only about 25 cello bridges, however, does not warrant a detailed account such as is given above which is based on the tuning of several hundred violin and viola bridges.

In this whole bridge tuning process it is well to remember that there is considerable force from the strings (about 20 lbs.) bearing down on this small piece of wood. Thus any cutting away of wood must take into account whether or not a bridge can support such a load.

For some years we have tried to obtain definitive measurements of the subtle changes in tone and playing qualities as a bridge is thus tuned to a particular violin or viola. The response curves and the input admittance curve (made using a single frequency (sinusoidal) input sweeping through the frequency range and the total response of the instrument and bridge recorded via a microphone pickup or an accelerometer on the bridge) show certain changes in the frequency spectrum particularly around 3000 Hz. The results of these tests are not as yet definitive or constant enough to provide reliable quantitative information on what is happening as a bridge is tuned to a given instrument. Also the results vary depending on the resonance spectrum of the instrument box itself. Just as in free plate tuning [6] we find that the process when properly applied works well over and over again, but so far we cannot satisfactorily identify the controlling mechanisms. Hopefully future measurement techniques will provide further insights.

In addition to bridge tuning, there is, of course, the art of bridge cutting which is not even considered here. One famous violin maker told me that it usually takes a new apprentice at least a year to be able to see how a proper bridge should be cut, much less be able to cut one! The more I experiment to try to understand some of the mechanisms which control certain tonal characteristics in violins the more respect I have for the art and skill of the master violin maker. Nowhere are these qualities more important than in the proper cutting of bridges.

REFERENCES:

1. Minnaert, M. and C.C.Vlam, "Vibrations of the Violin Bridge," Physica, 4(5), 361-372, (1937).
2. Bladier, B., "On the Bridge of the Violoncello," Compt. Rend., 250, 2161-2163, (March 1960). (English translation by R. Bruce Lindsay. See Benchmark Papers in Acoustics, Musical Acoustics, Part I,Violin Family Components,Ed.C.M.Hutchins pp 296-298. Dowden Hutchinson & Ross. 1975
3. Reinicke,W.,"Ubertragungseigenschafter des Streichinstrumentensteges," CAS NL #19, 26-34, (1973). (English translation by E. Wall available on request from CAS office).
4. Müller, H. A.,"The Function of the Violin Bridge," CAS NL # 31, 19-22, (May 1979). (English translation by E. Wall). (Originally published in German in Das Musikinstrumente, 1977).
5. Hacklinger, M., "Violin Adjustment - Strings and Bridges," CAS NL # 31, 17-19, (May 1979).
6. Hutchins, C.M., "Plate Tuning for the Violin Maker," CAS NL # 39, 25-32, (May 1983).

A PARAMETRIC STUDY OF THE BOWED STRING: THE VIOLINIST'S MENAGERIE

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1: Introduction

There are many ways to make an unsatisfactory sound on a violin. Some of these require extreme actions of one kind or another from the player, but there are a few undesirable regimes of oscillation which even the best players slip into from time to time when playing near the limits of normal performance. Indeed, those limits are determined by transitions to such undesirable regimes. We explore this "menagerie" of bowed string oscillations of direct interest to musicians, using simple theory complemented by observations and computer simulations of bowed string motion, in an attempt to understand under what conditions the usual regime for steady playing is accessible.

This usual regime is of course the Helmholtz motion [1,2,3], in which at any given instant the string lies in two more or less straight pieces separated by a sharp corner ('the Helmholtz corner'). This corner shuttles around the visible envelope of the string motion at the wave speed of the string, triggering the onset of slipping and sticking alternately as it passes the bow. Thus there is one period of sticking and one of slipping in each cycle.

A major reason for this study of the violinist's menagerie is that it might shed light on the harder problem of how the conditions under which the Helmholtz motion is possible vary among different violins. While the layman commonly supposes that violins are chosen solely on the basis of their sound qualities, the player may be at least as concerned about differences in "feel" which make one instrument more "docile" than another. Among the many things implied by such verbal expressions is surely a difference between instruments in the range of bowing parameters for which normal steady playing is possible. In any case, the tolerance problem for steady playing is the simplest problem for scientific study and forms a necessary first step in a more complete study.

We build upon the well-known work on this problem by Raman [4] and Schelleng [5]. We take as our starting point the last-named's diagrammatic representation of bowing tolerance for the Helmholtz regime. During steady bowing the player controls three parameters: bow speed v_b, normal force f_b and position of the bow on the string, which latter we describe by the parameter β denoting the distance of the bowed point from the bridge as a fraction of total string length. Schelleng held v_b constant, and plotted a first approximation (relying on smallness of β) to the region of the f_b-β plane in which the Helmnoltz motion could exist. He then indicated in the diagram some of the musical effects encountered in different regions. We shall show a version of Schelleng's diagram later, but with different labelling.

Schelleng considered two other types of motion to which the Helmholtz motion could give way. Following Raman, he calculated a minimum bow force based on transition to motion with two slip periods per cycle (the "double-slip" motion), and in addition calculated a maximum bow force where the Helmholtz corner is no longer strong enough to initiate slipping when it passes the bow. Above this maximum force, motion may be aperiodic ("raucous" motion), or it may be more or less periodic with a period substantially greater than the string's natural period.

A competent player will not stray into, or even close to, the raucous regime. His maximum usable bow force is usually determined by the need to avoid one of two other undesirable deviations from the Helmholtz motion with the natural string period - one or other of them occurs well before the Schelleng maximum is reached. When β is not too small (i.e. playing with the bow not too near the bridge), the limit is determined by the string playing unacceptably flat, as a result of an effect of hysteresis in the transitions between slipping and sticking discussed by us previously [3,6].

When playing nearer the bridge, a different effect sets the limit on bow force. As a result of the finite width of the ribbon of bow hair in contact with the string, some of the hairs slip, typically several times, during the nominal sticking period of the Helmholtz motion. These slips tend not to be accurately periodic, and give rise to a component of audible noise accompanying the note being played. Depending on the musical context, this eventually reaches an unacceptable level, so determining the maximum bow force in such cases [7,8].

A quite different member of our menagerie is encountered when playing well away from the bridge (sul tasto), at a point on the string close to a simple fractional subdivision of the length. The midpoint is the most extreme case, but is unusual in practice. The one-third, one-quarter and one-fifth points are more commonly encountered, and progressively less troublesome. What usually happens near one of these points is the onset of a totally different oscillation regime, described collectively by Lawergren [9] as "S-motion". The S-motion regimes form an interesting and important subset of the "higher types" classified by Raman in his monumental work on the bowed string. An audible characteristic of S-motion is the very strong presence in the note of the Nth harmonic, when playing close to the β=1/N point. S-motion is sometimes used, consciously or unconsciously, for colouristic effects in sul tasto playing.

It should be noted that S-motion does not occur exactly at the 1/N points: at those points, a different set of higher types is obtained which correspond simply to removing from the Helmholtz motion every Nth Fourier component. These motions, known as "Helmholtz's crumples", were observed by Helmholtz himself, but are not very important in practice since they require extremely accurate placement of the bow at the 1/N point (and a light bow force). They have a much lower maximum bow force than the adjacent S-motion.

The final character we need to include in the menagerie is another of Raman's higher types, which we have christened the "double flyback" motion. So far as we are aware, specific attention has not been drawn to this motion in the past, except briefly in Ref. [6]. The motion contains two slip periods per cycle, in close succession. This distinguishes it from what we have called the double slip motion above, where the two slips are roughly equally spaced. It is an entirely different oscillation regime from the double slip motion. In Raman's classification (by the number of "corners" propagating on the string), it is of the third type rather than the second type. The reason for the name will become clear below when an example of the motion will be given.

Raman's catalogue of higher types contains many other possible motions of a bowed string, a good number of which have been observed under special laboratory conditions using long, very thin strings on a monochord [10]. However, from the point of view of the player, and thus for our present purpose, the list given above seems substantially complete. Only under rather rare circumstances do competent players who are trying to produce the Helmholtz motion slip into any regime we have not mentioned. The only candidate known to us for addition to the menagerie is the E-string "whistle" to which certain instruments are prone, especially when doing a legato string-crossing onto the E-string. Unfortunately, we have never had access to a sufficiently repeatable example to pin down what motion is involved. We would be most interested to obtain access to an instrument which suffers seriously from this problem, to fill this gap in current knowledge. This empirical observation that our menagerie is now substantially complete is the result both of watching oscillograms of string motion during much playing, and of extensive experiments with the computer simulations which we have described previously [3,7,11].

2: The menagerie

The various desirable and undesirable regimes of oscillation described above are illustrated in Figure 1. These all show waveforms of transverse force exerted by the string on the bridge, observed by means of a piezoelectric transducer in the string notch. They were all obtained on the same open violin G string, bowed by hand with a conventional bow, and the time scale is the same in each case. The bridge force waveform for the Helmholtz motion is approximately a sawtooth. Since this is a real string, the rapid flyback (as the Helmholtz corner reflects from the bridge) is rather rounded. This is shown as Fig. 1(a). The other inmates of the menagerie appear in the other figures, with the exception of flattening, which even when clearly audible is virtually indistinguishable from the Helmholtz motion in such a small picture. Figure 1(e) shows a "double-flyback motion". The bridge-force waveform exhibits a pair of closely-spaced flybacks.

Schelleng's original study of tolerance considered the motions shown in Figs. 1(b) and 1(f). We are suggesting that a more refined version of his study should allow for the motions shown in Figs. 1(b)-(e) plus the flattening effect. Figure 1(f) is rarely relevant in practice; by the time this regime is reached, other undesirable things have happened. This middle road between allowing for Schelleng's two motions and Raman's vast collection seems not to have been pursued before. It provides a framework for a more complete study of the practical limits on bowing parameters for steady playing. Such a study is still under way, but it seemed of some interest to report the general findings here.

Figure 2 shows roughly where most of the oscillation regimes discussed above fall in Schelleng's diagram. The two slanting lines represent Schelleng's maximum and minimum bow forces for the Helmholtz motion. On the simplest theories, these vary like β^{-1} and β^{-2} respectively, giving rise to the two different slopes in the log-log graph. The vertical position of the lower line is sensitive to the strength of coupling of the string to the violin body, giving the simplest example of how the tolerance range for the Helmholtz motion can vary with frequency and among different instruments. The reader is referred to Schelleng [5] for the basic analysis, based on an approximation valid for small β.

The regions of spikes and flattening fall inside Schelleng's region, and vertical stripes surrounding the points β=1/N for low values of N indicate where S-motion is encountered (remember near, but not at 1/N). Schelleng's lines are defined by transition to double-slip motion and raucous motion respectively, so the double-slip and raucous regimes appear outside his tolerance region.

Double-flyback motion is not indicated at all on Fig. 2, and to explain why we need to examine another aspect of the behaviour of the different oscillation regimes as a function of bow force. Suppose we fix β at some typical, moderately small, value. Figure 3 then indicates the relationship between Helmholtz motion, double-slip motion and double-flyback motion. The first vertical bar indicates the Schelleng tolerance range for the Helmholtz motion at this β. Alongside is the corresponding range for the double-slip motion, which can be calculated readily by the same small-β approximation used by Schelleng (although this figure is purely a sketch to show qualitative behaviour, and does not claim to give quantitative force limits for any particular theory). If we start with a Helmholtz motion and slowly reduce the bow force, a transition to double-slip motion occurs roughly at the Raman-Schelleng minimum force as indicated by the arrow labelled "decreasing f_b". However, if we now increase the force again we do not immediately revert to Helmholtz motion. The bars overlap, and we have a hysteresis of regimes. The transition back to Helmholtz motion occurs at a far higher force, indicated by the arrow labelled "increasing f_b". This hysteresis of oscillation regimes will be familiar to players, and is also exhibited clearly by the computer simulations which we have described previously [3,7,11].

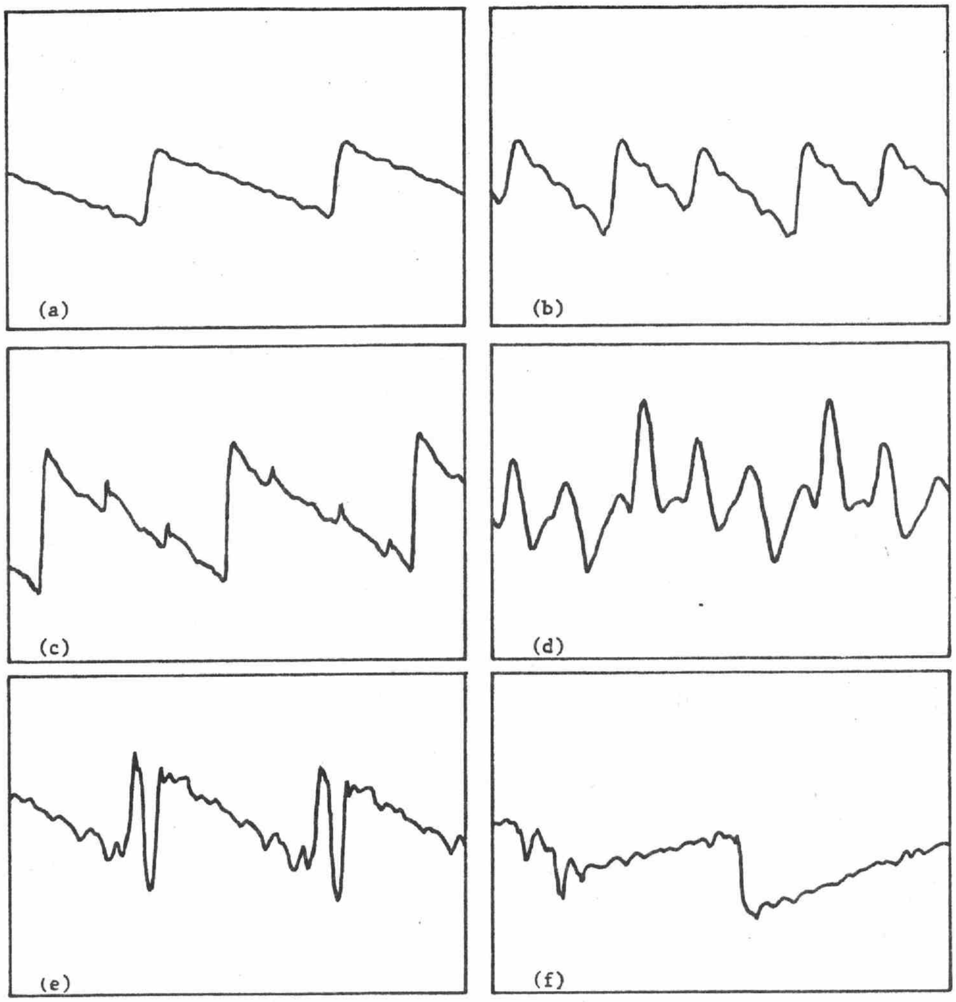

Figure 1. Steady oscillation regimes of a bowed violin G string (196 Hz). The time window is 13mS. in each case. (a) Helmholtz motion. (b) Double-slip motion. (c) Spikes. (d) S-motion for $\beta \approx 1/4$. (e) Double-flyback motion. (f) Slightly raucous motion; note the tendency toward longer intervals between slips.

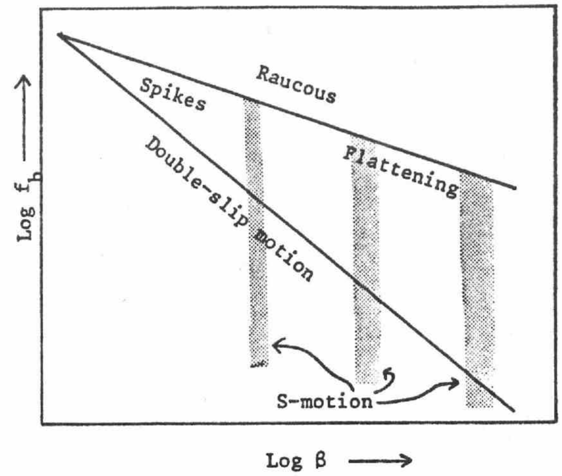

Figure 2. Schelleng's tolerance diagram showing the approximate positions of some of the regimes discussed.

The right-hand bar in Fig. 3 shows the tolerance range for the double-flyback motion according to the same small-β approximation. It is virtually identical to the Helmholtz range, as will be demonstrated in the next section. This means that there is unlikely to be a forced transition from Helmholtz to double-flyback or vice versa in the way we have just discussed for the double-slip motion. This is the reason that the double-flyback motion was not indicated in Fig. 2: its tolerance region is almost exactly coextensive with the Helmholtz region.

This behaviour of the double-flyback motion has both its good and its bad aspects, from the point of view of the player. Once a Helmholtz motion is established, gradual changes to bow force will not cause an unwanted transition to double-flyback motion. However, if the note is started with the wrong kind of transient so that double-flyback motion is established, then no small adjustment can change it to a Helmholtz motion. A new transient is required. Since the "shrieky" sound of the double-flyback motion is rather unpleasant, it is fortunate that most simple transients give rise to the Helmholtz motion rather than the double-flyback motion.

3: Some details of the double-flyback motion

Since the double-flyback regime seems not to have been described before, we give some details here of its kinematics. Among other things, this will show why its force tolerance range is the same as that for the Helmholtz motion, using Schelleng's approximation. The nature of the oscillation is most easily conveyed using

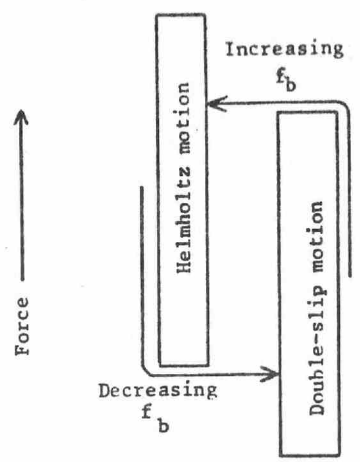

Figure 3. Force tolerance ranges for three oscillation regimes at fixed β, illustrating regime hysteresis.

a space-time diagram to show the behaviour of the three travelling "corners" on the string. We have used this diagram previously to illustrate the pattern of secondary waves during a Helmholtz motion, and also the subharmonic instability to which the Helmholtz motion is sometimes subject [7].

Figure 4 shows the version of the diagram appropriate to the double-flyback regime. Time is plotted horizontally, and distance along the string is plotted vertically. The slanting lines are the trajectories of the travelling corners, and the +/- signs against them indicate the signs of the associated velocity jumps: note that each one changes sign upon reflection from the ends of the string. The position of the bow appears as a horizontal line, solid when the string is sticking and dotted when it is slipping. It is apparent that this picture is consistent with the bridge-force waveform shown in Fig. 1(e), and that there are indeed two slips per cycle at the bow, as described above. Notice that while the character of the Helmholtz motion is substantially independent of the value of β, the spacing of the three corners in the double-flyback motion varies with β. It is necessary that the two "outer" corners of the group of three cross and cancel at the bow, as indicated in the figure.

Figure 4 shows that the total slipping time during one cycle is equal to that for the Helmholtz motion with the same value of β. Consequently, the slip velocity is the same as for the Helmholtz motion. Since Schelleng's maximum bow force depends only on the slip velocity (as well as the shape of the friction/velocity curve), this shows why the maximum force is the same for both motions when estimated in this way.

The Raman-Schelleng minimum force depends upon the length of the main sticking period in each cycle, as well as the magnitude of losses from the ends of the string, since it is an estimate of the force at which a second slip would occur in the middle of that sticking period. (In practice, a second slip tends to happen at a somewhat higher force and rather earlier in the sticking period, as a result of triggering by secondary waves [3].) Provided β is small, the length of sticking period is more or less equal to the period, for both Helmholtz and double-flyback motions. Thus the Raman-Schelleng minimum force is almost exactly the same for both.

It is perhaps at least intuitively apparent from Fig. 4 why the double-flyback motion should be less easy to set up by a suitable starting transient than the Helmholtz motion, as remarked upon earlier. It involves establishing a more complicated pattern of corners with particular relationships of timing and sign between them.

It is not easy to say any more than this about details of starting transients for any of these motions, although the recent article by Cremer [12] gives some very useful clues, further discussed in refs. [7] and [11]. It turns out (see especially ref. [11], §IID) that the scattering of transverse waves into torsional waves during a transient is qualitatively important for damping out the strong subharmonic components of motion which tend to be present initially. Further investigation of this effect may well prove to be an important line of future research.

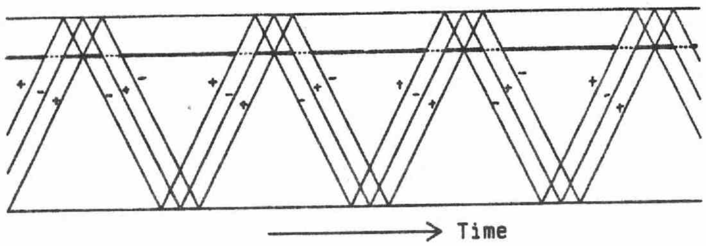

Figure 4. Space-time diagram illustrating the kinematics of the double-flyback oscillation regime.

4: Conclusions

In summary, we have identified a small subset of the large collection of possible steady oscillations of a bowed string which seems to include most regimes of interest to the player. We have indicated roughly where these occur in parameter space, using Schelleng's well-known diagram of the f_b-β plane. We have also drawn attention to the importance of hysteretic behaviour of the transitions between different oscillation regimes. A more detailed analysis of these effects should give a good basis of understanding of the parameter ranges in which a musically-acceptable Helmholtz motion can be sustained. It can also yield other incidental results of some interest, such as a more realistic criterion for the occurrence of wolf notes, based on an assumption of slowly-varying alternation between Helmholtz and double-slip motions. Such a description is now known to be needed to take into account (among other things) the strong non-linear variation of wolfing frequency with bow force [11], which disproves earlier ideas that the wolfing frequency is determined by the linear properties of the string and body, as a beat frequency.

References

1. H. Helmholtz 1877 On the sensations of tone. English translation of the German original, Dover, New York, 1954. See pages 80-88.
2. M. E. McIntyre and J. Woodhouse 1979 Interdisc. Sci. Rev. 3, 157-173. The acoustics of stringed musical instruments.
3. M. E. McIntyre and J. Woodhouse 1979 Acustica 43, 93-108. On the fundamentals of bowed-string dynamics.
4. C. V. Raman 1918 Indian Assoc. Cult. Sci. Bull. 15, 1-158. On the mechanical theory of vibrations of bowed strings.
5. J. C. Schelleng 1973 J. Acoust. Soc. Am. 53, 26. The bowed string and the player. Schelleng's main results on bow force limits are given in eqs. (1a) and (2) and footnote 10. See also the forthcoming English translation of L. Cremer, The Physics of the Violin, MIT Press Nov. 1984.
6. M. E. McIntyre, R. T. Schumacher and J. Woodhouse 1977 Catgut Acoust. Soc. Newsletter 28, 27-31. New results on the bowed string. A waveform of velocity at the bowed point is given here for a double flyback motion from a computer simulation. However, it appears that there is an error in the marking of the equivalent Helmholtz motion on this figure, since it is incorrect for the normal double flyback motion. The period of the two slips should occupy twice the Helmholtz slip time for the same value of β.
7. M. E. McIntyre, R. T. Schumacher and J. Woodhouse 1981 Acustica 49, 13-32. Aperiodicity in bowed-string motion. See also Ref. 8.
8. M. E. McIntyre, R. T. Schumacher and J. Woodhouse 1982 Acustica 50, 294-295. Aperiodicity in bowed-string motion: on the differential-slipping mechanism.
9. B. Lawergren 1980 Acustica 44, 194-206. On the motion of bowed violin strings.
10. O. Krigar-Menzel and A. Raps 1891 Ann. Phys. Chem. (later Ann. Phys. Leipzig) 44, 623. Ueber Saitenschwingungen.
11. M. E. McIntyre, R. T. Schumacher and J. Woodhouse 1983 J. Acoust. Soc. Am. 74, 1325-1345. On the oscillations of musical instruments.
12. L. Cremer 1982 Catgut Acoust. Soc. Newsletter 38, 13-18. Consideration of the duration of transients in bowed instruments. See also the forthcoming English translation of L. Cremer, The Physics of the Violin, MIT Press Nov. 1984.