No wonder that violin making lore indicates to "glue spring and fall" when ambient conditions are half way between the high humidity of summers and low humidity of our winter - heated houses!

Since the OCTET instruments were developed only a short 30 years ago and are still in a sense experimental, the Catgut Society would appreciate critical feed back information from those who wish to make the instruments as well as from players and listeners who are familiar with the OCTET. The OCTET has been characterized by the Royal Swedish Academy as "the most important new development in musical instruments of the 20th Century." We would appreciate your help to make the OCTET as musically effective as possible!

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THE VIOLIN OCTET - a brochure giving information for musicians, composers and violin makers is available for $1.50 from the CAS.

ON MEASURING WOOD PROPERTIES, PART 3

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1: Introduction

In the first part of this study [1], we have tried to indicate the importance for violin acoustics of a thorough quantitative knowledge of the elastic and damping constants of wood, more thorough than exists at present. By "wood" here, we mean primarily spruce and maple of a quality suitable for instrument making, but we are not only concerned with new wood. We are also interested in varnishes and other treatments applied to the wood, and in the effects of age and environment. In Part 2 [2] we described one approach to measuring some of these material parameters, and illustrated it with some preliminary measurements on a sample of spruce and a sample of carbon-fibre composite material (see also a corrigendum in Appendix B below). In this final part of the study we attempt to tie up loose ends, so far as is possible at present, by reviewing some of the existing literature, pointing out some advantages and problems associated with various experimental techniques, and looking for promising ways forward.

The literature on wood behaviour in general is quite extensive, although some of it is published in obscure places and rather hard to track down. In terms of our particular problem, the linear elastic and viscoelastic behaviour of "clear" (defect-free) wood, quite a lot is known about elastic constants. By contrast, very little is known about internal damping. Much of the most useful literature for our purpose is quite old, dating from the period between the two world wars when wood was an important material in aircraft manufacture. Fortunately, the requirements of aircraft designers were in some ways quite similar to those of instrument makers - in particular they wanted maximal strength-to-weight ratio, so they too were interested in high-quality spruce. An excellent, if rather hard to obtain, review covering this earlier literature is the report by Hearmon [3]. This contains much more material about wood than does his later but better-known book [4]. A good general reference for more recent work is the textbook by Bodig and Jayne [5].

As a starting point for our discussion, we draw attention to the very useful table given by Hearmon [3, Table 2] summarising all the then-known sets of measurements of all nine elastic constants, covering some 20 different types of wood. Since this reference is not widely available, we reproduce in Table 1 the entries for relevant woods - described by Hearmon as "spruce", Sitka spruce and "maple". Since he was writing in England, he probably refers to European spruce and maple as understood by instrument makers. The notation follows the conventional pattern: the three principal directions in the tree are called L(ongitudinal), R(adial) and T(angential), and the elastic constants are called E (Young's moduli), ν (Poisson's ratios) and G (shear moduli) as in the Appendix to our Part 1 [1]. (For example, E_R is the radial Young's modulus, ν_{LR} is the Poisson contraction in the radial direction for a longitudinal rod subjected to tension along its length, and G_{LR} is the shear modulus in the LR plane.) While we cannot tell whether the specimens tested to yield these results were of instrument quality, these figures at least give us a guide to the magnitudes to be expected for the different constants. So far as we are aware, there are no corresponding complete sets of measurements in existence for damping behaviour.

In Part 1 of this study [1], we examined the simplest class of problems relevant to musical instrument vibrations, the case of flat, quarter-cut plates. We saw that for this case, four elastic (and four damping) constants govern the plate bending behaviour. These were expressed

<u>Table 1</u> Elastic constants of woods relevant to violin makers, from Hearmon [3]. Young's moduli and shear moduli are expressed in GPa, densities in kg/m³.

Wood	Density	E_L	E_R	E_T	ν_{RT}	ν_{RL}	ν_{TR}	ν_{TL}	ν_{LR}	ν_{LT}	G_{LT}	G_{LR}	G_{RT}
Spruce	370	9.9	0.73	0.41	0.57	0.031	0.29	0.013	0.44	0.56	0.61	0.50	0.022
Spruce	500	16.6	0.85	0.69	0.43	0.018	0.33	0.023	0.36	0.52	0.84	0.63	0.037
Spruce	390	10.7	0.71	0.43	0.51	0.030	0.31	0.025	0.38	0.51	0.62	0.50	0.023
Spruce	390	10.9	0.64	0.42	0.64	0.029	0.32	0.019	0.39	0.49	0.59	0.58	0.026
Spruce	430	13.5	0.89	0.48	0.56	0.030	0.30	0.019	0.45	0.54	0.50	0.72	0.032
Spruce	440	15.9	0.69	0.39	0.47	0.028	0.25	0.013	0.44	0.38	0.77	0.62	0.036
Sitka spruce	390	11.6	0.90	0.50	0.43	0.029	0.25	0.020	0.37	0.47	0.72	0.75	0.039
Maple	590	10.0	1.52	0.87	0.82	0.093	0.40	0.038	0.46	0.50	1.10	1.22	0.29

Note: "The nine constants", for the purpose of calculating Figs. 1 and 2, is taken to mean the Young's moduli E, the shear moduli G, and the larger of each of the three pairs of Poisson's ratios ν (see below the last equation in Appendix A). The smaller of each pair is more subject to experimental error [3].

in terms of Young's moduli etc. in the Appendix of Part 1. It is obviously of interest to see what values of these four elastic constants can be deduced from the figures given in Table 1. We can conveniently combine this question with another, namely that of the effect on these four elastic constants of departures from precise quarter-cutting. For a flat plate cut at a completely general orientation from an orthotropic solid, <u>six</u> independent elastic constants are needed (and the calculation is very messy). However, if the plate contains one of the principal directions of the solid, four constants still suffice, and we now examine that case. We consider plates which depart from exact quartering <u>either</u> by having tilted annual rings <u>or</u> by having tilted grain lines, but not both.

In Figures 1(a) and (b) we show the four elastic constants D_1-D_4 (see refs. [1,2]) plotted as a function of ring tilt angle (measured in the TR plane) and grain tilt angle (measured in the LT plane) respectively, using the nine elastic constants taken from row 3 of Table 1, which we take to be a typical set of "spruce" values. Because of the large spread of values of these constants, a logarithmic vertical scale is used in both cases. The values for zero angle are the same in each case, and correspond to the ideal quarter-cut plate, with D_1 in the L direction and D_3 in the R direction. Figures 2(a) and (b) show the same quantities calculated from Hearmon's figures for "maple". The formulae from which Figs. 1 and 2 were plotted are given in Appendix A.

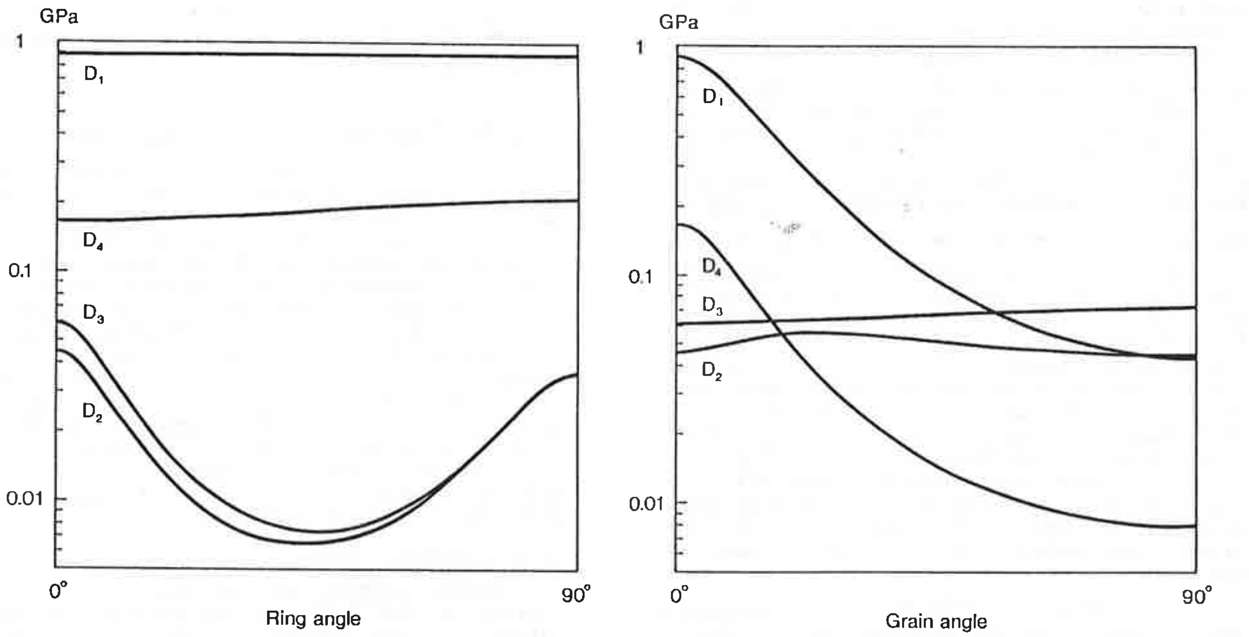

Figure 1. The four elastic constants D_1 to D_4 for a flat plate of spruce, calculated from the data in row 3 of Table 1. In each case the plate contains one of the symmetry axes of the material, and the constants are plotted as functions of the angle of rotation about that axis. In (a), the L axis (the 'grain') lies in the plate, while the angle (measured in the TR plane) of the annual rings varies linearly from zero (the ideal quarter-cut plate) to 90° (plate in the LT plane). In (b), the R axis lies in the plate, while the grain angle (measured in the LT plane) is varied. Zero angle again corresponds to the ideal quarter-cut plate, so that both graphs agree at their left-hand edges. The same logarithmic vertical scale is used in both cases.

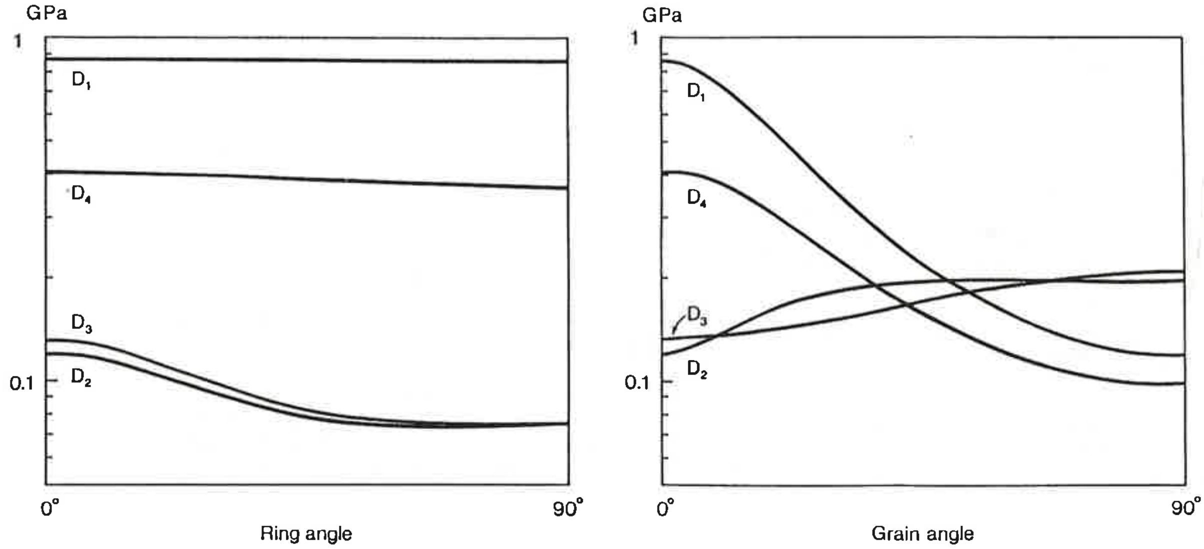

Figure 2. The four plate constants D_1 to D_4 are plotted (a) against ring angle and (b) against grain angle, using the "maple" data from Table 1. All details are as for Fig. 1, except that the range of values of the constants is smaller, so the vertical scale is different (but is the same for each of the two graphs here).

These graphs show the sensitivity of some of the plate constants to grain and ring angles. Most important are the very strong variations of D_2 and D_3 with ring angle for spruce - one reason why makers pay such careful attention to ring angle [6]. These results demonstrate that, even to understand flat plates properly, we need to know all nine constants of the solid (notice from the results of Appendix A that all nine do indeed enter, even in the simplest problems of pure ring tilt or pure grain tilt). Much of the wood sold for instrument making is not precisely quarter-cut, and we see from Fig. 1(a), for example, that D_3 can change by a factor of two or so with a ring angle of only 10°. The extent of such variations is governed by the other constants of the solid, which do not enter the calculation for the precisely quarter-cut case described earlier. The main culprit responsible for the most striking variation, in D_2 and D_3 for spruce, is the very low value of G_{RT} (making Δ large in formula (A2e) for ring angles in mid-range). For a rod cut in the T-R plane at about 45° to the rings, it is easy to visualise large shearing motions between the rings reducing the effective Young's modulus. (Schelleng [op. cit.] cites A. H. Benade as having made this point.) In practice, then, even to understand flat plates we need more complete measurements than are conventionally made. For violin plates, it is now more than plausible that all nine constants enter the problem, since arching gives different sections of the plates a wide range of orientations relative to the principal axes of the tree.

We should note here that Fig. 1(a) reveals a problem concerning the measurements reported by us in Part 2 for a spruce plate [2]. We obtained values of D_1, D_3 and D_4 which are plausibly close to those which we have just calculated from Hearmon's data for the case of zero tilt. However, in Part 2 we obtained a ratio D_2:D_3 of roughly two, whereas inspection of Fig. 1(a) shows at once that Hearmon's data imply a value just under unity. Furthermore, allowing for the possibility of slight ring tilt in our specimen does not help, since eqs. (A2b) and (A2c) show that this ratio never exceeds the greater of $2\nu_{LR}$ and $2\nu_{LT}$ for any ring angle. We shall see in section 3 that consideration of wood microstructure leads one to expect values no bigger than 0.5 or so for ν_{LR} or ν_{LT}, and the figures in Table 1 confirm this expectation. The explanation was found to be a flaw in the experimental technique used for one of the measurements reported in Part 2, and the results for the spruce plate have to be corrected somewhat. We give revised results in Appendix B: the only significant changes are in the results for D_2 and the associated damping constant η_2.

2: A brief survey of measurement methods

We now widen the discussion by examining the various methods of measuring the elastic and damping constants of wood which are to be found in the literature. These can be divided into three broad categories, based respectively on static deformations, low-frequency vibrations and ultrasonics. There are also some possible approaches which we shall call "indirect", which we leave until the next section. We shall cite examples of each of the three categories, and ask whether they are suitable for achieving our goal of measuring all 18 parameters. We shall find no easy answers to this problem, but at least we shall obtain a ranking order of more and less promising approaches, and suggest some definite lines of future research.

2(i): Static tests

Static methods are perhaps the first to spring to mind for measuring elastic constants. After all, the textbook definitions of Young's modulus, Poisson's ratio and shear modulus are given in terms of the deformation of the material in response to static stresses of various kinds. Thus static compression or shear of suitably-shaped specimens could be used to make direct measurements of the constants from their definitions. However, while this is true in principle, in practice it is not easy to carry out such measurements on wood with sufficient accuracy because the magnitudes of the moduli make the elastic strains very small.

This problem is exacerbated by the fact that in the case of wood, we need to work with rather small specimens, a constraint which affects all measurement methods. There are two reasons. First is the spatial inhomogeneity of the material (heartwood and sapwood, and the multifarious growth anomalies that wood is heir to). Second, and more serious, is the fact that the simple orthotropic theory we are using ignores the curvature of the annual rings. To cut samples for testing purposes in the T direction without significant problems arising from curvature requires very small dimensions (recall the extreme sensitivity of the constants to ring angle demonstrated in Fig. 1(a)).

On the other hand, specimens cannot be too small. We are using a continuum theory which ignores the ring structure of the wood. The conditions for this to be a reasonable approximation are not easy to specify succinctly, but the main requirement is that samples should not get so small that they have any radial dimension comparable to the ring spacing. Otherwise large fluctuations in results are to be expected, depending on precisely how much spring wood compared with summer wood is included. This constraint is again unavoidable, for any method, and together with the first one poses a serious dilemma regarding sample sizes. Some empirical work is needed, doing similar measurements on samples of different sizes to find out precisely what the constraints are in practice, for a given level of measurement accuracy.

A more useful form of static testing than simple compression or shear involves bending or torsion of rods or plates. For a given level of elastic strain, much larger measurable displacements are obtainable this way, with a corresponding improvement in accuracy. Such methods have been used extensively in the past. Hearmon [3] reviews several such approaches, and compares the results with those of other approaches. There are also more recent suggestions in this vein, for example Tsai [7].

For our purposes, we should note two serious difficulties with all static measurements which make it not worth our while to pursue the method in much detail. First, static tests cannot be used to give any of the damping constants. Even if energy loss in the static test were measured, there is no very good reason to expect this to relate closely to the vibration damping. The reason points up the second difficulty with static tests. It has been extensively reported in the literature [e.g. 3,4,5,8] that any static test involving shear strains relative to the principal axes of the wood is likely to exhibit creep. The deformation will continue to increase with time after the load is applied, and the specimen will not in general recover its original dimensions fully when the load is removed. This is the reason for the familiar permanent sagging or yielding of wood under conditions of sustained load. One result of this is that static measurements of shear moduli tend to give consistently lower values than do vibration tests, the precise value depending on the time taken to perform the measurement.

This creep behaviour involves energy loss, and thus makes a possible link with damping constants. For some models of viscoelastic materials, creep behaviour is indeed related to vibration damping [e.g. 9]. This might perhaps turn out to be the case for wood, but there is no obvious reason why, in a real material with complicated microstructure, the energy loss mechanism during creeping deformation should be the same as that operating during audio-frequency vibration. Of course, the question could be investigated empirically by making damping measurements both ways, and trying to relate them using viscoelastic material models.

In summary, static tests do not seem very promising for our purposes. However, our problem is sufficiently difficult that it is rash to disregard any possibilities. Static tests are easy to carry out, and at the very least they are probably worth doing to act as cross-checks on other methods. We have already seen in the case of our own measurement of D_2 reported in Part 2, and discussed above and in Appendix B, that any single measurement can be very fallible. Static tests might still turn out to have a valuable place in the armoury of cross-checks needed to reduce the error bars of other measurements, and to guard against outright mistakes!

2(ii): Low-frequency vibration tests

The second category of measurement methods is the broadest of the three. "Low-frequency vibration tests" means low audio frequencies, and includes the loaded cantilever and torsion-bar vibrations discussed by Hearmon [3], strip tests familiar to readers of these pages [e.g. 6], measurements on plates such as our own [2] and those of Caldersmith [10] and Molin et al. [11], and so on. In all of these cases, the procedure is essentially the same. One or more normal modes of vibration of the wood sample, perhaps with some added masses or constraints, are studied, to deduce the frequency, damping or Q factor, and perhaps the mode shape. To be useful, such measurements must be made on vibration modes which are well understood theoretically. . One can then relate the measured frequencies and Q-factors to the desired elastic and damping constants of the material.

Before we examine any particular techniques in detail, we should note that, at least in principle, low-frequency vibration tests do not suffer from the two problems noted below as being peculiar to static tests. First, low-frequency vibration tests represent the only approach likely to give damping constants reliably. Damping behaviour is in all probability significantly frequency-dependent, so one needs to measure at frequencies in the range of interest for the eventual application. One can at least imagine covering a useful fraction of the audio range by varying sample size and configuration (including, for example, added masses) and conducting resonance tests such as we are now considering. Second, the problem of creep is avoided by using vibration techniques: indeed, the problem was revealed by comparing static tests with vibration tests.

To determine the complete set of elastic and damping constants by studying low-frequency vibration modes, one obviously needs to measure at least nine different modes. For cross-checking one needs more. Ideally, one would like to think of sample geometries such that each constant is deduced directly from one measurement. In practice, however, this simplicity cannot be achieved for all the constants. In most cases, the frequency measured depends on a combination of the elastic constants (and similarly the Q-factor depends on a combination of the damping constants). One then has to solve an "inverse problem" to deduce the separate constants from the measurements. Such a process was illustrated for the four constants of a flat, quarter-cut plate in Part 2 [2]. We saw there that three of the four constants were relatively easy to determine, but that the fourth one (D_2) only ever had an influence in combination with the others, and its value (particularly the value of its imaginary part) was hard to determine accurately. For this reason, the results of a given inverse problem become more credible if some independent checks can be made by alternative methods. These check both the underlying theory and the experimental accuracy.

Certain commonly-used sample configurations avoid the difficulties of tricky inverse problems to a considerable extent. The Young's moduli can be measured directly (using strip bending tests or loaded cantilever tests, for example) because one-dimensional thin-beam bending depends only on Young's modulus as explained in Part 1 [1] and in standard texts. Similarly, the three shear moduli can be deduced almost independently from torsional vibration tests. A rod of some uniform cross-section is cut parallel to one of the principal axes of the material (subject of course to the sample-size constraints discussed above). One end is clamped in some suitable way, and a mass having a large moment of inertia is attached to the other end. The low frequency torsional oscillations of this system are then observed. The theory of such oscillations is quite well understood - it appears in many standard textbooks under the heading "Saint-Venant torsion" (see for example the classic work by Love [12]). The frequencies are governed by a combination of two of the shear moduli alone (in two planes, each perpendicular to the rod's cross-section). The precise combination depends on the cross-sectional shape of the rod. Further details necessary to use the method in practice will be omitted here: see the discussion by Hearmon [3].

The measurements reported by Hearmon and reproduced in Table 1 were mostly taken by a combination of static tests, cantilever bending and torsional vibration. Apparently the Poisson's ratios were measured statically. Perhaps this traditional combination of methods is still the most straightforward way to measure six of the nine elastic and damping constants. It is valuable at least as a check on any other methods which might be used. In the case of damping measurements, even this would represent a considerable advance on present knowledge. The best we have at present are the measurements by Haines of the imaginary parts of E_L and E_R, and his much less reliable determinations of G_{LT} and G_{RT} based on assuming Timoshenko beam theory to hold for high modes of strips [13]. (Even for elastic constants Timoshenko beam theory is a good approximation only for a rather narrow frequency range (see Cremer et al. [14], §II3(b)), while for the determination of damping constants it requires extreme care, as we have pointed out previously [15].)

However, for our full problem of determining all the constants, two difficulties are still evident. First, bending and torsion tests about principal axes still leave us with three of the nine constants to be measured. These are, for example, three of the Poisson's ratios. We have already noted that static methods are not desirable for the elastic constants, and are unlikely to be possible for the damping constants. Second, it would be better for experimental efficiency and economy of sample material if some way could be found to measure all the constants from a minimal number of separate samples, rather than having to cut a range of different samples for measurements of the different constants. As noted above, we also want in particular to avoid having to cut any shapes which are large in the T direction. For example, conventional strip-tests for E_T would be error-prone for this reason.

2(iii): Summary of requirements

It would perhaps be useful to pause here and summarise our "shopping list" of features which the elusive ideal measurement method should offer. It should involve the cutting of a few - say no more than two or three - specimens of wood. These will have all dimensions small compared with the radius of curvature of the annual rings, but large compared with the ring spacing; in other words of the order of a centimetre or two. The shapes should not present great technical difficulties in the cutting process (such as, for example, a requirement for exact spheres or hollow shells would pose). The specimens would then be used in such a way that they can be tuned to produce low vibration modes at a wide range of audio frequencies (see below), involving all nine of the independent elastic constants in such a way that, at each frequency separately, we can solve the inverse problem to give good accuracy on all the elastic and damping constants. These should, of course, be in agreement with results obtained by other reputable methods such as those discussed above. All of this makes a tall order, but it is as well to have the ideal clearly in mind when assessing practical possibilities.

None of the low-frequency resonance approaches known to us from the existing literature come close to satisfying the above shopping list. Space does not permit us to examine all methods in detail. Our own approach to obtaining some of the constants using flat plates has already been discussed at length [2]. Hearmon [3] mentions a variety of other methods, for example employing bending-strip and torsion-bar tests with samples cut at various angles to the principal axes, flat plates cut at 45° to the principal axes, and composite cantilever beams made by laminating thin strips cut at +45° and -45° to one principal axis. Some of these are ingenious, and as we have said before, all methods are of interest at least as cross-checks. The reader is referred to Hearmon's discussion for more detail. Note, however, that Hearmon hardly discusses damping constants.

Hearmon reports that these different methods do not always yield accurately consistent results for elastic constants. It is important to be aware of such disagreements. They may point merely to experimental inaccuracies, but two other possibilities may be important. First, methods involving the solving of complicated inverse problems may exhibit very high sensitivities of certain of the constants to the accuracy of the measurements. This can lead to large errors - see Appendix B for an example. Such sensitivity is reasonably straightforward to investigate theoretically, and a useful direction for research would be to carry out error analyses (sensitivity analyses) for the different measurement methods in use.

The second problem to bear in mind is that the orthotropic theory which we are using to interpret all measurements, and in particular to solve inverse problems, is only an approximation to the truth in a number of ways. Wood is not spatially homogeneous, rings are curved and have finite, irregular spacing, the angle between fibres and rings need not be accurately 90°, etc. This is one reason why we have stressed the need for cross-checks between methods throughout this article. Different methods, involving different measurement techniques and different inverse problems, provide a test of the underlying theory which is of fundamental interest in itself as well as helping to improve the accuracy of measurements of elastic and damping constants. Yet again, Hearmon has some relevant discussion: he reports some tests of the theory in the form of the variation of Young's modulus with direction relative to the principal axes [3], and for this case he finds that theory is quite well supported.

2(iv): An untried method

We now offer some speculations about a method which might possibly come close to satisfying our demanding shopping list above. These are untested ideas, and are intended more as a spur to further thought and experiment than a proven recipe for solving the problem. The hardest part of the problem as stated is perhaps the requirement for tunability of the frequencies of resonances over a significant part of the audio range without violating the stringent conditions on sample size. One solution to this might be make measurements off resonance, using forced vibration. A machine is apparently made by DuPont to do this for simple bar tests, but it is well beyond the means of the violin-making community! Resonance methods do not require high technology, and for such a method the only simple but practical way to achieve tunability seems to involve adding masses to the wooden specimen. The simplest configuration involves a cubic or rectangular block of wood with blocks of metal firmly attached to a pair of opposite faces - in other words a metal/wood/metal sandwich. Varying the masses of the metal blocks (which need to be effectively rigid in comparison with the wood for a tractable theory) could allow the lowest few resonances of the composite body to be tuned over a useful range.

These low resonances will fall into four distinct types - compressional modes, two orientations of shear modes, and twisting modes. Thus measurements on one configuration will yield information about several combinations of the elastic and damping constants of the solid wood. The hope would be that by using "meat" for the sandwich cut in more than one orientation relative to the principal axes of the tree, one might be able to deduce all the constants by solving the appropriate inverse problem. In terms of economy of specimens, cubes of wood have a lot to recommend them. Provided the metal blocks could be fixed in a way which can be undone without damaging the wood, a single cube could be used in all three orientations with the same set of metal weights. There is thus a faint possibility that one cube in principal axes might be sufficient to yield the full set of constants, elastic and damping. A stronger possibility is that two cubes, one principal and one skew, could do the job.

Some careful thought about both theory and experimental technique is needed before it becomes clear whether this hope is justified. The biggest problem, as in other methods, may come from the error sensitivity of the inverse problem. While it is plausible that Young's moduli and shear moduli might be determined this way, it remains to be seen whether the Poisson's ratios can be deduced reliably from such measurements. It might well turn out that significant improvements in accuracy could be gained by using rectangular blocks with particular aspect ratios, rather as in the case of flat plates in Part 2, where we found that by far the best accuracy for determining D_2 was obtained by shaping the plate to a particular aspect ratio to produce a ring mode and an X-mode [2]. It might also turn out that optical methods to measure accurately the amplitude and phase of displacements of the free sides of the samples could provide useful extra information. Experimentation with computer simulations is needed to assess the various possibilities, and indeed to evaluate the feasibility of the suggested method as a whole.

2(v): Ultrasonics

The use of ultrasonic testing to determine the elastic constants of wood is a relatively recent development. It is best known to readers of these pages through the work of Bucur [16], although some work has been done by other authors [e.g. 17]. The method is simple in concept, and shares some advantages with the speculative method outlined above. Small cubes of wood are cut at various orientations to the principal axes, and transmitting and receiving transducers are attached to a pair of opposite faces. A short pulse of high-frequency vibration is then transmitted through the sample, and its travel time measured.

By varying the type of transducers used, both compressional and shear wave speeds can be measured on the specimen. Provided the wavelength is short compared with the sample dimensions, the waves can be assumed to be plane. With that assumption, the measured wave speed can be used to deduce an appropriate stiffness of the material (since the density is known, and the square of the wave speed is just the ratio of stiffness to density). If enough orientations are measured to give the complete 6x6 stiffness matrix (see Bucur [16] or Hearmon [3,4]), this can be inverted to give the compliance matrix [op. cit.] from which the Young's moduli etc. may be deduced.

The method just described is attractive in many ways, and is the natural extension to wood of a method which works extremely well for homogeneous materials like metals. However, there are two likely drawbacks in applying it to wood. First, in common with static tests described earlier, it cannot be used to measure damping constants since it involves frequencies well outside the audio range. It would be surprising if the damping "constants" were in fact constant over this wide frequency range.

The second problem relates to the measurement of elastic constants, and is a new manifestation of the sample-size constraints mentioned earlier. It is most clearly illustrated for the case of compressional waves travelling in the R direction, perpendicular to the annual rings. We are using a continuum theory which ignores the ring structure of wood, but if one looks in more detail, of course the elastic behaviour and density of wood varies roughly periodically with radial distance, because of the rings. The continuum theory can be used safely only if the wavelength is long compared with the ring spacing.

Once the half-wavelength becomes comparable to the ring spacing, the wave-transmission properties of the wood will be quite different, in a way familiar to solid-state physicists who study the behaviour of electrons in crystals. The periodic structure of the wood will produce a series of "pass bands" and "stop bands". These are frequency bands within which waves can and cannot propagate respectively. (For an account of the solid-state applications, see for example the textbook of Ziman [18]. For applications to vibration problems, see for example Cremer, Heckl and Ungar [14,§V5].) A consequence of this is that the group velocity of the waves, which is what is actually being measured in the ultrasonic test, will vary with frequency, the variation being most rapid close to a stop band. Strong frequency variation would invalidate the assumptions made in solving the inverse problem described above. (For a review of theory and measurements of such effects in composite materials, see Bedford et al. [19].)

To be safe, the wavelength should be at least four times the ring spacing. Published data [13,16] give typical values of the sound speed in the R direction in spruce of 1400m/s. Thus the highest frequency one could use reliably is about 350kHz for fairly close rings of 1mm spacing, dropping to about 100kHz for ring spacing of 3mm common in spruce used for cello top plates. This is on the low side for conventional ultrasonic testing, where other problems (associated with longer pulse lengths) are beginning to appear which affect the measurement accuracy. Bucur does present some results measured at 100kHz [16], which perhaps represent the most promising line to follow.

In some circumstances (and it is not known at present whether these apply to wood), the method can be used, with caution, at higher frequencies. In the higher pass-bands, the fastest group velocity (around the pass-band centre frequency) may be quite close to the desired sound speed. Since what is actually measured in an ultrasonic test is presumably something like the fastest group velocity in the frequency band excited, the measurement might sometimes yield the right answer for excitation frequencies well above the limits given above. To know whether this is true to a useful level of accuracy for wood, some experimental work is needed, exciting each specimen radially with a variety of different frequencies to plot out a "dispersion relation" like Fig. 2 of Bedford et al. [19]. Such experiments would be of interest in themselves, but it should be borne in mind that even if certain higher frequencies do give acceptable answers for a particular specimen, one would expect these frequencies to vary significantly from sample to sample with variations in ring spacing and other physical properties of the wood. The microdensitometry measurements reviewed by Evertsen [20] might prove a useful source of data for a serious study of this problem.

3: Indirect methods, correlations and microstructure modelling

We have now surveyed briefly the range of methods available for direct measurement of the elastic and damping constants of wood. However, direct measurement is not the only way of gaining useful insight into the problem, and in this final section we look at what may prove to be a powerful alternative approach. The theory we have been using up to now deduces the existence of nine independent elastic constants from very general arguments about the mirror symmetry of the material in three mutually orthogonal planes [4]. However, these constants are "independent" only to the extent that we use no further knowledge of the structure of wood in building our theory.

If we understood the microscopic mechanics of wood better, we might find useful inter-relations between the nine constants. This could have two important benefits. First, it would reduce the number of degrees of freedom in the problem, so that fewer measurements would suffice. In view of the difficulty of finding reliable ways of measuring all the constants, this would obviously be very useful. However, it would require the inter-relations to be known rather accurately.

The second advantage does not rely so much on accuracy. Even approximate inter-relations backed by physical understanding can provide some valuable prejudices about the results of direct measurements. Since we have repeatedly stressed the difficulties of such measurements, any well-founded expectations we may have about the relative magnitudes of the various constants are likely to be valuable in helping to detect errors in experiments or their interpretation.

We have inadvertently provided an example of this process. When Part 2 was written [2], we had no clear idea of the values to be expected for the elusive D_2, so the mistake rectified here in Appendix B was not noticed. Only in the light of the new understanding resulting from discovery of the work we are about to describe did it become clear that the value of D_2 was implausibly large in comparison with D_3. Old experimental notebooks were checked, and sure enough a careless mistake in experimental technique was revealed.

One way in which inter-relations such as we are discussing might be revealed is through correlation analysis. If the various constants are not independent, then correlations between them must <u>ipso facto</u> exist. (But this does not necessarily mean that they will be revealed clearly by conventional regression analysis, which looks for <u>linear</u> relations between different quantities.) Correlation analyses have appeared in the literature over the years, including one recently in this Journal [21]. Statistically significant correlations between various pairs of variables have been found by these studies, and these provide a strong incentive to seek underlying physical explanations, since correlation without explanation is well known to be a dangerous thing to build upon (see for example the amusing little book "How to lie with statistics" by Huff [22]).

The microscopic structure of wood is described in many standard texts, for example Bodig and Jayne [5]. A useful short description has appeared in these pages, by Bucur [23]. For our present purposes it is fortunate that we are particularly interested in Norway spruce, since this has a rather simple structure (at least in a first approximation to what appears under the microscope). It consists of hollow, rather thin-walled cells which are very long compared with their cross-sectional dimensions. Most of these cells are aligned parallel to the axis of the tree - these are the grain fibres, and are known botanically as tracheids. A small proportion of the cells form medullary rays. For the present purpose ray cells are similar to the main fibres, except that the ray cells are aligned in the radial direction. No cells run in the transverse direction. The walls of all these cells have anisotropic mechanical properties, because they contain "microfibrils" whose orientations vary between cells of different types.

As long ago as 1928, this structure suggested a simple theoretical model to A. T. Price, who analysed it in a paper [8] which is highly significant for our purposes. He ignored the finite length of the cells, and considered them as indefinitely long tubes of constant cross-section. His model thus consisted of close-packed tubes aligned axially, interpenetrated by a small proportion of similar (but not necessarily mechanically identical) tubes running radially, to model the medullary rays. He analyses quantitatively the mechanics of the close-packed axial tubes, and comments qualitatively on the effect of the rays. This second stage of the calculation was later made more quantitative by Barkas [24]. Price devoted the rest of his paper to consideration of the effect of the radial modulation in cell size constituting the annual rings. We will not review all his work in detail, but we will sketch the clear physical understanding he gives of the relative magnitudes of the elastic constants. It seems that this simple model goes a long way toward explaining the measured values which we reproduced in Table 1 above, and thus appears to capture some of the essential properties of wood. In our account, we shall also draw on some much more recent work on similar lines [25,26], which fills out the picture given by Price and Barkas and provides further checks against measurements.

A qualitative explanation of the anisotropy of Young's modulus along and across the grain is the most immediate deduction from this model. For stretching along the grain, the individual tubes must be stretched. For stretching across the grain, on the other hand, the tube walls need only bend, approximately inextensionally, giving a much lower modulus. Price suggests that the smaller degree of anisotropy between the two cross-grain directions (T and R) is explained by the medullary rays. E_T is always smaller than E_R, usually by about a factor of two. The idea is that E_T represents just cell-wall bending as just described, whereas E_R involves extra stiffness from stretching of the small proportion of ray cells.

To make these ideas quantitative, Price analysed the simplest case of tube geometry. He imagined the tubes to have circular cross-sections, and used results from Love [12] for inextensional (bending) deformation of the cross-sectional shape. This model, combined with geometric information about cell configuration from microscopic examination, gave first estimates of the ratio $E_L:E_T$ which were rather larger than those observed [8, p9], but close enough to suggest that this theory was indeed modelling the dominant mechanism for L-T anisotropy. Effects of finite wall thickness and departures of geometry from that assumed could plausibly be believed to account for the deviation from observation.

The same theory of circular cylinder deformations also gives a value of the cross-grain Poisson's ratio ν_{RT} which is of interest. Price's prediction of this was 0.92, a larger number than is possible for an isotropic solid, but one which seems to be of the right order for ν_{RT} in softwoods [24]. In fact, Price's value of 0.92 seems to have been taken as gospel in much subsequent work, despite the fact that the precise value depends on the assumption of exactly circular cell geometry. Interesting light is shed on the range of possibilities for this Poisson's ratio by recent work of Gibson et al. [26]. They examined a family of two-dimensional cellular materials formed from various kinds of hexagons, as suggested by the sketches in Fig. 3. They give formulae for all four in-plane elastic constants for such materials, as functions of the angle θ shown in Fig. 3 and the cell wall dimensions. For the case of regular

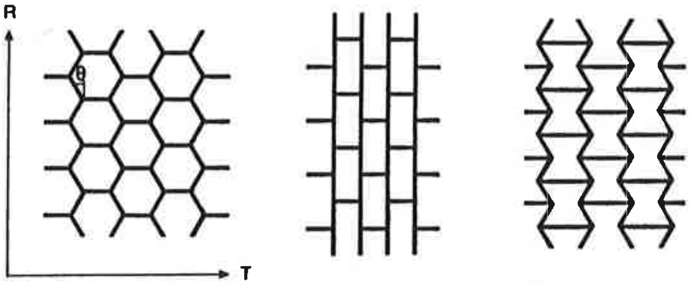

Figure 3. Three representatives of the family of two-dimensional honeycomb structures studied by Gibson et al. [26]. The heavy lines show the "cell walls", the cell interiors being empty space. Case (a) shows regular hexagons (θ=30°), (b) shows the case of "hexagons" with θ=0 so as to produce staggered rectangles, while (c) shows a negative angle θ=-30°. Directions R and T are indicated, showing the orientation which makes such idealised patterns correspond most closely to the cross-grain structure of spruce as revealed under the microscope. The in-plane Poisson's ratio ν_{RT} takes the values (a) 1; (b) ∞; and (c) -1 respectively.

hexagons sketched in Fig. 3(a), all the values are comparable to those calculated by Price. All the constants are θ-dependent, though, and the Poisson's ratio is particularly sensitive to θ. For the three cases sketched in Fig. 3, the values of ν_{RT} given by the formula of Gibson et al. [26] are respectively 1, ∞ and -1. The negative Poisson's ratio implies that a bar made of the third cellular material will curl proclastically rather than anticlastically, when bent - a prediction amusingly verified by the silicone rubber models used by Gibson et al. in their study. The value infinity should not be taken too seriously, except as an indication that values may cover a large range - Gibson et al's theory assumes inextensional cell walls, as did Price's, and within that theory this is a singular case.

Price's 0.92 is indeed seen to be similar to the value for regular hexagons, but the model examples suggest that ν_{RT} might well vary greatly between different species and specimens of tree, since small details of the cell geometry can have strong effects. Careful experiments on this, combining measurement of ν_{RT} with microscopic examination of the same wood specimens, have not yet been done to the best of our knowledge, and would surely be worthwhile.

Quantitative calculations of the effect of rays on R-T anisotropy of Young's modulus were made by Barkas [24]. By extending Price's cylindrical tube model to allow for the anisotropy of the cell-wall material and the difference in properties between grain fibres and ray cells, encouraging agreement with measurement was produced. The main purpose of Barkas' paper was in fact to investigate another problem, which is itself of interest, falling under our general heading of "indirect methods". He notes a strong correlation between the three principal Young's moduli and the shrinkage rates in the respective directions as the wood dried. He postulates a model of the effect of water involving a uniform hydrostatic pressure within the material to explain this correlation. If this could be believed, it might make another useful cross-check on vibration measurements of the Young's moduli.

The "tube model" of microstructure which we have outlined above can give estimates of the other elastic constants of the material, by similar arguments to those we have cited. These have apparently not all been studied quantitatively, and such a study might well make a very profitable research project, especially if realistic tube geometries were used (in view of the sensitivity noted above). As well as predicting typical values of the elastic constants, such a study might provide insights into the degree of scatter of experimental results due to sample inhomogeneity, and perhaps into other issues such as humidity sensitivity [3,24]. An extension of the work to investigate the damping constants might also be possible.

The final deduction from the tube model which is perhaps worth noting explicitly here concerns ν_{LR}, and is the one which led us to discover our own experimental error noted above. The model suggests a very simple answer for the value of this constant - it should be more or less the same as the Poisson's ratio for the <u>solid</u> material of which the cell walls are composed, since only a simple axial stretching of the tube walls is involved. Thus the value close to 1 suggested by our earlier measurements seemed rather implausible in the light of the above, whereas the value of about 0.47 given in Appendix B, following a more careful re-measurement, is more in line with this physical picture, as well as with the results reproduced in Table 1 above.

As a final note on microstructure modelling, we draw attention to the interesting paper on balsa wood by Easterling et al. [25]. They use arguments similar to those of Price, and combine them with more up-to-date direct measurements of cell-wall elastic properties [27] to produce (among other things) predictions of the three principal Young's moduli as functions of density of the wood. Using their own measurements on balsa specimens having a wide range of densities together with published measurements for other woods, they plot an interesting graph showing that the predicted correlations and values are encouragingly well supported by measurements ([25], Fig. 10). There is surely much more to be learnt about wood properties by developing this line of thinking.

Appendix A: Flat plate elastic constants for tilted rings or grain

In this Appendix we outline the method of calculating the elastic constants for a flat plate cut from an orthotropic solid. We give detailed formulae for the case considered in the text, where the plate is cut in a plane containing one of the symmetry axes of the material, so that only four constants D_1 to D_4 are needed to describe the plate behaviour. Our account is based on Hearmon's, which is given most clearly for our purposes in ref. [3], p.26, but ref. [4], §1.5 covers the same material in more general terms.

The starting point for analysis is the 6×6 "compliance matrix" S_{ij}, which transforms the six stress components into the six strain components according to Hearmon's eq. (7) [3]. The essential approximation of thin plate theory is that the deformed plate is in a state of "plane stress", in which the three stress components acting in the plane are much larger than the other three, cross-plane, stresses [e.g. 12]. It is thus a justifiable approximation to delete from the compliance matrix the rows and columns applying to the three cross-plane stress components, leaving a (symmetric) 3×3 matrix. The inverse of this 3×3 matrix gives the plate stiffnesses, which enter into the expression for the potential energy of bending which we discussed in Part 1 [1].

For a flat plate cut at a completely general angle to the principal axes, the 3×3 matrix will have no zeros in it, and we will thus need six independent elastic constants to describe the plate vibrations. However, when the plate contains one of the principal axes of the material, only four independent constants enter the matrix, in accordance with our earlier discussion and with the in-plane reflection symmetries of such a plate. Specifically, the matrix inversion to be carried out relates to our D's according to

$$\begin{pmatrix} S_{33} & S_{31} & 0 \\ S_{13} & S_{11} & 0 \\ 0 & 0 & S_{55} \end{pmatrix}^{-1} = \begin{pmatrix} 12D_1 & 6D_2 & 0 \\ 6D_2 & 12D_3 & 0 \\ 0 & 0 & 3D_4 \end{pmatrix} \qquad (A1)$$

where we have adhered to Hearmon's conventions appropriate to the case of ring tilt only, with the identifications R=1, T=2, L=3. For that case, Hearmon gives the values of S_{11}, $S_{13}(=S_{31})$, S_{33} and S_{55} in terms of the constants shown in Table 1 and the tilt angle θ, in his eq. (10) [3].* Rotation is about the 3-axis (i.e. the L-axis), with $\theta=0$ when the plate is quarter-cut, i.e. lying in the 1,3 or RL plane. Performing the matrix inversion of eq. (A1) then yields the formulae used to plot Figs. 1(a) and 2(a):

$$D_1 = \frac{1}{12\Delta}\left\{\frac{n^4}{E_T} + \left(\frac{1}{G_{RT}} - \frac{2\nu_{RT}}{E_R}\right)n^2m^2 + \frac{m^4}{E_R}\right\} \qquad (A2a)$$

$$D_2 = \frac{1}{6\Delta E_L}\left\{\nu_{LR}m^2 + \nu_{LT}n^2\right\} \qquad (A2b)$$

$$D_3 = \frac{1}{12\Delta E_L} \qquad (A2c)$$

$$D_4 = \frac{G_{LR}G_{LT}}{3[m^2G_{LT} + n^2G_{LR}]} \qquad (A2d)$$

where $n = \sin\theta$, $m = \cos\theta$, and

$$\Delta = (1-\nu_{LT}\nu_{TL})n^4/(E_TE_L) + (1-\nu_{RL}\nu_{LR})m^4/(E_RE_L) + \left\{\left(\frac{1}{G_{RT}} - \frac{2\nu_{RT}}{E_R}\right)\frac{1}{E_L} - \frac{2\nu_{RL}\nu_{LT}}{E_RE_L}\right\}n^2m^2 . \qquad (A2e)$$

(Recall that $\nu_{RT}/E_R = \nu_{TR}/E_T$, etc., as noted in Part 1 Appendix A.) Notice that setting $m = 1$, $n = 0$ recovers the formulae given in the Appendix of Part 1 [1], for the case of a plate cut in one of the principal planes of the material. A simple permutation of indices in these formulae (swapping R and L, and reversing the roles of D_1 and D_3) leads to the equivalent formulae for the case of grain tilt, as plotted in Figs. 1(b) and 2(b).

Appendix B: Correction to previous results

In this Appendix we give revised values of the measured constants for the spruce plate reported on in Part 2 [2], after correcting the experimental error noted earlier. The problem

*See also ref. [4], section 1.5.

lay in measuring the ratio of frequencies of ring mode to X-mode in the rectangular plate tuned to give these two modes. The old value was given as 1.19, whereas careful re-measurement gives 1.084. The reason for the error is of interest in itself. The plate in question was very light (see Part 2). The old measurements of these two frequencies were made with a very small attached accelerometer (Bruel and Kjaer type 4374). In an effort to minimise the perturbing effect of the accelerometer mass, it had been placed close to node lines of the two respective modes, but evidently not close enough - differential effects on the two modes were the source of the erroneous frequency ratio. Even the most careful measurements by this method, with such a light plate, do not seem able to give accurate values, and so the new values were taken with a frequency counter and a Chladni-pattern shaker table. Even there, it proved vital to use the very tiniest pieces of foam under the plate, and to adjust their position under the nodal lines very carefully, before consistent results were obtained.

Using this new ratio, Fig. 2 of Part 2 [2] yields a ratio D_2/D_3 of 0.95, and thus a Poisson's ratio ν_{RT} of about 0.47. Following through the procedure described in Part 2, we finally obtain corrected values for the elastic constants as follows:

D_1 (MPa) 1320 (was 1340)
D_2 (MPa) 78 (was 168)
D_3 (MPa) 82 (was 85.5)
D_4 (MPa) 227 (unchanged).

The extent of fitting of computed to measured frequencies is if anything slightly improved.

The only significant change to the damping constants is that η_2, which was not well-determined before, can no longer be determined at all - the errors are much greater than the predicted value, since all the values of J_2 for the different modes are reduced, and the particular inverse problem for this particular damping constant becomes completely ill-conditioned.

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