# New results on the bowed string

M.E.McIntyre, R.T.Schumacher and J.Woodhouse

## 1: Introduction

When a string player presses sufficiently hard with his bow, the musical note gives way either to the familiar raucous 'beginner's noise' or, if the bow arm is steady enough, to more or less stable oscillations at pitches much lower than the fundamental string pitch. The reason why some such catastrophic breakdown must occur was made clear by Schelleng in his penetrating paper on the bowed string[1]. As Schelleng realised, maximum bow force in practice is generally less than the force causing breakdown, and it is signalled by one of several less drastic phenomena.

Of these phenomena, two are particularly important. The first is a very slight deviation of pitch - almost always on the 'flat' side of the string tuning - as bow force increases. This flattening is easily demonstrated with the bow a moderate distance from the bridge at a low bow speed[2]. When flattening is audible, pitch is sensitive to bow force so that control of intonation becomes difficult. Players avoid this regime. The second phenomenon is the gradual buildup of noise accompanying the musical note, which is noticeable when trying to play more and more loudly near the bridge. This noisy regime is frequently used to deliberate musical effect, but the noise can reach an unacceptable level, depending on context. This imposes the other major limitation on bow force in practice.

The physical causes of neither phenomenon have been elucidated as far as we know, although mention of flattening in the scientific literature goes back at least as far as Raman[3](p135). In section 2 we present a theoretical model of the bowed string which predicts the flattening effect. The model also describes within a rational and self-consistent framework some of the effects of bow force upon waveform detail, vindicating ideas which were first put forward by Cremer and Lazarus[4,5] and further developed by Schelleng (op. cit.). In section 3 we report progress toward understanding the second phenomenon (the buildup of noise), and argue that its major cause is a mechanism depending on the finite width of the ribbon of bow-hair in contact with the string.

## 2: The flattening effect

When slight flattening occurs, observations show that the motion of the string remains qualitatively similar to the normal Helmholtz motion, with a single, somewhat rounded, 'corner' travelling back and forth along the string. Observations also show the expected drop in frequency, equivalent to a delay in the round-trip time of the corner. The delay can happen only at the bow, so we must study in detail the processes of capture and release of the string.

Previous studies of these processes yielded important insights into the changes in waveform as bow force is varied, but did not predict the flattening behaviour we have described. Cremer[5] followed the course of the Helmholtz corner, examining the detailed changes in its shape occurring at different points in the cycle and seeing how these various changes can reach equilibrium to give a precisely periodic motion. Schelleng[1] showed how Cremer's 'secondary waves' produce the pattern of 'ripples' observed in velocity waveforms. We extend their discussion in two stages. In the first instance we follow Cremer and neglect the secondary waves. We later give examples of computer solutions which take them fully into account, and in the process simulate Schelleng's ripples. We begin with a re-examination of exactly what happens at the bow; this will reveal a new phenomenon which appears to account for the flattening effect.

Suppose an ideal Helmholtz corner or velocity discontinuity, as shown in figure 1a, leaves the bow travelling towards the bridge. By the time it returns to the bow it will have been inverted and somewhat rounded, as shown by the solid curve in figure 1b. For the purposes of clarity in the discussion it is best to begin by assuming a form of rounding which is symmetric in time (i.e. reversible), and which is the same for the two sections of the string (although with different delays, depending on the distances to bridge and nut). A model string with these properties will be called 'quasi-symmetric'; it has harmonic overtones, but damping which increases with frequency.

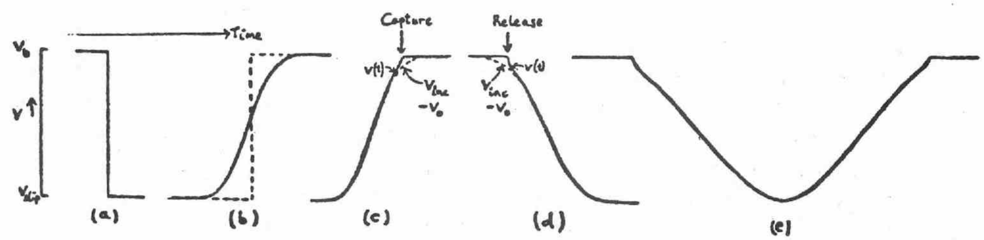



Figure 1. Successive waveforms for the Helmholtz corner, with dashed curves echoing the preceding stage, in b-d (after Cremer and Lazarus[4]). (In c and d, $v_o$ is a constant 'DC offset' arising from the fact that F(t) has a positive mean value.)

We now ask what happens to the rounded corner as it passes the bow. In the absence of the bow, the velocity v(t) at this point would simply be equal to the incident velocity wave $v_{inc}(t)$ which we have plotted in figure 1b. The frictional force F(t) exerted by the bow produces an additional contribution to v(t), so that

$$v(t) = v_{inc}(t) + F(t)/2Z \qquad (1)$$

where Z is the relevant wave impedance[6], a constant property of the string. F and v are also related by the friction law: in the usual idealisation, $F/F_b$, where $F_b$ is the normal component of bow force, has a functional dependence upon v of the kind sketched in figure 2[7]. From eq. (1) and figure 2 we can find F and v from the known $v_{inc}$ at any given instant: they may be read off the friction curve as its intersection with the straight line of slope $2Z/F_b$ shown in figure 2. Cremer and Lazarus[4,5] discussed the case in which the bow force is sufficiently small that the slope of the friction curve is everywhere less than $2Z/F_b$, so that there is a simple one-to-one correspondence between the point of intersection and $v_{inc}$. This requires $F_b<F_{crit}$, where

$$F_{crit} = \frac{2Z}{\text{max. slope of graph in fig. 2}} \qquad (2)$$

Figure 1c shows $v_{inc}(t)$ (dotted) and the corresponding v(t) (solid) when $F_b$ is well below $F_{crit}$. The wave transmitted past the bow towards the nut has the same shape v(t), and returns from the nut after being inverted and rounded again as in the dotted curve of figure 1d. This $v_{inc}(t)$ now produces the v(t) shown in the solid curve of figure 1d, and another round trip begins. The final curve (1e) shows the shape of the periodic solution which is closely approached after this chain of events has been repeated a few times. The period of the motion is precisely equal to the fundamental string period[8].

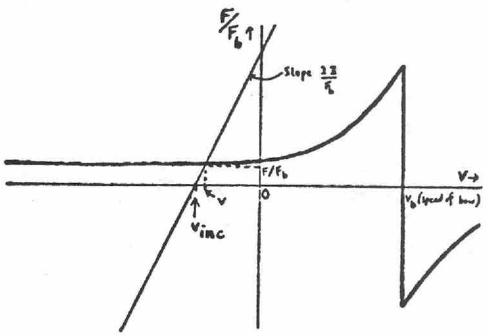



Figure 2

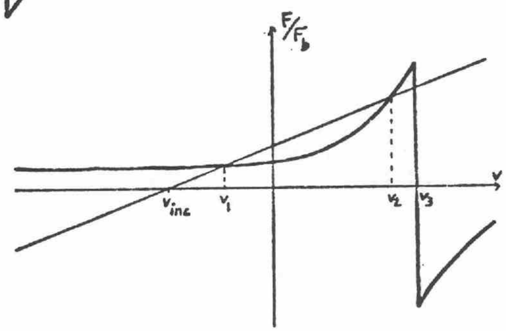



Figure 3

Now our proposed explanation of flattening hinges on a physically correct resolution of the ambiguity which arises when $F_b>F_{crit}$. The ambiguity, which was first pointed out by Friedlander[9], is illustrated in figure 3. There are now three intersections ($v_1$, $v_2$ and $v_3$) of the straight line and the friction curve, and we must decide which is appropriate at any given instant. It may be shown that the answer involves a kind of <u>hysteresis</u>, an essential difference between capture and release. The string chooses one of the two outer intersections ($v_1$ or $v_3$), according to the following rule:

> Slipping ($v=v_1$ in figure 3) will persist until $v_{inc}$ reaches the value $v_c$ shown in figure 4a; then capture occurs (v jumps to $v_3$). Sticking ($v=v_3$) will persist until $v_{inc}$ reaches the value $v_r$ shown in figure 4b; then release occurs (v jumps to $v_1$).[10]

The parts of the friction curve shown dotted in figure 4 are traversed instantaneously in any model where (1) is regarded as exact. For a real string, these sections are presumably traversed in a finite but very small time.

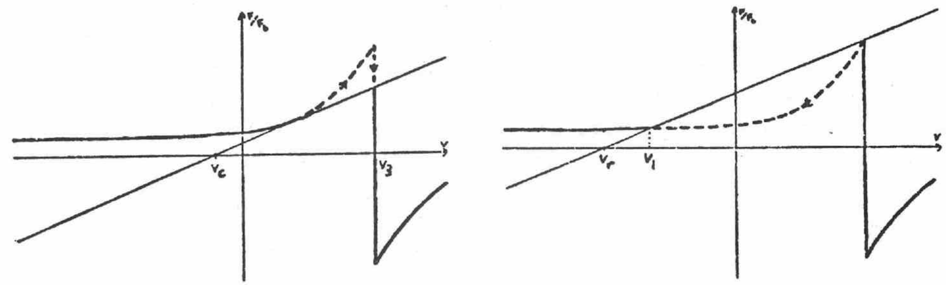



Figure 4 (a) Capture (b) Release

In figure 5, we plot the series of waveforms equivalent to those of figure 1 but with $F_b>F_{crit}$. Notice now that each time a rounded corner passes the bow, it has a 'bite' taken from it as the velocity jumps the relevant gap in figure 4, creating a discontinuity in v(t).

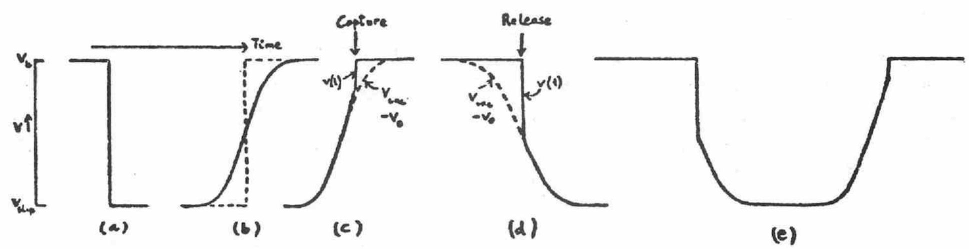



Figure 5. Same as figure 1, but with $F_b>F_{crit}$

Hysteresis means that the bite removed on release is bigger than that on capture. This produces a delay in the round-trip time of the Helmholtz corner, giving the flattening effect. The extent of flattening depends on the amount of hysteresis as well as on the amount of corner-rounding, so that the model predicts behaviour qualitatively similar to that commonly observed. For example, we now understand why notes high on a violin G string are particularly prone to flattening: corner-rounding is most drastic there.

We now verify that the same effects occur in fully consistent computer simulations for the same quasi-symmetric bowed string. Two methods have been used, one using an integral-equation formulation[11] to find periodic solutions, the other simulating transient motion directly and waiting for the solution to settle down to periodicity. The methods have been shown to produce identical solutions given the same conditions. Examples of the resulting v(t) waveforms are shown in figures 6a, b and c, the first being for $F_b=F_{crit}$ and the others for $F_b>F_{crit}$. In Figure 6c the value of $F_b$ is very close to that for breakdown to a raucous regime : the bite taken on release occupies the full height of the pulse[12]. That at capture is far smaller, and the note has flattened by about 50 cents (1/2 of a semitone).

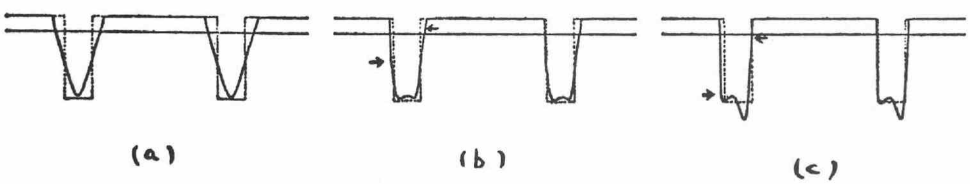



**Figure 6.** Velocity waveforms at the bow, from computer solutions for three values of $F_b$. The ideal Helmholtz pulse is shown dotted in each case, and the 'bites' are indicated by arrows.

Details of the pulse shapes are different from those in figures 1e and 5e, especially for large $F_b$, because the secondary waves are now accounted for. Their effects are more obvious in figure 7a, in which torsional motion (and string anharmonicity) is included. Here we plot the velocity of the centre of the string, corresponding to what is usually observed. The ripples toward the end of the sticking portion mark reflection from the bow of secondary waves generated on release, reverberating between nut and bow exactly as described by Schelleng[1]. Those reverberating between bridge and bow (generated at capture and manifest as the ripples near the beginning of sticking) make an important contribution to the change in tone colour as the player varies the position of the bow.

Figures 7b and c give two examples of the many other kinds of motion which have been numerically simulated and which correspond to what can be observed. 7b is an example of a corner-rounded modification to one of the 'higher types' of motion discussed comprehensively by

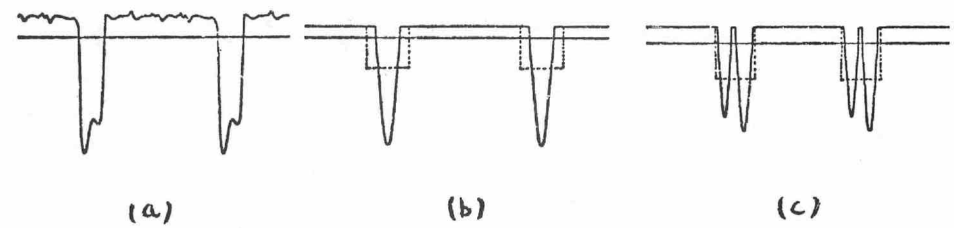



**Figure 7.** Computer simulations of bowed-string motions, (a) allowing for torsional waves, (b) showing a Raman 'higher type', and (c) a 'double-flyback' motion. The ideal Helmholtz pulse is shown dotted in (b) and (c).

Raman[3]. It has a pulse time about half the corresponding Helmholtz pulse time, and involves several 'corners' propagating on the string; these higher types are easily elicited, and are often used for colouristic effects in sul tasto playing. 7c shows a 'double-flyback' motion involving a short sticking period during what would normally be the Helmholtz slip period. It may be set up by pressing too hard during the starting transient, so that stray disturbances in $v_{inc}$ bring it close to $v_c$ (see figure 4a) during slipping.

## 3: The buildup of noise : 'spikes'

Noise presumably means some lack of periodicity in the bowed-string motion. The most obvious candidate is the previously reported cycle-by-cycle variation in period as measured by the timing of the Helmholtz corner at the bridge[5]; however, our measurements have shown that such 'jitter' does not increase significantly in loud playing near the bridge when the noise we are studying becomes audible[13]. Thus we must look for other kinds of aperiodicity. In figure 8 we show transverse force exerted by the string on the bridge during a noisy note. The most obviously aperiodic features in the picture are the sharp 'spikes' superimposed on the Helmholtz sawtooth. Observations have shown that these spikes are present whenever the noise is audible.

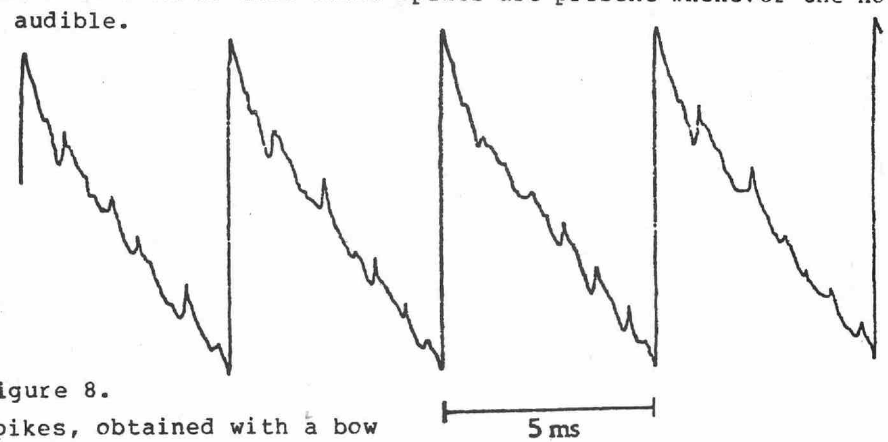
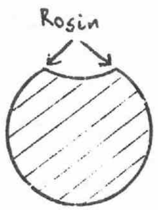



**Figure 8.**
Spikes, obtained with a bow

Our main clue to the source of spikes came from a pair of decisive experiments. First, we bowed the string with a smooth, round, rosined stick in place of a bow. No spikes could be obtained. Note that such a stick has only a very small length in contact with the string. Second, we modified the stick by cutting a groove along its length to give it the cross-section shown in figure 9. The two parallel edges of this groove were smoothed and rosined, and this 'double stick' used to bow the string, with both edges in contact. This produced spikes similar to those obtained with the bow: a typical example is shown in figure 10. The two experiments differ only in that the double stick spans a much greater length of the string than the original, single stick. For a bow, we infer that the finite width of hair in contact with the string is an essential ingredient for the production of spikes.



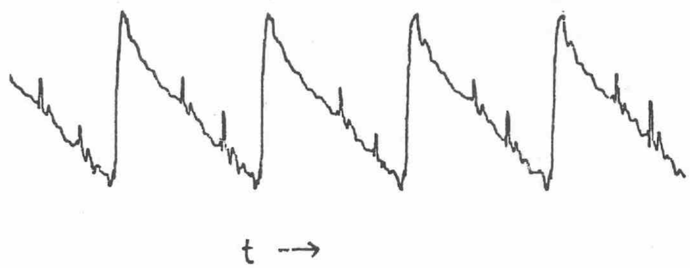




Figure 9
The double stick.

Figure 10
Spikes obtained with the double stick.

Now a simple calculation shows that within the tolerance range of normal bowing, one or other part of the short section of string in contact with the bow hair (or the double stick) must slip momentarily during the nominal 'sticking' part of the Helmholtz cycle. (This much was stated in 1918 by Raman[3], p115.) Moreover, such 'differential-slipping' events must occur many times if the bow force is well below Schelleng's maximum. As bow force is increased differential slipping need occur less often, but will then tend to be more violent since the short section of string will have been forced further out of line beforehand. We believe that spikes are the direct result of such violent differential-slipping events.

To check this idea in detail, a self-consistent theoretical model is required. An obvious candidate would be an extension of the corner-rounding model developed in the previous section to have two bow 'hairs' in place of one. We have not yet implemented such a model, but we have made a preliminary study of the differential-slipping effect using a two bow-hair version of the simple Raman model[3,9], which considers an ideal string with pure-resistive terminations. This model has no ripple-generating mechanism, and we can expect ripples to be important in triggering differential slipping; but the model is enough to confirm that differential slipping does take place, and to show that with equal forces on the two hairs, it is mainly the hair nearer the bridge which slips. An example is shown in figure 11A. The top curve shows transverse bridge force, to compare with figures 8 and 10. The other two curves show the velocities at the two bow hairs. Very crudely we may say that, while the outer hair is sticking, the short section of string between it and the bridge is being played in a very 'raucous' regime by the inner hair (bow force well in excess of Schelleng's maximum for the short section) - although this raucous regime is one which is continually being disturbed by other events. Figure 11b shows the same, with a larger $F_b$.

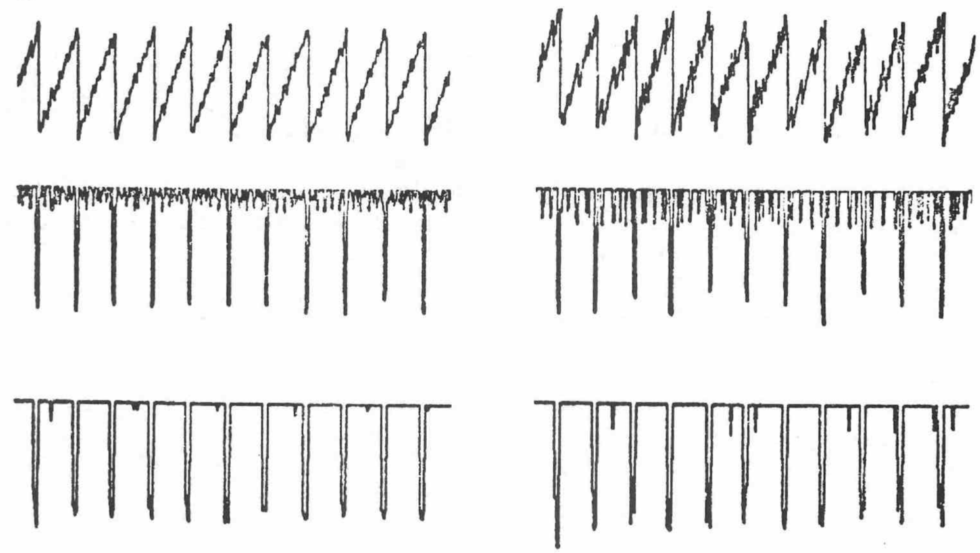


Figure 11: Computer solution for the Raman model with two bow-hairs: (a) $F_b=0.12F_{max}$, (b) $F_b=0.4F_{max}$, where $F_{max}$ is twice the force given by Schelleng's equation (2); see his footnote 10.

Many observations have not yet been explained in detail. An example is the fact that spikes seem to occur more easily on thick strings than thin ones. More realistic models should throw light on this, as soon as more experimental data on the damping and reflection of torsional waves on the string becomes available.

References

(1) Schelleng, J.C. The bowed string and the player. J.Acoust. Soc. Am. 53, 26-41, (1973).
(2) There is then no significant amplitude-dependent sharpening, due simply to increased mean string tension, to confuse the issue.
(3) Raman, C.V. On the mechanical theory of vibrations of bowed strings, etc. Indian Assoc. Cult. Sci. Bull. 15, 1-158, (1918).
(4) Cremer, L. and Lazarus, H. 6th ICA Congress, Tokyo, N.2-3 (1968).

(cont)

(5) Cremer, L. Der Einfluss des 'Bogendrucks' auf die selbsterregten Schwingungen der gestrichenen Saite. Acustica 30, 119-136, (1974). English translation of an earlier version in Catgut Acoustical Society Newsletters 18 and 19, pp. 13-19 and 21-25.

(6) For an ideal string Z is the usual wave impedance, defined as (string tension)/(wave speed). However, relation (1) holds even if torsional string motion is allowed, provided we redefine Z to have a smaller, but still constant, value[1], and take $v(t)$ to be the velocity of the string surface.

(7) see Cremer, L. Die Geige aus der Sicht des Physikers. Nachr. Akad. Wiss. Göttingen : II Math. Physik. Kl. 12, 223-259, (1971)

(8) In Cremer's detailed analysis[5] he obtained a slight sharpening, attributable entirely to different terminations at the two ends of the string. We are using a quasi-symmetric string model to separate hysteresis-induced flattening from such additional effects, although real strings are of course not quasi-symmetric.

(9) Friedlander, F.G. On the oscillations of the bowed string. Proc. Camb. Phil. Soc. 49, 3, 516, (1953).

(10) The simplest way of theoretically verifying this rule is to introduce artificially a point mass m at the bowed point on the string, solve for $v(t)$ given $v_{inc}(t)$ (so that $v(t)$ is now continuous and uniquely defined)[9], and then let m tend to zero. The motion then tends to that stated, and the time spent in the dotted parts of the friction curve in figure 4 tends to zero. The basic fact underlying this behaviour is the stability of the two outer solutions $v_1$ and $v_3$ in figure 3, which contrasts with the 'instability local to the contact' noticed by Schelleng[1] and occurring for any value v for which the slope of the friction curve exceeds $2Z/F_b$, such as $v_2$.

(11) An account of the integral-equation formalism for studying musical oscillators may be found in Schumacher, R.T. Self-sustained oscillations of organ flue pipes: an integral equation approach. In press with Acustica, (1977).

(12) cf. equation (2) and footnote 10 of Schelleng[1].

(13) Indeed it appears that jitter measured in this way is almost always below the audibility threshhold as measured in simple psychoacoustical experiments: see the review article by McIntyre, M.E. and Woodhouse, J., The acoustics of stringed musical instruments. In press with Interdisciplinary Science Reviews, (1977).

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## BOOK REVIEW

### Quieting: A Practical Guide to Noise Control

By Raymond D. Berendt, Edith L. R. Corliss and Morris S. Ojalvo

U.S.Government Printing Office, Washington, D.C. 20402
S.D.Cat. No. C13, 11:119    163 pp, Price $ 5.10

That reduction of noise is an old problem is proved by our authors' quoting from a Babylonian epic:

In those days the world teemed; the people multiplied, the world bellowed like a wild bull, and the great god was aroused by the clamour. Enlil heard the clamour and he said to the gods in council, "The uproar of mankind is intolerable and sleep is no longer possible by reason of the babel." So the gods agreed to exterminate mankind.

Fortunately there seem to be techniques today for dealing with the problem less drastic than that envisioned by the gods. At any rate, such is the hope of this new manual from the Bureau of Standards, the abstract of which follows.

This guide offers practical solutions for ordinary noise problems that a person is likely to meet. The discussion describes the ways in which sounds are generated, travel to the listener, and affect his hearing and well-being. Recommendations are given for controlling noise at the source and along its path of travel, and for protecting the listener. The guide instructs the reader by way of "Warning Signs" on how to determine whether he is being subjected in his environment to prolonged noise exposures that may prove hazardous to hearing. Remedies are given for noise problems that a person is likely to find in his home, at work and at school, while traveling, and in the growth and development of his community. The remedies include noise prevention techniques and selection of quiet alternatives to existing noise sources. General principles for selecting quiet appliances are given. Ways of searching for the sources of noise and for determining the paths over which they travel to the listener are described. A detailed index is given for individual noise sources describing specific solutions to the problems they present. General ways of looking for the inherently quiet homes and travel accomodations are described. In a final chapter, there are suggestions for enlisting community help where large external sources of noise must be quieted, such as those arising from public utilities and public transportation.

So condensed a statement however cannot do justice to the strategy of the book, which is to begin simply, to explain the "why" accurately, and to amplify the "how" not only in principle but in terms of "nuts and bolts". All told the book must contain hundreds of detailed drawings understandably captioned. Each item in the abstract undergoes elaboration.

The subject is obviously of general interest. Workers in musical acoustics have a special need for the knowledge that the book gives simply and authoritatively.

John C. Schelleng