

# CUSPARSE LIBRARY DU-06709-001\_v8.0 | January 2017

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# Chapter 1. INTRODUCTION

The cuSPARSE library contains a set of basic linear algebra subroutines used for handling sparse matrices. It is implemented on top of the NVIDIA $^{\mathbb{R}}$  CUDA $^{\mathbb{T}^{\mathsf{M}}}$  runtime (which is part of the CUDA Toolkit) and is designed to be called from C and C++. The library routines can be classified into four categories:

- Level 1: operations between a vector in sparse format and a vector in dense format
- Level 2: operations between a matrix in sparse format and a vector in dense format
- Level 3: operations between a matrix in sparse format and a set of vectors in dense format (which can also usually be viewed as a dense tall matrix)
- Conversion: operations that allow conversion between different matrix formats, and compression of csr matrices.

The cuSPARSE library allows developers to access the computational resources of the NVIDIA graphics processing unit (GPU), although it does not auto-parallelize across multiple GPUs. The cuSPARSE API assumes that input and output data reside in GPU (device) memory, unless it is explicitly indicated otherwise by the string <code>DevHostPtr</code> in a function <code>parameter</code>'s name (for example, the parameter <code>\*resultDevHostPtr</code> in the function <code>cusparse<t>doti()</code>).

It is the responsibility of the developer to allocate memory and to copy data between GPU memory and CPU memory using standard CUDA runtime API routines, such as cudaMalloc(), cudaFree(), cudaMemcpy(), and cudaMemcpyAsync().



The cuSPARSE library requires hardware with compute capability (CC) of at least 2.0 or higher. Please see the NVIDIA CUDA C Programming Guide, Appendix A for a list of the compute capabilities corresponding to all NVIDIA GPUs.

# 1.1. Naming Conventions

The cuSPARSE library functions are available for data types **float**, **double**, **cuComplex**, and **cuDoubleComplex**. The sparse Level 1, Level 2, and Level 3 functions follow this naming convention:

cusparse<t>[<matrix data format>]<operation>[<output matrix data
format>]

where <t> can be S, D, C, Z, or X, corresponding to the data types float, double, cuComplex, cuDoubleComplex, and the generic type, respectively.

The <matrix data format> can be dense, coo, csr, csc, or hyb, corresponding to the dense, coordinate, compressed sparse row, compressed sparse column, and hybrid storage formats, respectively.

Finally, the <operation> can be axpyi, doti, dotci, gthr, gthrz, roti, or sctr, corresponding to the Level 1 functions; it also can be mv or sv, corresponding to the Level 2 functions, as well as mm or sm, corresponding to the Level 3 functions.

All of the functions have the return type **cusparseStatus\_t** and are explained in more detail in the chapters that follow.

# 1.2. Asynchronous Execution

The cuSPARSE library functions are executed asynchronously with respect to the host and may return control to the application on the host before the result is ready. Developers can use the **cudaDeviceSynchronize()** function to ensure that the execution of a particular cuSPARSE library routine has completed.

A developer can also use the **cudaMemcpy()** routine to copy data from the device to the host and vice versa, using the **cudaMemcpyDeviceToHost** and **cudaMemcpyHostToDevice** parameters, respectively. In this case there is no need to add a call to **cudaDeviceSynchronize()** because the call to **cudaMemcpy()** with the above parameters is blocking and completes only when the results are ready on the host.

# 1.3. Static Library support

Starting with release 6.5, the cuSPARSE Library is also delivered in a static form as libcusparse\_static.a on Linux and Mac OSes. The static cuSPARSE library and all others static maths libraries depend on a common thread abstraction layer library called libculibos.a on Linux and Mac and culibos.lib on Windows.

For example, on linux, to compile a small application using cuSPARSE against the dynamic library, the following command can be used:

```
nvcc myCusparseApp.c -lcusparse -o myCusparseApp
```

Whereas to compile against the static cuSPARSE library, the following command has to be used:

```
nvcc myCusparseApp.c -lcusparse_static -lculibos -o myCusparseApp
```

It is also possible to use the native Host C++ compiler. Depending on the Host Operating system, some additional libraries like **pthread** or **d1** might be needed on the linking line. The following command on Linux is suggested:

```
g++ myCusparseApp.c -lcusparse_static -lculibos -lcudart_static -lpthread -ldl -I <cuda-toolkit-path>/include -L <cuda-toolkit-path>/lib64 -omyCusparseApp
```

Note that in the latter case, the library **cuda** is not needed. The CUDA Runtime will try to open explicitly the **cuda** library if needed. In the case of a system which does not have the CUDA driver installed, this allows the application to gracefully manage this issue and potentially run if a CPU-only path is available.

# Chapter 2. USING THE CUSPARSE API

This chapter describes how to use the cuSPARSE library API. It is not a reference for the cuSPARSE API data types and functions; that is provided in subsequent chapters.

# 2.1. Thread Safety

The library is thread safe and its functions can be called from multiple host threads.

## 2.2. Scalar Parameters

In the cuSPARSE API, the scalar parameters  $\alpha$  and  $\beta$  can be passed by reference on the host or the device.

The few functions that return a scalar result, such as doti() and nnz(), return the resulting value by reference on the host or the device. Even though these functions return immediately, similarly to those that return matrix and vector results, the scalar result is not ready until execution of the routine on the GPU completes. This requires proper synchronization be used when reading the result from the host.

This feature allows the cuSPARSE library functions to execute completely asynchronously using streams, even when  $\alpha$  and  $\beta$  are generated by a previous kernel. This situation arises, for example, when the library is used to implement iterative methods for the solution of linear systems and eigenvalue problems [3].

# 2.3. Parallelism with Streams

If the application performs several small independent computations, or if it makes data transfers in parallel with the computation, CUDA streams can be used to overlap these tasks.

The application can conceptually associate a stream with each task. To achieve the overlap of computation between the tasks, the developer should create CUDA streams using the function **cudaStreamCreate()** and set the stream to be used by each

individual cuSPARSE library routine by calling <code>cusparseSetStream()</code> just before calling the actual cuSPARSE routine. Then, computations performed in separate streams would be overlapped automatically on the GPU, when possible. This approach is especially useful when the computation performed by a single task is relatively small and is not enough to fill the GPU with work, or when there is a data transfer that can be performed in parallel with the computation.

When streams are used, we recommend using the new cuSPARSE API with scalar parameters and results passed by reference in the device memory to achieve maximum computational overlap.

Although a developer can create many streams, in practice it is not possible to have more than 16 concurrent kernels executing at the same time.

# Chapter 3. CUSPARSE INDEXING AND DATA FORMATS

The cuSPARSE library supports dense and sparse vector, and dense and sparse matrix formats.

## 3.1. Index Base Format

The library supports zero- and one-based indexing. The index base is selected through the <code>cusparseIndexBase\_t</code> type, which is passed as a standalone parameter or as a field in the matrix descriptor <code>cusparseMatDescr t</code> type.

## 3.2. Vector Formats

This section describes dense and sparse vector formats.

### 3.2.1. Dense Format

Dense vectors are represented with a single data array that is stored linearly in memory, such as the following  $7 \times 1$  dense vector.

[1.0 0.0 0.0 2.0 3.0 0.0 4.0]

(This vector is referenced again in the next section.)

## 3.2.2. Sparse Format

Sparse vectors are represented with two arrays.

- ▶ The *data array* has the nonzero values from the equivalent array in dense format.
- ► The *integer index array* has the positions of the corresponding nonzero values in the equivalent array in dense format.

For example, the dense vector in section 3.2.1 can be stored as a sparse vector with one-based indexing.

It can also be stored as a sparse vector with zero-based indexing.

In each example, the top row is the data array and the bottom row is the index array, and it is assumed that the indices are provided in increasing order and that each index appears only once.

## 3.3. Matrix Formats

Dense and several sparse formats for matrices are discussed in this section.

#### 3.3.1. Dense Format

The dense matrix  $\mathbf{x}$  is assumed to be stored in column-major format in memory and is represented by the following parameters.

m	(integer)	he number of rows in the matrix.	
n	(integer)	The number of columns in the matrix.	
ldx	(integer)	The leading dimension of $x$ , which must be greater than or equal to $\mathfrak m$ . It law is greater than $\mathfrak m$ , then $x$ represents a sub-matrix of a larger matrix stored in memory	
х	(pointer)	Points to the data array containing the matrix elements. It is assumed that enough storage is allocated for $\mathbf{x}$ to hold all of the matrix elements and that cuSPARSE library functions may access values outside of the sub-matrix, but will never overwrite them.	

For example,  $m \times n$  dense matrix x with leading dimension ldx can be stored with one-based indexing as shown.

$$X_{1,1}$$
  $X_{1,2}$  ...  $X_{1,n}$   $X_{2,1}$   $X_{2,2}$  ...  $X_{2,n}$  ...  $X_{m,1}$   $X_{m,2}$  ...  $X_{m,n}$  ...  $X_{m,n}$ 

Its elements are arranged linearly in memory in the order below.

$$\begin{bmatrix} X_{1,1} & X_{2,1} & \cdots & X_{m,1} & \cdots & X_{ldX,1} & \cdots & X_{1,n} & X_{2,n} & \cdots & X_{m,n} & \cdots & X_{ldX,n} \end{bmatrix}$$



This format and notation are similar to those used in the NVIDIA CUDA cuBLAS library.

# 3.3.2. Coordinate Format (COO)

The  $m \times n$  sparse matrix **A** is represented in COO format by the following parameters.

nnz	(integer)	ne number of nonzero elements in the matrix.	
cooValA	(pointer)	Points to the data array of length ${\tt nnz}$ that holds all nonzero values of a in row-major format.	
cooRowIndA	(pointer)	Points to the integer array of length nnz that contains the row indices of the corresponding elements in array coovala.	
cooColIndA	(pointer)	Points to the integer array of length nnz that contains the column indices of the corresponding elements in array coovala.	

A sparse matrix in COO format is assumed to be stored in row-major format: the index arrays are first sorted by row indices and then within the same row by compressed column indices. It is assumed that each pair of row and column indices appears only once.

For example, consider the following  $4 \times 5$  matrix **A**.

It is stored in COO format with zero-based indexing this way.

$$cooValA = [1.0 \ 4.0 \ 2.0 \ 3.0 \ 5.0 \ 7.0 \ 8.0 \ 9.0 \ 6.0]$$
  
 $cooRowIndA = [0 \ 0 \ 1 \ 1 \ 2 \ 2 \ 2 \ 3 \ 3 \ ]$   
 $cooColIndA = [0 \ 1 \ 1 \ 2 \ 0 \ 3 \ 4 \ 2 \ 4 \ ]$ 

In the COO format with one-based indexing, it is stored as shown.

$$cooValA = [1.0 \ 4.0 \ 2.0 \ 3.0 \ 5.0 \ 7.0 \ 8.0 \ 9.0 \ 6.0]$$
  
 $cooRowIndA = [1 \ 1 \ 2 \ 2 \ 3 \ 3 \ 4 \ 4 \ ]$   
 $cooColIndA = [1 \ 2 \ 2 \ 3 \ 1 \ 4 \ 5 \ 3 \ 5 \ ]$ 

# 3.3.3. Compressed Sparse Row Format (CSR)

The only way the CSR differs from the COO format is that the array containing the row indices is compressed in CSR format. The  $m \times n$  sparse matrix A is represented in CSR format by the following parameters.

nnz	(integer)	The number of nonzero elements in the matrix.	
csrValA	(pointer)	Points to the data array of length ${\tt nnz}$ that holds all nonzero values of A in row-major format.	
csrRowPtrA	(pointer)	Points to the integer array of length m+1 that holds indices into the arrays csrColIndA and csrValA. The first m entries of this array contain the indices of the first nonzero element in the ith row for i=i,,m, while the last entry contains nnz+csrRowPtrA(0). In general, csrRowPtrA(0) is 0 or 1 for zero- and one-based indexing, respectively.	

csrColIndA	Points to the integer array of length nnz that contains the column indices of the corresponding elements in array csrValA.
	matees of the corresponding elements in array estvara.

Sparse matrices in CSR format are assumed to be stored in row-major CSR format, in other words, the index arrays are first sorted by row indices and then within the same row by column indices. It is assumed that each pair of row and column indices appears only once.

Consider again the 4 × 5matrix**A**.

```
1.0 4.0 0.0 0.0 0.0
0.0 2.0 3.0 0.0 0.0
5.0 0.0 0.0 7.0 8.0
0.0 0.0 9.0 0.0 6.0
```

It is stored in CSR format with zero-based indexing as shown.

```
csrValA = [1.0 \ 4.0 \ 2.0 \ 3.0 \ 5.0 \ 7.0 \ 8.0 \ 9.0 \ 6.0]

csrRowPtrA = [0 \ 2 \ 4 \ 7 \ 9 \ ]

csrColIndA = [0 \ 1 \ 1 \ 2 \ 0 \ 3 \ 4 \ 2 \ 4 \ ]
```

This is how it is stored in CSR format with one-based indexing.

```
csrValA = [1.0 \ 4.0 \ 2.0 \ 3.0 \ 5.0 \ 7.0 \ 8.0 \ 9.0 \ 6.0]

csrRowPtrA = [1 \ 3 \ 5 \ 8 \ 10 \ ]

csrColIndA = [1 \ 2 \ 2 \ 3 \ 1 \ 4 \ 5 \ 3 \ 5 \ ]
```

# 3.3.4. Compressed Sparse Column Format (CSC)

The CSC format is different from the COO format in two ways: the matrix is stored in column-major format, and the array containing the column indices is compressed in CSC format. The m×n matrix A is represented in CSC format by the following parameters.

nnz	(integer)	The number of nonzero elements in the matrix.	
cscValA	(pointer)	Points to the data array of length ${\tt nnz}$ that holds all nonzero values of A in column-major format.	
cscRowIndA	(pointer)	pints to the integer array of length nnz that contains the row indices f the corresponding elements in array cscValA.	
cscColPtrA	(pointer)	Points to the integer array of length n+1 that holds indices into the arrays cscRowIndA and cscValA. The first n entries of this array contain the indices of the first nonzero element in the ith row for i=i,,n, while the last entry contains nnz+cscColPtrA(0). In general, cscColPtrA(0) is 0 or 1 for zero- and one-based indexing, respectively.	



The matrix  $\mathbf{A}$  in CSR format has exactly the same memory layout as its transpose in CSC format (and vice versa).

For example, consider once again the  $4 \times 5$  matrix **A**.

It is stored in CSC format with zero-based indexing this way.

In CSC format with one-based indexing, this is how it is stored.

$$cscValA = [1.0 5.0 4.0 2.0 3.0 9.0 7.0 8.0 6.0]$$
  
 $cscRowIndA = [1 3 1 2 2 4 3 3 4]$   
 $cscColPtrA = [1 3 5 7 8 10]$ 

Each pair of row and column indices appears only once.

# 3.3.5. Ellpack-Itpack Format (ELL)

An  $m \times n$  sparse matrix A with at most k nonzero elements per row is stored in the Ellpack-Itpack (ELL) format [2] using two dense arrays of dimension  $m \times k$ . The first data array contains the values of the nonzero elements in the matrix, while the second integer array contains the corresponding column indices.

For example, consider the  $4 \times 5$  matrix **A**.

This is how it is stored in ELL format with zero-based indexing.

$$data = \begin{bmatrix} 1.0 & 4.0 & 0.0 \\ 2.0 & 3.0 & 0.0 \\ 5.0 & 7.0 & 8.0 \\ 9.0 & 6.0 & 0.0 \end{bmatrix}$$

$$0 & 1 & -1 \\ 1 & 2 & -1 \\ 0 & 3 & 4 \\ 2 & 4 & -1 \end{bmatrix}$$

It is stored this way in ELL format with one-based indexing.

$$data = \begin{vmatrix} 1.0 & 4.0 & 0.0 \\ 2.0 & 3.0 & 0.0 \\ 5.0 & 7.0 & 8.0 \\ 9.0 & 6.0 & 0.0 \end{vmatrix}$$

$$indices = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & -1 \\ 1 & 4 & 5 \\ 3 & 5 & -1 \end{vmatrix}$$

Sparse matrices in ELL format are assumed to be stored in column-major format in memory. Also, rows with less than  $\mathbf{k}$  nonzero elements are padded in the **data** and **indices** arrays with zero and -1, respectively.

The ELL format is not supported directly, but it is used to store the regular part of the matrix in the HYB format that is described in the next section.

# 3.3.6. Hybrid Format (HYB)

The HYB sparse storage format is composed of a regular part, usually stored in ELL format, and an irregular part, usually stored in COO format [1]. The ELL and COO parts are always stored using zero-based indexing. HYB is implemented as an opaque data format that requires the use of a conversion operation to store a matrix in it. The conversion operation partitions the general matrix into the regular and irregular parts automatically or according to developer-specified criteria.

For more information, please refer to the description of **cusparseHybPartition\_t** type, as well as the description of the conversion routines **dense2hyb**, **csc2hyb** and **csr2hyb**.

# 3.3.7. Block Compressed Sparse Row Format (BSR)

The only difference between the CSR and BSR formats is the format of the storage element. The former stores primitive data types (**single**, **double**, **cuComplex**, and **cuDoubleComplex**) whereas the latter stores a two-dimensional square block of primitive data types. The dimension of the square block is blockDim. The **m×n** sparse matrix **A** is equivalent to a block sparse matrix  $A_b$  with  $mb = \frac{m + blockDim - 1}{blockDim}$  block rows and  $nb = \frac{n + blockDim - 1}{blockDim}$  block columns. If m or n is not multiple of blockDim, then zeros are filled into  $A_b$ .

**A** is represented in BSR format by the following parameters.

blockDim	(integer)	lock dimension of matrix A.		
mb	(integer)	he number of block rows of A.		
nb	(integer)	The number of block columns of A.		
nnzb	(integer)	he number of nonzero blocks in the matrix.		
bsrValA	(pointer)	Points to the data array of length $nnzb*blockDim^2$ that holds all elements of nonzero blocks of <b>A</b> . The block elements are stored in either column-major order or row-major order.		
bsrRowPtrA	(pointer)	Points to the integer array of length mb+1 that holds indices into the arrays bsrColIndA and bsrValA. The first mb entries of this array contain the indices of the first nonzero block in the ith block row for i=1,,mb, while the last entry contains nnzb+bsrRowPtrA(0). In general, bsrRowPtrA(0) is 0 or 1 for zero- and one-based indexing, respectively.		
bsrColIndA	(pointer)	Points to the integer array of length nnzb that contains the column indices of the corresponding blocks in array bsrValA.		

As with CSR format, (row, column) indices of BSR are stored in row-major order. The index arrays are first sorted by row indices and then within the same row by column indices.

For example, consider again the 4×5 matrix A.

If blockDim is equal to 2, then mb is 2, nb is 3, and matrix **A** is split into **2×3** block matrix  $A_b$ . The dimension of  $A_b$  is **4×6**, slightly bigger than matrix A, so zeros are filled in the last column of  $A_b$ . The element-wise view of  $A_b$  is this.

Based on zero-based indexing, the block-wise view of  $A_b$  can be represented as follows.

$$A_b = \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \end{bmatrix}$$

The basic element of BSR is a nonzero  $A_{ij}$  block, one that contains at least one nonzero element of **A**. Five of six blocks are nonzero in  $A_b$ .

$$A_{00} = \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}, A_{01} = \begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix}, A_{10} = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}, A_{11} = \begin{bmatrix} 0 & 7 \\ 9 & 0 \end{bmatrix}, A_{12} = \begin{bmatrix} 8 & 0 \\ 6 & 0 \end{bmatrix}$$

BSR format only stores the information of nonzero blocks, including block indices (i, j) and values  $A_{ij}$ . Also row indices are compressed in CSR format.

bsr ValA = 
$$\begin{bmatrix} A_{00} & A_{01} & A_{10} & A_{11} & A_{12} \end{bmatrix}$$
  
bsr Row Ptr A =  $\begin{bmatrix} 0 & 2 & 5 \end{bmatrix}$   
bsr CollndA =  $\begin{bmatrix} 0 & 1 & 0 & 1 & 2 \end{bmatrix}$ 

There are two ways to arrange the data element of block  $A_{ij}$ : row-major order and column-major order. Under column-major order, the physical storage of **bsrValA** is this.

Under row-major order, the physical storage of **bsrValA** is this.

Similarly, in BSR format with one-based indexing and column-major order, **A** can be represented by the following.

$$A_b = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$$

bsrValA=[1 0 4 2 | 0 3 0 0 | 5 0 0 0 | 0 9 7 0 | 8 6 0 0 ]

bsr Row Ptr A = 
$$[1 \ 3 \ 6]$$
  
bsr CollndA =  $[1 \ 2 \ 1 \ 2 \ 3]$ 



The general BSR format has two parameters, rowBlockDim and colBlockDim. rowBlockDim is number of rows within a block and colBlockDim is number of columns within a block. If rowBlockDim=colBlockDim, general BSR format is the same as BSR format. If rowBlockDim=colBlockDim=1, general BSR format is the same as CSR format. The conversion routine gebsr2gebsr is used to do conversion among CSR, BSR and general BSR.



In the cuSPARSE Library, the storage format of blocks in BSR format can be column-major or row-major, independently of the base index. However, if the developer uses BSR format from the Math Kernel Library (MKL) and wants to directly interface with the cuSPARSE Library, then cusparseDirection\_t CUSPARSE\_DIRECTION\_COLUMN should be used if the base index is one; otherwise, cusparseDirection\_t CUSPARSE DIRECTION ROW should be used.

# 3.3.8. Extended BSR Format (BSRX)

BSRX is the same as the BSR format, but the array **bsrRowPtrA** is separated into two parts. The first nonzero block of each row is still specified by the array **bsrRowPtrA**, which is the same as in BSR, but the position next to the last nonzero block of each row is specified by the array **bsrEndPtrA**. Briefly, BSRX format is simply like a 4-vector variant of BSR format.

Matrix <b>A</b> is re	presented in	BSRX format by	v the following	parameters.
1110001170 == 10 10				

blockDim	(integer)	Block dimension of matrix A.
mb	(integer)	The number of block rows of A.
nb	(integer)	The number of block columns of A.
nnzb	(integer)	number of nonzero blocks in the matrix A.
bsrValA	(pointer)	Points to the data array of length $nnzb*blockDim^2$ that holds all the elements of the nonzero blocks of $\bf A$ . The block elements are stored in either column-major order or row-major order.
bsrRowPtrA	(pointer)	Points to the integer array of length mb that holds indices into the arrays bsrColIndA and bsrValA; bsrRowPtrA(i) is the position of the first nonzero block of the ith block row in bsrColIndA and bsrValA.
bsrEndPtrA	(pointer)	Points to the integer array of length mb that holds indices into the arrays bsrColIndA and bsrValA; bsrRowPtrA(i) is the position next to the last nonzero block of the ith block row in bsrColIndA and bsrValA.
bsrColIndA	(pointer)	Points to the integer array of length nnzb that contains the column indices of the corresponding blocks in array bsrValA.

A simple conversion between BSR and BSRX can be done as follows. Suppose the developer has a  $2\times3$  block sparse matrix  $A_b$  represented as shown.

$$A_b = \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \end{bmatrix}$$

Assume it has this BSR format.

bsr ValA of BSR = 
$$\begin{bmatrix} A_{00} & A_{01} & A_{10} & A_{11} & A_{12} \end{bmatrix}$$
  
bsr Row Ptr A of BSR =  $\begin{bmatrix} 0 & 2 & 5 \end{bmatrix}$   
bsr CollndA of BSR =  $\begin{bmatrix} 0 & 1 & 0 & 1 & 2 \end{bmatrix}$ 

The **bsrRowPtrA** of the BSRX format is simply the first two elements of the **bsrRowPtrA** BSR format. The **bsrEndPtrA** of BSRX format is the last two elements of the **bsrRowPtrA** of BSR format.

The advantage of the BSRX format is that the developer can specify a submatrix in the original BSR format by modifying bsrRowPtrA and bsrEndPtrA while keeping bsrColIndA and bsrValA unchanged.

For example, to create another block matrix  $\tilde{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_{11} & 0 \end{bmatrix}$  that is slightly different from A, the developer can keep **bsrColIndA** and **bsrValA**, but reconstruct  $\tilde{A}$  by properly setting of **bsrRowPtrA** and **bsrEndPtrA**. The following 4-vector characterizes  $\tilde{A}$ .

bsr ValA of 
$$\tilde{A} = \begin{bmatrix} A_{00} & A_{01} & A_{10} & A_{11} & A_{12} \end{bmatrix}$$
  
bsr CollndA of  $\tilde{A} = \begin{bmatrix} 0 & 1 & 0 & 1 & 2 \end{bmatrix}$   
bsr Row Ptr A of  $\tilde{A} = \begin{bmatrix} 0 & 3 \end{bmatrix}$   
bsr EndPtr A of  $\tilde{A} = \begin{bmatrix} 0 & 4 \end{bmatrix}$ 

# Chapter 4. CUSPARSE TYPES REFERENCE

# 4.1. Data types

The **float**, **double**, **cuComplex**, and **cuDoubleComplex** data types are supported. The first two are standard C data types, while the last two are exported from **cuComplex**.h.

# 4.2. cusparseAction\_t

This type indicates whether the operation is performed only on indices or on data and indices.

Value	Meaning
CUSPARSE_ACTION_SYMBOLIC	the operation is performed only on indices.
CUSPARSE_ACTION_NUMERIC	the operation is performed on data and indices.

# 4.3. cusparseDirection\_t

This type indicates whether the elements of a dense matrix should be parsed by rows or by columns (assuming column-major storage in memory of the dense matrix) in function cusparse[S|D|C|Z]nnz. Besides storage format of blocks in BSR format is also controlled by this type.

Value	Meaning
CUSPARSE_DIRECTION_ROW	the matrix should be parsed by rows.
CUSPARSE_DIRECTION_COLUMN	the matrix should be parsed by columns.

# 4.4. cusparseHandle\_t

This is a pointer type to an opaque cuSPARSE context, which the user must initialize by calling prior to calling cusparseCreate() any other library function. The handle created and returned by cusparseCreate() must be passed to every cuSPARSE function.

# 4.5. cusparseHybMat\_t

This is a pointer type to an opaque structure holding the matrix in HYB format, which is created by **cusparseCreateHybMat** and destroyed by **cusparseDestroyHybMat**.

# 4.5.1. cusparseHybPartition\_t

This type indicates how to perform the partitioning of the matrix into regular (ELL) and irregular (COO) parts of the HYB format.

The partitioning is performed during the conversion of the matrix from a dense or sparse format into the HYB format and is governed by the following rules. When <code>CUSPARSE\_HYB\_PARTITION\_AUTO</code> is selected, the cuSPARSE library automatically decides how much data to put into the regular and irregular parts of the HYB format. When <code>CUSPARSE\_HYB\_PARTITION\_USER</code> is selected, the width of the regular part of the HYB format should be specified by the caller. When <code>CUSPARSE\_HYB\_PARTITION\_MAX</code> is selected, the width of the regular part of the HYB format equals to the maximum number of non-zero elements per row, in other words, the entire matrix is stored in the regular part of the HYB format.

The *default* is to let the library automatically decide how to split the data.

Value	Meaning
CUSPARSE_HYB_PARTITION_AUTO	the automatic partitioning is selected (default).
CUSPARSE_HYB_PARTITION_USER	the user specified treshold is used.
CUSPARSE_HYB_PARTITION_MAX	the data is stored in ELL format.

# 4.6. cusparseMatDescr\_t

This structure is used to describe the shape and properties of a matrix.

```
typedef struct {
    cusparseMatrixType_t MatrixType;
    cusparseFillMode_t FillMode;
    cusparseDiagType_t DiagType;
    cusparseIndexBase_t IndexBase;
} cusparseMatDescr t;
```

# 4.6.1. cusparseDiagType\_t

This type indicates if the matrix diagonal entries are unity. The diagonal elements are always assumed to be present, but if <code>CUSPARSE\_DIAG\_TYPE\_UNIT</code> is passed to an API routine, then the routine assumes that all diagonal entries are unity and will not read or modify those entries. Note that in this case the routine assumes the diagonal entries are equal to one, regardless of what those entries are actually set to in memory.

Value	Meaning
CUSPARSE_DIAG_TYPE_NON_UNIT	the matrix diagonal has non-unit elements.
CUSPARSE_DIAG_TYPE_UNIT	the matrix diagonal has unit elements.

# 4.6.2. cusparseFillMode\_t

This type indicates if the lower or upper part of a matrix is stored in sparse storage.

Value	Meaning
CUSPARSE_FILL_MODE_LOWER	the lower triangular part is stored.
CUSPARSE_FILL_MODE_UPPER	the upper triangular part is stored.

# 4.6.3. cusparseIndexBase\_t

This type indicates if the base of the matrix indices is zero or one.

Value	Meaning
CUSPARSE_INDEX_BASE_ZERO	the base index is zero.
CUSPARSE_INDEX_BASE_ONE	the base index is one.

# 4.6.4. cusparseMatrixType\_t

This type indicates the type of matrix stored in sparse storage. Notice that for symmetric, Hermitian and triangular matrices only their lower or upper part is assumed to be stored.

The whole idea of matrix type and fill mode is to keep minimum storage for symmetric/ Hermitian matrix, and also to take advantage of symmetric property on SpMV (Sparse Matrix Vector multiplication). To compute  $\mathbf{y}=\mathbf{A}*\mathbf{x}$  when  $\mathbf{A}$  is symmetric and only lower triangular part is stored, two steps are needed. First step is to compute  $\mathbf{y}=(\mathbf{L}+\mathbf{D})*\mathbf{x}$  and second step is to compute  $\mathbf{y}=\mathbf{L}^*\mathbf{T}*\mathbf{x} + \mathbf{y}$ . Given the fact that the transpose operation  $\mathbf{y}=\mathbf{L}^*\mathbf{T}*\mathbf{x}$  is  $10\mathbf{x}$  slower than non-transpose version  $\mathbf{y}=\mathbf{L}^*\mathbf{x}$ , the symmetric property does not show up any performance gain. It is better for the user to extend the symmetric matrix to a general matrix and apply  $\mathbf{y}=\mathbf{A}*\mathbf{x}$  with matrix type CUSPARSE\_MATRIX\_TYPE\_GENERAL.

In general, SpMV, preconditioners (incomplete Cholesky or incomplete LU) and triangular solver are combined together in iterative solvers, for example PCG and

GMRES. If the user always uses general matrix (instead of symmetric matrix), there is no need to support other than general matrix in preconditioners. Therefore the new routines, [bsr|csr]sv2 (triangular solver), [bsr|csr]ilu02 (incomplete LU) and [bsr|csr]ic02 (incomplete Cholesky), only support matrix type CUSPARSE\_MATRIX\_TYPE\_GENERAL.

Value	Meaning
CUSPARSE_MATRIX_TYPE_GENERAL	the matrix is general.
CUSPARSE_MATRIX_TYPE_SYMMETRIC	the matrix is symmetric.
CUSPARSE_MATRIX_TYPE_HERMITIAN	the matrix is Hermitian.
CUSPARSE_MATRIX_TYPE_TRIANGULAR	the matrix is triangular.

# 4.7. cusparseOperation\_t

This type indicates which operations need to be performed with the sparse matrix.

Value	Meaning
CUSPARSE_OPERATION_NON_TRANSPOSE	the non-transpose operation is selected.
CUSPARSE_OPERATION_TRANSPOSE	the transpose operation is selected.
CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE	the conjugate transpose operation is selected.

# 4.8. cusparsePointerMode\_t

This type indicates whether the scalar values are passed by reference on the host or device. It is important to point out that if several scalar values are passed by reference in the function call, all of them will conform to the same single pointer mode. The pointer mode can be set and retrieved using <code>cusparseSetPointerMode()</code> and <code>cusparseGetPointerMode()</code> routines, respectively.

Value	Meaning
CUSPARSE_POINTER_MODE_HOST	the scalars are passed by reference on the host.
CUSPARSE_POINTER_MODE_DEVICE	the scalars are passed by reference on the device.

# 4.9. cusparseSolvePolicy\_t

This type indicates whether level information is generated and used in csrsv2, csric02, csrilu02, bsrsv2, bsric02 and bsrilu02.

Value	Meaning
CUSPARSE_SOLVE_POLICY_NO_LEVEL	no level information is generated and used.

Value	Meaning
CUSPARSE_SOLVE_POLICY_USE_LEVEL	generate and use level information.

# 4.10. cusparseSolveAnalysisInfo\_t

This is a pointer type to an opaque structure holding the information collected in the analysis phase of the solution of the sparse triangular linear system. It is expected to be passed unchanged to the solution phase of the sparse triangular linear system.

# 4.11. cusparseSolveAnalysisInfo\_t

This is a pointer type to an opaque structure holding the information collected in the analysis phase of the solution of the sparse triangular linear system. It is expected to be passed unchanged to the solution phase of the sparse triangular linear system.

# 4.12. csrsv2lnfo\_t

This is a pointer type to an opaque structure holding the information used in csrsv2\_bufferSize(), csrsv2\_analysis(), and csrsv2\_solve().

# 4.13. csric02lnfo\_t

This is a pointer type to an opaque structure holding the information used in csric02\_bufferSize(), csric02\_analysis(), and csric02().

# 4.14. csrilu02Info\_t

This is a pointer type to an opaque structure holding the information used in csrilu02 bufferSize(), csrilu02 analysis(), and csrilu02().

# 4.15. bsrsv2lnfo\_t

This is a pointer type to an opaque structure holding the information used in bsrsv2\_bufferSize(), bsrsv2\_analysis(), and bsrsv2\_solve().

# 4.16. bsrsm2Info\_t

This is a pointer type to an opaque structure holding the information used in bsrsm2\_bufferSize(), bsrsm2\_analysis(), and bsrsm2\_solve().

# 4.17. bsric02Info\_t

This is a pointer type to an opaque structure holding the information used in bsric02\_bufferSize(), bsric02\_analysis(), and bsric02().

# 4.18. bsrilu02Info\_t

This is a pointer type to an opaque structure holding the information used in bsrilu02\_bufferSize(), bsrilu02\_analysis(), and bsrilu02().

# 4.19. csrgemm2Info\_t

This is a pointer type to an opaque structure holding the information used in csrgemm2 bufferSizeExt(), and csrgemm2().

# 4.20. cusparseStatus\_t

This is a status type returned by the library functions and it can have the following values.

#### CUSPARSE\_STATUS\_SUCCESS

The operation completed successfully.

#### CUSPARSE\_STATUS\_NOT\_INITIALIZED

The cuSPARSE library was not initialized. This is usually caused by the lack of a prior call, an error in the CUDA Runtime API called by the cuSPARSE routine, or an error in the hardware setup.

**To correct:** call cusparseCreate() prior to the function call; and check that the hardware, an appropriate version of the driver, and the cuSPARSE library are correctly installed.

#### CUSPARSE STATUS ALLOC FAILED

Resource allocation failed inside the cuSPARSE library. This is usually caused by a cudaMalloc() failure.

**To correct:** prior to the function call, deallocate previously allocated memory as much as possible.

#### CUSPARSE\_STATUS\_INVALID\_VALUE

An unsupported value or parameter was passed to the function (a negative vector size, for example).

To correct: ensure that all the parameters being passed have valid values.

#### CUSPARSE STATUS ARCH MISMATCH

The function requires a feature absent from the device architecture; usually caused by the lack of support for atomic operations or double precision.

**To correct:** compile and run the application on a device with appropriate compute capability, which is 1.1 for 32-bit atomic operations and 1.3 for double precision.

#### CUSPARSE STATUS MAPPING ERROR

An access to GPU memory space failed, which is usually caused by a failure to bind a texture.

**To correct:** prior to the function call, unbind any previously bound textures.

#### CUSPARSE\_STATUS\_EXECUTION\_FAILED

The GPU program failed to execute. This is often caused by a launch failure of the kernel on the GPU, which can be caused by multiple reasons.

**To correct:** check that the hardware, an appropriate version of the driver, and the cuSPARSE library are correctly installed.

#### CUSPARSE STATUS INTERNAL ERROR

An internal cuSPARSE operation failed. This error is usually caused by a cudaMemcpyAsync() failure.

**To correct:** check that the hardware, an appropriate version of the driver, and the cuSPARSE library are correctly installed. Also, check that the memory passed as a parameter to the routine is not being deallocated prior to the routine's completion.

#### CUSPARSE\_STATUS\_MATRIX\_TYPE\_NOT\_SUPPORTED

The matrix type is not supported by this function. This is usually caused by passing an invalid matrix descriptor to the function.

To correct: check that the fields in cusparseMatDescr\_t descrA were set correctly.

# Chapter 5. CUSPARSE HELPER FUNCTION REFERENCE

The cuSPARSE helper functions are described in this section.

# 5.1. cusparseCreate()

```
cusparseStatus_t
cusparseCreate(cusparseHandle t *handle)
```

This function initializes the cuSPARSE library and creates a handle on the cuSPARSE context. It must be called before any other cuSPARSE API function is invoked. It allocates hardware resources necessary for accessing the GPU.

#### Output

handle	the pointer to the handle to the cuSPARSE context.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the initialization succeeded.
CUSPARSE_STATUS_NOT_INITIALIZED	the CUDA Runtime initialization failed.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_ARCH_MISMATCH	the device compute capability (CC) is less than 1.1. The CC of at least 1.1 is required.

# 5.2. cusparseCreateSolveAnalysisInfo()

cusparseStatus\_t
cusparseCreateSolveAnalysisInfo(cusparseSolveAnalysisInfo t \*info)

This function creates and initializes the solve and analysis structure to *default* values.

#### Input

info	the pointer to the solve and analysis structure.

CUSPARSE_STATUS_SUCCESS	the structure was initialized successfully.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.

# 5.3. cusparseCreateHybMat()

cusparseStatus\_t
cusparseCreateHybMat(cusparseHybMat t \*hybA)

This function creates and initializes the hybA opaque data structure.

#### Input

hybA	the pointer to the hybrid format storage structure.
------	---

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the structure was initialized successfully.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.

# 5.4. cusparseCreateMatDescr()

cusparseStatus\_t
cusparseCreateMatDescr(cusparseMatDescr\_t \*descrA)

This function initializes the matrix descriptor. It sets the fields MatrixType and IndexBase to the *default* values CUSPARSE\_MATRIX\_TYPE\_GENERAL and CUSPARSE\_INDEX\_BASE\_ZERO, respectively, while leaving other fields uninitialized.

#### Input

descrA	the pointer to the matrix descriptor.
--------	---------------------------------------

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the descriptor was initialized successfully.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.

# 5.5. cusparseCreateSolveAnalysisInfo()

cusparseStatus\_t
cusparseCreateSolveAnalysisInfo(cusparseSolveAnalysisInfo\_t \*info)

This function creates and initializes the solve and analysis structure to *default* values.

#### Input

info	the pointer to the solve and analysis structure.
11110	the pointer to the solve and analysis structure.

CUSPARSE_STATUS_SUCCESS	the structure was initialized successfully.
	,

CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
------------------------------	---------------------------------------

# 5.6. cusparseDestroy()

cusparseStatus\_t
cusparseDestroy(cusparseHandle t handle)

This function releases CPU-side resources used by the cuSPARSE library. The release of GPU-side resources may be deferred until the application shuts down.

#### Input

handle	the handle to the cuSPARSE context.

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the shutdown succeeded.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.

# 5.7. cusparseDestroySolveAnalysisInfo()

cusparseStatus\_t
cusparseDestroySolveAnalysisInfo(cusparseSolveAnalysisInfo\_t info)

This function destroys and releases any memory required by the structure.

#### Input

info	the solve and analysis structure.
------	-----------------------------------

#### **Status Returened**

CUSPARSE_STATUS_SUCCESS	the resources were released successfully.
-------------------------	---

# 5.8. cusparseDestroyHybMat()

cusparseStatus\_t
cusparseDestroyHybMat(cusparseHybMat t hybA)

This function destroys and releases any memory required by the hybA structure.

#### Input

hybA	the hybrid format storage structure.

CUSPARSE_STATUS_SUCCESS	the resources were released successfully.

# 5.9. cusparseDestroyMatDescr()

```
cusparseStatus_t
cusparseDestroyMatDescr(cusparseMatDescr t descrA)
```

This function releases the memory allocated for the matrix descriptor.

#### Input

descrA	the matrix descriptor.
--------	------------------------

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the resources were released successfully.
-------------------------	---

# 5.10. cusparseDestroySolveAnalysisInfo()

```
cusparseStatus_t
cusparseDestroySolveAnalysisInfo(cusparseSolveAnalysisInfo t info)
```

This function destroys and releases any memory required by the structure.

#### Input

info	the solve and analysis structure.

#### **Status Returened**

CUSPARSE_STATUS_SUCCESS	the resources were released successfully.
-------------------------	---

# 5.11. cusparseGetLevelInfo()

This function returns the number of levels and the assignment of rows into the levels computed by either the csrsv\_analysis, csrsm\_analysis or hybsv\_analysis routines.

#### Input

handle	handle to the cuSPARSE library context.
info	the pointer to the solve and analysis structure.

#### Output

nlevels	number of levels.
nievers	number of tevets.

levelPtr	integer array of nlevels+1 elements that contains the start of every level and the end of the last level plus one.
levelInd	integer array of $m$ (number of rows in the matrix) elements that contains the row indices belonging to every level.

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the structure was initialized successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library or the solve analysis structure was not initialized.

# 5.12. cusparseGetMatDiagType()

cusparseDiagType\_t
cusparseGetMatDiagType(const cusparseMatDescr t descrA)

This function returns the DiagType field of the matrix descriptor descrA.

#### Input

descrA	the matrix descriptor.
·	

#### Returned

	One of the enumerated diagType types.
--	---------------------------------------

# 5.13. cusparseGetMatFillMode()

cusparseFillMode\_t
cusparseGetMatFillMode(const cusparseMatDescr t descrA)

This function returns the FillMode field of the matrix descriptor descrA.

#### Input

descrA	the matrix descriptor.
--------	------------------------

#### Returned

One of the enumerated fillMode types.

# 5.14. cusparseGetMatIndexBase()

cusparseIndexBase\_t
cusparseGetMatIndexBase(const cusparseMatDescr\_t descrA)

This function returns the IndexBase field of the matrix descriptor descrA.

#### Input

descrA	the matrix descriptor.
--------	------------------------

#### Returned

	One of the enumerated indexBase types.
--	--

# 5.15. cusparseGetMatType()

cusparseMatrixType\_t
cusparseGetMatType(const cusparseMatDescr t descrA)

This function returns the MatrixType field of the matrix descriptor descrA.

#### Input

descrA	the matrix descriptor.
--------	------------------------

#### Returned

One of the enumerated matrix types.

# 5.16. cusparseGetPointerMode()

This function obtains the pointer mode used by the cuSPARSE library. Please see the section on the **cusparsePointerMode** t type for more details.

#### Input

handle	the handle to the cuSPARSE context.

#### Output

mode One of the enumerated pointer mode types.
--

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the pointer mode was returned successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.

# 5.17. cusparseGetVersion()

```
cusparseStatus_t
cusparseGetVersion(cusparseHandle_t handle, int *version)
```

This function returns the version number of the cuSPARSE library.

#### Input

handle	the handle to the cuSPARSE context.

#### Output

version	the version number of the library.
---------	------------------------------------

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the version was returned successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.

# 5.18. cusparseSetMatDiagType()

This function sets the DiagType field of the matrix descriptor descrA.

#### Input

diagType	One of the enumerated diagType types.
----------	---------------------------------------

#### Output

descrA	the matrix descriptor.
--------	------------------------

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the field DiagType was set successfully.
CUSPARSE_STATUS_INVALID_VALUE	An invalid diagType parameter was passed.

# 5.19. cusparseSetMatFillMode()

This function sets the FillMode field of the matrix descriptor descrA.

#### Input

fillMode	One of the enumerated fillMode types.
----------	---------------------------------------

#### Output

descrA	the matrix descriptor.
--------	------------------------

CUSPARSE_STATUS_SUCCESS	the FillMode field was set successfully.
CUSPARSE_STATUS_INVALID_VALUE	An invalid fillMode parameter was passed.

# 5.20. cusparseSetMatIndexBase()

This function sets the IndexBase field of the matrix descriptor descrA.

#### Input

base	One of the enumerated indexBase types.

#### Output

descrA	the matrix descriptor.
--------	------------------------

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the IndexBase field was set successfully.
CUSPARSE_STATUS_INVALID_VALUE	An invalid base parameter was passed.

# 5.21. cusparseSetMatType()

cusparseStatus\_t
cusparseSetMatType(cusparseMatDescr t descrA, cusparseMatrixType t type)

This function sets the MatrixType field of the matrix descriptor descrA.

#### Input

type	One of the enumerated matrix types.
------	-------------------------------------

#### Output

descrA	the matrix descriptor.
--------	------------------------

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the MatrixType field was set successfully.
CUSPARSE_STATUS_INVALID_VALUE	An invalid type parameter was passed.

# 5.22. cusparseSetPointerMode()

This function sets the pointer mode used by the cuSPARSE library. The *default* is for the values to be passed by reference on the host. Please see the section on the cublasPointerMode\_t type for more details.

#### Input

handle	the handle to the cuSPARSE context.
mode	One of the enumerated pointer mode types.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the pointer mode was set successfully.
CUSPARSE_STATUS_INVALID_VALUE	the library was not initialized.

# 5.23. cusparseSetStream()

```
cusparseStatus_t
cusparseSetStream(cusparseHandle_t handle, cudaStream_t streamId)
```

This function sets the stream to be used by the cuSPARSE library to execute its routines.

#### Input

handle	the handle to the cuSPARSE context.
streamId	the stream to be used by the library.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the stream was set successfully.
CUSPARSE_STATUS_INVALID_VALUE	the library was not initialized.

# 5.24. cusparseGetStream()

```
cusparseStatus_t
cusparseGetStream(cusparseHandle_t handle, cudaStream_t *streamId)
```

This function gets the cuSPARSE library stream, which is being used to to execute all calls to the cuSPARSE library functions. If the cuSPARSE library stream is not set, all kernels use the default NULL stream.

#### Input

	handle	the handle to the cuSPARSE context.
--	--------	-------------------------------------

#### Output

streamId	the stream to be used by the library.
----------	---------------------------------------

CUSPARSE_STATUS_SUCCESS	the stream was returned successfully.
CUSPARSE_STATUS_INVALID_VALUE	the library was not initialized.

### 5.25. cusparseCreateCsrsv2Info()

```
cusparseStatus_t
cusparseCreateCsrsv2Info(csrsv2Info_t *info);
```

This function creates and initializes the solve and analysis structure of csrsv2 to *default* values.

#### Input

info	the pointer to the solve and analysis structure of csrsv2.
	*****

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the structure was initialized successfully.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.

# 5.26. cusparseDestroyCsrsv2Info()

```
cusparseStatus_t
cusparseDestroyCsrsv2Info(csrsv2Info_t info);
```

This function destroys and releases any memory required by the structure.

#### Input

info	the solve (csrsv2_solve) and analysis
	(csrsv2_analysis) structure.

#### **Status Returened**

CUSPARSE_STATUS_SUCCESS	the resources were released successfully.
-------------------------	---

### 5.27. cusparseCreateCsric02Info()

```
cusparseStatus_t
cusparseCreateCsric02Info(csric02Info_t *info);
```

This function creates and initializes the solve and analysis structure of incomplete Cholesky to *default* values.

#### Input

info	the pointer to the solve and analysis structure of incomplete Cholesky.

CUSPARSE_STATUS_SUCCESS	the structure was initialized successfully.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.

## 5.28. cusparseDestroyCsric02Info()

```
cusparseStatus_t
cusparseDestroyCsric02Info(csric02Info t info);
```

This function destroys and releases any memory required by the structure.

#### Input

info	the solve (csric02_solve) and analysis
	(csric02_analysis) Structure.

#### **Status Returened**

CUSPARSE_STATUS_SUCCESS	the resources were released successfully.
-------------------------	---

# 5.29. cusparseCreateCsrilu02Info()

```
cusparseStatus_t
cusparseCreateCsrilu02Info(csrilu02Info_t *info);
```

This function creates and initializes the solve and analysis structure of incomplete LU to *default* values.

#### Input

info	the pointer to the solve and analysis structure of
	incomplete LU.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the structure was initialized successfully.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.

# 5.30. cusparseDestroyCsrilu02Info()

```
cusparseStatus_t
cusparseDestroyCsrilu02Info(csrilu02Info_t info);
```

This function destroys and releases any memory required by the structure.

info	the solve (csrilu02_solve) and analysis
	(csrilu02_analysis) Structure.

#### **Status Returened**

CUSPARSE_STATUS_SUCCESS	the resources were released successfully.
-------------------------	---

## 5.31. cusparseCreateBsrsv2Info()

```
cusparseStatus_t
cusparseCreateBsrsv2Info(bsrsv2Info_t *info);
```

This function creates and initializes the solve and analysis structure of bsrsv2 to *default* values.

#### Input

info	the pointer to the solve and analysis structure of
	bsrsv2.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the structure was initialized successfully.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.

### 5.32. cusparseDestroyBsrsv2Info()

```
cusparseStatus_t
cusparseDestroyBsrsv2Info(bsrsv2Info_t info);
```

This function destroys and releases any memory required by the structure.

#### Input

info	the solve (bsrsv2_solve) and analysis
	(bsrsv2_analysis) Structure.

#### **Status Returened**

CUSPARSE_STATUS_SUCCESS	the resources were released successfully.
-------------------------	---

### 5.33. cusparseCreateBsrsm2Info()

```
cusparseStatus_t
cusparseCreateBsrsm2Info(bsrsm2Info_t *info);
```

This function creates and initializes the solve and analysis structure of bsrsm2 to *default* values.

info	the pointer to the solve and analysis structure of bsrsm2.

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the structure was initialized successfully.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.

### 5.34. cusparseDestroyBsrsm2Info()

```
cusparseStatus_t
cusparseDestroyBsrsm2Info(bsrsm2Info_t info);
```

This function destroys and releases any memory required by the structure.

#### Input

info	the solve (bsrsm2_solve) and analysis
	(bsrsm2_analysis) Structure.

#### **Status Returened**

CUSPARSE_STATUS_SUCCESS	the resources were released successfully.
-------------------------	---

# 5.35. cusparseCreateBsric02Info()

```
cusparseStatus_t
cusparseCreateBsric02Info(bsric02Info_t *info);
```

This function creates and initializes the solve and analysis structure of block incomplete Cholesky to *default* values.

#### Input

info	the pointer to the solve and analysis structure of block incomplete Cholesky.
	Stock mooniple to one tookly.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the structure was initialized successfully.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.

# 5.36. cusparseDestroyBsric02Info()

```
cusparseStatus_t
cusparseDestroyBsric02Info(bsric02Info_t info);
```

This function destroys and releases any memory required by the structure.

info	the solve (bsric02_solve) and analysis
	(bsric02_analysis) Structure.

#### **Status Returened**

CUSPARSE_STATUS_SUCCESS	the resources were released successfully.
-------------------------	---

## 5.37. cusparseCreateBsrilu02Info()

```
cusparseStatus_t
cusparseCreateBsrilu02Info(bsrilu02Info_t *info);
```

This function creates and initializes the solve and analysis structure of block incomplete LU to *default* values.

#### Input

info	the pointer to the solve and analysis structure of
	block incomplete LU.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the structure was initialized successfully.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.

### 5.38. cusparseDestroyBsrilu02Info()

```
cusparseStatus_t
cusparseDestroyBsrilu02Info(bsrilu02Info_t info);
```

This function destroys and releases any memory required by the structure.

#### Input

info	the solve (bsrilu02_solve) and analysis
	(bsrilu02_analysis) Structure.

#### **Status Returened**

CUSPARSE_STATUS_SUCCESS	the resources were released successfully.

# 5.39. cusparseCreateCsrgemm2Info()

```
cusparseStatus_t
cusparseCreateCsrgemm2Info(csrgemm2Info_t *info);
```

This function creates and initializes analysis structure of general sparse matrix-matrix multiplication.

info	the pointer to the analysis structure of general
	sparse matrix-matrix multiplication.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the structure was initialized successfully.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.

# 5.40. cusparseDestroyCsrgemm2Info()

cusparseStatus\_t
cusparseDestroyCsrgemm2Info(csrgemm2Info\_t info);

This function destroys and releases any memory required by the structure.

#### Input

info	opaque structure of csrgemm2.
------	-------------------------------

CUSPARSE_STATUS_SUCCESS	the resources were released successfully.
-------------------------	---

# Chapter 6. <a href="CUSPARSE LEVEL 1 FUNCTION REFERENCE">CUSPARSE LEVEL 1 FUNCTION REFERENCE</a>

This chapter describes sparse linear algebra functions that perform operations between dense and sparse vectors.

### 6.1. cusparse<t>axpyi()

This function multiplies the vector  $\mathbf{x}$  in sparse format by the constant  $\alpha$  and adds the result to the vector  $\mathbf{y}$  in dense format. This operation can be written as

```
y = y + a * x
```

In other words,

```
for i=0 to nnz-1
    y[xInd[i]-idxBase] = y[xInd[i]-idxBase] + alpha*xVal[i]
```

This function requires no extra storage. It is executed asynchronously with respect to the host, and it may return control to the application on the host before the result is ready.

handle	handle to the cuSPARSE library context.
nnz	number of elements in vector $\mathbf{x}$ .
alpha	<type> scalar used for multiplication.</type>
xVal	<pre><type> vector with <math>nnz</math> nonzero values of vector x.</type></pre>
*Ind	integer vector with nnz indices of the nonzero values of vector x.
У	<type> vector in dense format.</type>
idxBase	CUSPARSE_INDEX_BASE_ZERO OF CUSPARSE_INDEX_BASE_ONE.

#### Output

У	<type> updated vector in dense format (that is</type>
	unchanged if nnz == 0).

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

### 6.2. cusparse<t>doti()

```
cusparseStatus_t
*y,
            cusparseIndexBase_t idxBase)
cusparseStatus_t
*y,
            cusparseIndexBase t idxBase)
cusparseStatus t
cusparseCdoti(cusparseHandle t handle, int nnz,
            const cuComplex *xVal,
const int *xInd, const cuComplex
cuComplex *resultDevHostPtr,
                                              *y,
            cusparseIndexBase_t idxBase)
cusparseStatus t
cusparseZdoti(cusparseHandle t handle, int nnz, const
            cuDoubleComplex *xVal,
            const int *xInd, const cuDoubleComplex *y,
cuDoubleComplex *resultDevHostPtr,
            cusparseIndexBase t idxBase)
```

This function returns the dot product of a vector  $\mathbf{x}$  in sparse format and vector  $\mathbf{y}$  in dense format. This operation can be written as

$$result = y^Tx$$

#### In other words,

```
for i=0 to nnz-1
   resultDevHostPtr += xVal[i]*y[xInd[i-idxBase]]
```

This function requires some temporary extra storage that is allocated internally. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

#### Input

handle	handle to the cuSPARSE library context.
nnz	number of elements in vector x.
xVal	<type> vector with nnz nonzero values of vector x.</type>
*Ind	integer vector with nnz indices of the nonzero values of vector x.
У	<type> vector in dense format.</type>
resultDevHostPtr	pointer to the location of the result in the device or host memory.
idxBase	CUSPARSE_INDEX_BASE_ZERO OF CUSPARSE_INDEX_BASE_ONE.

#### Output

resultDevHostPtr	scalar result in the device or host memory (that is
	zero if nnz == 0).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the reduction buffer could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 6.3. cusparse<t>dotci()

This function returns the dot product of a complex conjugate of vector  $\mathbf{x}$  in sparse format and vector  $\mathbf{y}$  in dense format. This operation can be written as

result = xHy

#### In other words,

```
for i=0 to nnz-1
resultDevHostPtr += xVal[i]*y[xInd[i-idxBase]]
```

This function requires some temporary extra storage that is allocated internally. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

#### Input

handle	handle to the cuSPARSE library context.
nnz	number of elements in vector x.
xVal	<type> vector with nnz nonzero values of vector x.</type>
xInd	integer vector with nnz indices of the nonzero values of vector x.
У	<type> vector in dense format.</type>
resultDevHostPtr	pointer to the location of the result in the device or host memory.
idxBase	CUSPARSE_INDEX_BASE_ZERO OF CUSPARSE_INDEX_BASE_ONE.

#### Output

resultDevHostPtr	scalar result in the device or host memory (that is
	zero if nnz == 0).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the reduction buffer could not be allocated.

CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

### 6.4. cusparse<t>gthr()

```
cusparseStatus t
cusparseSgthr(cusparseHandle t handle, int nnz,
             cusparseIndexBase t idxBase)
cusparseStatus t
cusparseDgthr(cusparseHandle t handle, int nnz,
             const double *y,
double *xVal, const int *xInd,
             cusparseIndexBase t idxBase)
cusparseStatus t
cusparseCgthr(cusparseHandle t handle, int nnz,
             const cuComplex *y,
cuComplex *xVal, const int *xInd,
             cusparseIndexBase_t idxBase)
cusparseStatus t
cusparseZgthr(cusparseHandle t handle, int nnz,
             const cuDoubleComplex *y,
cuDoubleComplex *xVal, const int *xInd,
              cusparseIndexBase t idxBase)
```

This function gathers the elements of the vector **y** listed in the index array **xInd** into the data array **xVal**.

This function requires no extra storage. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

#### Input

handle	handle to the cuSPARSE library context.
nnz	number of elements in vector x.
У	<type> vector in dense format (of size≥max(xInd)-idxBase+1).</type>
*Ind	integer vector with nnz indices of the nonzero values of vector x.
idxBase	CUSPARSE_INDEX_BASE_ZERO OF CUSPARSE_INDEX_BASE_ONE.

#### Output

<pre><type> vector with nnz nonzero values that were gathered from vector y (that is unchanged if nnz</type></pre>
== 0).

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

### 6.5. cusparse<t>gthrz()

```
cusparseStatus t
cusparseSgthrz(cusparseHandle_t handle, int nnz, float
                                                               *y,
              float *xVal, const int *xInd,
              cusparseIndexBase t idxBase)
cusparseStatus t
cusparseDgthrz(cusparseHandle_t handle, int nnz, double
                                                               *y,
              double *xVal, const int *xInd,
              cusparseIndexBase t idxBase)
cusparseStatus t
cusparseCgthrz(cusparseHandle_t handle, int nnz, cuComplex
                                                               *y,
              cuComplex - *xVal, const int *xInd,
              cusparseIndexBase t idxBase)
cusparseStatus t
cusparseZgthrz(cusparseHandle t handle, int nnz, cuDoubleComplex *y,
              cuDoubleComplex *xVal, const int *xInd,
              cusparseIndexBase t idxBase)
```

This function gathers the elements of the vector **y** listed in the index array **xInd** into the data array **xVal**. Also, it zeros out the gathered elements in the vector **y**.

This function requires no extra storage. It is executed asynchronously with respect to the host, and it may return control to the application on the host before the result is ready.

#### Input

handle	handle to the cuSPARSE library context.
nnz	number of elements in vector $\mathbf{x}$ .
У	<type> vector in dense format (of size≥max(xInd)-idxBase+1).</type>
*Ind	integer vector with nnz indices of the nonzero values of vector x.
idxBase	CUSPARSE_INDEX_BASE_ZERO OF CUSPARSE_INDEX_BASE_ONE.

#### Output

xVal	<pre><type> vector with nnz nonzero values that were gathered from vector y (that is unchanged if nnz</type></pre>
	== 0).

<del>-</del>	<pre><type> vector in dense format with elements indexed by xInd set to zero (it is unchanged if nnz</type></pre>
	== 0).

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

### 6.6. cusparse<t>roti()

This function applies the Givens rotation matrix

$$G = \begin{pmatrix} C & S \\ -S & C \end{pmatrix}$$

to sparse **x** and dense **y** vectors. In other words,

```
for i=0 to nnz-1
    y[xInd[i]-idxBase] = c * y[xInd[i]-idxBase] - s*xVal[i]
    x[i] = c * xVal[i] + s * y[xInd[i]-idxBase]
```

handle	handle to the cuSPARSE library context.
nnz	number of elements in vector x.
xVal	<type> vector with nnz nonzero values of vector x.</type>
*Ind	integer vector with nnz indices of the nonzero values of vector x.
У	<type> vector in dense format.</type>
С	cosine element of the rotation matrix.
s	sine element of the rotation matrix.
idxBase	CUSPARSE_INDEX_BASE_ZERO OF CUSPARSE_INDEX_BASE_ONE.

#### Output

xVal	<pre><type> updated vector in sparse format (that is unchanged if nnz == 0).</type></pre>
У	<pre><type> updated vector in dense format (that is unchanged if nnz == 0).</type></pre>

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

# 6.7. cusparse<t>sctr()

```
cusparseStatus t
cusparseSsctr(cusparseHandle t handle, int nnz,
             const float
                           *xVal,
             const int *xInd, float
                                              *y,
             cusparseIndexBase t idxBase)
cusparseStatus t
cusparseDsctr(cusparseHandle_t handle, int nnz,
             const double
                                  *xVal,
             const int *xInd, double
                                              *y,
             cusparseIndexBase t idxBase)
cusparseStatus t
cusparseCsctr(cusparseHandle t handle, int nnz,
             const cuComplex *xVal,
             const int *xInd, cuComplex
                                              *y,
             cusparseIndexBase_t idxBase)
cusparseStatus t
cusparseZsctr(cusparseHandle t handle, int nnz,
             const cuDoubleComplex *xVal,
             const int *xInd, cuDoubleComplex *y,
             cusparseIndexBase t idxBase)
```

This function scatters the elements of the vector  $\mathbf{x}$  in sparse format into the vector  $\mathbf{y}$  in dense format. It modifies only the elements of  $\mathbf{y}$  whose indices are listed in the array  $\mathbf{xInd}$ .

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

handle	handle to the cuSPARSE library context.
nnz	number of elements in vector $\mathbf{x}$ .
xVal	<type> vector with <math>nnz</math> nonzero values of vector <math>x</math>.</type>

xInd	integer vector with nnz indices of the nonzero values of vector x.
У	<type> dense vector (of size≥max(xInd) - idxBase+1).</type>
idxBase	CUSPARSE_INDEX_BASE_ZERO OF CUSPARSE_INDEX_BASE_ONE.

### Output

<del>-</del>	<pre><type> vector with nnz nonzero values that were scattered from vector x (that is unchanged if nnz</type></pre>
	== 0).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

# Chapter 7. <a href="CUSPARSE\_LEVEL 2 FUNCTION REFERENCE">CUSPARSE\_LEVEL 2 FUNCTION REFERENCE</a>

This chapter describes the sparse linear algebra functions that perform operations between sparse matrices and dense vectors.

In particular, the solution of sparse triangular linear systems is implemented in two phases. First, during the analysis phase, the sparse triangular matrix is analyzed to determine the dependencies between its elements by calling the appropriate <code>csrsv\_analysis()</code> function. The analysis is specific to the sparsity pattern of the given matrix and to the selected <code>cusparseOperation\_t</code> type. The information from the analysis phase is stored in the parameter of type <code>cusparseSolveAnalysisInfo\_t</code> that has been initialized previously with a call to <code>cusparseCreateSolveAnalysisInfo()</code>.

Second, during the solve phase, the given sparse triangular linear system is solved using the information stored in the <code>cusparseSolveAnalysisInfo\_t</code> parameter by calling the appropriate <code>csrsv\_solve()</code> function. The solve phase may be performed multiple times with different right-hand sides, while the analysis phase needs to be performed only once. This is especially useful when a sparse triangular linear system must be solved for a set of different right-hand sides one at a time, while its coefficient matrix remains the same.

Finally, once all the solves have completed, the opaque data structure pointed to by the cusparseSolveAnalysisInfo\_t parameter can be released by calling cusparseDestroySolveAnalysisInfo(). For more information please refer to [3].

### 7.1. cusparse<t>bsrmv()

```
cusparseStatus t
cusparseSbsrmv(cusparseHandle t handle, cusparseDirection t dir,
   cusparseOperation t trans, int mb, int nb, int nnzb,
   const float *alpha, const cusparseMatDescr t descr,
   const float *bsrVal, const int *bsrRowPtr, const int *bsrColInd,
   int blockDim, const float *x,
   const float *beta, float *y)
cusparseStatus t
cusparseDbsrmv(cusparseHandle t handle, cusparseDirection t dir,
   cusparseOperation t trans, int mb, int nb, int nnzb,
   const double *alpha, const cusparseMatDescr t descr,
   const double *bsrVal, const int *bsrRowPtr, const int *bsrColInd,
   int blockDim, const double *x,
   const double *beta, double *y)
cusparseStatus t
cusparseCbsrmv(cusparseHandle_t handle, cusparseDirection_t dir,
   cusparseOperation_t trans, int mb, int nb, int nnzb,
   const cuComplex *alpha, const cusparseMatDescr_t descr,
   const cuComplex *bsrVal, const int *bsrRowPtr, const int *bsrColInd,
   int blockDim, const cuComplex *x,
   const cuComplex *beta, cuComplex *y)
cusparseStatus t
const cuDoubleComplex *alpha, const cusparseMatDescr_t descr,
   const cuDoubleComplex *bsrVal, const int *bsrRowPtr, const int
 *bsrColInd,
   int blockDim, const cuDoubleComplex *x,
   const cuDoubleComplex *beta, cuDoubleComplex *y)
```

This function performs the matrix-vector operation

$$y = a * op(A) * x + B * y$$

where A is an  $(mb*blockDim) \times (nb*blockDim)$  sparse matrix that is defined in BSR storage format by the three arrays **bsrVal**, **bsrRowPtr**, and **bsrColInd**); **x** and **y** are vectors;  $\alpha$  and  $\beta$  are scalars; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

Several comments on bsrmv():

▶ Only **cusparse operation non transpose** is supported, that is

$$V = a * A * X + B * V$$

- ▶ Only cusparse matrix type general is supported.
- ► The size of vector **x** should be (*nb* \* *blockDim*) at least, and the size of vector **y** should be (*mb* \* *blockDim*) at least; otherwise, the kernel may return **CUSPARSE STATUS EXECUTION FAILED** because of an out-of-bounds array.

For example, suppose the user has a CSR format and wants to try **bsrmv()**, the following code demonstrates how to use **csr2bsr()** conversion and **bsrmv()** multiplication in single precision.

```
// Suppose that A is m x n sparse matrix represented by CSR format,
// hx is a host vector of size n, and hy is also a host vector of size m.
// m and n are not multiple of blockDim.
// step 1: transform CSR to BSR with column-major order
int base, nnz;
int nnzb;
cusparseDirection t dirA = CUSPARSE DIRECTION COLUMN;
int mb = (m + blockDim-1)/blockDim;
int nb = (n + blockDim-1)/blockDim;
cudaMalloc((void**)&bsrRowPtrC, sizeof(int) *(mb+1));
cusparseXcsr2bsrNnz(handle, dirA, m, n,
        descrA, csrRowPtrA, csrColIndA, blockDim,
        descrC, bsrRowPtrC, &nnzb);
cudaMalloc((void**)&bsrColIndC, sizeof(int)*nnzb);
cudaMalloc((void**)&bsrValC, sizeof(float)*(blockDim*blockDim)*nnzb);
cusparseScsr2bsr(handle, dirA, m, n,
       descrA, csrValA, csrRowPtrA, csrColIndA, blockDim,
        descrC, bsrValC, bsrRowPtrC, bsrColIndC);
// step 2: allocate vector x and vector y large enough for bsrmv
cudaMalloc((void**)&x, sizeof(float)*(nb*blockDim));
cudaMalloc((void**)&y, sizeof(float)*(mb*blockDim));
cudaMemcpy(x, hx, sizeof(float)*n, cudaMemcpyHostToDevice);
cudaMemcpy(y, hy, sizeof(float)*m, cudaMemcpyHostToDevice);
// step 3: perform bsrmv
cusparseSbsrmv(handle, dirA, transA, mb, nb, nnzb, &alpha,
  descrC, bsrValC, bsrRowPtrC, bsrColIndC, blockDim, x, &beta, y);
```

handle	handle to the cuSPARSE library context.
dir	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
trans	the operation op(A). Only cusparse_operation_non_transpose is supported.
mb	number of block rows of matrix A.
nb	number of block columns of matrix A.
nnzb	number of nonzero blocks of matrix A.
alpha	<type> scalar used for multiplication.</type>
descr	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrVal	<pre><type> array of nnz (= csrRowPtrA(mb) - csrRowPtrA(0) ) nonzero blocks of matrix A.</type></pre>
bsrRowPtr	integer array of mb +1 elements that contains the start of every block row and the end of the last block row plus one.

bsrColInd	integer array of nnz (= csrRowPtrA(mb) - csrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A, larger than zero.
x	<type> vector of nb * blockDim elements.</type>
beta	<pre><type> scalar used for multiplication. If beta is zero, y does not have to be a valid input.</type></pre>
У	<type> vector of mb * blockDim elements.</type>

### Output

У	<type> updated vector.</type>
---	-------------------------------

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were  passed (m,n,nnz<0, trans !=  CUSPARSE_OPERATION_NON_TRANSPOSE,  blockDim<1, dir is not row-major or column- major, or IndexBase of descr is not base-0 or  base-1).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

### 7.2. cusparse<t>bsrxmv()

```
cusparseStatus t
cusparseSbsrxmv(cusparseHandle t handle,
                cusparseDirection t dir,
                cusparseOperation t trans,
                int sizeOfMask,
                int mb,
                int nb,
                int nnzb,
                const float *alpha,
                const cusparseMatDescr t descr,
                const float *bsrVal,
                const int *bsrMaskPtr,
                const int *bsrRowPtr,
                const int *bsrEndPtr,
                const int *bsrColInd,
                int blockDim,
                const float *x,
                const float *beta,
                float *y)
cusparseStatus t
cusparseDbsrxmv(cusparseHandle t handle,
                cusparseDirection t dir,
                cusparseOperation t trans,
                int sizeOfMask,
                int mb,
                int nb,
                int nnzb,
                const double *alpha,
                const cusparseMatDescr t descr,
                const double *bsrVal,
                const int *bsrMaskPtr,
                const int *bsrRowPtr,
                const int *bsrEndPtr,
                const int *bsrColInd,
                int blockDim,
                const double *x,
                const double *beta,
                double *y)
cusparseStatus t
cusparseCbsrxmv(cusparseHandle t handle,
                cusparseDirection t dir,
                cusparseOperation t trans,
                int sizeOfMask,
                int mb,
                int nb,
                int nnzb,
                const cuComplex *alpha,
                const cusparseMatDescr t descr,
                const cuComplex *bsrVal,
                const int *bsrMaskPtr,
                const int *bsrRowPtr,
                const int *bsrEndPtr,
                const int *bsrColInd,
                int blockDim,
                const cuComplex *x,
                const cuComplex *beta,
                cuComplex *y)
cusparseStatus t
cusparseZbsrxmv(cusparseHandle t handle,
                cusparseDirection_t dir,
                cusparseOperation t trans,
                int sizeOfMask,
                int mb.
```

This function performs a bsrmv and a mask operation

$$y (mask) = (a * op(A) * x + B * y)(mask)$$

where A is an  $(mb*blockDim) \times (nb*blockDim)$  sparse matrix that is defined in BSRX storage format by the four arrays **bsrVal**, **bsrRowPtr**, **bsrEndPtr**, and **bsrColInd**); **x** and **y** are vectors;  $\alpha$  and  $\beta$  are scalars; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

The mask operation is defined by array **bsrMaskPtr** which contains updated block row indices of y. If row i is not specified in **bsrMaskPtr**, then **bsrxmv()** does not touch row block i of A and y.

For example, consider the  $2 \times 3$  block matrix A:

$$A = \begin{bmatrix} A_{11} & A_{12} & O \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$$

and its one-based BSR format (three vector form) is

$$bsrVal = \begin{bmatrix} A_{11} & A_{12} & A_{21} & A_{22} & A_{23} \end{bmatrix}$$

$$bsrRowPtr = \begin{bmatrix} 1 & 3 & 6 \end{bmatrix}$$

$$bsrColInd = \begin{bmatrix} 1 & 2 & 1 & 2 & 3 \end{bmatrix}$$

Suppose we want to do the following **bsrmv** operation on a matrix  $\overline{A}$  which is slightly different from A.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} := alpha * (\tilde{A} = \begin{bmatrix} O & O & O \\ O & A_{22} & O \end{bmatrix}) * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} y_1 \\ beta * y_2 \end{bmatrix}$$

We don't need to create another BSR format for the new matrix  $\overline{A}$ , all that we should do is to keep **bsrVal** and **bsrColInd** unchanged, but modify **bsrRowPtr** and add an additional array **bsrEndPtr** which points to the last nonzero elements per row of  $\overline{A}$  plus 1.

For example, the following bsrRowPtr and bsrEndPtr can represent matrix  $\overline{A}$ :

$$bsrRowPtr = [1 4]$$
  
 $bsrEndPtr = [1 5]$ 

Further we can use a mask operator (specified by array **bsrMaskPtr**) to update particular block row indices of y only because  $y_1$  is never changed. In this case, **bsrMaskPtr** = [2] and **sizeOfMask=1**.

The mask operator is equivalent to the following operation:

$$\begin{bmatrix} ? \\ y_2 \end{bmatrix} := alpha * \begin{bmatrix} ? & ? & ? \\ O & A_{22} & O \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + beta * \begin{bmatrix} ? \\ y_2 \end{bmatrix}$$

If a block row is not present in the **bsrMaskPtr**, then no calculation is performed on that row, and the corresponding value in **y** is unmodified. The question mark "?" is used to inidcate row blocks not in **bsrMaskPtr**.

In this case, first row block is not present in **bsrMaskPtr**, so **bsrRowPtr[0]** and **bsrEndPtr[0]** are not touched also.

A couple of comments on bsrxmv():

- ► Only CUSPARSE\_OPERATION\_NON\_TRANSPOSE and CUSPARSE MATRIX TYPE GENERAL are supported.
- Parameters bsrMaskPtr, bsrRowPtr, bsrEndPtr and bsrColInd are consistent with base index, either one-based or zero-based. The above example is one-based.

	I II ( ) CDADCE III
handle	handle to the cuSPARSE library context.
dir	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
trans	the operation op(A). Only cusparse_operation_non_transpose is supported.
sizeOfMask	number of updated block rows of y.
mb	number of block rows of matrix A.
nb	number of block columns of matrix A.
nnzb	number of nonzero blocks of matrix A.
alpha	<type> scalar used for multiplication.</type>
descr	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrVal	<type> array of nnz nonzero blocks of matrix A.</type>
bsrMaskPtr	integer array of sizeOfMask elements that contains the indices corresponding to updated block rows.
bsrRowPtr	integer array of mb elements that contains the start of every block row.
bsrEndPtr	integer array of mb elements that contains the end of the every block row plus one.
bsrColInd	integer array of nnzb column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix <i>A</i> , larger than zero.

х	<type> vector of nb * blockDim elements.</type>
beta	<pre><type> scalar used for multiplication. If beta is zero, <math>\mathbf{y}</math> does not have to be a valid input.</type></pre>
У	<type> vector of mb * blockDim elements.</type>

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were  passed (m,n,nnz<0, trans !=  CUSPARSE_OPERATION_NON_TRANSPOSE,  blockDim<1, dir is not row-major or column- major, or IndexBase of descr is not base-0 or base-1).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

### 7.3. cusparse<t>csrmv()

```
cusparseStatus t
cusparseScsrmv(cusparseHandle t handle, cusparseOperation t transA,
            int m, int n, int nnz, const float
            const cusparseMatDescr t descrA,
            const float *csrValA,
            const int *csrRowPtrA, const int *csrColIndA,
            const float *x, const float
                                                     *beta,
            float
cusparseStatus t
cusparseDcsrmv(cusparseHandle t handle, cusparseOperation t transA,
            int m, int n, int nnz, const double
            const cusparseMatDescr t descrA,
            const double *x, const double
                                                     *beta,
            double
cusparseStatus t
cusparseCcsrmv(cusparseHandle_t handle, cusparseOperation_t transA,
            int m, int n, int nnz, const cuComplex *alpha,
            const cusparseMatDescr_t descrA,
            *beta,
            const cuComplex *x, const cuComplex
            cuComplex
cusparseStatus t
cusparseZcsrmv(cusparseHandle_t handle, cusparseOperation_t transA,
            int m, int n, int nnz, const cuDoubleComplex *alpha,
            const cusparseMatDescr_t descrA,
            const cuDoubleComplex *csrValA,
            const int *csrRowPtrA, const int *csrColIndA,
            const cuDoubleComplex *x, const cuDoubleComplex *beta,
            cuDoubleComplex *y)
```

This function performs the matrix-vector operation

$$y = a * op(A) * x + B * y$$

**A** is an  $m \times n$  sparse matrix that is defined in CSR storage format by the three arrays **csrValA**, **csrRowPtrA**, and **csrColIndA**); **x** and **y** are vectors;  $\alpha$  and  $\beta$  are scalars; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

When using the (conjugate) transpose of a general matrix or a Hermitian/symmetric matrix, this routine may produce slightly different results during different runs with the same input parameters. For these matrix types it uses atomic operations to compute the final result, consequently many threads may be adding floating point numbers to the same memory location without any specific ordering, which may produce slightly different results for each run.

If exactly the same output is required for any input when multiplying by the transpose of a general matrix, the following procedure can be used:

- 1. Convert the matrix from CSR to CSC format using one of the csr2csc() functions. Notice that by interchanging the rows and columns of the result you are implicitly transposing the matrix.
- 2. Call the csrmv() function with the cusparseOperation\_t parameter set to CUSPARSE\_OPERATION\_NON\_TRANSPOSE and with the interchanged rows and columns of the matrix stored in CSC format. This (implicitly) multiplies the vector by the transpose of the matrix in the original CSR format.

This function requires no extra storage for the general matrices when operation <code>CUSPARSE\_OPERATION\_NON\_TRANSPOSE</code> is selected. It requires some extra storage for Hermitian/symmetric matrices and for the general matrices when an operation different than <code>CUSPARSE\_OPERATION\_NON\_TRANSPOSE</code> is selected. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

#### Input

handle to the cuSPARSE library context.  trans the operation op(A).  m number of rows of matrix A.  n number of columns of matrix A.  nnz number of nonzero elements of matrix A.  alpha <type> scalar used for multiplication.  descrA the descriptor of matrix A. The supported matrix type is cusparse_matrix_Type_gemeral,</type>		
number of rows of matrix A.  number of columns of matrix A.  number of nonzero elements of matrix A.  alpha	handle	handle to the cuSPARSE library context.
number of columns of matrix A.  nnz  number of nonzero elements of matrix A.  alpha <type> scalar used for multiplication.  the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, CUSPARSE_MATRIX_TYPE_SYMMETRIC, and CUSPARSE_MATRIX_TYPE_BERMITIAN. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.  csrValA  <type> array of nnz (= csrRowPtrA (m) - csrRowPtrA (0) ) nonzero elements of matrix A.  csrRowPtrA  integer array of m+1 elements that contains the start of every row and the end of the last row plus one.  csrColIndA  integer array of nnz (= csrRowPtrA (m) - csrRowPtrA (0) ) column indices of the nonzero elements of matrix A.  x  <type> vector of n elements if op(A) = A, and m elements if op(A) = A, and m elements if op(A) = A<sup>H</sup>  beta  <type> scalar used for multiplication. If beta is zero, y does not have to be a valid input.  y  <type> vector of m elements if op(A) = A, and n</type></type></type></type></type>	trans	the operation op(A).
number of nonzero elements of matrix A.  alpha	m	number of rows of matrix A.
alpha <type> scalar used for multiplication.  the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GEMERAL, CUSPARSE_MATRIX_TYPE_SYMMETRIC, and CUSPARSE_MATRIX_TYPE_HERMITIAN. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.  csrValA  <type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) nonzero elements of matrix a.  csrRowPtrA  integer array of m+1 elements that contains the start of every row and the end of the last row plus one.  csrColIndA  integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column indices of the nonzero elements of matrix a.  x  <type> vector of n elements if op(A) = A, and m elements if op(A) = A, and m elements if op(A) = A for op(A) = A for op(A) = A, and n elements if op(A) = A, and n n  **Type&gt; vector of melements if op(A) = A, and n n  **Type&gt; vector of melements if op(A) = A, and n n  **Type&gt; vector of melements if op(A) = A, and n n  **Type&gt; vector of melements if op(A) = A, and n n</type></type></type>	n	number of columns of matrix A.
the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, CUSPARSE_MATRIX_TYPE_GENERAL, CUSPARSE_MATRIX_TYPE_HERMITIAN. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.  CSTVALA <pre></pre>	nnz	number of nonzero elements of matrix A.
type is CUSPARSE_MATRIX_TYPE_GENERAL, CUSPARSE_MATRIX_TYPE_SYMMETRIC, and CUSPARSE_MATRIX_TYPE_HERMITIAN. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.  CSTValA <pre></pre>	alpha	<type> scalar used for multiplication.</type>
csrRowPtrA (0) ) nonzero elements of matrix A.  csrRowPtrA integer array of m+1 elements that contains the start of every row and the end of the last row plus one.  csrColIndA integer array of nnz (= csrRowPtrA (m) - csrRowPtrA (0) ) column indices of the nonzero elements of matrix A.  x <pre></pre>	descrA	type is CUSPARSE MATRIX TYPE GENERAL, CUSPARSE MATRIX TYPE SYMMETRIC, and CUSPARSE MATRIX TYPE HERMITIAN. Also, the supported index bases are CUSPARSE INDEX BASE ZERO and
start of every row and the end of the last row plus one.  csrColIndA  integer array of nnz (= csrRowPtrA (m) - csrRowPtrA (0) ) column indices of the nonzero elements of matrix A.  x <type> vector of n elements if op(A) = A, and m elements if op(A) = A<sup>T</sup> or op(A) = A<sup>H</sup>  beta  <type> scalar used for multiplication. If beta is zero, y does not have to be a valid input.  y  <type> vector of m elements if op(A) = A, and n</type></type></type>	csrValA	, , , , , , , , , , , , , , , , , , , ,
csrRowPtrA(0)) column indices of the nonzero elements of matrix A.  * type> vector of n elements if op(A) = A, and m elements if op(A) = A <sup>T</sup> or op(A) = A <sup>H</sup> beta  * type> scalar used for multiplication. If beta is zero, y does not have to be a valid input.  * type> vector of m elements if op(A) = A, and n	csrRowPtrA	start of every row and the end of the last row plus
elements if $op(A) = A^{T}$ or $op(A) = A^{H}$ beta <pre></pre>	csrColIndA	csrRowPtrA(0)) column indices of the nonzero
zero, y does not have to be a valid input.  y $< type> vector of m elements if op(A) = A, and n$	x	
	beta	
	У	1

#### Output

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,nnz<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision (compute capability (c.c.) >= 1.3 required), symmetric/Hermitian matrix (c.c. >= 1.2 required), or transpose operation (c.c. >= 1.1 required).
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 7.4. cusparseCsrmvEx()

This function is an extended version of **cusparse<t>csrmv()** which performs the matrix-vector multiply operation. For detailed description of the functionality, see **cusparse<t>csrmv()**.

For half-precision execution type, the minimum GPU architecture is SM\_53. Also, for both half-precision IO and execution, only CUSPARSE\_MATRIX\_TYPE\_GENERAL and CUSPARSE\_OPERATION\_NON\_TRANSPOSE are supported.

All pointers should be aligned with 128 bytes.

#### Input specifically required by cusparseCsrmvEx

alg	Not currently used. Will be used for future algorithms such as MergePath or Multi-GPU.
alphatype	Data type of alpha.
csrValAtype	Data type of csrValA.
xtype	Data type of x.
betatype	Data type of beta.
ytype	Data type of y.

executiontype	Data type used for computation.
buffer	Pointer to workspace buffer. Not currently used, however must be allocated and be aligned with word boundaries. Recommended to always use cusparseCsrmvEx_bufferSize to obtain the right size for this buffer.

### 7.5. cusparseCsrmvEx\_bufferSize()

```
cusparseCsrmvEx bufferSize(cusparseHandle t handle,
                                                   cusparseAlgMode t alg,
                                                  cusparseOperation t
transA,
                                                  int m, int n, int nnz,
                                                  const void *alpha,
cudaDataType alphatype,
                                                  const
cusparseMatDescr t descrA,
                                                  const void *csrValA,
cudaDataType csrValAtype,
                                                  const int *csrRowPtrA,
                                                  const int *csrColIndA,
                                                  const void *x,
cudaDataType xtype,
                                                  const void *beta,
cudaDataType betatype,
                                                  void *y, cudaDataType
ytype,
                                                  cudaDataType
executiontype,
                                                  size t
*bufferSizeInBytes);
```

This function returns the size of the workspace needed by **cusparseCsrmvEx**. User needs to allocate a buffer of this size and give that buffer to **cusparseCsrmvEx** as an argument. All the arguments are similar to **cusparseCsrmvEx** except the following output argument.

#### Output specifically required by cusparseCsrmvEx\_bufferSize

bufferSizeInBytes	Pointer to a size_t variable, which will be assigned with the size of workspace needed by cusparseCsrmv. Since workspace is not currently used by cusparseCsrmv, this function returns the number 16 as an arbitrary number.
-------------------	--

### 7.6. cusparse<t>csrmv\_mp()

```
cusparseStatus t
cusparseScsrmv mp(cusparseHandle t handle, cusparseOperation t transA,
             int m, int n, int nnz,
              const float *alpha,
              const cusparseMatDescr t descrA,
              const float *csrValA,
              const int *csrRowPtrA, const int *csrColIndA,
                                                          *beta,
             cusparseStatus t
cusparseDcsrmv_mp(cusparseHandle_t handle, cusparseOperation t transA,
              int m, int n, int nnz,
             const double *alpha,
             const cusparseMatDescr t descrA,
              const double *csrValA,
              const int *csrRowPtrA, const int *csrColIndA,
                                                          *beta,
             const double *x, const double
             double
cusparseStatus_t
cusparseCcsrmv_mp(cusparseHandle_t handle, cusparseOperation_t transA,
              int m, int n, int nnz,
             const cuComplex *alpha,
              const cusparseMatDescr_t descrA,
              const cuComplex \overline{*}csrValA,
              const int *csrRowPtrA, const int *csrColIndA,
                                                          *beta,
              const cuComplex *x, const cuComplex
              cuComplex
cusparseStatus t
cusparseZcsrmv_mp(cusparseHandle t handle, cusparseOperation t transA,
              int m, int n, int nnz,
              const cuDoubleComplex *alpha,
              const cusparseMatDescr t descrA,
              const cuDoubleComplex *\overline{\text{*}}csrValA,
const int *csrRowPtrA, const int *csrColIndA,
              const cuDoubleComplex *x, const cuDoubleComplex *beta,
              cuDoubleComplex *y)
```

This function performs a load-balanced matrix-vector operation

$$y = a * op(A) * x + B * y$$

**A** is an  $m \times n$  sparse matrix that is defined in CSR storage format by the three arrays **csrValA**, **csrRowPtrA**, and **csrColIndA**); **x** and **y** are vectors;  $\alpha$  and  $\beta$  are scalars; and

$$\mathrm{op}(A) = \begin{cases} A & \text{if trans} == \mathrm{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \mathrm{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \mathrm{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

This routine was introduced specifically to address some of the loss of performance in the regular csrmv() code due to irregular sparsity patterns and transpose operations. The core kernels are based on the "MergePath" approach created by Duanne Merril. By using this approach, we are able to provide performance independent of a sparsity pattern, transpose or non-transpose, across data types.

handle	handle to the cuSPARSE library context.
trans	the operation op(A).
m	number of rows of matrix A.
n	number of columns of matrix A.
nnz	number of nonzero elements of matrix A.
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix a. The supported matrix type is cusparse_matrix_type_general, cusparse_matrix_type_symmetric, and cusparse_matrix_type_hermitian. Also, the supported index bases are cusparse_index_base_zero and cusparse_index_base_one.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
x	<type> vector of n elements if op(A) = A, and m elements if op(A) = <math>A^T</math> or op(A) = <math>A^H</math></type>
beta	<pre><type> scalar used for multiplication. If beta is zero, y does not have to be a valid input.</type></pre>
У	<type> vector of m elements if op(A) = A, and n elements if op(A) = <math>A^T</math> or op(A) = <math>A^H</math></type>

### Output

У	<type> updated vector.</type>
---	-------------------------------

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,nnz<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision (compute capability (c.c.) >= 1.3 required), symmetric/Hermitian matrix (c.c. >= 1.2 required), or transpose operation (c.c. >= 1.1 required).
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

### 7.7. cusparse<t>gemvi()

```
cusparseStatus t
cusparseSgemvi(cusparseHandle t handle, cusparseOperation t transA,
            const float *A,
            int lda, int nnz,
            const float
                             *x,
*xInd,
            const int
const float
float
                              *beta,
            cusparseIndexBase_t idxBase, void *pBuffer)
cusparseStatus t
cusparseDgemvi(cusparseHandle t handle, cusparseOperation t transA,
            int m, int n, const double *alpha,
            const double *A,
            int lda, int nnz,
            const double *x,
const int *xIn
const double *be
double *y,
                             *x,
*xInd,
*beta,
            cusparseStatus t
cusparseCgemvi(cusparseHandle t handle, cusparseOperation t transA,
            int m, int n, const cuComplex *alpha,
            const cuComplex *A,
            int lda, int nnz,
            cusparseStatus t
cusparseZgemvi(cusparseHandle t handle, cusparseOperation t transA,
            int m, int n, const cuDoubleComplex
            const cuDoubleComplex *A,
            int lda, int nnz,
            const cuDoubleComplex
            const int *xInd,
```

This function performs the matrix-vector operation

```
y = a * op(A) * x + B * y
```

**A** is an  $m \times n$  dense matrix and a sparse vector x that is defined in a sparse storage format by the two arrays **xVal**, **xInd** of length **nnz**, and **y** is a dense vector;  $\alpha$  and  $\beta$  are scalars; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

To simplify the implementation, we have not (yet) optimized the transpose multiple case. We recommend the following for users interested in this case.

- 1. Convert the matrix from CSR to CSC format using one of the csr2csc() functions. Notice that by interchanging the rows and columns of the result you are implicitly transposing the matrix.
- 2. Call the **gemvi()** function with the **cusparseOperation\_t** parameter set to **CUSPARSE\_OPERATION\_NON\_TRANSPOSE** and with the interchanged rows and columns of the matrix stored in CSC format. This (implicitly) multiplies the vector by the transpose of the matrix in the original CSR format.

This function requires no extra storage for the general matrices when operation <code>CUSPARSE\_OPERATION\_NON\_TRANSPOSE</code> is selected. It requires some extra storage for Hermitian/symmetric matrices and for the general matrices when an operation different than <code>CUSPARSE\_OPERATION\_NON\_TRANSPOSE</code> is selected. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

#### Input

	<del>-</del>
handle	handle to the cuSPARSE library context.
trans	the operation op(A).
m	number of rows of matrix A.
n	number of columns of matrix A.
alpha	<type> scalar used for multiplication.</type>
A	the pointer to dense matrix A.
lda	size of the leading dimension of A.
nnz	number of nonzero elements of vector x.
x	<pre><type> sparse vector of nnz elements of size n if op(A) = A, and size m if op(A) = <math>A^T</math> or op(A) = <math>A^H</math></type></pre>
xInd	Indices of non-zero values in x
beta	<pre><type> scalar used for multiplication. If beta is zero, y does not have to be a valid input.</type></pre>
У	<type> dense vector of m elements if op(A) = A, and n elements if op(A) = <math>A^T</math> or op(A) = <math>A^H</math></type>
idxBase	0 or 1, for 0 based or 1 based indexing, respectively
pBuffer	working space buffer, of size given by Xgemvi_getBufferSize()

#### Output

у	<type> updated dense vector.</type>
---	-------------------------------------

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
-------------------------	---------------------------------------

CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,nnz<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision (compute capability (c.c.) >= 1.3 required), symmetric/Hermitian matrix (c.c. >= 1.2 required), or transpose operation (c.c. >= 1.1 required).
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

### 7.8. cusparse<t>gemvi\_bufferSize()

```
cusparseStatus t
cusparseSgemvi bufferSize(cusparseHandle t handle,
                           cusparseOperation t transA,
                           int m,
                           int n,
                           int nnz,
                           int *pBufferSize)
cusparseStatus t
cusparseDgemvi bufferSize(cusparseHandle t handle,
                           cusparseOperation t transA,
                           int m,
                           int n,
                           int nnz,
                           int *pBufferSize)
cusparseStatus t
cusparseCgemvi bufferSize(cusparseHandle t handle,
                           cusparseOperation t transA,
                           int m,
                           int n,
                           int nnz,
                           int *pBufferSize)
cusparseStatus t
cusparseZgemvi bufferSize(cusparseHandle t handle,
                           cusparseOperation t transA,
                           int m,
                           int n,
                           int nnz,
                           int *pBufferSize)
```

This function returns size of buffer used in gemvi ()

A is an  $(m) \times (n)$  dense matrix.

```
op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \\ \textbf{Input} \end{cases}
```

handle	handle to the cuSPARSE library context.
transA	the operation op (A) .
m	number of rows of matrix A.
n	number of columns of matrix ¥.
nnz	number of nonzero entries of vector $\mathbf{x}$ multiplying $\mathbf{A}$ .

### Output

pBufferSize	number of elements needed the buffer used in
	gemvi().

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n, nnz<=0)
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

### 7.9. cusparse<t>bsrsv2\_bufferSize()

```
cusparseStatus t
cusparseSbsrsv2 bufferSize(cusparseHandle t handle,
                            cusparseDirection t dirA,
                            cusparseOperation_t transA,
                            int mb,
                            int nnzb,
                            const cusparseMatDescr t descrA,
                            float *bsrValA,
                            const int *bsrRowPtrA,
                            const int *bsrColIndA,
                            int blockDim,
                            bsrsv2Info t info,
                            int *pBufferSizeInBytes);
cusparseStatus_t
cusparseDbsrsv2 bufferSize(cusparseHandle_t handle,
                            cusparseDirection_t dirA,
                            cusparseOperation t transA,
                            int mb,
                            int nnzb,
                            const cusparseMatDescr t descrA,
                            double *bsrValA,
                            const int *bsrRowPtrA,
                            const int *bsrColIndA,
                            int blockDim,
                            bsrsv2Info t info,
                            int *pBufferSizeInBytes);
cusparseStatus t
cusparseCbsrsv2 bufferSize(cusparseHandle t handle,
                            cusparseDirection t dirA,
                            cusparseOperation t transA,
                            int mb,
                            int nnzb,
                            const cusparseMatDescr t descrA,
                            cuComplex *bsrValA,
const int *bsrRowPtrA,
                            const int *bsrColIndA,
                            int blockDim,
                            bsrsv2Info t info,
                            int *pBufferSizeInBytes);
cusparseStatus t
cusparseZbsrsv2 bufferSize(cusparseHandle_t handle,
                            cusparseDirection t dirA,
                            cusparseOperation t transA,
                            int mb,
                            int nnzb,
                            const cusparseMatDescr t descrA,
                            cuDoubleComplex *bsrValA,
                            const int *bsrRowPtrA,
                            const int *bsrColIndA,
                            int blockDim,
                            bsrsv2Info t info,
                            int *pBufferSizeInBytes);
```

This function returns size of the buffer used in **bsrsv2**, a new sparse triangular linear system **op** (A) \*y =  $\alpha$  x.

**A** is an (mb\*blockDim) **x** (mb\*blockDim) sparse matrix that is defined in BSR storage format by the three arrays **bsrValA**, **bsrRowPtrA**, and **bsrColIndA**); **x** and **y** are the right-hand-side and the solution vectors; *α* is a scalar; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

Although there are six combinations in terms of parameter trans and the upper (lower) triangular part of A, bsrsv2\_bufferSize() returns the maximum size buffer among these combinations. The buffer size depends on the dimensions mb, blockDim, and the number of nonzero blocks of the matrix nnzb. If the user changes the matrix, it is necessary to call bsrsv2\_bufferSize() again to have the correct buffer size; otherwise a segmentation fault may occur.

#### Input

handle to the cuSPARSE library context.
storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
the operation op(A).
number of block rows of matrix A.
number of nonzero blocks of matrix A.
the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.
<pre><type> array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) nonzero blocks of matrix A.</type></pre>
integer array of mb +1 elements that contains the start of every block row and the end of the last block row plus one.
integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.
block dimension of sparse matrix A; must be larger than zero.

#### Output

info	record of internal states based on different algorithms.
pBufferSizeInBytes	number of bytes of the buffer used in the bsrsv2_analysis() and bsrsv2_solve().

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb,nnzb<=0), base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

### 7.10. cusparse<t>bsrsv2\_analysis()

```
cusparseStatus t
cusparseSbsrsv2 analysis(cusparseHandle t handle,
                          cusparseDirection t dirA,
                          cusparseOperation t transA,
                          int mb,
                          int nnzb,
                          const cusparseMatDescr t descrA,
                          const float *bsrValA,
                          const int *bsrRowPtrA,
                          const int *bsrColIndA,
                          int blockDim,
                          bsrsv2Info t info,
                          cusparseSolvePolicy t policy,
                          void *pBuffer);
cusparseStatus_t
cusparseDbsrsv\overline{2} analysis(cusparseHandle_t handle,
                          cusparseDirection_t dirA,
                          cusparseOperation t transA,
                          int mb,
                          int nnzb,
                          const cusparseMatDescr_t descrA,
                          const double *bsrValA,
                          const int *bsrRowPtrA,
                          const int *bsrColIndA,
                          int blockDim,
                          bsrsv2Info t info,
                          cusparseSolvePolicy t policy,
                          void *pBuffer);
cusparseStatus t
cusparseCbsrsv2 analysis(cusparseHandle t handle,
                          cusparseDirection t dirA,
                          cusparseOperation t transA,
                          int mb,
                          int nnzb,
                          const cusparseMatDescr t descrA,
                          const cuComplex *bsrValA,
                          const int *bsrRowPtrA,
                          const int *bsrColIndA,
                          int blockDim,
                          bsrsv2Info t info,
                          cusparseSolvePolicy t policy,
                          void *pBuffer);
cusparseStatus t
cusparseZbsrsv\overline{2}_analysis(cusparseHandle_t handle,
                          cusparseDirection t dirA,
                          cusparseOperation_t transA,
                          int mb,
                          int nnzb,
                          const cusparseMatDescr t descrA,
                          const cuDoubleComplex \overline{*}bsrValA,
                          const int *bsrRowPtrA,
                          const int *bsrColIndA,
                          int blockDim,
                          bsrsv2Info t info,
                          cusparseSolvePolicy_t policy,
                          void *pBuffer);
```

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This function performs the analysis phase of **bsrsv2**, a new sparse triangular linear system **op** (A) \*y =  $\alpha$  x.

**A** is an (mb\*blockDim) **x** (mb\*blockDim) sparse matrix that is defined in BSR storage format by the three arrays **bsrValA**, **bsrRowPtrA**, and **bsrColIndA**); **x** and **y** are the right-hand side and the solution vectors; *α* is a scalar; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

The block of BSR format is of size blockDim\*blockDim, stored as column-major or row-major as determined by parameter dirA, which is either CUSPARSE\_DIRECTION\_COLUMN or CUSPARSE\_DIRECTION\_ROW. The matrix type must be CUSPARSE MATRIX\_TYPE\_GENERAL, and the fill mode and diagonal type are ignored.

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires a buffer size returned by **bsrsv2\_bufferSize()**. The address of **pBuffer** must be multiple of 128 bytes. If it is not, **CUSPARSE\_STATUS\_INVALID\_VALUE** is returned.

Function bsrsv2\_analysis() reports a structural zero and computes level information, which stored in the opaque structure info. The level information can extract more parallelism for a triangular solver. However bsrsv2\_solve() can be done without level information. To disable level information, the user needs to specify the policy of the triangular solver as CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL.

Function bsrsv2\_analysis() always reports the first structural zero, even when parameter policy is CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL. No structural zero is reported if CUSPARSE\_DIAG\_TYPE\_UNIT is specified, even if block A(j,j) is missing for some j. The user needs to call cusparseXbsrsv2\_zeroPivot() to know where the structural zero is.

It is the user's choice whether to call <code>bsrsv2\_solve()</code> if <code>bsrsv2\_analysis()</code> reports a structural zero. In this case, the user can still call <code>bsrsv2\_solve()</code>, which will return a numerical zero at the same position as a structural zero. However the result <code>x</code> is meaningless.

#### Input

handle	handle to the cuSPARSE library context.
dirA	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
transA	the operation op(A).
mb	number of block rows of matrix A.
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE MATRIX TYPE GENERAL, while the supported diagonal types

	are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.
bsrValA	<pre><type> array of nnzb(= bsrRowPtrA(mb) - bsrRowPtrA(0)) nonzero blocks of matrix A.</type></pre>
bsrRowPtrA	integer array of mb +1 elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A, larger than zero.
info	structure initialized using cusparseCreateBsrsv2Info().
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user, the size is return by bsrsv2_bufferSize().

### Output

structure filled with information collected during the analysis phase (that should be passed to the
solve phase unchanged).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb,nnzb<=0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 7.11. cusparse<t>bsrsv2\_solve()

```
cusparseStatus t
cusparseSbsrsv2 solve(cusparseHandle t handle,
                       cusparseDirection t dirA,
                       cusparseOperation t transA,
                       int mb,
                       int nnzb,
                       const float *alpha,
                       const cusparseMatDescr t descrA,
                       const float *bsrValA,
                       const int *bsrRowPtrA,
                       const int *bsrColIndA,
                       int blockDim,
                      bsrsv2Info t info,
                       const float *x,
                       float *y,
                       cusparseSolvePolicy_t policy,
                       void *pBuffer);
cusparseStatus_t
cusparseDbsrsv\overline{2}_solve(cusparseHandle_t handle,
                       cusparseDirection t dirA,
                       cusparseOperation t transA,
                       int mb,
                       int nnzb,
                       const double *alpha,
                       const cusparseMatDescr t descrA,
                       const double *bsrValA,
                       const int *bsrRowPtrA,
                       const int *bsrColIndA,
                       int blockDim,
                       bsrsv2Info t info,
                       const double *x,
                       double *y,
                       cusparseSolvePolicy t policy,
                       void *pBuffer);
cusparseStatus t
cusparseCbsrsv\overline{2} solve(cusparseHandle t handle,
                       cusparseDirection t dirA,
                       cusparseOperation t transA,
                       int mb,
                       int nnzb,
                       const cuComplex *alpha,
                       const cusparseMatDescr t descrA,
                       const cuComplex *bsrValA,
                       const int *bsrRowPtrA,
                       const int *bsrColIndA,
                       int blockDim,
                       bsrsv2Info_t info,
                       const cuComplex *x,
                       cuComplex *y,
                       cusparseSolvePolicy_t policy,
                       void *pBuffer);
cusparseStatus t
cusparseZbsrsv\overline{2} solve(cusparseHandle t handle,
                       cusparseDirection t dirA,
                       cusparseOperation_t transA,
                       int mb,
                       int nnzb,
                       const cuDoubleComplex *alpha,
                       const cusparseMatDescr t descrA,
                       const cuDoubleComplex *bsrValA.
```

This function performs the solve phase of **bsrsv2**, a new sparse triangular linear system op (A) \*y =  $\alpha$  x.

**A** is an (mb\*blockDim) **x** (mb\*blockDim) sparse matrix that is defined in BSR storage format by the three arrays **bsrValA**, **bsrRowPtrA**, and **bsrColIndA**); **x** and **y** are the right-hand-side and the solution vectors; *α* is a scalar; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

The block in BSR format is of size blockDim\*blockDim, stored as column-major or row-major as determined by parameter dirA, which is either CUSPARSE\_DIRECTION\_COLUMN or CUSPARSE\_DIRECTION\_ROW. The matrix type must be CUSPARSE\_MATRIX\_TYPE\_GENERAL, and the fill mode and diagonal type are ignored. Function bsrsv02 solve() can support an arbitrary blockDim.

This function may be executed multiple times for a given matrix and a particular operation type.

This function requires a buffer size returned by **bsrsv2\_bufferSize()**. The address of **pBuffer** must be multiple of 128 bytes. If it is not, **CUSPARSE\_STATUS\_INVALID\_VALUE** is returned.

Although bsrsv2\_solve() can be done without level information, the user still needs to be aware of consistency. If bsrsv2\_analysis() is called with policy CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL, bsrsv2\_solve() can be run with or without levels. On the other hand, if bsrsv2\_analysis() is called with CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL, bsrsv2\_solve() can only accept CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL; otherwise, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

The level information may not improve the performance, but may spend extra time doing analysis. For example, a tridiagonal matrix has no parallelism. In this case, <code>CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL</code> performs better than <code>CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL</code>. If the user has an iterative solver, the best approach is to do <code>bsrsv2\_analysis()</code> with <code>CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL</code> once. Then do <code>bsrsv2\_solve()</code> with <code>CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL</code> in the first run, and with <code>CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL</code> in the second run, and pick the fastest one to perform the remaining iterations.

Function bsrsv02\_solve() has the same behavior as csrsv02\_solve(). That is, bsr2csr(bsrsv02(A)) = csrsv02(bsr2csr(A)). The numerical zero of csrsv02\_solve() means there exists some zero A(j,j). The numerical zero of bsrsv02\_solve() means there exists some block A(j,j) that is not invertible.

Function bsrsv2\_solve() reports the first numerical zero, including a structural zero. No numerical zero is reported if CUSPARSE\_DIAG\_TYPE\_UNIT is specified, even if A(j,j) is not invertible for some j. The user needs to call cusparseXbsrsv2\_zeroPivot() to know where the numerical zero is.

For example, suppose L is a lower triangular matrix with unit diagonal, then the following code solves **L\*y=x** by level information.

```
// Suppose that L is m x m sparse matrix represented by BSR format,
// The number of block rows/columns is mb, and
// the number of nonzero blocks is nnzb.
// L is lower triangular with unit diagonal.
// Assumption:
// - dimension of matrix L is m(=mb*blockDim),
// - matrix L has nnz(=nnzb*blockDim*blockDim) nonzero elements,
// - handle is already created by cusparseCreate(),
// - (d bsrRowPtr, d bsrColInd, d bsrVal) is BSR of L on device memory,
// - d\bar{x} is right hand side vector on device memory.
// - d_y is solution vector on device memory.
// - d_x and d_y are of size m.
cusparseMatDescr t descr = 0;
bsrsv2Info t info = 0;
int pBufferSize;
void *pBuffer = 0;
int structural zero;
int numerical zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy = CUSPARSE_SOLVE_POLICY USE LEVEL;
const cusparseOperation_t trans = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseDirection t dir = CUSPARSE DIRECTION COLUMN;
// step 1: create a descriptor which contains
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has unit diagonal, specified by parameter CUSPARSE DIAG TYPE UNIT
// (L may not have all diagonal elements.)
cusparseCreateMatDescr(&descr);
cusparseSetMatIndexBase(descr, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatFillMode(descr, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr, CUSPARSE DIAG TYPE UNIT);
// step 2: create a empty info structure
cusparseCreateBsrsv2Info(&info);
// step 3: query how much memory used in bsrsv2, and allocate the buffer
cusparseDbsrsv2 bufferSize(handle, dir, trans, mb, nnzb, descr,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, &pBufferSize);
// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);
// step 4: perform analysis
cusparseDbsrsv2 analysis(handle, dir, trans, mb, nnzb, descr,
    d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim,
    info, policy, pBuffer);
// L has unit diagonal, so no structural zero is reported.
status = cusparseXbsrsv2 zeroPivot(handle, info, &structural zero);
if (CUSPARSE STATUS ZERO PIVOT == status) {
  printf("L(%d,%d) is missing\n", structural zero, structural zero);
// step 5: solve L*y = x
cusparseDbsrsv2 solve(handle, dir, trans, mb, nnzb, &alpha, descr,
   d bsrVal, d bsrRowPtr, d bsrColInd, blockDim, info,
   d_x, d_y, policy, pBuffer);
// L has unit diagonal, so no numerical zero is reported.
status = cusparseXbsrsv2 zeroPivot(handle, info, &numerical zero);
if (CUSPARSE STATUS ZERO PIVOT == status) {
  printf("L(%d,%d) is zero\n", numerical_zero, numerical_zero);
// step 6: free resources
cudaFree(pBuffer);
cusparseDestroyBsrsv2Info(info);
cusparseDestroyMatDescr(descr);
cusparseDestroy(handle);
```

### Input

handle	handle to the cuSPARSE library context.
dirA	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
transA	the operation op(A).
mb	number of block rows and block columns of matrix A.
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.
bsrValA	<pre><type> array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) nonzero blocks of matrix A.</type></pre>
bsrRowPtrA	integer array of mb +1 elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A, larger than zero.
info	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).
x	<type> right-hand-side vector of size m.</type>
policy	the supported policies are  CUSPARSE_SOLVE_POLICY_NO_LEVEL and  CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user, the size is returned by bsrsv2_bufferSize().

### Output

У	<type> solution vector of size m.</type>
---	--

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb,nnzb<=0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_MAPPING_ERROR	the texture binding failed.

CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 7.12. cusparseXbsrsv2\_zeroPivot()

If the returned error code is **CUSPARSE\_STATUS\_ZERO\_PIVOT**, **position=j** means **A**(j,j) is either structural zero or numerical zero (singular block). Otherwise **position=-1**.

The **position** can be 0-based or 1-based, the same as the matrix.

Function cusparseXbsrsv2\_zeroPivot() is a blocking call. It calls cudaDeviceSynchronize() to make sure all previous kernels are done.

The **position** can be in the host memory or device memory. The user can set the proper mode with **cusparseSetPointerMode()**.

#### Input

handle	handle to the cuSPARSE library context.
info	<pre>info contains a structural zero or numerical zero if the user already called bsrsv2_analysis() or bsrsv2_solve().</pre>

#### Output

if no structural or numerical zero, position is -1; otherwise if A(j,j) is missing or U(j,j) is zero,
position=j.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	info is not valid.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

### 7.13. cusparse<t>csrsv\_analysis()

```
cusparseStatus t
cusparseScsrsv analysis(cusparseHandle t handle,
                        cusparseOperation t transA,
                        int m,
                        int nnz,
                        const cusparseMatDescr t descrA,
                        const float *csrValA,
                        const int *csrRowPtrA,
                        const int *csrColIndA,
                        cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseDcsrsv analysis(cusparseHandle t handle,
                        cusparseOperation t transA,
                        int m,
                        int nnz,
                        const cusparseMatDescr_t descrA,
                        const double *csrValA,
                        const int *csrRowPtrA,
                        const int *csrColIndA,
                        cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseCcsrsv analysis(cusparseHandle t handle,
                        cusparseOperation t transA,
                        int m,
                        int nnz,
                        const cusparseMatDescr t descrA,
                        const cuComplex *csrValA,
                        const int *csrRowPtrA,
                        const int *csrColIndA,
                        cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseZcsrsv analysis(cusparseHandle t handle,
                        cusparseOperation t transA,
                        int m,
                        int nnz,
                        const cusparseMatDescr t descrA,
                        const cuDoubleComplex *csrValA,
                        const int *csrRowPtrA,
                        const int *csrColIndA,
                        cusparseSolveAnalysisInfo t info)
```

This function performs the analysis phase of the solution of a sparse triangular linear system

$$op(A) * y = a * x$$

where **A** is an **m**×**m** sparse matrix that is defined in CSR storage format by the three arrays **csrValA**, **csrRowPtrA**, and **csrColIndA**); **x** and **y** are the right-hand-side and the solution vectors; *α* is a scalar; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

The routine csrsv\_analysis supports analysis phase of csrsv\_solve, csric0 and csrilu0. The user has to be careful of which routine is called after csrsv\_analysis. The matrix descriptor must be the same for csrsv\_analysis and its subsequent call to csrsv\_solve, csric0 and csrilu0.

For csrsv\_solve, the matrix type must be CUSPARSE\_MATRIX\_TYPE\_TRIANGULAR or CUSPARSE MATRIX TYPE GENERAL.

For csrilu0, the matrix type must be CUSPARSE\_MATRIX\_TYPE\_GENERAL.

For csric0, the matrix type must be CUSPARSE\_MATRIX\_TYPE\_SYMMETRIC or CUSPARSE\_MATRIX\_TYPE\_HERMITIAN.

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires a significant amount of extra storage that is proportional to the matrix size. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

### Input

handle	handle to the cuSPARSE library context.
trans	the operation op(A)
m	number of rows of matrix A.
nnz	number of nonzero elements of matrix A.
descrA	the descriptor of matrix A. The supported matrix types are cusparse_matrix_type_triangular and cusparse_matrix_type_general for csrsv_solve,  cusparse_matrix_type_symmetric and cusparse_matrix_type_hermitian for csric0, cusparse_matrix_type_deneral for csrilu0, while the supported diagonal types are cusparse_diag_type_unit and cusparse_diag_type_non_unit.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $\tt m$ +1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
info	structure initialized using cusparseCreateSolveAnalysisInfo.

#### Output

info	structure filled with information collected during
	the analysis phase (that should be passed to the
	solve phase unchanged).

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,nnz<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 7.14. cusparseCsrsv\_analysisEx()

This function is an extended version of **cusparse<t>csrsv\_analysis()**. For detailed description of the functionality, see **cusparse<t>csrsv\_analysis()**.

This function does not support half-precision execution type, but it supports half-precision IO with single precision execution.

### Input specifically required by cusparseCsrsv\_analysisEx

csrSortedValAtype	Data type of csrSortedValA.
executiontype	Data type used for computation.

### 7.15. cusparse<t>csrsv\_solve()

```
cusparseStatus t cusparseScsrsv solve(cusparseHandle t handle,
                                       cusparseOperation t transA,
                                       int m, const float *alpha,
                                      const cusparseMatDescr t descrA,
                                      const float *csrSortedValA,
                                      const int *csrSortedRowPtrA, const
int *csrSortedColIndA,
                                      cusparseSolveAnalysisInfo t info,
                                      const float *f, float *x);
cusparseStatus t cusparseDcsrsv solve(cusparseHandle t handle,
                                      cusparseOperation t transA,
                                      int m, const double *alpha,
                                      const cusparseMatDescr_t descrA,
                                      const double *csrSortedValA,
                                      const int *csrSortedRowPtrA, const
int *csrSortedColIndA,
                                      cusparseSolveAnalysisInfo t info,
                                      const double *f, double (x, x);
cusparseStatus t cusparseCcsrsv solve(cusparseHandle t handle,
                                      cusparseOperation t transA,
                                      int m, const cuComplex *alpha,
                                      const cusparseMatDescr t descrA,
                                      const cuComplex *csrSortedValA,
                                      const int *csrSortedRowPtrA, const
int *csrSortedColIndA,
                                      cusparseSolveAnalysisInfo t info,
                                      const cuComplex *f, cuComplex *x);
cusparseStatus t cusparseZcsrsv solve(cusparseHandle t handle,
                                       cusparseOperation t transA,
                                       int m, const cuDoubleComplex
*alpha,
                                      const cusparseMatDescr t descrA,
                                      const cuDoubleComplex
 *csrSortedValA,
                                      const int *csrSortedRowPtrA, const
int *csrSortedColIndA,
                                      cusparseSolveAnalysisInfo t info,
                                      const cuDoubleComplex *f,
cuDoubleComplex *x);
```

This function performs the solve phase of the solution of a sparse triangular linear system

$$op(A) * x = a * f$$

where **A** is an  $m \times m$  sparse matrix that is defined in CSR storage format by the three arrays **csrSortedValA**, **csrSortedRowPtrA**, and **csrSortedColIndA**); **f** and **x** are the right-hand-side and the solution vectors;  $\alpha$  is a scalar; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

This function may be executed multiple times for a given matrix and a particular operation type.

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

### Input

handle	handle to the cuSPARSE library context.
	ŕ
trans	the operation op(A)
m	number of rows and columns of matrix A.
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix a. The supported matrix types are cusparse_matrix_type_triangular and cusparse_matrix_type_general, while the supported diagonal types are cusparse_diag_type_unit and cusparse_diag_type_non_unit.
csrSortedValA	<pre><type> array of nnz (= csrSortedRowPtrA(m)     csrSortedRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrSortedRowPtrA	integer array of $m + 1$ elements that contains the start of every row and the end of the last row plus one.
csrSortedColIndA	integer array of nnz (= csrSortedRowPtrA(m) - csrSortedRowPtrA(0) ) column indices of the nonzero elements of matrix A.
info	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).
f	<type> right-hand-side vector of size m.</type>

### Output

	<type> solution vector of size m.</type>
--	--

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_MAPPING_ERROR	the texture binding failed.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

CUSPARSE STATUS MATRIX TYPE NOT SUPPORTED the matrix type is not supported.

# 7.16. cusparseCsrsv\_solveEx()

```
cusparseStatus t cusparseCsrsv solveEx(cusparseHandle t handle,
                                       cusparseOperation t transA,
                                       int m,
                                       const void *alpha, cudaDataType
alphatype,
                                       const cusparseMatDescr t descrA,
                                       const void *csrSortedValA,
cudaDataType csrSortedValAtype,
                                       const int *csrSortedRowPtrA,
                                       const int *csrSortedColIndA,
                                       cusparseSolveAnalysisInfo t
info,
                                       const void *f, cudaDataType
ftype,
                                       void *x, cudaDataType xtype,
                                       cudaDataType executiontype);
```

This function is an extended version of **cusparse<t>csrsv\_solve()**. For detailed description of the functionality, see **cusparse<t>csrsv\_solve()**.

This function does not support half-precision execution type, but it supports half-precision IO with single precision execution.

#### Input specifically required by cusparseCsrsv solveEx

alphatype	Data type of alpha.
csrSortedValAtype	Data type of csrSortedValA.
ftype	Data type of £.
xtype	Data type of x.
executiontype	Data type used for computation.

### 7.17. cusparse<t>csrsv2\_bufferSize()

```
cusparseStatus t
cusparseScsrsv2 bufferSize(cusparseHandle t handle,
                            cusparseOperation t transA,
                            int m,
                            int nnz,
                            const cusparseMatDescr t descrA,
                            float *csrValA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            csrsv2Info t info,
                            int *pBufferSizeInBytes);
cusparseStatus t
cusparseDcsrsv\overline{2}_bufferSize(cusparseHandle_t handle,
                            cusparseOperation t transA,
                            int m,
                            int nnz,
                            const cusparseMatDescr t descrA,
                            double *csrValA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            csrsv2Info t info,
                            int *pBufferSizeInBytes);
cusparseStatus t
cusparseCcsrsv2 bufferSize(cusparseHandle t handle,
                            cusparseOperation t transA,
                            int m,
                            int nnz,
                            const cusparseMatDescr t descrA,
                            cuComplex *csrValA,
const int *csrRowPtrA,
                            const int *csrColIndA,
                            csrsv2Info t info,
                            int *pBufferSizeInBytes);
cusparseStatus t
cusparseZcsrsv2 bufferSize(cusparseHandle t handle,
                            cusparseOperation t transA,
                            int m,
                            int nnz,
                            const cusparseMatDescr t descrA,
                            cuDoubleComplex *csrValA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            csrsv2Info t info,
                            int *pBufferSizeInBytes);
```

This function returns the size of the buffer used in **csrsv2**, a new sparse triangular linear system **op** (A) \*y =  $\alpha$  x.

**A** is an  $m \times m$  sparse matrix that is defined in CSR storage format by the three arrays **csrValA**, **csrRowPtrA**, and **csrColIndA**); **x** and **y** are the right-hand-side and the solution vectors;  $\alpha$  is a scalar; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

Although there are six combinations in terms of the parameter **trans** and the upper (lower) triangular part of **A**, **csrsv2\_bufferSize()** returns the maximum size buffer of these combinations. The buffer size depends on the dimension and the number of nonzero elements of the matrix. If the user changes the matrix, it is necessary to call **csrsv2\_bufferSize()** again to have the correct buffer size; otherwise, a segmentation fault may occur.

### Input

handle	handle to the cuSPARSE library context.
transA	the operation op(A).
m	number of rows of matrix A.
nnz	number of nonzero elements of matrix A.
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m + 1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.

### Output

info	record of internal states based on different algorithms.
pBufferSizeInBytes	number of bytes of the buffer used in the csrsv2_analysis and csrsv2_solve.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,nnz<=0), base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED the matrix type is not supported.	

# 7.18. cusparse<t>csrsv2\_analysis()

```
cusparseStatus t
cusparseScsrsv2 analysis(cusparseHandle t handle,
                         cusparseOperation t transA,
                         int m,
                         int nnz,
                         const cusparseMatDescr t descrA,
                         const float *csrValA,
                         const int *csrRowPtrA,
                         const int *csrColIndA,
                         csrsv2Info t info,
                         cusparseSolvePolicy t policy,
                         void *pBuffer);
cusparseStatus_t
cusparseDcsrsv\overline{2}_analysis(cusparseHandle_t handle,
                         cusparseOperation t transA,
                          int m,
                         int nnz,
                         const cusparseMatDescr t descrA,
                         const double *csrValA,
                         const int *csrRowPtrA,
                         const int *csrColIndA,
                         csrsv2Info t info,
                         cusparseSolvePolicy_t policy,
                         void *pBuffer);
cusparseStatus t
cusparseCcsrsv2_analysis(cusparseHandle_t handle,
                         cusparseOperation t transA,
                          int m,
                          int nnz,
                         const cusparseMatDescr t descrA,
                          const cuComplex *csrValA,
                          const int *csrRowPtrA,
                          const int *csrColIndA,
                          csrsv2Info t info,
                          cusparseSolvePolicy t policy,
                         void *pBuffer);
cusparseStatus t
cusparseZcsrsv2 analysis(cusparseHandle t handle,
                          cusparseOperation t transA,
                          int m,
                          int nnz,
                          const cusparseMatDescr t descrA,
                         const cuDoubleComplex *csrValA,
                         const int *csrRowPtrA,
                         const int *csrColIndA,
                         csrsv2Info t info,
                         cusparseSolvePolicy_t policy,
                         void *pBuffer);
```

This function performs the analysis phase of **csrsv2**, a new sparse triangular linear system **op** (A) \* $\mathbf{y} = \alpha \mathbf{x}$ .

**A** is an m×m sparse matrix that is defined in CSR storage format by the three arrays **csrValA**, **csrRowPtrA**, and **csrColIndA**); **x** and **y** are the right-hand-side and the solution vectors; α is a scalar; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires a buffer size returned by csrsv2\_bufferSize(). The address of pBuffer must be multiple of 128 bytes. If it is not, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

Function csrsv2\_analysis() reports a structural zero and computes level information that is stored in opaque structure info. The level information can extract more parallelism for a triangular solver. However csrsv2\_solve() can be done without level information. To disable level information, the user needs to specify the policy of the triangular solver as CUSPARSE SOLVE POLICY NO LEVEL.

Function csrsv2\_analysis() always reports the first structural zero, even if the policy is CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL. No structural zero is reported if CUSPARSE\_DIAG\_TYPE\_UNIT is specified, even if A(j,j) is missing for some j. The user needs to call cusparseXcsrsv2\_zeroPivot() to know where the structural zero is.

It is the user's choice whether to call <code>csrsv2\_solve()</code> if <code>csrsv2\_analysis()</code> reports a structural zero. In this case, the user can still call <code>csrsv2\_solve()</code> which will return a numerical zero in the same position as the structural zero. However the result <code>x</code> is meaningless.

#### Input

handle	handle to the cuSPARSE library context.
transA	the operation op(A).
m	number of rows of matrix A.
nnz	number of nonzero elements of matrix A.
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.

info	structure initialized using cusparseCreateCsrsv2Info().
policy	The supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user, the size is returned by csrsv2_bufferSize().

### Output

info	structure filled with information collected during the analysis phase (that should be passed to the
	solve phase unchanged).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,nnz<=0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

### 7.19. cusparse<t>csrsv2\_solve()

```
cusparseStatus t
cusparseScsrsv2 solve(cusparseHandle t handle,
                      cusparseOperation t transA,
                      int m,
                      int nnz,
                      const float *alpha,
                      const cusparseMatDescr t descra,
                      const float *csrValA,
                      const int *csrRowPtrA,
                      const int *csrColIndA,
                      csrsv2Info t info,
                      const float *x,
                      float *y,
                      cusparseSolvePolicy_t policy,
                      void *pBuffer);
cusparseStatus_t
cusparseDcsrsv\overline{2}_solve(cusparseHandle_t handle,
                      cusparseOperation t transA,
                      int m,
                      int nnz,
                      const double *alpha,
                      const cusparseMatDescr t descra,
                      const double *csrValA,
                      const int *csrRowPtrA,
                      const int *csrColIndA,
                      csrsv2Info t info,
                      const double *x,
                      double *y,
                      cusparseSolvePolicy t policy,
                      void *pBuffer);
cusparseStatus t
cusparseCcsrsv2 solve(cusparseHandle t handle,
                      cusparseOperation t transA,
                      int m,
                      int nnz,
                      const cuComplex *alpha,
                      const cusparseMatDescr t descra,
                      const cuComplex *csrValA,
                      const int *csrRowPtrA,
                      const int *csrColIndA,
                      csrsv2Info t info,
                      const cuComplex *x,
                      cuComplex *y,
                      cusparseSolvePolicy t policy,
                      void *pBuffer);
cusparseStatus t
cusparseZcsrsv2 solve(cusparseHandle t handle,
                      cusparseOperation t transA,
                      int m,
                      int nnz,
                      const cuDoubleComplex *alpha,
                      const cusparseMatDescr t descra,
                      const cuDoubleComplex *csrValA,
                      const int *csrRowPtrA,
                      const int *csrColIndA,
                      csrsv2Info_t info,
                      const cuDoubleComplex *x,
                      cuDoubleComplex *y,
                      cusparseSolvePolicy_t policy,
                      void *pBuffer):
```

This function performs the solve phase of **csrsv2**, a new sparse triangular linear system op (A) \*y =  $\alpha$  x.

**A** is an m×m sparse matrix that is defined in CSR storage format by the three arrays **csrValA**, **csrRowPtrA**, and **csrColIndA**); **x** and **y** are the right-hand-side and the solution vectors; α is a scalar; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

This function may be executed multiple times for a given matrix and a particular operation type.

This function requires the buffer size returned by csrsv2\_bufferSize(). The address of pBuffer must be multiple of 128 bytes. If it is not, CUSPARSE STATUS INVALID VALUE is returned.

Although csrsv2\_solve() can be done without level information, the user still needs to be aware of consistency. If csrsv2\_analysis() is called with policy CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL, csrsv2\_solve() can be run with or without levels. On the contrary, if csrsv2\_analysis() is called with CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL, csrsv2\_solve() can only accept CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL; otherwise, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

The level information may not improve the performance but spend extra time doing analysis. For example, a tridiagonal matrix has no parallelism. In this case, <code>CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL</code> performs better than <code>CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL</code>. If the user has an iterative solver, the best approach is to do <code>csrsv2\_analysis()</code> with <code>CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL</code> once. Then do <code>csrsv2\_solve()</code> with <code>CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL</code> in the first run and with <code>CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL</code> in the second run, picking faster one to perform the remaining iterations.

Function <code>csrsv2\_solve()</code> reports the first numerical zero, including a structural zero. If <code>status</code> is 0, no numerical zero was found. Furthermore, no numerical zero is reported if <code>CUSPARSE\_DIAG\_TYPE\_UNIT</code> is specified, even if <code>A(j,j)</code> is zero for some <code>j</code>. The user needs to call <code>cusparseXcsrsv2\_zeroPivot()</code> to know where the numerical zero is.

For example, suppose L is a lower triangular matrix with unit diagonal, the following code solves **L\*y=x** by level information.

```
// Suppose that L is m x m sparse matrix represented by CSR format,
// L is lower triangular with unit diagonal.
// Assumption:
// - dimension of matrix L is m,
// - matrix L has nnz number zero elements,
// - handle is already created by cusparseCreate(),
// - (d csrRowPtr, d csrColInd, d csrVal) is CSR of L on device memory,
// - d \overline{x} is right hand side vector on device memory,
// - d y is solution vector on device memory.
cusparseMatDescr t descr = 0;
csrsv2Info t info = 0;
int pBufferSize;
void *pBuffer = 0;
int structural zero;
int numerical_zero;
const double alpha = 1.;
const cusparseSolvePolicy t policy = CUSPARSE SOLVE POLICY USE LEVEL;
const cusparseOperation t trans = CUSPARSE OPERATION NON TRANSPOSE;
// step 1: create a descriptor which contains
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has unit diagonal, specified by parameter CUSPARSE DIAG TYPE UNIT
// (L may not have all diagonal elements.)
cusparseCreateMatDescr(&descr);
cusparseSetMatIndexBase(descr, CUSPARSE INDEX BASE ONE);
cusparseSetMatFillMode(descr, CUSPARSE FILL MODE LOWER);
cusparseSetMatDiagType(descr, CUSPARSE DIAG TYPE UNIT);
// step 2: create a empty info structure
cusparseCreateCsrsv2Info(&info);
// step 3: query how much memory used in csrsv2, and allocate the buffer
cusparseDcsrsv2_bufferSize(handle, trans, m, nnz, descr,
    d_csrVal, d_csrRowPtr, d_csrColInd, &pBufferSize);
// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);
// step 4: perform analysis
cusparseDcsrsv2_analysis(handle, trans, m, nnz, descr,
    d_csrVal, d_csrRowPtr, d_csrColInd,
    info, policy, pBuffer);
// L has unit diagonal, so no structural zero is reported.
status = cusparseXcsrsv2 zeroPivot(handle, info, &structural zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status) {
  printf("L(%d,%d) is missing\n", structural_zero, structural_zero);
// step 5: solve L*y = x
cusparseDcsrsv2 solve(handle, trans, m, nnz, &alpha, descr,
   d csrVal, d csrRowPtr, d csrColInd, info,
   d x, d y, policy, pBuffer);
// L has unit diagonal, so no numerical zero is reported.
status = cusparseXcsrsv2 zeroPivot(handle, info, &numerical zero);
if (CUSPARSE_STATUS_ZERO_PIVOT == status) {
  printf("L(%d,%d) is zero\n", numerical_zero, numerical_zero);
// step 6: free resources
cudaFree (pBuffer);
cusparseDestroyCsrsv2Info(info);
cusparseDestroyMatDescr(descr);
cusparseDestroy(handle);
```

Remark: csrsv2\_solve() needs more nonzeros per row to achieve good performance. It would perform better if more than 16 nonzeros per row in average.

### Input

handle	handle to the cuSPARSE library context.
transA	the operation op(A).
m	number of rows and columns of matrix A.
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m + 1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
info	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).
x	<type> right-hand-side vector of size m.</type>
policy	The supported policies are  CUSPARSE_SOLVE_POLICY_NO_LEVEL and  CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user, the size is return by csrsv2_bufferSize.

### Output

V	<type> solution vector of size m.</type>
-	3,60 00.000.0000.0000.0000

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,nnz<=0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_MAPPING_ERROR	the texture binding failed.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 7.20. cusparseXcsrsv2\_zeroPivot()

If the returned error code is **CUSPARSE\_STATUS\_ZERO\_PIVOT**, **position=j** means **A**(j, j) has either a structural zero or a numerical zero. Otherwise **position=-1**.

The **position** can be 0-based or 1-based, the same as the matrix.

Function cusparseXcsrsv2\_zeroPivot() is a blocking call. It calls cudaDeviceSynchronize() to make sure all previous kernels are done.

The **position** can be in the host memory or device memory. The user can set the proper mode with **cusparseSetPointerMode()**.

#### Input

handle	handle to the cuSPARSE library context.
info	<pre>info contains structural zero or numerical zero if the user already called csrsv2_analysis() or csrsv2_solve().</pre>

### Output

1 •	if no structural or numerical zero, position is -1; otherwise, if A(j,j) is missing or U(j,j) is zero,
	position=j.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	info is not valid.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 7.21. cusparse<t>hybmv()

```
cusparseStatus t
cusparseShybmv(cusparseHandle t handle, cusparseOperation t transA,
             const float *alpha,
             const cusparseMatDescr t descrA,
             const cusparseHybMat_t hybA, const float
                                                            *x,
             const float
                                  *beta, float
cusparseStatus t
cusparseDhybmv(cusparseHandle t handle, cusparseOperation t transA,
             const double *alpha,
             const cusparseMatDescr t descrA,
             const cusparseHybMat_t hybA, const double
                                                            *x,
             const double
                                  *beta, double
cusparseStatus t
cusparseChybmv(cusparseHandle t handle, cusparseOperation t transA,
             const cuComplex *alpha,
             const cusparseMatDescr_t descrA,
             const cusparseHybMat_t hybA, const cuComplex
                                                            *x,
             const cuComplex
                                  *beta, cuComplex
cusparseStatus t
cusparseZhybmv(cusparseHandle_t handle, cusparseOperation_t transA,
             const cuDoubleComplex *alpha,
             const cusparseMatDescr_t descrA,
             const cusparseHybMat_t hybA, const cuDoubleComplex *x,
             const cuDoubleComplex *beta, cuDoubleComplex *y)
```

This function performs the matrix-vector operation

$$y = a * op(A) * x + B * y$$

**A** is an  $\mathbf{m} \times \mathbf{n}$  sparse matrix that is defined in the HYB storage format by an opaque data structure  $\mathbf{h} \mathbf{y} \mathbf{b} \mathbf{A}$ ,  $\mathbf{x}$  and  $\mathbf{y}$  are vectors,  $\alpha$  and  $\beta$  are scalars, and  $\mathbf{op}(A) = \{A \mid \text{if trans} A == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE}\}$ 

Notice that currently only op(A) = A is supported.

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

### Input

handle	handle to the cuSPARSE library context.
transA	the operation op( $A$ ) (currently only op( $A$ ) = $A$ is supported).
m	number of rows of matrix A.
n	number of columns of matrix A.
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix <b>A</b> . The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.
hybA	the matrix A in HYB storage format.
х	<type> vector of n elements.</type>

	<pre><type> scalar used for multiplication. If beta is zero, <math>\mathbf{y}</math> does not have to be a valid input.</type></pre>
У	<type> vector of m elements.</type>

#### Output

У	<type> updated vector.</type>
---	-------------------------------

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	the internally stored HYB format parameters are invalid.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 7.22. cusparse<t>hybsv\_analysis()

```
cusparseStatus t
cusparseShybsv analysis(cusparseHandle t handle,
                        cusparseOperation t transA,
                        const cusparseMatDescr t descrA,
                        cusparseHybMat t hybA,
                        cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseDhybsv analysis(cusparseHandle t handle,
                        cusparseOperation_t transA,
                        const cusparseMatDescr t descrA,
                        cusparseHybMat t hybA,
                        cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseChybsv analysis(cusparseHandle t handle,
                        cusparseOperation t transA,
                        const cusparseMatDescr t descrA,
                        cusparseHybMat t hybA,
                        cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseZhybsv analysis(cusparseHandle t handle,
                        cusparseOperation t transA,
                        const cusparseMatDescr t descrA,
                        cusparseHybMat t hybA,
                        cusparseSolveAnalysisInfo t info)
```

This function performs the analysis phase of the solution of a sparse triangular linear system

op(A) \* y = a \* x

**A** is an  $m \times m$  sparse matrix that is defined in HYB storage format by an opaque data structure **hybA**, **x** and **y** are the right-hand-side and the solution vectors,  $\alpha$  is a scalar, and

 $op(A) = \{A \text{ if transA} == CUSPARSE\_OPERATION\_NON\_TRANSPOSE}$ 

Notice that currently only op(A) = A is supported.

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires a significant amount of extra storage that is proportional to the matrix size. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

### Input

handle	handle to the cuSPARSE library context.
transA	the operation op(A) (currently only op(A) = A is supported).
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_TRIANGULAR and diagonal type USPARSE_DIAG_TYPE_NON_UNIT.
hybA	the matrix A in HYB storage format.
info	structure initialized using cusparseCreateSolveAnalysisInfo().

### Output

info	structure filled with information collected during
	the analysis phase (that should be passed to the solve phase unchanged).
	solve phase unchanged).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	the internally stored HYB format parameters are invalid.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 7.23. cusparse<t>hybsv\_solve()

```
cusparseStatus t
cusparseShybsv solve(cusparseHandle t handle,
                    cusparseOperation t transA,
                    const float
                                          *alpha,
                    const cusparseMatDescr t descrA,
                    cusparseHybMat t hybA,
                    cusparseSolveAnalysisInfo t info,
                    const float
                                          *x, float
                                                               *y)
cusparseStatus t
cusparseDhybsv solve(cusparseHandle t handle,
                    cusparseOperation t transA,
                    const double
                                          *alpha,
                    const cusparseMatDescr t descrA,
                    cusparseHybMat t hybA,
                    cusparseSolveAnalysisInfo t info,
                    const double
                                          *x, double
                                                               *V)
cusparseStatus t
cusparseChybsv_solve(cusparseHandle_t handle,
                    cusparseOperation_t transA,
                    const cuComplex *alpha,
                    const cusparseMatDescr t descrA,
                    cusparseHybMat t hybA,
                    cusparseSolveAnalysisInfo t info,
                                          *x, cuComplex
                    const cuComplex
                                                              *y)
cusparseStatus t
cusparseZhybsv_solve(cusparseHandle t handle,
                    cusparseOperation t transA,
                    const cuDoubleComplex *alpha,
                    const cusparseMatDescr t descrA,
                    cusparseHybMat t hybA,
                    cusparseSolveAnalysisInfo t info,
                    const cuDoubleComplex *x, cuDoubleComplex *y)
```

This function performs the solve phase of the solution of a sparse triangular linear system:

$$op(A) * y = a * x$$

**A** is an  $m \times m$  sparse matrix that is defined in HYB storage format by an opaque data structure **hybA**, **x** and **y** are the right-hand-side and the solution vectors,  $\alpha$  is a scalar, and

```
op(A) = \{A \text{ if trans } A == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE } \}
```

Notice that currently only op(A) = A is supported.

This function may be executed multiple times for a given matrix and a particular operation type.

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

#### Input

handle to the cuSPARSE library context.
---

transA	the operation op( $A$ ) (currently only op( $A$ ) = $A$ is supported).
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix A.  The supported matrix type is  CUSPARSE_MATRIX_TYPE_TRIANGULAR and the  diagonal type is CUSPARSE_DIAG_TYPE_NON_UNIT.
hybA	the matrix A in HYB storage format.
info	structure with information collected during the analysis phase (that should be passed to the solve phase unchanged).
x	<type> right-hand-side vector of size m.</type>

### Output

У	<type> solution vector of size m.</type>
---	--

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the internally stored hyb format parameters are invalid.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_MAPPING_ERROR	the texture binding failed.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# Chapter 8. CUSPARSE LEVEL 3 FUNCTION REFERENCE

This chapter describes sparse linear algebra functions that perform operations between sparse and (usually tall) dense matrices.

In particular, the solution of sparse triangular linear systems with multiple righthand sides is implemented in two phases. First, during the analysis phase, the sparse triangular matrix is analyzed to determine the dependencies between its elements by calling the appropriate <code>csrsm\_analysis()</code> function. The analysis is specific to the sparsity pattern of the given matrix and to the selected <code>cusparseOperation\_t</code> type. The information from the analysis phase is stored in the parameter of type <code>cusparseSolveAnalysisInfo\_t</code> that has been initialized previously with a call to <code>cusparseCreateSolveAnalysisInfo()</code>.

Second, during the solve phase, the given sparse triangular linear system is solved using the information stored in the <code>cusparseSolveAnalysisInfo\_t</code> parameter by calling the appropriate <code>csrsm\_solve()</code> function. The solve phase may be performed multiple times with different multiple right-hand sides, while the analysis phase needs to be performed only once. This is especially useful when a sparse triangular linear system must be solved for different sets of multiple right-hand sides one at a time, while its coefficient matrix remains the same.

Finally, once all the solves have completed, the opaque data structure pointed to by the cusparseSolveAnalysisInfo\_t parameter can be released by calling cusparseDestroySolveAnalysisInfo(). For more information please refer to [3].

# 8.1. cusparse<t>csrmm()

```
cusparseStatus t
cusparseScsrmm(cusparseHandle t handle,
   cusparseOperation t transA,
   int m,
   int n,
   int k,
   int nnz,
   const float *alpha,
   const cusparseMatDescr t descrA,
   const float *csrValA,
   const int *csrRowPtrA,
   const int *csrColIndA,
   const float *B,
   int ldb,
   const float *beta,
   float *C,
   int ldc)
cusparseStatus t
cusparseDcsrmm(cusparseHandle t handle,
   cusparseOperation_t transA,
   int m,
   int n,
   int k,
   int nnz,
   const double *alpha,
   const cusparseMatDescr t descrA,
   const double *csrValA,
   const int *csrRowPtrA,
   const int *csrColIndA,
   const double *B,
   int ldb,
   const double *beta,
   double *C,
   int ldc)
cusparseStatus t
cusparseCcsrmm(cusparseHandle t handle,
   cusparseOperation t transA,
    int m,
   int n,
   int k,
   int nnz,
   const cuComplex *alpha,
   const cusparseMatDescr_t descrA,
   const cuComplex *csrValA,
   const int *csrRowPtrA,
   const int *csrColIndA,
   const cuComplex *B,
   int ldb,
   const cuComplex *beta,
   cuComplex *C,
   int ldc)
cusparseStatus t
cusparseZcsrmm(cusparseHandle t handle,
   cusparseOperation t transA,
   int m,
   int n,
   int k,
   int nnz,
   const cuDoubleComplex *alpha,
   const cusparseMatDescr_t descrA,
   const cuDoubleComplex *csrValA,
   const int *csrRowPtrA,
    const int *csrColIndA,
    const cuDoubleComplex *P
```

This function performs one of the following matrix-matrix operations:

$$C = a * op(A) * B + B * C$$

**A** is an  $m \times k$  sparse matrix that is defined in CSR storage format by the three arrays **csrValA**, **csrRowPtrA**, and **csrColIndA**); **B** and **C** are dense matrices;  $\alpha$  and  $\beta$  are scalars; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

When using the (conjugate) transpose of a general matrix or a Hermitian/symmetric matrix, this routine may produce slightly different results with the same input parameters during different runs of this function. For these matrix types it uses atomic operations to compute the final result; consequently, many threads may be adding floating point numbers to the same memory location without any specific ordering, which may produce slightly different results for each run.

If exactly the same output is required for any input when multiplying by the transpose of a general matrix, the following procedure can be used:

- 1. Convert the matrix from CSR to CSC format using one of the csr2csc() functions. Notice that by interchanging the rows and columns of the result you are implicitly transposing the matrix.
- 2. Call the **csrmm()** function with the **cusparseOperation\_t** parameter set to **CUSPARSE\_OPERATION\_NON\_TRANSPOSE** and with the interchanged rows and columns of the matrix stored in CSC format. This (implicitly) multiplies the vector by the transpose of the matrix in the original CSR format.

This function requires no extra storage for the general matrices when operation <code>CUSPARSE\_OPERATION\_NON\_TRANSPOSE</code> is selected. It requires some extra storage for Hermitian/symmetric matrices and for the general matrices when an operation different from <code>CUSPARSE\_OPERATION\_NON\_TRANSPOSE</code> is selected. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

### Input

handle	handle to the cuSPARSE library context.
transA	the operation op(A)
m	number of rows of sparse matrix A.
n	number of columns of dense matrices <b>B</b> and <b>c</b> .
k	number of columns of sparse matrix A.
nnz	number of nonzero elements of sparse matrix <b>A</b> .
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix a. The supported matrix types are CUSPARSE_MATRIX_TYPE_GENERAL, CUSPARSE_MATRIX_TYPE_SYMMETRIC, and CUSPARSE_MATRIX_TYPE_HERMITIAN.  Also, the supported index bases are

	CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column indices of the nonzero elements of matrix A.
В	array of dimensions (1db, n).
1db	leading dimension of B. It must be at least $max(1, k)$ if $op(A) = A$ and at least $max(1, m)$ otherwise.
beta	<pre><type> scalar used for multiplication. If beta is zero, c does not have to be a valid input.</type></pre>
С	array of dimensions (ldc, n).
ldc	leading dimension of c. It must be at least max $(1, m)$ if op $(A) = A$ and at least max $(1, k)$ otherwise.

### Output

С	<type> updated array of dimensions (lde, n).</type>
---	---

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n, k, nnz<0 or ldb and ldc are incorrect).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 8.2. cusparse<t>csrmm2()

```
cusparseStatus t
cusparseScsrmm2(cusparseHandle t
                                         handle,
                cusparseOperation t
                                         transA,
                cusparseOperation t
                                         transB,
                int
                                         m,
                int
                                         n,
                int
                                         k,
                int
                                         nnz,
                                         *alpha,
                const float
                const cusparseMatDescr t descrA,
                const float
                                         *csrValA,
                                         *csrRowPtrA,
                const int
                const int
                                         *csrColIndA,
                const float
                                         *B,
                                         ldb,
                                         *beta,
                const float
                float
                                         *C,
                int
                                         ldc)
cusparseStatus_t
cusparseDcsrmm2(cusparseHandle t
                                         handle,
                cusparseOperation_t
                                         transA,
                cusparseOperation_t
                                         transB,
                int
                                         m,
                int
                                         n,
                int
                                         k,
                                         nnz,
                const double
                                          *alpha,
                const cusparseMatDescr_t descrA,
                const Guspared *csrvain, const double *csrRowPtrA,
                const int
                                         *csrColIndA,
                const double
                                         ldb,
                const double
                                         *beta,
                double
                                         *C,
                                         ldc)
cusparseStatus t
cusparseCcsrmm2(cusparseHandle t
                                        handle,
                cusparseOperation t
                                         transA,
                cusparseOperation t
                                         transB,
                int
                int
                                          n,
                int
                int
                                         nnz,
                const cuComplex
                                         *alpha,
                const cusparseMatDescr t descrA,
                const cuComplex
                                        *csrValA,
                const int
                                        *csrRowPtrA,
                const int
                                         *csrColIndA,
                                         *B,
                const cuComplex
                                         ldb,
                                         *beta,
                const cuComplex
                                         *C,
                cuComplex
                                         ldc)
                int
cusparseStatus t
cusparseZcsrmm2(cusparseHandle t
                                        handle,
                cusparseOperation t
                                         transA,
                                         transB,
                cusparseOperation t
                int
                                         m,
                int
                                         n,
                int
                                         nnz,
                const cuDoubleComplex
                                         *alpha,
                const cusparseMatDescr t descrA.
```

This function performs one of the following matrix-matrix operations:

$$C = a * op(A) * op(B) + B * C$$

**A** is an  $m \times k$  sparse matrix that is defined in CSR storage format by the three arrays **csrValA**, **csrRowPtrA**, and **csrColIndA**); **B** and **C** are dense matrices;  $\alpha$  and  $\beta$  are scalars; and

$$op(A) = \begin{cases} A & \text{if transA} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if transA} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if transA} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

and

$$op(B) = \begin{cases} B & \text{if transB} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ B^T & \text{if transB} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ B^H & \text{not supported} \end{cases}$$

If op (B) =B, cusparse<t>csrmm2() is the same as cusparse<t>csrmm(); otherwise, only op (A) =A is supported and the matrix type must be CUSPARSE MATRIX TYPE GENERAL.

The motivation of **transpose** (B) is to improve the memory access of matrix B. The computational pattern of **A\*transpose** (B) with matrix B in column-major order is equivalent to **A\*B** with matrix B in row-major order.

In practice, no operation in iterative solver or eigenvalue solver uses A\*transpose (B). However we can perform A\*transpose (transpose (B)) which is the same as A\*B. For example, suppose A is m\*k, B is k\*n and C is m\*n, the following code shows usage of cusparseDcsrmm().

Instead of using A\*B, our proposal is to transpose B to Bt first by calling cublas<t>geam(), then to perform A\*transpose (Bt).

```
// step 1: Bt := transpose(B)
   double *Bt;
   const int ldb_Bt = n; // leading dimension of Bt
   cudaMalloc((void**)&Bt, sizeof(double)*ldb_Bt*k);
   double one = 1.0;
   double zero = 0.0;
   cublasSetPointerMode(cublas handle, CUBLAS POINTER MODE HOST);
   cublasDgeam(cublas_handle, CUBLAS_OP_T, CUBLAS_OP_T,
       n, k, &one, B, int ldb B, &zero, B, int ldb B, Bt, ldb Bt);
// step 2: perform C:=alpha*A*transpose(Bt) + beta*C
   cusparseDcsrmm2(cusparse handle,
              CUSPARSE OPERATION NON TRANSPOSE,
              CUSPARSE OPERATION TRANSPOSE
              m, n, k, nnz, alpha,
              descrA, csrValA, csrRowPtrA, csrColIndA,
              Bt, ldb Bt,
              beta, C, ldc);
```

Remark 1: cublas<t>geam() and cusparse<t>csrmm2() are memory bound.

The complexity of cublas<t>geam() is 2\*n\*k, and the minimum complexity of cusparse<t>csrmm2() is about (nnz + nnz\*n + 2\*m\*n). If nnz per column (=nnz/k) is large, it is worth paying the extra cost on transposition because A\*transpose(B) may be 2× faster than A\*B if the sparsity pattern of A is not good.

Remark 2: A\*transpose (B) is only supported on compute capability 2.0 and above.

### Input

handle	handle to the cuSPARSE library context.
transA	the operation op(A)
transB	the operation op(B)
m	number of rows of sparse matrix A.
n	number of columns of dense matrix op(B) and c.
k	number of columns of sparse matrix A.
nnz	number of nonzero elements of sparse matrix A.
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix a. The supported matrix types is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	<pre>integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.</pre>

В	array of dimensions (ldb, n) if op(B)=B and (ldb, k) otherwise.
ldb	leading dimension of B. If op (B) =B, it must be at least max $(1, k)$ if op $(A) = A$ and at least max $(1, m)$ otherwise. If op $(B)$ !=B, it must be at least max $(1, n)$ .
beta	<pre><type> scalar used for multiplication. If beta is zero, c does not have to be a valid input.</type></pre>
С	array of dimensions (ldc, n).
lde	leading dimension of c. It must be at least max $(1, m)$ if op $(A) = A$ and at least max $(1, k)$ otherwise.

# Output

С	<type> updated array of dimensions (ldc, n).</type>
---	---

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n, k, nnz<0 or ldb and ldc are incorrect).
CUSPARSE_STATUS_ARCH_MISMATCH	if op (B) =B, the device does not support double precision or if op (B) =transpose (B) the device is below compute capability 2.0.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	only CUSPARSE_MATRIX_TYPE_GENERAL is supported otherwise.

# 8.3. cusparse<t>csrsm\_analysis()

```
cusparseStatus t
cusparseScsrsm analysis(cusparseHandle t handle,
                        cusparseOperation t transA,
                       int m, int nnz,
                       const cusparseMatDescr t descrA,
                       const float *csrValA,
                       const int *csrRowPtrA, const int *csrColIndA,
                       cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseDcsrsm analysis(cusparseHandle t handle,
                       cusparseOperation t transA,
                        int m, int nnz,
                       const cusparseMatDescr t descrA,
                       const double \overline{*}csrValA,
                       const int *csrRowPtrA, const int *csrColIndA,
                       cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseCcsrsm_analysis(cusparseHandle_t handle,
                       cusparseOperation t transA,
                        int m, int nnz,
                        const cusparseMatDescr t descrA,
                       cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseZcsrsm analysis(cusparseHandle t handle,
                        cusparseOperation t transA,
                        int m, int nnz,
                        const cusparseMatDescr t descrA,
                        const cuDoubleComplex *\overline{*}csrValA,
const int *csrRowPtrA, const int *csrColIndA,
                        cusparseSolveAnalysisInfo t info)
```

This function performs the analysis phase of the solution of a sparse triangular linear system

$$op(A) * Y = a * X$$

with multiple right-hand sides, where **A** is an **m×m** sparse matrix that is defined in CSR storage format by the three arrays **csrValA**, **csrRowPtrA**, and **csrColIndA**; **X** and **Y** are the right-hand-side and the solution dense matrices; α is a scalar; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires a significant amount of extra storage that is proportional to the matrix size. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

#### Input

handle	handle to the cuSPARSE library context.
transA	the operation op(A).
m	number of rows of matrix A.
nnz	number of nonzero elements of matrix A.
descrA	the descriptor of matrix a. The supported matrix types are cusparse_matrix_type_triangular and cusparse_matrix_type_general, while the supported diagonal types are cusparse_diag_type_unit and cusparse_diag_type_non_unit.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m$ +1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
info	structure initialized using cusparseCreateSolveAnalysisInfo().

# Output

info	structure filled with information collected during
	the analysis phase (that should be passed to the
	solve phase unchanged).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,nnz<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 8.4. cusparse<t>csrsm\_solve()

```
cusparseStatus t
cusparseScsrsm solve(cusparseHandle t handle,
                     cusparseOperation t transA,
                     int m, int n, const float *alpha,
                     const cusparseMatDescr t descrA,
                     const float *csrValA,
                     const int *csrRowPtrA, const int *csrColIndA,
                    cusparseSolveAnalysisInfo t info,
                    const float *X, int ldx,
                                    *Y, int ldy)
                    float
cusparseStatus t
cusparseDcsrsm solve(cusparseHandle t handle,
                     cusparseOperation t transA,
                     int m, int n, const double *alpha,
                     const cusparseMatDescr_t descrA,
                     const double \overline{*}csrValA,
                     const int *csrRowPtrA, const int *csrColIndA,
                     cusparseSolveAnalysisInfo_t info,
                    const double *X, int ldx, double *Y, int ldy)
cusparseStatus t
cusparseCcsrsm_solve(cusparseHandle_t handle,
                     cusparseOperation_t transA,
                     int m, int n, const cuComplex *alpha,
                     const cusparseMatDescr t descrA,
                     const cuComplex \overline{*}csrValA,
                     const int *csrRowPtrA, const int *csrColIndA,
                     cusparseSolveAnalysisInfo t info,
                     cusparseStatus t
cusparseZcsrsm solve(cusparseHandle t handle,
                     cusparseOperation t transA,
                     int m, int n, const cuDoubleComplex *alpha,
                     const cusparseMatDescr t descrA,
                     const cuDoubleComplex *csrValA,
const int *csrRowPtrA, const int *csrColIndA,
                     cusparseSolveAnalysisInfo t info,
                     const cuDoubleComplex *X, int ldx,
                     cuDoubleComplex *Y, int ldy)
```

This function performs the solve phase of the solution of a sparse triangular linear system

$$op(A) * Y = a * X$$

with multiple right-hand sides, where **A** is an **m×n** sparse matrix that is defined in CSR storage format by the three arrays **csrValA**, **csrRowPtrA**, and **csrColIndA**); **X** and **Y** are the right-hand-side and the solution dense matrices; α is a scalar; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

This function may be executed multiple times for a given matrix and a particular operation type.

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

### Input

	-
handle	handle to the cuSPARSE library context.
transA	the operation op (A).
m	number of rows and columns of matrix A.
n	number of columns of matrix $x$ and $y$ .
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix a. The supported matrix types are CUSPARSE_MATRIX_TYPE_TRIANGULAR and CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m + 1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
info	structure with information collected during the analysis phase (that should be passed to the solve phase unchanged).
х	<type> right-hand-side array of dimensions (<math>ldx</math>, <math>n</math>).</type>
ldx	leading dimension of $x$ (that is $\ge max(1, m)$ ).

### Output

Y	<type> solution array of dimensions (ldy, n).</type>
ldy	leading dimension of Y (that is ≥ max(1, m)).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_MAPPING_ERROR	the texture binding failed.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 8.5. cusparse<t>bsrmm()

```
cusparseStatus t
cusparseSbsrmm(cusparseHandle t handle,
               cusparseDirection t dirA,
               cusparseOperation t transA,
               cusparseOperation t transB,
               int mb,
               int n,
               int kb,
               int nnzb,
               const float *alpha,
               const cusparseMatDescr t descrA,
               const float *bsrValA,
               const int *bsrRowPtrA,
               const int *bsrColIndA,
               const int blockDim,
               const float *B,
               const int ldb,
               const float *beta,
               float *C,
               int ldc)
cusparseStatus t
cusparseDbsrmm(cusparseHandle t handle,
               cusparseDirection t dirA,
               cusparseOperation_t transA,
               cusparseOperation t transB,
               int mb,
               int n,
               int kb,
               int nnzb,
               const double *alpha,
               const cusparseMatDescr t descrA,
               const double *bsrValA,
               const int *bsrRowPtrA,
               const int *bsrColIndA,
const int blockDim,
               const double *B,
               const int ldb,
               const double *beta,
               double *C,
               int ldc)
cusparseStatus t
cusparseCbsrmm(cusparseHandle t handle,
               cusparseDirection t dirA,
               cusparseOperation_t transA,
               cusparseOperation_t transB,
               int mb,
               int n,
               int kb,
               int nnzb,
               const cuComplex *alpha,
               const cusparseMatDescr t descrA,
               const cuComplex *bsrValA,
               const int *bsrRowPtrA,
               const int *bsrColIndA,
               const int blockDim,
               const cuComplex *B,
               const int ldb,
               const cuComplex *beta,
               cuComplex *C,
               int ldc)
```

This function performs one of the following matrix-matrix operations:

$$C = a * op(A) * op(B) + B * C$$

**A** is an **mb×kb** sparse matrix that is defined in BSR storage format by the three arrays **bsrValA**, **bsrRowPtrA**, and **bsrColIndA**; **B** and **C** are dense matrices;  $\alpha$  and  $\beta$  are scalars; and

$$op(A) = \begin{cases} A & \text{if transA} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if transA} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \text{ (not supported)} \\ A^H & \text{if transA} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \text{ (not supported)} \end{cases}$$

and

$$op(B) = \begin{cases} B & \text{if transB} == CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ B^T & \text{if transB} == CUSPARSE\_OPERATION\_TRANSPOSE} \\ B^H & \text{if transB} == CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \text{ (not supported)} \end{cases}$$

The matrix type must be **CUSPARSE MATRIX TYPE GENERAL**.

The motivation of **transpose (B)** is to improve memory access of matrix **B**. The computational pattern of **A\*transpose (B)** with matrix **B** in column-major order is equivalent to **A\*B** with matrix **B** in row-major order.

In practice, no operation in an iterative solver or eigenvalue solver uses A\*transpose(B). However, we can perform A\*transpose(transpose(B)) which is the same as A\*B. For example, suppose A is mb\*kb, B is k\*n and C is m\*n, the following code shows usage of cusparseDbsrmm().

Instead of using A\*B, our proposal is to transpose B to Bt by first calling cublas<t>geam(), and then to perform A\*transpose (Bt).

```
// step 1: Bt := transpose(B)
   const int m = mb*blockSize;
    const int k = kb*blockSize;
   double *Bt;
   const int ldb_Bt = n; // leading dimension of Bt
cudaMalloc((void**)&Bt, sizeof(double)*ldb_Bt*k);
   double one = 1.0;
   double zero = 0.0;
    cublasSetPointerMode(cublas handle, CUBLAS POINTER MODE HOST);
    cublasDgeam(cublas_handle, CUBLAS_OP_T, CUBLAS_OP_T,
        n, k, &one, B, int ldb_B, &zero, B, int ldb_B, Bt, ldb_Bt);
// step 2: perform C:=alpha*A*transpose(Bt) + beta*C
    cusparseDbsrmm(cusparse handle,
                CUSPARSE_DIRECTION_COLUMN,
                CUSPARSE OPERATION NON TRANSPOSE, CUSPARSE OPERATION TRANSPOSE,
                mb, n, kb, nnzb, alpha,
                descrA, bsrValA, bsrRowPtrA, bsrColIndA, blockSize,
                Bt, ldb Bt,
                beta, C, ldc);
```

Function **bsrmm()** is only supported on compute capability 2.0 and above.

#### Input

handle	handle to the cuSPARSE library context.
dir	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
transA	the operation op (A).
transB	the operation op (B).
mb	number of block rows of sparse matrix A.
n	number of columns of dense matrix $op(B)$ and A.
kb	number of block columns of sparse matrix A.
nnzb	number of non-zero blocks of sparse matrix A.
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<pre><type> array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) nonzero blocks of matrix A.</type></pre>
bsrRowPtrA	integer array of mb +1 elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.

blockDim	block dimension of sparse matrix A, larger than zero.
В	array of dimensions (ldb, n) if op(B)=B and (ldb, k) otherwise.
ldb	leading dimension of B. If op (B) =B, it must be at least max $(1, k)$ If op (B) $!= B$ , it must be at least max $(1, n)$ .
beta	<pre><type> scalar used for multiplication. If beta is zero, c does not have to be a valid input.</type></pre>
С	array of dimensions (ldc, n).
lde	leading dimension of c. It must be at least $max(1, m)$ if op (A) = A and at least $max(1, k)$ otherwise.

# Output

С	<type> updated array of dimensions (ldc, n).</type>
---	---

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	Either invalid parameters were passed (mb, n, kb, nnzb<0; or ldb and ldc are incorrect).
	Either invalid or unsupported operations were passed (op (A) is different from CUSPARSE_OPERATION_NON_TRANSPOSE, or op (B) is different from CUSPARSE_OPERATION_NON_TRANSPOSE or CUSPARSE_OPERATION_TRANSPOSE).
CUSPARSE_STATUS_ARCH_MISMATCH	if device is below compute capability 2.0.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	only CUSPARSE_MATRIX_TYPE_GENERAL is supported otherwise.

# 8.6. cusparse<t>bsrsm2\_bufferSize()

```
cusparseStatus t
cusparseSbsrsm\overline{2} bufferSize(cusparseHandle t handle,
                            cusparseDirection t dirA,
                            cusparseOperation t transA,
                            cusparseOperation t transX,
                            int mb,
                            int n,
                            int nnzb,
                            const cusparseMatDescr t descrA,
                            float *bsrValA,
                            const int *bsrRowPtrA,
                            const int *bsrColIndA,
                            int blockDim,
                            bsrsm2Info_t info,
                            int *pBufferSizeInBytes)
cusparseStatus_t
cusparseDbsrsm2 bufferSize(cusparseHandle_t handle,
                            cusparseDirection_t dirA,
                            cusparseOperation_t transA,
                            cusparseOperation t transX,
                            int mb,
                            int n,
                            int nnzb,
                            const cusparseMatDescr t descrA,
                            double *bsrValA,
                            const int *bsrRowPtrA,
                            const int *bsrColIndA,
                            int blockDim,
                            bsrsm2Info t info,
                            int *pBufferSizeInBytes)
cusparseStatus t
cusparseCbsrsm2 bufferSize(cusparseHandle t handle,
                            cusparseDirection t dirA,
                            cusparseOperation t transA,
                            cusparseOperation t transX,
                            int mb,
                            int n,
                            int nnzb,
                            const cusparseMatDescr t descrA,
                            cuComplex *bsrValA,
const int *bsrRowPtrA,
                            const int *bsrColIndA,
                            int blockDim,
                            bsrsm2Info t info,
                            int *pBufferSizeInBytes)
cusparseStatus t
cusparseZbsrsm2 bufferSize(cusparseHandle t handle,
                            cusparseDirection t dirA,
                            cusparseOperation t transA,
                            cusparseOperation t transX,
                            int mb,
                            int n,
                            int nnzb,
                            const cusparseMatDescr t descrA,
                            cuDoubleComplex *bsrValA,
                            const int *bsrRowPtrA,
                            const int *bsrColIndA,
                            int blockDim,
                            bsrsm2Info t info,
                            int *pBufferSizeInBytes)
```

This function returns size of buffer used in **bsrsm2()**, a new sparse triangular linear system **op(A)**\*Y =  $\alpha$  **op(X)**.

**A** is an  $(mb*blockDim) \times (mb*blockDim)$  sparse matrix that is defined in BSR storage format by the three arrays **bsrValA**, **bsrRowPtrA**, and **bsrColIndA**); **X** and **Y** are the right-hand-side and the solution matrices;  $\alpha$  is a scalar; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

Although there are six combinations in terms of parameter trans and the upper (and lower) triangular part of A, bsrsm2\_bufferSize() returns the maximum size of the buffer among these combinations. The buffer size depends on dimension mb,blockDim and the number of nonzeros of the matrix, nnzb. If the user changes the matrix, it is necessary to call bsrsm2\_bufferSize() again to get the correct buffer size, otherwise a segmentation fault may occur.

#### Input

·	·
handle	handle to the cuSPARSE library context.
dirA	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
transA	the operation op (A).
transX	the operation $op(x)$ .
mb	number of block rows of matrix <b>A</b> .
n	number of columns of matrix $\mathbf{Y}$ and $\mathbf{op}(\mathbf{X})$ .
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.
bsrValA	<pre><type> array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) nonzero blocks of matrix A.</type></pre>
bsrRowPtrA	integer array of mb +1 elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A; larger than zero.

#### Output

info	record internal states based on different
	algorithms.

pBufferSizeInBytes	number of bytes of the buffer used in
	bsrsm2_analysis() and bsrsm2_solve().

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb, n, nnzb<=0); base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 8.7. cusparse<t>bsrsm2\_analysis()

```
cusparseStatus t
cusparseSbsrsm2 analysis(cusparseHandle t handle,
                         cusparseDirection t dirA,
                         cusparseOperation t transA,
                         cusparseOperation t transXY,
                         int mb,
                         int n,
                         int nnzb,
                         const cusparseMatDescr t descrA,
                         const float *bsrVal,
                         const int *bsrRowPtr,
                         const int *bsrColInd,
                         int blockDim,
                         bsrsm2Info t info,
                         cusparseSolvePolicy_t policy,
                         void *pBuffer)
cusparseStatus_t
cusparseDbsrsm2_analysis(cusparseHandle_t handle,
                         cusparseDirection_t dirA,
                         cusparseOperation_t transA,
                         cusparseOperation t transXY,
                         int mb,
                         int n,
                         int nnzb,
                         const cusparseMatDescr t descrA,
                         const double *bsrVal,
                         const int *bsrRowPtr,
                         const int *bsrColInd,
                         int blockDim,
                         bsrsm2Info t info,
                         cusparseSolvePolicy t policy,
                         void *pBuffer)
cusparseStatus t
cusparseCbsrsm2_analysis(cusparseHandle_t handle,
                         cusparseDirection t dirA,
                         cusparseOperation t transA,
                         cusparseOperation t transXY,
                         int mb,
                         int n,
                         int nnzb,
                         const cusparseMatDescr t descrA,
                         const cuComplex *bsrVal,
                         const int *bsrRowPtr,
                         const int *bsrColInd,
                         int blockDim,
                         bsrsm2Info t info,
                         cusparseSolvePolicy_t policy,
                         void *pBuffer)
cusparseStatus t
cusparseZbsrsm2 analysis(cusparseHandle t handle,
                         cusparseDirection t dirA,
                         cusparseOperation t transA,
                         cusparseOperation t transXY,
                         int mb,
                         int n,
                         int nnzb,
                         const cusparseMatDescr t descrA,
                         const cuDoubleComplex *bsrVal,
                         const int *bsrRowPtr,
                         const int *bsrColInd.
```

This function performs the analysis phase of **bsrsm2()**, a new sparse triangular linear system **op(A)** \***op(Y)** =  $\alpha$  **op(X)**.

**A** is an  $(mb*blockDim) \times (mb*blockDim)$  sparse matrix that is defined in BSR storage format by the three arrays **bsrValA**, **bsrRowPtrA**, and **bsrColIndA**); **X** and **Y** are the right-hand-side and the solution matrices;  $\alpha$  is a scalar; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

and

$$op(X) = \begin{cases} X & \text{if transXY} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ X^T & \text{if transX} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ X^H & \text{if transX} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \text{ (not supported)} \end{cases}$$

and op(X) and op(Y) are equal.

The block of BSR format is of size blockDim\*blockDim, stored in column-major or row-major as determined by parameter dirA, which is either CUSPARSE\_DIRECTION\_ROW OR CUSPARSE\_DIRECTION\_COLUMN. The matrix type must be CUSPARSE MATRIX\_TYPE\_GENERAL, and the fill mode and diagonal type are ignored.

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires the buffer size returned by **bsrsm2\_bufferSize()**. The address of **pBuffer** must be multiple of 128 bytes. If not, **CUSPARSE\_STATUS\_INVALID\_VALUE** is returned.

Function bsrsm2\_analysis() reports a structural zero and computes the level information stored in opaque structure info. The level information can extract more parallelism during a triangular solver. However bsrsm2\_solve() can be done without level information. To disable level information, the user needs to specify the policy of the triangular solver as CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL.

Function bsrsm2\_analysis() always reports the first structural zero, even if the parameter policy is CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL. Besides, no structural zero is reported if CUSPARSE\_DIAG\_TYPE\_UNIT is specified, even if block A(j,j) is missing for some j. The user must call cusparseXbsrsm2\_query\_zero\_pivot() to know where the structural zero is.

Even when **bsrsm2\_analysis()** reports a structural zero, the user can still call asynchronously **bsrsm2\_solve()**. In this case, the solve will return a numerical zero in the same position as the structural zero but this result **x** is meaningless.

### Input

handle	handle to the cuSPARSE library context.
dirA	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
transA	the operation op (A).
transXY	the operation op $(X)$ and op $(Y)$ .

mb	number of block rows of matrix A.
n	number of columns of matrix $\mathbf{Y}$ and $\mathbf{op}(\mathbf{X})$ .
nnzb	number of non-zero blocks of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.
bsrValA	<pre><type> array of nnzb (= bsrRowPtrA (mb) - bsrRowPtrA(0) ) nonzero blocks of matrix A.</type></pre>
bsrRowPtrA	integer array of mb +1 elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A; larger than zero.
info	structure initialized using cusparseCreateBsrsm2Info.
policy	The supported policies are  CUSPARSE_SOLVE_POLICY_NO_LEVEL and  CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user; the size is return by bsrsm2_bufferSize().

### Output

info	structure filled with information collected during
	the analysis phase (that should be passed to the
	solve phase unchanged).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	Either invalid parameters were passed (mb, n, nnzb<=0).
	Either invalid or unsupported operations were passed (op (X) and op (Y) are different from CUSPARSE_OPERATION_NON_TRANSPOSE or CUSPARSE_OPERATION_TRANSPOSE).
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

CUSPARSE\_STATUS\_MATRIX\_TYPE\_NOT\_SUPPORTED the matrix type is not supported.

# 8.8. cusparse<t>bsrsm2\_solve()

```
cusparseStatus t
cusparseSbsrsm2 solve(cusparseHandle t handle,
                      cusparseDirection t dirA,
                      cusparseOperation t transA,
                      cusparseOperation t transXY,
                      int mb,
                      int n,
                      int nnzb,
                      const float *alpha,
                      const cusparseMatDescr t descrA,
                      const float *bsrVal,
                      const int *bsrRowPtr,
                      const int *bsrColInd,
                      int blockDim,
                      bsrsm2Info_t info,
                      const float *X,
                      int ldx,
                      float *Y,
                      int ldy,
                      cusparseSolvePolicy_t policy,
                      void *pBuffer)
cusparseStatus t
cusparseDbsrsm2 solve(cusparseHandle t handle,
                      cusparseDirection t dirA,
                      cusparseOperation_t transA,
                      cusparseOperation t transXY,
                      int mb,
                      int n,
                      int nnzb,
                      const double *alpha,
                      const cusparseMatDescr t descrA,
                      const double *bsrVal,
                      const int *bsrRowPtr,
                      const int *bsrColInd,
                      int blockDim,
                      bsrsm2Info t info,
                      const double *X,
                      int ldx,
                      double *Y,
                      int ldy,
                      cusparseSolvePolicy t policy,
                      void *pBuffer)
cusparseStatus t
cusparseCbsrsm2 solve(cusparseHandle_t handle,
                      cusparseDirection_t dirA,
                      cusparseOperation_t transA,
                      cusparseOperation_t transXY,
                      int mb,
                      int n,
                      int nnzb,
                      const cuComplex *alpha,
                      const cusparseMatDescr t descrA,
                      const cuComplex *bsrVal,
                      const int *bsrRowPtr,
                      const int *bsrColInd,
                      int blockDim,
                      bsrsm2Info_t info,
                      const cuComplex *X,
                      int ldx,
                      cuComplex *Y,
                      int ldv.
```

This function performs the solve phase of the solution of a sparse triangular linear system:

$$op(A) * op(Y) = a * op(X)$$

**A** is an  $(mb*blockDim) \times (mb*blockDim)$  sparse matrix that is defined in BSR storage format by the three arrays **bsrValA**, **bsrRowPtrA**, and **bsrColIndA**); **X** and **Y** are the right-hand-side and the solution matrices;  $\alpha$  is a scalar, and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

and

$$op(X) = \begin{cases} X & \text{if transX} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ X^T & \text{if transX} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ X^H & \text{not supported} \end{cases}$$

Only op (A) =A is supported.

op (X) and op (Y) must be performed in the same way. In other words, if op (X) = X, op (Y) = Y.

The block of BSR format is of size blockDim\*blockDim, stored as column-major or row-major as determined by parameter dirA, which is either CUSPARSE\_DIRECTION\_ROW OR CUSPARSE\_DIRECTION\_COLUMN. The matrix type must be CUSPARSE\_MATRIX\_TYPE\_GENERAL, and the fill mode and diagonal type are ignored. Function bsrsm02 solve() can support an arbitrary blockDim.

This function may be executed multiple times for a given matrix and a particular operation type.

This function requires the buffer size returned by bsrsm2\_bufferSize(). The address of pBuffer must be multiple of 128 bytes. If it is not, CUSPARSE STATUS INVALID VALUE is returned.

Although bsrsm2\_solve() can be done without level information, the user still needs to be aware of consistency. If bsrsm2\_analysis() is called with policy CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL, bsrsm2\_solve() can be run with or without levels. On the other hand, if bsrsm2\_analysis() is called with CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL, bsrsm2\_solve() can only accept CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL; otherwise, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

Function bsrsm02\_solve() has the same behavior as bsrsv02\_solve(), reporting the first numerical zero, including a structural zero. The user must call cusparseXbsrsm2 query zero pivot() to know where the numerical zero is.

The motivation of **transpose** (**X**) is to improve the memory access of matrix **X**. The computational pattern of **transpose** (**X**) with matrix **X** in column-major order is equivalent to **X** with matrix **X** in row-major order.

In-place is supported and requires that **x** and **y** point to the same memory block, and **ldx=ldy**.

Input

handle	handle to the cuSPARSE library context.
dirA	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
transA	the operation op (A).
transXY	the operation $op(X)$ and $op(Y)$ .
mb	number of block rows of matrix A.
n	number of columns of matrix $\mathbf{Y}$ and $\mathbf{op}(\mathbf{X})$ .
nnzb	number of non-zero blocks of matrix A.
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, while the supported diagonal types are CUSPARSE_DIAG_TYPE_UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.
bsrValA	<pre><type> array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) non-zero blocks of matrix A.</type></pre>
bsrRowPtrA	integer array of mb +1elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A; larger than zero.
info	structure initialized using cusparseCreateBsrsm2Info().
x	<type> right-hand-side array.</type>
ldx	leading dimension of x. If op (x) = x, $1dx > = k$ ; otherwise, $1dx > = n$ .
ldy	leading dimension of Y. If op (A) =A, then $1dc \ge m$ . If op (A) !=A, then $1dx \ge k$ .
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user; the size is returned by bsrsm2_bufferSize().

# Output

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.

CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_MAPPING_ERROR	the texture binding failed.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 8.9. cusparseXbsrsm2\_zeroPivot()

If the returned error code is **CUSPARSE\_STATUS\_ZERO\_PIVOT**, **position=j** means **A**(j,j) is either a structural zero or a numerical zero (singular block). Otherwise **position=-1**.

The **position** can be 0-base or 1-base, the same as the matrix.

Function cusparseXbsrsm2\_zeroPivot() is a blocking call. It calls cudaDeviceSynchronize() to make sure all previous kernels are done.

The **position** can be in the host memory or device memory. The user can set the proper mode with **cusparseSetPointerMode()**.

### Input

handle	handle to the cuSPARSE library context.
info	<pre>info contains a structural zero or a numerical zero if the user already called bsrsm2_analysis() Or bsrsm2_solve().</pre>

#### Output

<del>-</del>	if no structural or numerical zero, position is -1; otherwise, if A(j,j) is missing or U(j,j) is zero,
	position=j.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	info is not valid.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# Chapter 9. CUSPARSE EXTRA FUNCTION REFERENCE

This chapter describes the extra routines used to manipulate sparse matrices.

# 9.1. cusparse<t>csrgeam()

```
cusparseStatus t
cusparseXcsrgeamNnz(cusparseHandle t handle,
                    int m,
                    int n,
                    const cusparseMatDescr t descrA,
                    int nnzA,
                    const int *csrRowPtrA,
                    const int *csrColIndA,
                    const cusparseMatDescr t descrB,
                    int nnzB,
                    const int *csrRowPtrB,
                    const int *csrColIndB,
                    const cusparseMatDescr t descrC,
                    int *csrRowPtrC,
                    int *nnzTotalDevHostPtr)
cusparseStatus_t
cusparseScsrgeam(cusparseHandle_t handle,
                 int m,
                 int n,
                 const float *alpha,
                 const cusparseMatDescr t descrA,
                 int nnzA,
                 const float *csrValA,
                 const int *csrRowPtrA,
                 const int *csrColIndA,
                 const float *beta,
                 const cusparseMatDescr t descrB,
                 int nnzB,
                 const float *csrValB,
                 const int *csrRowPtrB,
                 const int *csrColIndB,
                 const cusparseMatDescr t descrC,
                 float *csrValC,
                 int *csrRowPtrC
                 int *csrColIndC)
cusparseStatus t
cusparseDcsrgeam(cusparseHandle t handle,
                 int m,
                 int n,
                 const double *alpha,
                 const cusparseMatDescr t descrA,
                 int nnzA,
                 const double *csrValA,
                 const int *csrRowPtrA,
                 const int *csrColIndA,
                 const double *beta,
                 const cusparseMatDescr_t descrB,
                 int nnzB,
                 const double *csrValB,
                 const int *csrRowPtrB,
                 const int *csrColIndB,
                 const cusparseMatDescr t descrC,
                 double *csrValC,
                 int *csrRowPtrC,
                 int *csrColIndC)
cusparseStatus t
cusparseCcsrgeam(cusparseHandle t handle,
                 int m,
                 int n,
                 const cuComplex *alpha,
                 const cusparseMatDescr t descrA,
                 int nnzA,
                 const cuComplex *csrValA.
```

This function performs following matrix-matrix operation

```
C = a * A + B * B
```

where A, B, and C are m×n sparse matrices (defined in CSR storage format by the three arrays csrValA|csrValB|csrValC, csrRowPtrA|csrRowPtrB|csrRowPtrC, and csrColIndA|csrColIndB|csrcolIndC respectively), and α and β are scalars. Since A and B have different sparsity patterns, cuSPARSE adopts a two-step approach to complete sparse matrix C. In the first step, the user allocates csrRowPtrC of m +1elements and uses function cusparseXcsrgeamNnz() to determine csrRowPtrC and the total number of nonzero elements. In the second step, the user gathers nnzC (number of nonzero elements of matrix C) from either (nnzC=\*nnzTotalDevHostPtr) or (nnzC=csrRowPtrC(m)-csrRowPtrC(0)) and allocates csrValC, csrColIndC of nnzC elements respectively, then finally calls function cusparse[S|D|C|Z]csrgeam() to complete matrix C.

The general procedure is as follows:

```
int baseC, nnzC;
// nnzTotalDevHostPtr points to host memory
int *nnzTotalDevHostPtr = &nnzC;
cusparseSetPointerMode(handle, CUSPARSE POINTER MODE HOST);
cudaMalloc((void**)&csrRowPtrC, sizeof(\overline{i}nt)*(m+\overline{1}));
cusparseXcsrgeamNnz(handle, m, n,
       descrA, nnzA, csrRowPtrA, csrColIndA,
       descrB, nnzB, csrRowPtrB, csrColIndB,
       descrC, csrRowPtrC, nnzTotalDevHostPtr);
if (NULL != nnzTotalDevHostPtr) {
   nnzC = *nnzTotalDevHostPtr;
   cudaMemcpy(&nnzC, csrRowPtrC+m, sizeof(int), cudaMemcpyDeviceToHost);
   cudaMemcpy(&baseC, csrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
   nnzC -= baseC;
cudaMalloc((void**)&csrColIndC, sizeof(int)*nnzC);
cudaMalloc((void**)&csrValC, sizeof(float)*nnzC);
cusparseScsrgeam(handle, m, n,
       alpha,
       descrA, nnzA,
        csrValA, csrRowPtrA, csrColIndA,
       beta.
       descrB, nnzB,
        csrValB, csrRowPtrB, csrColIndB,
        descrC,
        csrValC, csrRowPtrC, csrColIndC);
```

#### Several comments on csrgeam():

- ► The other three combinations, NT, TN, and TT, are not supported by cuSPARSE. In order to do any one of the three, the user should use the routine **csr2csc()** to convert  $A \mid B$  to  $A^T \mid B^T$ .
- ► Only **CUSPARSE\_MATRIX\_TYPE\_GENERAL** is supported. If either **A** or **B** is symmetric or Hermitian, then the user must extend the matrix to a full one and reconfigure the **MatrixType** field of the descriptor to **CUSPARSE\_MATRIX\_TYPE\_GENERAL**.
- If the sparsity pattern of matrix **C** is known, the user can skip the call to function **cusparseXcsrgeamNnz()**. For example, suppose that the user has an iterative algorithm which would update **A** and **B** iteratively but keep the sparsity patterns.

- The user can call function **cusparseXcsrgeamNnz()** once to set up the sparsity pattern of **C**, then call function **cusparse[S|D|C|Z]geam()** only for each iteration.
- ▶ The pointers alpha and beta must be valid.
- When alpha or beta is zero, it is not considered a special case by cuSPARSE. The sparsity pattern of c is independent of the value of alpha and beta. If the user wants  $C = 0 \times A + 1 \times B^T$ , then csr2csc() is better than csrgeam().

### Input

handle	handle to the cuSPARSE library context.
m	number of rows of sparse matrix A,B,C.
n	number of columns of sparse matrix A,B,C.
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.
nnzA	number of nonzero elements of sparse matrix A.
csrValA	<pre><type> array of nnzA(= csrRowPtrA(m) - csrRowPtrA(0)) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m + 1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnzA (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
beta	<pre><type> scalar used for multiplication. If beta is zero, y does not have to be a valid input.</type></pre>
descrB	the descriptor of matrix B. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.
nnzB	number of nonzero elements of sparse matrix B.
csrValB	<pre><type> array of nnzB(= csrRowPtrB(m) - csrRowPtrB(0)) nonzero elements of matrix B.</type></pre>
csrRowPtrB	integer array of $m + 1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndB	<pre>integer array of nnzB(= csrRowPtrB(m) - csrRowPtrB(0) ) column indices of the nonzero elements of matrix B.</pre>
descrC	the descriptor of matrix c. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.

### Output

csrValC	<pre><type> array of nnzC(= csrRowPtrC(m) - csrRowPtrC(0)) nonzero elements of matrix c.</type></pre>
csrRowPtrC	integer array of $\mathfrak{m}$ +1 elements that contains the start of every row and the end of the last row plus one.

csrColIndC	<pre>integer array of nnzC(= csrRowPtrC(m) - csrRowPtrC(0) ) column indices of the nonzero elements of matrixc.</pre>
nnzTotalDevHostPtr	total number of nonzero elements in device or host memory. It is equal to (csrRowPtrC(m) - csrRowPtrC(0)).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,nnz<0, IndexBase Of descrA, descrB, descrC is not base-0 or base-1, or alpha or beta is nil )).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 9.2. cusparse<t>csrgemm()

```
cusparseStatus t
cusparseXcsrgemmNnz(cusparseHandle t handle,
                    cusparseOperation t transA,
                    cusparseOperation t transB,
                    int n,
                    int k,
                    const cusparseMatDescr t descrA,
                    const int nnzA,
                    const int *csrRowPtrA,
                    const int *csrColIndA,
                    const cusparseMatDescr t descrB,
                    const int nnzB,
                    const int *csrRowPtrB,
                    const int *csrColIndB,
                    const cusparseMatDescr t descrC,
                    int *csrRowPtrC,
                    int *nnzTotalDevHostPtr )
cusparseStatus_t
cusparseScsrgemm(cusparseHandle_t handle,
                 cusparseOperation_t transA,
                 cusparseOperation_t transB,
                 int m,
                 int n,
                 int k,
                 const cusparseMatDescr t descrA,
                 const int nnzA,
                 const float *csrValA,
                 const int *csrRowPtrA,
                 const int *csrColIndA,
                 const cusparseMatDescr t descrB,
                 const int nnzB,
                 const float *csrValB,
                 const int *csrRowPtrB,
                 const int *csrColIndB,
                 const cusparseMatDescr t descrC,
                 float *csrValC,
                 const int *csrRowPtrC,
                 int *csrColIndC )
cusparseStatus t
cusparseDcsrgemm (cusparseHandle t handle,
                 cusparseOperation t transA,
                 cusparseOperation t transB,
                 int m,
                 int n,
                 int k,
                 const cusparseMatDescr t descrA,
                 const int nnzA,
                 const double *csrValA,
                 const int *csrRowPtrA,
                 const int *csrColIndA,
                 const cusparseMatDescr t descrB,
                 const int nnzB,
                 const double *csrValB,
                 const int *csrRowPtrB,
                 const int *csrColIndB,
                 const cusparseMatDescr t descrC,
                 double *csrValC,
                 const int *csrRowPtrC,
                 int *csrColIndC )
cusparseStatus t
cusparseCcsrgemm (cusparseHandle t handle,
                 cusparseOperation t transA
```

This function performs following matrix-matrix operation:

```
C = op(A) * op(B)
```

where op(A), op(B) and C are m×k, k×n, and m×n sparse matrices (defined in CSR storage format by the three arrays csrValA|csrValB|csrValC, csrRowPtrA|csrRowPtrB|csrRowPtrC, and csrColIndA|csrColIndB|csrcolIndC respectively. The operation is defined by

```
op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} != \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \end{cases}
```

There are four versions, NN, NT, TN, and TT. NN stands for C = A\*B, NT stands for  $C = A*B^T$ , TN stands for  $C = A^T*B$  and TT stands for  $C = A^T*B^T$ .

The cuSPARSE library adopts a two-step approach to complete sparse matrix. In the first step, the user allocates <code>csrRowPtrC</code> of <code>m+1</code> elements and uses the function <code>cusparseXcsrgemmNnz()</code> to determine <code>csrRowPtrC</code> and the total number of nonzero elements. In the second step, the user gathers <code>nnzC</code> (the number of nonzero elements of matrix C) from either <code>(nnzC=\*nnzTotalDevHostPtr)</code> or <code>(nnzC=csrRowPtrC(m)-csrRowPtrC(0))</code> and allocates <code>csrValC</code> and <code>csrColIndC</code> of <code>nnzC</code> elements respectively, then finally calls function <code>cusparse[SIDICIZ]csrgemm()</code> to complete matrix <code>C</code>.

The general procedure is as follows:

```
int baseC, nnzC;
// nnzTotalDevHostPtr points to host memory
int *nnzTotalDevHostPtr = &nnzC;
cusparseSetPointerMode(handle, CUSPARSE POINTER MODE HOST);
cudaMalloc((void**)&csrRowPtrC, sizeof(\overline{i}nt)*(m+\overline{1}));
cusparseXcsrgemmNnz(handle, transA, transB, m, n, k,
       descrA, nnzA, csrRowPtrA, csrColIndA,
       descrB, nnzB, csrRowPtrB, csrColIndB,
       descrC, csrRowPtrC, nnzTotalDevHostPtr );
if (NULL != nnzTotalDevHostPtr) {
   nnzC = *nnzTotalDevHostPtr;
}else{
   cudaMemcpy(&nnzC, csrRowPtrC+m, sizeof(int), cudaMemcpyDeviceToHost);
   cudaMemcpy(&baseC, csrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
   nnzC -= baseC;
cudaMalloc((void**)&csrColIndC, sizeof(int)*nnzC);
cudaMalloc((void**)&csrValC, sizeof(float)*nnzC);
cusparseScsrgemm(handle, transA, transB, m, n, k,
       descrA, nnzA,
       csrValA, csrRowPtrA, csrColIndA,
       descrB, nnzB,
       csrValB, csrRowPtrB, csrColIndB,
        descrC,
       csrValC, csrRowPtrC, csrColIndC);
```

Several comments on csrgemm():

- Although NN, NT, TN and TT are supported, only the NN version is implemented. For the NT, TN and TT versions, csr2csc() is used to transpose the relevant matrices, followed by a call to the NN version of csrgemm().
- ► The NN version needs working space of size nnzA integers at least.

- ► Only CUSPARSE\_MATRIX\_TYPE\_GENERAL is supported. If either A or B is symmetric or Hermitian, the user must extend the matrix to a full one and reconfigure the MatrixType field descriptor to CUSPARSE\_MATRIX\_TYPE\_GENERAL.
- ▶ Only devices of compute capability 2.0 or above are supported.

### Input

r · ·	
handle	handle to the cuSPARSE library context.
transA	the operation op(A)
transB	the operation op(B)
m	number of rows of sparse matrix op(A) and c.
n	number of columns of sparse matrix op(B) and c.
k	number of columns/rows of sparse matrix op(A) / op(B).
descrA	the descriptor of matrix <b>A</b> . The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.
nnzA	number of nonzero elements of sparse matrix A.
csrValA	<pre><type> array of nnzA(= csrRowPtrA(m) - csrRowPtrA(0)) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $\tilde{m}+1$ elements that contains the start of every row and the end of the last row plus one. $\tilde{m}=m$ if transA == CUSPARSE_OPERATION_NON_TRANSPOSE, otherwise $\tilde{m}=k$ .
csrColIndA	integer array of nnzA (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
descrB	the descriptor of matrix B. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.
nnzB	number of nonzero elements of sparse matrix <b>B</b> .
csrValB	<type> array of nnzB nonzero elements of matrix B.</type>
csrRowPtrB	integer array of $\tilde{k}$ + 1 elements that contains the start of every row and the end of the last row plus one. $\tilde{k} = k$ if transB == CUSPARSE_OPERATION_NON_TRANSPOSE, otherwise $\tilde{k} = n$
csrColIndB	integer array of nnzB column indices of the nonzero elements of matrix B.
descrC	the descriptor of matrix c. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.

### Output

	1
csrValC	<pre><type> array of nnzC (= csrRowPtrC(m) -</type></pre>
	csrRowPtrC(0)) nonzero elements of matrix c.

csrRowPtrC	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndC	<pre>integer array of nnzC(= csrRowPtrC(m) - csrRowPtrC(0) ) column indices of the nonzero elements of matrix c.</pre>
nnzTotalDevHostPtr	total number of nonzero elements in device or host memory. It is equal to (csrRowPtrC(m) - csrRowPtrC(0)).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,k<0; IndexBase of descrA,descrB,descrC is not base-0 or base-1; or alpha or beta is nil )).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 9.3. cusparse<t>csrgemm2()

```
cusparseStatus t
cusparseScsrgemm2 bufferSizeExt(cusparseHandle t handle,
                                int m,
                                int n,
                                int k,
                                const float *alpha,
                                const cusparseMatDescr t descrA,
                                int nnzA,
                                const int *csrRowPtrA,
                                const int *csrColIndA,
                                const cusparseMatDescr t descrB,
                                int nnzB,
                                const int *csrRowPtrB,
                                const int *csrColIndB,
                                const float *beta,
                                const cusparseMatDescr t descrD,
                                int nnzD,
                                const int *csrRowPtrD,
                                const int *csrColIndD,
                                csrgemm2Info t info,
                                size t *pBufferSizeInBytes );
cusparseStatus t
cusparseDcsrgemm2 bufferSizeExt(cusparseHandle t handle,
                                int m,
                                int n,
                                int k,
                                const double *alpha,
                                 const cusparseMatDescr t descrA,
                                 int nnzA,
                                const int *csrRowPtrA,
                                const int *csrColIndA,
                                 const cusparseMatDescr t descrB,
                                 int nnzB,
                                const int *csrRowPtrB,
                                 const int *csrColIndB,
                                const double *beta,
                                 const cusparseMatDescr t descrD,
                                 int nnzD,
                                 const int *csrRowPtrD,
                                const int *csrColIndD,
                                csrgemm2Info t info,
                                 size t *pBufferSizeInBytes );
cusparseStatus t
cusparseCcsrgemm2 bufferSizeExt(cusparseHandle t handle,
                                int m,
                                int n,
                                int k,
                                const cuComplex *alpha,
                                const cusparseMatDescr t descrA,
                                int nnzA,
                                const int *csrRowPtrA,
                                const int *csrColIndA,
                                const cusparseMatDescr t descrB,
                                int nnzB,
                                const int *csrRowPtrB,
                                const int *csrColIndB,
                                const cuComplex *beta,
                                const cusparseMatDescr_t descrD,
                                int nnzD,
                                const int *csrRowPtrD,
                                 const int *csrColIndD.
```

This function performs following matrix-matrix operation:

$$C = alpha * A * B + beta * D$$

where A, B, D and C are m×k, k×n, m×n and m×n sparse matrices (defined in CSR storage format by the three arrays csrValA|csrValB|csrValD|csrValC, csrRowPtrA|csrRowPtrB|csrRowPtrD|csrRowPtrC, and csrColIndA|csrColIndB|csrColIndC respectively.

We provide csrgemm2 as a generalization of csrgemm. It provides more operations in terms of alpha and beta. For example, C = -A\*B+D can be done by csrgemm2.

The csrgemm2 uses alpha and beta to support the following operations:

alpha	beta	operation
NULL	NULL	invalid
NULL	!NULL	C = beta*D, A and B are not used
! NULL	NULL	C = alpha*A*B, D is not used
! NULL	!NULL	C = alpha*A*B + beta*D

The numerical value of alpha and beta only affects the numerical values of C, not its sparsity pattern. For example, if alpha and beta are not zero, the sparsity pattern of C is union of A\*B and D, independent of numerical value of alpha and beta.

The following table shows different operations according to the value of m, n and k

m,n,k	operation
m<0 or n <0 or k<0	invalid
m is 0 or n is 0	do nothing
m >0 and n >0 and k is 0	invalid if beta is zero;
	C = beta*D if beta is not zero.
m >0 and n >0 and k >0	C = beta*D if alpha is zero.
	C = alpha*A*B if beta is zero.
	C = alpha*A*B + beta*D if alpha and beta are not zero.

This function requires the buffer size returned by csrgemm2\_bufferSizeExt(). The address of pBuffer must be multiple of 128 bytes. If it is not, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

The cuSPARSE library adopts a two-step approach to complete sparse matrix. In the first step, the user allocates csrRowPtrC of m+1 elements and uses the function cusparseXcsrgemm2Nnz() to determine csrRowPtrC and the total number of nonzero elements. In the second step, the user gathers nnzC (the number of nonzero elements of matrix C) from either (nnzC=\*nnzTotalDevHostPtr) or (nnzC=csrRowPtrC(m) - csrRowPtrC(0)) and allocates csrValC and csrColIndC of nnzC elements respectively, then finally calls function cusparse[S|D|C|Z]csrgemm2() to evaluate matrix C.

#### The general procedure of **C=-A\*B+D** is as follows:

```
// assume matrices A, B and D are ready.
int baseC, nnzC;
csrgemm2Info t info = NULL;
size t bufferSize;
void *buffer = NULL;
// nnzTotalDevHostPtr points to host memory
int *nnzTotalDevHostPtr = &nnzC;
double alpha = -1.0;
double beta = 1.0;
cusparseSetPointerMode(handle, CUSPARSE POINTER MODE HOST);
// step 1: create an opaque structure
cusparseCreateCsrgemm2Info(&info);
// step 2: allocate buffer for csrgemm2Nnz and csrgemm2
cusparseDcsrqemm2 bufferSizeExt(handle, m, n, k, &alpha,
    descrA, nnzA, csrRowPtrA, csrColIndA,
    descrB, nnzB, csrRowPtrB, csrColIndB,
    descrD, nnzD, csrRowPtrD, csrColIndD,
    &beta.
    info,
    &bufferSize);
cudaMalloc(&buffer, bufferSize);
// step 3: compute csrRowPtrC
cudaMalloc((void**)&csrRowPtrC, sizeof(int)*(m+1));
cusparseXcsrgemm2Nnz(handle, m, n, k,
        descrA, nnzA, csrRowPtrA, csrColIndA,
        descrB, nnzB, csrRowPtrB, csrColIndB,
        descrD, nnzD, csrRowPtrD, csrColIndD,
       descrC, csrRowPtrC, nnzTotalDevHostPtr,
       info, buffer );
if (NULL != nnzTotalDevHostPtr) {
    nnzC = *nnzTotalDevHostPtr;
    cudaMemcpy(&nnzC, csrRowPtrC+m, sizeof(int), cudaMemcpyDeviceToHost);
    cudaMemcpy(&baseC, csrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
    nnzC -= baseC;
// step 4: finish sparsity pattern and value of C
cudaMalloc((void**)&csrColIndC, sizeof(int)*nnzC);
cudaMalloc((void**)&csrValC, sizeof(double)*nnzC);
// Remark: set csrValC to null if only sparsity pattern is required.
cusparseDcsrgemm2(handle, m, n, k, &alpha,
        descrA, nnzA, csrValA, csrRowPtrA, csrColIndA,
        descrB, nnzB, csrValB, csrRowPtrB, csrColIndB,
        descrD, nnzD, csrValD, csrRowPtrD, csrColIndD,
        descrC, csrValC, csrRowPtrC, csrColIndC,
        info, buffer);
// step 5: destroy the opaque structure
cusparseDestroyCsrgemm2Info(info);
```

#### Several comments on csrgemm2():

Only the NN version is supported. For other modes, the user has to transpose **A** or **B** explicitly.

- ► Only CUSPARSE\_MATRIX\_TYPE\_GENERAL is supported. If either A or B is symmetric or Hermitian, the user must extend the matrix to a full one and reconfigure the MatrixType field descriptor to CUSPARSE\_MATRIX\_TYPE\_GENERAL.
- if csrValC is zero, only sparisty pattern of C is calculated.
- ▶ Only devices of compute capability 2.0 or above are supported.

### Input

handle	handle to the cuSPARSE library context.
m	number of rows of sparse matrix A, D and C.
n	number of columns of sparse matrix в, р and с.
k	number of columns/rows of sparse matrix A / B.
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix <b>A</b> . The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.
nnzA	number of nonzero elements of sparse matrix A.
csrValA	<type> array of nnzA nonzero elements of matrix A.</type>
csrRowPtrA	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnzA column indices of the nonzero elements of matrix A.
descrB	the descriptor of matrix B. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.
nnzB	number of nonzero elements of sparse matrix <b>B</b> .
csrValB	<type> array of nnzB nonzero elements of matrix B.</type>
csrRowPtrB	integer array of k+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndB	integer array of nnzB column indices of the nonzero elements of matrix B.
descrD	the descriptor of matrix D. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.
nnzD	number of nonzero elements of sparse matrix D.
csrValD	<type> array of nnzD nonzero elements of matrix D.</type>
csrRowPtrD	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndD	integer array of nnzD column indices of the nonzero elements of matrix D.
beta	<type> scalar used for multiplication.</type>

descrC	the descriptor of matrix c. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL only.
info	structure with information used in csrgemm2Nnz and csrgemm2.
pBuffer	buffer allocated by the user; the size is returned by csrgemm2_bufferSizeExt.

# Output

csrValC	<type> array of nnzC nonzero elements of matrix C.</type>
csrRowPtrC	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndC	integer array of nnzc column indices of the nonzero elements of matrix c.
pBufferSizeInBytes	number of bytes of the buffer used in csrgemm2Nnnz and csrgemm2.
nnzTotalDevHostPtr	total number of nonzero elements in device or host memory. It is equal to (csrRowPtrC(m) - csrRowPtrC(0)).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,k<0; IndexBase Of descrA,descrB,descrD,descrC is not base-0 or base-1).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# Chapter 10. CUSPARSE PRECONDITIONERS REFERENCE

This chapter describes the routines that implement different preconditioners.

In particular, the incomplete factorizations are implemented in two phases. First, during the analysis phase, the sparse triangular matrix is analyzed to determine the dependencies between its elements by calling the appropriate <code>csrsv\_analysis()</code> function. The analysis is specific to the sparsity pattern of the given matrix and the selected <code>cusparseOperation\_t</code> type. The information from the analysis phase is stored in the parameter of type <code>cusparseSolveAnalysisInfo\_t</code> that has been initialized previously with a call to <code>cusparseCreateSolveAnalysisInfo()</code>.

Second, during the numerical factorization phase, the given coefficient matrix is factorized using the information stored in the **cusparseSolveAnalysisInfo\_t** parameter by calling the appropriate **csrilu0()** or **csric0()** function.

The analysis phase is shared across the sparse triangular solve, and the incomplete factorization and must be performed only once. The resulting information can be passed to the numerical factorization and the sparse triangular solve multiple times.

Finally, once the incomplete factorization and all the sparse triangular solves have completed, the opaque data structure pointed to by the cusparseSolveAnalysisInfo\_t parameter can be released by calling cusparseDestroySolveAnalysisInfo().

# 10.1. cusparse<t>csric0()

```
cusparseStatus t
cusparseScsricO(cusparseHandle t handle,
                cusparseOperation t trans,
                int m,
                const cusparseMatDescr t descrA,
                float *csrValM,
                const int *csrRowPtrA,
                const int *csrColIndA,
                cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseDcsricO(cusparseHandle t handle,
                cusparseOperation t trans,
                int m,
                const cusparseMatDescr t descrA,
                double *csrValM,
                const int *csrRowPtrA,
                const int *csrColIndA,
                cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseCcsric0(cusparseHandle t handle,
                cusparseOperation t trans,
                int m,
                const cusparseMatDescr t descrA,
                cuComplex *csrValM,
                const int *csrRowPtrA,
                const int *csrColIndA,
                cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseZcsric0(cusparseHandle t handle,
                cusparseOperation t trans,
                int m,
                const cusparseMatDescr t descrA,
                cuDoubleComplex *csrValM,
                const int *csrRowPtrA,
                const int *csrColIndA,
                cusparseSolveAnalysisInfo t info)
```

This function computes the incomplete-Cholesky factorization with 0 fill-in and no pivoting:

$$op(A) \approx R^T R$$

A is an  $m \times m$  Hermitian/symmetric positive definite sparse matrix that is defined in CSR storage format by the three arrays csrValM, csrRowPtrA, and csrColIndA; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

Notice that only a lower or upper Hermitian/symmetric part of the matrix  $\mathbf{A}$  is actually stored. It is overwritten by the lower or upper triangular factors  $R^T$  and R, respectively.

A call to this routine must be preceded by a call to the csrsv analysis() routine.

The matrix descriptor for csrsv\_analysis() and csric0() must be the same. Otherwise, runtime error would occur.

This function requires some extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

#### Input

handle	handle to the cuSPARSE library context.
trans	the operation op (A).
m	number of rows and columns of matrix A.
descrA	the descriptor of matrix A. The supported matrix types are CUSPARSE_MATRIX_TYPE_SYMMETRIC and CUSPARSE_MATRIX_TYPE_HERMITIAN.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValM	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $\mathfrak{m}$ +1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
info	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).

#### Output

csrValM	<type> matrix containing the incomplete-Cholesky</type>
	lower or upper triangular factor.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.2. cusparse<t>csric02\_bufferSize()

```
cusparseStatus t
cusparseScsric02 bufferSize(cusparseHandle t handle,
                             int m,
                            int nnz,
                            const cusparseMatDescr t descrA,
                            float *csrValA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            csric02Info t info,
                            int *pBufferSizeInBytes);
cusparseStatus t
cusparseDcsric02 bufferSize(cusparseHandle t handle,
                            int m,
                            int nnz,
                            const cusparseMatDescr t descrA,
                            double *csrValA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            csric02Info t info,
                            int *pBufferSizeInBytes);
cusparseStatus t
cusparseCcsric02 bufferSize(cusparseHandle t handle,
                             int m,
                            int nnz,
                            const cusparseMatDescr t descrA,
                            cuComplex *csrValA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            csric02Info t info,
                            int *pBufferSizeInBytes);
cusparseStatus t
cusparseZcsric02 bufferSize(cusparseHandle t handle,
                             int m,
                            int nnz,
                            const cusparseMatDescr t descrA,
                            cuDoubleComplex *csrValA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            csric02Info t info,
                             int *pBufferSizeInBytes);
```

This function returns size of buffer used in computing the incomplete-Cholesky factorization with 0 fill-in and no pivoting:

$$A \approx LL^H$$

A is an mxm sparse matrix that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA.

The buffer size depends on dimension **m** and **nnz**, the number of nonzeros of the matrix. If the user changes the matrix, it is necessary to call **csric02\_bufferSize()** again to have the correct buffer size; otherwise, a segmentation fault may occur.

### Input

handle	handle to the cuSPARSE library context.
m	number of rows and columns of matrix A.
nnz	number of nonzeros of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $\mathfrak{m}$ +1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.

### Output

info	record internal states based on different algorithms.
pBufferSizeInBytes	number of bytes of the buffer used in csric02_analysis() and csric02().

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,nnz<=0), base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.3. cusparse<t>csric02\_analysis()

```
cusparseStatus t
cusparseScsric02 analysis(cusparseHandle t handle,
                           int m,
                           int nnz,
                           const cusparseMatDescr t descrA,
                           const float *csrValA,
                           const int *csrRowPtrA,
                          const int *csrColIndA,
                          csric02Info t info,
                          cusparseSolvePolicy t policy,
                           void *pBuffer);
cusparseStatus t
cusparseDcsric\overline{0}2 analysis(cusparseHandle t handle,
                           int m,
                           int nnz,
                           const cusparseMatDescr t descrA,
                           const double *csrValA,
                           const int *csrRowPtrA,
                           const int *csrColIndA,
                           csric02Info t info,
                           cusparseSolvePolicy t policy,
                           void *pBuffer);
cusparseStatus t
cusparseCcsric02_analysis(cusparseHandle_t handle,
                           int m,
                           int nnz,
                           const cusparseMatDescr t descrA,
                           const cuComplex *csrValA,
                           const int *csrRowPtrA,
                           const int *csrColIndA,
                           csric02Info t info,
                           cusparseSolvePolicy t policy,
                           void *pBuffer);
cusparseStatus t
cusparseZcsric02 analysis(cusparseHandle t handle,
                           int m,
                           int nnz,
                           const cusparseMatDescr t descrA,
                           const cuDoubleComplex *csrValA,
                           const int *csrRowPtrA,
                           const int *csrColIndA,
                           csric02Info t info,
                           cusparseSolvePolicy_t policy,
                           void *pBuffer);
```

This function performs the analysis phase of the incomplete-Cholesky factorization with 0 fill-in and no pivoting:

 $A \approx II^H$ 

A is an mxm sparse matrix that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA.

This function requires a buffer size returned by csric02\_bufferSize(). The address of pBuffer must be multiple of 128 bytes. If not, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

Function csric02\_analysis() reports a structural zero and computes level information stored in the opaque structure info. The level information can extract more parallelism during incomplete Cholesky factorization. However csric02() can be done without level information. To disable level information, the user must specify the policy of csric02 analysis() and csric02() as CUSPARSE SOLVE POLICY NO LEVEL.

Function csric02\_analysis() always reports the first structural zero, even if the policy is CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL. The user needs to call cusparseXcsric02 zeroPivot() to know where the structural zero is.

It is the user's choice whether to call <code>csric02()</code> if <code>csric02\_analysis()</code> reports a structural zero. In this case, the user can still call <code>csric02()</code>, which will return a numerical zero at the same position as the structural zero. However the result is meaningless.

#### Input

handle	handle to the cuSPARSE library context.
m	number of rows and columns of matrix ${f a}$ .
nnz	number of nonzeros of matrix A.
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m + 1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
info	structure initialized using cusparseCreateCsric02Info().
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user; the size is returned by csric02_bufferSize().

#### Output

info	number of bytes of the buffer used in
	csric02_analysis() and csric02().

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,nnz<=0), base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.4. cusparse<t>csric02()

```
cusparseStatus t
cusparseScsric02(cusparseHandle t handle,
                 int m,
                 int nnz,
                 const cusparseMatDescr t descrA,
                 float *csrValA valM,
                 const int *csrRowPtrA,
                 const int *csrColIndA,
                 csric02Info t info,
                 cusparseSolvePolicy t policy,
                 void *pBuffer);
cusparseStatus t
cusparseDcsric02(cusparseHandle_t handle,
                 int m,
                 int nnz,
                 const cusparseMatDescr t descrA,
                 double *csrValA_valM,
                 const int *csrRowPtrA,
                 const int *csrColIndA,
                 csric02Info t info,
                 cusparseSolvePolicy_t policy,
                 void *pBuffer);
cusparseStatus t
cusparseCcsric02(cusparseHandle t handle,
                 int m,
                 int nnz,
                 const cusparseMatDescr t descrA,
                 cuComplex *csrValA valM,
                 const int *csrRowPtrA,
                 const int *csrColIndA,
                 csric02Info t info,
                 cusparseSolvePolicy t policy,
                 void *pBuffer);
cusparseStatus t
cusparseZcsric02(cusparseHandle t handle,
                 int m,
                 int nnz,
                 const cusparseMatDescr t descrA,
                 cuDoubleComplex *csrValA valM,
                 const int *csrRowPtrA,
                 const int *csrColIndA,
                 csric02Info t info,
                 cusparseSolvePolicy_t policy,
                 void *pBuffer);
```

This function performs the solve phase of the computing the incomplete-Cholesky factorization with 0 fill-in and no pivoting:

 $A \approx LL^H$ 

This function requires a buffer size returned by csric02\_bufferSize(). The address of pBuffer must be a multiple of 128 bytes. If not, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

Although csric02() can be done without level information, the user still needs to be aware of consistency. If csric02\_analysis() is called with policy CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL, csric02() can be run with or without levels. On the other hand, if csric02\_analysis() is called with CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL, csric02() can only accept CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL; otherwise, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

Function csric02() reports the first numerical zero, including a structural zero. The user must call cusparseXcsric02\_zeroPivot() to know where the numerical zero is.

Function <code>csric02()</code> only takes the lower triangular part of matrix <code>A</code> to perform factorization. The matrix type must be <code>CUSPARSE\_MATRIX\_TYPE\_GENERAL</code>, the fill mode and diagonal type are ignored, and the strictly upper triangular part is ignored and never touched. It does not matter if <code>A</code> is Hermitian or not. In other words, from the point of view of <code>csric02()</code> <code>A</code> is Hermitian and only the lower triangular part is provided.



In practice, a positive definite matrix may not have incomplete cholesky factorization. To the best of our knowledge, only matrix M can guarantee the existence of incomplete cholesky factorization. If csric02() failed cholesky factorization and reported a numerical zero, it is possible that incomplete cholesky factorization does not exist.

For example, suppose  $\mathbf{A}$  is a real  $\mathbf{m} \times \mathbf{m}$  matrix, the following code solves the precondition system  $\mathbf{M} \star \mathbf{y} = \mathbf{x}$  where  $\mathbf{M}$  is the product of Cholesky factorization  $\mathbf{L}$  and its transpose.

#### $M = LL^H$

```
// Suppose that A is m x m sparse matrix represented by CSR format,
// Assumption:
// - handle is already created by cusparseCreate(),
// - (d_csrRowPtr, d_csrColInd, d_csrVal) is CSR of A on device memory,
// - d\bar{x} is right hand side vector on device memory,
// - d_y is solution vector on device memory.
// - d^{-}z is intermediate result on device memory.
cusparseMatDescr t descr M = 0;
cusparseMatDescr_t descr_L = 0;
csric02Info_t info_M = 0;
csrsv2Info_t info_L = 0;
csrsv2Info_t info_Lt = 0;
int pBufferSize M;
int pBufferSize L;
int pBufferSize_Lt;
int pBufferSize;
void *pBuffer = 0;
int structural zero;
int numerical zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy_M = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_L = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_Lt = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation t trans L = CUSPARSE OPERATION NON TRANSPOSE;
const cusparseOperation_t trans_Lt = CUSPARSE_OPERATION_TRANSPOSE;
// step 1: create a descriptor which contains
// - matrix M is base-1
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has non-unit diagonal
cusparseCreateMatDescr(&descr_M);
cusparseSetMatIndexBase(descr_M, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr M, CUSPARSE MATRIX TYPE GENERAL);
cusparseCreateMatDescr(&descr_L);
cusparseSetMatIndexBase(descr_L, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_L, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_L, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr_L, CUSPARSE_DIAG_TYPE_NON_UNIT);
// step 2: create a empty info structure
// we need one info for csric02 and two info's for csrsv2
cusparseCreateCsricO2Info(&info M);
cusparseCreateCsrsv2Info(&info L);
cusparseCreateCsrsv2Info(&info Lt);
// step 3: query how much memory used in csric02 and csrsv2, and allocate the
buffer
cusparseDcsric02_bufferSize(handle, m, nnz,
    descr_M, d_csrVal, d_csrRowPtr, d_csrColInd, info_M, &bufferSize_M);
cusparseDcsrsv2 bufferSize(handle, trans_L, m, nnz,
    descr L, d csrVal, d csrRowPtr, d csrColInd, info L, &pBufferSize L);
cusparseDcsrsv2 bufferSize(handle, trans_Lt, m, nnz,
    descr L, d csrVal, d csrRowPtr, d csrColInd, info Lt,&pBufferSize Lt);
pBufferSize = max(bufferSize M, max(pBufferSize L, pBufferSize Lt));
// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);
// step 4: perform analysis of incomplete Cholesky on M
// perform analysis of triangular solve on L
           perform analysis of triangular solve on L'
// The lower triangular part of M has the same sparsity pattern as L, so
// we can do analysis of csric02 and csrsv2 simultaneously.
cusparseDcsric02_analysis(handle, m, nnz, descr_M,
    d_csrVal, d_csrRowPtr, d_csrColInd, info_M,
    policy M, pBuffer);
status = cusparseXcsric02 zeroPivot(handle, info M, &structural zero);
```

### Input

	1
handle	handle to the cuSPARSE library context.
m	number of rows and columns of matrix A.
nnz	number of nonzeros of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA_valM	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $\mathfrak{m}$ +1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
info	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user; the size is returned by csric02_bufferSize().

### Output

csrValA_valM	<type> matrix containing the incomplete-Cholesky</type>
	lower triangular factor.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,nnz<=0), base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.5. cusparseXcsric02\_zeroPivot()

If the returned error code is **CUSPARSE\_STATUS\_ZERO\_PIVOT**, **position=j** means **A**(j, j) has either a structural zero or a numerical zero; otherwise, **position=-1**.

The **position** can be 0-based or 1-based, the same as the matrix.

Function cusparseXcsric02\_zeroPivot() is a blocking call. It calls cudaDeviceSynchronize() to make sure all previous kernels are done.

The **position** can be in the host memory or device memory. The user can set proper mode with **cusparseSetPointerMode()**.

#### Input

handle	handle to the cuSPARSE library context.
info	<pre>info contains structural zero or numerical zero if the user already called csric02_analysis() or csric02().</pre>

#### Output

•	if no structural or numerical zero, position is -1; otherwise, if A(j,j) is missing or L(j,j) is zero,
	position=j.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	info is not valid.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 10.6. cusparse<t>csrilu0()

```
cusparseStatus t
cusparseScsrilu0(cusparseHandle t handle,
                 cusparseOperation t trans,
                 int m,
                 const cusparseMatDescr t descrA,
                 float *csrValM,
                 const int *csrRowPtrA,
                 const int *csrColIndA,
                 cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseDcsrilu0(cusparseHandle t handle,
                 cusparseOperation t trans,
                 int m,
                 const cusparseMatDescr_t descrA,
                 double *csrValM,
                 const int *csrRowPtrA,
                 const int *csrColIndA,
                 cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseCcsrilu0(cusparseHandle t handle,
                 cusparseOperation t trans,
                 int m,
                 const cusparseMatDescr t descrA,
                 cuComplex *csrValM,
                 const int *csrRowPtrA,
                 const int *csrColIndA,
                 cusparseSolveAnalysisInfo t info)
cusparseStatus t
cusparseZcsrilu0(cusparseHandle t handle,
                 cusparseOperation t trans,
                 int m,
                 const cusparseMatDescr t descrA,
                 cuDoubleComplex *csrValM,
                 const int *csrRowPtrA,
                 const int *csrColIndA,
                 cusparseSolveAnalysisInfo t info)
```

This function computes the incomplete-LU factorization with 0 fill-in and no pivoting:

$$op(A) \approx LU$$

A is an  $m \times m$  sparse matrix that is defined in CSR storage format by the three arrays csrValM, csrRowPtrA, and csrColIndA; and

$$op(A) = \begin{cases} A & \text{if trans} == \text{CUSPARSE\_OPERATION\_NON\_TRANSPOSE} \\ A^T & \text{if trans} == \text{CUSPARSE\_OPERATION\_TRANSPOSE} \\ A^H & \text{if trans} == \text{CUSPARSE\_OPERATION\_CONJUGATE\_TRANSPOSE} \end{cases}$$

Notice that the diagonal of lower triangular factor L is unitary and need not be stored. Therefore, the input matrix is overwritten with the resulting lower and upper triangular factors L and U, respectively.

A call to this routine must be preceded by a call to the csrsv analysis() routine.

The matrix descriptor for <code>csrsv\_analysis()</code> and <code>csrilu0()</code> must be the same. Otherwise, runtime error would occur.

This function requires some extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

### Input

handle	handle to the cuSPARSE library context.
trans	the operation op (A).
m	number of rows and columns of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValM	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
info	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).

### Output

csrValM	<type> matrix containing the incomplete-LU lower</type>
	and upper triangular factors.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.7. cusparseCsrilu0Ex()

This function is an extended version of **cusparse<t>csrilu0()**. For detailed description of the functionality, see **cusparse<t>csrilu0()**.

This function does not support half-precision execution type, but it supports half-precision IO with single precision execution.

#### Input specifically required by cusparseCsrilu0Ex

csrSortedValA_ValMtype	Data type of csrSortedValA_ValM.
executiontype	Data type used for computation.

# 10.8. cusparse<t>csrilu02\_numericBoost()

```
cusparseStatus t
cusparseScsrilu02 numericBoost(cusparseHandle t handle,
                                csrilu02Info t info,
                                int enable boost,
                                double *to\overline{l},
                                float *boost val);
cusparseStatus t
cusparseDcsrilu02 numericBoost(cusparseHandle t handle,
                               csrilu02Info t info,
                                int enable boost,
                                double *tol,
                                double *boost val);
cusparseStatus t
cusparseCcsrilu02_numericBoost(cusparseHandle_t handle,
                               csrilu02Info t info,
                                int enable boost,
                                double *tol,
                                cuComplex *boost val);
cusparseStatus t
cusparseZcsrilu02 numericBoost(cusparseHandle_t handle,
                               csrilu02Info t info,
                                int enable boost,
                                double *tol,
                                cuDoubleComplex *boost val);
```

The user can use a boost value to replace a numerical value in incomplete LU factorization. The tol is used to determine a numerical zero, and the boost\_val is used to replace a numerical zero. The behavior is

```
if tol >= fabs(A(j,j)), then A(j,j)=boost_val.
```

To enable a boost value, the user has to set parameter <code>enable\_boost</code> to 1 before calling <code>csrilu02()</code>. To disable a boost value, the user can call <code>csrilu02\_numericBoost()</code> again with parameter <code>enable\_boost=0</code>.

If enable\_boost=0, tol and boost\_val are ignored.

Both tol and boost\_val can be in the host memory or device memory. The user can set the proper mode with cusparseSetPointerMode().

#### Input

handle	handle to the cuSPARSE library context.
info	structure initialized using cusparseCreateCsrilu02Info().
enable_boost	disable boost by enable_boost=0; otherwise, boost is enabled.
tol	tolerance to determine a numerical zero.

boost_val	boost value to replace a numerical zero.
-----------	--

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	info or pointer mode is not valid.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 10.9. cusparse<t>csrilu02\_bufferSize()

```
cusparseStatus t
cusparseScsrilu02 bufferSize(cusparseHandle t handle,
                             int m,
                             const cusparseMatDescr t descrA,
                             float *csrValA,
                             const int *csrRowPtrA,
                             const int *csrColIndA,
                             csrilu02Info t info,
                             int *pBufferSizeInBytes);
cusparseStatus t
cusparseDcsrilu02 bufferSize(cusparseHandle t handle,
                             int m,
                             int nnz,
                             const cusparseMatDescr t descrA,
                             double *csrValA,
                             const int *csrRowPtrA,
                             const int *csrColIndA,
                             csrilu02Info t info,
                             int *pBufferSizeInBytes);
cusparseStatus t
cusparseCcsrilu02 bufferSize(cusparseHandle t handle,
                             int m,
                             int nnz,
                             const cusparseMatDescr t descrA,
                             cuComplex *csrValA,
                             const int *csrRowPtrA,
                             const int *csrColIndA,
                             csrilu02Info t info,
                             int *pBufferSizeInBytes);
cusparseStatus t
cusparseZcsrilu02 bufferSize(cusparseHandle t handle,
                             int m,
                             int nnz,
                             const cusparseMatDescr t descrA,
                             cuDoubleComplex *csrValA,
                             const int *csrRowPtrA,
                             const int *csrColIndA,
                             csrilu02Info_t info,
                             int *pBufferSizeInBytes);
```

This function returns size of the buffer used in computing the incomplete-LU factorization with 0 fill-in and no pivoting:

A≈LU

A is an mxm sparse matrix that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA.

The buffer size depends on the dimension m and nnz, the number of nonzeros of the matrix. If the user changes the matrix, it is necessary to call csrilu02\_bufferSize() again to have the correct buffer size; otherwise, a segmentation fault may occur.

#### Input

handle	handle to the cuSPARSE library context.
m	number of rows and columns of matrix A.
nnz	number of nonzeros of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.

#### Output

info	record internal states based on different algorithms.
pBufferSizeInBytes	number of bytes of the buffer used in csrilu02_analysis() and csrilu02().

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,nnz<=0), base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.10. cusparse<t>csrilu02\_analysis()

```
cusparseStatus t
cusparseScsrilu02 analysis(cusparseHandle t handle,
                            int m,
                            int nnz,
                            const cusparseMatDescr t descrA,
                           const float *csrValA,
                           const int *csrRowPtrA,
                           const int *csrColIndA,
                           csrilu02Info t info,
                            cusparseSolvePolicy t policy,
                            void *pBuffer);
cusparseStatus t
cusparseDcsrilu02_analysis(cusparseHandle_t handle,
                            int m,
                            int nnz,
                            const cusparseMatDescr t descrA,
                            const double *csrValA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            csrilu02Info t info,
                            cusparseSolvePolicy_t policy,
                            void *pBuffer);
cusparseStatus t
cusparseCcsrilu02_analysis(cusparseHandle_t handle,
                            int m,
                            int nnz,
                            const cusparseMatDescr t descrA,
                            const cuComplex *csrValA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            csrilu02Info t info,
                            cusparseSolvePolicy t policy,
                            void *pBuffer);
cusparseStatus t
cusparseZcsrilu02_analysis(cusparseHandle_t handle,
                            int m,
                            int nnz,
                            const cusparseMatDescr t descrA,
                            const cuDoubleComplex \overline{*}csrValA,
                            const int *csrRowPtrA,
                            const int *csrColIndA,
                            csrilu02Info t info,
                            cusparseSolvePolicy t policy,
                            void *pBuffer);
```

This function performs the analysis phase of the incomplete-LU factorization with 0 fill-in and no pivoting:

A≈LU

A is an mxm sparse matrix that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA.

This function requires the buffer size returned by csrilu02\_bufferSize(). The address of pBuffer must be a multiple of 128 bytes. If not, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

Function csrilu02\_analysis() reports a structural zero and computes level information stored in the opaque structure info. The level information can extract more parallelism during incomplete LU factorization; however csrilu02() can be done without level information. To disable level information, the user must specify the policy of csrilu02() as CUSPARSE SOLVE POLICY NO LEVEL.

It is the user's choice whether to call <code>csrilu02()</code> if <code>csrilu02\_analysis()</code> reports a structural zero. In this case, the user can still call <code>csrilu02()</code>, which will return a numerical zero at the same position as the structural zero. However the result is meaningless.

#### Input

handle	handle to the cuSPARSE library context.
m	number of rows and columns of matrix ${f a}$ .
nnz	number of nonzeros of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $\mathfrak{m}$ +1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
info	structure initialized using cusparseCreateCsrilu02Info().
policy	the supported policies are  CUSPARSE_SOLVE_POLICY_NO_LEVEL and  CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user, the size is returned by csrilu02_bufferSize().

### Output

structure filled with information collected during the analysis phase (that should be passed to the
solve phase unchanged).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,nnz<=0), base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.11. cusparse<t>csrilu02()

```
cusparseStatus t
cusparseScsrilu02(cusparseHandle t handle,
                   int m,
                   int nnz,
                   const cusparseMatDescr t descrA,
                   float *csrValA valM,
                   const int *csrRowPtrA,
                   const int *csrColIndA,
                  csrilu02Info t info,
                  cusparseSolvePolicy t policy,
                  void *pBuffer);
cusparseStatus t
cusparseDcsrilu02(cusparseHandle_t handle,
                   int m,
                   int nnz,
                   const cusparseMatDescr t descrA,
                   double *csrValA valM,
                   const int *csrRowPtrA,
                   const int *csrColIndA,
                   csrilu02Info t info,
                  cusparseSolvePolicy_t policy,
                   void *pBuffer);
cusparseStatus t
cusparseCcsrilu02(cusparseHandle t handle,
                   int m,
                   int nnz,
                   const cusparseMatDescr t descrA,
                  cuComplex *csrValA_valM,
const int *csrRowPtrA,
                   const int *csrColIndA,
                   csrilu02Info t info,
                   cusparseSolvePolicy t policy,
                   void *pBuffer);
cusparseStatus t
cusparseZcsrilu02(cusparseHandle t handle,
                   int m,
                   int nnz,
                   const cusparseMatDescr t descrA,
                   cuDoubleComplex *csrValA valM,
                   const int *csrRowPtrA,
                   const int *csrColIndA,
                   csrilu02Info t info,
                   cusparseSolvePolicy t policy,
                   void *pBuffer);
```

This function performs the solve phase of the incomplete-LU factorization with 0 fill-in and no pivoting:

A≈LU

A is an mxm sparse matrix that is defined in CSR storage format by the three arrays csrValA valM, csrRowPtrA, and csrColIndA.

This function requires a buffer size returned by csrilu02\_bufferSize(). The address of pBuffer must be a multiple of 128 bytes. If not, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

The matrix type must be **CUSPARSE\_MATRIX\_TYPE\_GENERAL**. The fill mode and diagonal type are ignored.

Although csrilu02() can be done without level information, the user still needs to be aware of consistency. If csrilu02\_analysis() is called with policy CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL, csrilu02() can be run with or without levels. On the other hand, if csrilu02\_analysis() is called with CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL, csrilu02() can only accept CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL; otherwise, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

Function csrilu02() reports the first numerical zero, including a structural zero. The user must call cusparseXcsrilu02\_zeroPivot() to know where the numerical zero is.

For example, suppose **A** is a real  $\mathbf{m} \times \mathbf{m}$  matrix, the following code solves precondition system  $\mathbf{M} \times \mathbf{y} = \mathbf{x}$  where **M** is the product of LU factors **L** and **U**.

```
// Suppose that A is m x m sparse matrix represented by CSR format,
// Assumption:
// - handle is already created by cusparseCreate(),
// - (d_csrRowPtr, d_csrColInd, d_csrVal) is CSR of A on device memory,
// - d x is right hand side vector on device memory,
// - d_y is solution vector on device memory.
// - d z is intermediate result on device memory.
cusparseMatDescr t descr M = 0;
cusparseMatDescr t descr L = 0;
cusparseMatDescr_t descr_U = 0;
csrilu02Info_t info_M = 0;
csrsv2Info_t info_L = 0;
csrsv2Info_t info_U = 0;
int pBufferSize M;
int pBufferSize L;
int pBufferSize_U;
int pBufferSize;
void *pBuffer = 0;
int structural zero;
int numerical zero;
const double alpha = 1.;
const cusparseSolvePolicy t policy M = CUSPARSE SOLVE POLICY NO LEVEL;
const cusparseSolvePolicy t policy L = CUSPARSE SOLVE POLICY NO LEVEL;
const cusparseSolvePolicy_t policy_U = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation t trans L = CUSPARSE OPERATION NON TRANSPOSE;
const cusparseOperation t trans U = CUSPARSE OPERATION NON TRANSPOSE;
// step 1: create a descriptor which contains
// - matrix M is base-1
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has unit diagonal
// - matrix U is base-1
// - matrix U is upper triangular
// - matrix U has non-unit diagonal
cusparseCreateMatDescr(&descr_M);
cusparseSetMatIndexBase(descr_M, CUSPARSE_INDEX BASE ONE);
cusparseSetMatType(descr M, CUSPARSE MATRIX TYPE GENERAL);
cusparseCreateMatDescr(&descr L);
cusparseSetMatIndexBase(descr L, CUSPARSE INDEX BASE ONE);
cusparseSetMatType(descr_L, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_L, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr_L, CUSPARSE_DIAG_TYPE_UNIT);
cusparseCreateMatDescr(&descr_U);
cusparseSetMatIndexBase(descr_U, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr U, CUSPARSE MATRIX TYPE GENERAL);
cusparseSetMatFillMode(descr_U, CUSPARSE_FILL_MODE_UPPER);
cusparseSetMatDiagType(descr U, CUSPARSE DIAG TYPE NON UNIT);
// step 2: create a empty info structure
// we need one info for csrilu02 and two info's for csrsv2
cusparseCreateCsrilu02Info(&info M);
cusparseCreateCsrsv2Info(&info L);
cusparseCreateCsrsv2Info(&info U);
// step 3: query how much memory used in csrilu02 and csrsv2, and allocate the
cusparseDcsrilu02 bufferSize(handle, m, nnz,
    descr M, d csrVal, d csrRowPtr, d csrColInd, info M, &pBufferSize M);
cusparseDcsrsv2_bufferSize(handle, trans_L, m, nnz,
descr_L, d_csrVal, d_csrRowPtr, d_csrColInd, info_L, &pBufferSize_L);
cusparseDcsrsv2_bufferSize(handle, trans_U, m, nnz,
    descr_U, d_csrVal, d_csrRowPtr, d_csrColInd, info_U, &pBufferSize_U);
pBufferSize = max(pBufferSize M, max(pBufferSize L, pBufferSize U));
// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);
```

## Input

handle	handle to the cuSPARSE library context.
m	number of rows and columns of matrix A.
nnz	number of nonzeros of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA_valM	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $\mathfrak{m}$ +1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
info	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user; the size is returned by csrilu02_bufferSize().

### Output

csrValA_valM	<type> matrix containing the incomplete-LU lower</type>
	and upper triangular factors.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,nnz<=0), base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.12. cusparseXcsrilu02\_zeroPivot()

If the returned error code is **CUSPARSE\_STATUS\_ZERO\_PIVOT**, **position=j** means **A**(j, j) has either a structural zero or a numerical zero; otherwise, **position=-1**.

The **position** can be 0-based or 1-based, the same as the matrix.

Function cusparseXcsrilu02\_zeroPivot() is a blocking call. It calls cudaDeviceSynchronize() to make sure all previous kernels are done.

The **position** can be in the host memory or device memory. The user can set proper mode with **cusparseSetPointerMode()**.

#### Input

handle	handle to the cuSPARSE library context.
info	<pre>info contains structural zero or numerical zero if the user already called csrilu02_analysis() or csrilu02().</pre>

#### Output

<del>-</del>	if no structural or numerical zero, position is -1; otherwise if A(j,j) is missing or U(j,j) is zero,
	position=j.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	info is not valid.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 10.13. cusparse<t>bsric02\_bufferSize()

```
cusparseStatus t
cusparseSbsric02 bufferSize(cusparseHandle t handle,
                             cusparseDirection t dirA,
                             int mb,
                             int nnzb,
                             const cusparseMatDescr t descrA,
                             float *bsrValA,
                             const int *bsrRowPtrA,
                             const int *bsrColIndA,
                             int blockDim,
                             bsric02Info t info,
                             int *pBufferSizeInBytes);
cusparseStatus t
cusparseDbsric\overline{0}2_bufferSize(cusparseHandle_t handle,
                             cusparseDirection t dirA,
                             int mb,
                             int nnzb,
                             const cusparseMatDescr t descrA,
                             double *bsrValA,
                             const int *bsrRowPtrA,
                             const int *bsrColIndA,
                             int blockDim,
                             bsric02Info t info,
                             int *pBufferSizeInBytes);
cusparseStatus t
cusparseCbsric02 bufferSize(cusparseHandle t handle,
                             cusparseDirection t dirA,
                             int mb,
                             int nnzb,
                             const cusparseMatDescr t descrA,
                             cuComplex *bsrValA,
const int *bsrRowPtrA,
                             const int *bsrColIndA,
                             int blockDim,
                             bsric02Info t info,
                             int *pBufferSizeInBytes);
cusparseStatus t
cusparseZbsric02 bufferSize(cusparseHandle t handle,
                             cusparseDirection t dirA,
                             int mb,
                             int nnzb,
                             const cusparseMatDescr t descrA,
                             cuDoubleComplex *bsrValA,
                             const int *bsrRowPtrA,
                             const int *bsrColIndA,
                             int blockDim,
                             bsric02Info t info,
                             int *pBufferSizeInBytes);
```

This function returns the size of a buffer used in computing the incomplete-Cholesky factorization with 0 fill-in and no pivoting

$$A \approx LL^H$$

A is an (mb\*blockDim) \* (mb\*blockDim) sparse matrix that is defined in BSR storage format by the three arrays bsrValA, bsrRowPtrA, and bsrColIndA.

The buffer size depends on the dimensions of mb, blockDim, and the number of nonzero blocks of the matrix nnzb. If the user changes the matrix, it is necessary to call bsric02\_bufferSize() again to have the correct buffer size; otherwise, a segmentation fault may occur.

#### Input

handle	handle to the cuSPARSE library context.
dirA	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
mb	number of block rows and block columns of matrix A.
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<pre><type> array of nnzb(= bsrRowPtrA(mb) - bsrRowPtrA(0) ) nonzero blocks of matrix A.</type></pre>
bsrRowPtrA	integer array of mb +1 elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A, larger than zero.

### Output

info	record internal states based on different algorithms.
pBufferSizeInBytes	number of bytes of the buffer used in bsric02_analysis() and bsric02().

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb,nnzb<=0); the base index is not 0 or 1.

CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.14. cusparse<t>bsric02\_analysis()

```
cusparseStatus t
cusparseSbsric02 analysis(cusparseHandle t handle,
                          cusparseDirection t dirA,
                          int mb,
                          int nnzb,
                          const cusparseMatDescr t descrA,
                          const float *bsrValA,
                          const int *bsrRowPtrA,
                          const int *bsrColIndA,
                          int blockDim,
                          bsric02Info t info,
                          cusparseSolvePolicy t policy,
                          void *pBuffer);
cusparseStatus t
cusparseDbsric02_analysis(cusparseHandle_t handle,
                          cusparseDirection t dirA,
                          int mb,
                          int nnzb,
                          const cusparseMatDescr t descrA,
                          const double *bsrValA,
                          const int *bsrRowPtrA,
                          const int *bsrColIndA,
                          int blockDim,
                          bsric02Info t info,
                          cusparseSolvePolicy_t policy,
                          void *pBuffer);
cusparseStatus t
cusparseCbsric02 analysis(cusparseHandle t handle,
                          cusparseDirection t dirA,
                          int mb,
                          int nnzb,
                          const cusparseMatDescr t descrA,
                          const cuComplex *bsrValA,
                          const int *bsrRowPtrA,
                          const int *bsrColIndA,
                          int blockDim,
                          bsric02Info t info,
                          cusparseSolvePolicy_t policy,
                          void *pBuffer);
cusparseStatus t
cusparseZbsric02 analysis(cusparseHandle t handle,
                          cusparseDirection_t dirA,
                          int mb,
                          int nnzb,
                          const cusparseMatDescr t descrA,
                          const cuDoubleComplex *bsrValA,
                          const int *bsrRowPtrA,
                          const int *bsrColIndA,
                          int blockDim,
                          bsric02Info t info,
                          cusparseSolvePolicy_t policy,
                          void *pBuffer);
```

This function performs the analysis phase of the incomplete-Cholesky factorization with 0 fill-in and no pivoting

$$A \approx LL^H$$

A is an (mb\*blockDim) x (mb\*blockDim) sparse matrix that is defined in BSR storage format by the three arrays bsrValA, bsrRowPtrA, and bsrColIndA.

The block in BSR format is of size blockDim\*blockDim, stored as column-major or row-major as determined by parameter dirA, which is either CUSPARSE\_DIRECTION\_COLUMN or CUSPARSE\_DIRECTION\_ROW. The matrix type must be CUSPARSE MATRIX TYPE GENERAL, and the fill mode and diagonal type are ignored.

This function requires a buffer size returned by **bsric02\_bufferSize90**. The address of **pBuffer** must be a multiple of 128 bytes. If it is not, **CUSPARSE STATUS INVALID VALUE** is returned.

Functionbsric02\_analysis() reports structural zero and computes level information stored in the opaque structure info. The level information can extract more parallelism during incomplete Cholesky factorization. However bsric02() can be done without level information. To disable level information, the user needs to specify the parameter policy of bsric02[ analysis| ] as CUSPARSE SOLVE POLICY NO LEVEL.

Function bsric02\_analysis always reports the first structural zero, even when parameter policy is CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL. The user must call cusparseXbsric02 zeroPivot() to know where the structural zero is.

It is the user's choice whether to call <code>bsric02()</code> if <code>bsric02\_analysis()</code> reports a structural zero. In this case, the user can still call <code>bsric02()</code>, which returns a numerical zero in the same position as the structural zero. However the result is meaningless.

#### Input

handle	handle to the cuSPARSE library context.
dirA	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
mb	number of block rows and block columns of matrix A.
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<pre><type> array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) nonzero blocks of matrix A.</type></pre>
bsrRowPtrA	integer array of mb +1 elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.

blockDim	block dimension of sparse matrix A; must be larger than zero.
info	structure initialized using cusparseCreateBsric02Info().
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user; the size is returned by bsric02_bufferSize().

## Output

info	structure filled with information collected during the analysis phase (that should be passed to the
	solve phase unchanged).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb,nnzb<=0); the base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.15. cusparse<t>bsric02()

```
cusparseStatus t
cusparseSbsric02(cusparseHandle t handle,
                 cusparseDirection t dirA,
                 int mb,
                 int nnzb,
                 const cusparseMatDescr t descrA,
                 float *bsrValA,
                 const int *bsrRowPtrA,
                 const int *bsrColIndA,
                 int blockDim,
                 bsric02Info t info,
                 cusparseSolvePolicy t policy,
                 void *pBuffer);
cusparseStatus_t
cusparseDbsric02(cusparseHandle_t handle,
                 cusparseDirection t dirA,
                 int mb,
                 int nnzb,
                 const cusparseMatDescr t descrA,
                 double *bsrValA,
                 const int *bsrRowPtrA,
                 const int *bsrColIndA,
                 int blockDim,
                 bsric02Info t info,
                 cusparseSolvePolicy_t policy,
                 void *pBuffer);
cusparseStatus t
cusparseCbsric02(cusparseHandle t handle,
                 cusparseDirection t dirA,
                 int mb,
                 int nnzb,
                 const cusparseMatDescr t descrA,
                 cuComplex *bsrValA,
const int *bsrRowPtrA,
                 const int *bsrColIndA,
                 int blockDim,
                 bsric02Info t info,
                 cusparseSolvePolicy t policy,
                 void *pBuffer);
cusparseStatus t
cusparseZbsric02(cusparseHandle t handle,
                 cusparseDirection t dirA,
                 int mb,
                 int nnzb,
                 const cusparseMatDescr t descrA,
                 cuDoubleComplex *bsrValA,
                 const int *bsrRowPtrA,
                 const int *bsrColIndA,
                 int blockDim,
                 bsric02Info t info,
                 cusparseSolvePolicy t policy,
                 void *pBuffer);
```

This function performs the solve phase of the incomplete-Cholesky factorization with 0 fill-in and no pivoting

$$A \approx LL^H$$

A is an (mb\*blockDim) × (mb\*blockDim) sparse matrix that is defined in BSR storage format by the three arrays bsrValA, bsrRowPtrA, and bsrColIndA.

The block in BSR format is of size blockDim\*blockDim, stored as column-major or row-major as determined by parameter dirA, which is either CUSPARSE\_DIRECTION\_COLUMN or CUSPARSE\_DIRECTION\_ROW. The matrix type must be CUSPARSE\_MATRIX\_TYPE\_GENERAL, and the fill mode and diagonal type are ignored.

This function requires a buffer size returned by bsric02\_bufferSize(). The address of pBuffer must be a multiple of 128 bytes. If it is not, CUSPARSE STATUS INVALID VALUE is returned.

Although bsric02() can be done without level information, the user must be aware of consistency. If bsric02\_analysis() is called with policy CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL, bsric02() can be run with or without levels. On the other hand, if bsric02\_analysis() is called with CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL, bsric02() can only accept CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL; otherwise, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

Function bsric02() has the same behavior as csric02(). That is, bsr2csr(bsric02(A)) = csric02(bsr2csr(A)). The numerical zero of csric02() means there exists some zero L(j,j). The numerical zero of bsric02() means there exists some block Lj,j) that is not invertible.

Function **bsric02** reports the first numerical zero, including a structural zero. The user must call **cusparseXbsric02\_zeroPivot()** to know where the numerical zero is.

The bsric02() function only takes the lower triangular part of matrix A to perform factorization. The strictly upper triangular part is ignored and never touched. It does not matter if A is Hermitian or not. In other words, from the point of view of bsric02(), A is Hermitian and only the lower triangular part is provided. Moreover, the imaginary part of diagonal elements of diagonal blocks is ignored.

For example, suppose **A** is a real m-by-m matrix, where  $\mathbf{m} = \mathbf{mb*blockDim}$ . The following code solves precondition system  $\mathbf{M*y} = \mathbf{x}$ , where **M** is the product of Cholesky factorization **L** and its transpose.

#### $M = LL^H$

```
// Suppose that A is m x m sparse matrix represented by BSR format,
// The number of block rows/columns is mb, and
// the number of nonzero blocks is nnzb.
// Assumption:
// - handle is already created by cusparseCreate(),
// - (d_bsrRowPtr, d_bsrColInd, d_bsrVal) is BSR of A on device memory,
// - d \overline{x} is right hand side vector on device memory,
// - d y is solution vector on device memory.
// - d z is intermediate result on device memory.
// - d_x, d_y and d_z are of size m.
cusparseMatDescr t descr M = 0;
cusparseMatDescr_t descr_L = 0;
bsric02Info t info M = \overline{0};
bsrsv2Info_t info_L = 0;
bsrsv2Info_t info_Lt = 0;
int pBufferSize M;
int pBufferSize L;
int pBufferSize Lt;
int pBufferSize;
void *pBuffer = 0;
int structural zero;
int numerical zero;
const double alpha = 1.;
const cusparseSolvePolicy t policy M = CUSPARSE SOLVE POLICY NO LEVEL;
const cusparseSolvePolicy_t policy_L = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_Lt = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation_t trans_L = CUSPARSE OPERATION NON TRANSPOSE;
const cusparseOperation_t trans_Lt = CUSPARSE OPERATION TRANSPOSE;
const cusparseDirection t dir = CUSPARSE DIRECTION COLUMN;
// step 1: create a descriptor which contains
// - matrix M is base-1
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has non-unit diagonal
cusparseCreateMatDescr(&descr_M);
cusparseSetMatIndexBase(descr_M, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr_M, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseCreateMatDescr(&descr L);
cusparseSetMatIndexBase(descr_L, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr L, CUSPARSE MATRIX TYPE GENERAL);
cusparseSetMatFillMode(descr_L, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr_L, CUSPARSE_DIAG_TYPE_NON_UNIT);
// step 2: create a empty info structure
// we need one info for bsric02 and two info's for bsrsv2
cusparseCreateBsric02Info(&info M);
cusparseCreateBsrsv2Info(&info L);
cusparseCreateBsrsv2Info(&info_Lt);
// step 3: query how much memory used in bsric02 and bsrsv2, and allocate the
cusparseDbsric02 bufferSize(handle, dir, mb, nnzb,
    descr M, d bsrVal, d bsrRowPtr, d bsrColInd, blockDim, info M,
 &bufferSize M);
cusparseDbsrsv2_bufferSize(handle, dir, trans_L, mb, nnzb,
    descr L, d bsrVal, d bsrRowPtr, d bsrColInd, blockDim, info L,
 &pBufferSize L);
cusparseDbsrsv2 bufferSize(handle, dir, trans Lt, mb, nnzb,
    descr_L, d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_Lt,
 &pBufferSize Lt);
pBufferSize = max(bufferSize M, max(pBufferSize L, pBufferSize Lt));
// pBuffer returned by cudaMalloc is automatically aligned to 128 bytes.
cudaMalloc((void**)&pBuffer, pBufferSize);
// step 4: perform analysis of incomplete Cholesky on M
           perform analysis of triangular solve on L
          perform analysis of triangular solve on L'
```

## Input

handle	handle to the cuSPARSE library context.
dirA	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
mb	number of block rows and block columns of matrix <b>A</b> .
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<pre><type> array of nnzb(= bsrRowPtrA(mb) - bsrRowPtrA(0) ) nonzero blocks of matrix A.</type></pre>
bsrRowPtrA	integer array of mb +1 elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A, larger than zero.
info	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user, the size is returned by bsric02_bufferSize().

## Output

bsrValA	<type> matrix containing the incomplete-Cholesky</type>
	lower triangular factor.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb,nnzb<=0); the base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

	CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.
--	---	-----------------------------------

# 10.16. cusparseXbsric02\_zeroPivot()

If the returned error code is **CUSPARSE\_STATUS\_ZERO\_PIVOT**, **position=j** means **A**(j,j) has either a structural zero or a numerical zero (the block is not positive definite). Otherwise **position=-1**.

The **position** can be 0-based or 1-based, the same as the matrix.

Function cusparseXbsric02\_zeroPivot() is a blocking call. It calls cudaDeviceSynchronize() to make sure all previous kernels are done.

The **position** can be in the host memory or device memory. The user can set the proper mode with **cusparseSetPointerMode()**.

### Input

handle	handle to the cuSPARSE library context.
info	<pre>info contains a structural zero or a numerical zero if the user already called bsric02_analysis() Of bsric02().</pre>

### Output

•	if no structural or numerical zero, position is -1, otherwise if A(j,j) is missing or L(j,j) is not
	positive definite, position=j.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	info is not valid.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 10.17. cusparse<t>bsrilu02\_numericBoost()

```
cusparseStatus t
cusparseSbsrilu02 numericBoost(cusparseHandle t handle,
                                bsrilu02Info t info,
                                int enable boost,
                                double *to\overline{l},
                                float *boost val);
cusparseStatus t
cusparseDbsrilu02 numericBoost(cusparseHandle t handle,
                                bsrilu02Info t info,
                                int enable boost,
                                double *tol,
                                double *boost val);
cusparseStatus t
{\tt cusparseCbsrilu02\_numericBoost(cusparseHandle\_t\ handle,}
                                bsrilu02Info t info,
                                int enable boost,
                                double *tol,
                                cuComplex *boost val);
cusparseStatus t
cusparseZbsrilu02 numericBoost(cusparseHandle_t handle,
                                bsrilu02Info t info,
                                int enable boost,
                                double *tol,
                                cuDoubleComplex *boost val);
```

The user can use a boost value to replace a numerical value in incomplete LU factorization. Parameter tol is used to determine a numerical zero, and boost\_val is used to replace a numerical zero. The behavior is as follows:

if tol >= fabs(A(j,j)), then reset each diagonal element of block A(j,j) by boost\_val.

To enable a boost value, the user sets parameter <code>enable\_boost</code> to 1 before calling <code>bsrilu02()</code>. To disable the boost value, the user can call <code>bsrilu02\_numericBoost()</code> with parameter <code>enable\_boost=0</code>.

If enable\_boost=0, tol and boost\_val are ignored.

Both tol and boost\_val can be in host memory or device memory. The user can set the proper mode with cusparseSetPointerMode().

### Input

handle	handle to the cuSPARSE library context.
info	structure initialized using cusparseCreateBsrilu02Info().
enable_boost	disable boost by setting enable_boost=0. Otherwise, boost is enabled.

tol	tolerance to determine a numerical zero.
boost_val	boost value to replace a numerical zero.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	info or pointer mode is not valid.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 10.18. cusparse<t>bsrilu02\_bufferSize()

```
cusparseStatus t
cusparseSbsrilu02 bufferSize(cusparseHandle t handle,
                              cusparseDirection t dirA,
                              int mb,
                              int nnzb,
                              const cusparseMatDescr t descrA,
                              float *bsrValA,
                              const int *bsrRowPtrA,
                              const int *bsrColIndA,
                              int blockDim,
                              bsrilu02Info t info,
                              int *pBufferSizeInBytes);
cusparseStatus t
cusparseDbsrilu02_bufferSize(cusparseHandle_t handle,
                              cusparseDirection t dirA,
                              int mb,
                              int nnzb,
                              const cusparseMatDescr t descrA,
                              double *bsrValA,
                              const int *bsrRowPtrA,
                              const int *bsrColIndA,
                              int blockDim,
                              bsrilu02Info t info,
                              int *pBufferSizeInBytes);
cusparseStatus t
cusparseCbsrilu02 bufferSize(cusparseHandle t handle,
                              cusparseDirection t dirA,
                              int mb,
                              int nnzb,
                              const cusparseMatDescr t descrA,
                              cuComplex *bsrValA,
const int *bsrRowPtrA,
                              const int *bsrColIndA,
                              int blockDim,
                              bsrilu02Info t info,
                              int *pBufferSizeInBytes);
cusparseStatus t
cusparseZbsrilu02 bufferSize(cusparseHandle t handle,
                              cusparseDirection t dirA,
                              int mb,
                              int nnzb,
                              const cusparseMatDescr t descrA,
                              cuDoubleComplex *bsrValA,
                              const int *bsrRowPtrA,
                              const int *bsrColIndA,
                              int blockDim,
                              bsrilu02Info t info,
                              int *pBufferSizeInBytes);
```

This function returns the size of the buffer used in computing the incomplete-LU factorization with 0 fill-in and no pivoting

A≈LU

A is an (mb\*blockDim) \* (mb\*blockDim) sparse matrix that is defined in BSR storage format by the three arrays bsrValA, bsrRowPtrA, and bsrColIndA.

The buffer size depends on the dimensions of mb, blockDim, and the number of nonzero blocks of the matrix nnzb. If the user changes the matrix, it is necessary to call bsrilu02\_bufferSize() again to have the correct buffer size; otherwise, a segmentation fault may occur.

# Input

handle	handle to the cuSPARSE library context.
dirA	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
mb	number of block rows and columns of matrix A.
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<pre><type> array of nnzb(= bsrRowPtrA(mb) - bsrRowPtrA(0)) nonzero blocks of matrix A.</type></pre>
bsrRowPtrA	integer array of mb +1 elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A, larger than zero.

# Output

info	record internal states based on different algorithms.
pBufferSizeInBytes	number of bytes of the buffer used in bsrilu02_analysis() and bsrilu02().

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb,nnzb<=0), base index is not 0 or 1.

CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.19. cusparse<t>bsrilu02\_analysis()

```
cusparseStatus t
cusparseSbsrilu02 analysis(cusparseHandle t handle,
                            cusparseDirection t dirA,
                            int mb,
                            int nnzb,
                            const cusparseMatDescr t descrA,
                            float *bsrValA,
                            const int *bsrRowPtrA,
                            const int *bsrColIndA,
                            int blockDim,
                            bsrilu02Info t info,
                            cusparseSolvePolicy t policy,
                            void *pBuffer);
cusparseStatus_t
cusparseDbsrilu02_analysis(cusparseHandle_t handle,
                            cusparseDirection t dirA,
                            int mb,
                            int nnzb,
                            const cusparseMatDescr t descrA,
                            double *bsrValA,
                            const int *bsrRowPtrA,
                            const int *bsrColIndA,
                            int blockDim,
                            bsrilu02Info_t info,
                            cusparseSolvePolicy_t policy,
                            void *pBuffer);
cusparseStatus t
cusparseCbsrilu02_analysis(cusparseHandle_t handle,
                            cusparseDirection t dirA,
                            int mb,
                            int nnzb,
                            const cusparseMatDescr t descrA,
                            cuComplex *bsrValA,
const int *bsrRowPtrA,
const int *bsrColIndA,
                            int blockDim,
                            bsrilu02Info t info,
                            cusparseSolvePolicy t policy,
                            void *pBuffer);
cusparseStatus t
cusparseZbsrilu02 analysis(cusparseHandle t handle,
                            cusparseDirection_t dirA,
                            int mb,
                            int nnzb,
                            const cusparseMatDescr t descrA,
                            cuDoubleComplex *bsrValA,
                            const int *bsrRowPtrA,
                            const int *bsrColIndA,
                            int blockDim,
                            bsrilu02Info t info,
                            cusparseSolvePolicy_t policy,
                            void *pBuffer);
```

This function performs the analysis phase of the incomplete-LU factorization with 0 fill-in and no pivoting

A≈LU

A is an (mb\*blockDim) × (mb\*blockDim) sparse matrix that is defined in BSR storage format by the three arrays bsrValA, bsrRowPtrA, and bsrColIndA.

The block in BSR format is of size blockDim\*blockDim, stored as column-major or row-major as determined by parameter dirA, which is either CUSPARSE\_DIRECTION\_COLUMN or CUSPARSE\_DIRECTION\_ROW. The matrix type must be CUSPARSE MATRIX TYPE GENERAL, and the fill mode and diagonal type are ignored.

This function requires a buffer size returned by bsrilu02\_bufferSize(). The address of pBuffer must be multiple of 128 bytes. If it is not, CUSPARSE STATUS INVALID VALUE is returned.

Function bsrilu02\_analysis() reports a structural zero and computes level information stored in the opaque structure info. The level information can extract more parallelism during incomplete LU factorization. However bsrilu02() can be done without level information. To disable level information, the user needs to specify the parameter policy of bsrilu02[\_analysis|] as CUSPARSE SOLVE POLICY NO LEVEL.

Function bsrilu02\_analysis() always reports the first structural zero, even with parameter policy is CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL. The user must call cusparseXbsrilu02 zeroPivot() to know where the structural zero is.

It is the user's choice whether to call <code>bsrilu02()</code> if <code>bsrilu02\_analysis()</code> reports a structural zero. In this case, the user can still call <code>bsrilu02()</code>, which will return a numerical zero at the same position as the structural zero. However the result is meaningless.

### Input

handle	handle to the cuSPARSE library context.
dirA	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
mb	number of block rows and block columns of matrix A.
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<pre><type> array of nnzb(= bsrRowPtrA(mb) - bsrRowPtrA(0) ) nonzero blocks of matrix A.</type></pre>
bsrRowPtrA	integer array of mb +1elements that contains the start of every block row and the end of the last block row plus one.

bsrColIndA	integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A, larger than zero.
info	structure initialized using cusparseCreateBsrilu02Info().
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user, the size is returned by bsrilu02_bufferSize().

# Output

info	structure filled with information collected during
	the analysis phase (that should be passed to the
	solve phase unchanged).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb,nnzb<=0); the base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.20. cusparse<t>bsrilu02()

```
cusparseStatus t
cusparseSbsrilu02(cusparseHandle t handle,
                  cusparseDirection t dirA,
                  int mb,
                  int nnzb,
                  const cusparseMatDescr t descry,
                  float *bsrValA,
                  const int *bsrRowPtrA,
                  const int *bsrColIndA,
                  int blockDim,
                  bsrilu02Info t info,
                  cusparseSolvePolicy t policy,
                  void *pBuffer);
cusparseStatus_t
cusparseDbsrilu02(cusparseHandle_t handle,
                  cusparseDirection t dirA,
                  int mb,
                  int nnzb,
                  const cusparseMatDescr_t descry,
                  double *bsrValA,
                  const int *bsrRowPtrA,
                  const int *bsrColIndA,
                  int blockDim,
                  bsrilu02Info_t info,
                  cusparseSolvePolicy_t policy,
                  void *pBuffer);
cusparseStatus t
cusparseCbsrilu02(cusparseHandle t handle,
                  cusparseDirection t dirA,
                  int mb,
                  int nnzb,
                  const cusparseMatDescr t descry,
                  cuComplex *bsrValA,
const int *bsrRowPtrA,
                  const int *bsrColIndA,
                  int blockDim,
                  bsrilu02Info t info,
                  cusparseSolvePolicy t policy,
                  void *pBuffer);
cusparseStatus t
cusparseZbsrilu02(cusparseHandle t handle,
                  cusparseDirection t dirA,
                  int mb,
                  int nnzb,
                  const cusparseMatDescr t descry,
                  cuDoubleComplex *bsrValA,
                  const int *bsrRowPtrA,
                  const int *bsrColIndA,
                  int blockDim,
                  bsrilu02Info t info,
                  cusparseSolvePolicy_t policy,
                  void *pBuffer);
```

This function performs the solve phase of the incomplete-LU factorization with 0 fill-in and no pivoting

A≈LU

A is an (mb\*blockDim) × (mb\*blockDim) sparse matrix that is defined in BSR storage format by the three arrays bsrValA, bsrRowPtrA, and bsrColIndA. The block in BSR format is of size blockDim\*blockDim, stored as column-major or row-major determined by parameter dirA, which is either CUSPARSE\_DIRECTION\_COLUMN or CUSPARSE\_DIRECTION\_ROW. The matrix type must be CUSPARSE\_MATRIX\_TYPE\_GENERAL, and the fill mode and diagonal type are ignored. Function bsrilu02() supports an arbitrary blockDim.

This function requires a buffer size returned by bsrilu02\_bufferSize(). The address of pBuffer must be a multiple of 128 bytes. If it is not, CUSPARSE STATUS INVALID VALUE is returned.

Although bsrilu02() can be used without level information, the user must be aware of consistency. If bsrilu02\_analysis() is called with policy CUSPARSE\_SOLVE\_POLICY\_USE\_LEVEL, bsrilu02() can be run with or without levels. On the other hand, if bsrilu02\_analysis() is called with CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL, bsrilu02() can only accept CUSPARSE\_SOLVE\_POLICY\_NO\_LEVEL; otherwise, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

Function bsrilu02() has the same behavior as csrilu02(). That is, bsr2csr(bsrilu02(A)) = csrilu02(bsr2csr(A)). The numerical zero of csrilu02() means there exists some zero U(j,j). The numerical zero of bsrilu02() means there exists some block U(j,j) that is not invertible.

Function **bsrilu02** reports the first numerical zero, including a structural zero. The user must call **cusparseXbsrilu02 zeroPivot()** to know where the numerical zero is.

For example, suppose  $\mathbf{A}$  is a real m-by-m matrix where  $\mathbf{m}=\mathbf{mb*blockDim}$ . The following code solves precondition system  $\mathbf{M*y} = \mathbf{x}$ , where  $\mathbf{M}$  is the product of LU factors  $\mathbf{L}$  and  $\mathbf{U}$ .

```
// Suppose that A is m x m sparse matrix represented by BSR format,
// The number of block rows/columns is mb, and
// the number of nonzero blocks is nnzb.
// Assumption:
// - handle is already created by cusparseCreate(),
// - (d_bsrRowPtr, d_bsrColInd, d_bsrVal) is BSR of A on device memory, // - d_x is right hand side vector on device memory.
// - d y is solution vector on device memory.
// - d z is intermediate result on device memory.
// - d_x, d_y and d_z are of size m.
cusparseMatDescr_t descr_M = 0;
cusparseMatDescr_t descr_L = 0;
cusparseMatDescr t descr U = 0;
bsrilu02Info t info M = 0;
bsrsv2Info_t info_L = 0;
bsrsv2Info_t info_U = 0;
int pBufferSize M;
int pBufferSize L;
int pBufferSize U;
int pBufferSize;
void *pBuffer = 0;
int structural_zero;
int numerical zero;
const double alpha = 1.;
const cusparseSolvePolicy_t policy_M = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_L = CUSPARSE_SOLVE_POLICY_NO_LEVEL;
const cusparseSolvePolicy_t policy_U = CUSPARSE_SOLVE_POLICY_USE_LEVEL;
const cusparseOperation_t trans_L = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseOperation_t trans_U = CUSPARSE_OPERATION_NON_TRANSPOSE;
const cusparseDirection_t dir = CUSPARSE DIRECTION COLUMN;
// step 1: create a descriptor which contains
// - matrix M is base-1
// - matrix L is base-1
// - matrix L is lower triangular
// - matrix L has unit diagonal
// - matrix U is base-1
// - matrix U is upper triangular
// - matrix U has non-unit diagonal
cusparseCreateMatDescr(&descr M);
cusparseSetMatIndexBase(descr_M, CUSPARSE_INDEX_BASE_ONE);
cusparseSetMatType(descr M, CUSPARSE MATRIX TYPE GENERAL);
cusparseCreateMatDescr(&descr L);
cusparseSetMatIndexBase(descr L, CUSPARSE INDEX BASE ONE);
cusparseSetMatType(descr_L, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_L, CUSPARSE_FILL_MODE_LOWER);
cusparseSetMatDiagType(descr_L, CUSPARSE_DIAG_TYPE_UNIT);
cusparseCreateMatDescr(&descr U);
cusparseSetMatIndexBase(descr U, CUSPARSE INDEX BASE ONE);
cusparseSetMatType(descr_U, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatFillMode(descr_U, CUSPARSE_FILL_MODE_UPPER);
cusparseSetMatDiagType(descr_U, CUSPARSE_DIAG_TYPE_NON_UNIT);
// step 2: create a empty info structure
// we need one info for bsrilu02 and two info's for bsrsv2
cusparseCreateBsrilu02Info(&info M);
cusparseCreateBsrsv2Info(&info L);
cusparseCreateBsrsv2Info(&info U);
// step 3: query how much memory used in bsrilu02 and bsrsv2, and allocate the
cusparseDbsrilu02 bufferSize(handle, dir, mb, nnzb,
     descr_M, d_bsrVal, d_bsrRowPtr, d_bsrColInd, blockDim, info_M,
 &pBufferSize M);
cusparseDbsrsv2_bufferSize(handle, dir, trans_L, mb, nnzb,
     descr L, d bsrVal, d bsrRowPtr, d bsrColInd, blockDim, info L,
 &pBuffer\overline{S}ize \overline{L});
cusparseDbsrsv2_bufferSize(handle, dir, trans_U, mb, nnzb,
     descr U, d bsrVal, d bsrRowPtr, d bsrColInd, blockDim, info U,
```

# Input

	I II ( ) ( CD) DCE ( )
handle	handle to the cuSPARSE library context.
dirA	storage format of blocks: either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
mb	number of block rows and block columns of matrix A.
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<pre><type> array of nnzb(= bsrRowPtrA(mb) - bsrRowPtrA(0)) nonzero blocks of matrix A.</type></pre>
bsrRowPtrA	integer array of mb +1 elements that contains the start of every block row and the end of the last block row plus one.
bsrColIndA	integer array of nnzb (= bsrRowPtrA(mb) - bsrRowPtrA(0) ) column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A; must be larger than zero.
info	structure with information collected during the analysis phase (that should have been passed to the solve phase unchanged).
policy	the supported policies are CUSPARSE_SOLVE_POLICY_NO_LEVEL and CUSPARSE_SOLVE_POLICY_USE_LEVEL.
pBuffer	buffer allocated by the user; the size is returned by bsrilu02_bufferSize().

# Output

bsrValA	<type> matrix containing the incomplete-LU lower</type>
	and upper triangular factors.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb,nnzb<=0); the base index is not 0 or 1.
CUSPARSE_STATUS_ARCH_MISMATCH	the device only supports compute capability 2.0 and above.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

	CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.
--	---	-----------------------------------

# 10.21. cusparseXbsrilu02\_zeroPivot()

If the returned error code is **CUSPARSE\_STATUS\_ZERO\_PIVOT**, **position=j** means **A**(j,j) has either a structural zero or a numerical zero (the block is not invertible). Otherwise **position=-1**.

The **position** can be 0-based or 1-based, the same as the matrix.

Function cusparseXbsrilu02\_zeroPivot() is a blocking call. It calls cudaDeviceSynchronize() to make sure all previous kernels are done.

The **position** can be in the host memory or device memory. The user can set proper the mode with **cusparseSetPointerMode()**.

# Input

handle	handle to the cuSPARSE library context.
info	<pre>info contains structural zero or numerical zero if the user already called bsrilu02_analysis() or bsrilu02().</pre>

### Output

if no structural or numerical zero, position is -1; otherwise if $A(j,j)$ is missing or $U(j,j)$ is not
invertible, position=j.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	info is not valid.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 10.22. cusparse<t>gtsv()

```
cusparseStatus t
cusparseSgtsv(cusparseHandle t handle,
              int m,
              int n,
              const float *dl,
              const float *d,
              const float *du,
              float *B,
              int ldb)
cusparseStatus t
cusparseDgtsv(cusparseHandle t handle,
              int m,
              int n,
              const double *dl,
              const double *d,
              const double *du,
              double *B,
              int ldb)
cusparseStatus t
cusparseCgtsv(cusparseHandle t handle,
              int m,
              int n,
              const cuComplex *dl,
              const cuComplex *d,
              const cuComplex *du,
              cuComplex *B,
              int ldb)
cusparseStatus t
cusparseZgtsv(cusparseHandle t handle,
              int m,
              int n,
              const cuDoubleComplex *dl,
              const cuDoubleComplex *d,
              const cuDoubleComplex *du,
              cuDoubleComplex *B,
              int ldb)
```

This function computes the solution of a tridiagonal linear system with multiple righthand sides:

$$A * Y = a * X$$

The coefficient matrix  $\mathbf{A}$  of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower ( $\mathbf{d}\mathbf{l}$ ), main ( $\mathbf{d}$ ), and upper ( $\mathbf{d}\mathbf{u}$ ) matrix diagonals; the right-hand sides are stored in the dense matrix  $\mathbf{x}$ . Notice that solution  $\mathbf{y}$  overwrites right-hand-side matrix  $\mathbf{x}$  on exit.

Assuming **A** is of size **m** and base-1, **dl**, **d** and **du** are defined by the following formula:

```
dl(i) := A(i, i-1) for i=1,2,...,m
The first element of dl is out-of-bound (dl(1) := A(1,0)), so dl(1) = 0.
d(i) = A(i,i) for i=1,2,...,m
```

```
du(i) = A(i,i+1) \text{ for } i=1,2,...,m
```

The last element of du is out-of-bound (du(m) := A(m,m+1)), so du(m) = 0.

The routine does perform pivoting, which usually results in more accurate and more stable results than <code>cusparse<t>gtsv\_nopivot()</code> at the expense of some execution time

This routine requires significant amount of temporary extra storage (min(m,8) × (3+n) ×sizeof (<type>)). It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

# Input

handle	handle to the cuSPARSE library context.
m	the size of the linear system (must be $\geq$ 3).
n	number of right-hand sides, columns of matrix B.
d1	<type> dense array containing the lower diagonal of the tri-diagonal linear system. The first element of each lower diagonal must be zero.</type>
d	<type> dense array containing the main diagonal of the tri-diagonal linear system.</type>
du	<type> dense array containing the upper diagonal of the tri-diagonal linear system. The last element of each upper diagonal must be zero.</type>
В	<pre><type> dense right-hand-side array of dimensions (ldb, <math>n</math>).</type></pre>
ldb	leading dimension of B (that is $\geq \max(1, m)$ ).

# Output

В	<type> dense solution array of dimensions (1db,</type>
	n).

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m<3, n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.23. cusparse<t>gtsv\_nopivot()

```
cusparseStatus t
cusparseSgtsv nopivot(cusparseHandle t handle, int m, int n,
              const float *dl, const float const float *du, float *B, int ldb)
                                                                      *d,
cusparseStatus t
cusparseDgtsv nopivot(cusparseHandle t handle, int m, int n,
             const double *dl, const double const double *du, double *B, int ldb)
                                                                      *d.
cusparseStatus t
cusparseCgtsv nopivot(cusparseHandle t handle, int m, int n,
             const cuComplex *d1, const cuComplex *d, const cuComplex *b, int ldb)
                                                                     *d,
cusparseStatus t
cusparseZgtsv_nopivot(cusparseHandle_t handle, int m, int n,
               const cuDoubleComplex *dl, const cuDoubleComplex *d,
               const cuDoubleComplex *du, cuDoubleComplex *B, int ldb)
```

This function computes the solution of a tridiagonal linear system with multiple righthand sides:

$$A * Y = a * X$$

The coefficient matrix  $\mathbf{A}$  of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower ( $\mathtt{dl}$ ), main ( $\mathtt{d}$ ), and upper ( $\mathtt{du}$ ) matrix diagonals; the right-hand sides are stored in the dense matrix  $\mathbf{x}$ . Notice that solution  $\mathbf{y}$  overwrites right-hand-side matrix  $\mathbf{x}$  on exit.

The routine does not perform any pivoting and uses a combination of the Cyclic Reduction (CR) and the Parallel Cyclic Reduction (PCR) algorithms to find the solution. It achieves better performance when **m** is a power of 2.

This routine requires a significant amount of temporary extra storage (mx (3+n) xsizeof (<type>)). It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

### Input

handle	handle to the cuSPARSE library context.
m	the size of the linear system (must be $\geq$ 3).
n	number of right-hand sides, columns of matrix B.
d1	<type> dense array containing the lower diagonal of the tri-diagonal linear system. The first element of each lower diagonal must be zero.</type>
d	<type> dense array containing the main diagonal of the tri-diagonal linear system.</type>
du	<type> dense array containing the upper diagonal of the tri-diagonal linear system. The last element of each upper diagonal must be zero.</type>

В	<pre><type> dense right-hand-side array of dimensions (ldb, <math>n</math>).</type></pre>
ldb	leading dimension of B. (that is $\geq \max(1, m)$ ).

### Output

В	<type> dense solution array of dimensions (1db,</type>
	n).

### Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m<3, n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 10.24. cusparse<t>gtsvStridedBatch()

```
cusparseStatus t
cusparseSgtsvStridedBatch(cusparseHandle t handle, int m,
                         const float *dl,
                         const float
                                                *d,
                                               *du, float
                                                                     *x,
                         const float
                         int batchCount, int batchStride)
cusparseStatus t
cusparseDgtsvStridedBatch(cusparseHandle_t handle, int m,
                         const double *d, const double *d, const double *du,
                                              *du, double
                                                                     *x,
                          int batchCount, int batchStride)
cusparseStatus t
cusparseCgtsvStridedBatch(cusparseHandle t handle, int m,
                         const cuComplex *dl,
                         *x,
                         int batchCount, int batchStride)
cusparseStatus t
cusparseZgtsvStridedBatch(cusparseHandle t handle, int m,
                         const cuDoubleComplex *d,
const cuDoubleComplex *d,
                          const cuDoubleComplex *du, cuDoubleComplex *x,
                          int batchCount, int batchStride)
```

This function computes the solution of multiple tridiagonal linear systems for i=0, ...,batchCount:

$$A^{(i)} * y^{(i)} = a * x^{(i)}$$

The coefficient matrix **A** of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (**d1**), main (**d**), and upper (**du**) matrix diagonals; the right-hand sides are stored in the dense matrix **x**. Notice that solution **y** overwrites right-hand-side matrix **x** on exit. The different matrices are assumed to be of the same size and are stored with a fixed **batchStride** in memory.

The routine does not perform any pivoting and uses a combination of the Cyclic Reduction (CR) and the Parallel Cyclic Reduction (PCR) algorithms to find the solution. It achieves better performance when **m** is a power of 2.

This routine requires a significant amount of temporary extra storage ((batchCount× (4×m+2048) ×sizeof (<type>))). It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

### Input

handle	handle to the cuSPARSE library context.
m	the size of the linear system (must be $\geq 3$ ).
d1	<type> dense array containing the lower diagonal of the tri-diagonal linear system. The lower diagonal <math>d^{(i)}</math> that corresponds to the <math>i^{th}</math> linear system starts at location <math>dl+batchStride\times i</math> in memory. Also, the first element of each lower diagonal must be zero.</type>
d	<type> dense array containing the main diagonal of the tri-diagonal linear system. The main diagonal <math>d^{(i)}</math> that corresponds to the <math>i^{th}</math> linear system starts at location d+batchStride×i in memory.</type>
du	<type> dense array containing the upper diagonal of the tri-diagonal linear system. The upper diagonal <math>du^{(i)}</math> that corresponds to the <math>i^{th}</math> linear system starts at location <math>du+batchStride \times i</math> in memory. Also, the last element of each upper diagonal must be zero.</type>
x	<type> dense array that contains the right-hand- side of the tri-diagonal linear system. The right- hand-side <math>x^{(i)}</math> that corresponds to the <math>i^{th}</math> linear system starts at location <math>x+batchStride \times iin</math> memory.</type>
batchCount	number of systems to solve.
batchStride	stride (number of elements) that separates the vectors of every system (must be at least m).

### Output

x	<type> dense array that contains the solution of</type>
	the tri-diagonal linear system. The solution $x^{(i)}$ that corresponds to the $i^{th}$ linear system starts at location $x+batchStride\times in$ memory.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m<3, batchCount≤0, batchStride <m).< td=""></m).<>
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# Chapter 11. CUSPARSE REORDERINGS REFERENCE

This chapter describes the reordering routines used to manipulate sparse matrices.

# 11.1. cusparse<t>csrcolor()

```
cusparseStatus t
cusparseScsrcolor(cusparseHandle t handle, int m, int nnz,
             const cusparseMatDescr_t descrA, const float *csrValA,
            const int *csrRowPtrA, const int *csrColIndA,
            const float *fractionToColor, int *ncolors, int *coloring,
            int *reordering, cusparseColorInfo t info);
cusparseStatus t
cusparseDcsrcolor(cusparseHandle t handle, int m, int nnz,
            const cusparseMatDescr t descrA, const double *csrValA,
            const int *csrRowPtrA, const int *csrColIndA,
            const double *fractionToColor,int *ncolors, int *coloring,
            int *reordering, cusparseColorInfo t info);
cusparseStatus t
cusparseCcsrcolor(cusparseHandle t handle, int m, int nnz,
             const cusparseMatDescr_t descrA, const cuComplex *csrValA,
            const int *csrRowPtrA, const int *csrColIndA,
            const float *fractionToColor, int *ncolors, int *coloring,
            int *reordering, cusparseColorInfo t info);
cusparseStatus t
cusparseZcsrcolor(cusparseHandle t handle, int m, int nnz,
      const cusparseMatDescr t descrA, const cuDoubleComplex *csrValA,
             const int *csrRowPtrA, const int *csrColIndA,
            const double *fractionToColor,int *ncolors, int *coloring,
            int *reordering, cusparseColorInfo t info);
```

This function performs the coloring of the adjacency graph associated with the matrix A stored in CSR format. The coloring is an assignment of colors (integer numbers) to nodes, such that neighboring nodes have distinct colors. An approximate coloring algorithm is used in this routine, and is stopped when a certain percentage of nodes has been colored. The rest of the nodes are assigned distinct colors (an increasing sequence of integers numbers, starting from the last integer used previously). The last two auxiliary routines can be used to extract the resulting number of colors, their assignment

and the associated reordering. The reordering is such that nodes that have been assigned the same color are reordered to be next to each other.

The matrix A passed to this routine, must be stored as a general matrix and have a symmetric sparsity pattern. If the matrix is nonsymmetric the user should pass A+A^T as a parameter to this routine.

# Input

handle	handle to the cuSPARSE library context.
m	number of rows of matrix A.
nnz	number of nonzero elements of matrix A.
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
fractionToColor	fraction of nodes to be colored, which should be in the interval [0.0,1.0], for example 0.8 implies that 80 percent of nodes will be colored.
info	structure with information to be passed to the coloring.

# Output

ncolors	The number of distinct colors used (at most the size of the matrix, but likely much smaller).
coloring	The resulting coloring permutation
reordering	The resulting reordering permutation (untouched if NULL)

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,nnz<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision (compute capability (c.c.) >= 1.3 required).
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

CUSPARSE\_STATUS\_MATRIX\_TYPE\_NOT\_SUPPORTED the matrix type is not supported.

# Chapter 12. CUSPARSE FORMAT CONVERSION REFERENCE

This chapter describes the conversion routines between different sparse and dense storage formats.

coosort, csrsort, cscsort, csru2csr and csr2csc\_indexOnly are sorting routines
without malloc inside, the following table estimates the buffer size

routine	buffer size	maximum problem size if buffer is limited by 2GB
coosort	> 16*n bytes	125M
csrsort or cscsort	> 20*n bytes	100M
csru2csr	'd' > 28*n bytes ; 'z' > 36*n bytes	71M for 'd' and 55M for 'z'
csr2csc_indexOnl	y> 16*n bytes	125M

# 12.1. cusparse<t>bsr2csr()

```
cusparseStatus t
cusparseSbsr2csr(cusparseHandle t handle,
   cusparseDirection t dir,
   int mb,
   int nb,
   const cusparseMatDescr t descrA,
   const float *bsrValA,
   const int *bsrRowPtrA,
   const int *bsrColIndA,
   int blockDim,
   const cusparseMatDescr t descrC,
   float *csrValC,
   int *csrRowPtrC,
   int *csrColIndC)
cusparseStatus_t
cusparseDbsr2csr(cusparseHandle_t handle,
   cusparseDirection t dir,
   int mb,
   int nb,
   const cusparseMatDescr t descrA,
   const double *bsrValA,
   const int *bsrRowPtrA,
   const int *bsrColIndA,
   int blockDim,
   const cusparseMatDescr t descrC,
   double *csrValC,
   int *csrRowPtrC,
   int *csrColIndC)
cusparseStatus t
cusparseCbsr2csr(cusparseHandle t handle,
   cusparseDirection t dir,
    int mb,
   int nb,
   const cusparseMatDescr t descrA,
   const cuComplex *bsrValA,
   const int *bsrRowPtrA,
    const int *bsrColIndA,
   int blockDim,
    const cusparseMatDescr t descrC,
    cuComplex *csrValC,
    int *csrRowPtrC,
   int *csrColIndC)
cusparseStatus t
cusparseZbsr2csr(cusparseHandle t handle,
   cusparseDirection t dir,
    int mb,
   int nb,
   const cusparseMatDescr t descrA,
   const cuDoubleComplex *bsrValA,
   const int *bsrRowPtrA,
   const int *bsrColIndA,
   int blockDim,
   const cusparseMatDescr t descrC,
   cuDoubleComplex *csrValC,
   int *csrRowPtrC,
   int *csrColIndC)
```

This function converts a sparse matrix in BSR format that is defined by the three arrays bsrValA, bsrRowPtrA, and bsrColIndA) into a sparse matrix in CSR format that is defined by arrays csrValC, csrRowPtrC, and csrColIndC.

Let m(=mb\*blockDim) be the number of rows of A and n(=nb\*blockDim) be number of columns of A, then A and C are m\*n sparse matrices. The BSR format of A contains nnzb(=bsrRowPtrA[mb] - bsrRowPtrA[0]) nonzero blocks, whereas the sparse matrix A contains nnz(=nnzb\*blockDim\*blockDim) elements. The user must allocate enough space for arrays csrRowPtrC, csrColIndC, and csrValC. The requirements are as follows:

csrRowPtrC of m+1 elements

csrValC of nnz elements

csrColIndC of nnz elements

The general procedure is as follows:

### Input

handle	handle to the cuSPARSE library context.
dir	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
mb	number of block rows of sparse matrix A.
nb	number of block columns of sparse matrix A.
descrA	the descriptor of matrix A.
bsrValA	<pre><type> array of nnzb*blockDim*blockDim nonzero elements of matrix A.</type></pre>
bsrRowPtrA	integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one of matrix A.
bsrColIndA	integer array of nnzb column indices of the nonzero blocks of matrix A.
blockDim	block dimension of sparse matrix A.
descrC	the descriptor of matrix c.

### Output

csrValC	<pre><type> array of nnz (=csrRowPtrC[m] - csrRowPtrC[0]) nonzero elements of matrix c.</type></pre>
csrRowPtrC	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one of matrix $c$ .
csrColIndC	integer array of nnz column indices of the nonzero elements of matrix c.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb,nb<0, IndexBase Of descrA, descrC is not base-0 or base-1, dir is not row-major or column-major, or blockDim<1).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 12.2. cusparse<t>gebsr2gebsc\_bufferSize()

```
cusparseStatus t
cusparseSgebsr2gebsc bufferSize(cusparseHandle t handle,
   int mb,
   int nb,
   int nnzb,
   const float *bsrVal,
   const int *bsrRowPtr,
   const int *bsrColInd,
   int rowBlockDim,
   int colBlockDim,
   int *pBufferSize)
cusparseStatus t
cusparseDgebsr2gebsc bufferSize(cusparseHandle t handle,
   int mb,
   int nb,
   int nnzb,
   const double *bsrVal,
   const int *bsrRowPtr,
   const int *bsrColInd,
   int rowBlockDim,
   int colBlockDim,
   int *pBufferSize)
cusparseStatus t
cusparseCgebsr2gebsc bufferSize(cusparseHandle t handle,
   int mb,
    int nb,
   int nnzb,
   const cuComplex *bsrVal,
   const int *bsrRowPtr,
   const int *bsrColInd,
    int rowBlockDim,
    int colBlockDim,
   int *pBufferSize)
cusparseStatus t
cusparseZgebsr2gebsc bufferSize(cusparseHandle t handle,
   int mb,
   int nb,
   int nnzb,
   const cuDoubleComplex *bsrVal,
   const int *bsrRowPtr,
   const int *bsrColInd,
   int rowBlockDim,
    int colBlockDim,
    int *pBufferSize)
```

This function returns size of buffer used in computing gebsr2gebsc().

## Input

handle	handle to the cuSPARSE library context.
mb	number of block rows of sparse matrix A.

nb	number of block columns of sparse matrix A.
nnzb	number of nonzero blocks of matrix A.
bsrVal	<pre><type> array of nnzb*rowBlockDim*colBlockDim non-zero elements of matrix A.</type></pre>
bsrRowPtr	integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one.
bsrColInd	integer array of nnzb column indices of the non-zero blocks of matrix A.
rowBlockDim	number of rows within a block of A.
colBlockDim	number of columns within a block of A.

# Output

pBufferSize	host pointer containing number of bytes of the
	buffer used in gebsr2gebsc().

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb, nb, nnzb<0, Or rowBlockDim, colBlockDim<1).
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 12.3. cusparse<t>gebsr2gebsc()

```
cusparseStatus t
cusparseSgebsr2gebsc(cusparseHandle t handle,
   int mb,
   int nb,
   int nnzb,
   const float *bsrVal,
   const int *bsrRowPtr,
   const int *bsrColInd,
   int rowBlockDim,
   int colBlockDim,
   float *bscVal,
   int *bscRowInd,
   int *bscColPtr,
   cusparseAction_t copyValues,
   cusparseIndexBase_t baseIdx,
   void *pBuffer)
cusparseStatus t
cusparseDgebsr2gebsc(cusparseHandle_t handle,
   int mb,
   int nb,
   int nnzb,
   const double *bsrVal,
   const int *bsrRowPtr,
   const int *bsrColInd,
   int rowBlockDim,
   int colBlockDim,
   double *bscVal,
   int *bscRowInd,
   int *bscColPtr,
   cusparseAction t copyValues,
   cusparseIndexBase t baseIdx,
   void *pBuffer)
cusparseStatus t
cusparseCgebsr2gebsc(cusparseHandle t handle,
    int mb,
    int nb,
   int nnzb,
   const cuComplex *bsrVal,
   const int *bsrRowPtr,
   const int *bsrColInd,
   int rowBlockDim,
   int colBlockDim,
   cuComplex *bscVal,
   int *bscRowInd,
   int *bscColPtr,
   cusparseAction_t copyValues,
   cusparseIndexBase_t baseIdx,
   void *pBuffer)
cusparseStatus t
cusparseZgebsr2gebsc(cusparseHandle t handle,
   int mb,
   int nb,
   int nnzb,
   const cuDoubleComplex *bsrVal,
   const int *bsrRowPtr,
   const int *bsrColInd,
   int rowBlockDim,
   int colBlockDim,
    cuDoubleComplex *bscVal,
    int *hscRowInd.
```

This function can be seen as the same as **csr2csc()** when each block of size **rowBlockDim\*colBlockDim** is regarded as a scalar.

This sparsity pattern of the result matrix can also be seen as the transpose of the original sparse matrix, but the memory layout of a block does not change.

The user must call <code>gebsr2gebsc\_bufferSize()</code> to determine the size of the buffer required by <code>gebsr2gebsc()</code>, allocate the buffer, and pass the buffer pointer to <code>gebsr2gebsc()</code>.

# Input

handle	handle to the cuSPARSE library context.
nanare	Handle to the cust ANSE tibrary context.
mb	number of block rows of sparse matrix A.
nb	number of block columns of sparse matrix A.
nnzb	number of nonzero blocks of matrix A.
bsrVal	<pre><type> array of nnzb*rowBlockDim*colBlockDim nonzero elements of matrix A.</type></pre>
bsrRowPtr	integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one.
bsrColInd	integer array of nnzb column indices of the non-zero blocks of matrix A.
rowBlockDim	number of rows within a block of A.
colBlockDim	number of columns within a block of A.
copyValues	CUSPARSE_ACTION_SYMBOLIC OF CUSPARSE_ACTION_NUMERIC.
baseIdx	CUSPARSE_INDEX_BASE_ZERO OF CUSPARSE_INDEX_BASE_ONE.
pBuffer	buffer allocated by the user; the size is return by gebsr2gebsc_bufferSize().

# Output

bscVal	<pre><type> array of nnzb*rowBlockDim*colBlockDim non-zero elements of matrix A. It is only filled-in if copyValues is set to CUSPARSE_ACTION_NUMERIC.</type></pre>
bscRowInd	integer array of nnzb row indices of the non-zero blocks of matrix A.
bscColPtr	integer array of nb+1 elements that contains the start of every block column and the end of the last block column plus one.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.

CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb,nb,nnzb<0, baseIdx is not base-0 or base-1, or rowBlockDim, colBlockDim<1).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 12.4. cusparse<t>gebsr2gebsr\_bufferSize()

```
cusparseStatus t
cusparseSgebsr\(\bar{2}\)gebsr bufferSize(cusparseHandle t handle,
   cusparseDirection t dir,
   int mb,
   int nb,
   int nnzb,
   const cusparseMatDescr t descrA,
   const float *bsrValA,
   const int *bsrRowPtrA,
   const int *bsrColIndA,
   int rowBlockDimA,
   int colBlockDimA,
   int rowBlockDimC,
   int colBlockDimC,
   int *pBufferSize )
cusparseStatus t
cusparseDgebsr2gebsr_bufferSize(cusparseHandle_t handle,
   cusparseDirection t dir,
   int mb,
   int nb,
   int nnzb,
   const cusparseMatDescr t descrA,
   const double *bsrValA,
   const int *bsrRowPtrA,
   const int *bsrColIndA,
   int rowBlockDimA,
   int colBlockDimA,
   int rowBlockDimC,
    int colBlockDimC,
    int *pBufferSize )
cusparseStatus t
cusparseCqebsr2qebsr bufferSize(cusparseHandle t handle,
   cusparseDirection t dir,
    int mb,
    int nb,
    const cusparseMatDescr t descrA,
   const cuComplex *bsrValA,
   const int *bsrRowPtrA,
   const int *bsrColIndA,
   int rowBlockDimA,
   int colBlockDimA,
    int rowBlockDimC,
    int colBlockDimC,
   int *pBufferSize )
cusparseStatus t
cusparseZgebsr2gebsr bufferSize(cusparseHandle t handle,
   cusparseDirection t dir,
   int mb,
   int nb,
   int nnzb,
   const cusparseMatDescr t descrA,
   const cuDoubleComplex *bsrValA,
   const int *bsrRowPtrA,
   const int *bsrColIndA,
   int rowBlockDimA,
   int colBlockDimA,
   int rowBlockDimC,
    int colBlockDimC,
    int *pBufferSize )
```

This function returns size of the buffer used in computing  ${\tt gebsr2gebsrNnz}$ () and  ${\tt gebsr2gebsr()}$ .

# Input

handle	handle to the cuSPARSE library context.
dir	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
mb	number of block rows of sparse matrix A.
nb	number of block columns of sparse matrix A.
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<pre><type> array of nnzb*rowBlockDimA*colBlockDimA non-zero elements of matrix A.</type></pre>
bsrRowPtrA	integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one of matrix A.
bsrColIndA	integer array of nnzb column indices of the nonzero blocks of matrix A.
rowBlockDimA	number of rows within a block of A.
colBlockDimA	number of columns within a block of A.
rowBlockDimC	number of rows within a block of c.
colBlockDimC	number of columns within a block of c

# Output

pBufferSize	host pointer containing number of bytes of the
	buffer used in gebsr2gebsr().

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	<pre>invalid parameters were passed (mb, nb, nnzb&lt;0; Or rowBlockDimA, colBlockDimA, rowBlockDimC, colBlockDimC&lt;1).</pre>
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 12.5. cusparse<t>gebsr2gebsr()

```
cusparseStatus t
cusparseXgebsr2gebsrNnz(cusparseHandle t handle,
   cusparseDirection t dir,
   int mb,
   int nb,
   int nnzb,
   const cusparseMatDescr t descrA,
   const int *bsrRowPtrA,
   const int *bsrColIndA,
   int rowBlockDimA,
   int colBlockDimA,
   const cusparseMatDescr t descrC,
   int *bsrRowPtrC,
   int rowBlockDimC,
   int colBlockDimC,
   int *nnzTotalDevHostPtr,
   void *pBuffer)
cusparseStatus t
cusparseSgebsr2gebsr(cusparseHandle t handle,
   cusparseDirection t dir,
   int mb,
   int nb,
   int nnzb,
   const cusparseMatDescr t descrA,
   const float *bsrValA,
   const int *bsrRowPtrA,
   const int *bsrColIndA,
   int rowBlockDimA,
   int colBlockDimA,
   const cusparseMatDescr t descrC,
    float *bsrValC,
   int *bsrRowPtrC,
   int *bsrColIndC,
    int rowBlockDimC,
   int colBlockDimC,
    void *pBuffer)
cusparseStatus t
cusparseDgebsr2gebsr(cusparseHandle t handle,
    cusparseDirection t dir,
    int mb,
   int nb,
   int nnzb,
   const cusparseMatDescr t descrA,
   const double *bsrValA,
   const int *bsrRowPtrA,
   const int *bsrColIndA,
   int rowBlockDimA,
   int colBlockDimA,
   const cusparseMatDescr t descrC,
   double *bsrValC,
   int *bsrRowPtrC,
   int *bsrColIndC,
   int rowBlockDimC,
   int colBlockDimC,
   void *pBuffer)
cusparseStatus t
cusparseCgebsr2gebsr(cusparseHandle_t handle,
    cusparseDirection t dir,
    int mb,
   int nb,
   int nnzh
```

This function converts a sparse matrix in general BSR format that is defined by the three arrays bsrValA, bsrRowPtrA, and bsrColIndA into a sparse matrix in another general BSR format that is defined by arrays bsrValC, bsrRowPtrC, and bsrColIndC.

If rowBlockDimA=1 and colBlockDimA=1, cusparse[S|D|C|Z]gebsr2gebsr() is the same as cusparse[S|D|C|Z]csr2gebsr().

If rowBlockDimC=1 and colBlockDimC=1, cusparse[S|D|C|Z]gebsr2gebsr() is the same as cusparse[S|D|C|Z]gebsr2csr().

A is an m\*n sparse matrix where m (=mb\*rowBlockDim) is the number of rows of A, and n (=nb\*colBlockDim) is the number of columns of A. The general BSR format of A contains nnzb(=bsrRowPtrA[mb] - bsrRowPtrA[0]) nonzero blocks. The matrix C is also general BSR format with a different block size, rowBlockDimC\*colBlockDimC. If m is not a multiple of rowBlockDimC, or n is not a multiple of colBlockDimC, zeros are filled in. The number of block rows of C is mc (= (m+rowBlockDimC-1) / rowBlockDimC). The number of block rows of C is nc (= (n+colBlockDimC-1) / colBlockDimC). The number of nonzero blocks of C is nnzc.

The implementation adopts a two-step approach to do the conversion.

First, the user allocates bsrRowPtrC of mc+1 elements and uses function cusparseXgebsr2gebsrNnz() to determine the number of nonzero block columns per block row of matrix C. Second, the user gathers nnzc (number of nonzero block columns of matrix C) from either (nnzc=\*nnzTotalDevHostPtr) or (nnzc=bsrRowPtrC[mc]-bsrRowPtrC[0]) and allocates bsrValC of nnzc\*rowBlockDimC\*colBlockDimC elements and bsrColIndC of nnzc integers. Finally the function cusparse[S|D|C|Z]gebsr2gebsr() is called to complete the conversion.

The user must call <code>gebsr2gebsr\_bufferSize()</code> to know the size of the buffer required by <code>gebsr2gebsr()</code>, allocate the buffer, and pass the buffer pointer to <code>gebsr2gebsr()</code>.

### The general procedure is as follows:

```
// Given general BSR format (bsrRowPtrA, bsrColIndA, bsrValA) and
// blocks of BSR format are stored in column-major order.
cusparseDirection t dir = CUSPARSE DIRECTION COLUMN;
int base, nnzc;
int m = mb*rowBlockDimA;
int n = nb*colBlockDimA;
int mc = (m+rowBlockDimC-1)/rowBlockDimC;
int nc = (n+colBlockDimC-1)/colBlockDimC;
int bufferSize;
void *pBuffer;
cusparseSgebsr2gebsr bufferSize(handle, dir, mb, nb, nnzb,
   descrA, bsrValA, bsrRowPtrA, bsrColIndA,
   rowBlockDimA, colBlockDimA,
   rowBlockDimC, colBlockDimC,
    &bufferSize);
cudaMalloc((void**)&pBuffer, bufferSize);
cudaMalloc((void**) &bsrRowPtrC, sizeof(int)*(mc+1));
// nnzTotalDevHostPtr points to host memory
int *nnzTotalDevHostPtr = &nnzc;
cusparseXgebsr2gebsrNnz(handle, dir, mb, nb, nnzb,
   descrA, bsrRowPtrA, bsrColIndA,
   rowBlockDimA, colBlockDimA,
   descrC, bsrRowPtrC,
   rowBlockDimC, colBlockDimC,
   nnzTotalDevHostPtr,
   pBuffer);
if (NULL != nnzTotalDevHostPtr) {
   nnzc = *nnzTotalDevHostPtr;
}else{
   cudaMemcpy(&nnzc, bsrRowPtrC+mc, sizeof(int), cudaMemcpyDeviceToHost);
   cudaMemcpy(&base, bsrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
   nnzc -= base;
cudaMalloc((void**)&bsrColIndC, sizeof(int)*nnzc);
cudaMalloc((void**)&bsrValC, sizeof(float)*(rowBlockDimC*colBlockDimC)*nnzc);
cusparseSgebsr2gebsr(handle, dir, mb, nb, nnzb,
   descrA, bsrValA, bsrRowPtrA, bsrColIndA,
   rowBlockDimA, colBlockDimA,
   descrC, bsrValC, bsrRowPtrC, bsrColIndC,
    rowBlockDimC, colBlockDimC,
   pBuffer);
```

handle	handle to the cuSPARSE library context.
dir	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
mb	number of block rows of sparse matrix A.
nb	number of block columns of sparse matrix A.
nnzb	number of nonzero blocks of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<pre><type> array of nnzb*rowBlockDimA*colBlockDimA non-zero elements of matrix A.</type></pre>

bsrRowPtrA	integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one of matrix A.
bsrColIndA	integer array of nnzb column indices of the non-zero blocks of matrix A.
rowBlockDimA	number of rows within a block of A.
colBlockDimA	number of columns within a block of A.
descrC	the descriptor of matrix c. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
rowBlockDimC	number of rows within a block of c.
colBlockDimC	number of columns within a block of c.
pBuffer	buffer allocated by the user; the size is return by gebsr2gebsr_bufferSize().

bsrValC	<pre><type> array of nnzc*rowBlockDimC*colBlockDimC non-zero elements of matrix c.</type></pre>
bsrRowPtrC	integer array of mc+1 elements that contains the start of every block row and the end of the last block row plus one of matrix c.
bsrColIndC	integer array of nnzc block column indices of the nonzero blocks of matrix c.
nnzTotalDevHostPtr	total number of nonzero blocks of c.  *nnzTotalDevHostPtr is the same as bsrRowPtrC[mc]-bsrRowPtrC[0].

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb, nb, nnzb<0, baseIdx is not base-0 or base-1; or rowBlockDimA, colBlockDimA, rowBlockDimC, colBlockDimC<1).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 12.6. cusparse<t>gebsr2csr()

```
cusparseStatus t
cusparseSgebsr2csr(cusparseHandle t handle,
   cusparseDirection t dir,
   int mb,
   int nb,
   const cusparseMatDescr t descrA,
   const float *bsrValA,
   int rowBlockDim,
   int colBlockDim,
   const cusparseMatDescr t descrC,
   float *csrValC,
   int    *csrRowPtrC,
int    *csrColIndC)
cusparseStatus t
cusparseDgebsr2csr(cusparseHandle_t handle,
   cusparseDirection t dir,
   int mb,
   int nb,
   const cusparseMatDescr t descrA,
   const double *bsrValA,
   int rowBlockDim,
         colBlockDim,
   const cusparseMatDescr t descrC,
   double *csrValC,
   int *csrcolIndC)
cusparseStatus t
cusparseCgebsr2csr(cusparseHandle t handle,
   cusparseDirection t dir,
   int mb,
   int nb,
   const cusparseMatDescr t descrA,
   const cuComplex *bsrValA,
   const int *bsrRowPtrA,
               *bsrColIndA,
   const int
   int rowBlockDim,
int colBlockDim,
   const cusparseMatDescr t descrC,
   cuComplex *csrValC,
        *csrRowPtrC,
   int
         *csrColIndC )
   int
cusparseStatus t
cusparseZgebsr\overline{2}csr(cusparseHandle_t handle,
   cusparseDirection_t dir,
   int mb,
   int nb,
   const cusparseMatDescr t descrA,
   const cuDoubleComplex *bsrValA,
   const int *bsrRowPtrA,
               *bsrColIndA,
   const int
   int rowBlockDim,
   int colBlockDim,
   const cusparseMatDescr t descrC,
   cuDoubleComplex *csrValC,
   int *csrRowPtrC,
          *csrColIndC )
   int
```

This function converts a sparse matrix in general BSR format that is defined by the three arrays bsrValA, bsrRowPtrA, and bsrColIndA into a sparse matrix in CSR format that is defined by arrays csrValC, csrRowPtrC, and csrColIndC.

Let m (=mb\*rowBlockDim) be number of rows of A and n (=nb\*colBlockDim) be number of columns of A, then A and C are m\*n sparse matrices. The general BSR format of A contains nnzb(=bsrRowPtrA[mb] - bsrRowPtrA[0]) non-zero blocks, whereas sparse matrix A contains nnz (=nnzb\*rowBlockDim\*colBlockDim) elements. The user must allocate enough space for arrays csrRowPtrC, csrColIndC, and csrValC. The requirements are as follows:

csrRowPtrC of m+1 elements

csrValC of nnz elements

csrColIndC of nnz elements

The general procedure is as follows:

handle	handle to the cuSPARSE library context.
dir	storage format of blocks, either CUSPARSE_DIRECTION_ROW OF CUSPARSE_DIRECTION_COLUMN.
mb	number of block rows of sparse matrix A.
nb	number of block columns of sparse matrix ${f a}$ .
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
bsrValA	<pre><type> array of nnzb*rowBlockDim*colBlockDim non-zero elements of matrix A.</type></pre>
bsrRowPtrA	integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one of matrix A.
bsrColIndA	integer array of nnzb column indices of the non-zero blocks of matrix A.

rowBlockDim	number of rows within a block of A.
colBlockDim	number of columns within a block of A.
descrC	the descriptor of matrix c. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.

csrValC	<pre><type> array of <math>nnz</math> non-zero elements of matrix c.</type></pre>
csrRowPtrC	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one of matrix $c$ .
csrColIndC	integer array of nnz column indices of the non-zero elements of matrix c.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (mb, nb<0 is not base-0 or base-1, or rowBlockDim, colBlockDim<1).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 12.7. cusparse<t>csr2gebsr\_bufferSize()

```
cusparseStatus t
cusparseScsr2gebsr bufferSize(cusparseHandle t handle,
   cusparseDirection t dir,
   int m,
   int n,
   const cusparseMatDescr t descrA,
   const float *csrValA,
   const int *csrRowPtrA,
   const int *csrColIndA,
   int rowBlockDim,
   int colBlockDim,
   int *pBufferSize)
cusparseStatus t
cusparseDcsr2gebsr_bufferSize(cusparseHandle_t handle,
   cusparseDirection_t dir,
   int m,
   int n,
   const cusparseMatDescr t descrA,
   const double *csrValA,
   const int *csrRowPtrA,
   const int *csrColIndA,
   int rowBlockDim,
   int colBlockDim,
   int *pBufferSize)
cusparseStatus t
cusparseCcsr2qebsr bufferSize(cusparseHandle t handle,
   cusparseDirection t dir,
    int m,
   int n,
   const cusparseMatDescr t descrA,
   const cuComplex *csrValA,
   const int *csrRowPtrA,
   const int *csrColIndA,
   int rowBlockDim,
    int colBlockDim,
   int *pBufferSize)
cusparseStatus t
cusparseZcsr2gebsr bufferSize(cusparseHandle t handle,
   cusparseDirection t dir,
   int m,
   int n,
   const cusparseMatDescr t descrA,
   const cuDoubleComplex *csrValA,
   const int *csrRowPtrA,
   const int *csrColIndA,
   int rowBlockDim,
    int colBlockDim,
   int *pBufferSize)
```

This function returns the size of the buffer used in computing csr2gebsrNnz and csr2gebsr.

## Input

handle	handle to the cuSPARSE library context.
dir	storage format of blocks, either CUSPARSE_DIRECTION_ROW OR CUSPARSE_DIRECTION_COLUMN.
m	number of rows of sparse matrix A.
n	number of columns of sparse matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<type> array of nnz nonzero elements of matrix A.</type>
csrRowPtrA	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one of matrix $A$ .
csrColIndA	integer array of nnz column indices of the nonzero elements of matrix A.
descrC	the descriptor of matrix c. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
rowBlockDim	number of rows within a block of c.
colBlockDim	number of columns within a block of c.

# Output

pBufferSize	host pointer containing number of bytes of the buffer used in csr2gebsrNnz() and
	csr2gebsr().

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n<0, or rowBlockDim, colBlockDim<1).
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 12.8. cusparse<t>csr2gebsr()

```
cusparseStatus t
cusparseXcsr2gebsrNnz(cusparseHandle t handle,
   cusparseDirection t dir,
   int m,
   int n,
   const cusparseMatDescr t descrA,
   const int *csrRowPtrA,
   const int *csrColIndA,
   const cusparseMatDescr t descrC,
   int *bsrRowPtrC,
   int rowBlockDim,
   int colBlockDim,
   int *nnzTotalDevHostPtr,
   void *pBuffer )
cusparseStatus t
cusparseScsr2gebsr(cusparseHandle_t handle,
   cusparseDirection t dir,
   int m,
   int n,
   const cusparseMatDescr t descrA,
   const float *csrValA,
   const int *csrRowPtrA,
   const int *csrColIndA,
   const cusparseMatDescr_t descrC,
   float *bsrValC,
   int *bsrRowPtrC,
   int *bsrColIndC,
   int rowBlockDim,
   int colBlockDim,
   void *pBuffer)
cusparseStatus t
cusparseDcsr2gebsr(cusparseHandle t handle,
   cusparseDirection t dir,
    int m,
   int n,
   const cusparseMatDescr t descrA,
    const double *csrValA,
   const int *csrRowPtrA,
   const int *csrColIndA,
   const cusparseMatDescr t descrC,
   double *bsrValC,
   int *bsrRowPtrC,
   int *bsrColIndC,
   int rowBlockDim,
   int colBlockDim,
   void *pBuffer)
cusparseStatus t
cusparseCcsr2gebsr(cusparseHandle t handle,
   cusparseDirection t dir,
   int m,
   int n,
   const cusparseMatDescr t descrA,
   const cuComplex *csrValA,
   const int *csrRowPtrA,
   const int *csrColIndA,
   const cusparseMatDescr_t descrC,
   cuComplex *bsrValC,
   int *bsrRowPtrC,
   int *bsrColIndC,
    int rowBlockDim.
```

This function converts a sparse matrix **A** in CSR format (that is defined by arrays **csrValA**, **csrRowPtrA**, and **csrColIndA**) into a sparse matrix **C** in general BSR format (that is defined by the three arrays **bsrValC**, **bsrRowPtrC**, and **bsrColIndC**).

The matrix A is a m\*n sparse matrix and matrix C is a (mb\*rowBlockDim) \* (nb\*colBlockDim) sparse matrix, where mb (= (m +rowBlockDim-1) /rowBlockDim) is the number of block rows of C, and nb (= (n +colBlockDim-1) /colBlockDim) is the number of block columns of C.

The block of **C** is of size **rowBlockDim\*colBlockDim**. If **m** is not multiple of **rowBlockDim** or **n** is not multiple of **colBlockDim**, zeros are filled in.

The implementation adopts a two-step approach to do the conversion. First, the user allocates <code>bsrRowPtrC</code> of <code>mb+1</code> elements and uses function <code>cusparseXcsr2gebsrNnz()</code> to determine the number of nonzero block columns per block row. Second, the user gathers <code>nnzb</code> (number of nonzero block columns of matrix <code>C</code>) from either (<code>nnzb=\*nnzTotalDevHostPtr</code>) or (<code>nnzb=bsrRowPtrC[mb]-bsrRowPtrC[0]</code>) and allocates <code>bsrValC</code> of <code>nnzb\*rowBlockDim\*colBlockDim</code> elements and <code>bsrColIndC</code> of <code>nnzb</code> integers. Finally function <code>cusparse[S|D|C|Z]csr2gebsr()</code> is called to complete the conversion.

The user must obtain the size of the buffer required by csr2gebsr() by calling csr2gebsr\_bufferSize(), allocate the buffer, and pass the buffer pointer to csr2gebsr().

### The general procedure is as follows:

```
// Given CSR format (csrRowPtrA, csrColIndA, csrValA) and
// blocks of BSR format are stored in column-major order.
cusparseDirection_t dir = CUSPARSE_DIRECTION_COLUMN;
int base, nnzb;
int mb = (m + rowBlockDim-1)/rowBlockDim;
int nb = (n + colBlockDim-1)/colBlockDim;
int bufferSize;
void *pBuffer;
cusparseScsr2gebsr bufferSize(handle, dir, m, n,
   descrA, csrValA, csrRowPtrA, csrColIndA,
   rowBlockDim, colBlockDim,
   &bufferSize);
cudaMalloc((void**)&pBuffer, bufferSize);
cudaMalloc((void**)&bsrRowPtrC, sizeof(int) *(mb+1));
// nnzTotalDevHostPtr points to host memory
int *nnzTotalDevHostPtr = &nnzb;
cusparseXcsr2gebsrNnz(handle, dir, m, n,
   descrA, csrRowPtrA, csrColIndA,
   descrC, bsrRowPtrC, rowBlockDim, colBlockDim,
   nnzTotalDevHostPtr,
   pBuffer);
if (NULL != nnzTotalDevHostPtr) {
   nnzb = *nnzTotalDevHostPtr;
}else{
   cudaMemcpy(&nnzb, bsrRowPtrC+mb, sizeof(int), cudaMemcpyDeviceToHost);
   cudaMemcpy(&base, bsrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
   nnzb -= base;
cudaMalloc((void**)&bsrColIndC, sizeof(int)*nnzb);
cudaMalloc((void**)&bsrValC, sizeof(float)*(rowBlockDim*colBlockDim)*nnzb);
cusparseScsr2gebsr(handle, dir, m, n,
       descrA,
       csrValA, csrRowPtrA, csrColIndA,
        descrC,
       bsrValC, bsrRowPtrC, bsrColIndC,
        rowBlockDim, colBlockDim,
       pBuffer);
```

handle	handle to the cuSPARSE library context.
dir	storage format of blocks, either CUSPARSE_DIRECTION_ROW OR CUSPARSE_DIRECTION_COLUMN.
m	number of rows of sparse matrix A.
n	number of columns of sparse matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<type> array of <math>\mathtt{nnz}</math> nonzero elements of matrix <math>\mathtt{A}</math>.</type>
csrRowPtrA	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one of matrix $A$ .
csrColIndA	integer array of nnz column indices of the nonzero elements of matrix A.

descrC	the descriptor of matrix c. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
rowBlockDim	number of rows within a block of c.
colBlockDim	number of columns within a block of c.
pBuffer	buffer allocated by the user, the size is return by csr2gebsr_bufferSize().

bsrValC	<pre><type> array of nnzb*rowBlockDim*colBlockDim nonzero elements of matrix c.</type></pre>
bsrRowPtrC	integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one of matrix c.
bsrColIndC	integer array of nnzb column indices of the nonzero blocks of matrix c.
nnzTotalDevHostPtr	total number of nonzero blocks of matrix c. Pointer nnzTotalDevHostPtr can point to a device memory or host memory.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n<0, baseIdx is not base-0 or base-1, or rowBlockDim, colBlockDim<1).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 12.9. cusparse<t>coo2csr()

This function converts the array containing the uncompressed row indices (corresponding to COO format) into an array of compressed row pointers (corresponding to CSR format).

It can also be used to convert the array containing the uncompressed column indices (corresponding to COO format) into an array of column pointers (corresponding to CSC format).

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

### Input

handle	handle to the cuSPARSE library context.
cooRowInd	integer array of nnz uncompressed row indices.
nnz	number of non-zeros of the sparse matrix (that is also the length of array cooRowInd).
m	number of rows of matrix A.
idxBase	CUSPARSE_INDEX_BASE_ZERO OF CUSPARSE_INDEX_BASE_ONE.

## Output

integer array of m+1 elements that contains the start of every row and the end of the last row plus
one.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	idxBase is neither CUSPARSE_INDEX_BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

# 12.10. cusparse<t>csc2dense()

```
cusparseStatus t
cusparseScsc2dense(cusparseHandle t handle, int m, int n,
                  const cusparseMatDescr t descrA,
                  const float *cscValA,
                  const int *cscRowIndA, const int *cscColPtrA,
                                 *A, int lda)
cusparseStatus t
cusparseDcsc2dense(cusparseHandle t handle, int m, int n,
                  const cusparseMatDescr t descrA,
                  const double \overline{\phantom{a}}cscValA,
                  const int *cscRowIndA, const int *cscColPtrA,
                 double
                                *A, int lda)
cusparseStatus t
cusparseCcsc2dense(cusparseHandle t handle, int m, int n,
                  const cusparseMatDescr_t descrA,
                  const cuComplex *cscValA,
                  const int *cscRowIndA, const int *cscColPtrA,
                                 *A, int lda)
                  cuComplex
cusparseStatus t
cusparseZcsc2dense(cusparseHandle_t handle, int m, int n,
                  const cusparseMatDescr_t descrA,
                  const cuDoubleComplex *cscValA,
                  const int *cscRowIndA, const int *cscColPtrA,
                  cuDoubleComplex *A, int lda)
```

This function converts the sparse matrix in CSC format that is defined by the three arrays cscValA, cscColPtrA, and cscRowIndA into the matrix A in dense format. The dense matrix A is filled in with the values of the sparse matrix and with zeros elsewhere.

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

handle	handle to the cuSPARSE library context.
m	number of rows of matrix A.
n	number of columns of matrix A.
descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
cscValA	<pre><type> array of nnz (= cscColPtrA(m) - cscColPtrA(0) ) nonzero elements of matrix A.</type></pre>
cscRowIndA	integer array of nnz (= cscColPtrA(m) - cscColPtrA(0) ) row indices of the nonzero elements of matrix A.
cscColPtrA	integer array of n+1 elements that contains the start of every row and the end of the last column plus one.
lda	leading dimension of dense array A.

A	array of dimensions (1da, n) that is filled in with
	the values of the sparse matrix.

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 12.11. cusparse<t>csc2hyb()

```
cusparseStatus t
cusparseScsc2hyb(cusparseHandle t handle, int m, int n,
           const cusparseMatDescr t descrA,
           const float *cscValA,
           const int *cscRowIndA, const int *cscColPtrA,
           cusparseHybMat_t hybA, int userEllWidth,
           cusparseHybPartition t partitionType)
cusparseStatus t
cusparseDcsc2hyb(cusparseHandle t handle, int m, int n,
           const cusparseMatDescr t descrA,
           const double \overline{*}cscValA,
           const int *cscRowIndA, const int *cscColPtrA,
           cusparseHybMat t hybA, int userEllWidth,
           cusparseHybPartition t partitionType)
cusparseStatus t
cusparseCcsc2hyb(cusparseHandle t handle, int m, int n,
           const cusparseMatDescr t descrA,
           const cuComplex *cscValA,
           const int *cscRowIndA, const int *cscColPtrA,
           cusparseHybMat t hybA, int userEllWidth,
           cusparseHybPartition t partitionType)
cusparseStatus t
cusparseZcsc2hyb(cusparseHandle t handle, int m, int n,
           const cusparseMatDescr t descrA,
           const cuDoubleComplex *cscValA,
           const int *cscRowIndA, const int *cscColPtrA,
           cusparseHybMat t hybA, int userEllWidth,
           cusparseHybPartition t partitionType)
```

This function converts a sparse matrix in CSC format into a sparse matrix in HYB format. It assumes that the hybA parameter has been initialized with the cusparseCreateHybMat() routine before calling this function.

This function requires some amount of temporary storage and a significant amount of storage for the matrix in HYB format. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

handle	handle to the cuSPARSE library context.
m	number of rows of matrix A.
n	number of columns of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
cscValA	<pre><type> array of nnz (= cscColPtrA(m) - cscColPtrA(0) ) nonzero elements of matrix A.</type></pre>
cscRowIndA	integer array of nnz (= cscColPtrA(m) - cscColPtrA(0) ) column indices of the nonzero elements of matrix A.
cscColPtrA	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
userEllWidth	width of the regular (ELL) part of the matrix in HYB format, which should be less than the maximum number of nonzeros per row and is only required if partitionType == CUSPARSE_HYB_PARTITION_USER.
partitionType	partitioning method to be used in the conversion (please refer to cusparseHybPartition_t for details).

hybA
------

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 12.12. cusparse<t>csr2bsr()

```
cusparseStatus t
cusparseXcsr2bsrNnz(cusparseHandle t handle,
      cusparseDirection t dir,
      int m,
      int n,
      const cusparseMatDescr t descrA,
      const int *csrRowPtrA,
      const int *csrColIndA,
      int blockDim,
      const cusparseMatDescr t descrC,
      int *bsrRowPtrC,
      int *nnzTotalDevHostPtr)
cusparseStatus t
cusparseScsr2bsr(cusparseHandle t handle,
      cusparseDirection_t dir,
      int m,
      int n,
      const cusparseMatDescr_t descrA,
      const float *csrValA,
     const int *csrRowPtrA,
      const int *csrColIndA,
      int blockDim,
      const cusparseMatDescr t descrC,
      float *bsrValC,
      int *bsrRowPtrC,
      int *bsrColIndC)
cusparseStatus_t
cusparseDcsr2bsr(cusparseHandle t handle,
      cusparseDirection t dir,
      int m,
      int n,
      const cusparseMatDescr t descrA,
      const double *csrValA,
      const int *csrRowPtrA,
      const int *csrColIndA,
      int blockDim,
      const cusparseMatDescr t descrC,
      double *bsrValC,
      int *bsrRowPtrC
      int *bsrColIndC)
cusparseStatus t
cusparseCcsr2bsr(cusparseHandle t handle,
      cusparseDirection t dir,
      int m,
      int n,
      const cusparseMatDescr t descrA,
      const cuComplex *csrValA,
      const int *csrRowPtrA,
      const int *csrColIndA,
      int blockDim,
      const cusparseMatDescr t descrC,
      cuComplex *bsrValC,
      int *bsrRowPtrC,
      int *bsrColIndC)
cusparseStatus t
cusparseZcsr2bsr(cusparseHandle t handle,
      cusparseDirection t dir,
      int m,
      int n,
      const cusparseMatDescr_t descrA,
      const cuDoubleComplex *csrValA,
      const int *csrRowPtrA,
      const int *csrColIndA,
      int blockDim.
```

This function converts a sparse matrix in CSR format that is defined by the three arrays csrValA, csrRowPtrA, and csrColIndA into a sparse matrix in BSR format that is defined by arrays bsrValC, bsrRowPtrC, and bsrColIndC.

A is an m\*n sparse matrix. The BSR format of A has mb block rows, nb block columns, and nnzb nonzero blocks, where mb=((m+blockDim-1)/blockDim) and nb=(n+blockDim-1)/blockDim.

If **m** or **n** is not multiple of **blockDim**, zeros are filled in.

The conversion in cuSPARSE entails a two-step approach. First, the user allocates bsrRowPtrC of mb+1 elements and uses function cusparseXcsr2bsrNnz() to determine the number of nonzero block columns per block row. Second, the user gathers nnzb (number of non-zero block columns of matrix C) from either (nnzb=\*nnzTotalDevHostPtr) or (nnzb=bsrRowPtrC[mb]-bsrRowPtrC[0]) and allocates bsrValC of nnzb\*blockDim\*blockDim elements and bsrColIndC of nnzb elements. Finally function cusparse[S|D|C|Z]csr2bsr90 is called to complete the conversion.

The general procedure is as follows:

```
// Given CSR format (csrRowPtrA, csrcolIndA, csrValA) and
// blocks of BSR format are stored in column-major order.
cusparseDirection t dir = CUSPARSE DIRECTION COLUMN;
int base, nnzb;
int mb = (m + blockDim-1)/blockDim;
cudaMalloc((void**)&bsrRowPtrC, sizeof(int) *(mb+1));
// nnzTotalDevHostPtr points to host memory
int *nnzTotalDevHostPtr = &nnzb;
cusparseXcsr2bsrNnz(handle, dir, m, n,
       descrA, csrRowPtrA, csrColIndA,
       blockDim,
       descrC, bsrRowPtrC,
      nnzTotalDevHostPtr);
if (NULL != nnzTotalDevHostPtr) {
   nnzb = *nnzTotalDevHostPtr;
   cudaMemcpy(&nnzb, bsrRowPtrC+mb, sizeof(int), cudaMemcpyDeviceToHost);
   cudaMemcpy(&base, bsrRowPtrC, sizeof(int), cudaMemcpyDeviceToHost);
   nnzb -= base;
cudaMalloc((void**)&bsrColIndC, sizeof(int)*nnzb);
cudaMalloc((void**)&bsrValC, sizeof(float)*(blockDim*blockDim)*nnzb);
cusparseScsr2bsr(handle, dir, m, n,
       descrA,
       csrValA, csrRowPtrA, csrColIndA,
       blockDim,
       descrC,
       bsrValC, bsrRowPtrC, bsrColIndC);
```

If blockDim is large (typically, a block cannot fit into shared memory), cusparse[S] D|C|Z]csr2bsr() allocates a temporary integer array of size mb\*blockDim integers. If device memory is not available, CUSPARSE STATUS ALLOC FAILED is returned.

handle	handle to the cuSPARSE library context.
dir	storage format of blocks, either CUSPARSE_DIRECTION_ROW OR CUSPARSE_DIRECTION_COLUMN.

m	number of rows of sparse matrix A.
n	number of columns of sparse matrix A.
descrA	the descriptor of matrix A.
csrValA	<pre><type> array of nnz (=csrRowPtrA[m] - csrRowPtr[0]) non-zero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz column indices of the non-zero elements of matrix A.
blockDim	block dimension of sparse matrix $A$ . The range of blockDim is between 1 and min $(m,n)$ .
descrC	the descriptor of matrix c.

bsrValC	<pre><type> array of nnzb*blockDim*blockDim nonzero elements of matrix c.</type></pre>
bsrRowPtrC	integer array of mb+1 elements that contains the start of every block row and the end of the last block row plus one of matrix c.
bsrColIndC	integer array of nnzb column indices of the non-zero blocks of matrix c.
nnzTotalDevHostPtr	total number of nonzero elements in device or host memory. It is equal to (bsrRowPtrC[mb]-bsrRowPtrC[0]).

## **Status Returned**

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n<0).  IndexBase field of descrA, descrC is not base-0 or base-1, dir is not row-major or column-major, or blockDim is not between 1 and min (m,n).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED the matrix type is not supported.	
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 12.13. cusparse<t>csr2coo()

This function converts the array containing the compressed row pointers (corresponding to CSR format) into an array of uncompressed row indices (corresponding to COO format).

It can also be used to convert the array containing the compressed column indices (corresponding to CSC format) into an array of uncompressed column indices (corresponding to COO format).

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

### Input

handle	handle to the cuSPARSE library context.
csrRowPtr	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.
nnz	number of nonzeros of the sparse matrix (that is also the length of array cooRowInd).
m	number of rows of matrix A.
idxBase	CUSPARSE_INDEX_BASE_ZERO OF CUSPARSE_INDEX_BASE_ONE.

## Output

cooRowInd	integer array of nnz uncompressed row indices.
-----------	--

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	idxBase is neither CUSPARSE_INDEX_BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

# 12.14. cusparse<t>csr2csc()

```
cusparseStatus t
cusparseScsr2csc(cusparseHandle t handle, int m, int n, int nnz,
                const float *csrVal, const int *csrRowPtr,
                                             *cscVal,
                const int *csrColInd, float
                int *cscRowInd, int *cscColPtr,
                cusparseAction t copyValues,
                cusparseIndexBase t idxBase)
cusparseStatus t
cusparseDcsr2csc(cusparseHandle t handle, int m, int n, int nnz,
                const double *csrVal, const int *csrRowPtr,
                const int *csrColInd, double
                                               *cscVal,
                int *cscRowInd, int *cscColPtr,
                cusparseAction t copyValues,
                cusparseIndexBase t idxBase)
cusparseStatus t
cusparseCcsr2csc(cusparseHandle_t handle, int m, int n, int nnz,
                const cuComplex *csrVal, const int *csrRowPtr,
                const int *csrColInd, cuComplex *cscVal,
                int *cscRowInd, int *cscColPtr,
                cusparseAction_t copyValues,
                cusparseIndexBase t idxBase)
cusparseStatus t
cusparseZcsr2csc(cusparseHandle t handle, int m, int nnz,
                const cuDoubleComplex *csrVal, const int *csrRowPtr,
                const int *csrColInd, cuDoubleComplex *cscVal,
                int *cscRowInd, int *cscColPtr,
                cusparseAction_t copyValues,
                cusparseIndexBase t idxBase)
```

This function converts a sparse matrix in CSR format (that is defined by the three arrays csrVal, csrRowPtr, and csrColInd) into a sparse matrix in CSC format (that is defined by arrays cscVal, cscRowInd, and cscColPtr). The resulting matrix can also be seen as the transpose of the original sparse matrix. Notice that this routine can also be used to convert a matrix in CSC format into a matrix in CSR format.

This function requires a significant amount of extra storage that is proportional to the matrix size. It is executed asynchronously with respect to the host, and it may return control to the application on the host before the result is ready.

handle	handle to the cuSPARSE library context.
m	number of rows of matrix A.
n	number of columns of matrix A.
nnz	number of nonzero elements of matrix A.
csrVal	<pre><type> array of nnz (= csrRowPtr(m) - csrRowPtr(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtr	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.

csrColInd	integer array of nnz (= csrRowPtr(m) - csrRowPtr(0) ) column indices of the nonzero elements of matrix A.
copyValues	CUSPARSE_ACTION_SYMBOLIC OF CUSPARSE_ACTION_NUMERIC.
idxBase	CUSPARSE_INDEX_BASE_ZERO OF CUSPARSE_INDEX_BASE_ONE.

cscVal	<pre><type> array of nnz (= cscColPtr(n) - cscColPtr(0) ) nonzero elements of matrix A. It is only filled in if copyValues is set to CUSPARSE_ACTION_NUMERIC.</type></pre>
cscRowInd	integer array of nnz (= cscColPtr(n) - cscColPtr(0) ) column indices of the nonzero elements of matrix A.
cscColPtr	integer array of n+1 elements that contains the start of every column and the end of the last column plus one.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,nnz<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

# 12.15. cusparseCsr2cscEx()

This function is an extended version of **cusparse<t>csr2csc()**. For detailed description of the functionality, see **cusparse<t>csr2csc()**.

This function does not support half-precision execution type, but it supports half-precision IO with single precision execution.

### Input specifically required by cusparseCsr2cscEx

csrSortedValAtype	Data type of csrSortedValA.
cscSortedValAtype	Data type of cscSortedValA.
executiontype	Data type used for computation.

# 12.16. cusparse<t>csr2dense()

```
cusparseStatus t cusparseScsr2dense(cusparseHandle t handle,
                                     int m,
                                     int n,
                                     const cusparseMatDescr t descrA,
                                     const float *csrValA,
                                     const int *csrRowPtrA,
                                     const int *csrColIndA,
                                     float *A,
                                     int lda)
cusparseStatus t cusparseDcsr2dense(cusparseHandle t handle,
                                    int m,
                                     int n,
                                     const cusparseMatDescr t descrA,
                                     const double *csrValA,
                                     const int *csrRowPtrA,
                                     const int *csrColIndA,
                                     double *A,
                                     int lda)
cusparseStatus t cusparseCcsr2dense(cusparseHandle t handle,
                                     int m,
                                     int n,
                                     const cusparseMatDescr t descrA,
                                     const cuComplex *csrValA,
                                     const int *csrRowPtrA,
                                     const int *csrColIndA,
                                     cuComplex *A,
                                     int lda)
cusparseStatus t cusparseZcsr2dense(cusparseHandle t handle,
                                     int m,
                                     int n,
                                     const cusparseMatDescr t descrA,
                                     const cuDoubleComplex *csrValA,
                                     const int *csrRowPtrA,
                                     const int *csrColIndA,
                                     cuDoubleComplex *A,
                                     int lda)
```

This function converts the sparse matrix in CSR format (that is defined by the three arrays csrValA, csrRowPtrA, and csrColIndA) into the matrix A in dense format. The dense matrix A is filled in with the values of the sparse matrix and with zeros elsewhere.

This function requires no extra storage. It is executed asynchronously with respect to the host, and it may return control to the application on the host before the result is ready.

## Input

handle	handle to the cuSPARSE library context.
m	number of rows of matrix A.
n	number of columns of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of m+1 elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column indices of the nonzero elements of matrix A.
lda	leading dimension of array matrixA.

## Output

A	array of dimensions (lda,n) that is filled in with
	the values of the sparse matrix.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED the function failed to launch on the GPU.	
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED the matrix type is not supported.	

# 12.17. cusparse<t>csr2csr\_compress()

```
cusparseStatus t cusparseScsr2csr compress(cusparseHandle t handle,
                                     int m,
                                     const cusparseMatDescr t descrA,
                                     const float *csrValA,
                                     const int *csrColIndA,
                                     const int *csrRowPtrA,
                                     int inNnz,
                                     int *nnzPerRow,
                                     float *outVal,
                                     int *outColInd,
                                     int *outRowPtr,
                                     float tol);
cusparseStatus t cusparseDcsr2csr compress(cusparseHandle t handle,
                                     int m,
                                     int n,
                                     const cusparseMatDescr_t descrA,
                                     const double *csrValA,
                                     const int *csrColIndA,
                                     const int *csrRowPtrA,
                                     int inNnz,
                                     int *nnzPerRow,
                                     double *outVal,
                                     int *outColInd,
                                     int *outRowPtr,
                                     double tol);
cusparseStatus t cusparseCcsr2csr compress(cusparseHandle t handle,
                                     int m,
                                     int n,
                                     const cusparseMatDescr t descrA,
                                     const cuComplex *csrValA,
                                     const int *csrColIndA,
                                     const int *csrRowPtrA,
                                     int inNnz,
                                     int *nnzPerRow,
                                     cuComplex *outVal,
                                     int *outColInd,
                                     int *outRowPtr,
                                     cuComplex tol);
cusparseStatus t cusparseZcsr2csr compress(cusparseHandle t handle,
                                     int m,
                                     int n,
                                     const cusparseMatDescr t descrA,
                                     const cuDoubleComplex *csrValA,
                                     const int *csrColIndA,
                                     const int *csrRowPtrA,
                                     int inNnz,
                                     int *nnzPerRow, //output: returns
number of nonzeros per row
                                     cuDoubleComplex *outVal,
                                     int *outColInd,
                                     int *outRowPtr,
                                     cuDoubleComplex tol);
```

This function compresses the sparse matrix in CSR format (that is defined by the three arrays csrValA, csrRowPtrA, and csrColIndA) into the matrix A in csr format, but with small values removed. The csr matrix on output is filled with the values of the sparse matrix which have magnitude exceeding the tol threshold. Note, in the case of complex values, only the magnitude of the real part is used in the check.

This function requires the user to allocate one integer for the output nnzPerRow value, a typed value array for the outVal array, and two integer arrays for the column indices and row pointer arrays. It is executed asynchronously with respect to the host, and it may return control to the application on the host before the result is ready.

### Input

handle	handle to the cuSPARSE library context.
m	number of rows of matrix A.
n	number of columns of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) elements of matrix A.</type></pre>
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column indices of the elements of matrix A.
csrRowPtrA	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.
inNnz	number of elements in matrixa.
tol	On input, this contains the non-negative tolerance value used for compression. Any values in matrix A smaller than this value will be dropped during compression.

## Output

nnzPerRow	On output, this array contains the number of elements kept in the compressed matrix, by row. An associated totalNnz is used internally to size the return values and column indices arrays.
outVal	On output, this array contains the typed values of elements kept in the compressed matrix. Size = totalNnz
outColInd	On output, this integer array contains the column indices of elements kept in the compressed matrix. Size = totalNnz
outRowPtr	On output, this integer array contains the row pointers for elements kept in the compressed matrix. Size = m+1

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED the matrix type is not supported.	

# 12.18. cusparse<t>csr2hyb()

```
cusparseStatus t
cusparseScsr2hyb(cusparseHandle t handle, int m, int n,
            const cusparseMatDescr t descrA,
            const float
                                    *csrValA,
            const int *csrRowPtrA, const int *csrColIndA, cusparseHybMat_t hybA, int userEllWidth,
            cusparseHybPartition t partitionType)
cusparseStatus t
cusparseDcsr2hyb(cusparseHandle t handle, int m, int n,
            const cusparseMatDescr t descrA,
            const double *csrValA,
            const int *csrRowPtrA, const int *csrColIndA,
            cusparseHybMat_t hybA, int userEllWidth,
            cusparseHybPartition t partitionType)
cusparseStatus t
cusparseCcsr2hyb(cusparseHandle t handle, int m, int n,
            const cusparseMatDescr t descrA,
            const cuComplex *csrValA,
            const int *csrRowPtrA, const int *csrColIndA,
            cusparseHybMat t hybA, int userEllWidth,
            cusparseHybPartition t partitionType)
cusparseStatus t
cusparseZcsr2hyb(cusparseHandle t handle, int m, int n,
            const cusparseMatDescr_t descrA,
            const cuDoubleComplex *csrValA,
const int *csrRowPtrA, const int *csrColIndA,
            cusparseHybMat t hybA, int userEllWidth,
            cusparseHybPartition_t partitionType)
```

This function converts a sparse matrix in CSR format into a sparse matrix in HYB format. It assumes that the hybA parameter has been initialized with cusparseCreateHybMat() routine before calling this function.

This function requires some amount of temporary storage and a significant amount of storage for the matrix in HYB format. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

handle	handle to the cuSPARSE library context.
m	number of rows of matrix A.
n	number of columns of matrix A.

descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.
userEllWidth	width of the regular (ELL) part of the matrix in HYB format, which should be less than maximum number of nonzeros per row and is only required if partitionType == CUSPARSE_HYB_PARTITION_USER.
partitionType	partitioning method to be used in the conversion (please refer to cusparseHybPartition_t for details).

hybA	the matrix A in HYB storage format.
------	-------------------------------------

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 12.19. cusparse<t>dense2csc()

```
cusparseStatus t
cusparseSdense2csc(cusparseHandle t handle, int m, int n,
                 const cusparseMatDescr t descrA,
                 const float \overline{*}A,
                 int lda, const int *nnzPerCol,
                 float *cscValA,
                 int *cscRowIndA, int *cscColPtrA)
cusparseStatus t
cusparseDdense2csc(cusparseHandle t handle, int m, int n,
                 const cusparseMatDescr t descrA,
                 const double \overline{{}^*A},
                 int lda, const int *nnzPerCol,
                 double *cscValA,
                 int *cscRowIndA, int *cscColPtrA)
cusparseStatus_t
cusparseCdense\overline{2}csc(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr t descrA,
                 const cuComplex \overline{*}A,
                 int lda, const int *nnzPerCol,
                 cuComplex *cscValA,
int *cscRowIndA, int *cscColPtrA)
cusparseStatus t
cusparseZdense2csc(cusparseHandle t handle, int m, int n,
                 const cusparseMatDescr t descrA,
                 const cuDoubleComplex \overline{*}A,
                 int lda, const int *nnzPerCol,
                 cuDoubleComplex *cscValA,
int *cscRowIndA, int *cscColPtrA)
```

This function converts the matrix **A** in dense format into a sparse matrix in CSC format. All the parameters are assumed to have been pre-allocated by the user, and the arrays are filled in based on nnzPerCol, which can be precomputed with cusparse<t>nnz().

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

handle	handle to the cuSPARSE library context.
m	number of rows of matrix A.
n	number of columns of matrix <b>A</b> .
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
A	array of dimensions (lda, n).
lda	leading dimension of dense array A.
nnzPerCol	array of size n containing the number of nonzero elements per column.

cscValA	<pre><type> array of nnz (= cscRowPtrA(m) - cscRowPtrA(0) ) nonzero elements of matrix A. It is only filled in if copyValues is set to CUSPARSE_ACTION_NUMERIC.</type></pre>
cscRowIndA	integer array of nnz (= cscRowPtrA(m) - cscRowPtrA(0) ) row indices of the nonzero elements of matrix A.
cscColPtrA	integer array of n+1 elements that contains the start of every column and the end of the last column plus one.

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED the matrix type is not supported.	

# 12.20. cusparse<t>dense2csr()

```
cusparseStatus t
cusparseSdense2csr(cusparseHandle t handle, int m, int n,
                const cusparseMatDescr_t descrA,
const float *A,
                int lda, const int *nnzPerRow,
                         *csrValA,
                float
                int *csrRowPtrA, int *csrColIndA)
cusparseStatus t
cusparseDdense2csr(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA,
                const double \overline{\phantom{a}}^*A,
                int lda, const int *nnzPerRow,
                double *csrValA,
                int *csrRowPtrA, int *csrColIndA)
cusparseStatus t
cusparseCdense2csr(cusparseHandle t handle, int m, int n,
               const cusparseMatDescr_t descrA,
                const cuComplex \overline{*}A,
                int lda, const int *nnzPerRow,
                cuComplex *csrValA,
                int *csrRowPtrA, int *csrColIndA)
cusparseStatus t
cusparseZdense2csr(cusparseHandle t handle, int m, int n,
                const cusparseMatDescr t descrA,
                const cuDoubleComplex *A,
                int lda, const int *nnzPerRow,
                cuDoubleComplex *csrValA,
                int *csrRowPtrA, int *csrColIndA)
```

This function converts the matrix **A** in dense format into a sparse matrix in CSR format. All the parameters are assumed to have been pre-allocated by the user and the arrays are filled in based on nnzPerRow, which can be pre-computed with cusparse<t>nnz().

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

### Input

handle	handle to the cuSPARSE library context.
m	number of rows of matrix A.
n	number of columns of matrix <b>A</b> .
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.  Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
A	array of dimensions (lda, n).
lda	leading dimension of dense array ${f a}.$
nnzPerRow	array of size ${\tt n}$ containing the number of non-zero elements per row.

## Output

csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m+1$ elements that contains the start of every column and the end of the last column plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the non-zero elements of matrix A.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 12.21. cusparse<t>dense2hyb()

```
cusparseStatus t
cusparseSdense2hyb(cusparseHandle t handle, int m, int n,
                   const cusparseMatDescr t descrA,
                   const float
                                         *A,
                   int lda, const int *nnzPerRow, cusparseHybMat t hybA,
                   int userEllWidth,
                   cusparseHybPartition t partitionType)
cusparseStatus t
cusparseDdense2hyb(cusparseHandle t handle, int m, int n,
                   const cusparseMatDescr t descrA,
                   const double
                                         *A,
                   int lda, const int *nnzPerRow, cusparseHybMat t
hybA,
                   int userEllWidth,
                   cusparseHybPartition_t partitionType)
cusparseStatus t
cusparseCdense\overline{2}hyb(cusparseHandle t handle, int m, int n,
                   const cusparseMatDescr_t descrA,
                   const cuComplex
                                          *A,
                   int lda, const int *nnzPerRow, cusparseHybMat t hybA,
                   int userEllWidth,
                   cusparseHybPartition t partitionType)
cusparseStatus t
cusparseZdenseZhyb(cusparseHandle t handle, int m, int n,
                   const cusparseMatDescr t descrA,
                   const cuDoubleComplex \overline{*}A,
                   int lda, const int *nnzPerRow, cusparseHybMat t hybA,
                   int userEllWidth,
                   cusparseHybPartition t partitionType)
```

This function converts matrix **A** in dense format into a sparse matrix in HYB format. It assumes that the routine **cusparseCreateHybMat()** was used to initialize the opaque structure **hybA** and that the array **nnzPerRow** was pre-computed with **cusparse<t>nnz()**.

This function requires some amount of temporary storage and a significant amount of storage for the matrix in HYB format. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

handle	handle to the cuSPARSE library context.
m	number of rows of matrix A.
n	number of columns of matrix A.
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.
A	array of dimensions (lda, n).
lda	leading dimension of dense array A.
nnzPerRow	array of size ${\tt m}$ containing the number of nonzero elements per row.

userEllWidth	width of the regular (ELL) part of the matrix in HYB format, which should be less than maximum number of nonzeros per row and is only required if partitionType == CUSPARSE_HYB_PARTITION_USER.
partitionType	partitioning method to be used in the conversion (please refer to cusparseHybPartition_t for details).

hybA	the matrix A in HYB storage format.
------	-------------------------------------

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 12.22. cusparse<t>hyb2csc()

```
cusparseStatus t
cusparseShyb2csc(cusparseHandle t handle,
                const cusparseMatDescr t descrA,
                const cusparseHybMat t hybA,
                float
                           *cscValA, int *cscRowIndA, int *cscColPtrA)
cusparseStatus t
cusparseDhyb2csc(cusparseHandle t handle,
                const cusparseMatDescr t descrA,
                const cusparseHybMat t hybA,
                double *cscValA, int *cscRowIndA, int *cscColPtrA)
cusparseStatus t
cusparseChyb2csc(cusparseHandle t handle,
                const cusparseMatDescr_t descrA,
                const cusparseHybMat_t hybA,
                cuComplex *cscValA, int *cscRowIndA, int *cscColPtrA)
cusparseStatus t
cusparseZhyb2csc(cusparseHandle t handle,
                const cusparseMatDescr_t descrA,
                const cusparseHybMat t hybA,
                cuDoubleComplex *cscValA, int *cscRowIndA, int
*cscColPtrA)
```

This function converts a sparse matrix in HYB format into a sparse matrix in CSC format.

This function requires some amount of temporary storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

## Input

handle	handle to the cuSPARSE library context.
descrA	the descriptor of matrix <b>A</b> in Hyb format. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.
hybA	the matrix <b>A</b> in HYB storage format.

## Output

cscValA	<pre><type> array of nnz (= cscColPtrA(m) - cscColPtrA(0) ) nonzero elements of matrix A.</type></pre>
cscRowIndA	integer array of nnz (= cscColPtrA(m) - cscColPtrA(0) ) column indices of the non-zero elements of matrix A.
cscColPtrA	integer array of $m+1$ elements that contains the start of every column and the end of the last row plus one.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 12.23. cusparse<t>hyb2csr()

```
cusparseStatus t
cusparseShyb2csr(cusparseHandle t handle,
                 const cusparseMatDescr t descrA,
                 const cusparseHybMat t hybA,
                            *csrValA, int *csrRowPtrA, int *csrColIndA)
cusparseStatus t
cusparseDhyb2csr(cusparseHandle t handle,
                 const cusparseMatDescr t descrA,
                 const cusparseHybMat t hybA,
                 double *csrValA, int *csrRowPtrA, int *csrColIndA)
cusparseStatus t
cusparseChyb2csr(cusparseHandle t handle,
                const cusparseMatDescr t descrA,
                const cusparseHybMat_t hybA,
                 cuComplex *csrValA, int *csrRowPtrA, int *csrColIndA)
cusparseStatus_t
cusparseZhyb2csr(cusparseHandle t handle,
                const cusparseMatDescr_t descrA,
                 const cusparseHybMat_t hybA,
                cuDoubleComplex *csrValA, int *csrRowPtrA, int
*csrColIndA)
```

This function converts a sparse matrix in HYB format into a sparse matrix in CSR format.

This function requires some amount of temporary storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

## Input

handle	handle to the cuSPARSE library context.
descrA	the descriptor of matrix A in Hyb format. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.
hybA	the matrix <b>A</b> in HYB storage format.

### Output

csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) nonzero elements of matrix A.</type></pre>
csrRowPtrA	integer array of $m+1$ elements that contains the start of every column and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0) ) column indices of the nonzero elements of matrix A.

CUSPARSE_STATUS_SUCCESS   the operation completed successfully.
---

CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 12.24. cusparse<t>hyb2dense()

```
cusparseStatus t
cusparseShyb2dense(cusparseHandle t handle,
                   const cusparseMatDescr t descrA,
                   const cusparseHybMat t hybA,
                   float *A,
                   int lda)
cusparseStatus t
cusparseDhyb2dense(cusparseHandle t handle,
                   const cusparseMatDescr t descrA,
                   const cusparseHybMat t hybA,
                   double *A,
                   int lda)
cusparseStatus t
cusparseChyb2dense(cusparseHandle t handle,
                   const cusparseMatDescr t descrA,
                   const cusparseHybMat t hybA,
                   cuComplex *A,
                   int lda)
cusparseStatus t
cusparseZhyb2dense(cusparseHandle t handle,
                   const cusparseMatDescr t descrA,
                   const cusparseHybMat t hybA,
                   cuDoubleComplex *A,
                   int lda)
```

This function converts a sparse matrix in HYB format (contained in the opaque structure) into matrix  $\mathbf{A}$  in dense format. The dense matrix  $\mathbf{A}$  is filled in with the values of the sparse matrix and with zeros elsewhere.

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

handle	handle to the cuSPARSE library context.
descrA	the descriptor of matrix <b>A</b> in Hyb format. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL.
hybA	the matrix <b>A</b> in HYB storage format.
lda	leading dimension of dense array A.

A	array of dimensions (lda, n) that is filled in with
	the values of the sparse matrix.

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the internally stored hyb format parameters are invalid.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED the matrix type is not supported.	

# 12.25. cusparse<t>nnz()

```
cusparseStatus t
cusparseSnnz(cusparseHandle t handle, cusparseDirection t dirA, int m,
             int n, const cusparseMatDescr t descrA,
            const float
            int lda, int *nnzPerRowColumn, int *nnzTotalDevHostPtr)
cusparseStatus t
cusparseDnnz(cusparseHandle t handle, cusparseDirection t dirA, int m,
            int n, const cusparseMatDescr t descrA,
            const double *A,
            int lda, int *nnzPerRowColumn, int *nnzTotalDevHostPtr)
cusparseStatus t
cusparseCnnz(cusparseHandle t handle, cusparseDirection t dirA, int m,
            int n, const cusparseMatDescr t descrA,
            const cuComplex *A,
            int lda, int *nnzPerRowColumn, int *nnzTotalDevHostPtr)
cusparseStatus t
cusparseZnnz(cusparseHandle t handle, cusparseDirection t dirA, int m,
            int n, const cusparseMatDescr t descrA,
            const cuDoubleComplex *A,
            int lda, int *nnzPerRowColumn, int *nnzTotalDevHostPtr)
```

This function computes the number of nonzero elements per row or column and the total number of nonzero elements in a dense matrix.

This function requires no extra storage. It is executed asynchronously with respect to the host and may return control to the application on the host before the result is ready.

handle	handle to the cuSPARSE library context.
dirA	direction that specifies whether to count nonzero elements by CUSPARSE_DIRECTION_ROW or by CUSPARSE_DIRECTION_COLUMN.
m	number of rows of matrix A.
n	number of columns of matrix A.

descrA	the descriptor of matrix a. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
A	array of dimensions (lda, n).
lda	leading dimension of dense array A.

#### Output

nnzPerRowColumn	array of size $m$ or $n$ containing the number of nonzero elements per row or column, respectively.
nnzTotalDevHostPtr	total number of nonzero elements in device or host memory.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m, n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED the matrix type is not supported.	

# 12.26. cusparseCreateIdentityPermutation()

This function creates an identity map. The output parameter p represents such map by p = 0:1:(n-1).

This function is typically used with coosort, csrsort, cscsort, csr2csc\_indexOnly. Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
n	host	size of the map.

#### Output

parameter	device or host	description
1 -		_

p devi	ce	integer array of dimensions n.
--------	----	--------------------------------

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

## 12.27. cusparseXcoosort()

```
cusparseStatus t
cusparseXcoosort_bufferSizeExt(
                      cusparseHandle_t handle,
                      int m,
                      int n,
                      int nnz,
                      const int *cooRows,
                      const int *cooCols,
                      size t *pBufferSizeInBytes);
cusparseStatus t
cusparseXcoosortByRow(cusparseHandle_t handle,
                      int m,
                       int n,
                      int nnz,
                      int *cooRows,
                      int *cooCols,
                      int *P,
                      void *pBuffer);
cusparseStatus t
cusparseXcoosortByColumn(cusparseHandle t handle,
                          int m,
                          int n,
                          int nnz,
                          int *cooRows,
                          int *cooCols,
int *P,
                          void *pBuffer);
```

This function sorts COO format. The stable sorting is in-place. Also the user can sort by row or sort by column.

A is an m×n sparse matrix that is defined in COO storage format by the three arrays cooVals, cooRows, and cooCols.

The matrix must be base 0.

The matrix type is regarded as **CUSPARSE\_MATRIX\_TYPE\_GENERAL** implicitly. In other words, any symmetric property is ignored.

This function coosort() requires buffer size returned by coosort\_bufferSizeExt(). The address of pBuffer must be multiple of 128 bytes. If not, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

The parameter P is both input and output. If the user wants to compute sorted cooVal, P must be set as 0:1:(nnz-1) before coosort(), and after coosort(), new sorted value array satisfies cooVal\_sorted = cooVal(P).

The following code shows how to sort a COO format by row.

```
// A is a 3x3 sparse matrix, base-0
// A = | 1 2 0 | // A = | 0 5 0 |
// | 0 8 0 |
const int m = 3;
const int n = 3;
const int nnz = 4;
cooRows[nnz] = {2, 1, 0, 0}; // on device
cooCols[nnz] = {1, 1, 0, 1}; // on device
cooVals[nnz] = {8.0, 5.0, 1.0, 2.0}; // on device
size_t pBufferSizeInBytes = 0;
void *pBuffer = NULL;
int *P = NULL;
// step 1: allocate buffer
cusparseXcoosort bufferSizeExt(handle, m, n, nnz, cooRows, cooCols,
&pBufferSizeInBytes);
cudaMalloc( &pBuffer, sizeof(char)* pBufferSizeInBytes);
// step 2: setup permutation vector P to identity
cudaMalloc( (void**)&P, sizeof(int)*nnz);
cusparseCreateIdentityPermutation(handle, nnz, P);
// step 3: sort COO format by Row
cusparseXcoosortByRow(handle, m, n, nnz, cooRows, cooCols, P, pBuffer);
// step 4: gather sorted cooVals
cusparseDgthr(handle, nnz, cooVals, cooVals sorted, P,
CUSPARSE INDEX BASE ZERO);
```

#### Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
m	host	number of rows of matrix A.
n	host	number of columns of matrix A.
nnz	host	number of nonzero elements of matrix A.
cooRows	device	integer array of nnz unsorted row indices of A.
cooCols	device	integer array of nnz unsorted column indices of A.
Р	device	integer array of nnz unsorted map indices. To construct cooVal, the user has to set P=0:1: (nnz-1).
pBuffer	device	buffer allocated by the user; the size is returned by coosort_bufferSizeExt().

#### Output

parameter	device or host	description
cooRows	device	integer array of nnz sorted row indices of A.
cooCols	device	integer array of $\mathtt{nnz}$ sorted column indices of A.
P	device	integer array of nnz sorted map indices.
pBufferSizeInBytes	host	number of bytes of the buffer.

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

## 12.28. cusparseXcsrsort()

```
cusparseStatus t
cusparseXcsrsort bufferSizeExt(
                 cusparseHandle t handle,
                 int m,
                 int n,
                 int nnz,
                 const int *csrRowPtr,
                 const int *csrColInd,
                 size t *pBufferSizeInBytes);
cusparseStatus t
cusparseXcsrsort(cusparseHandle t handle,
                 int m,
                 int n,
                 int nnz,
                 const cusparseMatDescr t descrA,
                 const int *csrRowPtr,
                 int *csrColInd,
                 int *P,
                 void *pBuffer);
```

This function sorts CSR format. The stable sorting is in-place.

The matrix type is regarded as **CUSPARSE\_MATRIX\_TYPE\_GENERAL** implicitly. In other words, any symmetric property is ignored.

This function csrsort() requires buffer size returned by csrsort\_bufferSizeExt(). The address of pBuffer must be multiple of 128 bytes. If not, CUSPARSE\_STATUS\_INVALID\_VALUE is returned.

The parameter P is both input and output. If the user wants to compute sorted csrVal, P must be set as 0:1:(nnz-1) before csrsort(), and after csrsort(), new sorted value array satisfies csrVal\_sorted = csrVal(P).

The general procedure is as follows:

```
// A is a 3x3 sparse matrix, base-0
    | 1 2 3 |
// A = | 4 5 6
// | 7 8 9 |
const int m = 3;
const int n = 3;
const int nnz = 9;
csrRowPtr[m+1] = { 0, 3, 6, 9}; // on device}
csrColInd[nnz] = { 2, 1, 0, 0, 2,1, 1, 2, 0}; // on device
csrVal[nnz] = { 3, 2, 1, 4, 6, 5, 8, 9, 7}; // on device
size_t pBufferSizeInBytes = 0;
void *pBuffer = NULL;
int *P = NULL;
// step 1: allocate buffer
cusparseXcsrsort_bufferSizeExt(handle, m, n, nnz, csrRowPtr, csrColInd,
&pBufferSizeInBytes);
cudaMalloc( &pBuffer, sizeof(char)* pBufferSizeInBytes);
// step 2: setup permutation vector P to identity
cudaMalloc( (void**)&P, sizeof(int)*nnz);
cusparseCreateIdentityPermutation(handle, nnz, P);
// step 3: sort CSR format
cusparseXcsrsort(handle, m, n, nnz, descrA, csrRowPtr, csrColInd, P, pBuffer);
// step 4: gather sorted csrVal
cusparseDgthr(handle, nnz, csrVal, csrVal sorted, P, CUSPARSE INDEX BASE ZERO);
```

#### Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
m	host	number of rows of matrix A.
n	host	number of columns of matrix A.
nnz	host	number of nonzero elements of matrix A.
csrRowsPtr	device	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.
csrColInd	device	integer array of nnz unsorted column indices of A.
Р	device	integer array of nnz unsorted map indices. To construct csrVal, the user has to set P=0:1: (nnz-1).
pBuffer	device	buffer allocated by the user; the size is returned by csrsort_bufferSizeExt().

#### Output

parameter	device or host	description
csrColInd	device	integer array of nnz sorted column indices of A.

P	device	integer array of nnz sorted map indices.
pBufferSizeInBytes	host	number of bytes of the buffer.

#### Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,nnz<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED the matrix type is not supported.	

## 12.29. cusparseXcscsort()

```
cusparseStatus t
cusparseXcscsort bufferSizeExt(
                 cusparseHandle t handle,
                 int m,
                 int n,
                 int nnz,
                 const int *cscColPtr,
                 const int *cscRowInd,
                 size t *pBufferSizeInBytes);
cusparseStatus t
cusparseXcscsort(cusparseHandle t handle,
                 int m,
                 int n,
                 int nnz,
                 const cusparseMatDescr t descrA,
                 const int *cscColPtr,
                 int *cscRowInd,
                 int *P,
                 void *pBuffer);
```

This function sorts CSC format. The stable sorting is in-place.

The matrix type is regarded as **CUSPARSE\_MATRIX\_TYPE\_GENERAL** implicitly. In other words, any symmetric property is ignored.

This function cscsort() requires buffer size returned by cscsort\_bufferSizeExt(). The address of pBuffer must be multiple of 128 bytes. If not, CUSPARSE STATUS INVALID VALUE is returned.

The parameter P is both input and output. If the user wants to compute sorted cscVal, P must be set as 0:1:(nnz-1) before cscsort(), and after cscsort(), new sorted value array satisfies cscVal\_sorted = cscVal(P).

The general procedure is as follows:

```
// A is a 3x3 sparse matrix, base-0
// | 1 2 |
// A = | 4 0
// | 0 8
const int m = 3;
const int n = 2;
const int nnz = 4;
cscColPtr[n+1] = { 0, 2, 4}; // on device
cscRowInd[nnz] = { 1, 0, 2, 0}; // on device
cscVal[nnz] = { 4.0, 1.0, 8.0, 2.0 }; // on device
size_t pBufferSizeInBytes = 0;
void *pBuffer = NULL;
int *P = NULL;
// step 1: allocate buffer
cusparseXcscsort bufferSizeExt(handle, m, n, nnz, cscColPtr, cscRowInd,
&pBufferSizeInBytes);
cudaMalloc( &pBuffer, sizeof(char)* pBufferSizeInBytes);
// step 2: setup permutation vector P to identity
cudaMalloc( (void**) &P, sizeof(int)*nnz);
cusparseCreateIdentityPermutation(handle, nnz, P);
// step 3: sort CSC format
cusparseXcscsort(handle, m, n, nnz, descrA, cscColPtr, cscRowInd, P, pBuffer);
// step 4: gather sorted cscVal
cusparseDgthr(handle, nnz, cscVal, cscVal sorted, P, CUSPARSE INDEX BASE ZERO);
```

#### Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
m	host	number of rows of matrix A.
n	host	number of columns of matrix A.
nnz	host	number of nonzero elements of matrix A.
cscColPtr	device	integer array of $n+1$ elements that contains the start of every column and the end of the last column plus one.
cscRowInd	device	integer array of nnz unsorted row indices of A.
Р	device	integer array of nnz unsorted map indices. To construct cscVal, the user has to set P=0:1: (nnz-1).
pBuffer	device	buffer allocated by the user; the size is returned by cscsort_bufferSizeExt().

#### Output

parameter	device or host	description
cscRowInd	device	integer array of nnz sorted row indices of A.
P	device	integer array of nnz sorted map indices.
pBufferSizeInBytes	host	number of bytes of the buffer.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,nnz<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	the matrix type is not supported.

# 12.30. cusparseXcsru2csr()

```
cusparseStatus t cusparseCreateCsru2csrInfo(csru2csrInfo t *info);
cusparseStatus t cusparseDestroyCsru2csrInfo(csru2csrInfo t info);
cusparseStatus t
cusparseScsru2csr bufferSizeExt(
                          cusparseHandle t handle,
                          int m,
                          int n,
                          int nnz,
                          float *csrVal,
                          const int *csrRowPtr,
                          int *csrColInd,
                          csru2csrInfo_t info,
                          size_t *pBufferSizeInBytes);
cusparseStatus_t
cusparseDcsru2csr_bufferSizeExt(
                          cusparseHandle t handle,
                          int m,
                          int n,
                          int nnz,
                          double *csrVal,
                          const int *csrRowPtr,
                          int *csrColInd,
                          csru2csrInfo t
                                          info,
                          size t *pBufferSizeInBytes);
cusparseStatus t
cusparseCcsru2csr bufferSizeExt(
                          cusparseHandle t handle,
                          int m,
                          int n,
                          int nnz,
                          cuComplex *csrVal,
                          const int *csrRowPtr,
                          int *csrColInd,
                          csru2csrInfo t
                          size t *pBufferSizeInBytes);
cusparseStatus t
cusparseZcsru2csr bufferSizeExt(
                          cusparseHandle t handle,
                          int m,
                          int n,
                          int nnz,
                          cuDoubleComplex *csrVal,
                          const int *csrRowPtr,
                          int *csrColInd,
                          csru2csrInfo t
                                          info,
                          size t *pBufferSizeInBytes);
cusparseStatus t
cusparseScsru2csr(cusparseHandle t handle,
                  int m,
                  int n,
                  int nnz,
                  const cusparseMatDescr_t descrA,
                  float *csrVal,
                  const int *csrRowPtr,
                  int *csrColInd,
                  csru2csrInfo t
                                  info.
```

This function transfers unsorted CSR format to CSR format, and vice versa. The operation is in-place.

This function is a wrapper of **csrsort** and **gthr**. The usecase is the following scenario.

If the user has a matrix **A** of CSR format which is unsorted, and implements his own code (which can be CPU or GPU kernel) based on this special order (for example, diagonal first, then lower triangle, then upper triangle), and wants to convert it to CSR format when calling CUSPARSE library, and then convert it back when doing something else on his/her kernel. For example, suppose the user wants to solve a linear system **Ax=b** by the following iterative scheme

$$x^{(k+1)} = x^{(k)} + L^{(-1)} * (b - Ax^{(k)})$$

The code heavily uses SpMv and triangular solve. Assume that the user has an inhouse design of SpMV (Sparse Matrix-Vector multiplication) based on special order of **A**. However the user wants to use CUSAPRSE library for triangular solver. Then the following code can work.

The requirements of step 2 and step 5 are

- 1. In-place operation.
- 2. The permutation vector **P** is hidden in an opaque structure.
- 3. No **cudaMalloc** inside the conversion routine. Instead, the user has to provide the buffer explicitly.
- 4. The conversion between unsorted CSR and sorted CSR may needs several times, but the function only generates the permutation vector  $\mathbf{p}$  once.
- 5. The function is based on csrsort, gather and scatter operations.

The operation is called **csru2csr**, which means unsorted CSR to sorted CSR. Also we provide the inverse operation, called **csr2csru**.

In order to keep the permutation vector invisible, we need an opaque structure called <code>csru2csrInfo</code>. Then two functions (<code>cusparseCreateCsru2csrInfo</code>, <code>cusparseDestroyCsru2csrInfo</code>) are used to initialize and to destroy the opaque structure.

cusparse[S|D|C|Z]csru2csr\_bufferSizeExt returns the size of the buffer. The permutation vector **P** is also allcated inside csru2csrInfo. The lifetime of the permutation vector is the same as the lifetime of csru2csrInfo.

**cusparse[S|D|C|Z]csru2csr** performs forward transformation from unsorted CSR to sorted CSR. First call uses csrsort to generate the permutation vector **P**, and subsequent call uses **P** to do transformation.

**cusparse[S|D|C|Z]csr2csru** performs backward transformation from sorted CSR to unsorted CSR. **P** is used to get unsorted form back.

The following tables describe parameters of csr2csru\_bufferSizeExt and csr2csru.

#### Input

parameter	device or host	description
handle	host	handle to the cuSPARSE library context.
m	host	number of rows of matrix A.
n	host	number of columns of matrix A.
nnz	host	number of nonzero elements of matrix A.
descrA	host	the descriptor of matrix A. The supported matrix type is CUSPARSE_MATRIX_TYPE_GENERAL, Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrVal	device	<type> array of nnz unsorted nonzero elements of matrix A.</type>
csrRowsPtr	device	integer array of $m+1$ elements that contains the start of every row and the end of the last row plus one.
csrColInd	device	integer array of nnz unsorted column indices of A.
info	host	opaque structure initialized using cusparseCreateCsru2csrInfo().
pBuffer	device	buffer allocated by the user; the size is returned by csru2csr_bufferSizeExt().

#### Output

parameter	device or host	description
csrVal	device	<type> array of nnz sorted nonzero elements of matrix A.</type>
csrColInd	device	integer array of nnz sorted column indices of A.
pBufferSizeInBytes	host	number of bytes of the buffer.

#### **Status Returned**

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,nnz<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

CUSPARSE\_STATUS\_MATRIX\_TYPE\_NOT\_SUPPORTED the matrix type is not supported.

# Chapter 13. APPENDIX A: CUSPARSE LIBRARY C++ EXAMPLE

For sample code reference please see the example code below. It shows an application written in C++ using the cuSPARSE library API. The code performs the following actions:

- 1. Creates a sparse test matrix in COO format.
- 2. Creates a sparse and dense vector.
- 3. Allocates GPU memory and copies the matrix and vectors into it.
- 4. Initializes the cuSPARSE library.
- 5. Creates and sets up the matrix descriptor.
- 6. Converts the matrix from COO to CSR format.
- 7. Exercises Level 1 routines.
- 8. Exercises Level 2 routines.
- 9. Exercises Level 3 routines.
- 10. Destroys the matrix descriptor.

#### 11. Releases resources allocated for the cuSPARSE library.

```
//Example: Application using C++ and the CUSPARSE library
#include <stdio.h>
#include <stdlib.h>
#include <cuda runtime.h>
#include "cusparse.h"
#define CLEANUP(s)
    printf ("%s\n", s);
    if (zHostPtr)
                            free(yHostPtr);
                            free(zHostPtr);
    if (xIndHostPtr) free(xIndHostPtr);
if (xValHostPtr) free(xValHostPtr);
if (cooRowIndexHostPtr) free(cooRowIndexHostPtr);\
    if (cooColIndexHostPtr) free(cooColIndexHostPtr);\
    if (cooValHostPtr) free(cooValHostPtr);
if (y) cudaFree(y);
                            cudaFree(z);
    if(z)
    if (xInd)
                             cudaFree(xInd);
                             cudaFree(xVal);
    if (xVal)
   if (cooVal)
                            cudaFree(cooVal);
cusparseDestroyMatDescr(descr);\
    if (descr)
    if (desc!)
if (handle)
cudaDeviceReset();
                            cusparseDestroy(handle); \
    fflush (stdout);
} while (0)
int main(){
    cudaError t cudaStat1,cudaStat2,cudaStat3,cudaStat4,cudaStat5,cudaStat6;
    cusparseStatus_t status;
    cusparseHandle_t handle=0;
    cusparseMatDescr t descr=0;
    int * cooRowIndexHostPtr=0;
int * cooColIndexHostPtr=0;
    double * cooValHostPtr=0;
    int * cooRowIndex=0;
int * cooColIndex=0;
    double * cooVal=0;
    int * xIndHostPtr=0;
    double * xValHostPtr=0;
    double * yHostPtr=0;
int * xInd=0;
double * xVal=0;
    double * y=0;
    int * csrRowPtr=0;
    double * zHostPtr=0;
    double dzero =0.0;
    double dtwo =2.0;
    double dthree=3.0;
    double dfive =5.0;
    printf("testing example\n");
    /* create the following sparse test matrix in COO format */
    /* |1.0 2.0 3.0|
            4.0
       15.0 6.0 7.01
        8.0 9.0 | */
    n=4; nnz=9;
    cooRowIndexHostPtr = (int *) malloc(nnz*sizeof(cooRowIndexHostPtr[0]));
cooColIndexHostPtr = (int *) malloc(nnz*sizeof(cooColIndexHostPtr[0]));
    cooValHostPtr = (double *) malloc(nnz*sizeof(cooValHostPtr[0]));
    if ((!cooRowIndexHostPtr) || (!cooColIndexHostPtr) || (!cooValHostPtr)) {
        CLEANUP("Host malloc failed (matrix)");
    cooRowIndexHostPtr[0]=0; cooColIndexHostPtr[0]=0; cooValHostPtr[0]=1.0;
    cooRowIndexHostPtr[1]=0; cooColIndexHostPtr[1]=2; cooValHostPtr[1]=2.0;
    cooRowIndexHostPtr[2]=0: cooColIndexHostPtr[2]=3: cooValHostPtr[2]=3.0:
```

# Chapter 14. APPENDIX B: CUSPARSE FORTRAN BINDINGS

The cuSPARSE library is implemented using the C-based CUDA toolchain, and it thus provides a C-style API that makes interfacing to applications written in C or C++ trivial. There are also many applications implemented in Fortran that would benefit from using cuSPARSE, and therefore a cuSPARSE Fortran interface has been developed.

Unfortunately, Fortran-to-C calling conventions are not standardized and differ by platform and toolchain. In particular, differences may exist in the following areas:

Symbol names (capitalization, name decoration)

Argument passing (by value or reference)

Passing of pointer arguments (size of the pointer)

To provide maximum flexibility in addressing those differences, the cuSPARSE Fortran interface is provided in the form of wrapper functions, which are written in C and are located in the file <code>cusparse\_fortran.c</code>. This file also contains a few additional wrapper functions (for <code>cudaMalloc()</code>, <code>cudaMemset</code>, and so on) that can be used to allocate memory on the GPU.

The cuSPARSE Fortran wrapper code is provided as an example only and needs to be compiled into an application for it to call the cuSPARSE API functions. Providing this source code allows users to make any changes necessary for a particular platform and toolchain.

The cuSPARSE Fortran wrapper code has been used to demonstrate interoperability with the compilers g95 0.91 (on 32-bit and 64-bit Linux) and g95 0.92 (on 32-bit and 64-bit Mac OS X). In order to use other compilers, users have to make any changes to the wrapper code that may be required.

The direct wrappers, intended for production code, substitute device pointers for vector and matrix arguments in all cuSPARSE functions. To use these interfaces, existing applications need to be modified slightly to allocate and deallocate data structures in GPU memory space (using CUDA\_MALLOC() and CUDA\_FREE()) and to copy data between GPU and CPU memory spaces (using the CUDA\_MEMCPY() routines). The sample wrappers provided in cusparse fortran.c map device pointers to the OS-

dependent type **size\_t**, which is 32 bits wide on 32-bit platforms and 64 bits wide on a 64-bit platforms.

One approach to dealing with index arithmetic on device pointers in Fortran code is to use C-style macros and to use the C preprocessor to expand them. On Linux and Mac OS X, preprocessing can be done by using the option '-cpp' with g95 or gfortran. The function GET\_SHIFTED\_ADDRESS (), provided with the cuSPARSE Fortran wrappers, can also be used, as shown in example B.

Example B shows the C++ of example A implemented in Fortran 77 on the host. This example should be compiled with **ARCH\_64** defined as 1 on a 64-bit OS system and as undefined on a 32-bit OS system. For example, on g95 or gfortran, it can be done directly on the command line using the option **-cpp -DARCH 64=1**.

### 14.1. Example B, Fortran Application

```
#define ARCH 64 0
      #define ARCH 64 1
      program cusparse fortran example
      implicit none
      integer cuda malloc
      external cuda free
     integer cuda memcpy c2fort int
      integer cuda memcpy c2fort real
      integer cuda_memcpy_fort2c_int
      integer cuda_memcpy_fort2c_real
      integer cuda memset
      integer cusparse_create
      external cusparse destroy
      integer cusparse_get_version
      integer cusparse_create_mat_descr
      external cusparse destroy mat descr
     integer cusparse_set_mat_type
     integer cusparse get mat type
     integer cusparse_get_mat_fill_mode
      integer cusparse_get_mat_diag_type
      integer cusparse_set_mat_index_base
integer cusparse_get_mat_index_base
     integer cusparse xcoo2csr
      integer cusparse dsctr
      integer cusparse_dcsrmv
      integer cusparse dcsrmm
      external get_shifted_address
#if ARCH 64
     integer*8 handle
      integer*8 descrA
      integer*8 cooRowIndex
      integer*8 cooColIndex
     integer*8 cooVal
     integer*8 xInd
     integer*8 xVal
      integer*8 y
      integer*8 z
      integer*8 csrRowPtr
     integer*8 ynp1
#else
      integer*4 handle
      integer*4 descrA
     integer*4 cooRowIndex
     integer*4 cooColIndex
     integer*4 cooVal
      integer*4 xInd
      integer*4 xVal
     integer*4 y
     integer*4 z
      integer*4 csrRowPtr
      integer*4 ynp1
#endif
      integer status
      integer cudaStat1, cudaStat2, cudaStat3
      integer cudaStat4, cudaStat5, cudaStat6
     integer n, nnz, nnz_vector
      parameter (n=4, nnz=9, nnz_vector=3)
      integer cooRowIndexHostPtr(nnz)
     integer cooColIndexHostPtr(nnz)
     real*8 cooValHostPtr(nnz)
     integer xIndHostPtr(nnz_vector)
     real*8  xValHostPtr(nnz_vector)
real*8  yHostPtr(2*n)
      real*8 zHostPtr(2*(n+1))
      integer i, j
      integer version, mtype, fmode, dtype, ibase
      real*8 dzero,dtwo,dthree,dfive real*8 epsilon
```

# Chapter 15. APPENDIX C: ACKNOWLEDGEMENTS

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► The cusparse<t>gtsv implementation is derived from a version developed by Li-Wen Chang from the University of Illinois.

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