md5: 2a9a3a

ICPC Notebook

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template

Otemplate.hpp

```
#include <bits/stdc++.h>
using namespace std;
#define ll long long
#define pii pair<int, int>
#define pll pair<ll, ll>
#define vi vector<int>
#define vl vector<ll>
#define ov3(a, b, c, name, ...) name
#define rep0(n) for(ll aaaaa = 0; aaaaa < n; ++aaaaa)</pre>
#define rep1(i, n) for(ll i = 0; i < (n); i++)
#define rep2(i, a, b) for(ll i = (a); i < (b); i++)</pre>
#define rep(...) ov3(__VA_ARGS__, rep2, rep1, rep0)(__VA_ARGS__)
#define per(i, a, b) for(ll i = (a)-1; i >= (b); i--)
#define fore(e, v) for(auto&& e : v)
#define all(a) begin(a), end(a)
#define si(a) (int)(size(a))
#define lb(v, x) (lower_bound(all(v), x) - begin(v))
#define eb emplace_back
bool chmin(auto& a, auto b) { return a > b ? a = b, 1 : 0; }
bool chmax(auto& a, auto b) { return a < b ? a = b, 1 : 0; }</pre>
const int INF = 1e9 + 100;
const ll INFL = 3e18 + 100;
#define i128 __int128_t
struct .
  _() { cin.tie(0)->sync_with_stdio(0), cout.tie(0); }
```

hash.sh

__;

```
# 使い方: sh hash.sh -> コピペ -> Ctrl + D
# コメント・空白・改行を削除して md5 でハッシュする
g++ -dD -E -P -fpreprocessed - | tr -d '[:space:]' | md5sum |
cut -c-6
```

random_test.sh

```
# correct wrong generate
make $1
make $2
make $3
while true; do
./$3 > input.txt
c=$(./$1 < input.txt)</pre>
w=$(./$2 < input.txt)
if [[ $c = $w ]]; then
echo "pass"
else
echo "WA"
echo "$(cat input.txt)"
echo "o:"
echo "$c"
echo "x:"
echo "$w"
exit
fi
done
```

rnd.hpp

sort(all(res));

md5: e7a280

```
ll rnd(ll l, ll r) { //[l, r)
    static mt19937_64 gen(time(NULL));
    return uniform_int_distribution<ll>(l, r - 1)(gen);
}
template<typename T> void rndshuf(vector<T>& v) { rep(i, 1,
    si(v)) swap(v[i], v[rnd(0, i)]); }
template<class T> vector<T> rvi(int n, T l, T r, bool unique =
false) {
    if(unique) {
        assert(r - l >= n);
        vector<T> res;
        rep(i, n) res.eb(rnd(l, r - n + 1));
```

```
Page 2 of 20
```

```
Speed Star (The University of Tokyo)
    rep(i, n) res[i] += i;
    rndshuf(res);
    return res;
}
vector<T> v(n);
fore(e, v) e = rnd(l, r);
    return v;
}
```

settings.sh

```
# makefile
export CXXFLAGS='-03 -std=c++2a -D_GLIBCXX_DEBUG -
D_GLIBCXX_DEBUG_PEDANTIC -Wfatal-errors'
```

data-structure

2d-segtree.hpp

// DS ... data_structure_type

md5: b23446

```
// S ... size_type
// T ... value_type
template<typename DS, typename T> struct RangeTree {
   using NEW = function<DS*(int)>;
   using ADD = function<void(DS&, int, T)>;
   using SUM = function<T(DS&, int, int)>;
   using MRG = function<T(T, T)>;
   int n, m;
   NEW nw;
   ADD ad;
   SUM sm;
   MRG mg;
   T ti;
   vector<DS*> ds;
   vector<vi> ys;
   vector<pii> ps;
   RangeTree(NEW nw, ADD ad, SUM sm, MRG mg, T ti) : nw(nw),
ad(ad), sm(sm), mg(mg), ti(ti) {}
   void add_point(int x, int y) { ps.eb(x, y); }
   void build() {
      sort(all(ps));
      ps.erase(unique(all(ps)), end(ps));
      n = si(ps);
      ds.resize(2 * n, nullptr);
      ys.resize(2 * n);
      rep(i, n) {
         ys[i + n].eb(ps[i].second);
         ds[i + n] = nw(1);
      per(i, n, 1) {
         ys[i].resize(si(ys[i << 1]) + si(ys[(i << 1) | 1]));</pre>
         merge(all(ys[i << 1]), all(ys[(i << 1) | 1]),
begin(ys[i]));
         ys[i].erase(unique(all(ys[i])), end(ys[i]));
         ds[i] = nw(si(ys[i]));
      }
   }
   int id(int x) const { return lower_bound(all(ps), pii(x, 0))
- begin(ps); }
   int id(int i, int y) const { return lower_bound(all(ys[i]),
y) - begin(ys[i]); }
   void add(int x, int y, T a) {
      int i = lower_bound(all(ps), pii(x, y)) - begin(ps);
      assert(ps[i] == make_pair(x, y));
      for(i += n; i; i >>= 1) ad(*ds[i], id(i, y), a);
   T sum(int xl, int yl, int xr, int yr) {
      T L = ti, R = ti;
      int a = id(xl), b = id(xr);
      for(a += n, b += n; a < b; a >>= 1, b >>= 1) {
         if(a & 1) L = mg(L, sm(*ds[a], id(a, yl), id(a, yr))),
++a;
         if(b \& 1) --b, R = mg(sm(*ds[b], id(b, yl), id(b, yr)),
```

BIT.hpp md5: 362531

```
struct BIT {
  vl a;
  BIT(ll n) : a(n + 1) {}
  void add(ll i, ll x) {
    i++;
    while(i < si(a)) a[i] += x, i += i & -i;
  }
  ll sum(ll r) {
    ll s = 0;
    while(r) s += a[r], r -= r & -r;
    return s;
  }
  ll sum(ll l, ll r) { return sum(r) - sum(l); }
};</pre>
```

FastSet.hpp

md5: 9dd1e2

```
using U = uint64_t;
const U B = 64;
struct FS {
   Un;
   vector<vector<U>> a;
   FS(U n) : n(n) {
      do a.eb(n = (n + B - 1) / B);
      while(n > 1);
   }
   bool operator[](ll i) const { return a[0][i / B] >> (i % B) &
1; }
   void set(ll i) {
      for(auto& v : a) {
         v[i / B] |= 1ULL << (i % B);
         i /= B;
      }
   }
   void erase(ll i) {
      for(auto& v : a) {
         v[i / B] &= ~(1ULL << (i % B));
         if(v[i / B]) break;
         i /= B;
      }
   ll next(ll i) {
      rep(h, si(a)) {
         i++:
         if(i / B >= si(a[h])) break;
         U d = a[h][i / B] >> (i % B);
         if(d) {
            i += countr_zero(d);
            while(h--) i = i * B + countr_zero(a[h][i]);
            return i:
         i /= B;
      }
      return n;
   ll prev(ll i) {
      rep(h, si(a)) {
         i--:
         if(i < 0) break;</pre>
         U d = a[h][i / B] << (~i % B);
         if(d) {
            i -= countl_zero(d);
            while(h--) i = i * B + __lg(a[h][i]);
            return i;
         }
         i /= B;
      }
      return -1;
   }
};
```

```
md5: 38dad3
```

```
template<typename T, bool isMin = true> struct SkewHeap {
   struct Node {
      T key, laz;
      Node *l, *r;
      int idx;
      Node() = default;
      Node(const T& k, int i = -1) : key(k), laz(0), l(nullptr),
r(nullptr), idx(i) {}
   };
   using P = Node*;
   static void propagate(P x) {
      if(x->laz == 0) return;
      if(x->l) x->l->laz += x->laz;
      if(x->r) x->r->laz += x->laz;
      x\rightarrow key += x\rightarrow laz;
      x->laz = 0;
   }
   static P meld(P x, P y) {
      if(!x || !y) return x ? x : y;
      if(!comp(x, y)) swap(x, y);
      propagate(x);
      x->r = meld(x->r, y);
      swap(x->l, x->r);
      return x;
   }
   static P alloc(const T& key, int idx = -1) { return new
Node(key, idx); }
   static P pop(P x) {
      propagate(x);
      return meld(x->l, x->r);
   static P push(P x, const T& key, int idx = -1) { return
meld(x, alloc(key, idx)); }
   static void apply(P x, const T& laz) {
      x->laz += laz;
      propagate(x);
   }
   private:
   static inline bool comp(P x, P y) {
      if constexpr(isMin) {
         return x->key + x->laz < y->key + y->laz;
      } else {
         return x->key + x->laz > y->key + y->laz;
  }
```

cht.hpp

md5: a05621

```
template<bool isMin = true> struct CHT {
#define x first
#define y second
   CHT() = default;
   deque<pll> v;
   bool empty() { return v.empty(); }
   void clear() { return v.clear(); }
   inline int sgn(ll x) \{ return ! x ? 0 : (x < 0 ? -1 : 1); \}
   using D = long double;
   inline bool check(const pll& a, const pll& b, const pll& c) {
      if(b.y == a.y or c.y == b.y) return sgn(b.x - a.x) *
sgn(c.y - b.y) >= sgn(c.x - b.x) * sgn(b.y - a.y);
      return D(b.x - a.x) * sgn(c.y - b.y) / D(abs(b.y - a.y))
\rightarrow= D(c.x - b.x) * sgn(b.y - a.y) / D(abs(c.y - b.y));
   }
   void add(ll a, ll b) {
      if(!isMin) a *= -1, b *= -1;
      pll line(a, b);
      if(empty()) v.emplace_front(line);
      else {
         if(ll c = v[0].x; c <= a) {
            if(c == a) {
               if(v[0].y <= b) return;</pre>
               v.pop_front();
            }
            while(si(v) >= 2 and check(line, v[0], v[1]))
v.pop_front();
            v.emplace_front(line);
         } else {
```

```
assert(a <= v.back().x);</pre>
            if(v.back().x == a) {
               if(v.back().y <= b) return;</pre>
               v.pop_back();
            while(si(v) >= 2 and check(v[si(v) - 2], v.back(),
line)) v.pop_back();
            v.emplace_back(line);
      }
   ll get_y(const pll\& a, const ll\& x) { return a.x * x + a.y; }
   ll query(ll x) {
      assert(!empty());
      int l = -1, r = si(v) - 1;
      while(l + 1 < r) {
         int m = (l + r) >> 1;
         if(get_y(v[m], x) \ge get_y(v[m + 1], x)) l = m;
         else r = m;
      }
      return get_y(v[r], x) * (isMin ? 1 : -1);
   ll query_monotone_inc(ll x) {
      assert(!empty());
      while(si(v) \ge 2 and get_y(v[0], x) \ge get_y(v[1], x))
v.pop_front();
      return get_y(v[0], x) * (isMin ? 1 : -1);
   ll query_monotone_dec(ll x) {
      assert(!empty());
      while(si(v) >= 2 and get_y(v.back(), x) >= get_y(v.end()
[-2], x)) v.pop_back();
      return get_y(v.back(), x) * (isMin ? 1 : -1);
   }
#undef x
#undef y
};
```

hash_map.hpp

md5: 1893ff

```
#include <bits/extc++.h>
struct chash {
    const uint64_t C = (ll)(4e18 * acos(0)) | 71;
    ll operator()(ll x) const { return __builtin_bswap64(x * C);
};
using namespace __gnu_pbds;
template<class T, class S> using hash_map = gp_hash_table<T, S, chash>;
```

lazy-segtree.hpp

md5: e79596

```
using U = uint64_t;
template<class S, S (*op)(S, S), S (*e)(), class F, S (*mpp)(F,
S), F (*cmpo)(F, F), F (*id)()> struct lazy_segtree {
   lazy_segtree() : lazy_segtree(0) {}
   explicit lazy_segtree(int n) : lazy_segtree(vector<S>(n,
e())) {}
   explicit lazy_segtree(const vector<S>& v) : n(si(v)) {
      s = bit_ceil(U(n));
      log = countr_zero(U(s));
      d = vector < S > (2 * s, e());
      lz = vector<F>(s, id());
      rep(i, n) d[s + i] = v[i];
      per(i, s, 1) update(i);
  }
   void set(int p, S x) {
      p += s;
      PUSH(p);
      d[p] = x;
      rep(i, 1, log + 1) update(p >> i);
   }
   S get(int p) {
      p += s;
      PUSH(p);
      return d[p];
   S prod(int l, int r) {
      if(l == r) return e();
      l += s, r += s;
      per(i, log + 1, 1) {
         if(((l >> i) << i) != l) push(l >> i);
```

```
Speed Star (The University of Tokyo)
```

```
Page 4 of 20
```

```
if(((r >> i) << i) != r) push((r - 1) >> i);
   S sml = e(), smr = e();
   while(l < r) {</pre>
      if(l \& 1) sml = op(sml, d[l++]);
      if(r & 1) smr = op(d[--r], smr);
      l >>= 1, r >>= 1;
   }
   return op(sml, smr);
S all_prod() { return d[1]; }
void apply(int p, F f) {
   // assert(0 <= p && p < n);
   p += s;
   PUSH(p);
   d[p] = mpp(f, d[p]);
   rep(i, 1, log + 1) update(p >> i);
void apply(int l, int r, F f) {
   // assert(0 <= l && l <= r && r <= _n);
   if(l == r) return;
   l += s, r += s;
   per(i, log + 1, 1) {
      if(((l >> i) << i) != l) push(l >> i);
      if(((r >> i) << i) != r) push((r - 1) >> i);
   int ml = l, mr = r;
   while(l < r) {</pre>
      if(l & 1) all_apply(l++, f);
      if(r & 1) all_apply(--r, f);
      l >>= 1, r >>= 1;
   }
   l = ml, r = mr;
   rep(i, 1, log + 1) {
      if(((l >> i) << i) != l) update(l >> i);
      if(((r >> i) << i) != r) update((r - 1) >> i);
}
template<class G> int max_right(int l, G g) {
   assert(g(e()));
   if(l == n) return n;
   l += s;
   PUSH(l);
   S sm = e();
      while(~l & 1) l >>= 1;
      if(!g(op(sm, d[l]))) {
         while(l < s) {</pre>
            push(l);
            l <<= 1:
            if(g(op(sm, d[l]))) {
               sm = op(sm, d[l]);
            }
         }
         return l - s;
      sm = op(sm, d[l]);
   } while((l & -l) != l);
   return n;
}
template<class G> int min_left(int r, G g) {
   assert(g(e()));
   if(r == 0) return 0;
   r += s;
   PUSH(r - 1);
   S sm = e();
   do {
      while(r > 1 && r & 1) r >>= 1;
      if(!g(op(d[r], sm))) {
         while(r < s) {</pre>
            push(r);
            r = (2 * r + 1);
            if(g(op(d[r], sm))) {
               sm = op(d[r], sm);
         }
         return r + 1 - s;
```

```
sm = op(d[r], sm);
  } while((r & -r) != r);
  return 0;
S operator[](int k) { return get(k); }
int len() { return n; }
private:
int n, s, log;
vector<S> d;
vector<F> lz;
void update(int k) { d[k] = op(d[2 * k], d[2 * k + 1]); }
void all_apply(int k, F f) {
  d[k] = mpp(f, d[k]);
  if(k < s) lz[k] = cmpo(f, lz[k]);
void push(int k) {
  all_apply(2 \star k, lz[k]);
  all_apply(2 * k + 1, lz[k]);
  lz[k] = id();
void PUSH(int k) { per(i, log + 1, 1) push(k >> i); }
```

li-chao.hpp

md5: ca57d5

```
struct lctree {
   struct line {
      ll a, b;
      line(): a(0), b(INFL) {}
      line(ll a, ll b) : a(a), b(b) {}
      ll get(ll x) { return a * x + b; }
      inline bool over(line r, ll x) { return get(x) < r.get(x);</pre>
}
   };
   int n;
   vector<ll> x;
   vector<line> seg;
   lctree() {}
   lctree(const vector<ll>& _x): x(_x) \{
      sort(all(x));
      int n2 = si(x);
      n = 1;
      while(n < n2) n <<= 1;</pre>
      x.resize(n);
      rep(i, n2, n) x[i] = x[n2 - 1];
      seg = vector<line>(n * 2);
   void upd(line L, int i, int l, int r) {
      while(true) {
         int mid = l + r >> 1;
         bool lov = L.over(seg[i], x[l]);
         bool rov = L.over(seg[i], x[r - 1]);
         if(lov == rov) {
            if(lov) swap(seg[i], L);
            return:
         bool mov = L.over(seq[i], x[mid]);
         if(mov) swap(seg[i], L);
         if(lov != mov) {
            i = (i << 1), r = mid;
         } else {
            i = (i << 1) + 1, l = mid;
     }
   void upd(line L, unsigned i) {
      int ub = bit_width(i) - 1;
      int l = (n >> ub) * (i - (1 << ub));
      int r = l + (n \gg ub);
      upd(L, i, l, r);
   void update(ll a, ll b) { upd(line(a, b), 1, 0, n); }
   void update_segment(ll l, ll r, ll a, ll b) {
      l = lb(x, l) + n, r = lb(x, r) + n;
      line L(a, b);
      for(; l < r; l >>= 1, r >>= 1) {
         if(l & 1) upd(L, l++);
         if(r & 1) upd(L, --r);
```

```
}
}
ll query(ll t) {
    ll k = lb(x, t);
    k += n;
    ll res = seg[k].get(t);
    while(k > 1) {
        k >>= 1;
        chmin(res, seg[k].get(t));
    }
    return res;
}
```

line_container.hpp

md5: b018d9

```
struct Line {
   mutable ll k, m, p;
   bool operator<(const Line& o) const { return k < o.k; }</pre>
   bool operator<(ll x) const { return p < x; }</pre>
template<bool ismin = true> struct LineContainer :
multiset<Line, less<>>> {
   // (for doubles, use inf = 1/.0, div(a,b) = a/b)
   const ll inf = LLONG_MAX / 2;
   ll div(ll a, ll b) { // floored division
      return a / b - ((a ^ b) < 0 && a % b);
   bool isect(iterator x, iterator y) {
      if(y == end()) {
         x->p = inf;
         return false;
      }
      if(x->k == y->k) x->p = x->m > y->m ? inf : -inf;
      else x->p = div(y->m - x->m, x->k - y->k);
      return x->p >= y->p;
   }
   void add(ll k, ll m) {
      if(ismin) k = -k, m = -m;
      auto z = insert(\{k, m, 0\}), y = z++, x = y;
      while(isect(y, z)) z = erase(z);
      if(x != begin() && isect(--x, y)) isect(x, y = erase(y)); while((y = x) != begin() && (--x)->p >= y->p) isect(x,
erase(y));
   }
   ll query(ll x) {
      auto l = *lower_bound(x);
      ll s = 1;
      if(ismin) s = -1;
      return s * (l.k * x + l.m);
};
```

link-cut.hpp

md5: e9b023

```
struct Node {
  typedef Node* NP;
  NP l, r, p;
  bool rev;
  int v, mx, lz;
  Node(): l(NULL), r(NULL), p(NULL), rev(false), v(-inf), mx(-
inf), lz(-inf) {}
  void Propagate() {
     if(rev) {
         swap(l, r);
         if(l) l->rev ^= true;
         if(r) r->rev ^= true;
         rev = false;
      if(l) chmax(l->lz, lz);
      if(r) chmax(r->lz, lz);
      chmax(v, lz);
      chmax(mx, lz);
      lz = -inf;
  int GetMax() { return max(mx, lz); }
  int GetVert() { return max(v, lz); }
  void Update() {
      assert(lz == -inf);
      mx = v;
      if(l) { chmax(mx, l->GetMax()); }
```

```
if(r) { chmax(mx, r->GetMax()); }
   }
   int Pos() {
      if(p && p->l == this) return -1;
      if(p && p->r == this) return 1;
      return 0:
   void Prepare() {
      if(Pos()) p->Prepare();
      Propagate();
   void Rotate() {
      NP q = p, c;
      if(Pos() == 1) {
         c = 1;
         l = p;
         p->r = c;
      } else {
         c = r;
         r = p;
         p->l = c;
      if(c) c \rightarrow p = p;
      p = p \rightarrow p;
      q->p = this;
      if(p && p->l == q) p->l = this;
      if(p \&\& p->r == q) p->r = this;
      q->Update();
   void Splay() {
      Prepare();
      while(Pos()) {
         int a = Pos(), b = p \rightarrow Pos();
         if(b && a == b) p->Rotate();
         if(b && a != b) Rotate();
         Rotate();
      }
      Update();
   }
   void Expose() {
      for(NP x = this; x; x = x->p) x->Splay();
      for(NP x = this; x->p; x = x->p) {
         x \rightarrow p \rightarrow r = x;
         x->p->Update();
      }
      Splay();
   }
   void Evert() {
      Expose();
      if(l) {
         l->rev ^= true;
         l = NULL;
         Update();
      }
   void Link(NP x) {
      Evert();
      p = x;
   void Set(int q) {
      Expose();
      r = NULL;
      chmax(lz, q);
   void Cut() {
      Expose();
      assert(l):
      1->p = NULL;
      l = NULL;
      Update();
   }-
   int Get() {
      Expose();
      r = NULL;
      Update();
      return GetMax();
  }
Node* LCA(Node* a, Node* b) {
  a->Expose();
   b->Expose();
   if(!a->p) { return NULL; }
```

```
Speed Star (The University of Tokyo)
```

```
Page 6 of 20
```

md5: 2fbe43

```
Node* d = a;
while(a->p != b) {
    if(a->Pos() == 0) { d = a->p; }
    a = a->p;
}
if(a == b->l) {
    return d;
} else {
    return b;
}
```

pbds.hpp

md5: 6ac09d

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
// not a multiset
// find_by_order(k) -> itr of k-th(0-based) element
// order_of_key(k) -> index of lower_bound(k)
using ordered_set = tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update>;
```

rbst.hpp

md5: b392ca

```
template<typename T, T (*f)(T, T), T (*e)()> struct RBST {
  inline int rnd() {
      static int x = 123456789;
      static int y = 362436069;
      static int z = 521288629;
      static int w = 88675123;
      int t;
      t = x ^ (x << 11);
     x = y;
      y = z;
     z = w:
      return w = (w ^ (w >> 19)) ^ (t ^ (t >> 8));
  }
  struct node {
     node *l, *r;
      int cnt;
      T x, sum;
      node() = default;
      node(T x) : x(x), sum(x), l(0), r(0) { cnt = 1; }
  RBST(int n) : pool(n) {}
  int cnt(const node* t) { return t ? t->cnt : 0; }
  T sum(const node* t) { return t ? t->sum : e(); }
  node* update(node* t) {
      t\rightarrow cnt = cnt(t\rightarrow l) + cnt(t\rightarrow r) + 1;
      t\rightarrow sum = f(f(sum(t\rightarrow l), t\rightarrow x), sum(t\rightarrow r));
      return t;
  }
  vector<node> pool;
   int ptr = 0;
  inline node* alloc(const T& v) {
      if(si(pool) == ptr) pool.resize(si(pool) * 2);
      return &(pool[ptr++] = node(v));
  node* merge(node* l, node* r) {
      if(!l or !r) return l ? l : r;
      if(rnd() % (cnt(l) + cnt(r)) < cnt(l)) {
         l->r = merge(l->r, r);
         return update(l);
      }
      r->l = merge(l, r->l);
      return update(r);
  pair<node*, node*> split(node* t, int k) {
      if(!t) return {t, t};
      if(k <= cnt(t->l)) {
         auto [l, r] = split(t->l, k);
         t->l = r;
         return {l, update(t)};
      }
      auto [l, r] = split(t->r, k - cnt(t->l) - 1);
      t->r = 1;
      return {update(t), r};
```

```
void insert(node*& t, int k, const T& v) {
   auto [l, r] = split(t, k);
   t = merge(merge(l, alloc(v)), r);
}
};
```

segbeats.hpp

```
struct Segtree_beats {
   ll op(int type, ll x, ll y) { return type ? min(x, y) :
max(x, v); }
   bool cmp(int type, ll x, ll y) { return type ? x < y : x > y;
   struct alignas(32) Node {
      ll sum = 0:
      11 a1[2] = {}, a2[2] = {}-INFL, INFL}, ac[2] = {1, 1}, add
= 0:
   };
   vector<Node> v;
   ll n, log, e[3] = {-INFL, INFL, 0};
   Segtree_beats() {}
   Segtree_beats(int n) : Segtree_beats(vl(n)) {}
   Segtree_beats(const vl& a) {
      n = 1, log = 0;
      while(n < si(a)) n <<= 1, log++;</pre>
      v.resize(2 * n);
      rep(i, si(a)) { v[i + n].sum = v[i + n].a1[0] = v[i + n]}
n].a1[1] = a[i]; }
      per(i, n, 1) update(i);
   // 0 : add, 1 : chmin, 2 : chmax, 3 : update
   template<int cmd> void apply(int l, int r, ll x) {
      if(l == r) return;
      l += n, r += n;
      per(i, log + 1, 1) {
         if(((l >> i) << i) != l) push(l >> i);
         if(((r >> i) << i) != r) push((r - 1) >> i);
         int l2 = l, r2 = r;
         while(l < r) {</pre>
            if(l \& 1) _apply<cmd>(l++, x);
            if(r & 1) _apply<cmd>(--r, x);
            l >>= 1;
            r >>= 1;
         }
         1 = 12;
         r = r2;
      rep(i, 1, log + 1) {
         if(((l >> i) << i) != l) update(l >> i);
         if(((r >> i) << i) != r) update((r - 1) >> i);
   }
   // 0 : max, 1 : min, 2 : sum
   template<int cmd> ll fold(int l, int r) {
      if(l == r) return e[cmd];
      l += n, r += n;
      per(i, log + 1, 1) {
         if(((l >> i) << i) != l) push(l >> i);
         if(((r >> i) << i) != r) push((r - 1) >> i);
      ll lx = e[cmd], rx = e[cmd];
      while(l < r) {</pre>
         if(l & 1) op<cmd>(lx, v[l++]);
         if(r & 1) op<cmd>(rx, v[--r]);
         1 >>= 1:
         r >>= 1;
      if constexpr(cmd <= 1) lx = op(cmd, lx, rx);</pre>
      if constexpr(cmd == 2) lx += rx;
      return lx;
   }
   private:
   void update(int k) {
      Node \delta \rho = v[k]:
      Node& l = v[k * 2 + 0];
```

```
Node& r = v[k * 2 + 1];
      p.sum = l.sum + r.sum;
      rep(t, 2) {
         if(l.a1[t] == r.a1[t]) {
            p.a1[t] = l.a1[t];
            p.a2[t] = op(t, l.a2[t], r.a2[t]);
            p.ac[t] = l.ac[t] + r.ac[t];
         } else {
            bool f = cmp(t, l.a1[t], r.a1[t]);
            p.a1[t] = f ? l.a1[t] : r.a1[t];
            p.ac[t] = f ? l.ac[t] : r.ac[t];
            p.a2[t] = op(t, f? r.a1[t] : l.a1[t], f? l.a2[t] :
r.a2[t]);
      }
   }
   void push_add(int k, ll x) {
      Node& p = v[k];
      p.sum += x \ll (log + \_builtin\_clz(k) - 31);
      rep(t, 2) {
         p.a1[t] += x;
         if(p.a2[t] != e[t]) p.a2[t] += x;
      }
      p.add += x;
   void push(int cmd, int k, ll x) {
      Node p = v[k];
      p.sum += (x - p.a1[cmd]) * p.ac[cmd];
      if(p.a1[cmd ^ 1] == p.a1[cmd]) p.a1[cmd ^ 1] = x;
      if(p.a2[cmd ^ 1] == p.a1[cmd]) p.a2[cmd ^ 1] = x;
      p.a1[cmd] = x;
   void push(int k) {
      Node& p = v[k];
      if(p.add) {
         rep(t, 2) push_add(k * 2 + t, p.add);
         p.add = 0;
      }
      rep(t, 2) rep(s, 2) if(cmp(t, v[k * 2 + s].a1[t],
p.a1[t])) push(t, k * 2 + s, p.a1[t]);
  }
   void subtree_ch(int cmd, int k, ll x) {
      if(!cmp(cmd, v[k].a1[cmd], x)) return;
      if(cmp(cmd, x, v[k].a2[cmd])) { return push(cmd, k, x); }
      rep(t, 2) subtree_ch(cmd, k * 2 + t, x);
      update(k);
   }
   template<int cmd> inline void _apply(int k, ll x) {
      rep(i, 2) if(cmd >> i & 1) subtree_ch(i, k, x);
      if constexpr(cmd == 0) push_add(k, x);
   template<int cmd> inline void op(ll& a, const Node& b) {
      if constexpr(cmd <= 1) a = op(cmd, a, b.a1[cmd]);</pre>
      if constexpr(cmd == 2) a += b.sum;
   }
};
```

segtree.hpp

```
md5: 2472c3
```

```
template<class S, S (*op)(S, S), S (*e)()> struct segtree {
  segtree(int n) : segtree(vector<S>(n)) {}
   segtree(const vector<S>& v) : n(si(v)) {
      s = bit_ceil(uint(n));
      log = countr_zero(uint(s));
     d = vector < S > (2 * s, e());
      rep(i, n) d[s + i] = v[i];
     per(i, s, 1) update(i);
  }
  void set(int p, S x) {
      d[p += s] = x;
      rep(i, 1, log + 1) update(p >> i);
  S prod(int l, int r) const {
      S sml = e(), smr = e();
     l += s, r += s;
      while(l < r) {</pre>
```

```
if(l \& 1) sml = op(sml, d[l++]);
      if(r \& 1) smr = op(d[--r], smr);
      l >>= 1, r >>= 1;
   }
   return op(sml, smr);
}
S all_prod() const { return d[1]; }
template<typename F> int max_right(int l, F f) const {
   if(l == n) return n;
   l += s;
   S sm = e();
   do {
      while(~l & 1) l >>= 1;
      if(!f(op(sm, d[l]))) {
         while(l < s) {
            l <<= 1:
            if(f(op(sm, d[l]))) sm = op(sm, d[l++]);
         }
         return 1 - s;
      }
      sm = op(sm, d[l++]);
   } while((l & -l) != l);
   return n;
template<typename F> int min_left(int r, F f) const {
   if(!r) return 0;
   r += s;
   S sm = e();
   do {
      r--;
      while(r > 1 and r & 1) r >>= 1;
      if(!f(op(d[r], sm))) {
         while(r < s) {</pre>
            r = (2 * r + 1);
            if(f(op(d[r], sm))) sm = op(d[r--], sm);
         }
         return r + 1 - s;
      }
      sm = op(d[r], sm);
   } while((r & -r) != r);
   return 0;
}
private:
int n, s, log;
vector<S> d;
void update(int k) { d[k] = op(d[k * 2], d[k * 2 + 1]); }
```

sparse-table-disjoint.hpp

md5: 198e80

```
template<typename T, typename F> struct sptable {
   const F f;
   vector<vector<T>> a;
   vi l;
   sptable(const vector<T>& v, F f) : f(f) {
      int m = 0;
      while((1 << m) <= si(v)) ++m;</pre>
      a.resize(m, vector<T>(si(v), T()));
      rep(i, si(v)) a[0][i] = v[i];
      rep(i, 1, m) {
         int s = 1 << i;
         for(int j = 0; j < si(v); j += s * 2) {
            int t = min(j + s, si(v));
            a[i][t - 1] = v[t - 1];
            per(k, t - 1, j) a[i][k] = f(v[k], a[i][k + 1]);
            if(si(v) <= t) break;</pre>
            a[i][t] = v[t];
            int r = min(t + s, si(v));
            rep(k, t + 1, r) a[i][k] = f(a[i][k - 1], v[k]);
         }
      l.resize(1 << m);
      rep(i, 2, si(l)) l[i] = l[i >> 1] + 1;
   T query(int x, int y) {
      if(x \ge --y) return a[0][x];
      int p = l[x ^ y];
      return f(a[p][x], a[p][y]);
```

```
}
};
```

swag.hpp md5: 85c3df

```
template<typename T, typename F> struct SWAG {
  using vp = vector<pair<T, T>>;
  vp a, b:
  Ff;
  TI;
  SWAG(F f, T i) : f(f), I(i) {}
  private:
  T get(vp& v) { return empty(v) ? I : v.back().second; }
  void pusha(T x) { a.eb(x, f(x, get(a))); }
  void pushb(T x) { b.eb(x, f(get(b), x)); } // reversed!!
  void rebalance() {
      int n = si(a) + si(b);
      int s0 = n / 2 + (empty(a) ? n & 1 : 0);
      vp v{a}:
      reverse(all(v));
     copy(all(b), back_inserter(v));
      a.clear(), b.clear();
     per(i, s0, 0) pusha(v[i].first);
      rep(i, s0, n) pushb(v[i].first);
  }
  nublic:
  T front() { return (a.empty() ? b.front() : a.back()).first;
  T back() { return (b.empty() ? a.front() : b.back()).first; }
  void pop_front() {
      if(empty(a)) rebalance();
      a.pop_back();
  void pop_back() {
      if(empty(b)) rebalance();
      b.pop_back();
  T query() { return f(get(a), get(b)); }
```

wavelet_matrix.hpp

md5: dec827

```
#define U uint32_t
#define L uint64 t
struct bit_vector {
   static constexpr U w = 64;
   vector<L> block;
   vector<U> count:
   int n, zeros;
   inline U get(U i) const { return U(block[i / w] >> (i % w)) &
1; }
   inline void set(U i) { block[i / w] |= 1LL << (i % w); }</pre>
   bit_vector() {}
   bit_vector(int n) { init(n); }
   void init(int _n) {
      n = zeros = _n;
      block.resize(n / w + 1, 0);
      count.resize(si(block), 0);
   void build() {
     rep(i, 1, si(block)) count[i] = count[i - 1] +
popcount(block[i - 1]);
      zeros = rank0(n);
   inline U rank0(U i) const { return i - rank1(i); }
   inline U rank1(U i) const { return count[i / w] +
popcount(block[i / w] & ((1ULL << i % w) - 1)); }</pre>
template<typename T, const int lg = 31> struct WaveletMatrix {
   int n;
   vector<T> a;
   array<bit_vector, lg> bv;
   WaveletMatrix(const vector<T>& _a) : n(_a.size()), a(_a) {
build2(); }
```

```
void build() {
      rep(i, lg) bv[i] = bit_vector(n);
      vector<T> cur = a, nxt(n);
      per(h, lg, 0) {
         rep(i, n) if(cur[i] >> h & 1) bv[h].set(i);
         bv[h].build();
         array<decltype(begin(nxt)), 2> it{begin(nxt),
begin(nxt) + bv[h].zeros};
         rep(i, n) * it[bv[h].get(i)]++ = cur[i];
         swap(cur, nxt);
      }
      return;
   }
   inline pair<U, U> succ0(int l, int r, int h) const { return
make_pair(bv[h].rank0(l), bv[h].rank0(r)); }
   inline pair<U, U> succ1(int l, int r, int h) const {
      U l0 = bv[h].rank0(l);
      U r0 = bv[h].rank0(r);
      U zeros = bv[h].zeros;
      return make_pair(l + zeros - l0, r + zeros - r0);
   T access(U k) const {
      T ret = 0;
      per(h, lg, 0) {
         U f = bv[h].get(k);
         ret |= f ? T(1) << h : 0;
         k = f ? bv[h].rank1(k) + bv[h].zeros : bv[h].rank0(k);
      return ret;
   }
   T kth_smallest(U l, U r, U k) const {
      T res = 0;
      for(int h = lg - 1; h >= 0; --h) {
         U l0 = bv[h].rank0(l), r0 = bv[h].rank0(r);
         if(k < r0 - l0) l = l0, r = r0;
         else {
            k = r0 - 10;
            res |= (T)1 << h;
            l += bv[h].zeros - l0, r += bv[h].zeros - r0;
         }
     }
      return res;
   T kth_largest(int l, int r, int k) { return kth_smallest(l,
r, r - l - k - 1); }
   int range_freq(int l, int r, T upper) {
      if(upper >= (T(1) << lg)) return r - l;
      int ret = 0;
      per(h, lg, 0) {
         bool f = (upper >> h) & 1;
         U l0 = bv[h].rank0(l), r0 = bv[h].rank0(r);
         if(f) {
            ret += r0 - l0;
            l += bv[h].zeros - l0;
            r += bv[h].zeros - r0;
         } else {
            1 = 10;
            r = r0;
         }
      }
      return ret;
   int range_freq(int l, int r, T lower, T upper) { return
range_freq(l, r, upper) - range_freq(l, r, lower); }
   array<vector<ll>, lg> sums;
   vector<ll> acc;
   void build2() {
      rep(i, lg) bv[i] = bit_vector(n), sums[i].assign(n + 1,
      acc.resize(si(a) + 1);
      vector<T> cur = a, nxt(n);
      per(h, lg, 0) {
         rep(i, n) if((cur[i] >> h) & 1) bv[h].set(i);
         bv[h].build();
         array<decltype(begin(nxt)), 2> it{begin(nxt),
begin(nxt) + bv[h].zeros};
```

```
Speed Star (The University of Tokyo)
                     rep(i, n) * it[bv[h].get(i)]++ = cur[i];
                     swap(cur, nxt);
                     rep(i, n) sums[h][i + 1] = sums[h][i] + cur[i];
              }
              rep(i, n) acc[i + 1] = acc[i] + a[i];
       }
       ll bottom_k_sum(int l, int r, int k) {
              ll res = 0;
              per(h, lg, 0) {
                     U l0 = bv[h].rank0(l), r0 = bv[h].rank0(r);
                     if(k < r0 - l0) {
                            l = 10, r = r0;
                     } else {
                            res += sums[h][r0] - sums[h][l0];
                            k = r0 - 10;
                            l += bv[h].zeros - l0;
                            r += bv[h].zeros - r0;
              }
              res += sums[0][l + k] - sums[0][l];
              return res;
       ll top_k_sum(int l, int r, int k) { return acc[r] - acc[l] -
bottom_k_sum(l, r, r - l - k); }
};
#undef U
#undef L
  dp
d-edge-monge.hpp
                                                                                                                            md5: 4ab7ee
template<class C, class T = decltype(std::declval<C>().get())> T
incremental_monge_shortest_path(const int n, C init) {
       class env {
     public:
              C mid;
              C last;
              int prev;
       };
       std::vector<env> nodes;
              int n_ = n;
              int d = 0;
              while(n_ != 0) {
                    n_ /= 2;
                     d += 1;
              }
              nodes.assign(d, {init, init, 0});
       }
       std::vector<T> dp(n + 1, static_cast<T>(0));
       const auto f = [&](const auto& f, const int d, const int r) -
              auto& [mid, last, prev] = nodes[d];
              const int w = 1 \ll d;
              if((r >> d) \% 2 == 1) {
                      for(int i = std::max(0, r - 2 * w); i != r; i += 1) {
mid.push_back(i); }
                     const int next = r + w \le n ? f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r + w) : r - f(f, d + 1, r +
w;
                     int argmin = prev;
                     dp[r] = dp[argmin] + mid.get();
                      for(int i = prev; i != next;) {
                            mid.pop_front(i);
                            i += 1;
                            const T t = dp[i] + mid.get();
                            if(dp[r] > t) {
                                   dp[r] = t;
                                   argmin = i;
                            }
                     }
                     prev = next;
                     return argmin;
              } else {
                     for(int i = std::max(0, r - 2 * w); i != r; i += 1) {
last.push_back(i); }
```

for(int i = std::max(0, r - 3 * w); i != r - 2 * w; i

+= 1) { last.pop_front(i); }

```
int argmin = prev;
         for(int i = r - 2 * w; i != r - w;) {
            last.pop_front(i);
            i += 1;
            const T t = dp[i] + last.get();
            if(dp[r] > t) {
               dp[r] = t;
               argmin = i;
         }
         return argmin;
     }
  };
   for(int i = 1; i != n + 1; i += 1) { f(f, 0, i); }
   return dp[n];
}
namespace golden_section_search_impl {
using i64 = std::int64_t;
template<class F, class T = decltype(std::declval<F>()
(std::declval<i64>())), class Compare = std::less<T>>
std::pair<i64, T> golden_section_search(F f, i64 min, i64 max,
Compare comp = Compare()) {
   assert(min <= max);</pre>
   i64 = min - 1, x, b;
   {
      i64 s = 1, t = 2;
      while(t < max - min + 2) { std::swap(s += t, t); }
      x = a + t - s;
      b = a + t;
   }
   T fx = f(x), fy;
   while(a + b != 2 * x) {
      const i64 y = a + b - x;
      if(max < y \mid | comp(fx, (fy = f(y)))) {
         b = a;
         a = y;
      } else {
         a = x;
         x = y;
         fx = fy;
   return {x, fx};
  // namespace golden_section_search_impl
using golden_section_search_impl::golden_section_search;
struct cost {
   const vector<ll>* a;
   ll lambda;
  ll cost:
   void pop_front(int l) {}
   void push_back(int r) {}
   ll get() { return lambda + c } // 最小化なら -
};
// k : 使う辺の本数
const auto f = [\&](ll \ l) \rightarrow ll \ {}
   auto res = incremental_monge_shortest_path(n + 1, cost{l, 0,
0}) - l * (k + 1);
   return res:
// L = - max(|e|) * 3, R = max(|e|) * 3
OUT(golden_section_search(f, L, R, greater<ll>()).se);
mo-rollback.hpp
                                                      md5: 5737bf
struct MoRollBack {
```

using ADD = function<void(int)>;

using REM = function<void(int)>;

using ROLLBACK = function<void()>;

using RESET = function<void()>;

using SNAP = function<void()>;

```
Speed Star (The University of Tokyo)
   vector<int> l, r, ord;
   MoRollBack(int n, int q) : w((int)sqrt(n)), ord(q) {
iota(all(ord), 0); }
   void add(int a, int b) { /* [l, r) */
      l.emplace_back(a);
      r.emplace_back(b);
   void run(const ADD& add, const REM& rem, const RESET& reset,
const SNAP& snap, const ROLLBACK& rollback) {
      sort(begin(ord), end(ord), [&](int a, int b) {
         int ab = l[a] / w, bb = l[b] / w;
         if(ab != bb) return ab < bb;</pre>
         return r[a] < r[b];</pre>
      }):
      reset();
      for(auto idx : ord) {
         if(r[idx] - l[idx] < w) {</pre>
            rep(i, l[idx], r[idx]) add(i);
            rem(idx);
            rollback();
         }
      }
      int nr = 0, lb = -1;
      for(auto idx : ord) {
   if(r[idx] - l[idx] < w) continue;</pre>
         int b = l[idx] / w;
         if(lb != b) {
            reset();
            lb = b;
            nr = (b + 1) * w;
         while(nr < r[idx]) add(nr++);</pre>
         per(j, (b + 1) * w, l[idx]) add(j);
         rem(idx);
         rollback();
      }
  }
};
mo.hpp
                                                        md5: 6ff6db
struct Mo {
   int n:
   vector<pii> lr;
   Mo(int n) : n(n) {}
   void add(int l, int r) { lr.eb(l, r); }
   template<typename AL, typename AR, typename EL, typename ER,
typename 0>
   void build(const AL& add_left, const AR& add_right, const EL&
erase_left, const ER& erase_right, const O& out) {
      int q = (int)lr.size();
      int bs = n / min<int>(n, sqrt(q));
      vector<int> ord(q);
      iota(all(ord), 0);
      sort(all(ord), [&](int a, int b) {
         int ab = lr[a].first / bs, bb = lr[b].first / bs;
         if(ab != bb) return ab < bb;</pre>
         return (ab & 1) ? lr[a].second > lr[b].second :
```

```
lr[a].second < lr[b].second;</pre>
      });
      int l = 0, r = 0;
      for(auto idx : ord) {
         while(l > lr[idx].first) add_left(--l);
         while(r < lr[idx].second) add_right(r++);</pre>
         while(l < lr[idx].first) erase_left(l++);</pre>
         while(r > lr[idx].second) erase_right(--r);
         out(idx):
   }
   template<typename A, typename E, typename O> void build(const
A& add, const E& erase, const O& out) {
      build(add, add, erase, erase, out);
   }
```

monge-incremental-rowmin.hpp

```
md5: 2cff0f
```

```
..., N)行目の最小値を返す
// f(i, j, v) で、j 行目の最小値が求まっている v を用いて、A[i][j] にア
template<typename T, typename F> vector<T> monge_rowmin(int n,
const F& f) {
   vector<T> mi(n + 1, numeric_limits<T>::max());
   mi[0] = 0;
   vector<int> amin(n + 1);
   auto check = [&](int i, int j) {
     if(chmin(mi[i], f(i, j, mi))) { amin[i] = j; }
   }:
   check(n, 0);
   auto solve = [&](auto&& self, int l, int r) {
     if(r - l == 1) return;
     int mid = l + r >> 1;
     rep(k, amin[l], amin[r] + 1) check(mid, k);
     self(self, l, mid);
     rep(k, l + 1, mid + 1) check(r, k);
     self(self, mid, r);
   };
   solve(solve, 0, n);
   return mi;
```

monotone-minima.hpp

md5: 187a2d

```
// monotone 行列の各行について、最小値を取る場所とその値を返す
template<typename T, typename F> vector<pair<int, T>>
monotone_minima(int h, int w, const F& f) {
   vector<pair<int, T>> dp(h, pair(-1, T()));
   auto rec = [&](auto&& rec, int u, int d, int l, int r) {
     if(u > d) return;
     int mid = u + d \gg 1;
     auto& [idx, mi] = dp[mid];
     idx = l, mi = f(mid, l);
     rep(i, l + 1, r + 1) if(chmin(mi, f(mid, i))) idx = i;
     rec(rec, u, mid - 1, l, idx);
     rec(rec, mid + 1, d, idx, r);
   rec(rec, 0, h - 1, 0, w - 1);
   return dp;
```

math

ExtGCD.hpp

md5: 88cb1c

```
// returns gcd(a, b) and assign x, y to integers
// s.t. ax + by = gcd(a, b) and |x| + |y| is minimized
ll extgcd(ll a, ll b, ll& x, ll& y) {
   // assert(a >= 0 && b >= 0);
   if(!b) return x = 1, y = 0, a;
   ll d = extgcd(b, a \% b, y, x);
   y -= a / b * x;
   return d;
ll inv_{mod}(ll x, ll md) {
  ll y, z;
   extgcd(x, md, y, z);
   return (y % md + md) % md;
```

and-or-convolution.hpp

md5: da6157

```
// and / or convolution
template<bool isOR, typename T> void fzt(vector<T>& a, bool inv
= false) {
   int n = si(a);
   int m = __lg(n);
   rep(i, m) {
      rep(b, n) {
         if((b >> i \& 1) == isOR) a[b] += a[b ^ 1 << i] * (inv ?)
-1:1);
      }
  }
```

```
crt.hpp
```

```
md5: 0e9c10
```

```
// (rem, mod)
pll crt(const vl& b, const vl& c) {
   int n = si(b);
   11 r = 0, m = 1;
   rep(i, n) {
      ll g, im, x;
      g = extgcd(m, c[i], im, x);
      if((b[i] - r) % g) return {0, -1};
      ll tmp = (b[i] - r) / g * im % (c[i] / g);
      r += m * tmp;
      m \star = c[i] / g;
   return {(r % m + m) % m, m};
```

floor_sum.hpp

md5: 930ca0

```
// x_i=floor((a*i+b)/c), i=0,1,..n-1
// a.c>0. b>=0
ll floor_sum(ll n, ll a, ll b, ll c) {
   if(n == 0) return 0;
   ll res = 0;
   res += n * (n - 1) / 2 * (a / c);
   a %= c;
   res += n \star (b / c);
   b %= c;
   if(a == 0) return res;
   ll top = (a * (n - 1) + b) / c;
   res += top * n;
   ll h = (b + 1 + c - 1) / c;
   if(h <= top) res -= floor_sum(top - h + 1, c, c * h - (b +
1), a) + top - h + 1;
   return res:
```

lagrange-hokan.hpp

md5: 22b9e3

```
template<typename T> T lagrange_polynomial(const vector<T>& y,
ll t) {
   int n = si(y) - 1;
   if(t <= n) return y[t];</pre>
   T ret(0);
   vector<T> dp(n + 1, 1), pd(n + 1, 1);
   rep(i, n) dp[i + 1] = dp[i] * (t - i);
   per(i, n + 1, 1) pd[i - 1] = pd[i] * (t - i);
   rep(i, n + 1) {
      T tmp = y[i] * dp[i] * pd[i] * ifact[i] * ifact[n - i];
      ret -= ((n - i) & 1 ? tmp : -tmp);
   return ret;
```

matrix.hpp

md5: 270f1c

```
template<typename T> struct M {
  vector<vector<T>> a;
  int n, m;
  M(int n, int m) : n(n), m(m), a(n, vector<T>(m)) {}
  M(int n = 0) : M<T>(n, n) {}
  vector<T>& operator[](int k) { return a[k]; }
  const vector<T>& operator[](int k) const { return a[k]; }
  static M I(int n) {
     M mat(n);
     rep(i, n) mat[i][i] = 1;
     return mat;
  M& operator+=(const M& b) {
     rep(i, n) rep(j, m)(*this)[i][j] += b[i][j];
      return *this;
  M& operator-=(const M& b) {
     rep(i, n) rep(j, m)(*this)[i][j] -= b[i][j];
      return *this;
  M& operator*=(const M& b) {
     int l = b.m;
      vector c(n, vector<T>(l));
     rep(i, n) rep(j, m) rep(k, l) c[i][k] += (*this)[i][j] *
```

```
b[j][k];
      a.swap(c);
      return *this;
   M& operator^=(ll k) {
      M b = M::I(n);
      while(k) {
         if(k & 1) b *= *this;
         *this *= *this;
         k >>= 1;
      a.swap(b.a);
      return *this;
  M operator+(const M& b) const { return (M(*this) += b); }
  M operator-(const M& b) const { return (M(*this) -= b); }
   M operator*(const M& b) const { return (M(*this) *= b); }
   M operator^(const M& b) const { return (M(*this) ^= b); }
template<typename T> pair<int, T> GaussElimination(M<T>& a, bool
LE = false) {
   int n = a.n, m = a.m;
   int rank = 0, je = LE ? m - 1 : m;
   mint det = 1;
   rep(j, je) {
      int idx = -1;
      rep(i, rank, n) {
         if(a[i][j].x) {
            idx = i;
            break;
         }
      }
      if(idx == -1) {
         det = 0;
         continue;
      if(rank != idx) {
         det = -det;
         swap(a[rank], a[idx]);
      det *= a[rank][i];
      if(LE && a[rank][j].x != 1) {
         mint coeff = a[rank][j].inv();
         rep(k, j, m) a[rank][k] *= coeff;
      int is = LE ? 0 : rank + 1;
      rep(i, is, n) {
         if(i == rank) continue;
         if(a[i][j].x) {
            mint coeff = a[i][j] / a[rank][j];
            rep(k, j, m) a[i][k] -= a[rank][k] * coeff;
     }
      rank++;
   }
   return make_pair(rank, det);
template<typename T> vector<vector<T>> LinearEquation(M<T> a,
vector<T> b) {
   int n = a.n, m = a.m;
   rep(i, n) a[i].eb(b[i]);
   a.m++;
   auto p = GaussElimination(a, true);
   int rank = p.first;
   rep(i, rank, n) {
      if(a[i][m].x != 0) return {};
   vector<vector<T>> res(1, vector<T>(m));
   vi piv(m, -1);
   int j = 0;
   rep(i, rank) {
      while(a[i][j].x == 0) ++j;
      res[0][j] = a[i][m], piv[j] = i;
   rep(j, m) {
      if(piv[j] == -1) {
         vector<T> x(m);
         x[i] = 1;
         rep(k, j) {
            if(piv[k] != -1) x[k] = -a[piv[k]][j];
```

```
Speed Star (The University of Tokyo)
```

```
Page 12 of 20
```

md5: f5168d

```
res.eb(x);
   }
   return res;
prime.hpp
                                                       md5: 94a4a8
template<class T, class U> T pow_mod(T x, U n, T md) {
   T r = 1 \% md:
   x \% = md;
   while(n) {
      if(n & 1) r = (r * x) % md;
      x = (x * x) % md;
      n >>= 1;
   }
   return r;
}
bool is_prime(ll n) {
   if(n <= 1) return false;</pre>
   if(n == 2) return true;
   if(n % 2 == 0) return false;
   ll d = n - 1;
   while(d % 2 == 0) d /= 2;
   for(ll a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
      if(n <= a) break;</pre>
      ll y = pow_mod < i128 > (a, t, n); // over
      while(t != n - 1 && y != 1 && y != n - 1) {
         y = i128(y) * y % n; // flow
      }
      if(y != n - 1 && t % 2 == 0) { return false; }
   }
   return true;
ll pollard_single(ll n) {
   ll R;
   auto f = [\&](ll x) \{ return (i128(x) * x + R) % n; \};
   if(is_prime(n)) return n;
   if(n % 2 == 0) return 2;
   ll st = 0;
   while(true) {
      R = rnd(1, n);
      st++;
      ll x = st, y = f(x);
      while(true) {
         ll p = gcd((y - x + n), n);
         if(p == 0 \mid \mid p == n) break;
         if(p != 1) return p;
         x = f(x);
         y = f(f(y));
      }
  }
vl factor(ll n) {
   if(n == 1) return {};
   ll x = pollard_single(n);
   if(x == n) return {x};
   vl l = factor(x), r = factor(n / x);
   return l.insert(end(l), all(r)), l;
```

primitive-root.hpp

md5: 4088f5

```
ll primitive_root(ll p) {
    auto v = factor(p - 1);
    sort(all(v)), v.erase(unique(all(v)), end(v));
    while(true) {
        ll g = rnd(1, p);
        bool ok = true;
        for(auto d : v) {
            ll f = (p - 1) / d;
            if(pow_mod<i128>(g, f, p) == 1) {
                 ok = false;
                break;
        }
        if(ok) return g;
    }
}
```

xor-convolution.hpp

```
template<typename T> void fwt(vector<T>& f, bool inv = false) {
   int n = si(f), m = __lg(n);
   rep(i, m) {
      rep(b, n) {
        if(~b >> i & 1) {
            T x = f[b], y = f[b ^ 1 << i];
            f[b] = x + y, f[b ^ 1 << i] = x - y;
        }
    }
   if(inv) {
      T iz = T(1) / T(si(f));
      fore(e, f) e *= iz;
}</pre>
```

graph

bcc.hpp

md5: 3df588

```
template<typename G> struct BCC : LL<G> {
   vi used;
   vector<vector<pii>>> bc;
   vector<pii> tmp;
   using L = LL<G>;
   using L::g;
   using L::low;
   using L::ord;
   BCC(G g) : L(g) { build(); }
   void build() {
      used.assign(si(g), 0);
      rep(i, si(used)) if(!used[i]) dfs(i, -1);
   void dfs(int x, int p) {
      used[x] = true;
      fore(e, g[x]) {
         if(e == p) continue;
         if(!used[e] || ord[e] < ord[x]) tmp.eb(minmax(x, e));</pre>
         if(!used[e]) {
            dfs(e, x);
            if(low[e] >= ord[x]) {
               bc.eb();
               while(true) {
                  auto p = tmp.back();
                  bc.back().eb(p);
                  tmp.pop_back();
                  if(p.first == min(x, e) and p.second == max(x, e)
e)) break;
               }
            }
         }
      }
  }
};
```

eulerian-trail.hpp

md5: 5d421d

```
struct edge {
   int x, v, idx:
vector<edge> eulerian_path(vector<edge> es, int s, bool directed
= false) {
   if(es.empty()) return {};
   int n = 0;
   fore(e, es) chmax(n, max(e.x, e.y) + 1);
   vector<vector<pair<edge, int>>> g(n);
   for(auto& e : es) {
      int p = si(g[e.y]);
      g[e.x].emplace_back(e, p);
      if(!directed) {
         int q = si(g[e.x]) - 1;
         swap(e.x, e.y);
         g[e.x].emplace_back(e, q);
      }
   }
```

```
vector<edge> ord;
stack<pair<int, edge>> st;
st.emplace(s, edge{-1, -1, -1});
while(st.size()) {
   int x = st.top().first;
   if(empty(g[x])) {
      ord.eb(st.top().second);
      st.pop();
   } else {
      auto e = g[x].back();
      g[x].pop_back();
      if(e.second == -1) continue;
      if(!directed) g[e.first.y][e.second].second = -1;
      st.emplace(e.first.y, e.first);
  }
}
ord.pop_back();
reverse(begin(ord), end(ord));
if(si(ord) != si(es)) return {};
return ord;
```

lowlink.hpp

md5: e3987c

```
template<typename G> struct LL {
   int n;
   const G g;
   vi ord, low, arti;
   vector<pii> bridge;
   LL(G g) : n(si(g)), g(g), ord(si(g), -1), low(si(g), -1) {
      int k = 0;
      rep(i, n) {
         if(ord[i] == -1) k = dfs(i, k, -1);
   }
   int dfs(int x, int k, int p) {
      low[x] = (ord[x] = k++);
      int cnt = 0:
      bool is_arti = false, second = false;
      fore(e, g[x]) {
         if(ord[e] == -1) {
            cnt++:
            k = dfs(e, k, x);
            chmin(low[x], low[e]);
            is_arti |= (p != -1) && (low[e] >= ord[x]);
            if(ord[x] < low[e]) bridge.eb(minmax(x, e));</pre>
         } else if(e != p or second) {
            chmin(low[x], ord[e]);
         } else {
            second = true;
      }
      is_arti |= p == -1 && cnt > 1;
      if(is_arti) arti.eb(x);
      return k;
   }
};
```

max_matching.hpp

md5: 2ece25

```
struct Matching {
  int n;
  vector<vi> g;
  vi mt;
  vi is_ev, gr_buf;
  vector<pii> nx;
  int st;
  int group(int x) {
      if(gr_buf[x] == -1 || is_ev[gr_buf[x]] != st) return
qr_buf[x];
     return gr_buf[x] = group(gr_buf[x]);
  void match(int p, int b) {
      int d = mt[p];
      mt[p] = b;
      if(d == -1 || mt[d] != p) return;
      if(nx[p].second == -1) {
         mt[d] = nx[p].first;
         match(nx[p].first, d);
     } else {
```

```
match(nx[p].first, nx[p].second);
         match(nx[p].second, nx[p].first);
      }
   }
   bool arg() {
      is_ev[st] = st;
      gr_buf[st] = -1;
      nx[st] = pii(-1, -1);
      queue<int> q;
      q.push(st);
      while(q.size()) {
         int a = q.front();
         q.pop();
         for(auto b : g[a]) {
            if(b == st) continue;
            if(mt[b] == -1) {
                mt[b] = a;
                match(a, b);
                return true;
            }
            if(is_ev[b] == st) {
                int x = group(a), y = group(b);
                if(x == y) continue;
                int z = -1;
                while(x != -1 || y != -1) {
                   if(y != -1) swap(x, y);
                   if(nx[x] == pii(a, b)) {
                      z = x;
                      break:
                   }
                   nx[x] = pii(a, b);
                   x = group(nx[mt[x]].first);
                for(int v : {group(a), group(b)}) {
                   while(v != z) {
                      q.push(v);
                      is_ev[v] = st;
                      gr_buf[v] = z;
                      v = group(nx[mt[v]].first);
                   }
                }
            } else if(is_ev[mt[b]] != st) {
                is_ev[mt[b]] = st;
                nx[b] = pii(-1, -1);
                nx[mt[b]] = pii(a, -1);
                gr_buf[mt[b]] = b;
                q.push(mt[b]);
            }
         }
      }
      return false;
   \label{eq:matching} \mbox{Matching(const vector<vi>\& _g) : n(int(_g.size())), g(_g),}
mt(n, -1), is_{ev}(n, -1), gr_{buf}(n), nx(n) {
      for(st = 0; st < n; st++)</pre>
         if(mt[st] == -1) arg();
   vector<pii> max_match() {
      vector<pii> res:
      rep(i, n) if(i < mt[i]) res.eb(i, mt[i]);</pre>
      return res;
   }
};
```

maximum-independent-set.hpp

md5: ac1384

```
unsigned ll maximum_independent_set(vector<vi> q) {
   using U = unsigned long long;
   int n = si(g);
   vector<U> nbd(n);
   rep(i, n) fore(e, g[i]) nbd[i] = 1ULL << e;
   int best = 0;
   U res = 0;
   auto dfs = [\&](auto\&\& dfs, U now, U rest) -> void {
      pii p(-1, -1);
      while(true) {
         bool upd = 0;
         rep(v, n) {
            if(rest >> v & 1) {
               int d = popcount(nbd[v] & rest);
               if(chmax(p.second, d)) p.first = v;
               if(d <= 1) rest ^= 1ULL << v, rest &= ^-nbd[v],
```

```
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```

```
Page 14 of 20
```

scc.hpp

md5: 73554b

```
template<typename G> struct SCC {
  Gq;
  vector<vi> rg;
  vi comp, ord, used;
  int num; // 連結成分の数
  SCC(G g) : g(g), rg(si(g)), comp(si(g), -1), ord(si(g)),
used(si(g)) {
     rep(i, si(g)) fore(e, g[i]) rg[e].eb(i);
      build();
  };
  int operator[](int k) { return comp[k]; }
  void dfs(int x) {
      if(used[x]) return;
      used[x] = true;
      fore(e, g[x]) if(!used[e]) dfs(e);
      ord.eb(x);
  }
  void rdfs(int x, int cnt) {
      if(comp[x] != -1) return;
      comp[x] = cnt;
      fore(e, rg[x]) if(comp[e] == -1) rdfs(e, cnt);
  void build() {
      rep(i, g.size()) dfs(i);
      reverse(all(ord));
     num = 0;
      fore(i, ord) if(comp[i] == -1) { rdfs(i, num), num++; }
  }
```

tecc.hpp

md5: 17c69f

```
template<typename G> struct TCC : LL<G> {
  using L = LL<G>;
  using L::bridge;
  using L::g;
  using L::low;
  using L::ord;
  vi cmp;
  vector<vi> tree, group;
  void build() {
     cmp.assign(si(g), -1);
      int k = 0;
     rep(i, si(cmp)) if(cmp[i] == -1) dfs(i, -1, k);
     group.resize(k);
     rep(i, si(g)) group[cmp[i]].eb(i);
     tree.resize(k);
     for(auto [a, b] : bridge) {
        tree[cmp[a]].eb(cmp[b]);
        tree[cmp[b]].eb(cmp[a]);
     }
  TCC(const G& g) : L(g) { build(); }
  void dfs(int x, int p, int& k) {
     if(p >= 0 and ord[p] >= low[x]) cmp[x] = cmp[p];
     else cmp[x] = k++;
     fore(e, g[x]) if(cmp[e] == -1) dfs(e, x, k);
```

```
modint
```

BarrettReduction.hpp

```
md5: 651912
```

```
using U = uint64_t;
struct Barret {
   U m, im;
   Barret(U mod): m(mod), im(-1ULL / m + 1) {}
   U mul(U a, U b) const {
      a *= b;
      U x = ((\_uint128\_t)a * im) >> 64;
      a -= x * m;
      if((ll)a < 0) a += m;</pre>
      return a;
};
constexpr ll mod = 998244353;
static Barret b(mod);
struct mint {
   int x;
   mint(ll x_{=} = 0) : x((x_{=} \% mod) + mod) {
      if(x >= mod) x -= mod;
  mint& s(uint xx) { return x = xx < mod ? xx : xx - mod,}
*this; }
   mint operator-() { return mint(-x); }
   mint& operator+=(const mint& r) { return s(x + r.x); }
   mint& operator-=(const mint& r) { return s(x + mod - r.x); }
   mint\& operator*=(const mint\& r) { return x = b.mul(x, r.x),}
*this; }
   mint& operator/=(const mint& r) { return *this *= r.inv(); }
   friend mint operator+(mint l, mint r) { return l += r; }
   friend mint operator-(mint l, mint r) { return l -= r; }
   friend mint operator*(mint l, mint r) { return l *= r; }
   friend mint operator/(mint l, mint r) { return l /= r; }
   mint inv() const { return pow(mod - 2); }
   mint pow(ll b) const {
      mint a = *this, c = 1;
      while(b) {
         if(b & 1) c *= a;
         a *= a;
         b >>= 1;
      }
      return c;
  }
}:
using vm = vector<mint>;
```

modint.hpp

md5: 492829

```
constexpr ll mod = 998244353;
struct mint {
   ll x;
   mint(ll x_ = 0) : x(x_ \% mod) {
     if(x < 0) x += mod;
  mint operator-() { return mint(-x); }
   friend mint operator+(mint a, mint b) { return a.x + b.x; }
   friend mint operator-(mint a, mint b) { return a.x - b.x; }
   friend mint operator*(mint a, mint b) { return a.x * b.x; }
   friend mint operator/(mint a, mint b) { return a * b.inv(); }
   // 4 行コピペ Alt + Shift + クリックで複数カーソル
  friend mint& operator+=(mint& a, mint b) { return a = a.x +
b.x; }
  friend mint& operator-=(mint& a, mint b) { return a = a.x -
b.x; }
  friend mint& operator*=(mint& a, mint b) { return a = a.x *
b.x; }
   friend mint& operator/=(mint& a, mint b) { return a = a *
b.inv(); }
  mint inv() const { return pow(mod - 2); }
   mint pow(ll b) const {
     mint a = *this, c = 1;
     while(b) {
         if(b & 1) c *= a;
         a *= a;
```

```
b >>= 1;
}
return c;
}
};
using vm = vector<mint>;
```

FPS

```
FFT.hpp
```

md5: f769b5

```
mint g = 3;
             // 原始根
void fft(vm& a, bool inv = false) {
   int n = si(a), s = __lg(n);
   static vm z, iz;
   while(si(z) <= s) {</pre>
      z.eb(g.pow(mint(-1).x / (1 << si(z))));
      iz.eb(z.back().inv());
   vm b(n);
   rep(i, 1, s + 1) {
      int w = 1 << s - i;
      mint base = inv ? iz[i] : z[i], now = 1;
      for(int y = 0; y < n / 2; y += w) {
         rep(x, w) {
            auto l = a[y << 1 | x], r = now * a[y << 1 | x | w];
            b[y \mid x] = l + r, b[y \mid x \mid n >> 1] = l - r;
         now *= base;
      }
      swap(a, b);
}
vm mul(vm a, vm b) {
   int n = si(a), m = si(b);
   if(!n or !m) return {};
   if(min(n, m) \le 30) {
      vm ans(n + m - 1);
      rep(i, n) rep(j, m) ans[i + j] += a[i] * b[j];
      return ans;
   int N = n + m - 1;
   int z = bit_ceil(unsigned(N));
   a.resize(z), b.resize(z);
   fft(a), fft(b);
   rep(i, z) a[i] *= b[i];
   fft(a, true);
   a.resize(n + m - 1);
   mint iz = mint(z).inv();
   fore(e, a) e *= iz;
   return a;
```

linear-recurrence.hpp

md5: 7ef16a

```
// [x ^ k] p / q
mint LinearRecurrence(ll k, fps q, fps p) {
   q.shrink();
   mint ret = 0;
   if(si(p) >= si(q)) {
      auto r = p / q;
      p -= r * q;
      p.shrink();
      if(k < r.size()) ret += r[k];</pre>
   if(p.size() == 0) return ret;
   p.resize(q.size() - 1);
   while(k) {
      auto q2 = q;
      for(int i = 1; i < q2.size(); i += 2) q2[i] = -q2[i];</pre>
      auto s = p * q2, t = q * q2;
      for(int i = (k \& 1); i < s.size(); i += 2) p[i >> 1] =
s[i];
      for(int i = 0; i < t.size(); i += 2) q[i >> 1] = t[i];
      k >>= 1;
   }
   return ret + p[0]:
// a * q = 0
mint kitamasa(ll n, fps q, fps a) {
```

```
if(n < si(a)) return a[n];
auto p = a.pre(si(q) - 1) * q;
p.resize(si(q) - 1);
return LinearRecurrence(n, q, p);</pre>
```

md5: 8da6ee

poly.hpp

```
struct fps {
   vm v:
   fps(const vm& v = \{\}) : v(v) \{\}
   fps(int n) : v(n) {}
   void shrink() {
     while(v.size() && !v.back().x) v.pop_back();
   }
   void resize(int n) { v.resize(n); }
   int size() const { return int(v.size()); }
   mint freq(int p) const { return (p < size()) ? v[p] : 0; }</pre>
   mint& operator[](int k) { return v[k]; }
   void emplace_back(mint x) { v.eb(x); }
   fps pre(int le) const { return {{v.begin(), v.begin() +
min(size(), le)}}; }
   fps operator-() const {
     vm res{v};
      fore(e, res) e = -e;
      return res;
   fps operator+(const fps& r) const {
      auto n = max(size(), r.size());
      vm res(n);
      rep(i, n) res[i] = freq(i) + r.freq(i);
      return res;
  fps operator-(const fps& r) const { return (*this) + (-r); }
   fps operator*(const fps& r) const { return {mul(v, r.v)}; }
   fps operator*(const mint& r) const {
      int n = size();
      vm res(n);
      for(int i = 0; i < n; i++) res[i] = v[i] * r;</pre>
      return res;
   fps operator/(const mint& r) const { return *this * r.inv();
   fps operator/(const fps& r) const {
      if(size() < r.size()) return {{}};</pre>
      int n = size() - r.size() + 1;
      return (rev().pre(n) * r.rev().inv(n)).pre(n).rev();
  fps operator%(const fps& r) const { return *this - *this / r
* r; }
  fps operator<<(int s) const {</pre>
      vm res(size() + s);
      rep(i, size()) res[i + s] = v[i];
      return res;
   fps operator>>(int s) const {
      if(size() <= s) return fps();</pre>
      vm res(size() - s);
      rep(i, size() - s) res[i] = v[i + s];
      return res;
   fps& operator+=(const fps& r) { return *this = *this + r; }
   fps& operator-=(const fps& r) { return *this = *this - r; }
   fps& operator*=(const fps& r) { return *this = *this * r; }
   fps& operator*=(const mint& r) { return *this = *this * r; }
   fps& operator/=(const fps& r) { return *this = *this / r; }
   fps& operator/=(const mint& r) { return *this = *this / r; }
   fps& operator%=(const fps& r) { return *this = *this % r; }
   fps& operator<<=(int n) { return *this = *this << n; }</pre>
   fps& operator>>=(int n) { return *this = *this >> n; }
   fps rev(int n = -1) const {
      vm res = v;
      if(n != -1) res.resize(n);
      reverse(res.begin(), res.end());
      return res:
   fps diff() const {
      vm res(max(0, size() - 1));
      rep(i, 1, size()) res[i - 1] = freq(i) * i;
      return res;
   fps integ() const {
```

```
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vm res(size() + 1);
rep(i, size()) res[i + 1] = freq(i) / (i + 1);
return res;

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sub[x] += sub[e];
if(sub[g[x][0]] < sub[e]) swap(g[x][0], e);
}</pre>
```

```
}
   // f * f.inv() = 1 + g(x)x^m
   fps inv(int m) const {
      fps res = fps(vm{mint(1) / freq(0)});
      for(int i = 1; i < m; i *= 2) { res = (res * mint(2) - res
* res * pre(2 * i)).pre(2 * i); }
      return res.pre(m);
   fps exp(int n) const {
      assert(freq(0).x == 0);
      fps g = fps(vm{1});
      for(int i = 1; i < n; i *= 2) { g = (g * (pre(i * 2) +
fps(vm{1}) - g.log(i * 2))).pre(i * 2); }
      return g.pre(n);
   fps log(int n) const {
      assert(freq(0).x == 1);
      auto f = pre(n);
      return (f.diff() * f.inv(n - 1)).pre(n - 1).integ();
   fps sqrt(int n) const {
      assert(freq(0).x == 1);
      fps f = pre(n + 1);
      fps g({1});
      for(int i = 1; i < n; i *= 2) { g = (g + f.pre(2 * i) *
g.inv(2 * i)) * mint((mod + 1) / 2); }
      return g.pre(n + 1);
   fps pow(ll k, ll n) {
      if(k == 0) {
         fps res(n);
         res[0] = 1;
         return res;
      rep(i, size()) {
         if((*this)[i].x) {
            mint rev = mint(1) / (*this)[i];
            fps ret = (((*this * rev) >> i).log(n) *
mint(k)).exp(n);
            ret *= (*this)[i].pow(k);
            ret = (ret << (i * k)).pre(n);
            if(ret.size() < n) ret.resize(n);</pre>
            return ret;
         if(i128(i + 1) * k >= n) return fps(n);
      }
      return fps(n);
   fps pow_mod(ll n, const fps& mod) {
      fps x = *this, r = {\{1\}};
      while(n) {
        if(n \& 1) r = r * x % mod;
         x = x * x % mod;
         n >>= 1;
      return r;
  }
};
```

tree

hld.hpp

md5: fa40a1

```
template<typename G> struct HLD {
   int n;
   6& g;
   vector<int> sub, in, out, head, rev, par, d;
   HLD(G& g) : n(si(g)), g(g), sub(n), in(n), out(n), head(n),
   rev(n), par(n), d(n) {}
   void dfs1(int x, int p) {
      par[x] = p;
      sub[x] = 1;
      if(g[x].size() and g[x][0] == p) swap(g[x][0],
   g[x].back());
   fore(e, g[x]) {
      if(e == p) continue;
      d[e] = d[x] + 1;
      dfs1(e, x);
```

```
}
   void dfs2(int x, int p, int& t) {
      in[x] = t++;
      rev[in[x]] = x;
      fore(e, g[x]) {
         if(e == p) continue;
         head[e] = (g[x][0] == e ? head[x] : e);
         dfs2(e, x, t);
      }
      out[x] = t;
  }
   void build() {
      int t = 0;
      head[0] = 0;
      dfs1(0, -1);
      dfs2(0, -1, t);
   }
   int la(int v, int k) {
      while(1) {
         int u = head[v];
         if(in[v] - k >= in[u]) return rev[in[v] - k];
         k = in[v] - in[u] + 1;
         v = par[u];
   }
   int lca(int u, int v) {
      for(;; v = par[head[v]]) {
         if(in[u] > in[v]) swap(u, v);
         if(head[v] == head[v]) return u;
      }
   template<typename T, typename Q, typename F>
   T query(int u, int v, const T& e, const Q& q, const F& f,
bool edge = false) {
      Tl = e, r = e;
      for(;; v = par[head[v]]) {
         if(in[u] > in[v]) swap(u, v), swap(l, r);
         if(head[u] == head[v]) break;
         l = f(q(in[head[v]], in[v] + 1), l);
      return f(f(q(in[u] + edge, in[v] + 1), l), r);
   }
   int dist(int u, int v) { return d[u] + d[v] - 2 * d[lca(u, v)]
v)]; }
   int jump(int s, int t, int i) {
      if(!i) return s;
      int l = lca(s, t);
      int dst = d[s] + d[t] - d[l] * 2;
      if(dst < i) return -1;</pre>
      if(d[s] - d[l] >= i) return la(s, i);
      i -= d[s] - d[l];
      return la(t, d[t] - d[l] - i);
  }
};
```

flow

bipartite-matching.hpp

md5: 2ffb05

```
struct Bimatch {
   vector<vi> q;
   vi d, mc, used, vv;
   Bimatch(int n, int m) : g(n), mc(m, -1), used(n) {}
   void add(int u, int v) { g[u].eb(v); }
   void bfs() H
      d.assign(si(g), -1);
      queue<int> q;
      rep(i, si(g)) {
         if(!used[i]) {
            q.emplace(i);
            d[i] = 0;
         }
      }
      while(!q.empty()) {
         int x = q.front();
         q.pop();
         fore(e, g[x]) {
            int c = mc[e];
```

```
Speed Star (The University of Tokyo)
            if(c >= 0 \text{ and } d[c] == -1) {
               d[c] = d[x] + 1;
                q.emplace(c);
      }
   bool dfs(int x) {
      vv[x] = true;
      fore(e, g[x]) {
         int c = mc[e];
         if(c < 0 or (!vv[c] and d[c] == d[x] + 1 and dfs(c))) {
            mc[e] = x;
            used[x] = true;
            return true;
      }
      return false;
   }
   int match() {
      int ret = 0;
      while(true) {
         bfs();
         vv.assign(si(g), false);
         int f = 0;
         rep(i, si(g)) if(!used[i] and dfs(i)) f++;
         if(!f) return ret;
         ret += f;
      }
   }
};
flow.hpp
                                                       md5: e7b79f
```

```
// struct E {
//
       int to, rev, cap;
// };
// vector<vector<E>> g(n);
// auto add_edge = [\&](int u, int v, int c) {
       g[v].push_back(E\{v, si(g[v]), c\});
//
//
       g[v].push_back(E\{u, si(g[u]) - 1, 0\});
// }:
template<class C> struct MaxFlow {
   C flow;
   vector<char> dual;
   // false: S-side true: T-side
template<class C, class E> struct MFExec {
   vector<vector<E>>& g;
   int s, t;
   vi level, iter;
   C dfs(int v, C f) {
      if(v == t) return f;
      C res = 0;
      for(int& i = iter[v]; i < si(g[v]); i++) {</pre>
         E\& e = g[v][i];
         if(!e.cap || level[v] >= level[e.to]) continue;
         C d = dfs(e.to, min(f, e.cap));
         e.cap -= d;
         g[e.to][e.rev].cap += d;
         res += d, f -= d;
         if(!f) break;
      }
      return res;
   MaxFlow<C> info;
   MFExec(vector<vector<E>>& g, int s, int t) : g(g), s(s), t(t)
      int n = si(g);
      C& flow = (info.flow = 0);
      while(true) {
         queue<int> q;
         level = vi(n, -1);
         level[s] = 0;
         a.push(s):
         while(!q.empty()) {
            int v = q.front();
            q.pop();
            fore(e, g[v]) {
               if(!e.cap or level[e.to] >= 0) continue;
```

```
level[e.to] = level[v] + 1;
               q.emplace(e.to);
         }
         if(level[t] == -1) break;
         iter = vi(n, 0);
         while(true) {
            C f = dfs(s, INF);
            if(!f) break;
            flow += f;
     }
      rep(i, n) info.dual.eb(level[i] == -1);
  }
}:
template<class C, class E> MaxFlow<C> get_mf(vector<vector<E>>&
g, int s, int t) { return MFExec<C, E>(g, s, t).info; }
```

mcf.hpp

md5: 96eeaa

```
struct MCF {
   struct edge {
      int to;
      ll cap, cost;
      int rev;
      bool isrev;
  }:
   vector<vector<edge>> g;
   vl pot, cost;
   vi pv, pe;
   MCF(int n) : g(n) {}
   void add(int u, int v, ll cap, ll cost) {
      g[u].eb(v, cap, cost, si(g[v]), false);
      g[v].eb(u, 0, -cost, si(g[u]) - 1, true);
  ll mcf(int s, int t, ll f) {
      int n = si(g);
      ll ret = 0;
      using P = pair<ll, int>;
      priority_queue<P, vector<P>, greater<P>> pq;
      pot.assign(n, 0), pe.assign(n, -1), pv.assign(n, -1);
      while(f) {
         cost.assign(n, INFL);
         pq.emplace(0, s);
         cost[s] = 0;
         while(!pq.empty()) {
            auto [c, x] = pq.top();
            pq.pop();
            if(cost[x] < c) continue;</pre>
            rep(i, si(g[x])) {
               edge\& e = g[x][i];
               ll ncost = cost[x] + e.cost + pot[x] - pot[e.to];
               if(e.cap and chmin(cost[e.to], ncost)) {
                  pv[e.to] = x, pe[e.to] = i;
                  pq.emplace(cost[e.to], e.to);
            }
         if(cost[t] == INFL) return -1;
         rep(i, n) pot[i] += cost[i];
         ll addflow = f;
         for(int v = t; v != s; v = pv[v]) chmin(addflow,
g[pv[v]][pe[v]].cap);
         f -= addflow;
         ret += addflow * pot[t];
         for(int v = t; v != s; v = pv[v]) {
            auto& e = g[pv[v]][pe[v]];
            e.cap -= addflow;
            g[v][e.rev].cap += addflow;
     }-
     return ret;
  }
};
```

md5: 1d70ce

二部グラフ.md

|| サイズ | 構成||最大マッチング||M|||最小点被覆||M||L到達不可能 + R到達可能||最大安定集合||V| - |M||上の補グラフ||最小辺被覆|孤立点がないなら|V| - |M||最大マッチング + 含まれない点 greedy|

燃やす埋める.md

変形前の制約	変形後の制約
xが 0 のとき z 失う	(x,T,z)
x が 0 のとき z 得る	無条件で z 得る; (S,x,z)
xが 1 のとき z 失う	(S,x,z)
x が 1 のとき z 得る	無条件で z 得る; (x,T,z)
x,y,\dots がすべて 0 のとき z 得る	無条件で z 得る; $(S,w,z),(w,x,\infty),(w,y,\infty)$
x,y,\dots がすべて 1 のとき z 得る	無条件で z 得る; $(w,T,z),(x,w,\infty),(y,w,\infty)$

string

KMP.hpp

md5: 886c63

```
// kmp[i] := max{ l ≤ i | s[:l] == s[(i+1)-l:i+1] }
// abacaba -> 0010123
auto KMP(string s) {
    vector<ll> p(sz(s));
    rep(i, 1, sz(s)) {
        ll g = p[i - 1];
        while(g && s[i] != s[g]) g = p[g - 1];
        p[i] = g + (s[i] == s[g]);
    }
    return p;
}
```

Manacher.hpp

md5: 5882fb

```
// 各位置での回文半径を求める
// aaabaaa -> 1214121
// 偶数長の回文を含めて直径を知るには, N+1 個の $ を挿入して 1 を引く
// $a$a$a$b$a$a$a$ -> 123432181234321
auto manacher(string s) {
  ll n = sz(s), i = 0, j = 0;
  vector<ll> r(n);
  while(i < n) {
     while(i >= j && i + j < n && s[i - j] == s[i + j]) j++;
     r[i] = j;
     11 k = 1:
     while(i >= k && i + k < n && k + r[i - k] < j) {
        r[i + k] = r[i - k];
        k++;
     }
     i += k, j -= k;
  }
  return r:
```

RollingHash.hpp

md5: adb8d3

```
// using u64 = uint64_t;
const u64 mod = INF;
u64 add(u64 a, u64 b) {
    a += b;
    if(a >= mod) a -= mod;
    return a;
}
u64 mul(u64 a, u64 b) {
    auto c = (__uint128_t)a * b;
    return add(c >> 61, c & mod);
}
random_device rnd;
const u64 r = ((u64)rnd() << 32 | rnd()) % mod;</pre>
```

```
struct RH {
    ll n;
    vector<u64> hs, pw;
    RH(string s) : n(sz(s)), hs(n + 1), pw(n + 1, 1) {
        rep(i, 0, n) {
            pw[i + 1] = mul(pw[i], r);
            hs[i + 1] = add(mul(hs[i], r), s[i]);
        }
    }
    u64 get(ll l, ll r) const { return add(hs[r], mod -mul(hs[l], pw[r - l])); }
};
```

SuffixArray.hpp

```
// returns pair{sa, lcp}
// sa 長さ n : s[sa[0]:] < s[sa[1]:] < … < s[sa[n-1]:]
// lcp 長さ n-1 : lcp[i] = LCP(s[sa[i]:], s[sa[i+1]:])
auto SA(string s) {
   ll n = sz(s) + 1, lim = 256;
   // assert(lim > ranges::max(s));
   vector<ll> sa(n), lcp(n), x(all(s) + 1), y(n), ws(max(n, s))
lim)), rk(n);
   iota(all(sa), 0);
   for(ll j = 0, p = 0; p < n; j = max(1LL, j * 2), lim = p) {
      p = j;
      iota(all(y), n - j);
      rep(i, 0, n) if(sa[i] >= j) y[p++] = sa[i] - j;
      fill(all(ws), 0);
      rep(i, 0, n) ws[x[i]]++;
      rep(i, 1, lim) ws[i] += ws[i - 1];
      for(ll i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
      swap(x, y);
      p = 1;
      x[sa[0]] = 0;
      rep(i, 1, n) {
         ll a = sa[i - 1], b = sa[i];
         x[b] = (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 :
p++;
      }
   }
   rep(i, 1, n) rk[sa[i]] = i;
   for(ll i = 0, k = 0; i < n - 1; lcp[rk[i++]] = k) {
      if(k) k--:
      while(s[i + k] == s[sa[rk[i] - 1] + k]) k++;
   sa.erase(begin(sa));
   lcp.erase(begin(lcp));
   return pair{sa, lcp};
```

Zalgorithm.hpp

md5: b20b04

```
// Z[i] := LCP(s, s[i:])
// abacaba -> 7010301
auto Z(string s) {
    ll n = sz(s), l = -1, r = -1;
    vector<ll> z(n, n);
    rep(i, 1, n) {
        ll& x = z[i] = i < r ? min(r - i, z[i - l]) : 0;
        while(i + x < n && s[i + x] == s[x]) x++;
        if(i + x > r) l = i, r = i + x;
    }
    return z;
}
```

geometry

argument-sort.hpp

md5: 26b1fa

```
bool operator<(point P, point Q) {
   long long C = cross(P, Q);
   if(C == 0 && dot(P, Q) > 0) {
      return false;
   } else if(P.x < 0 && P.y == 0) {
      return true;
   } else if(Q.x < 0 && Q.y == 0) {
      return false;
   } else if(P.y * Q.y <= 0) {
      return P.y < Q.y;
   }
}</pre>
```

```
Speed Star (The University of Tokyo)
} else {
   return C > 0;
```

```
Page 19 of 20
```

```
circle.hpp
                                                      md5: 514ea6
struct circle {
   point C;
   double r:
   circle() {}
   circle(point C, double r) : C(C), r(r) {}
pair<point, point> line_circle_intersection(line L, circle C) {
   point P = projection(C.C, L);
   double d = point_line_distance(C.C, L);
   double h = sqrt(C.r * C.r - d * d);
   point A = P + vec(L) / abs(vec(L)) * h;
   point B = P - vec(L) / abs(vec(L)) * h;
   return make_pair(A, B);
pair<point, point> circle_intersection(circle C1, circle C2) {
   double d = dist(C1.C, C2.C);
   double m = (C1.r * C1.r - C2.r * C2.r + d * d) / (d * 2);
   point M = C1.C + (C2.C - C1.C) / d * m;
   double h = sqrt(C1.r * C1.r - m * m);
   point H = rotate90(C2.C - C1.C) / d * h;
   return make_pair(M - H, M + H);
pair<point, point> circle_tangent(point P, circle C) {
   double d = dist(P, C.C);
   double r = sqrt(d * d - C.r * C.r);
   return circle_intersection(C, circle(P, r));
vector<line> common_tangent(circle C1, circle C2) {
   if(C1.r < C2.r) { swap(C1, C2); }</pre>
   double d = dist(C1.C, C2.C);
   vector<line> L;
   if(C1.r - C2.r <= d + eps) {
      if(C1.r - C2.r <= eps) {</pre>
         point D = rotate90(C2.C - C1.C) / d \star C1.r;
         L.push_back(line(C1.C + D, C2.C + D));
         L.push_back(line(C1.C - D, C2.C - D));
      } else {
         double m = (C1.r - C2.r) * (C1.r - C2.r) / d;
         point M = C1.C + (C2.C - C1.C) / d * m;
         double h = sqrt((C1.r - C2.r) * (C1.r - C2.r) - m * m);
         point H1 = M + rotate90(C2.C - C1.C) / d * h;
         point D1 = (H1 - C1.C) / dist(H1, C1.C) * C2.r;
         L.push_back(line(H1 + D1, C2.C + D1));
         point H2 = M - rotate90(C2.C - C1.C) / d * h;
         point D2 = (H2 - C1.C) / dist(H2, C1.C) * C2.r;
         L.push_back(line(H2 + D2, C2.C + D2));
     }
```

line.hpp

}

return L:

 $if(C1.r + C2.r \le d + eps)$ {

double m = (C1.r + C2.r) * (C1.r + C2.r) / d;point M = C1.C + (C2.C - C1.C) / d * m;

point H1 = M + rotate90(C2.C - C1.C) / d * h;
point D1 = (H1 - C1.C) / dist(H1, C1.C) * C2.r;

L.push_back(line(H1 - D1, C2.C - D1));
point H2 = M - rotate90(C2.C - C1.C) / d * h;
point D2 = (H2 - C1.C) / dist(H2, C1.C) * C2.r;

 $L.push_back(line(H2 - D2, C2.C - D2));$

double h = sqrt((C1.r + C2.r) * (C1.r + C2.r) - m * m);

md5: 447fab

```
bool point_on_segment(point P, line L) { return dot(P - L.A,
vec(L)) > -eps && dot(P - L.B, vec(L)) < eps; }
point projection(point P, line L) { return L.A + vec(L) /
abs(vec(L)) * dot(P - L.A, vec(L)) / abs(vec(L)); }
point reflection(point P, line L) { return projection(P, L) * 2
- P; }
double point_line_distance(point P, line L) { return abs(cross(P
- L.A, vec(L))) / abs(vec(L)); }
double point_segment_distance(point P, line L) {
   if(dot(P - L.A, vec(L)) < 0) {
      return dist(P, L.A);
}</pre>
```

```
} else if(dot(P - L.B, vec(L)) > 0) {
      return dist(P, L.B);
      return point_line_distance(P, L);
}
bool is_parallel(line L1, line L2) { return abs(cross(vec(L1),
vec(L2))) < eps; }</pre>
point line_intersection(line L1, line L2) {
   return L1.A + vec(L1) * cross(L2.A - L1.A, vec(L2)) /
cross(vec(L1), vec(L2));
bool segment_intersect(line L1, line L2) {
  return cross(L1.A - L2.A, vec(L2)) * cross(L1.B - L2.A,
vec(L2)) < eps
          && cross(L2.A - L1.A, vec(L1)) * cross(L2.B - L1.A,
vec(L1)) < eps;
double segment_distance(line L1, line L2) {
   if(segment_intersect(L1, L2)) {
     return 0;
   } else {
     double ans = INF;
      ans = min(ans, point_segment_distance(L1.A, L2));
      ans = min(ans, point_segment_distance(L1.B, L2));
      ans = min(ans, point_segment_distance(L2.A, L1));
      ans = min(ans, point_segment_distance(L2.B, L1));
      return ans:
  }
```

memo

Primes.md

素数の個数

n	1	10^2	10^3	10^4	10^5	10^{6}	10^{7}	10^{8}	10^{9}	10^{10}
$\pi(r)$	2) 2	25	168	1229	9592	78498	664579	5.76e+6	5.08e+7	4.55e+8

高度合成数

$\leq n$	10^3	10^4	10^5	10^{6}	107	7		10^{8}	10^{9}		
\boldsymbol{x}	840	7560	83160	720720	86486	640	735	13440	735134	400	
$d^0(x)$	32	64	128	240	448		768	3	1344		
$\leq n$	10^{10}	10^{11}	10^{12}	10^{13}	10^{14}	10	$)^{15}$	10^{16}	10^{17}	10^{18}	8
$d^0(x)$	2304	4032	6720	10752	17280	268	380	41472	64512	1036	80

素数階乗

n	2	3	5	7	11	13	17	19	23	29
n#	2	6	30	210	2310	30030	510510	9.70e+6	2.23e+8	6.47e+9

階乗

4!	5!	6!	7!	8!	9!	10!	11!	12!	13!
24	120	720	5040	40320	362880	3.63e+6	3.99e+7	4.79e+8	6.23e+9

math.md

二項係数

n\k	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			

n\k	0	1	2	3	4	5	6	7	8	9	10
8	1	8	8	28	56	70	56	28	8	1	
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1a716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003
16	1	16	120	560	1820	4368	8008	11440	12870	11440	8008
17	1	17	136	680	2380	6188	12376	19448	24310	24310	19448
18	1	18	153	816	3060	8568	18564	31824	43758	48620	43758
19	1	19	171	969	3876	11628	27132	50388	75582	92378	92378
20	1	20	190	1140	4845	15504	38760	77520	125970	167960	184756

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \binom{n}{k} = \frac{n}{k} \binom{n-1}{k-1} \binom{L}{k} + \dots + \binom{R-1}{k} = \binom{R}{k+1} - \binom{L}{k+1}$$

第一種スターリング数

c(n,k): $1,2,\ldots,n$ の順列で巡回置換 k 個に分割できるものの個数

$n \setminus k$	0	1	2	3	4	5	6	7
0	1							
1	0	1						
2	0	1	1					
3	0	2	3	1				
4	0	6	11	6	1			
5	0	24	50	35	10	1		
6	0	120	274	225	85	15	1	
7	0	720	1764	1624	735	175	21	1

$$\begin{array}{l} c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k) \\ x(x+1) \dots (x+n-1) = \sum_{k=0}^n c(n,k) x^k \sum_{k=0}^n c(n,k) = n! \\ \sum_{k=0}^n 2^k c(n,k) = (n+1)! \sum_{k=0}^n (-1)^k c(n,k) = 0 \end{array}$$

 $\sum_{k=0}^n c(n,k) x^k = x(x+1)\dots(x+n-1)$ を用いて分割統治し,片方の計算を polynomial taylor shift で再利用すると,c(N,k) の k に関する列挙が $O(N\log N)$ 時間でできる.

第二種スターリング数

S(n,k): $1,2,\ldots,n$ を k 個の区別しない集合に分割する方法の数

$n \setminus k$	0	1	2	3	4	5	6	7
0	1							
1	0	1						
2	0	1	1					
3	0	1	3	1				
4	0	1	7	6	1			
5	0	1	15	25	10	1		
6	0	1	31	90	65	15	1	
7	0	1	63	301	350	140	21	1
	-		-	-		-		-

$$S(n,k) = S(n-1,k-1) + kS(n-1,k) \ x^n = \sum_{k=0}^n S(n,k) x(x-1) \dots (x-k+1) \ S(n,k) = rac{1}{k!} \sum_{m=1}^k (-1)^{k-m} inom{k}{m} m^n$$

最後の式と畳み込みを使うと S(N,k) の k に関する列挙が $O(N\log N)$ 時間でできる.

n	0	1	2	3	4	5	6	7	8	9	10
B_n	1	1	2	5	15	52	203	877	4140	21147	115975

$$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k B_n = \frac{1}{e} \sum_{k=0}^{\infty} \frac{k^n}{k!}$$

指数型母関数 $\exp(\exp x-1)=\sum_{n=0}^\infty B_n rac{x^n}{n!}$ を使うと B_0,B_1,\dots,B_n の計算が $O(N\log N)$ でできる.

カタラン数

 C_n : n 個の (と)を括弧列になるように並べる方法の数

	n	0	1	2	3	4	5	6	7	8	9	10
ľ	\overline{C}_n	1	1	2	5	14	42	132	429	1430	4862	16796

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!} \; C_{n+1} = \frac{2(2n+1)}{n+2} C_n \; C_{n+1} = \sum_{k=0}^n C_k C_{n-k}$$

モンモール数

 a_n : $1,2,\ldots,n$ の順列 P で $P_i
eq i$ となるものの個数

n	0	1	2	3	4	5	6	7	8	9	10
a_n		0	1	2	9	44	265	1854	14833	133496	1334961

$$a_n = (n-1)(a_{n-1} + a_{n-2}) a_n = na_{n-1} + (-1)^n$$

分割数

 P_n : n を正の整数の和として表す方法の数

n	0	1	2	3	4	5	6	7	8	9	10
P_n	1	1	2	3	5	7	11	15	22	30	42

母関数は $\prod_{n=1}^\infty rac{1}{1-x^n} = \sum_{n=0}^\infty P_n x^n$ である.五角数定理より $\prod_{n=1}^\infty (1-x^n) = \sum_{n=-\infty}^\infty (-1)^n x^{n(3n+1)/2}$ なので, P_0,P_1,\ldots,P_N を $O(N\log N)$ 時間で列挙できる.

母関数

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots = \sum_{n=0}^{\infty} x^n$$

$$rac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots = \sum_{n=0}^{\infty} (n+1)x^n$$

$$\frac{1}{(1-x)^3} = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + \dots = \sum_{n=0}^{\infty} \frac{1}{2}(n+1)(n+2)x^n$$

$$\frac{1}{(1-x)^d} = \sum_{n=0}^{\infty} {n+d-1 \choose n} x^n$$

$$\sqrt{1-x} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \frac{5}{128}x^4 - \dots = 1 - \sum_{n=1}^{\infty} \frac{(2n-2)!}{2^{2n-1}n!(n-1)!}x^n$$

$$\frac{1}{\sqrt{1-x}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4 - \dots = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n n!} x^n$$

$$rac{1-\sqrt{1-4x}}{2x}=1+x+2x^2+5x^3+14x^4+\cdots=\sum_{n=0}^{\infty}C_nx^n=\sum_{n=0}^{\infty}rac{(2n)!}{(n+1)!n!}x^n$$
 (カタラン数)

$$rac{1}{\sqrt{1-4x}} = \sum_{n=0}^{\infty} inom{2n}{n} x^n$$

$$\frac{1}{1-x-x^2}=1+x+2x^2+3x^3+5x^4+\cdots=\sum_{n=0}^{\infty}F_nx^n$$
 (フィボナッチ数)

$$\log(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \dots = \sum_{n=1}^{\infty} \frac{1}{n}x^n$$

$$\exp(\exp x - 1) = \sum_{n=0}^{\infty} B_n \frac{x^n}{n}$$
 (ベル数)

$$\frac{1}{k}(\exp x - 1)^k = \sum_{n=0}^{\infty} S(n,k) \frac{x^n}{n!}$$
 (第二種スターリング数)

$$\frac{\exp(-x)}{1-x} = 1 + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{3}{8}x^4 + \dots = \sum_{n=0}^{\infty} a_n \frac{x^n}{n!}$$
 (モンモール数)

$$C(x)^k = \left(rac{1-\sqrt{1-4x}}{2x}
ight)^k = \sum_{n=0}^\infty rac{k}{n+k} inom{2n+k-1}{n}$$
 (カタラン数の母関数の累乗)