

WonderMarket

Section A – Internal Report

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MATH3202 Assignment 2
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Abstract

We have been approached by WonderMarket to optimise their supply chain logistics. Given information about their costs and needs, we created and optimised a model using Gurobi. This report describes the model and its solution.

Model Definition

WonderMarket operates a number of stores and distribution centres (DCs). Each week, each store requires some amount of goods (measured in truckloads) to be delivered from one or more of their distribution centres (DC).

After communication 9, we developed a detailed model to mathematically represent their objectives. This includes handling surge scenarios and predicting the total yearly cost of their operations.

Sets

- S set of stores.
- D set of distribution centres (this includes proposed new DCs).
- U set of surge scenarios.

Data

$Cost_{d,s}$	cost of moving one truckload of goods from DC d to store s (comm 1). (this includes cost of transport from new DCs after communication 6).
$Demand_s$	number of truckloads required at store s per week normally (comm 1).
$Capacity_d$	maximum capacity (truckloads) which can be sent from d per week (comm 2). (this includes capacities of new DCs after communication 6).
Northside	set of distribution centres on the north side (comm 3).
NorthsideMax	maximum combined capacity of northside DCs (comm 3).
$SurgeDemand_{u,s}$	surge demand during scenario u at store s (comm 4).
NewDCs	set of DCs which are not currently built (comm 6).
FTCost	cost per week of hiring one full-time team (comm 8).
PTCost	cost per week of hiring one part-time team (comm 8).
FTCapacity	capacity per week handled by one full-time team (comm 8).
PTCapacity	capacity per week handled by one part-time team (comm 8).
CasualCost	cost per week of hiring one casual employee (comm 9).
$SurgeWeeks_u$	number of weeks surge scenario u occurs per year (comm 9).

Derived Data

To simplify later calculations, we derived some data from the data given above.

SurgeMultiplier _{<i>u,s</i>}	surge demand at store <i>s</i> during surge <i>u</i> , divided by regular demand at store <i>s</i> . SurgeMultiplier _{<i>u,s</i>} = SurgeDemand _{<i>u,s</i>} /Demand _{<i>s</i>}
NormalWeeks	number of weeks in the year which have no surge scenario. NormalWeeks = 52 - $\sum_{u \in U} \text{SurgeWeeks}_u$

Variables

The following variables were used in the Gurobi model.

B_d	binary variables for whether DC <i>d</i> is active. (1 means if <i>d</i> is new then <i>d</i> is built; if <i>d</i> already exists then <i>d</i> is not closed).
P_d	integer number of part-time teams employed at DC <i>d</i> year-round.
F_d	integer number of full-time teams employed at DC <i>d</i> year-round.
$C_{u,d}$	integer number of casual employee employed at DC <i>d</i> during surge <i>u</i> . (each casual employee is only employed for the duration of the surge).
$A_{d,s}$	binary variable for whether DC <i>d</i> delivers to store <i>s</i> .
$X_{d,s}$	integer truckloads to be sent from DC <i>d</i> to store <i>s</i> during normal demand.
$Y_{d,s,u}$	integer truckloads to be sent from DC <i>d</i> to store <i>s</i> during surge scenario <i>u</i> . (<i>X</i> and <i>Y</i> will be 0 or exactly match demand, so will be integers.)

Objective

This calculates the total yearly cost, considering transport and labour costs. The objective is to *minimise* the following function.

$$\begin{aligned}
 & \text{(normal transport cost)} && \sum_{s \in S} \sum_{d \in D} \text{NormalWeeks} \cdot \text{Cost}_{d,s} X_{d,s} \\
 & \text{(surge transport costs)} && + \sum_{u \in U} \sum_{s \in S} \sum_{d \in D} \text{SurgeWeeks}_u \text{Cost}_{d,s} Y_{d,s,u} \\
 & \text{(full/part-time labour costs)} && + 52 \sum_{d \in D} \text{PTCost} P_d + 52 \sum_{d \in D} \text{FTCost} F_d \\
 & \text{(casual labour costs)} && + \sum_{u \in U} \sum_{d \in D} \text{CasualCost} \text{SurgeWeeks}_u C_{u,d}
 \end{aligned}$$

Constraints

First, we have the constraint that all variables are non-negative and certain variables are integers or binary. For all $d \in D$, $s \in S$, $u \in U$, we have

$$\begin{aligned}
 B_d, P_d, F_d, C_{u,d}, A_{d,s}, X_{d,s}, Y_{d,s,u} &\geq 0 \\
 B_d, A_{d,s} &\in \{0, 1\} \\
 P_d, F_d, C_{u,d} &\in \mathbb{Z}
 \end{aligned}$$

We link *X* and *Y* variables via *A* and the known demand at each store. This ensures each store receives all its supplies from one DC.

$$X_{d,s} = \text{Demand}_s A_{d,s} \quad \forall d \in D, s \in S$$

We link X and Y via the surge multipliers data. This ensures for each store, it receives deliveries from the same DCs in each scenario as during normal demand.

$$Y_{d,s,u} = \text{SurgeMultiplier}_{u,s} X_{d,s} \quad \forall d \in D, s \in S, u \in U$$

We ensure the solution is valid for normal demand considering truckloads and capacities. Note that the northside capacity limit has been removed.

$$\begin{aligned} \sum_{d \in D} X_{d,s} &\geq \text{Demand}_s & \forall s \in S \\ \sum_{s \in S} X_{d,s} &\leq \text{Capacity}_d & \forall d \in D \end{aligned}$$

Ensuring at most 2 new DCs are built and there are 4 DCs in total (3 or 2 old).

$$\begin{aligned} \sum_{d \in \text{NewDCs}} B_d &\leq 2 \\ \sum_{d \in D} B_d &= 4 \end{aligned}$$

Ensuring enough teams are employed to meet normal demand at each DC.

$$\sum_{s \in S} X_{d,s} \leq \text{FTCapacity}_d F_d + \text{PTCapacity}_d P_d \quad \forall d \in D$$

We ensure our assignments can scale up to each surge scenario while remaining feasible with the given constraints. For each $u \in U$,

$$\begin{aligned} \sum_{d \in D} Y_{d,s,u} &\geq \text{SurgeDemand}_{u,s} & \forall s \in S \\ \sum_{s \in S} Y_{d,s,u} &\leq \text{Capacity}_d & \forall d \in D \\ \sum_{s \in S} Y_{d,s,u} &\leq \text{FTCapacity}_d F_d + \text{PTCapacity}_d P_d + C_{u,d} & \forall d \in D \end{aligned}$$

Solution

Solving this MILP model in Gurobi returns the following solution. This would cost WonderMarket \$12576018.00 each year (\$241846.50 per week).

Store Assignments

Store	DC0	DC1	DC2	DC3	DC4	DC5	DC6
S0				100.0%			
S1				100.0%			
S2				100.0%			
S3						100.0%	
S4				100.0%			
S5		100.0%					
S6		100.0%					
S7			100.0%				
S8			100.0%				
S9				100.0%			

Distribution Centres

WonderMarket should close DC0 and build DC3 and DC5. DC1 and DC2 should remain open.

Labour

WonderMarket should hire the following teams for the whole year.

DC	Part-time	Full-time
DC1	0	2
DC2	0	2
DC3	0	8
DC5	0	2

Additionally, they should hire the following casual staff for certain surges.

Surge	DC	Casual
Surge 0	DC5	11
Surge 1	DC2	20
Surge 3	DC2	20
Surge 4	DC1	42
Surge 4	DC3	2