

Probing the Playability of Violins by Supercomputer

By R.T. Schumacher and J. Woodhouse

This month, for the refined members of our readership, we have an article that sets in science the issue of what makes for a good violin. It reveals just how complicated a complete and objective analysis of a violin can be. What is confirmed, via supercomputer calculations, is that the interaction of the bow and string makes a major contribution to the quality of the sound produced. Still unanswered is the question of what makes a Stradivarius the ultimate violin.—GA

The violins made by Antonio Stradivari and a few other luthiers from the 16th, 17th, and 18th centuries are surrounded with an air of

mystery, and they command staggeringly high prices. It may seem surprising that this should continue to be the case, given the advances in both theoretical and experimental techniques for studying sound and vibration. But researchers who make a serious effort to apply these techniques find time and again that they are pushing against the limits of what can be achieved.

There are two related reasons for this situation. First, the assessment of a violin as "good" is based on subjective impressions of players and listeners, and before any useful physics can be done, it is necessary to try to pin down physical correlates of those impressions. Second, the violin, in common with any other successful musical instrument, has evolved to take the best possible advantage of human abilities: It allows motor actions, up to the limit of what we can achieve, to be turned into a range of sounds that we can process most acutely. The result is that the all-important nuances distinguishing a great violin from a moderate one may stem from rather small and subtle physical differences.

The most reliable judgments of violins, with respect to physical correlates of "quality," are obtained not from listeners but from players. A good player may be able to make compensations that mask the inadequacies of a poor instrument to a considerable extent, so that a listener is hardly aware of them. But the player, being inside the feedback loop of those compensations, will be quite well aware that the instrument is a poor one, precisely because it calls for such compensation. This suggests that physical correlates be sought for differences of "playability" between instruments and between notes on the same instrument. Such differences certainly exist, and they point to the possibility that the "stick-slip" oscillations produced by bowing a string are somehow influenced in their details by the acoustical behavior of the wooden box, which is the violin body, to which the string is attached.

This is a matter suitable for investigation by the physicist. Theoretical and experimental study of the bowed string has a long history,

and the physical basis for much of the observed behavior is believed to be fairly well understood. A bowed string is a self-sustained oscillator, in which a complicated linear system (the string, with attached violin body, which is in turn weakly coupled to the acoustics of the auditorium) is driven by the friction force from the bow. The dependence of this friction force on the string motion under the bow is strongly nonlinear. Because this description is generically similar to that of various other nonlinear systems that have been much studied in the last 15 years or so, complicated behavior involving the possibility of many periodic and nonperiodic

the violin body might influence this capability, we must resort to simulation. Certain knowledge, especially about the regimes of self-sustained oscillation that are possible under given conditions, can be gained by analytic calculation. Because of the strongly nonlinear character of the system, however, it is very hard to make progress on the questions of starting transients and of the choice among the possible regimes from a given bowing transient.

An efficient simulation scheme, based on the simplest physical model of a bowed string that seems to allow the main observed effects, has existed for some time [2, 3]. This scheme has been used to explore various questions of bowed-string behavior and has also penetrated the commercial world, where musical synthesizers based on this technique have been reported to be under development for replacing or augmenting the standard FM-synthesis paradigm that has long dominated synthesizers and computer music. (No commercial products had been produced when this article was written, but the expected products were implicit as well as explicit at the International Conference on Physical Modeling, Grenoble, France, September 1990.)

Early implementations of the simulation scheme took the form of interactive programs, in which the playing parameters could be varied during a run so that the program could be "played" roughly like the real string. This yielded many valuable insights into the bowing process and the strengths and weaknesses of the particular model used. The parameter space explored in this way is so large, however, that it is extremely difficult to discern any structure in the overall behavior by watching individual interactive runs of the program. A more organized use of simulation is needed.

The aim of the project described here is to use simulations to map out some part of the player's parameter space and then represent the results in diagrammatic form so that any interesting structure can be readily discerned. Study of a two-dimensional subspace is suggested as it is hard to convey results in more than two dimensions. The choice of a suitable

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Applications on Advanced Architecture Computers

Greg Astfalk, Editor

("chaotic") regimes might be anticipated. This expectation is in qualitative agreement with the wide range of unmusical noises that can be elicited from a violin, especially in the hands of a novice.

Out of that multiplicity, the violinist is almost always trying to achieve one particular regime, which was first described by Helmholtz in the 19th century and is thus known as Helmholtz motion. It is a periodic, or at least approximately periodic, regime in which the string sticks to the bow for most of the time, slipping rapidly backward relative to the bow motion just once per vibration period. Many issues of playability therefore depend on how readily Helmholtz motion can be initiated and maintained, by means of various bowing techniques that the player wants to be free to use for musical reasons. Perhaps an instrument considered "easy to play" or to "speak easily" is one that readily yields a Helmholtz motion, with an acceptably short starting transient, under a wide range of bowing conditions.

To investigate how the vibration behavior of

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subspace for a preliminary investigation requires some care, since the computational cost is quite high and there are many possibilities to consider.

Previous discussions of regimes of bowed-string motion focused mainly on the influence of the "bow force," the normal force with which the bow is pressed against the string. Players know very well that this force must lie within certain limits for a Helmholtz motion to be obtained: Too little force produces a "surface sound," which involves more than one slip per cycle of oscillation, while too much force can produce a raucous "crunch," in which the motion is not periodic at all. It therefore seems natural that the steady bow force at the end of the transient should be one variable to consider.

A second variable is then wanted, one that can be used to specify a range of different starting transients, all of which share the same eventual bow force. It was decided to use a second force-like variable. Starting transients can be simulated in which the initial bow force is different from the asymptotic value, with the offset decaying exponentially with time. In this way a range of somewhat plausible bowing transients can be simulated.

If the force starts from zero and increases to the final level, the result is a simple representation of a string-crossing transient in which the bow alights on the string and the force takes a finite time to build up. If the initial force is the same as the final value, a switch-on transient is produced. This probably does not represent anything done in normal playing, but it was a favorite condition for previous simulations and is useful for comparison. Finally, if the force starts high and decreases to the final value, we obtain at least a crude representation of a martelé transient. (Martelé bowing, a common way to give an emphatic start to a note, involves "digging in" to the string with the bow at the initial instant.) This transient will be less accurate than the string-crossing transient, because the bow speed will also vary significantly during a martelé. Despite this reservation, the two-dimensional space of transients in which the initial and final forces are varied (keeping the exponential time scale and the rest of the model constant) seems to be a promising candidate for a first study.

The calculation to be done is now analogous to the computation of the famous Mandelbrot set: For each point in the parameter plane, a

shorter convolutions with the "reflection functions" of the two sections of string on either side of the bow, the impulse responses $h_1(t)$ and $h_2(t)$, which would apply if one or the other section of string were replaced by a semi-infinite string. It is readily shown that:

$$g(t) = \frac{Y_0}{2} [h_1(t) + h_2(t) + 2h_1 * h_2 \\ + h_1 * h_2 * h_1 + h_2 * h_1 * h_2 + \dots], \quad (2)$$

where $*$ denotes the operation of convolution.

The second relation between $v(t)$ and $f(t)$ is a nonlinear function, $f = F(v)$, giving the velocity dependence of the frictional force between bow-hair and string. The adequacy of such a function to characterize the physics of rosin friction is by no means obvious, and exploration of a more complete characterization is in the forefront of modern tribological (frictional) research. Here we simply assume a function $F(v)$ with a plausible and mathematically tractable form loosely based on experiment, which is known to give fairly realistic results when applied to the bowed-string problem [2, 3]. The function is plotted as the heavy curve in Figure 1. If the convolution integral from (1) is denoted by $v_h(t)$, it is plain that the values of $f(t)$ and $v(t)$ are found at the intersection of this curve with a straight line with slope $2/Y_0$ and intercept $v_h(t)$, as shown.

The two parts of the $F(v)$ curve require different calculations to solve for the intersection point, depending on whether the string is slipping or sticking at the given time step in the calculation. Because the CM2 is a SIMD (single instruction, multiple data) machine, every processor is executing the same instruction at a given time. Because each processor has a different set of bowing force parameters, as described earlier, it is clear that not all processors are slipping or sticking at the same time. In fact, because of the possibility of multiple intersections of $F(v)$ and the straight line, an ambiguity whose correct resolution is described in [3], there are four separate branches in the computation. On a serial machine, they are handled straightforwardly with Fortran IF THEN statements. On a CM2, they are dealt with by means of the WHERE statement. Each processor executes every instruction, whether or not it is appropriate to the particular state of the processor at that time, and the processors for which the instruction is inappropriate for their current states throw away the results of

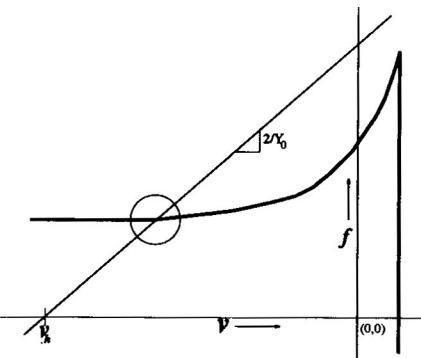


Figure 1. Friction force as a function of string velocity at the bowed point (*heavy curve*). The vertical portion represents sticking, which occurs when the string velocity equals that of the bow. The curved portion represents sliding. The sloping line and ringed intersection illustrate the solution procedure for the governing equations at a given moment t .

Helmholtz oscillation are that the motion be periodic and that the number of stick-slip transitions and the number of kinks per period be equal to one. These tests must be fine-tuned to work adequately, which is not easy as the main object of the simulations is to cover a wide range of parameter values, to ensure that the Helmholtz motion itself varies quite widely: Both the waveform and frequency vary with bow force within a given model. There is no doubt that further effort will be needed in this area if a more sophisticated "expert system" is to be developed for regime classification.

From the tests just described, we construct a 128×128 pixel plane in which white space is Helmholtz motion and black space is not, for one reason or another. By saving the equivalent, color-mapped planes for autocorrelation, number of slips, and number of kinks, we can also make informed guesses about the nature of the oscillations in the non-Helmholtz regions. Some of the interesting ones can then be examined directly by reading out those portions of the simulation from selected processors in a separate run.

Another useful technique is to examine the output at periodic intervals, looking at the last five nominal periods at, say, 30-period inter-

vals. After 100 period-lengths (Figure 2c), the diagram shows a quite large area within which Helmholtz motion has been produced. The three regions of different non-Helmholtz behavior that can be distinguished in this final picture correspond well, qualitatively at least, to what happens in real playing. At low bow forces, the Helmholtz motion gives way to other periodic regimes with more than one slip per period, known to players as "surface sound." At very high bow forces, the figure shows a rather fuzzy vertical stripe. Here, the destabilizing effect of "negative resistance" [4] at the bow during slipping prevents periodic motion of any kind being established.

In between these regimes is a large area of white space traversed by streaks of black. These streaks indicate a quite different periodic oscillation regime, which we have christened "multiple-flyback motion." That motion has more than one slip per period, but instead of being spread through the whole period, as in the surface sound obtained at low bow force, they appear in a tight cluster. This regime has been observed on real violin strings and is recognized by players as an undesirable sound, but it has not been the subject of any detailed investigation. The diagrams shown in Figure 2 are the first definite indication of any coherent structure in the parameter-dependence of this regime, and they provide encouragement for continuing investigations along the lines described here.

It is very easy to think of further models and cases about which much might be learned by applying this technique, and we intend to explore at least some of them in the near future. Examples of model enhancements include:

- incorporation of flexural stiffness in the string model,
- more correct allowance for torsional motion of the string (a major energy-loss mechanism in the bowed string),
- use of measured or predicted data on the reflection behavior of a player's finger, and
- inclusion of one or more resonances of the violin body.

This last possibility is of the most obvious interest: Knowledge of the influence of instru-



set: For each point in the parameter plane, a nonlinear process is simulated with the coordinates of the point used as input data. The simulation is continued long enough to indicate the eventual outcome (in our case, whether a periodic Helmholtz motion is or is not produced); that point can be colored in some way to represent this outcome, and the calculation then moves on to the next point. When a reasonable area has been covered in this way, a picture will have been built up of the region of the parameter subspace in which Helmholtz motion actually occurs from a starting transient.

This computation lends itself very easily to implementation on a parallel computer. Values of the asymptotic force and the initial force may be assigned to different processors, which can simulate the string motion independently of one another. This suits the architecture of the Connection Machine-2 (CM2) perfectly; in fact, it is the simplest possible application for such a machine. (The CM2 used for the work described here is operated by the Pittsburgh Supercomputer Center.) We are interested, typically, in the performance of 16,384 processors running simulations that do not interact with one another.

The basic algorithm is described in [2, 3]. Two quantities are involved, the string velocity at the bowed point, $v(t)$, and the force applied to the string by the bow, $f(t)$. These quantities are connected in two different ways; the combination gives the governing equation for the process.

First, when the string and attached violin body are considered as a linear system:

$$v(t) = \frac{Y_0}{2} f(t) + \int_{-\infty}^t g(t-\tau) f(\tau) d\tau, \quad (1)$$

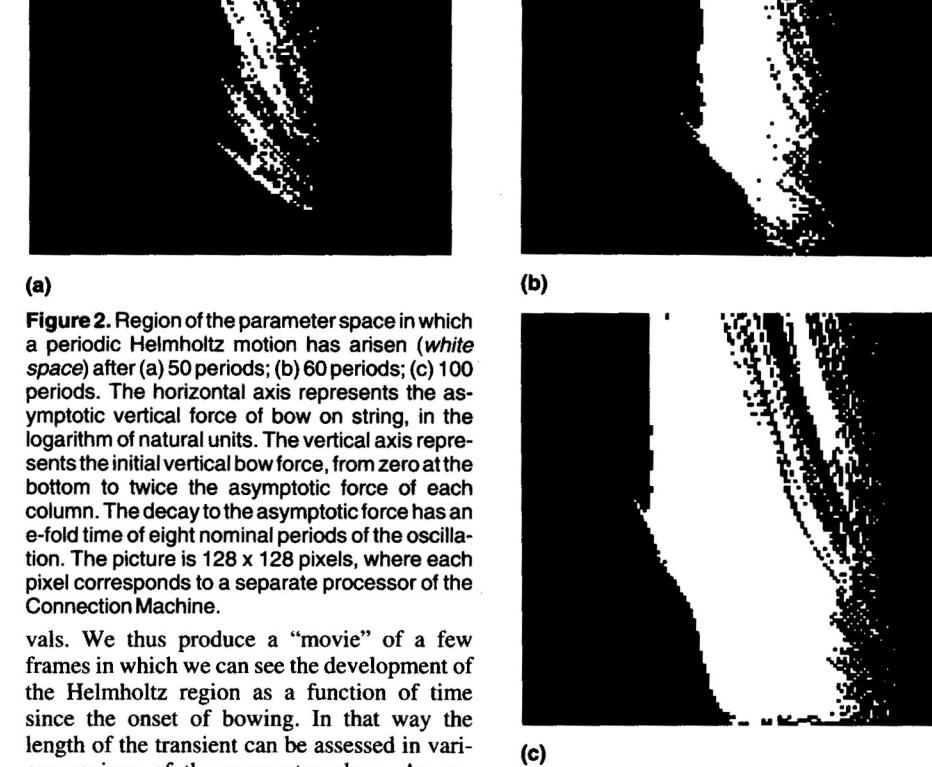
where Y_0 is the characteristic admittance of the string. The first term on the right-hand side of this equation represents the instantaneous response to the force (as would be found on an infinite string); the second represents the combined effect of all reflected velocity waves arriving back at the bowed point at time t . The latter is expressed as a convolution integral involving the impulse response function of the system, $g(t)$. For computational efficiency, this integral is best evaluated via two much

their current states throw away the results of that step of the calculation.

Thus, the price paid for the parallelism is a certain inefficiency; each branch of the code written for a serial machine is executed on every processor. The advantage of the large number of processors far outweighs this inefficiency, however. A single period of oscillation takes about 0.4 second to execute. A comparison of speed with that of a serial machine running in "real time" (1/440 second for a period of oscillation, at a sampling rate corresponding to commercial CD recording, 44,100 samples per second) can be made by dividing the per period execution time by the number of processors. When we use 16K processors (half of the processors of the Pittsburgh Supercomputer Center's CM2), the arithmetic shows a speedup of approximately 100 over the same mapping of the two-dimensional subspace of parameter space by a serial machine running in real time. That factor spells the difference between an investigation that is doable and one that would never be attempted.

Perhaps the most interesting technical part of the problem lies in the interpretation and classification of the output. The player is interested in how quickly an acceptable oscillation is established. An acceptable oscillation is that originally described by Helmholtz, with a single slip and recapture to the sticking state per period of oscillation. A great many other forms of periodic oscillation are possible, as is the nonperiodic noise that sometimes seems to be the most common result of a beginner's efforts. From this "zoo" of oscillations, periodic and nonperiodic, an experienced listener has no trouble distinguishing the Helmholtz motion from other regimes. Similarly, when watching a single simulation run on the screen, the human eye has no trouble classifying regimes into "Helmholtz" versus others. But it is not so easy to find a robust algorithm that allows each processor to recognize the characteristic Helmholtz pattern, so as to tell the experimenter which processors are oscillating properly.

For the first trials, we developed a detection criterion based on several simple tests. After a given number of iterations, we examine five nominal periods of the oscillation for periodicity by computing the autocorrelation function. We also keep track of the number of stick-slip transitions during that time, and of the number of "kinks," or velocity discontinuities, traveling on the string. The criteria for a successful



(a)

Figure 2. Region of the parameter space in which a periodic Helmholtz motion has arisen (white space) after (a) 50 periods; (b) 60 periods; (c) 100 periods. The horizontal axis represents the asymptotic vertical force of bow on string, in the logarithm of natural units. The vertical axis represents the initial vertical bow force, from zero at the bottom to twice the asymptotic force of each column. The decay to the asymptotic force has an e-fold time of eight nominal periods of the oscillation. The picture is 128 x 128 pixels, where each pixel corresponds to a separate processor of the Connection Machine.

vals. We thus produce a "movie" of a few frames in which we can see the development of the Helmholtz region as a function of time since the onset of bowing. In that way the length of the transient can be assessed in various regions of the parameter plane. An example of the output is shown in Figure 2.

In each diagram in Figure 2 the horizontal axis shows the asymptotic bowing force, on a logarithmic scale. The vertical axis shows the initial force as a multiple of the asymptotic force, on a linear scale. The force begins as the specified multiple of the asymptotic force and decays to the asymptotic force with an e-fold time of about eight nominal periods.

A very simple model, developed by Cremer [1], is used for the rest of the system. It allows for the frequency-dependent energy dissipation of a string on a real violin, but it does not attempt to model the reactive, resonant nature of the response of the violin body. By separating these two effects, this model makes a very useful benchmark case against which the changes produced by more sophisticated models may be judged.

For the example illustrated here, there is no "white space" in the diagrams until about 35 period-lengths have passed. After 50 period-lengths, as shown in Figure 2a, some small areas of the parameter plane have settled into periodic Helmholtz motion. By 60 period-lengths (Figure 2b), the white region has broad-

ment-body behavior on playability would immediately open the way to an investigation of how playability is influenced by specific constructional details (such as choice of wood, arching profile, and thickness distribution for the front and back plates of the violin). This would relate to physical measurements that can be made on old instruments and also to the concerns of present-day instrument makers.

References

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