Cal Poly SLO Hyperloop Propulsion CDR

Kent Roberts

Propellant Selection

Gas	Molecular Weight	Specific Impulse (sec)	∝ Total Impulse per unit Volume
Air	28.9	74	2138.6
Argon	39.9	57	2274.3
CO2	44	67	2948
Helium	4	179	716
Hydrogen	2	296	592
Nitrogen	28	80	2240
Methane	16	114	1824

- Air is widely available for a test campaign, performance runs where $T<0^{\circ}C$, use nitrogen

Rocket Equation

$$\Delta v = I_{sp} g_0 ln \frac{m_0}{m_f}$$

m_F=Inert Pod Mass (Non-Tank Pod mass + Tank Mass + Residual propellant)
m_0=Wet Pod Mass (m_F + Propellant Mass)

$$m_0 = m_f + \Delta m$$

$$\Delta m = V(\rho_1 - \rho_2)$$

Isentropic Processes and Tank Discharge Limitations

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{\rho_2}{\rho_1}\right)^{(\gamma-1)}$$

$$\rho_2 = \rho_1 max\left[\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}, \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right]$$

T_2=Lower Temperature Limit of the Tank & Valve hardware

P_2=Lower Pressure Limit (function of P_{nozzle_inlet} and Flow Coefficient of Hardware C_v)

*Most systems with T_2~-40F and P_2~500psi are Temperature limited

Rocket Eq. & Isentropic Process

$$\Delta v = I_{sp} g_0 \ln\left(1 + \frac{V\rho_1\left(1 - max\left[\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}, \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right]\right)}{m_f}\right)$$

Use the Ideal Gas Law to find ρ_1

$$\rho_1 = \frac{P_1(MM)}{RT_1}$$

$$\Delta v = I_{sp} g_0 \ln\left(1 + \frac{V\frac{P_1(MM)}{RT_1}\left(1 - max\left[\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}, \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right]\right)}{m_f}\right)$$

ΔV as a Function of Pod Inert Mass and Tank Specifications

$$m_f = m_{InertPod} + m_{Tank} + m_{ResidualProp}$$

$$m_{ResidualProp} = \rho_2 V$$

$$\Delta v = I_{sp} g_0 \ln\left(1 + \frac{V \frac{P_1 (MM)}{R T_1}\left(1 - max\left[\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}, \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right]\right)}{m_{InertPod} + m_{Tank} + V \frac{P_1 (MM)}{R T_1} max\left[\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}, \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right]}\right)$$

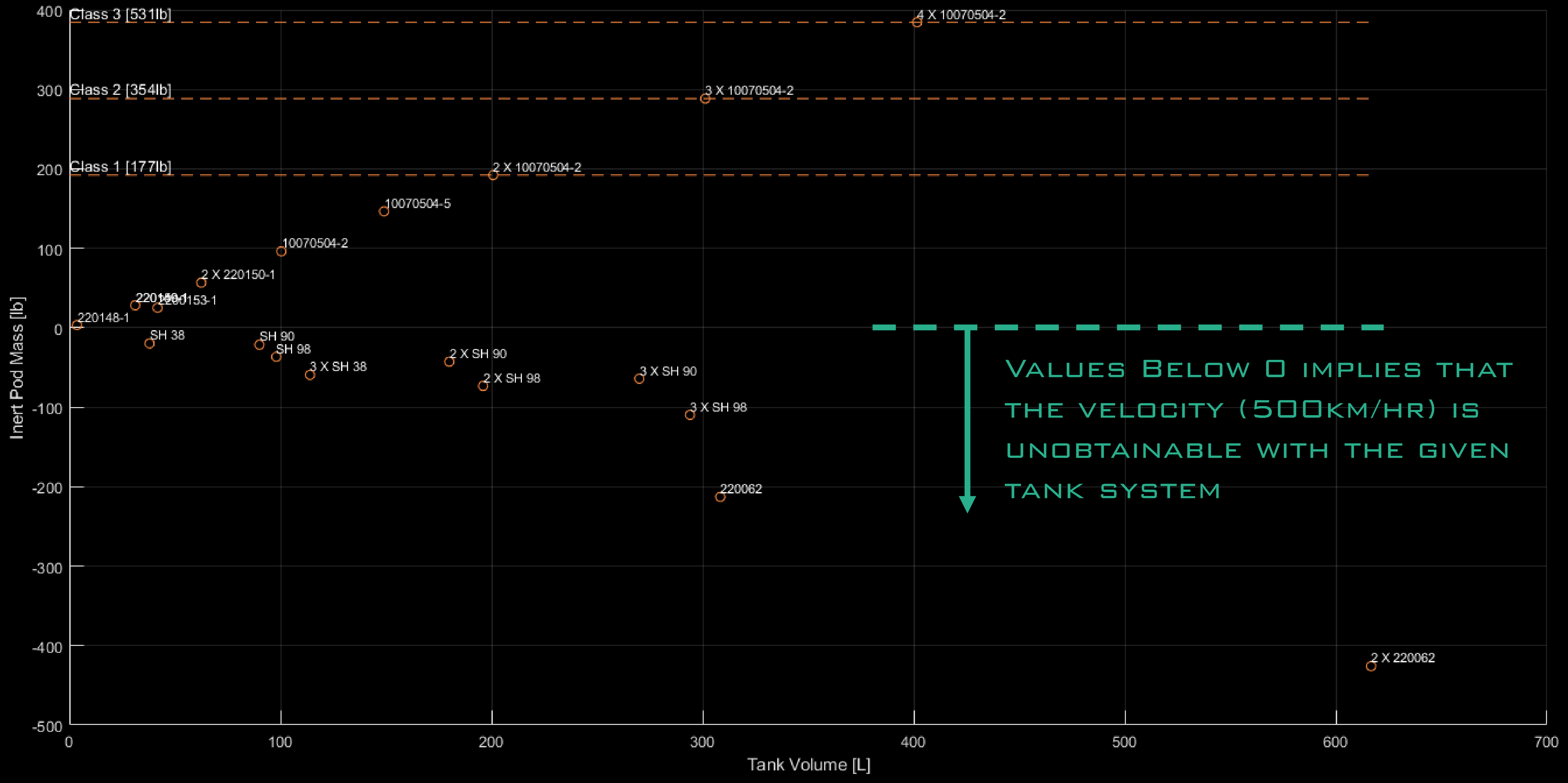
$$\Delta v = I_{sp} g_0 \ln\left(1 + \frac{V(\rho_1 - \rho_2)}{m_{InertPod} + m_{Tank} + V \rho_2}\right)$$

Pod Inert mass as a function of ΔV and Tank Specifications

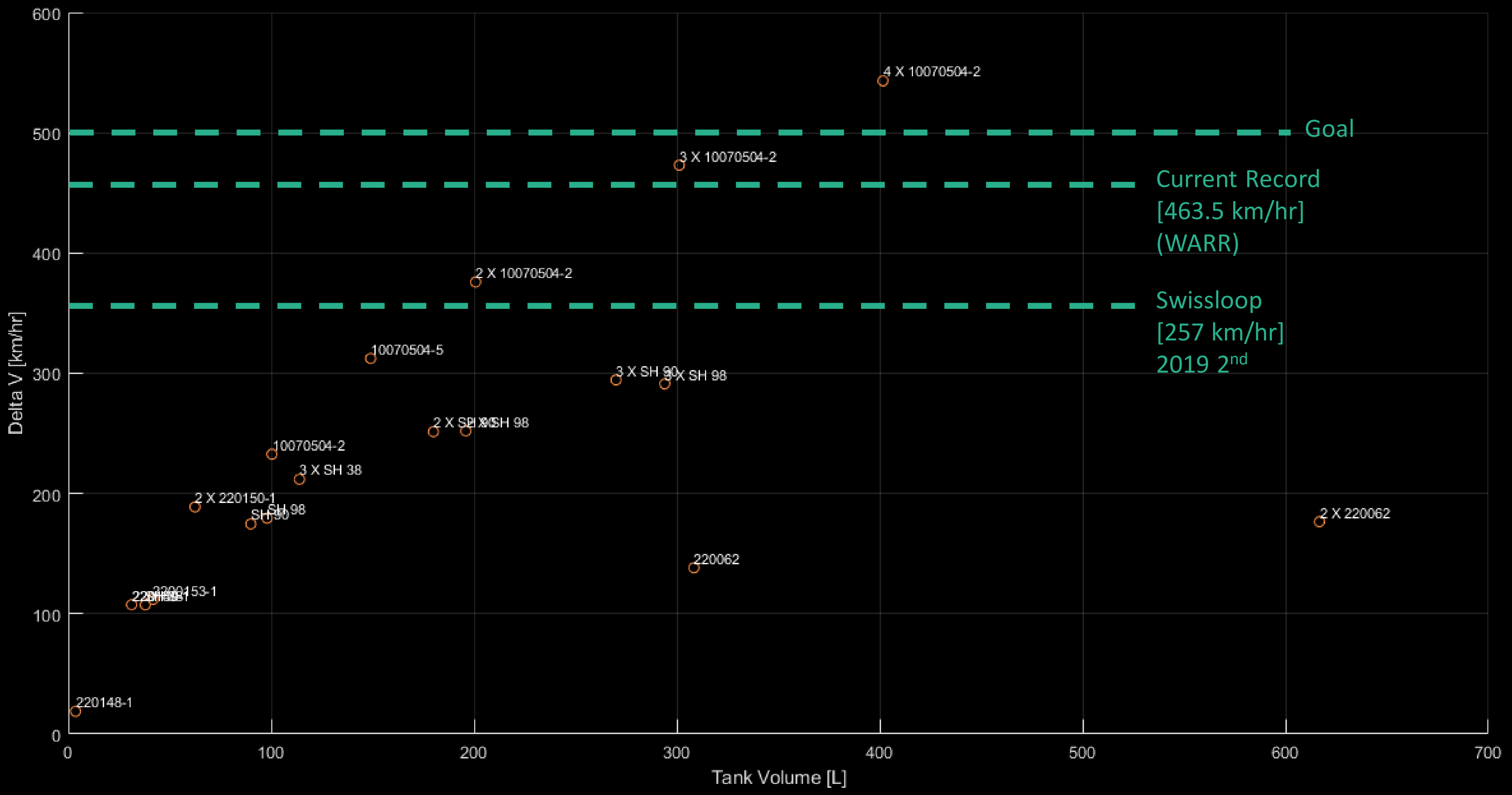
$$m_{InertPod} = V\frac{P_1(MM)}{RT_1}\left(1 - max\left[\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}, \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right]\right)\left(e^{\frac{\Delta v}{I_{sp}g_0}} - 1\right)^{-1} - m_{tank} - V\frac{P_1(MM)}{RT_1}\left(max\left[\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}, \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right]\right)$$

$$m_{InertPod} = \left(e^{\frac{\Delta v}{I_{sp}g_0}} - 1\right)^{-1} V(\rho_1 - \rho_2) - m_{tank} - V\rho_2$$

Pod Inert Mass v. Tank System ΔV=500km/hr



ΔV v. Tank System $M_{InertPod}$=322[lb]



Tank Dimensions

- Generally, tank systems were only considered if they were within reasonable dimensions
- For a more detailed breakdown, reference the google sheet document.
- An initial design is further presented with the 2X10070504-2 configuration and a 2 X SH 90 alternative (Former sourced from General Dynamics, later from Steelhead Composites)
  - 10070504-2 [15.2inØ X 42.3in]
  - SH 90 [17.1inØ X 39in]

Mass Flow Rate

Assume a constant flow rate

$$\dot{m} = \frac{\Delta m}{t_{MECO}}$$

$$m(t) = m_0 - \dot{m}t$$

$$a(t) = \frac{F}{m(t)} = \frac{I_{sp} g_0 \Delta m}{t_{MECO} m(t)}$$

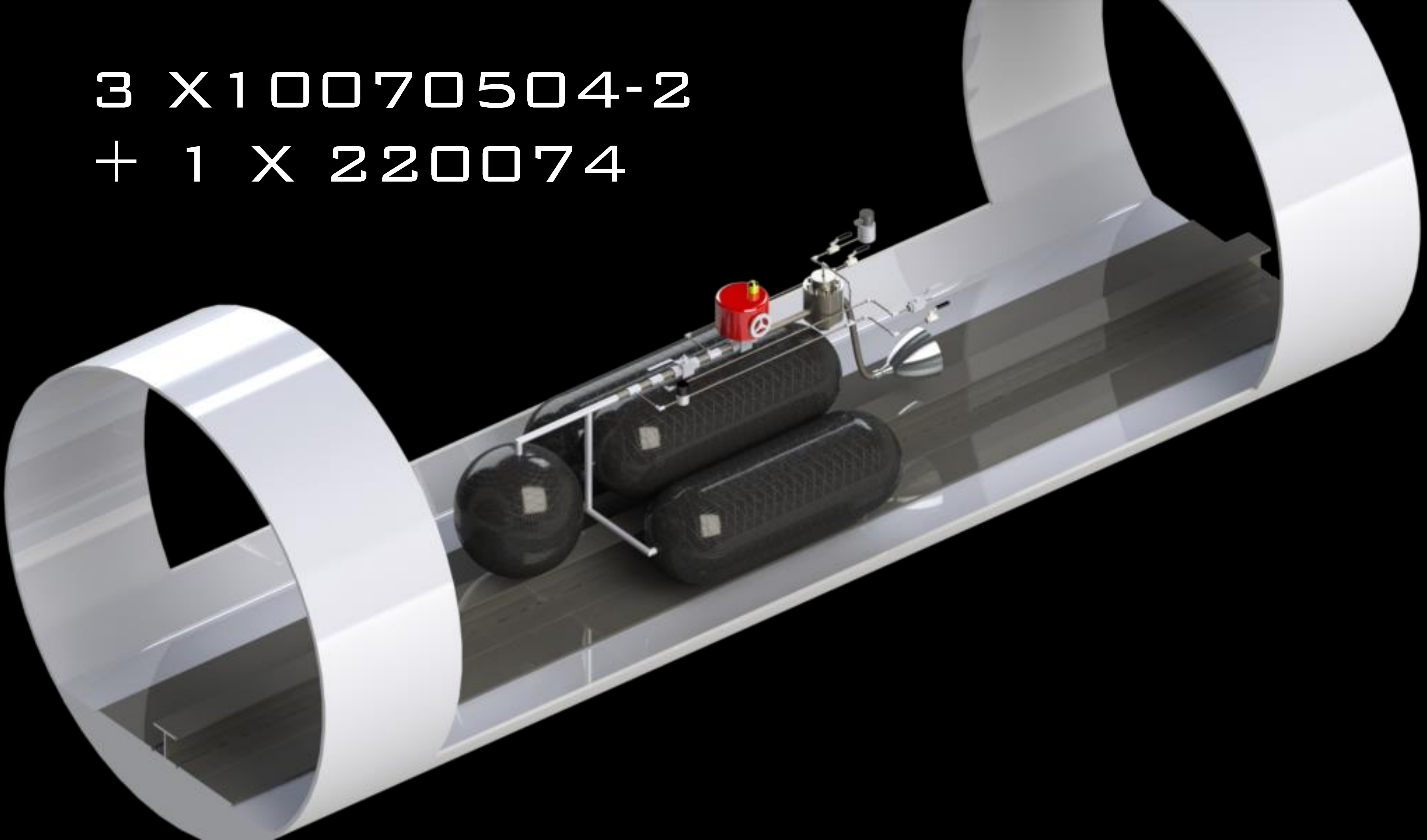
$$v(t) = -I_{sp} g_0 \ln\left[m(t)\right] + I_{sp} g_0 ln(m_0)$$

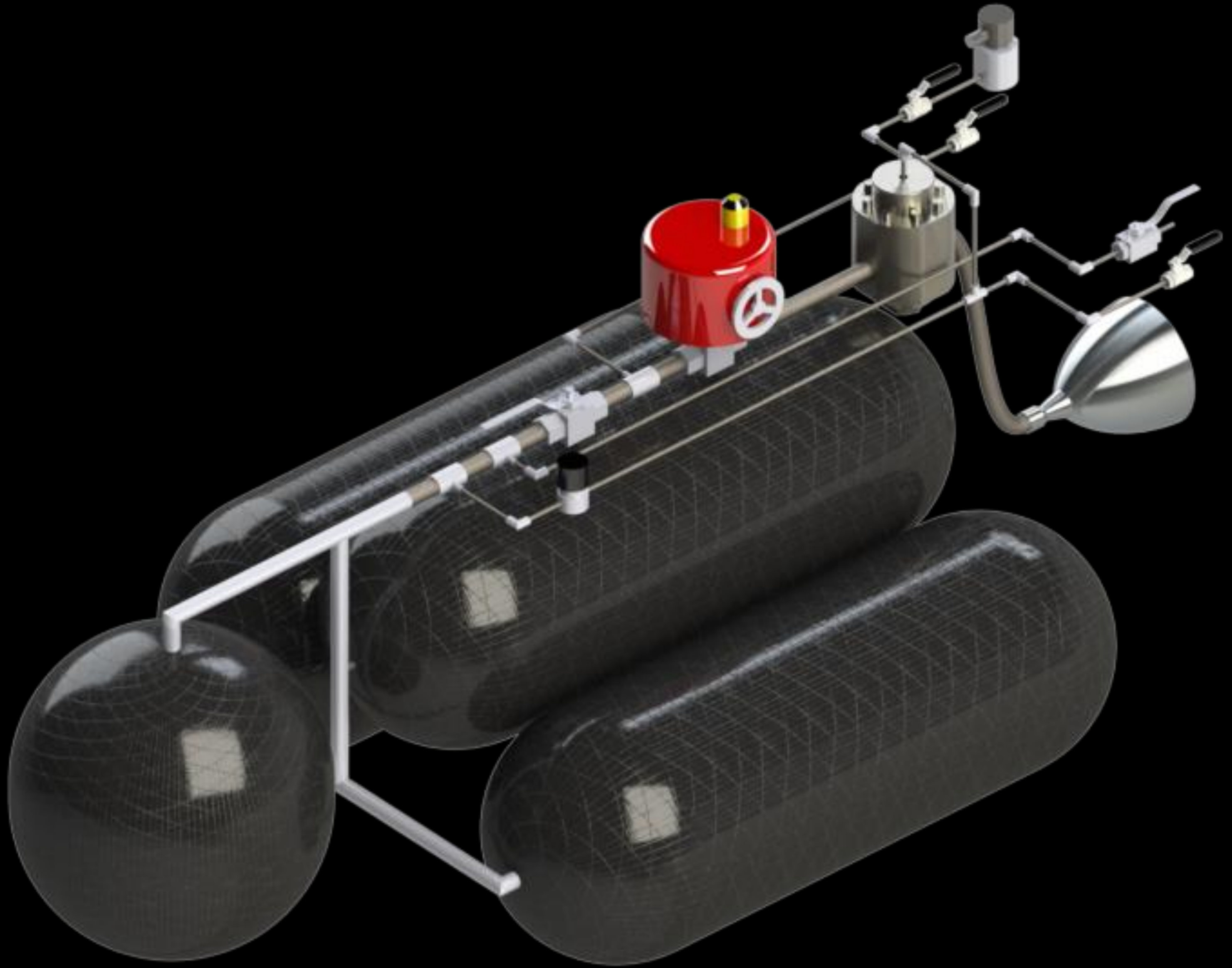
$$x(t) = I_{sp} g_0 \frac{t_{MECO}}{\Delta m}\left[m(t)\ln[m(t)] - m(t)\right] + I_{sp} g_0 \ln(m_0) t - \frac{I_{sp} g_0 t_{MECO}}{\Delta m}\left[m_0 \ln(m_0) - m_0\right]$$

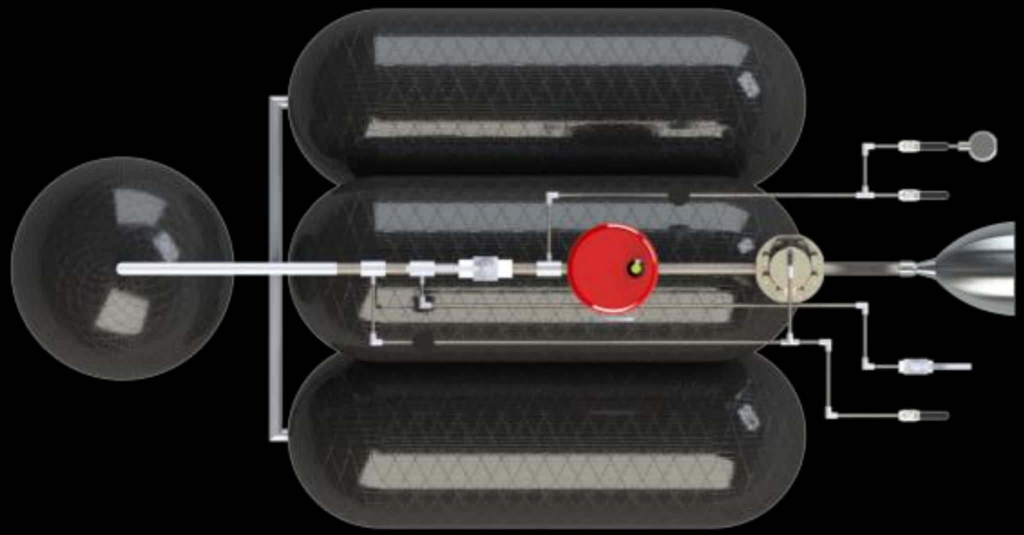
$$t_{MECO} = x_{ThrustLen}\left[I_{sp} g_0 \left(\frac{1}{\Delta m}\left(m_f \ln(m_f) - (m_f - \Delta m)\ln(m_f - \Delta m) - \Delta m\right) + \ln(m_f - \Delta m)\right)\right]^{-1}$$

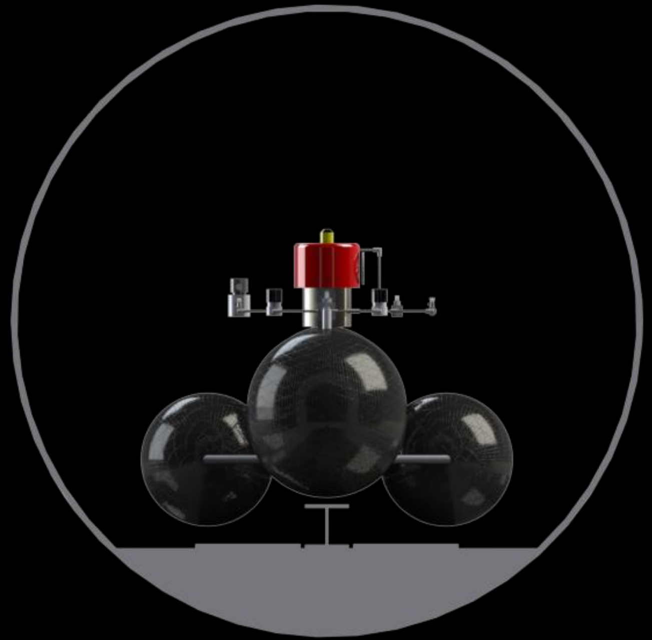
3 X10070504-2

+ 1 X 220074









3 X 10070504-2 + 1 X 220074 Performance

322 lb Inert Pod mass, 600m thrusting length, 493 km/hr max V

$V_{max} = 493$ km/hr

$L_{thrust} = 600$m

$m_{inert} = 322$ lb

$t_{MECO} = 9.03$sec

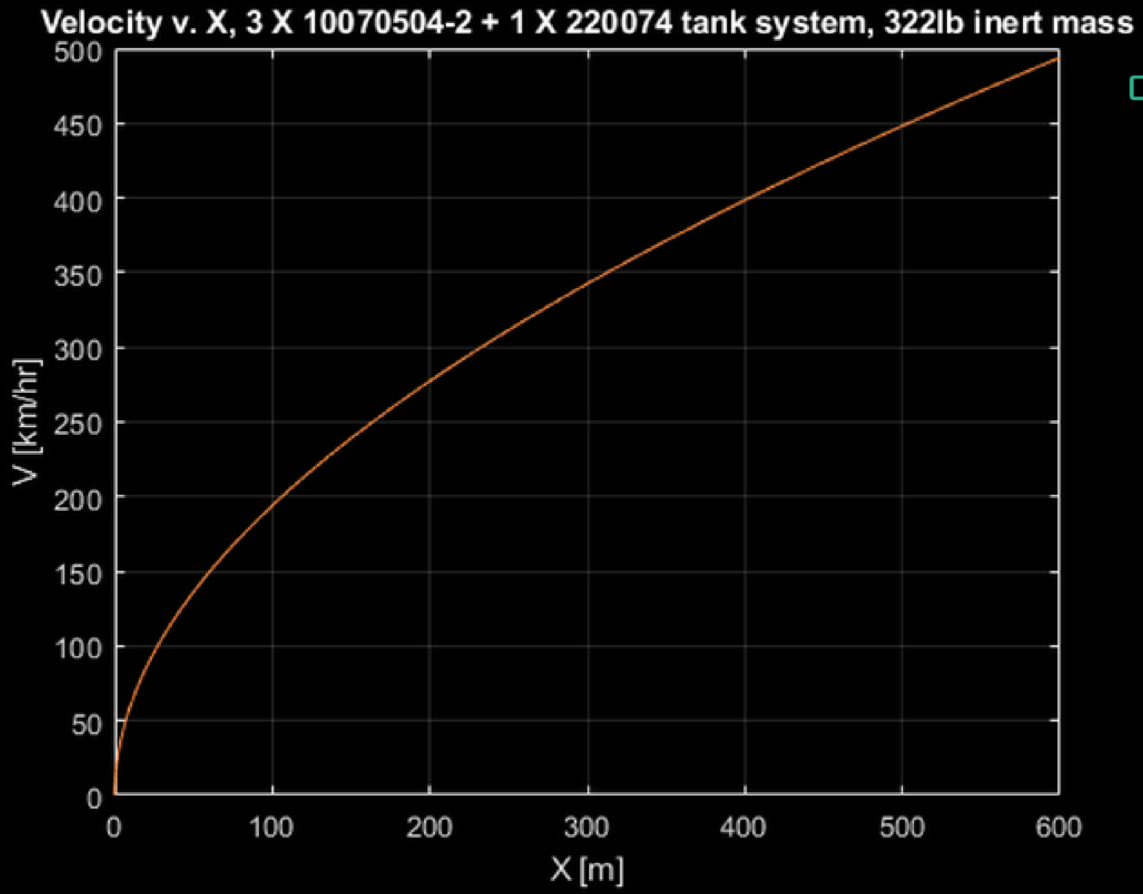
$\dot{m} = 6.59$ kg/sec

$F_{thrust} = 4721$ N

$a_{max} = 1.7$g

$m_{0\_loaded\_tanks} = 430$ lb

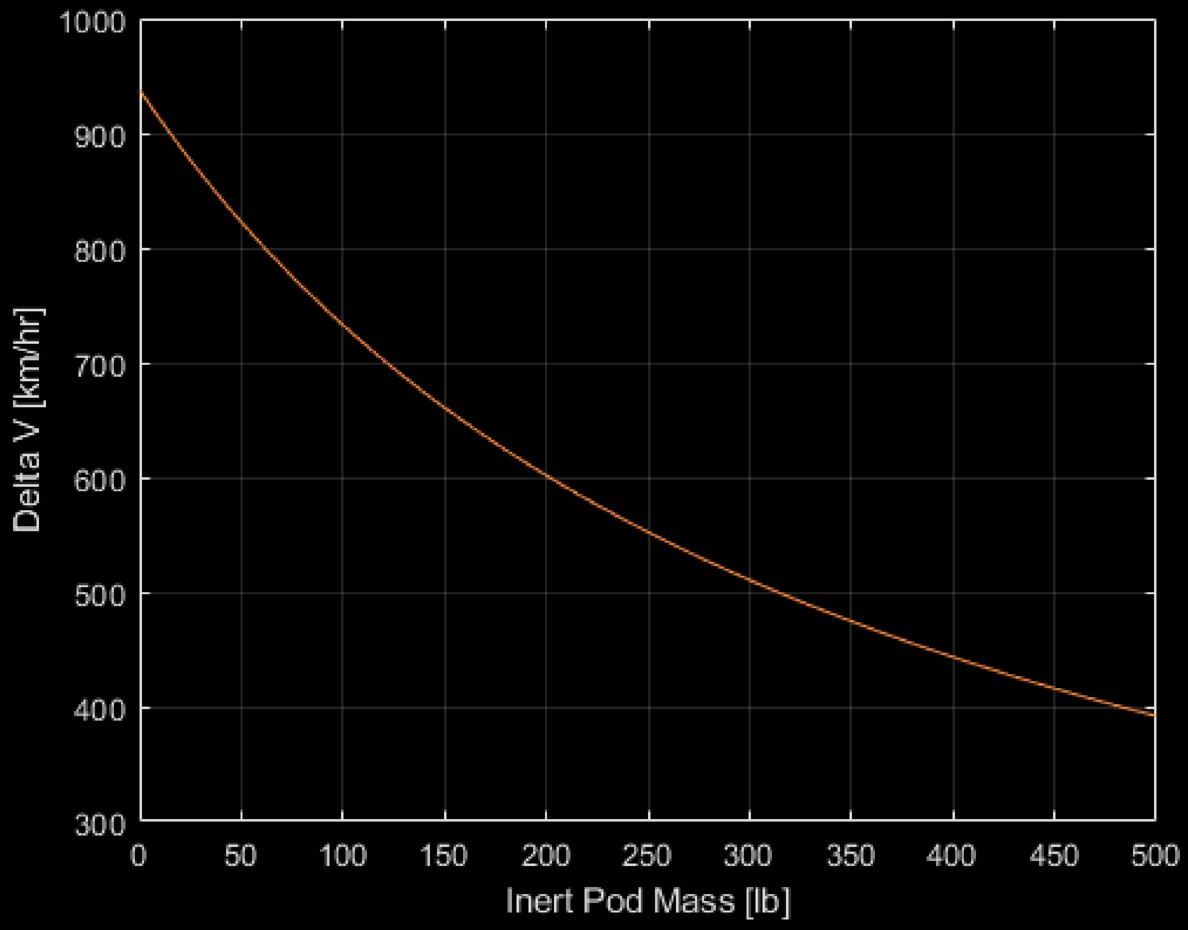
$m_{0\_loaded\_pod} = 751$ lb



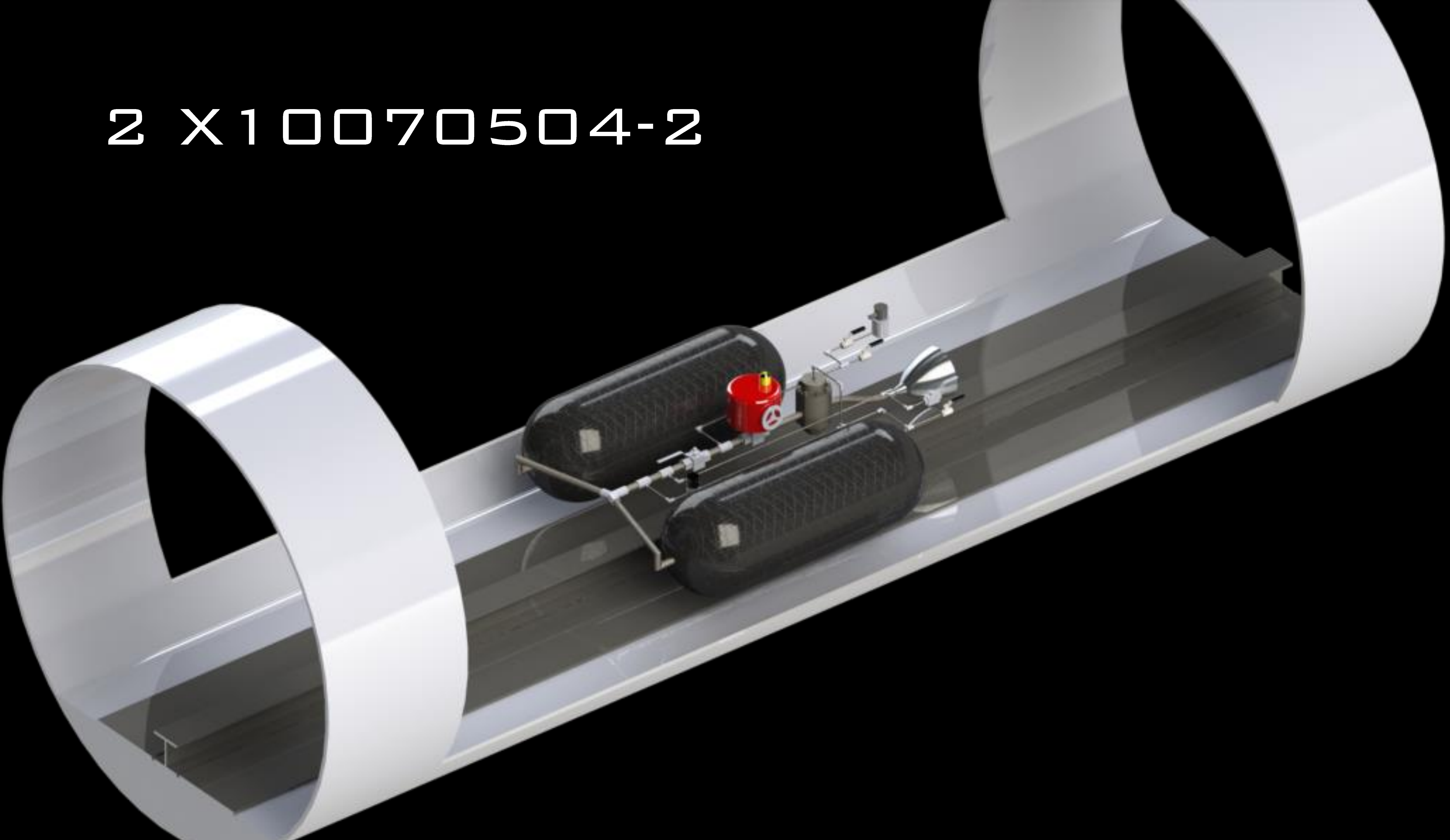
Coast/Braking =>

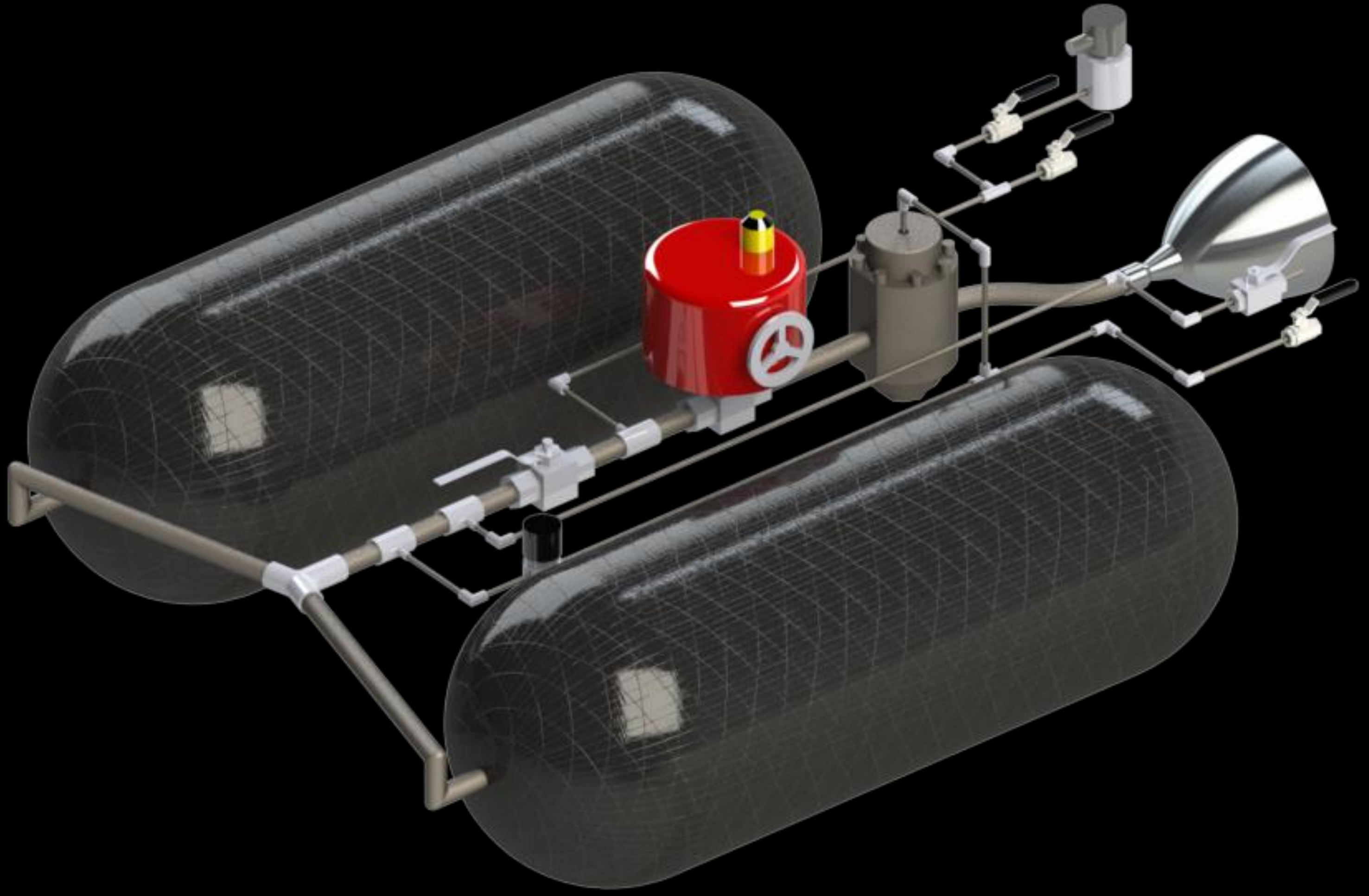
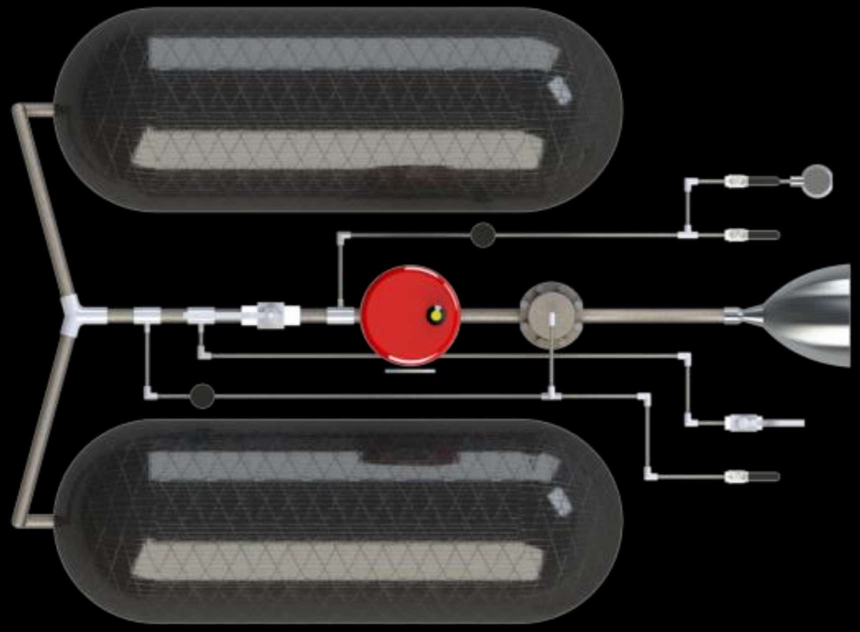
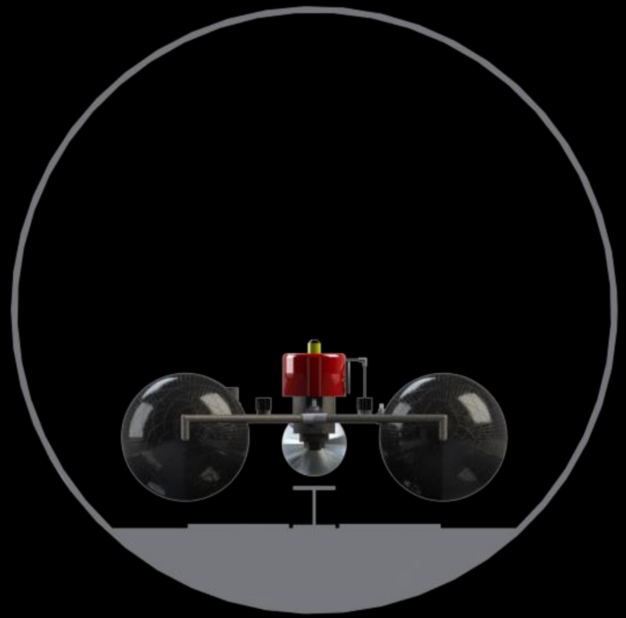
The inert mass (everything except for the tanks and propellent) is probably significantly more than the 322lb assumption, give a 430lb loaded tank system...

3 X10070504-2 + 1 X 220074 Performance



2 X10070504-2



2 X10070504-2 Performance

322 lb Inert Pod mass, 500m thrusting length, 377 km/hr max V

V_{max} = 377 km/hr

L_{thrust} = 500m

m_{inert} = 322 lb

t_{MECO} = 9.7sec

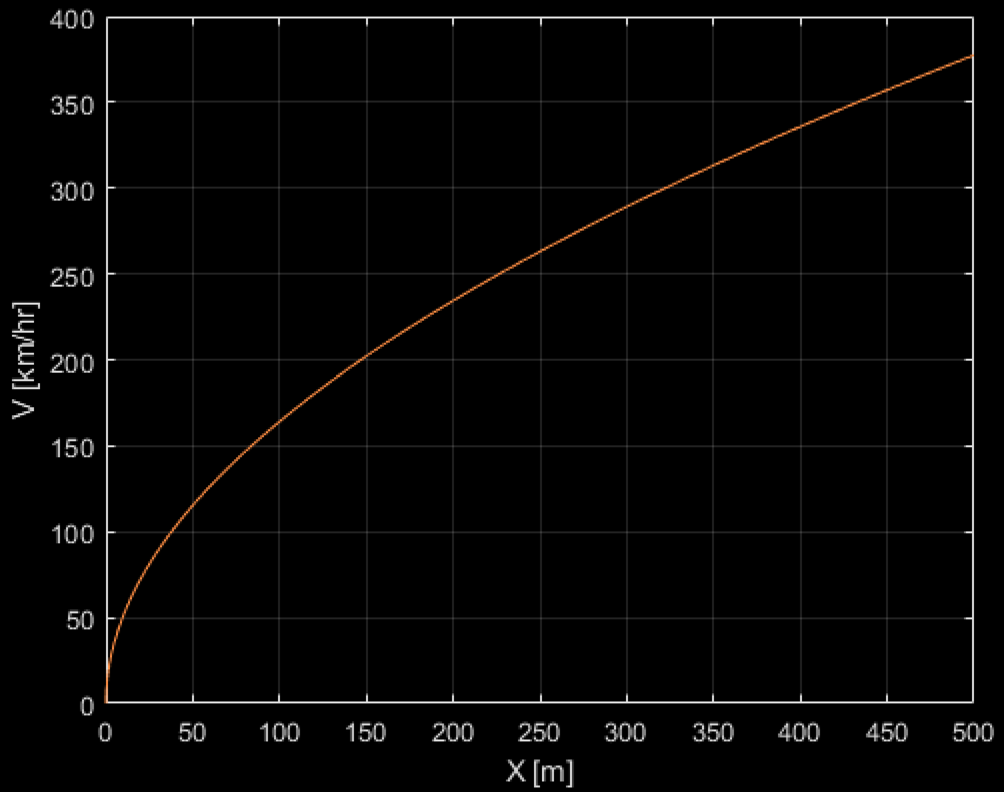
$\dot{m}$ = 3.56 kg/sec

F_{thrust} = 2550 N

a_{max} = 1.18g

$m_{0\_loaded\_tanks}$ = 241 lb

$m_{0\_loaded\_pod}$ = 563 lb

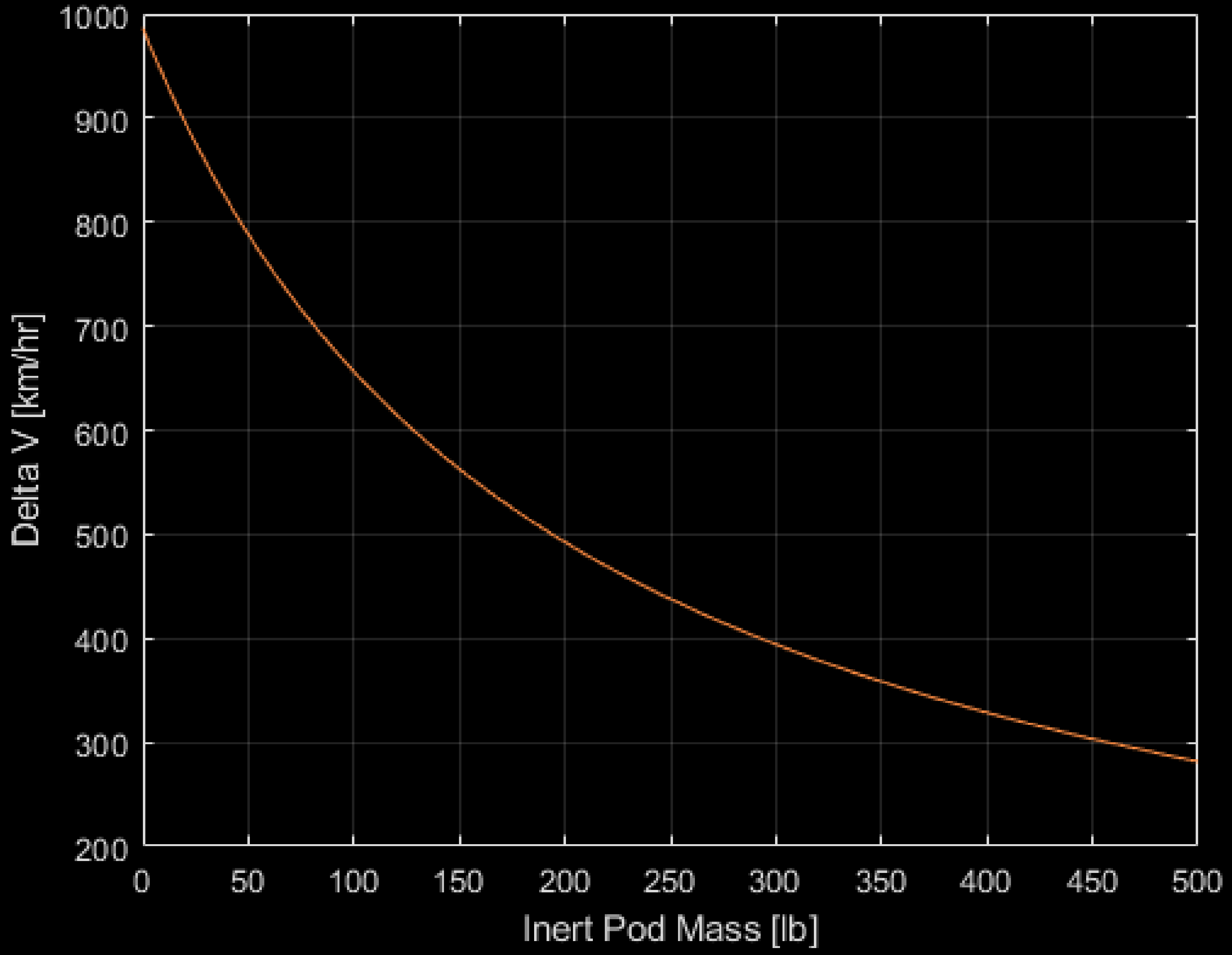


Coast/Braking =>

p_{Nozzle_inlet} = 466psi

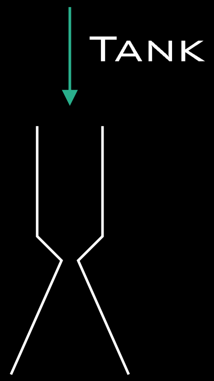
The inert mass (everything except for the tanks and propellent) is probably significantly more than the 322lb assumption give a 430lb loaded tank system...

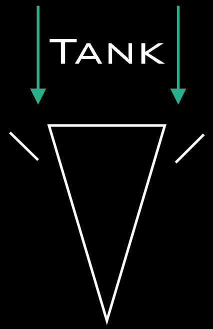
2 X10070504-2 Performance

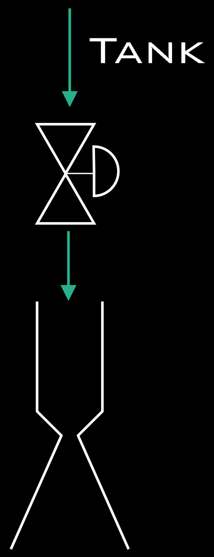


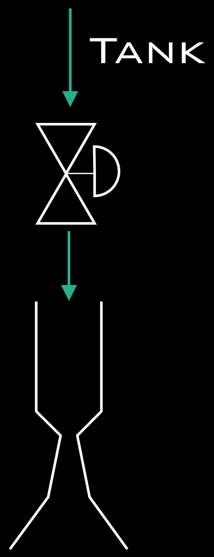
Nozzle Design

Configuration options:









Straight dump

- Phase through Under/Over expanded
- No pressure regulation
- Dynamics?

Aerospike

- Variable expansion
- No pressure regulator
- Thermal
- Manufacturing
- Cool factor

Regulated

- Optimal expansion
- Requires pressure regulation
- Additional set of nozzles for testing

Dual-Bell

- One nozzle with optimal expansion for the tube & 1 atm testing
- Requires pressure regulation

Nozzle Design

$$\dot{m}_{FlowCoefficient} > \dot{m}_{Nozzle} > \dot{m}_{Physics}$$

The nozzle mass flow rate must be above the physics requirement (to accelerate within the thrust length) and within the limits of the flow hardware

Avoid the possibility of supersonic flow in the hardware

$$\dot{m}_{FlowCoefficient} = C_v \sqrt{\rho \rho_{water} \Delta p}$$

$$\dot{m}_{Nozzle} = A_t p_1 k \frac{\sqrt{\left[\frac{2}{k+1}\right]^{(k+1)/(k-1)}}}{\sqrt{kRT_1}}$$

Nozzle Design

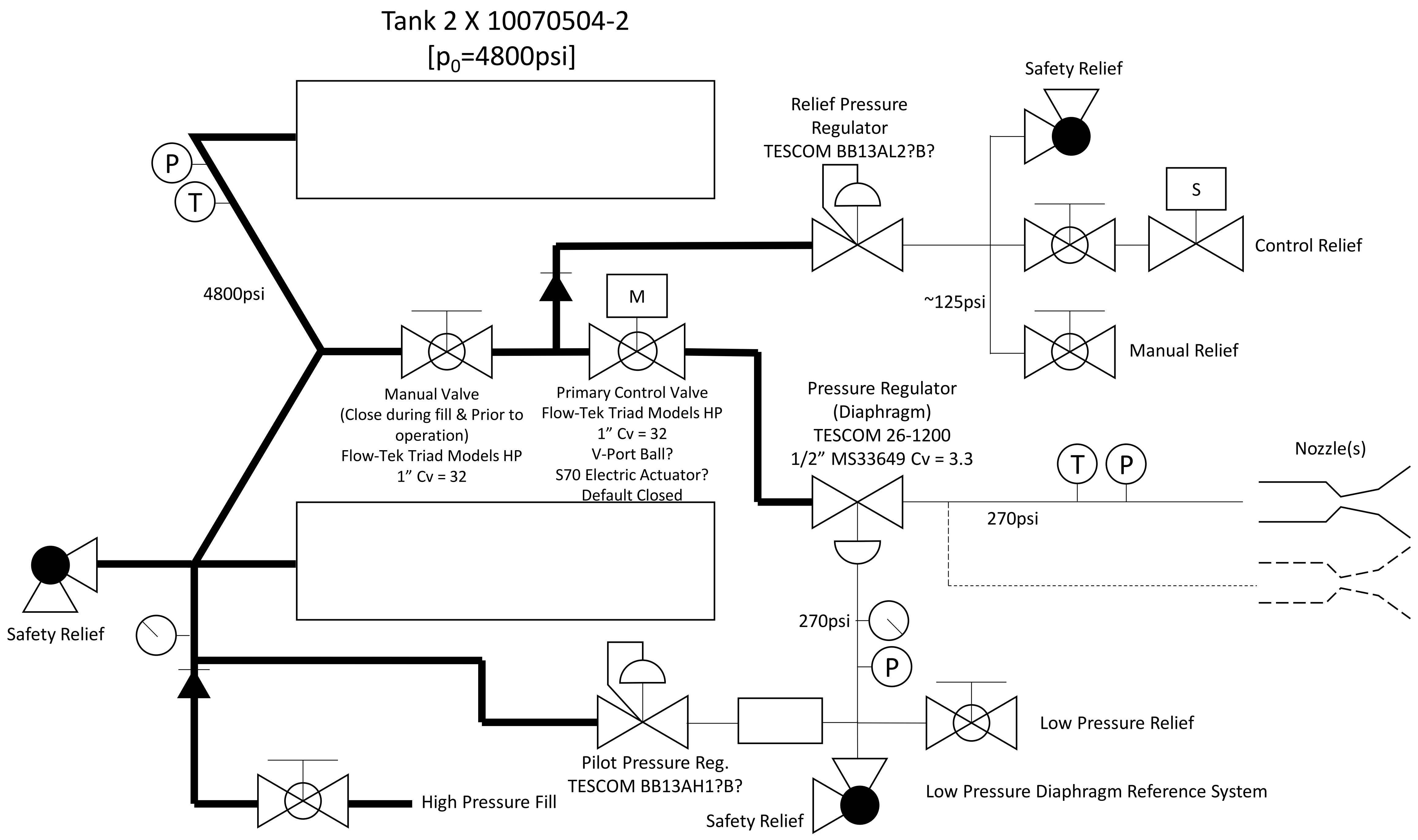
Minimum orifice

$$d_{O-min} = 2\left(\frac{1}{\pi}\frac{\dot{m}}{p_1 k}\frac{\sqrt{kRT_1}}{\sqrt{\left[\frac{2}{k+1}\right]^{(k+1)/(k-1)}}}\right)^{\frac{1}{2}}$$

2 X 10070504-2 System (322lb):

$$d_{O-min} = 0.304[in]$$

This is for a combination of extrema of P_1 and T_1, a condition which is not predicted. (A more refined approach will be implemented later)


Tank 2 X 10070504-2
[p0=4800psi]
Safety Relief
Relief Pressure
Regulator
TESCOM BB13AL2?B?
S
Control Relief
M
4800psi
~125psi
Manual Relief
P
T
Manual Valve
(Close during fill & Prior to
operation)
Flow-Tek Triad Models HP
1" Cv = 32
Primary Control Valve
Flow-Tek Triad Models HP
1" Cv = 32
V-Port Ball?
S70 Electric Actuator?
Default Closed
Pressure Regulator
(Diaphragm)
TESCOM 26-1200
1/2" MS33649 Cv = 3.3
Nozzle(s)
T
P
270psi
Safety Relief
270psi
P
Low Pressure Relief
Pilot Pressure Reg.
TESCOM BB13AH1?B?
High Pressure Fill
Safety Relief
Low Pressure Diaphragm Reference System