Playability of the wolf note of bowed string instruments

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Playability is an important aspect of the evaluation of bowed string instruments. The well-known "wolf note" of a cello is a particularly obvious playability issue, and it has been suggested that susceptibility to wolfiness might be deduced directly from a measurement of the Schelleng minimum bow force for the playing of a steady note. This prediction is explored by comparing physical measurements with the experience of players after making controlled mechanical changes to a cello. Experienced luthiers and musicians made subjective judgements of changes in the severity of the wolf note, under blinded conditions. The results strongly suggest a direct and intimate link between the measurable acoustical parameter and perceptual discrimination. This simple acoustical measurement can help instrument makers to identify problem notes, and to assess the effectiveness of different possible interventions. © *2018 Acoustical Society of America*. https://doi.org/10.1121/1.5079317

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I. INTRODUCTION

Players of bowed-string instruments do not only rely on their sense of hearing when judging quality and comparing instruments; they also monitor the haptic response provided by the instrument. There are thus two important aspects of discrimination between instruments (Türckheim *et al.*, 2009). Properties related directly to acoustical response are usually described in terms of "sound quality," which is normally assessed by the subjective impressions of listeners. On the other hand, "playability" also incorporates haptic feedback, in other words the mechanical interactions between the instrument and the player: it describes whether an instrument is easy to play from the perspective of the musician.

In contrast to sound quality, playability can only be judged by a player, not by a listener. During a performance, experienced players can detect quirks and inadequacies of instruments immediately, since these deficiencies require extra effort and attention in order to make appropriate compensations through careful bowing technique. For this reason, when it comes to comparing instruments, judgements by players are more reliable than judgements by external listeners because listeners cannot be aware of these compensations (Woodhouse, 1993a).

Playability undoubtedly involves a range of phenomena. As has been revealed by interviews with professional luthiers and players, many terms are used to describe the playing qualities of the instrument: examples include "resistance," "cushion," "range of tone," "good ring," and "wolf note" (Zhang, 2015). The challenge for the scientific study of playability is to pin down terms like these to reproducible effects via psychoacoustic tests, and to correlate them with physical measurements on instruments so that the effect might be controlled in a systematic way by an instrument maker. As a contribution to that agenda, this paper focuses on a particular physical quantity, the Schelleng minimum bow force computed from a determination of the admittance at the cello bridge. It has been proposed in earlier literature that this measure is a good candidate to capture some aspects of playability (Woodhouse, 1993b). Experimental results will be presented to test whether this measure has predictive potential in relation to subjective judgements of an obvious playability issue known as the "wolf note," which is a common problem experienced by many players, especially of the cello.

A. Wolf note

In the musical world, a wolf note is an unpleasant warbling note most commonly found around F3 or F#3 and manifesting most strongly on the heavier strings of a cello (Cremer, 1984). For a musician, it is difficult to produce a steady tone of good quality when playing at the pitch of the wolf note; rather, the tone is inherently unstable and changes with the ambient conditions, string tuning or setup adjustments. Precautions are generally taken to control this particular note through playing techniques or the use of a "wolf eliminator" (which usually takes the form of a tuned mass damper of one kind or another). There can be no doubt that the wolf note is one of the specific problems related to "ease of playing" or playability.

Although the wolf note is not popular among musicians, this phenomenon has often attracted the attention of researchers. Over the past century there have been several attempted explanations which contribute to knowledge of the basic characteristics of the wolf tone. White (1915) was one of the first to confirm that a wolf tone often occurs when a played note matches a strong body resonance of a bowed string instrument. He argued that the periodic fluctuation in the intensity of the wolf note is the result of "beating" between two adjacent frequencies.

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In 1916, Raman (Raman, 1916) applied optical levers to record the motion of both the top plate and string of the instrument during a wolf note, and raised doubts about White's conjecture. He suggested as an explanation of the wolf note that the cyclic intensity variation is due to an alternation of types of forced vibration of the bowed string. One type of motion, known as "Helmholtz motion," is the usual regime of string vibration that a player is aiming for [see, for example, Cremer (1984)]. It involves single episodes of sticking and slipping friction at the bow during each cycle. The second regime involves two (or sometimes more) episodes of slipping in each cycle, with an undesirable result often described by players as "surface sound." This "double-slipping motion" usually arises when the force between the bow and string is too low, so it is intimately associated with the notion of a minimum bow force, as Raman was the first to explore.

In the 1960s, Schelleng (1963) revisited the idea of beating. As an electrical engineer and a keen amateur cellist, he discussed the nature of the wolf note by analogy with equivalent coupled electrical resonant circuits. Schelleng pointed out that the cyclic intensity fluctuation at the wolf note could indeed arise as a result of the beating of two equally forced oscillations. Viewed in the frequency domain, the body resonance at the wolf note pitch is split by coupling to the string motion into a pair of peaks. Firth and Buchanan (1971) extended the work of Schelleng to provide additional data for this theory.

McIntyre and Woodhouse (1978) measured the transverse waveform of the bridge force and corresponding frequency spectrum of the wolf tone by using a piezoelectric transducer. The results verified Raman's explanation by showing an alternation of string oscillation regimes. Schelleng's description was also supported, at least qualitatively: viewed in the frequency domain, the measured spectrum of string velocity does indeed show a split pair of strong peaks. However, the idea of "beating" is at heart a linear concept, whereas McIntyre and Woodhouse showed that nonlinearity is crucial to the phenomenon. By modelling the nonlinear characteristics associated with the frictional excitation, they developed simulations of string and body vibration during a wolf note. The results showed that the period of the "warbling" wolf can be varied by changing the bow force, whereas a beat frequency is a property of the coupled string and body resonances and would be unaffected by bow force (Cremer, 1984).

B. Minimum bow force

Research accompanying the further development of computer simulation models for bowed-string motion has suggested strongly that the prominence of a wolf note should relate to variation of the minimum bow force for the playing of a steady note (Woodhouse, 1993b). The aim of this paper is to investigate that link. To address this issue, three stages are necessary: (i) to correlate small constructional changes on the instrument to acoustical measurements of minimum bow force; (ii) to relate the same changes to perceptual effects on the wolf note; and (iii) to examine the correlation between the acoustical changes and the player's perception.

The maintenance of Helmholtz motion (Helmholtz, 1954) in a steady state requires a balance between three bowing parameters: the bow speed, the bow force and the location of the bow on the string. Based on early work by Raman (1920), the first systematic study of the required balance was made by Schelleng (1973). He summarised his results related to the bow force and the bowing position at a fixed bowing velocity in a famous diagram shown schematically in Fig. 1. The force applied on the string and the location of the bowing point must be kept within the triangular area between two lines indicating the maximum bow force and minimum bow force. Of the two force limits, only the minimum bow force is strongly influenced by the vibrational characteristics of the body of the instrument (Woodhouse, 2014).

Schelleng derived his formula for minimum bow force based on Raman's model, in which the complexity of the body vibration is simplified into a single mechanical resistance R. His resulting formula cannot be made quantitative because of uncertainty about the value of R. Woodhouse (1993b) extended Schelleng's theoretical analysis of minimum bow force by eliminating this restrictive assumption and instead making direct use of the measured vibration response of the body. The resulting equation can be written in terms of the following parameters: f_0 is the fundamental frequency of the played note on the string, β the bowing position as a fraction of the vibrating string length, v_b the bow velocity, Y_0 the characteristic admittance of the string, $Y(\omega)$ the measured mechanical admittance at the bridge, and μ_s and μ_d respectively the coefficients of sticking friction and sliding friction. The predicted minimum bow force can then be written in the form

$$f_{min} = \frac{2v_b}{\pi^2\beta^2Y_0^2(\mu_s-\mu_d)} \cdot \max_t\left\{Re\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}Y(n\omega_0)e^{in\omega_0 t} + Re\sum_{n=1}^{\infty}\frac{Y(n\omega_0)}{n^2}\right\}, \quad (1)$$

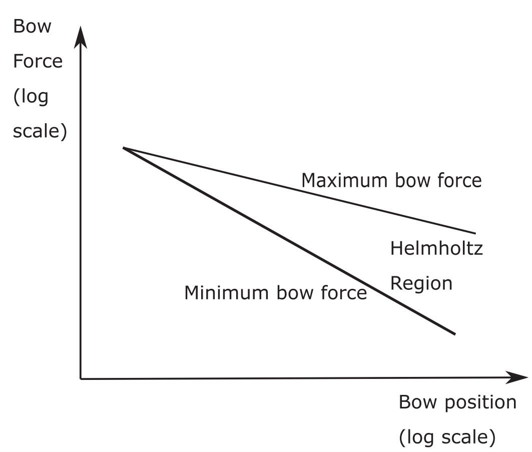


FIG. 1. Schelleng's diagram in schematic form, showing the region in which Helmholtz motion can be sustained bounded above and below, respectively, by the maximum and minimum bow force lines.

where $\omega_0 = 2\pi f_0$. The minimum bow force is thus predicted to vary from note to note on a given instrument, in a way that can be estimated quite straightforwardly from a measurement of $Y(\omega)$.

Recently this analysis has been further extended by Mansour *et al.* (2017). The original analysis assumed a perfect Helmholtz motion in the string, but close to a wolf note there are small but systematic deviations from that assumption. These were allowed for in the extended model, and the results revealed the possibility of more complicated dependence of minimum bow force on the bow position β, at least under extreme circumstances. These results were validated using a computer simulation model able to take into account the influence of a number of physical details, such as the torsional string motion, the vibration of sympathetic strings, and the second polarisation of transverse string vibration. However, the extended prediction does not have any simple closed-form expression similar to Eq. (1). For the purposes of the present study Woodhouse's simpler form will be used since the more detailed computations confirmed that it gives a good first approximation to the expected behaviour under all musically realistic conditions, and so it still seems a good candidate to attempt to correlate with subjective judgements of playability.

It is easy to link the idea of minimum bow force to the physical mechanism of a wolf note. While playing at the wolf pitch, suppose a Helmholtz motion is established before the body vibration has a chance to grow large. As the body response grows (because the played note is close to a strong resonance), the effective minimum bow force also increases because the rate of energy transfer to the instrument body increases. When this effective minimum bow force exceeds the actual bowing force, a second slip occurs near the middle of the sticking period of the Helmholtz motion. The string then slips twice or more per period, as Raman predicted: "surface sound." Now the second slip interacts with the body vibration in a different phase to the original slip, and the result is that it grows while the original slip gets smaller. The energy of the body vibration can then be transferred back into the string, and the effective minimum bow force value falls down again. A new Helmholtz motion may eventually be established when the second slip replaces the original one entirely. But then the cycle repeats as the body resonance builds up in the new phase. The resulting cyclical behaviour gives the annoying wolf note. This argument suggests that the peak value of the minimum bow force predicted by Eq. (1) may be a good candidate for correlating with judgments of wolf note location and severity.

II. ACOUSTICAL MEASUREMENTS AND PSYCHOACOUSTIC TESTS

This study thus addresses the following question: how sensitive is a skilled player to differences in ease of playing near the wolf note, when small controlled changes are made to the cello, and do their subjective ratings of variations in wolf severity correlate with the prediction associated with Eq. (1)? The experiment consists of two independent sessions (see Fig. 2 for a schematic). In the first session (acoustical measurements), small physical changes were made to a cello and the corresponding changes of minimum bow force as determined by Eq. (1) were measured. In the second session (psychoacoustic tests), the participants were asked to rate and describe the ease of playing near the pitch of the wolf note of the cello, when the same changes were made (in a randomised sequence).

In two respects the cello is more suitable for this experiment than the violin. First, in the cello the wolf note is generally more severe. Second, it is much easier to make the measurements and to make controlled changes in behaviour because both the size and weight of the cello are considerably greater than those of the violin. Preliminary playing tests were used to select a cello of reasonable quality (made by the second author in 1976) which showed a clear wolf tone. This cello was then used consistently in all tests. In the interests of demonstrating the clearest possible link between small physical changes and differences of playability, the experiments concentrated on the cello C string because this string is most susceptible to the wolf note, usually around F3 or F#3. This note would not normally be played on the C string, precisely because it exacerbates the wolf, but it is ideal for the purposes of this investigation.

A. Acoustical measurements and results

In order to obtain a prediction of the minimum bow force of the cello, an estimate of the driving-point admittance (or mobility) at the bridge must be measured, in the direction parallel to bowing. As indicated in the sketch in Fig. 2, the acoustical measurements were made with the cello held by a steel support frame with a firm base, and steadied by two soft foam pads. This fixture was designed to mimic the normal holding method used by players: the cello rests on its endpin, is held in a foam-padded support at the neck, and the two foam pieces at the sides replicate the action of the player's knees.

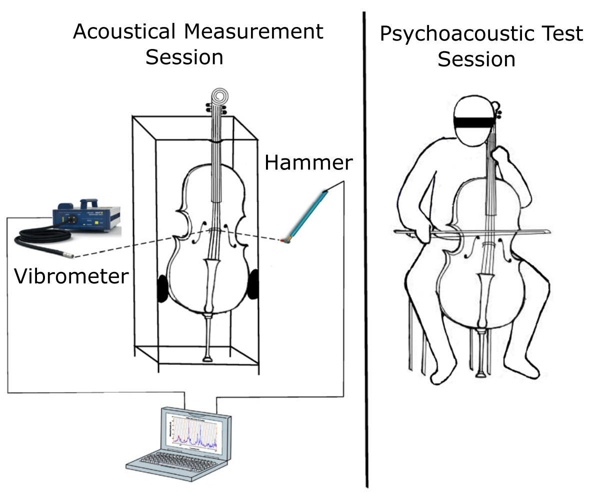
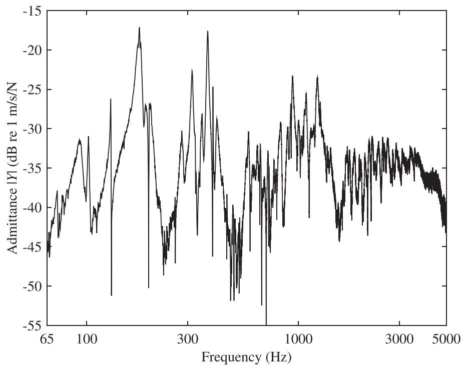
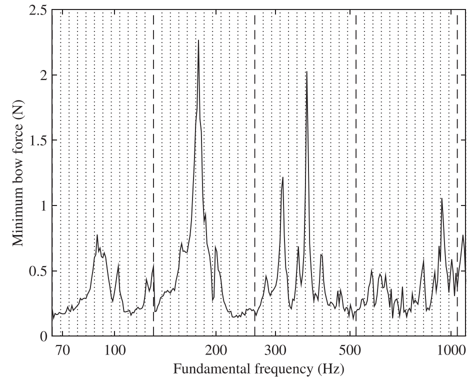


FIG. 2. (Color online) Experimental methodology in diagrammatic form. The first stage involved measuring the driving-point admittance $Y(\omega)$ at the cello bridge, when small physical changes were made. The second stage involved making the same small physical changes and asking a blindfolded player to describe and rate the changes in playability.



(a)



(b)

FIG. 3. (a) Measured bridge admittance $|Y|$ for the test cello; (b) minimum bow force deduced from this admittance, for the C string of the cello with a bow speed 0.1 m/s and bow position $\beta = 0.1$. In (b), vertical dotted lines denote semitones, with successive notes C highlighted by dashed lines.

An input force was applied to the A-string corner of the bridge by a miniature instrumented hammer (PCB 086D80) in the direction of bowing on the adjacent string. A laser-Doppler vibrometer (Polytec OFV-056) was used to measure the velocity response at the C-string corner, in the direction of bowing on the C string. The signals from hammer and vibrometer were sampled at a rate of 40 kHz by a PC-based datalogger, and analysed using software written in Matlab. The test procedure is described in more detail in an earlier paper (Zhang and Woodhouse, 2014). That paper, together with Zhang *et al.* (2016), also examines carefully the question of how accurately this "corner-to-corner" measurement actually approximates the true drive-point admittance at the individual string notches on the bridge. An example of measured admittance $Y(\omega)$ for the test cello is shown in Fig. 3(a). This measurement was taken with strings other than the C string well damped.

From the input admittance, the minimum bow force can be calculated using Eq. (1). The values depend on the prefactor in that equation, and throughout this paper that factor has been calculated using the characteristic admittance $Y_0 = 0.71$ m/s/N, a bow speed $v_b = 0.1$ m/s, $\beta = 0.1$ and $\mu_s - \mu_d = 0.5$. However, these parameter values and the resulting absolute values of the minimum bow force do not matter for the purposes of this study since the only important thing will be the pattern of variation when changes are made to the vibration behaviour of the cello. The predicted minimum bow force corresponding to the admittance from Fig. 3(a) is shown in Fig. 3(b). The horizontal axis shows the fundamental frequency of the played note, over a range of 4 octaves from the lowest note of the cello (C_2, 65 Hz). Vertical dotted lines mark semitones, with octaves of the low C indicated by dashed lines. Peaks in the plot indicate problematic notes that are likely to cause difficulties when playing this cello. In particular, the highest peak at 175 Hz, around note F_3, marks the wolf note.

The results of preliminary tests showed that the amplitude of the wolf note peak was sensitive to the moment of inertia of the cello bridge. Consequently, a clip was designed to allow controllable changes to this moment of inertia. Figure 4 shows a sketch of this clip: it has a total mass 6.2 g, and is made from a wooden clothes peg and an adjustable mass which can be rotated 360° around a connecting bolt. The value of the angle α between the centre line of the added mass and a fixed reference line could be read out by means of a paper protractor. For effective and repeatable operation, it was essential that during vibration of the cello body the clip and the bridge move as a unit. To minimise frictional slipping, two pieces of coarse sandpaper were glued to the clamping surfaces of the peg: it was verified that there was indeed no slipping under the conditions of the experiment (Zhang, 2015).

It should be emphasised that this clip is simply a means to an end: it satisfies the requirements for something that was non-damaging to the cello, and that allowed a quick and accurately repeatable change which (as will be seen shortly) had a significant effect on the admittance. There is no suggestion that the device "improves" the sound of the cello, or that it might have any direct musical application as, for example, a new kind of wolf eliminator. Indeed, it is essentially an adjustable mute, and the effect on the sound of the cello is as one would expect for any mute. None of that is important for the question under investigation: no comparison is sought between the behaviour of the cello with and without the clip, simply between different positions of the adjustable arm.

The input admittance was measured with this clip on the cello bridge, with a range of setting angles α covering the full circle. To ensure reliability, the admittance test for each

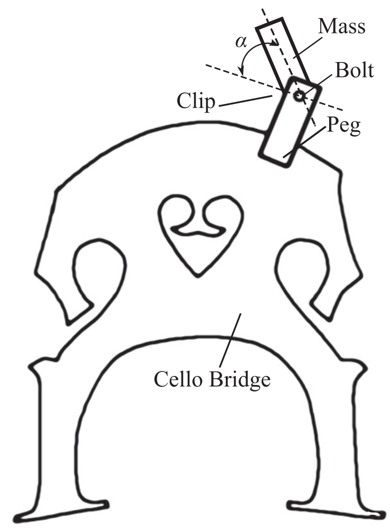


FIG. 4. The cello bridge with the attached clip (see text for details).

α was repeated four times. The averaged amplitudes of the wolf note peak were calculated, and as had been hoped the wolf note peak was found to vary markedly with the angle α. The lowest wolf note peak occurs when $\alpha = 180°$: at this angle the wolf was predicted to be least troublesome. The opposite extreme occurred at $\alpha = 225°$, where the most serious wolf note was predicted. In the light of these results, it was decided that the perceptual experiments would employ 10 values of the angle α from 80° to 225°. The measured pattern of heights of the wolf-note peak over this range of angles is plotted in Fig. 5. The values have been converted to non-dimensional form by dividing them all by the maximum value. Significant variation as a function of the angle α can be seen, covering roughly a factor of 2 and with a pattern which is perhaps unexpectedly complicated.

B. Psychoacoustic tests and results

In order to explore the link between acoustical measurements and the perceptions of players, corresponding playing tests were carried out.

1. Participants

Seven participants, having an average playing experience of 20 years, were included in this study. They were all right-handed with normal hearing and tactile sensation. None had suffered from psychiatric or neurological diseases. All participants received academic credits for their participation and provided written consent. The experiments were performed in accordance with the relevant guidelines and regulations. The participants were blindfolded for the duration of the playing tests. Six participants were instrument makers, including three experienced cello makers and tone-adjusters; the seventh was an experienced amateur musician. This information was ascertained through a detailed questionnaire completed before the playing tests started. It is accepted that a larger number of participants would have been desirable, but the authors wanted to maximise participation by experienced cello makers and tonal adjusters, and such people are not common.

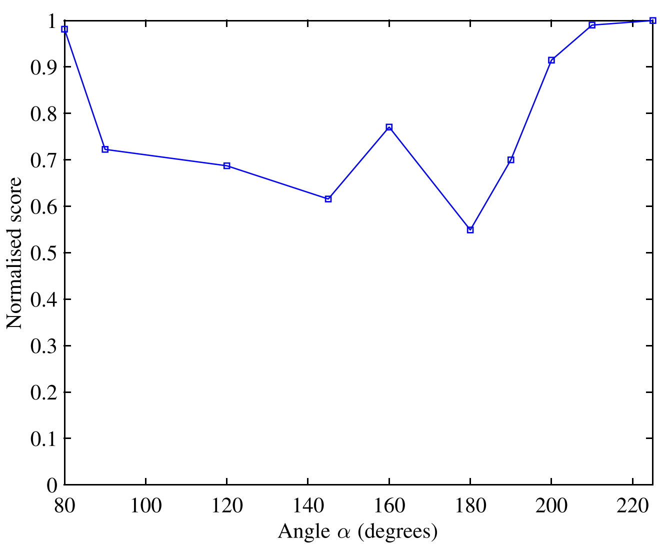


FIG. 5. (Color online) Normalised height of peak in minimum bow force measurement, as in Fig. 3(b), plotted against the angle of the clip. High values connote conditions that are harder to play, low values are easier. The data has been normalised to a maximum value of unity.

2. Test conditions

The playing tests took place in an acoustically dry room and were performed on the same cello used in the acoustical experiments. The strings of the cello were tuned optimally and other set-up details were adjusted properly prior to the experiment. Since the bow might influence the playing techniques used by players, all subjects were invited to use their own bows if they preferred. The 10 different clip angles between $\alpha = 80°$ and $\alpha = 225°$ were used, each subject being exposed to all 10 angles twice in the course of the test, in a randomised sequence that was different for each participant.

3. Procedure

Before the formal tests, the subjects were asked to play and assess the cello in the bare state, with no added clip. They were asked to find the wolf tone (or other problematic notes) in the cello. These problematic notes and their other comments on difficulties of playing were recorded. The subjects were then allowed to calibrate their responses to the adjustable clip by being exposed to the cases predicted to be easiest to play (180°) and hardest to play (225°) according to the results of acoustical measurements.

During the whole process of the formal playing tests, including the "calibration" session at the start, the subjects wore a lightweight eye mask, so that they had no idea what kind of change had been made on the cello. The task requirement was to play on the C string in the vicinity of the wolf note and rate how hard to play the wolf note was. In order to keep the experience as close as possible to normal musical practice, no further detail was specified: players were left free to choose all aspects of playing style, and they were allowed as much time as they wished for each case.

Once subjects were acquainted with the task requirements, the formal playing tests started. Modifications were made using the clip described previously: the adjustable mass in the clip was rotated to one of 10 angles from 80° to 225° on each trial. The subjects were asked to give an absolute rating of the severity of the wolf, rather than a relative judgement compared to the previous state. To test the reliability of results of this perceptual experiment, each case was tested twice so that there were a total of 20 trials, in randomized order. The numerical rating scale for every player was different because it was chosen according to personal preference.

4. Results

The results showed that all participants were satisfyingly self-consistent when they were rating the difficulty of playing at the same α, so it was decided to average the ratings for each angle. In order to allow results to be compared despite the individual choices of rating scale, each set of averaged ratings was then normalised to a maximum value of unity.

One of the subjects repeated the playing test a second time, several months after their first trial, so eight sets of ratings were recorded in total. The results of all eight trials are shown in Fig. 6, alongside the predicted minimum bow force values reproduced from Fig. 5. The two trials for the repeat participant are shown as red dashed lines, and show reasonable consistency. But at first glance the full set of results shows wide disparities, and none of them follows the predicted pattern with any convincing accuracy.

However, a more encouraging picture emerges when the results are aggregated. Figure 7 shows the average of the normalised perceptual judgements, as the dashed red line. It also shows as dotted lines the mean plus and minus one standard deviation. When the value of mean plus standard deviation exceeded unity, the result has been truncated in the plot. Although the wide spread in the individual results is still reflected in the wide band between the dotted lines, the dashed mean line follows the main trends of the measured curve for minimum bow force remarkably convincingly.

The conclusion, at least tentatively, is that the proposed prediction of minimum bow force from the measured bridge admittance of an instrument really does capture some details of perceived variations in playability. First, it successfully identifies problem notes. Second, it captures variations of severity of these problem notes when relatively minor mechanical changes are made to the instrument. Although the adjustable clip used here is not directly representative of the things an instrument maker would normally do to alleviate a wolf note, the level of mechanical influence probably is comparable to other control strategies, and so the results here hold out considerable hope that this simple measurement might be used to guide a maker to optimise such interventions. For a maker equipped to make some version of this measurement, it very quickly gives a global picture of the effect of a change, not only on the wolf note itself but on other possible problem notes. Eliciting that information directly by playing the instrument takes far longer, and is less well-controlled, objective and repeatable.

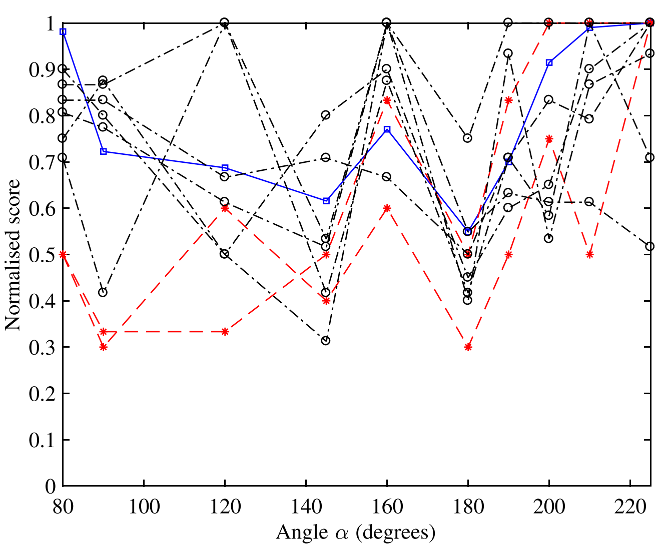


FIG. 6. (Color online) Normalised scores for "ease of playing" near the wolf note, plotted against the angle of the clip. High values connote conditions that are harder to play, low values are easier. Each data set has been normalised to a maximum value of unity. Heavy blue curve: height of peak in minimum bow force measurement, as in Fig. 3(b); red dashed curves: subjective judgements from two separate test sessions by a single participant; black dashed-dotted curves: results from the other participants.

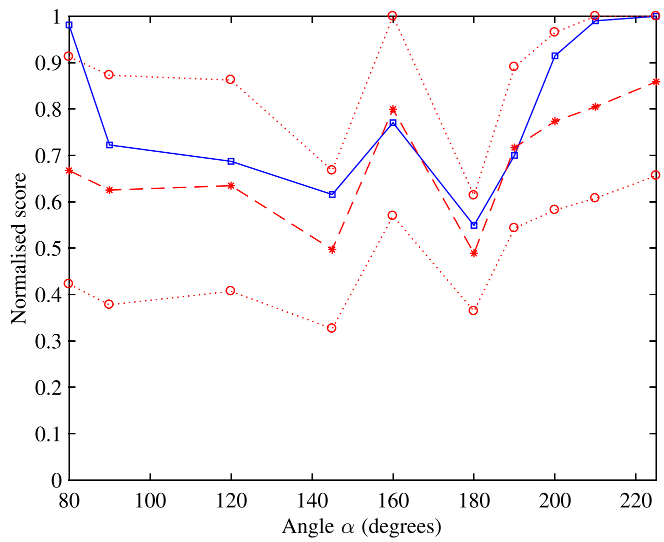


FIG. 7. (Color online) Normalised scores for "ease of playing" near the wolf note, plotted against the angle of the clip. High values connote conditions that are harder to play, low values are easier. Each data set has been normalised to a maximum value of unity. Blue curve: height of peak in minimum bow force measurement, as in Fig. 3(b); red dashed curve: mean of all subjective judgements from the psychoacoustics tests; dotted curves: mean subjective judgement, plus and minus one standard deviation (with values greater than unity truncated).

III. DISCUSSION

The work described in this paper does not claim to be more than a preliminary investigation of the link between minimum bow force and playability problems associated with wolf notes, but the results are very encouraging. The variation of peak heights in minimum bow force plots has been shown to reflect, at least in an average sense, the pattern of changes of players' perception of the severity of the wolf note. This provides a direct link between a measurable acoustical parameter, constructional changes and perceptual preferences.

It should be emphasised that even this limited and preliminary success is a rare achievement in musical acoustics. As has been discussed in more detail elsewhere (Woodhouse, 2014), there is often a conflict between the requirements of scientific experiment design and the need for musicians taking part in a test to retain some sense of musical context. The effects being examined are usually rather subtle, although musically important. Trials involving many repeats in order to obtain reliable statistical information can be mind-numbing for the participants, and the result may be to blunt the subtle musical judgements that are of interest. Furthermore, confounding factors not intended by the experiment-designers can distract the attention of players: see, for example, the discussion by Fritz *et al.* (2010) about the difficulties of designing a test to probe the apparently uncontroversial link between violin body response and sensitivity to vibrato.

In the perceptual experiment described here, a deliberate choice was made to design a procedure to satisfy the needs of the players. They were given considerable freedom; to choose their own rating scale, and to play whatever they chose for as long as they wished before giving a judgement. Each player was exposed to only two repeats of 10 test conditions. These conditions carry costs, and it would be beneficial to carry out further experiments to test the robustness of the conclusions reached here. One obvious limitation of this study is that all the results were obtained using only one cello. It is a possibility that this particular instrument has a significant impact on players' preferences. In order to test the reliability of these results, acoustic experiments and playing tests should be carried out with other cellos, and perhaps other bowed instruments which have a severe wolf note such as the bass viol and the Chinese erhu.

The particular mechanical change made during these tests was rather special and unusual, involving a kind of adjustable mute. In principle, it would be interesting to do a similar investigation using structural modifications closer to those that an instrument maker might use in practice. However, it is not easy to make such changes quickly and repeatably so that they could be incorporated into the kind of experiment described here. Some informal investigations along these lines have in fact already been made by technically minded instrument makers. They report encouraging results, but at present the evidence is purely anecdotal.

Questions could be asked about the physical mechanism by which the adjustable clip added to the bridge had such a complicated influence on the wolf note. Presumably the nodal lines of the mode causing the wolf note were being shifted, in such a way as to alter the strength of coupling to the bowed string. To examine this possibility would require further testing to investigate the motion of the bridge and the associated vibration behaviour of the cello body [see, for example, Zhang and Woodhouse (2016)]. Such testing could be done, but in the opinion of the authors this would not be a very profitable exercise since the clip used in these experiments is so non-standard as an instrument maker's intervention.

The results from individual test subjects in this study showed considerable variation. The authors do not by any means suggest that these variations are simply "noise." For example, one subject in the playing test, an experienced luthier and player, showed very "poor" performance when compared with the minimum bow force prediction. This highlights the fact that playability is a multi-faceted and subjective judgement, likely to vary with personal factors such as age, gender, training, playing style, and so on. Also, players might ask themselves different questions and assign different priorities while judging "ease of playing."

In fact, the individual responses of participants, both in the preliminary study and the adjustable-clip study reported in detail, led the authors to suspect that the better the player, the worse the correlation of their judgements with the physical predictions. Tentatively, it might be suggested that the best players are so well practised at compensating for things like wolf notes that they adjust their bowing rapidly and perhaps subconsciously when changes are made. But they are sensitive to more subtle effects, perhaps influencing their ability to achieve the quality of tone they are aiming for, so their judgements recorded in the tests reflect a different aspect of the physical behaviour of the instrument.

There are many possible directions for future work along the lines presented here. It is very interesting to note the detailed comments made by the test subjects. Terms like "more even" and "shifted a little bit" were used by players to describe variations in the wolf note during the test. These comments might suggest more subtle changes in the minimum bow force, or other acoustical parameters. Also, it would be of obvious interest to obtain quantitative judgements of preference in respect of other playability issues. A possible target might be the term "range of tone" used frequently by string players: see, for example, the discussion by Woodhouse (2014).

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Cremer, L. (**1984**). *The Physics of the Violin* (MIT Press, Cambridge, MA).

Firth, I. M., and Buchanan, J. M. (**1971**). "The wolf in the cello," J. Acoust. Soc. Am. **53**, 457–463.

Fritz, C., Woodhouse, J., Cheng, F. P. H., Cross, I., Blackwell, A. F., and Moore, B. C. J. (**2010**). "Perceptual studies of violin body damping and vibrato," J. Acoust. Soc. Am. **127**, 513–524.

Helmholtz, H. von. (**1954**). *On the Sensations of Tone* (Dover, New York).

Mansour, H., Woodhouse, J., and Scavone, G. P. (**2017**). "On minimum bow force for bowed strings," Acta Acust. Acust. **103**, 317–330.

McIntyre, M. E., and Woodhouse, J. (**1978**). "The acoustics of stringed musical instruments," Interdiscip. Sci. Rev. **3**(2), 157–173.

Raman, C. V. (**1916**). "On the wolf-note of the violin and cello," Nature **97**, 362–363.

Raman, C. V. (**1920**). "Experiments with mechanically-played violins," Proc. Indian Assoc. Cultiv. Sci. **6**, 19–36.

Schelleng, J. C. (**1963**). "The violin as a circuit," J. Acoust. Soc. Am. **35**, 326–338.

Schelleng, J. C. (**1973**). "The bowed string and the player," J. Acoust. Soc. Am. **53**, 26–41.

Türckheim, F., Smit, T., and Mores, R. (**2009**). "An experimental musician-based study on playability and responsiveness of violins," in *Proceedings of International Conference on Acoustics (NAG/DAGA09)*, Rotterdam, Netherlands, pp. 1474–1477.

White, G. W. (**1915**). "The origin of the wolf-note in bowed string instruments," Proc. Cambridge Philos. Soc. **18**, 85–87.

Woodhouse, J. (**1993a**). "On the playability of violins: Part 1 Reflection functions," Acustica **78**, 125–136.

Woodhouse, J. (**1993b**). "On the playability of violins: Part 2 Minimum bow force and transients," Acustica **78**, 137–153.

Woodhouse, J. (**2014**). "The acoustics of the violin: A review," Rep. Prog. Phys. **77**(11), 115901.

Zhang, A. (**2015**). "Playability of bowed string instruments," Doctoral dissertation, University of Cambridge.

Zhang, A., and Woodhouse, J. (**2014**). "Reliability of the input admittance of bowed-string instruments measured by the hammer method," J. Acoust. Soc. Am. **136**, 3371–3381.

Zhang, A., and Woodhouse, J. (**2016**). "Motion of the cello bridge," J. Acoust. Soc. Am. **140**(4), 2636–2675.