



Mapua University
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COE60/B2

Machine Problem 3
User Manual

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2014150748

COE60/B2

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Muller's Method

Muller's method is a root-finding algorithm, a numerical method for solving equations of the form $f(x) = 0$. It was first presented by David E. Muller in 1956.

Muller's method is based on the secant method, which constructs at every iteration a line through two points on the graph of f . Instead, Muller's method uses three points, constructs the parabola through these three points, and takes the intersection of the x -axis with the parabola to be the next approximation. Muller's method is a generalization of the secant method. Instead of starting with two initial values and then joining them with a straight line in secant method, Muller's method starts with three initial approximations to the root and then join them with a second-degree polynomial (a parabola), then the quadratic formula is used to find a root of the quadratic for the next approximation. That is if x_0 , x_1 and x_2 are the initial approximations then x_3 is obtained by solving the quadratic which is obtained by means of x_0 , x_1 and x_2 . Then two values among x_0 , x_1 and x_2 which are close to x_3 are chosen for the next iteration.

$$x_3 = x_2 + z$$

where $z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$a = \frac{D_1}{D_2}, b = \frac{D_1^2}{D_2} \quad \text{and} \quad c = f(x_2)$$

$$D = h_0 h_1 (h_0 - h_1), D_1 = (f_0 - c) h_1 - (f_1 - c) h_0, D_2 = (f_1 - c) h_0^2 - (f_0 - c) h_1^2$$

$$h_0 = x_0 - x_2, h_1 = x_1 - x_2$$

Crout's Method

Crout's method also called as Cholesky Method. In linear algebra, the Cholesky decomposition or Cholesky factorization is a decomposition of a Hermitian, positive-definite matrix into the product of a lower triangular matrix and its conjugate transpose, which is useful e.g. for efficient numerical solutions and Monte Carlo simulations. It was discovered by André-Louis Cholesky for real matrices. When it is applicable, the Cholesky decomposition is roughly twice as efficient as LU decomposition in linear algebra, the Crout matrix decomposition is an LU decomposition which decomposes a matrix into a lower triangular matrix (L), an upper triangular matrix (U) and, although not always needed, a permutation matrix (P). It was developed by Prescott Durand Crout. [1]

The Crout matrix decomposition algorithm differs slightly from the Doolittle method. Doolittle's method returns a unit lower triangular matrix and an upper triangular matrix, while the Crout method returns a lower triangular matrix and a unit upper triangular matrix.

So, if a matrix decomposition of a matrix A is such that:

$$A = LDU$$

being L a unit lower triangular matrix, D a diagonal matrix and U a unit upper triangular matrix, then Doolittle's method produces

$$A = L(DU)$$

and Crout's method produces

$$A = (LD)U.$$

being L a lower triangular matrix, D a diagonal matrix and U a normalised upper triangular matrix efficient as the LU decomposition for solving systems of linear equations.

Gauss – Seidel Method

In numerical methods, the Gauss–Seidel method, also known as the Liebman method or the method of successive displacement, is an iterative method used to solve a linear system of equations. It is named after the German mathematicians Carl Friedrich Gauss and Philipp Ludwig von Seidel, and is like the Jacobi method. Though it can be applied to any matrix with non-zero elements on the diagonals, convergence is only guaranteed if the matrix is either diagonally dominant, or symmetric and positive definite. It was only mentioned in a private letter from Gauss to his student Gerling in 1823. A publication was not delivered before 1874 by Seidel. The element-wise formula for the Gauss–Seidel method is extremely like that of the Jacobi method.

The computation of $x_i(k+1)$ uses only the elements of $x(k+1)$ that have already been computed, and only the elements of $x(k)$ that have not yet to be advanced to iteration $k+1$. This means that, unlike the Jacobi method, only one storage vector is required as elements can be overwritten as they are computed, which can be advantageous for very large problems.

However, unlike the Jacobi method, the computations for each element cannot be done in parallel. Furthermore, the values at each iteration are dependent on the order of the original equations.

Gauss-Seidel is the same as SOR (successive over-relaxation)

Parts of the Program:

I. Numerical method selection for MP3 (Main Window)

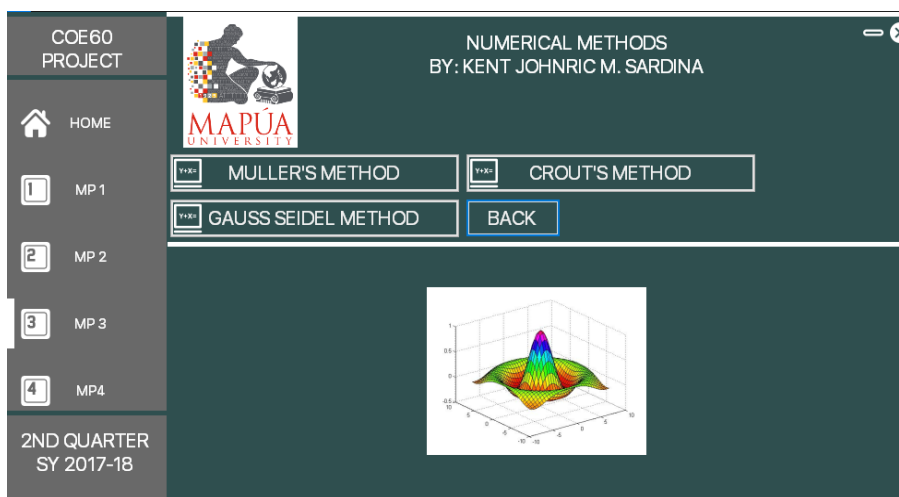


Figure 1. Main Menu

This depicts the available numerical methods to be used by the user. For Machine Problem 3, Muller's, Crout's, and Gauss Seidel is available.

II. Muller's Method Window

The screenshot shows a web application interface for numerical methods. On the left is a sidebar with a 'COE60 PROJECT' header and a 'HOME' button. Below it are four menu items labeled 'MP 1', 'MP 2', 'MP 3', and 'MP 4'. At the bottom of the sidebar is '2ND QUARTER SY 2017-18'. The main content area has a dark green header with the 'MAPUA UNIVERSITY' logo and the text 'NUMERICAL METHODS BY: KENT JOHN RIC M. SARDINA'. Below the header are four buttons: 'MULLER'S METHOD' (highlighted with a blue border), 'CROUT'S METHOD', 'GAUSS SEIDEL METHOD', and 'BACK'. The 'MULLER'S METHOD' section contains a 'SOLVE' button and a 'CLEAR' button. Underneath, it lists the 'Degree of the Function' with three options: $ax^2 + bx + c$, $ax^3 + bx^2 + cx + d$, and $ax^4 + bx^3 + cx^2 + dx + e$. Below these options is a section for 'Enter Coefficients' with input fields for 'a', 'b', 'c', 'd', and 'e'. To the right of these fields are labels 'x0' and 'x1' with corresponding input fields.

Figure 2. Window for Muller's Method.

III. Crout's Method Window

The screenshot shows the same web application interface as Figure 2, but with 'CROUT'S METHOD' selected. The 'CROUT'S METHOD' section contains a 'SOLVE' button and a 'CLEAR' button. Below these buttons is a table for solving a system of linear equations. The table has columns for variables x_3 , x_2 , x_1 , and 'Value'. The first row shows the equation $x_3 + x_2 + x_1 =$ followed by an input field, a 'Do' button, and an input field labeled 'x1'. The second row shows $x_3 + x_2 + x_1 =$ followed by an input field, a 'Do' button, and an input field labeled 'x2'. The third row is partially visible, showing $x_3 + x_2 + x_1 =$ followed by an input field, a 'Do' button, and an input field labeled 'x3'.

Figure 3. Window for Crout's Method Program.

IV. Gauss-Seidel Window

COE60 PROJECT

HOME

MP 1

MP 2

MP 3

MP 4

2ND QUARTER SY 2017-18

NUMERICAL METHODS
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GAUSS SEIDEL METHOD

BACK

GAUSS SEIDEL METHOD

x_3 x_2 x_1

$\square + \square + \square = \square$ Do

$\square + \square + \square = \square$ Do

$\square + \square + \square = \square$ Do

SOLVE CLEAR

Terminating Value

Iteration x_1 x_2 x_3 x_1

x_2

x_3

Figure 4. Window for Gauss-Seidel Program

Steps to use the program:

- I. Muller's Method
 1. First you must choose your desired degree of function, may it be a quadratic, cubic or polynomial function.
 2. Enter the coefficient of a, b, c, d, and e. Enter also any initial value.
 3. Click Solve.
 4. If the coefficient you've entered is correct, then it will successfully show you all iterations.

COE60 PROJECT

HOME

MP 1

MP 2

MP 3

MP 4

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MULLER'S METHOD

SOLVE CLEAR

Degree of the Function

- ☐ $ax^2 + bx + c$
- ☐ $ax^3 + bx^2 + cx + d$
- ☐ $ax^4 + bx^3 + cx^2 + dx + e$

Enter Coefficients

a b c d e

1 -4 1 -10

x_0 x_1 x_2

1 2 3

k	x_0	x_1	x_2
1	1	2	3
2	2	3	4.357417562100...
3	3	4.357417562100...	4.313707386391...
4	4.357417562100...	4.313707386391...	4.309227782771...
5	4.313707386391...	4.309227782771...	4.306848...

5. You may click "Clear" if you wanted to enter another set of value for coefficient or the initial values.
6. Click "Back" if you want to try another method.

II. Crout's Method

1. In Crout's Method, 3 equations are always given to solve for the value of X_1, X_2 , and X_3 . First, enter the coefficients of the 3 equations that will be used for the Matrix Decomposition.
2. Click the "DO" box when you've filled all the boxes.

COE60 PROJECT

HOME

1 MP 1

2 MP 2

3 MP 3

4 MP 4

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CROUTS METHOD

SOLVE CLEAR

	x3	x2	x1	=		Do		Value
1	6	-2	-3	-11			x1	0
2	-2	5	-2	12			x2	0
3	-3	-2	8	13			x3	0

3. Click Solve.

4. Then the value for X_1, X_2 , and X_3 will appear

5. You may click "Clear" if you wanted to enter another set of value for coefficient or the initial values. Click "Back" if you want to try another method.

III. Gauss-Seidel Method

1. In Gauss-Seidel Method, 3 equations are always given to solve for the value of X_1, X_2 , and X_3 . First, enter the coefficients of the 3 equations that will be used. Make sure that X_1 is the highest for the first equation, X_2 for the second equation and X_3 for the last equation.
2. Click the "DO" box once you've filled all the boxes.
3. Enter your desired terminating condition.
4. Click Solve

COE60 PROJECT

HOME

1 MP 1

2 MP 2

3 MP 3

4 MP 4

2ND QUARTER SY 2017-18

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GAUSS SEIDEL METHOD

SOLVE CLEAR

	x3	x2	x1	=		Do
1	7	-3	-4	-11		
2	-3	6	-2	3		
3	-4	-2	11	25		

0.001
Terminating Value

Iteration	x1	x2	x3
1	-1.5714	-0.2857	1.6494
2	-0.7514	0.6741	2.1221
3	-0.0599	1.1724	2.4605
4	0.3370	1.4867	2.6659
5	0.5900	1.6836	2.7934
6	0.7463	1.8043	2.8722
7	0.8431	1.8789	2.9209
8	0.9029	1.9251	2.9511
9	0.9400	1.9537	2.9697

x1 0.9979

x2 1.9984

x3 2.999

5. Then it will show the total number of iterations and the iteration the equation performed.

6. You may click "Clear" if you wanted to enter another set of value for coefficient or the initial values. Click "Back" if you want to try another method.

References:

https://en.wikipedia.org/wiki/Cholesky_decomposition

https://en.wikipedia.org/wiki/Muller%27s_method

https://en.wikipedia.org/wiki/Gauss%E2%80%93Seidel_method