Exhaustification algorithm

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The algorithm used by this package is described here, with a proof of correction.

1 Definitions

Write p for the prejacent, and $\mathscr{A}$ the set of alternatives.

Definition: A set of alternative $\mathscr{A}'$ is consistent with the prejacent iff p $\wedge \bigwedge_{q \in \mathscr{A}'} \neg q$ is consistent.

Definition: A set of alternative $\mathscr{A}'$ is maximal iff it is consistent with the prejacent and there is no set $\mathscr{A}' \subsetneq \mathscr{A}''$ consistent with the prejacent.

Definition: A world is minimal if p is true in w and there is no other world w' such that the set of alternatives false in w' strictly contains the set of alternatives false in w''.

Definition An alternative $q \in \mathscr{A}$ is innocently excludable (*IE*) iff it belongs to every maximal set of alternatives.

2 Description of the algorithm

1. Find, for every minimal world w, the set of alternatives false in w.
   (a) Initialize an empty list L of boolean arrays of length $\#\mathscr{A}$.
   (b) For each world w true of p, add the array v_i - which maps i to true if alternative a_i is true in w - if there isn't a vector with more false values in L.
   (c) By construction, the list L contains, for every minimal world w, the set of alternatives false in w.
2. The IE alternative are the intersection of the vectors in L.

3 Proof of correction

Claim: The set of alternatives false in a minimal world is a maximal set. Reciprocally, every maximal set is the set of false alternatives of some minimal world.

Proof: If w is a world where p is true, call $alt(w)$ the set of alternatives false in w. Now, if w is a minimal world, then for all worlds w', $alt(w')$ does not strictly contain $alt(w)$.This means that there is no set of alternatives $\mathscr{A}'$ strictly containing $alt(w')$ such that $p \wedge \bigwedge_{q \in \mathscr{A}'} \neg q$ is consistent, i.e. true of some world w'. If there were, then $alt(w')$ would contain $\mathscr{A}'$ and thus $alt(w)$ itself. So no such $\mathscr{A}'$ exists. In other words, $alt(w)$ is maximal
Reciprocally, if $\mathscr{A}'$ is a maximal set and w a world where $p \wedge \bigwedge_{q \in \mathscr{A}'} \neg q$ is true, then w is minimal. If w weren't, then we could find a world w' where p is true and where a bigger set of alternatives $\mathscr{A}''$ than $\mathscr{A}'$ is false. But then $\mathscr{A}'$ wouldn't be maximal.

Claim: The set of alternatives false in every minimal worlds is the set of IE alternatives.

Proof: From the claim above, it follows that the set of maximal sets is the set $\{alt(w) \mid w \text{ is a minimal world}\}$. Therefore, the set of alternatives false in every minimal world is the set of alternatives in every maximal set, i.e. the IE alternatives.