0274 | 0.3091 | (1.5282, 2.5346) |
| $\psi_2$ | 0.9192 | 0.3555 | (0.3976, 1.5794) |
| $r_A$ | 0.2703 | 0.1990 | (0.0193, 0.6441) |
| $\pi^*$ | 1.4275 | 0.2922 | (0.9913, 1.9448) |
| $\gamma_Q$ | 0.7650 | 0.1321 | (0.5532, 0.9833) |
| $\rho_R$ | 0.8544 | 0.0255 | (0.8114, 0.8946) |
| $\rho_g$ | 0.9891 | 0.0073 | (0.9763, 0.999) |
| $\rho_z$ | 0.9529 | 0.0148 | (0.9268, 0.9752) |
| $\sigma_R$ | 0.1685 | 0.0148 | (0.1454, 0.1928) |
| $\sigma_g$ | 0.6757 | 0.0493 | (0.5944, 0.7612) |
| $\sigma_z$ | 0.1896 | 0.0177 | (0.1625, 0.2203) |

**Table 5:** Estimated Posterior Distribution for HANK model

| Parameter | Mean | Std.Dev | 90% Confidence Interval |
|:---:|:---:|:---:|:---:|
| $\phi_1$ | 0.5305 | 0.0922 | (0.374, 0.6851) |
| $\theta$ | 100.6775 | 10.2741 | (83.7623, 117.4115) |
| $\psi_1$ | 1.2692 | 0.2100 | (0.8844, 1.5926) |
| $\psi_2$ | 0.1051 | 0.0465 | (0.031, 0.1843) |
| $\sigma_R$ | 0.4535 | 0.1847 | (0.2567, 0.7995) |
| $\theta_R$ | 0.2935 | 0.0978 | (0.1427, 0.4613) |

# References

**Ahn, SeHyoun, Greg Kaplan, Benjamin Moll, Thomas Winberry, and Christian Wolf**, "When Inequality Matters for Macro and Macro Matters for Inequality," *National Bureau of Economic Research Working Paper*, 2017, (No.23494).

**An, Sungbae and Frank Schorfheide**, "Bayesian analysis of DSGE models," *Econometric reviews*, 2007, *26* (2-4), 113–172.

**Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante**, "Monetary policy according to HANK," *American Economic Review*, 2018, *108* (3), 697–743.

**McKay, Alisdair, Emi Nakamura, and Jón Steinsson**, "The power of forward guidance revisited," *American Economic Review*, 2016, *106* (10), 3133–3158.

**Sims, Christopher A.**, "Solving Linear Rational Expectations Models," *Computational Economics*, oct 2002, *20* (1-2), 1–20.

# APPENDIX

## A-1   Description of Parameters

**Table 6:** Description of Structure parameters in RANK model

| Parameter | Description |
|:---:|:---|
| $\tau$ | The intemporal elasticity of substitution |
| $\kappa$ | Composite parameter in NKPC |
| $\psi_1$ | Weight on inflation in monetary policy rule |
| $\psi_2$ | Weight on the output gap in monetary policy rule |
| $r_A$ | Discount factor |
| $\pi^*$ | Target inflation rate |
| $\gamma_Q$ | Steady state growth rate of technology |
| $\rho_R$ | AR(1) coefficient on interest rate |
| $\rho_g$ | AR(1) coefficient on government spending |
| $\rho_z$ | AR(1) coefficient on shocks to the technology |
| $\sigma_R$ | Std.dev of shocks to the nominal interest rate. |
| $\sigma_g$ | Std.dev of shocks to the gov. spending process. |
| $\sigma_z$ | Std.dev of shocks to the technology process. |

**Table 7:** Description of Structure parameters in HANK model

| Parameter | Description |
|:---:|:---|
| $\phi_1$ | Frisch elasticity of labor supply |
| $\theta$ | Price stickiness |
| $\psi_1$ | Weight on inflation in monetary policy rule |
| $\psi_2$ | Weight on the output gap in monetary policy rule |
| $\sigma_R$ | Volatility of monetary policy shocks |
| $\theta_R$ | Rate of mean reversion in monetary policy shocks |

## A-2   Markov chain Monte Carlo Algorithm

TODO: add details

**Metropolis–Hastings algorithm (MH)**

- Set prior distributions and start initial values of parameters. Then, draw proposed parameters from normal distribution with previous parameters as mean and a certain variance (random work sampling).

- If the proposed parameters are satisfied with the model conditions (e.g., parameter boundary and rational expectations solutions condition), calculate the

acceptance probability defined as the following equation

$$q = \min\left[\frac{f(\theta_n^{\text{proposal}}|Y)}{g(\theta_{n-1}|Y)}, 1\right] \qquad \text{(A.2.1)}$$

- The algorithm accept the proposed parameters $\theta_n^{\text{proposal}}$ in the probability $q$, if the parameters are accepted, update it. Recursive this procedure until reaching maximized iterations.

**Sequential Monte Carlo algorithm (SMC)**

- SMC, or also known Particle filters, is an alternative to Metropolis Hastings. First, create a sequence of intermediate distributions $\{\pi_n(\theta)\}_{n=0}^{N_\phi}$ that converge to the target posterior distribution, $\pi(\theta)$, where we call a sequence.

- Suppose $\phi_n$ for $n = 0, \cdots, N_\phi$ is a sequence that slowly increases from zero to one. Define a sequence of bridge distributions, $\{\pi_n(\theta)\}_{n=0}^{N_\phi}$ for $n = N_\phi$ and $\phi_n = 1$ as

$$\pi_n(\theta) = \frac{[f(Y|\theta)]^{\phi_n} f(\theta)}{\int [f(Y|\theta)]^{\phi_n} f(\theta)d\theta} \text{ for } n = 0, \cdots, N_\phi \qquad \text{(A.2.2)}$$

- The rest of procedure is similar to MH algorithm.

TODO?: elaborate HANK's linearization procedure?