Word count: 15582

Cumulativity from the perspective of homogeneity

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Abstract

Cumulative readings of quantifiers like *every* (Champollion, 2010; Haslinger and Schmitt, 2018; Kratzer, 2003; Schein, 1993) seem to defy traditional rules of composition. This paper offers a new analysis of such readings which aims to preserve both the semantics of the quantifiers and traditional compositional rules. It starts with the observation that the truth-conditions of the negation of these sentences are unproblematic: a single stipulation about the meaning of the verb yields appropriate truth-conditions for the examples considered. Taking this as a starting point, this paper then extends the analysis to positive sentences using mechanisms for strengthening akin to those proposed by Bar-Lev (2018b) in the context of homogeneity. The resulting analysis captures not only cumulative readings of *every* and other quantifiers but also subject/object asymmetries regarding the presence of these readings (Haslinger and Schmitt, 2018; Kratzer, 2003).

1 The problem of cumulative readings of quantifiers

1.1 Ordinary cumulativity

When two or more plural referential expressions are arguments of the same verb, they often give rise to the so-called cumulative reading. In (1), the cumulative reading asserts that the cooks and the oysters were involved in some opening but does not specifically say which of the cooks opened which of the oysters. Assuming that an oyster can only be opened by one cook[1], (1b) is the paraphrase of the truth-conditions:

(1) a. The 10 cooks opened the 15 oysters.
    b. **Truth-conditions:**
       Every cook opened an oyster.
       Every oyster was opened by a cook.

Cumulative sentences of the form in (1a), with two referential arguments and a transitive verb, will be referred to as *ordinary cumulative sentences*.

[1] This paraphrase, found in Scha (1984) but made prominent in Sternefeld (1998) isn't adequate when collective actions are possible, for instance if more than one cook collaborate to open one oyster. For most of the article, I will set aside the possibility of collective action ; this possibility is properly addressed in section 6.3.

The cumulative truth-conditions of these sentences can be explained in various ways. A simple analysis would treat (1a) as a simple predication, as in (2a). Under that view, the cumulative truth-conditions observed in (1b) would be attributed to the meaning of the word *open*. Specifically, we would assume that *open* obeys what we can call *the cumulative stipulation* given in (2b). This analysis of the cumulative truth-conditions is *prima facie* plausible and has been pursued in Roberts (1987); Scha (1984).

(2) a. $[\![\text{opened}]\!](\iota\text{15-oysters})(\iota\text{10-cooks})$

b. **Cumulative lexical stipulation:**
$[\![\text{opened}]\!](X)(Y)$ iff
every one of Y opened one of X
and every one of X was by opened one of Y

1.2 The problem of cumulative readings of *every*

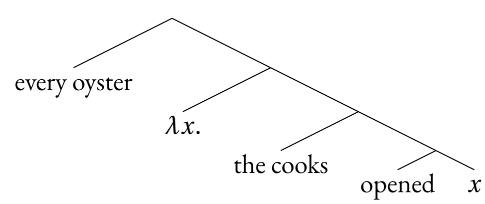
The cumulative stipulation in (2b)[2] generates problematic predictions outside of ordinary cumulative sentences. Consider (3a), where the object argument is replaced with the quantifier *every*. This sentence has a cumulative reading, like the original sentence in (1), i.e. the reading in (3b).

(3) a. The ten cooks opened every oyster.

b. **Truth-conditions:**
Every cook opened an oyster.
Every oyster was opened by a cook.

The problem is that the cumulative stipulation, together with very natural assumptions about composition, doesn't derive this cumulative reading. Since *every* is a universal quantifier, one expects (3a) to be paraphrasable as: "*for every oyster x, the cooks opened x*". This is indeed what one derives by applying the compositional rules of e.g. Heim and Kratzer (1998), as is done in (4b). Because of the cumulative lexical stipulation, this paraphrase is in turn equivalent to "*every cook opened every oyster*", which is not the cumulative reading of (3b) we are interested in.

(4) a.



b. $[\![(4a)]\!]$ is true
iff $\forall x \in \text{oyster}, \text{open}(x)(\iota\text{cooks})$
(by compositional rules)
iff $\forall x \in \text{oyster}, \big(\forall y < \iota\text{cooks}, \text{open}(x)(y)\big) \wedge \big(\exists y < \iota\text{cooks}, \text{open}(x)(y)\big)$

[2] All examples reported in this work are either adaptations of examples from the literature (positive cumulative sentences and non-cumulative sentences) or constructed English sentences checked with four native speakers of English (negative cumulative sentences).

(by cumulative denotation of *open*)
iff $\forall x \in \mathsf{oyster}, \mathsf{open}(x)(\iota\mathsf{cooks})$
(by simplification)
i.e. "*every cook opened every oyster*"

This problem of cumulative sentences with *every* was brought to the attention of the semantics literature by Schein (1993) and has been studied in many other works such as Brasoveanu (2013); Champollion (2010, 2016b); Ferreira (2005); Haslinger and Schmitt (2018); Ivlieva (2013); Kratzer (2000); Lasersohn (1990).

This paper seeks to address this basic puzzle that this paper seeks to address. I will try to explain why sentences like (3a) come to have the cumulative truth-conditions that they do, when natural assumptions would lead us to predict that they only yield strong doubly-distributive readings (e.g. *every cook opened every oyster*).

My strategy will be to investigate the truth-conditions of the negative counterparts of these sentences. Indeed, as we will see, the paradox of cumulative readings of *every* does not arise in negative sentences, where normal compositional assumptions can be maintained. From there on, I will build an analysis of negative sentences and then extend it to positive sentences.

1.3 A broader perspective

Before explaining the strategy, I will present three additional facts to broaden our perspective on the puzzle: collective readings of *every*, asymmetries in cumulative readings and cumulative readings of other quantifiers.

Collective readings of *every* First off, it is known that *every* can yield collective readings, incompatible with the distributive meaning traditionally ascribed to it. For some speakers[3], (5) is acceptable:

(5) Every revolutionary gathered at *Café Musain*.

This fact invites a natural response to the challenge of cumulative readings of *every*. Assume that, in some circumstances, *every oyster* denotes the same object that *the fifteen oysters* denotes, a plurality of fifteen oysters (Landman, 2000), as evidenced by (5). Under this assumption, the cumulative sentence with *every* would, under some construal, be semantically equivalent to an ordinary cumulative sentence (i.e. *the ten cooks opened the fifteen oysters*). As we saw, there is no particular problem in accounting for the readings of ordinary cumulative sentences.

However, with his video-game example in (6), Schein (1993) provides an argument against that response:

(6) a. The ten video-games taught every quarterback four new plays.
b. **Truth-conditions:**
Every quarterback was taught two new plays by some of the video-games
Every video-game taught a quarterback some plays.

[3] 2 out of 4 consulted.

(6a) can only be true when four new plays are taught to *each* player. By contrast, this distributive reading is only optional when *every* is replaced with a definite, as in (7). If *every quarterback* has to be semantically equivalent to *the quarterbacks* for the cumulative reading to arise, the difference between (6a) and (7) is unexplained.

(7) The ten video-games taught the quarterbacks four new plays.
(ok) four new plays in total

This shows that even in cumulative sentences with *every*, elements in the scope of *every* continue to be read distributively.

Asymmetries in cumulative readings A major puzzle connected to the cumulative readings is the presence of asymmetries. Indeed, the cumulative reading of *every* does not obtain when *every* is in the subject position. (8) is an example: this sentence is perceived as strange and seems to imply that oysters were resealed. Under a cumulative reading, (8) should be as natural as the cumulative sentences seen above.

(8) a. # Every cook opened the 15 oysters.
b. **Truth-conditions:**
Every cook opened every oyster.

To my knowledge, Kratzer (2003) was the first to explicitly note this fact. It is also discussed in Champollion (2010); Ferreira (2005); Haslinger and Schmitt (2018); Ivlieva (2013). These asymmetries constitute an important aspect of the problem which should be accounted for. I will however set it aside for the time being, focusing on explaining the cumulative reading in object position. With a more complete analysis, we will be able to return to these asymmetries in section 5.2.

Other quantifiers than *every*. A final relevant fact is that *every* is not an exception among quantifiers in giving rise to cumulative readings. The examples in (9-10) are all examples of cumulative readings with various quantifiers.

(9) a. The ten cooks opened between 28 and 35 oysters.
b. **Truth-conditions:**
Between 28 and 35 oysters were opened by some of the ten cooks.
Every one of the ten cooks opened an oyster.

(10) a. The ten cooks opened a prime number of oysters.
b. **Truth-conditions:**
A prime number of oysters were opened by some of the ten cooks.
Every one of the ten cooks opened an oyster.

These examples are less interesting to the theorist[4]. The quantifiers involved are quantifiers over pluralities and the truth-conditions of these sentences arise naturally from composition

[4]Similar sentences are more interesting. Particularly well-studied is the case of cumulative readings of two modified numerals (Brasoveanu, 2013; Buccola and Spector, 2016). Dealing with such examples is outside the scope of this paper.

and the cumulative meaning postulate of (2b). They are also different from *every* in that they do not yield asymmetries ; a cumulative reading is equally available when the quantifier stands in subject position:

(11) a. A prime number of cooks opened the ten oysters
b. Between 28 and 35 cooks opened the ten oysters.

There are, however, some suggestive analogues to the case of *every* with plural quantifiers. This is for instance the case of the non-partitive quantifier *most Ns*. While being marked in the plural, this quantifier is often[5] read as if it ranged over singularities (like *every*). This is illustrated in (13) (adapted from Crnič (2010)): in both sentences, only a distributive reading is possible. Note, in particular, that (13a) is a configuration from which a cumulative reading might be expected to arise.

(13) a. # Most cooks opened the ten oysters.
≈ *a majority of cooks is such that* ***each*** *of them opened the oysters.*
≠ *a majority of cooks is such that they opened the 10 oysters (together).*
b. Most lawyers$_1$ hired a secretary they$_1$ liked.
≈ *each member of a majority of lawyers is such that they hired a secretary they liked.*
≠ *the members of a majority of lawyers are such that they hired a secretary they all liked.*

In spite of this, cumulative readings of *most Ns* are possible when the latter stands in object position, just like *every*.

(14) The ten cooks opened most oysters.
≈ *there is majority of oysters such every cook opened one of them and each one of them was opened by a cook*

These facts suggest that the problem of cumulative readings in object position is not restricted to *every*. Consequently, a solution to this puzzle had better not rely on a particular semantics of *every* but should apply fully generally to other quantifiers. The analysis provided in this paper aims for this level of generality.

1.4 Outlook and roadmap

Quantifiers in object position give rise to cumulative readings. For a distributive quantifier such as *every*, this reading is not expected to arise given traditional denotations and rules of composition. The problem is most clearly seen with *every* but extends to other quantifiers ; a general solution is desired.

In this paper, I propose a solution to the problem of cumulative readings of *every* which relies on another property of plural interpretation: homogeneity. To motivate this, I will start by

[5] This isn't always so. As noted by Kamp and Reyle (2013), collective predicates can combine with non-partitive *most* with varying degrees of acceptability. Just as with *every*, we can construct a video-game example to show that *most* retains its distributive semantics even when construed cumulatively:

(12) The ten video-games taught most quarterbacks three new plays.

making the observation (section 2) that the truth-conditions of the negation of cumulative denotation can be accounted for, without altering our compositional assumptions. While successful on negative sentences, this analysis will miss some of the inferences attested in positive cumulative sentences. In section 3, I will then propose to model the missed inferences after well-known implicatures, namely Free Choice and distributive implicatures. I explain how the latter two implicatures can be derived, compiling the analyses of previous work. Section 4 simply transposes this analysis to cumulative sentences providing a full derivation of the truth-conditions of both the positive and the negative cumulative sentences. Section 5 fleshes out the analysis on several more cases: asymmetries, cumulative readings of other quantifiers than *every*, including non-partitive *most*, ordinary cumulative sentences. Finally, in section 6, I locate this contribution within the broader context of existing proposals on cumulative readings of *every*, while tying some loose ends.

2 Generalizations about plural interpretations: the dual perspective of homogeneity and cumulativity

The previous section presented the problem of cumulative readings of quantifiers. We will now present the phenomenon of homogeneity and study the homogeneity properties of the sentences from the previous section. I will show that a straightforward account of the truth-conditions of all the negative sentences can be obtained if we replace the cumulative stipulation for *open* by an existential stipulation. This will form the first pillar of the analysis to be presented.

2.1 Homogeneity and the homogeneity properties of cumulative sentences

There are situations that neither a sentence containing a plural nor its negation can appropriately describe. For instance, in the context of (15), neither (15a) nor (15b) can be truthfully asserted.

(15) **Context:** *Half of the dancers are smiling and the other half is crying*

a. # The ten dancers are smiling.

b. # The ten dancers aren't smiling.

This "*truth-value gap*" that plural-referring expressions give rise to is what we call *homogeneity*[6] (Bar-Lev, 2018a,b; Kriz, 2015; Križ, 2016; Križ and Spector, 2021; Löbner, 2000; Magri, 2014; Malamud, 2012; Schwarzschild, 1993). To be more precise, *the ten dancers* in the positive sentence (16a) seems to have a universal or quasi-universal meaning, being roughly equivalent to *every dancer*[7], whereas it only has an existential meaning in the negative sentence (i.e. *one of the dancers*).

[6] By extension, the name is also applied to truth-value gaps exhibited by other constructions such as conditionals (Bassi and Bar-Lev, 2018), embedded questions (Kriz, 2015).

[7] I set aside the possibility of exceptions to these universal meaning (non-maximality). The analysis proposed which draws on Bar-Lev (2018b) can import his account of non-maximality.

(16) a. The ten dancers smiled.
⇝ ***every*** *one of the ten dancers smiled*

b. The ten dancers didn't smile.
⇝ *it's not the case that* ***one*** *of the ten dancers smiled*

The phenomenon of homogeneity extends to most sentences with plural arguments. This includes cumulative sentences (Kriz, 2015). By taking the complement of the cumulative truth-conditions, one may expect (18a) to be true in the circumstances described in (18b). But the observed truth-conditions of (18c) are stronger and require that no oysters whatsoever have been opened (Kriz, 2015, ex. 17-18).

(17) a. The ten cooks opened the fifteen oysters.

b. **Cumulative truth-conditions:**
Every cook opened an oyster.
Every oyster was opened by a cook.

(18) a. The ten cooks didn't open the fifteen oysters.

b. **Complement of the cumulative truth-conditions:**
Either not every cook opened an oyster. or not every oyster was opened by a cook.

c. **Attested truth-conditions:**
No cook opened any oyster.

Likewise, cumulative sentences with *every* also display homogeneity. The negative sentence in (20a) is true in just the case outlined in (20b). By simply taking the complement of the cumulative truth-conditions, one might have expected the truth-conditions to be as in (18b). But these truth-conditions are too weak: the sentence is not simply true when every oyster was opened but some cook didn't contribute to the opening.

(19) a. The ten cooks opened every oyster.

b. **Truth-conditions:**
Every cook opened an oyster.
Every oyster was opened by a cook.

(20) a. The ten cooks didn't open every oyster.

b. **Truth-conditions:**
Not every oyster was opened by a cook.

To say it concisely, we may say that (20a) does not negate exhaustive participation of the cooks, but simply exhaustive opening of the oysters.

As for *every*, the negation of a cumulative sentence with *most* does not negate exhaustive participation of the cooks. It simply negates the fact that a majority of oysters were opened.

(21) a. The ten cooks didn't open most oysters.

b. **Truth-conditions:**
The number of oysters opened by a cook is less than half.

To summarize, homogeneity is also a property of cumulative sentences. What is the relevance of homogeneity to an account of cumulative readings? The important point is that since the truth-conditions of negative sentences are not straightforwardly deducible from the truth-conditions of positive sentences, we now have to account for the truth-conditions of negative sentences as well.

The task of accounting for the truth-conditions of negative sentences turns out to be easy. As the next section shows, there is no problem of cumulative readings of quantifiers as far as negative sentences are concerned: one and the same lexical stipulation on the verbs involved accounts for all cases of cumulative readings of quantifiers.

2.2 A simple analysis of the truth-conditions of negative sentences

In section 1, a cumulative stipulation on the verb *open* was made to account for the cumulative reading of ordinary cumulative sentences:

(22) a. The ten cooks opened the fifteen oysters.
b. **Cumulative lexical stipulation:**
$[\![\text{opened}]\!](X)(Y)$ iff
every one of Y opened one of X
and every one of X was opened by one of Y

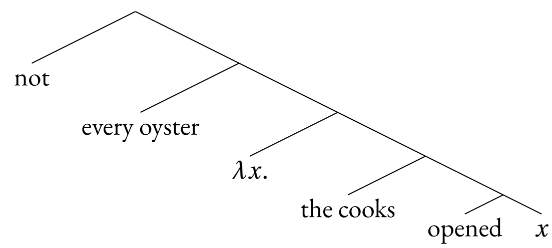
What stipulation would we have made if we had started our investigation with negative sentences? To make the observed truth-conditions in (23b) equivalent to the truth-conditions derived by composition in (23c), we would need *an existential stipulation* on the meaning of *open*. This stipulation is given in (24).

(23) a. The ten cooks didn't open the fifteen oysters.
b. **Observed truth-conditions:**
No cook opened any oysters.
$\Leftrightarrow \neg\exists x \prec \iota\mathsf{cooks}, \exists y \prec \iota\mathsf{oysters}, \mathsf{opened}(y)(x)$
c. $[\![(23a)]\!]$ is true iff $\neg\mathsf{opened}(\iota\mathsf{oysters})(\iota\mathsf{cooks})$

(24) **Existential lexical stipulation:**
$[\![\text{opened}]\!](X)(Y)$ iff
one of Y opened one of X

In section 1, we saw that, for the positive case, the cumulative stipulation, adequate for ordinary cumulative sentences, yielded stronger readings than desired for cumulative sentences with *every*. Here, the existential denotation is both adequate for negative counterparts of ordinary cumulative sentences and cumulative sentences with quantifiers. Let us illustrate this by looking at the case of cumulative readings of *every*.

(25) a.



b. The cooks didn't open every oyster.

c. $[\![(25a)]\!]$ is true
$\Leftrightarrow \neg\forall y \in \mathsf{oyster},\ \mathsf{opened}(y)(\iota\mathsf{cooks})$
(by composition)
$\Leftrightarrow \neg\forall y \in \mathsf{oyster}, \exists x \prec \iota\mathsf{cooks}, \mathsf{opened}(y)(x)$
(by existential lexical stipulation)
= attested truth-conditions

The existential denotation is not only adequate for deriving cumulative readings of *every* but it is also adequate for deriving cumulative readings of *most*. For instance, assuming a simple-minded semantics for *most* as in (26b), we derive the correct truth-conditions in (26c)

(26) a. The cooks didn't open most oysters.

b. $[\![\text{most oysters}]\!](q) = \#\{x \in \mathsf{oyster} \mid \mathsf{q}(x)\} > \frac{1}{2}\#\mathsf{oyster}$

c. $[\![(26a)]\!]$ is true
$\Leftrightarrow \neg\#\{x \in \mathsf{oyster} \middle| \mathsf{open}(x)(\iota\mathsf{cook})\} > \frac{1}{2}\#\mathsf{oyster}$
(by composition)
$\Leftrightarrow \#\{x \in \mathsf{oyster} \middle| \exists y \prec \iota\mathsf{cooks}, \mathsf{open}(x)(y)\} \leq \frac{1}{2}\#\mathsf{oyster}$
(by existential lexical stipulation)
"Less than half the oysters were opened by a cook."
= attested truth-conditions

2.3 Summary

In conclusion, the existential stipulation enjoys considerably more success on negative sentences than the cumulative stipulation did on positive sentences. I therefore suggest to use the existential lexical stipulation as the foundation of an account of cumulative readings of quantifiers. From now on, I will assume that verbs like *open*[8] have an existential meaning by default.

(27) **Existential lexical stipulation:**
$[\![\text{opened}]\!](X)(Y)$ iff
one of Y opened one of X

[8] Some verbs like *outweigh* don't seem to yield cumulative readings. We may assume these verbs are not subject to the existential denotation. Although an interesting topic, I have nothing to say here about what determines which verb yield cumulative readings or not or, as the theory here would have it, which verb the existential lexical stipulation is true of.

This is the first piece of the account: an underlying existential meaning for verbs. On its own, the existential stipulation will not be enough to derive the truth-conditions of positive cumulative sentences. In the next section, I show how the missed predictions can be corrected by extending Bar-Lev (2018b)'s account of homogeneity to cumulative sentences.

3 Missed inferences in positive sentences: analogy with implicatures

3.1 Missed inferences in positive sentences

In the last section, we posited an existential denotation for *open*, repeated in (28), to predict the truth-conditions for negative sentences.

(28) **Existential lexical stipulation:**
$[\![\text{opened}]\!]\,(X)(Y)$ iff
one of Y opened one of X

To highlight this lexical stipulation, I will from now on represent *open* and its meta-language denotation as ∃-`open`[9].

In positive sentences, this assumption makes overly weak predictions. If *open* has the existential denotation in (28), then (29a) should be equivalent to the first line of (29b) (represented in black). But (28a) entails not only the black line of (28b) but also the propositions in gray.

(29) a. The ten cooks opened the fifteen oysters.
b. **Truth-conditions:**
Some cooks opened some oysters
Every cook opened an oyster
Every oyster was opened by a cook

In all other examples, we find that the existential denotation of verbs consistently predicts weaker readings than attested. I continue to use gray to represent attested inferences which are not predicted by the existential stipulation and normal rules of composition.

(30) a. The ten cooks opened every oyster.
b. **Truth-conditions:**
Every oyster was opened by a cook.
Every cook opened an oyster

(31) a. The ten cooks opened most oysters.
b. **Truth-conditions:**
Most oysters were opened by a cook.
Every cook opened an oyster

[9] In much the same way that Kratzer (2003) writes *`open` to highlight her assumption that lexical denotations are closed under sums.

This may seem like a bad outcome. But, in fact, we can observe that empirically, the missed inferences in gray have a special status. Namely, speakers can accept the cumulative sentences, even when these inferences are not met. By contrast, our inferences in black must be met for the sentence to be assertable. This is illustrated in (32): the continuation that denies the predicted inference (e.g. (32a)) is odd but the continuation that denies the missed inference is acceptable.

(32) The cooks on staff that night opened every oyster in this bag ...
  a. # Of course, following a widespread superstition, they didn't open the last three oysters.
  b. Of course, the more experienced cooks, as usual, found excuses not to do anything so it was mostly the rookies.

Following the terminology of the literature (Brisson, 1997; Križ and Spector, 2021), this is a case of *non-maximality* in cumulative sentences.

This observation begs a natural question: if the gray inferences do not have to hold for the sentence to be utterable, why were they included in the truth-conditions of these sentences in the first place? As it turns out, there are certain circumstances in which the gray inferences must hold. This happens in (at least) two cases: with small domains (33a) or when work attributions are deemed important (33b).

(33) a. Joana and Marius opened every oyster. #Of course, Marius didn't do anything.
  b. # These 10 physicists and these 4 actors discovered every major theory in physics. (adapted from Kratzer (2003))

In short, the missed inferences in gray not currently predicted have two important properties. First, they are not systematically present. Second, as we saw in the previous section, they are not part of what is negated in negative sentences. Because they assert that all members of a plurality took part in the action of the verb, I will label the missing inferences *participation inferences*.

Following Bar-Lev (2018b)'s account of homogeneity, I propose to treat participation inferences as implicatures. This explains their particular status: absence of participation inferences in certain contexts can be treated as a case of non-derivation of implicatures ; absence of these inferences under negation is equated to the disappearance of implicatures under negation.

To bolster the analogy with implicatures, I will show that the missing inferences bear similarity to two known types of implicatures: Free Choice and distributive implicatures. The analogy with Free Choice is developed in Bar-Lev (2018b)'s ; the analogy to distributive implicatures is new to this work. With the help of these analogies, it will be possible to adapt an account of these implicatures into an account of participation inferences, giving a full account of the cumulative truth-conditions in positive sentences.

3.2 Bar-Lev's account of homogeneity and analogy to Free Choice

Bar-Lev (2018b)'s goal is to account for the truth-conditions of positive and negative non-cumulative examples like (34a) and (34b).

(34) a. The ten dancers didn't smile.
b. The ten dancers smiled.

Just like the present account, Bar-Lev gives the verb an existential meaning. In his system, the existential meaning is not obtained as a lexical stipulation on the verb but is the result of applying an existential distributivity operator. The difference, for present purposes, is immaterial ; we return to these differences later in section 6.2.

Thus, the underlying truth-conditions of the examples in (34) are predicted to be as in (35). As with cumulative sentences, a participation inference is missed in positive sentences (in gray).

(35) a. **Truth-conditions of (34a):**
It's not the case that any dancer smiled.
b. **Truth-conditions of (34b):**
Some dancers smiled.
Every dancer smiled.

Bar-Lev (2018b) compares the missing participation inference of (35b) to a Free Choice implicature. Let us explore this analogy.

Parallel to Free Choice Free Choice implicatures (illustrated in (36)) are cases where a disjunction embedded under an existential modal is interpreted as a wide-scope conjunction.

(36) a. You are allowed to eat apple or cake.
b. **Truth-conditions:**
$\Diamond(\mathsf{cake} \vee \mathsf{apple})$
$\wedge \Diamond \mathsf{cake} \wedge \Diamond \mathsf{apple}$

Seeing existentials and universals as generalized counterparts of disjunctions and conjunctions, we can see a parallel between Free Choice implicatures and sentences like (37a): in both cases, an existential or disjunctive meaning is converted to a universal or conjunctive meaning[10].

(37) a. The dancers ∃-smiled.
b. $\exists x \prec \iota\mathsf{dancers},\ \mathsf{smiled}(x)$
$\forall x \prec \iota\mathsf{dancers},\ \mathsf{smiled}(x)$

This leads Bar-Lev (2018b) to derive the participation inference of non-cumulative sentences as Free Choice implicatures.

[10] This analogy is somewhat loose for Free Choice, since the typical free choice inference only occurs in the scope of a possibility modal (cf (36)), and the universal inference we observed can occur in the absence of a modal. Still, this configuration has been argued to not be critical for Free Choice-like implicatures: the strengthening of unembedded disjunctions to conjunctions has proven useful to account for properties of Warlpiri connectives *manu* (Bowler, 2014)[11] and children's conjunctive interpretation of *or* (Singh et al., 2016)[12].

3.3 Cumulative sentences and analogy to distributive implicatures

With cumulative sentences, the analogy to Free Choice does not seem to hold. Consider (38). The assumed underlying meaning asserts that all oysters were opened by a cook. The participation inference to be derived asserts that all cooks opened a oyster. This inference does not look like a Free Choice inference: an existential in the underlying meaning (*an oyster*) is indeed converted to a universal, but in addition, the universal meaning of *every* in the underlying meaning is "*weakened*" to an existential.

(38) a. The cooks ∃-opened every oyster.

b. **Attested meaning:**
$\forall x \in \mathsf{oyster}, \exists y \prec \iota\mathrm{cooks}, \mathrm{open}(x)(y)$
$\forall y \prec \iota\mathsf{cooks}, \exists y \in [\![\mathrm{oyster}]\!], \mathrm{open}(x)(y)$

However, another type of implicatures gives an adequate parallel: distributive implicatures. Distributive implicatures occur when a disjunction is embedded under some quantifiers such as *every*[13] in (39). In this case, an implicature arise that every disjunct is true of at least one individual in the domain of the quantifier.

(39) Every ambassador speaks Arabic, English or Mandarin.
⇝ at least one ambassador speaks Arabic.
⇝ at least one ambassador speaks English.
⇝ at least one ambassador speaks Mandarin.

This inference has a similar shape to the participation inference of cumulative sentences. To see this, consider a case where there are three cooks: Joana, Marius and Becky. In this case, the underlying meaning of (38a) simplifies to (40a) and the participation inference to be derived can be written as in (40b). The inferences are isomorphic to the inferences in (39).

(40) a. $\forall x \in \mathsf{oyster}, \mathsf{open}(x)(\mathsf{Joana}) \vee \mathsf{open}(x)(\mathsf{Marius}) \vee \mathsf{open}(x)(\mathsf{Becky})$
⇝ *every oyster was opened by Joana, Marius or Becky*

b. $\exists x \in \mathsf{oyster}, \mathsf{open}(x)(\mathsf{Joana})$
$\exists x \in \mathsf{oyster}, \mathsf{open}(x)(\mathsf{Marius})$
$\exists x \in \mathsf{oyster}, \mathsf{open}(x)(\mathsf{Becky})$

The parallel does not stop at cumulative readings of *every*. Other quantifiers as well give rise to distributive implicatures. For instance, non-partitive *most*[14]:

(41) Most ambassadors speak Arabic, English or Mandarin.
⇝ at least one ambassador speaks Arabic.
⇝ at least one ambassador speaks English.
⇝ at least one ambassador speaks Mandarin.

Correspondingly, these inferences match the participation inferences of cumulative sentences with *most*:

[13] We come back to the question of exactly which quantifiers give rise to these inferences in section 5.1.
[14] Credits for this observation goes to F. Hisao Kobayashi (p.c.).

(42) a. The cooks ∃-opened most oysters.

b. Most $x \in \mathsf{oyster}, \mathsf{open}(x)(\mathsf{Joana}) \vee \mathsf{open}(x)(\mathsf{Marius}) \vee \mathsf{open}(x)(\mathsf{Becky})$
$\rightsquigarrow$ *most oysters were opened by Joana, Marius or Becky*

c. $\exists x \in \mathsf{oyster}, \mathrm{open}(x)(\mathsf{Joana})$
$\exists x \in \mathsf{oyster}, \mathrm{open}(x)(\mathsf{Marius})$
$\exists x \in \mathsf{oyster}, \mathrm{open}(x)(\mathsf{Becky})$

3.4 Summary: Generalized distributive implicatures.

The existential stipulation made to derive the truth-conditions of negative sentences systematically misses some inferences observed in positive sentences. These missed inferences match the signature of implicatures: they are polarity-sensitive and context-sensitive. They also are formally isomorphic to Free Choice implicatures in the case of non-cumulative sentences and distributive implicatures in the case of cumulative sentences. The parallel is summarized in the chart below in (43).

(43)

Sentence	You can have ice-cream or cake.	The dancers smiled.
Underlying meaning	$\Diamond a \vee \Diamond b$	$\exists x \prec \iota\mathsf{dancers}, \mathsf{smiled}(x)$
Implicature	$\Diamond a \wedge \Diamond b$	$\forall x \prec \iota\mathsf{dancers}, \mathsf{smiled}(x)$
Sentence	Everyone speaks Mandarin or Tagalog.	The cooks opened every oyster.
Underlying meaning	$\forall x, m(x) \vee t(x)$	$\forall y \in \mathsf{oyster}, \exists x \prec \iota\mathsf{cooks}, \mathsf{opened}(x)(y)$
Implicature	$\exists x, m(x) \wedge \exists x, t(x)$	$\forall x \prec \iota\mathsf{cooks}, \exists y \in \mathsf{oyster}, \mathsf{opened}(x)(y)$

The next section presents an account of participation inferences in cumulative sentences. Building off of this analogy, the account simply adapts existing accounts of Free Choice and distributive implicatures to cumulative sentences.

4 Accounting for participation inferences: recursive exhaustification

In this section, I present a unified account of free choice/distributive implicatures based on Bar-Lev and Fox (2016); Fox (2007), using recursive exhaustification. I will then apply this account *mutatis mutandis* to sentences with plural arguments to derive the missing participation inferences.

Following the grammatical tradition (Chierchia, 2013; Chierchia et al., 2012; Fox, 2007), this account assumes that the scalar implicatures of interest are derived in the grammar by application of a covert operator Exh. This operator strengthens the meaning of a prejacent by comparing it to a set of alternatives.

I specifically assume the *innocent exclusion exhaustification* of Fox (2007), whose definition is given below in (44). To put it concisely, this operator negates as many alternatives as possible while (1) not creating logical contradictions, (2) treating all alternatives symmetrically.

(44) a. **Maximal sets:**

$$\text{Max}(p)(\mathsf{alts}) := \left\{ S \subset \mathsf{alts} \;\middle|\; \begin{array}{l} p \wedge \bigwedge_{\mathsf{alt} \in S} \neg \mathsf{alt} \text{ is not contradictory} \\ \neg \exists S' \supset S, p \wedge \bigwedge_{\mathsf{alt} \in S'} \neg \mathsf{alt} \text{ is not contradictory} \end{array} \right\}$$

b. **Innocently excludable alternatives:**

$IE(p)(\mathsf{alts}) := \bigcap \text{Max}(p)(\mathsf{alts})$

c. $[\![\text{Exh}_{\ \mathsf{alts}}]\!]\,(p) = p \wedge \bigwedge_{\mathsf{alt} \in \mathsf{IE}(p)(\mathsf{alts})} alt$

(45) illustrates how application of Exh can derive the *not-and* inference of simple disjunctions.

(45) a. Exh [Joana or Marius came]

b. **Alternatives:**
- $\mathsf{came(Joana)}$
- $\mathsf{came(Marius)}$
- $\mathsf{came(Joana)} \wedge \mathsf{came(Marius)}$

c. **Maximal sets:**
- $\{\mathsf{came(Joana)}, \mathsf{came(Joana)} \wedge \mathsf{came(Marius)}\}$
- $\{\mathsf{came(Marius)}, \mathsf{came(Joana)} \wedge \mathsf{came(Marius)}\}$

d. **Innocently excludable alternatives:**

$\{\mathsf{came(Joana)} \wedge \mathsf{came(Marius)}\}$

In this section, I show, following previous literature, how applying this operator recursively can derive Free Choice and distributive implicatures and adapt the account to sentences with plural arguments.

4.1 Inclusivity and Free Choice

In the last section, we presented Bar-Lev (2018b)'s analogy between Free Choice and the universal meaning of positive intransitive sentences.

Unfortunately, the Free Choice implicature, repeated in (46), cannot be derived by applying one Exh operator as we did above for ordinary disjunctions. In the case of disjunction, a stronger alternative to the sentence was negated (e.g. the *and* alternative). Here however, the gray inferences of (46b) could only be obtained as the negation of "*You're not allowed to eat apples*" or the negation of "*You're not allowed to eat cake*". It is difficult to see how these could be alternatives to the sentence, making Free Choice a tricky implicature to derive.

(46) a. You are allowed to eat apples or cake.

b. **Paraphrase:**

You're allowed to eat apples or cake

You're allowed to eat apples

You're allowed to eat cake

This informal reasoning suggests that something extra is needed to derive Free Choice implicatures. Fox (2007) proposes to derive these implicatures by applying two Exh operators to the sentence (i.e. recursive exhaustification). Specifically, he assumes the sentence (46) has the structure in (47).

(47) $\textsc{Exh}_2$ $\textsc{Exh}_1$ you are allowed to eat apple or cake.

To guide intuitions about what this is meant to accomplish, we can reason along the following Gricean lines (cf. Kratzer and Shimoyama (2017)): the speaker chose to assert "apples or cake" and not either of the disjuncts. This means she was probably not in a position to assert one of the disjuncts. One reason might be that by uttering e.g. (48a), she may have conveyed that "apple" was the only option (by way of implicature), likewise for (48b). From this, we conclude that the speaker believes neither "apple" nor "cake" is the only option, i.e. both are in fact allowed.

(48) a. You are allowed to eat apples.
$\rightsquigarrow$ *you're allowed to eat apples and nothing else*
b. You are allowed to eat cake.
$\rightsquigarrow$ *you're allowed to eat cake and nothing else*

This reasoning relied on negating alternatives (e.g. (48a) and (48b)) *along with their implicatures*. In a grammatical tradition, this means negating alternatives which themselves contain and $\textsc{Exh}$ operator, something that can be achieved with the recursive structure in (47).

To compute the truth-conditions that such a structure yield, one needs to compute the result of applying $\textsc{Exh}_2$ to a structure like "*$\textsc{Exh}_1$ you are allowed to eat apples or cake*". This means comparing the sentence in (49a) to alternatives of the form (49b).

(49) a. **Prejacent:**
$\textsc{Exh}_1\left(\Diamond(\mathsf{cake} \vee \mathsf{apples}), \mathsf{alts}_1\right)$
where $\mathsf{alts}_1 = \{\Diamond\mathsf{cake}, \Diamond\mathsf{apples}\}$
b. **Alternatives for Exh $_2$:**
$\mathsf{alts}_2 = \{\textsc{Exh}_1(\Diamond\mathsf{cake}, \mathsf{alts}_1), \textsc{Exh}_1(\Diamond\mathsf{apples}, \mathsf{alts}_1)\}$

These alternatives in (50b) are themselves exhaustive statements. By assumption, Fox (2007) assume that these alternatives are all exhaustified with respect to the same set of alternatives, the alternatives to "*You are allowed to eat apples or cake*" (alt_1). Their truth-conditions of the exhaustified alternatives is given in (50):

(50) **Alternatives for Exh $_1$:**
$\mathsf{alts}_1 = \{\Diamond\mathsf{cake}, \Diamond\mathsf{apples})\}$
a. $\textsc{Exh}(\Diamond\mathsf{cake}, \mathsf{alts}_1) = \Diamond\mathsf{cake} \wedge \neg\Diamond\mathsf{apples}$
$\rightsquigarrow$ *you're allowed to eat cake and not apples*
b. $\textsc{Exh}(\Diamond\mathsf{apples}, \mathsf{alts}_1) = \Diamond\mathsf{apples} \wedge \neg\Diamond\mathsf{cake}$
$\rightsquigarrow$ *you're allowed to eat apples and not cake*

Negating these alternatives yields the attested FC inference[15]:

[15] This recursive reasoning is very powerful. The same reasoning, applied to the simple unembedded disjunction, would derive that you ate both apple and cake. In unembedded cases, it is assumed that the exclusive inference (*not and*) derived by competition with *and* in $\textsc{Exh}_1$ block the Free Choice inference. This safeguard does not apply when a modal intervenes, as in the case given, or when there is no *and* implicature, as in the plural case we now turn to.

(51) $\textsc{Exh}_2(\textsc{Exh}_1(\Diamond(\mathsf{cake} \vee \mathsf{apples}, \mathsf{alts}_1)))$
$= \textsc{Exh}_1(\Diamond(\mathsf{cake} \vee \mathsf{apples}, \mathsf{alts}_1)) \wedge \neg\textsc{Exh}_1(\Diamond\mathsf{cake}, \mathsf{alts}_1) \wedge \neg\textsc{Exh}_1(\Diamond\mathsf{apples}, \mathsf{alts}_1)$
$= \Diamond(\mathsf{cake} \vee \mathsf{apples}) \wedge \neg(\Diamond\mathsf{cake} \wedge \neg\Diamond\mathsf{apples}) \wedge \neg(\Diamond\mathsf{apples} \wedge \neg\Diamond\mathsf{cake})$
$= \Diamond\mathsf{cake} \wedge \Diamond\mathsf{apples}$

(52) $\textsc{Exh}_2$ $\textsc{Exh}_1$ You ate apple or cake.
   a. You ate apple.
      $\rightsquigarrow$ *You only ate apple.*
   b. You ate cake.
      $\rightsquigarrow$ *You only ate cake.*

This reasoning offers a mechanism to turn disjunctive meanings into conjunctive meanings. The sequel shows how Bar-Lev (2018b) adapts the recipe to the plural case and derives an account of participation inferences in positive sentences.

Applying Free Choice Reasoning to plural sentences Recall the non-cumulative sentence in (53a). As seen in section 3, Bar-Lev (2018b) assumes this sentence's underlying reading to be existential, e.g. (53a):

(53) a. The dancers smiled.
   b. **Unstrengthened meaning:**
      $\exists x \prec \iota\mathsf{dancers},\ \mathsf{smiled}(x)$
   c. **Attested meaning:**
      $\exists x \prec \iota\mathsf{dancers},\ \mathsf{smiled}(x)$
      $\wedge \forall x \prec \iota\mathsf{dancers},\ \mathsf{smiled}(x)$

To make this case completely parallel to the case of Free Choice and deliver the attested universal truth-conditions, we need a counterpart to the "*individual disjunct*" alternative of disjunction, seen in (52a) and (52b). Thinking of an existential as a grand disjunction, as in (54), these alternatives find a parallel in the *sub-domain alternatives* of the existential: alternatives where the existential is constrained to range over a smaller set of entities. In our specific case, where the domain of the existential are the atomic parts of *the dancers*, the sub-domain alternatives are simply sentences where *the dancers* is replaced by a plurality of smaller size X[16], as in (54).

(54) a. $\exists x \prec \iota\mathsf{dancers},\ \mathsf{smiled}(x)$
      $\leftrightsquigarrow \mathsf{smiled}(\mathsf{dancer\ 1}) \vee \mathsf{smiled}(\mathsf{dancer\ 2}) \vee \mathsf{smiled}(\mathsf{dancer\ 3}) \vee \ldots$
   b. $\exists x \prec X, \mathsf{smiled}(x)$
      where $X \prec \iota\mathsf{dancers}$

Assumptions about alternatives (to be modified)

1. $[\![\text{the NP}]\!]$ has as alternatives all pluralities X, such that $X \prec [\![\text{the NP}]\!]$

[16] Similar alternatives are needed in the literature on exceptives (Crnič, 2018; Hirsch, 2016).

Everything is in place to derive the universal inference of the non-cumulative sentence in (53). The recursive exhaustification structure is given in (55).

(55) Exh_1 Exh_2 [The dancers ∃-smiled.]$_\alpha$
Alts. to α:
$\{[\![\exists\text{-smiled}]\!](X) \mid X \prec [\![\text{the dancers}]\!]\}$

By recursive exhaustification, (55) will be strengthened to a universal meaning, just as in the Free Choice case (details in (56)). Informally, the alternatives to the constituent that Exh_2 heads can be paraphrased as *among the dancers, only X smiled*, where X is strict sub-plurality of *the dancers*. They can all be negated consistently by Exh_1 ; negating these alternatives is equivalent to asserting that either all the dancers smiled or none of them did. Together with the prejacent, this entails that all the dancers smiled, the desired result. (I use ∃-smile in the the LF representations below to emphasize the existential stipulation associated with the verb.)

(56) a. **Alternatives to** Exh_2 [The dancers ∃-smiled.]$_\alpha$:
- Only Marie-Lou ∃-smiled.
- Only the dancers who are not Marie-Lou ∃-smiled.
- Only Marius ∃-smiled.
- Only the dancers who are not Marius ∃-smiled.
- …

b. **Implicatures generated by Exh_2:**
1. Not only Marie-Lou ∃-smiled.
$\rightsquigarrow$ either Marie-Lou didn't smile or someone who wasn't Marie-Lou smiled.
2. Not only the dancers who aren't Marie-Lou ∃-smiled.
$\rightsquigarrow$ either Marie-Lou smiled or no one who wasn't Marie-Lou smiled.
$\rightsquigarrow$ either (Marie-Lou smiled and someone other than her did as well) or no one smiled*(together with 1)*
3. …

In conclusion, both Free Choice implicatures and the isomorphic case of non-cumulative sentence are accounted for by recursive exhaustification. As we will now see, the same process can account for distributive implicatures and cumulative sentences.

4.2 Distributive implicatures.

Recursive exhaustification can also account for distributive implicatures, repeated below in (57a). As we saw, these inferences serve as our model for the participation inferences of cumulative sentences, repeated in (57b).

(57) a. Every ambassador speaks Arabic, English or Mandarin.
$\rightsquigarrow$ at least one ambassador speaks Arabic.
$\rightsquigarrow$ at least one ambassador speaks English.
$\rightsquigarrow$ at least one ambassador speaks Mandarin.

b. The cooks ∃-opened every oyster.
$\rightsquigarrow$ Susan opened at least one oyster.

$\rightsquigarrow$ Adrian opened at least one oyster.
$\rightsquigarrow$ Walter opened at least one oyster.

Let me start by motivating the use of recursive exhaustification to account for distributive implicatures. Traditionally[17], distributive implicatures are obtained by negating alternatives where the disjunction is replaced by a smaller one, as in (58a). The inferences in (58a), together with the prejacent, correctly entail that at least one ambassador speaks Arabic, at least one English, etc.

(58) Exh Every ambassador speaks Arabic, English or Mandarin.

a. **Negated alternatives:**
   - not every ambassador speaks Arabic or English
     $\rightsquigarrow$ *some ambassador doesn't speak Arabic or English*
   - not every ambassador speaks Arabic or Mandarin
     $\rightsquigarrow$ *some ambassador doesn't speak Arabic or Mandarin*
   - not every ambassador speaks Mandarin or English
     $\rightsquigarrow$ *some ambassador doesn't speak Mandarin or English*

As Crnič et al. (2015) notes on similar examples, it also predicts, incorrectly, that one ambassador *only* speaks Arabic. Yet, the sentence can be uttered if all ambassadors are bilingual in two of the languages, so long as all languages are spoken by at least one ambassador[18]. Crnič et al. (2015) provide experimental evidence against the existence of strong implicatures such as (58b). Rather, the distributive implicatures derived by speakers are nothing more and nothing less than (59).

(59) Every ambassador speaks Arabic, English or Mandarin.
$\rightsquigarrow$ *some ambassador speaks Arabic*
$\rightsquigarrow$ *some ambassador speaks English*
$\rightsquigarrow$ *some ambassador speaks Mandarin*

As it turns out, a parallel discussion independently occurs in Kratzer (2003)'s discussion of cumulative readings of *every*. Consider the cumulative sentence with *every* in (60a). With the existential meanings posited in section 3, (60a) would be equivalent to "*every mistake was caught by copy-editor 1, copy-editor 2 or copy-editor 3*". Kratzer, working in an event semantics, also derives these readings. To obtain the missing inference that all copy-editors contributed, she considers adding a minimality condition, asserting that the event described is minimal. Translated to our system, this means exhaustifying the sentence against alternatives where "*the 3 copy-editors*" is replaced by smaller pluralities (e.g. *copy-editor 1 and copy-editor 2 caught every mistake*). In either system, the strong implicatures in (60b) are generated.

(60) a. The three copy-editors caught every mistake.
b. **Strong Dist implicatures:**
for every copy-editor, there is a mistake that only they caught.
c. Two mistakes: Add and Omit

[17] In either the Gricean tradition (Sauerland, 2004) or the grammatical tradition.
[18] And not every ambassador speaks all three languages. The latter inference comes from the *and* implicature which I haven't shown.

Copy-editor 1 caught Add
Copy-editor 2 caught Omit
Copy-editor 3 caught Omit

But Kratzer notes that this way of deriving participation inferences incorrectly predicts that (60a) is false in the scenario in (60b), since there isn't a mistake that either of the last two copy-editors caught alone.

Crnič et al. (2015)'s problem shows that the traditional account, which derives distributive implicatures from simple exhaustification, is inadequate. The reasons for this failure is that the distributive implicatures, like the Free Choice implicatures, are positive (cf (61)). It is not obvious how to derive them as the negation of an alternative to the sentence, or even a conjunction of such alternatives.

(61) Every ambassador speaks Arabic, English or Mandarin.
$\rightsquigarrow$ at least one ambassador speaks Arabic.
$\rightsquigarrow$ at least one ambassador speaks English.
$\rightsquigarrow$ at least one ambassador speaks Mandarin.

Just like Free Choice then, these implicatures seem to require something beyond simple exhaustification. As Bar-Lev and Fox (2016) show, recursive exhaustification can be applied to this case as well. Specifically, they propose the structure below:

(62) Exh Exh every ambassador speaks Arabic, English or Mandarin.

To understand the effect of the second layer of exhaustification informally, I will capitalize on the Gricean intuition already used in the case of Free Choice (c.f. section 4.1). Consider how hearers may interpret the alternative in (63) if it is relevant which languages are represented among the ambassadors.

(63) *Of these three languages, which are spoken by the ambassadors?*
Every ambassador speaks Arabic or Mandarin.
$\rightsquigarrow$ no ambassador speaks English.

By uttering (64), the speaker seems to convey that the other language, English, is not spoken at all. A speaker who does not utter (64) would thus convey that the alternative and its implicatures are false, namely that either not every ambassador speaks Arabic or Mandarin or some ambassador speaks English. Either way, this entails that some ambassador speaks English, which is the desired implicature.

(64) Every ambassador speaks Arabic, English or Mandarin.

Note that the implicature of (63) seems derived by negating "*some ambassador speaks English*". It must be assumed that this is an alternative to the original sentence, since by assumption, the alternatives and the prejacent have the same alternatives. This alternative can be obtained by replacing *every* with *some* and simplifying the disjunction to just the second disjunct "*English*". In particular, the *some/every* scale is a necessary ingredient of this computation. Though standard, I will list this assumption because it will be generalized to other quantifiers, in section 5.1.

Assumptions about alternatives (to be modified)

1. ⟦the NP⟧ has as alternatives all pluralities X, such that $X \prec$ ⟦the NP⟧
2. "*every*" has *some* as an alternative

The recursive exhaustification is a direct formal rendition of the Gricean intuition (cf). At a high level, the higher Exh will negate alternatives of the form in (63b), the counterpart of the alternatives that (64b) represents. Just as above, negating these inferences, together with the contribution of the prejacent will result in the attested distributive implicatures: some ambassador speaks Arabic, some ambassador speaks English, some ambassador speaks Mandarin.

(65) a. Exh Exh Every ambassador speaks Arabic, English or Mandarin.

b. **Excludable alternatives:**
   - Exh (Every ambassador speaks Arabic or Mandarin)
     $\leftrightarrow$ *every ambassador speaks Arabic or Mandarin and it is not the case that some ambassador speaks English.*
   - Exh (Every ambassador speaks Arabic or English)
   - Exh (Every ambassador speaks English or Mandarin)
   - …

Cumulative readings of *every*. Let us transpose this analysis to cumulative sentences with *every*. Remember that with an existential denotation for *open*, the underlying truth-conditions of (66a) are as in the black line of (66b).

(66) a. The three cooks opened every oyster.

b. **Truth-conditions:**
Every oyster was opened by a cook.
Every cook opened an oyster

I claim that this sentence, parsed as in (67), delivers the attested truth-conditions, including the participation inferences.

(67) Exh_1 Exh_2 The cooks ∃-opened every oyster

For simplicity, imagine the three cooks are Susan, Adrian and Walter. Then the alternatives must be as in (68a). The first six alternatives assert the cooks that opened an oyster are among some sub-group of Susan, Adrian and Walter and no one outside this group opened an oyster. It is possible for Exh_1 to negate all of these alternatives consistently. The resulting meaning is that all three cooks are oyster openers, the desired participation inference.

(68) a. **Alternatives:**

- Exh (S and W ∃-opened every oyster.)
  = *every oyster was opened by S or W*
  and no oyster was opened by A
- Exh (A and W ∃-opened every oyster.)
- Exh (S and A ∃-opened every oyster.)
- Exh (W ∃-opened every oyster.)
- Exh (S ∃-opened every oyster.)
- Exh (A ∃-opened every oyster.)
- Exh (S and W ∃-opened some oyster.)
- Exh (A and W ∃-opened some oyster.)
- Exh (S and A ∃-opened some oyster.)
- Exh (S ∃-opened some oyster.)
- Exh (A ∃-opened some oyster.)
- Exh (W ∃-opened some oyster.)

b. **Predicted implicatures:**
$\rightsquigarrow$ Susan opened an oyster.
$\rightsquigarrow$ Adrian opened an oyster.
$\rightsquigarrow$ Walter opened an oyster.

(69) a. Exh Exh Every oyster was opened by Susan, Adrian or Walter

b. **Predicted implicatures:**
$\rightsquigarrow$ Susan opened at least one oyster.
$\rightsquigarrow$ Adrian opened at least one oyster.
$\rightsquigarrow$ Walter opened at least one oyster.

This provides the basic account of participation sentences in cumulative sentences with *every*. In the next section, we will generalize the account to the other sentences in our dataset, including cumulative reading of *most* and ordinary cumulative readings.

4.3 Summary

This section provided a formal analysis of the participation inferences of cumulative readings of *every* and intransitive sentences, building on the analogies developed in the previous section. We saw how to derive both Free Choice and distributive implicatures through the procedure of recursive exhaustification. To transpose to the case of plural sentences, we assumed that sub-pluralities are alternatives to a given plurality. The next section simply applies the recursive exhaustification analysis to the other sentences in our dataset.

5 Extending to asymmetries, ordinary cumulative sentences and other quantifiers

The last section presented an account of participation inferences in simple non-cumulative sentences and cumulative readings of *every*, modeled after a similar account of Free Choice and distributive implicatures respectively. We assumed recursive exhaustification at the root of the tree and that alternatives were constructed following the principles below.

Assumptions about alternatives

1. $[\![\text{the NP}]\!]$ has as alternatives all pluralities X, such that $X \prec [\![\text{the NP}]\!]$
2. "*every*" has *some* as an alternative

However, only a limited portion of our dataset has been covered by the analysis. In this section, we derive participation inferences for other quantifiers than *every*, for ordinary cumulative sentences and provide an account of subject/object asymmetries. Doing so will give the opportunity to flesh out some more assumptions about alternatives and the position of exhaustification.

5.1 What about other quantifiers?

Upward-entailingness entails participation. The existential denotation of *open* also predicted too weak a meaning for other quantifiers than *every*:

(70) a. The ten cooks opened most oysters.
  b. **Truth-conditions:**
  Most oysters was opened by a cook.
  Every cook opened an oyster

(71) a. The ten cooks opened many oysters.
  b. **Truth-conditions:**
  Many oysters was opened by a cook.
  Every cook opened an oyster

These participation inferences mirror the distributive implicatures of the corresponding sentences:

(72) a. Most oysters were opened by Susan, Adrian or Walter.
  $\rightsquigarrow$ *Susan opened an oyster*
  $\rightsquigarrow$ *Adrian opened an oyster*
  $\rightsquigarrow$ *Walter opened an oyster*
  b. Many oysters were opened by Susan, Adrian or Walter.
  $\rightsquigarrow$ *Susan opened an oyster*
  $\rightsquigarrow$ *Adrian opened an oyster*
  $\rightsquigarrow$ *Walter opened an oyster*

Recursive exhaustification does derive the missing inference in all three cases above. Let me illustrate on the case of *most*. Just as with *every*, it must be assumed that *most* has *some* as an alternative[19] and that the cumulative sentence is parsed with two Exh, as in (73a). Some of the critical alternatives to the topmost Exh are given in (73b).

[19]Such an alternative is needed to account for indirect implicatures: *Joana didn't open most oysters* $\rightsquigarrow$ *Joana opened some oysters*.

(73) a. Exh Exh The cooks ∃-opened most oysters

b. **Alternatives:**

- Exh (Susan and Walter opened most oysters)
  ≈ *most oysters were opened by Susan or Walter and none by Adrian*
- Exh (Adrian and Walter opened most oysters)
- Exh (Susan and Adrian opened most oysters)
- Exh (Susan and Walter opened some oysters)
  ≈ *some but not most oysters were opened by Susan or Walter and none by Adrian*
- Exh (Adrian and Walter opened some oysters)
- Exh (Susan and Adrian opened some oysters)
- † Exh (Susan, Adrian and Adrian opened some oysters)
  ≈ *some but not most oysters were opened by Susan or Walter or Adrian*

All alternatives to the sentence but the one marked with † can be negated. Indeed, these alternatives are all false in a world where all three cooks opened an oyster, as can be seen from the paraphrases provided. Reciprocally, in any world where all these alternatives are false and the prejacent is true, all cooks opened an oyster. To see this, consider what would happen if Adrian, one of the cooks, didn't open any oyster. Then it would be true that most oysters were opened by Susan or Walter, since the prejacent assert that most oysters were opened by one of the cooks and we know Adrian didn't contribute to the collective effort. That would make the first alternative of (74b) true. Because this alternative is innocently excludable, we know that it can't possibly be true. By *reductio ad absurdum*, we show that Adrian must have opened an oyster. By symmetry, all cooks must have opened an oyster.

More generally, one can prove that recursive exhaustification will derive participation inferences for upward-entailing quantifiers quantifiers, provided they have *some* as an alternative:

The "*UE entails participation*" guarantee.

Let $\mathcal{Q}$ be a non-trivial quantifier, $\exists_C$ an existential quantifier with sub-domain alternatives.

If the following conditions hold:

- $\mathcal{Q}$ is upward-entailing,
- $\mathcal{Q}$ has *some* as its *only* alternative

then Exh Exh$(\mathcal{Q}x, \exists_C y, R(x,y))$ will be equivalent to the conjunction of the prejacent and the inference that $\forall y, \exists x, R(x,y)$

The formal proof of the result is complex and deferred to appendix A. Provided we ignore other alternatives these quantifiers might have, this result guarantees that all of the sentences in (74) will yield the participation inferences:

(74) a. The cooks opened most oysters.

b. The cooks opened many oysters.

Outside the guarantee's jurisdiction: downward-entailing quantifiers. The "*UE entails participation*" guarantee has clauses and it is natural to wonder what happens when one of these clauses is not met. For instance, what about participation inferences with quantifiers which are not upward entailing? It turns out that such quantifiers don't seem to have participation inferences to start with.

Take the case of *no* in (75a). Participation inferences are empirically not attested ; they would, in this case, contradict the assertion. The process of recursive exhaustification is sensitive to this logical property: because the prejacent's meaning in (75b) is stronger than all of the alternatives and no exclusion is possible.

(75) a. The three cooks opened no oysters.
$\not\rightsquigarrow$ *Adrian opened an oyster*

b. **Underlying meaning:**
$\neg\exists x \in \mathsf{oyster}, \exists y \prec \mathsf{Susan} + \mathsf{Walter} + \mathsf{Adrian}, \mathsf{opened}(x)(y)$

c. **Alternatives:**
- Susan and Walter opened no oysters
$\rightsquigarrow \neg\exists x \in \mathsf{oyster}, \exists y \prec \mathsf{Susan} + \mathsf{Walter}, \mathsf{opened}(x)(y)$
- …

For the case of downward-entailing quantifiers such as *less than 10*, the empirical picture is more difficult. (Bayer, 2013, p. 198) reports that (76) does not have a participation inference it can be uttered even when Michael didn't wash a car.

(76) Michael and LaToya (together) washed fewer than three cars.
$\overset{?}{\rightsquigarrow}$ *Michael washed a car.*

Correspondingly, whether these quantifiers do give rise to distributive implicatures is not clear:

(77) Less than 10 ambassadors speak Arabic, English or Mandarin.
$\overset{?}{\rightsquigarrow}$ *one ambassador speaks Arabic*

To decide whether participation inferences are predicted in this case, we must solve a very simple problem: unlike upward-entailing quantifiers, we cannot assume that "*less than 3 oysters*" has "*some oysters*" as its sole alternative. If that were the case, it would be possible to negate the latter and strengthen "*fewer than 3 cars*" to "*no cars*". There are several options to confront that problem: one could assume that *some* is not an alternative to *fewer than three* or one could assume that *some* is not the only alternative to *fewer than 3*.

For simplicity, I take the latter route: I assume more generally that quantifiers can only have quantifiers of the same monotonicity as alternatives. This is probably an oversimplification[20] but I believe the lack of participation inference will be predicted under a more sophisticated set of assumptions.

[20] The question of the alternatives to *fewer than n* is complex. First, it is likely that *fewer than n* has other numbers as alternatives (e.g. *fewer than m*). But on their own, these alternatives generate unattested implicatures (*fewer than n but no fewer than n-1*) (Fox and Hackl, 2006). To avoid implicatures, one may invoke alternatives of the form *more than n* (Mayr, 2013). These alternatives create symmetries which make exhaustification vacuous. It may make the

If *some* is not an alternative to *less than 3 cars*, then all alternatives, when exhaustified as in (78), contradict the prejacent: they can all be negated but they don't yield a stronger meaning.

(78) **Alternatives:**

- Exh (LaToya washed fewer than 3 cars)
  LaToya washed fewer than 3 cars and not (Michael washed fewer than 3 cars)
- Exh (Michael washed fewer than 3 cars.)
  Micheal washed fewer than 3 cars and not (LaToya washed fewer than 3 cars).

We can summarize the discussion in two points: first, downward-entailing quantifiers do not seem to give rise to participation inferences, following Bayer (2013) ; second, the recursive exhaustification mechanism can derive this result, assuming that quantifiers only have alternatives of the same monotonicity.

In short, we have the following assumptions about alternatives:

Assumptions about alternatives

1. $[\![$the NP$]\!]$ has as alternatives all pluralities X, such that $X < [\![$the NP$]\!]$
2. upward-entailing quantifiers have *some* as an alternative.
3. quantifiers' alternatives must be of the same monotonicity.

5.2 Asymmetries in cumulative readings

Asymmetries with *every*: the data As we saw in section 1.3, the cumulative reading of *every*, *most* and other singular quantifiers is unavailable if the quantifier stands in subject position.

(79) a. Every cook opened the ten oysters. (#cumulative)
b. The ten cooks opened every oyster. (✓ cumulative)

Although Kratzer (2003) initially described the asymmetry as an asymmetry in thematic positions, Champollion (2010); Zweig (2008) shows that the asymmetry is one of c-command as expressed in the generalization below:

Generalization

A cumulative reading between *every* and a plural-referring expression is only available when *every* is c-commanded by the plural-referring expression's base position.

Can this generalization be captured in the analysis of the present work?

Analysis. Within the theory of this chapter, the underlying meanings of sentences with *every* in subject position and the sentences with *every* in object position are parallel: the plural-referring expression, translated as an existential, takes scope under the universal quantifier.

set of alternatives too symmetrical: *fewer than 3* has an existence implicature, which should not be overlooked.

(80) a. Every cook ∃-opened the three oysters.
b. **Underlying meaning:**
$\forall y \in \mathsf{cook}, \exists x \prec \iota\mathsf{oysters}, \mathsf{open}(x)(y)$

(81) a. The three cooks ∃-opened every oyster.
b. **Underlying meaning:**
$\forall y \in \mathsf{oyster}, \exists x \prec \iota\mathsf{cooks}, \mathsf{open}(y)(x)$

This lack of asymmetry in underlying meanings means that the sentence will have parallel truth-conditions (*mutatis mutandis*) under negation. This prediction is borne out: as reported in Križ and Chemla (2015), the negation of sentences like (80) have the truth-conditions in (83a). These truth-conditions mirror[21] the truth-conditions of the negation of (81), which we already discussed in section 2, repeated in (83b).

(83) a. Not every cook opened the three oysters.
=*not every cook opened an oyster*
=*some cook opened no oyster*
b. The three cooks didn't open every oyster.
=*not every oyster was opened by a cook*
=*some oyster wasn't opened by any cook*

The parallel in underlying meanings suggest that any difference between subject *every* sentences and object *every* sentences is due to the way the two sentences are strengthened in positive environments.

Problematically, using the recursive exhaustification at root that we have been using so far, as in (84), is bound to deliver the same strengthening for both sentences. (I will use the symbol Exh^2 as an abbreviation for recursive exhaustification). Indeed, these sentences have the same underlying meaning and identical alternatives. Both sentences, as it stands, will receive a cumulative reading, contrary to fact.

(84) a. Exh^2 Every cook ∃-opened the three oysters.
$\rightsquigarrow$ *every cook opened an oyster and every oyster was opened by a cook*.
b. Exh^2 The three cooks ∃-opened every oyster.
$\rightsquigarrow$ *every cook opened an oyster and every oyster was opened by a cook*

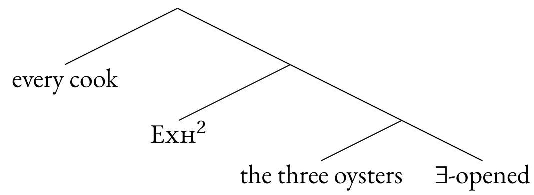
The reason for the asymmetry stems, I contend, from the scope of Exh. So far, I have assumed that all exhaustification happens at root. Consider what would happen for different placements of Exh^2. When *every* is in subject position, Exh^2 can be inserted in the scope of *every*, while still c-commanding *the three oysters*, as shown in (85).

[21] Interestingly, these sentences seem to differ in their implicatures. While (82a) implicates that some cook opened the three oysters, (82b) implicates that some cooks opened some oysters. If the treatment of participation inferences that I propose is correct, these implicatures would parallel the strengthened indirect implicature of (82):

(82) I didn't show every boy some of my paintings. $\rightsquigarrow$ *I showed some boy some but not all of my paintings.*

To my knowledge, these types of implicatures are not accounted for, or discussed by previous literature.

(85)

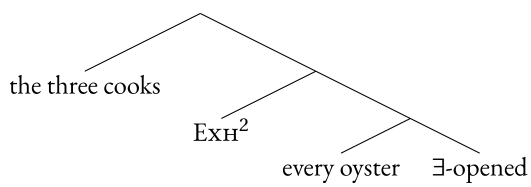


In this position, Exh^2 applies directly to the existential over subsets of oysters, without intervening quantifers. As we saw in section 4, this is precisely the configuration in which a Free Choice-like inference is generated. Concretely, this means that *the three oysters* is strengthened to a universal. As a result, the sentence receives a doubly distributive reading.

(86) $[\![(85a)]\!] = \forall y \in \mathsf{cook},\ \text{Exh}^2(\exists x \prec \mathsf{oyster}_1 + \mathsf{oyster}_2 + \mathsf{oyster}_3,\ \mathsf{opened}(x)(y))$
$\quad\quad$ **alts:** $\exists y \prec \mathsf{oyster}_1 + \mathsf{oyster}_2,\ \mathsf{opened}(x)(y)), \ldots$
$\quad = \forall x \in \mathsf{cook}, \forall y \prec \mathsf{oyster}_1 + \mathsf{oyster}_2 + \mathsf{oyster}_3,\ \mathsf{opened}(x)(y)$

By contrast, when *every* is in object position, there is no position where Exh^2 can be placed in which it both c-commands the plural and is c-commanded by *every*. There are embedded positions but they fail to c-command the plural-referring expressions *the three cooks*. Failing to c-command *the three cooks* means that the alternatives which Exh compares will not contain any sub-domain alternatives (e.g. *the first two cooks*) ; no cumulative strengthening is derived. This is illustrated in (87).

(87) a.



b. $[\![(86a)]\!] = \iota\mathsf{cooks}\ \lambda y.\ \text{Exh}^2(\forall x \in \mathsf{oysters},\ \mathsf{opened}(x)(y))$
$\quad\quad$ **alts:** $\exists y \in [\![\text{oyster}]\!],\ \mathsf{opened}(x)(y))$
$\quad = \iota\mathsf{cooks}\ \lambda y.\ \forall x \in \mathsf{oysters},\ \mathsf{opened}(x)(y)$

The following chart summarizes our discussion so far:

	subject *every*	object *every*
root Exh^2	cumulative	cumulative
embedded Exh^2	doubly-distributive	vacuous

From this chart, I make the following simple proposal: Exh^2 must apply at all positions (Magri, 2011). This type of structure is summarized in (88) below.

(88) a. Exh^2 the three cooks Exh^2 ∃-opened every oyster.
b. Exh^2 every cook Exh^2 ∃-opened the three oysters.

With object *every*, this assumption is innocuous because embedded Exh^2 does not result in strengthening as seen above. The composition proceeds as if there was only a root Exh^2 ; as we saw, this is how the cumulative reading is generated. With subject *every*, an embedded

$\textsc{Exh}^2$ results in the attested doubly distributive reading. The root $\textsc{Exh}^2$ cannot strengthen the meaning beyond further.

In this theory, the reason for the asymmetries between the cumulative reading of object *every* and the doubly distributive reading of subject *every* is that only in the latter case, there is a position below the quantifier where the sub-domain alternatives are visible and strengthening can happen. Assuming that strengthening not only can but must happen, through the postulate that exhaustification applies in all positions, creates the split between subject and object *every* in positive environments.

The assumption that recursive exhaustification must apply in all positions is the last assumption that I will make in this chapter about cumulativity. Adding it to the list of assumptions, we arrive at the following final set of assumptions:

Assumptions

1. **assumptions about composition**
   - verbs have existential meanings (e.g. ∃-*opened*)
   - recursive Exh in positive environments *in all positions.*.
2. **assumptions about alternatives**
   - $[\![\text{the NP}]\!]$ has as alternatives all pluralities X, such that $X \prec [\![\text{the NP}]\!]$
   - "*every*", *many*, *most* have *some* as an alternative.
   - quantifiers' alternatives must be of the same monotonicity.

5.3 Ordinary cumulative readings

With these assumptions, we are ready to come back to ordinary cumulative readings, like (89). Ordinary cumulative readings do not raise particular issues for the theory, but the full set of assumptions made in this chapter are necessary to account for it.

(89) a. The ten cooks opened the fifteen oysters.
  b. The ten cooks didn't open the fifteen oysters.

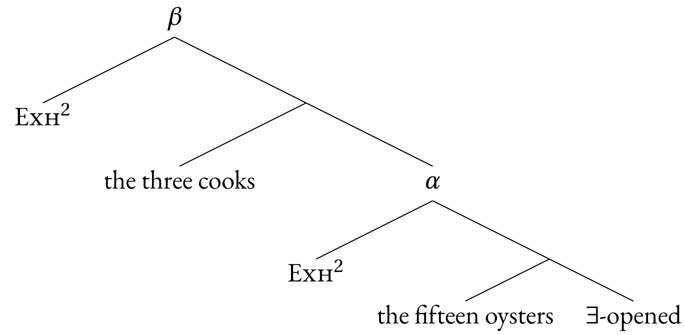
The negative case in (89b) is the simplest. In the scope of negation, no strengthening through $\textsc{Exh}^2$ occurs ; by the existential meaning of *open*, both plural-referring expressions are interpreted as existentials in the scope of negation. The predicted meaning matches the attested meaning: *no cook opened any oyster*.

(90) a. not [the ten cooks ∃-opened the fifteen oysters]
  b. $[\![(90a)]\!] = \neg\exists x \prec \iota\mathsf{cooks}, \exists y \prec \iota\mathsf{oysters},\ \mathsf{opened}(y)(x)$
  $\leftrightsquigarrow$ *no cook opened any oyster*

In the positive case of (91a), exhaustification is active. As seen in the last section, we need to include one $\textsc{Exh}^2$ in all positions.

(91) a. The ten cooks opened the fifteen oysters.

b.



The computation is arduous , but we can develop a simple intuition for how it will run. In the embedded position α, $\textsc{Exh}^2$ operates over the sub-domain alternatives of the existential represented by *the oysters*, cf (92). Because there is no intervening quantifier, *the oysters* will be strengthened to a universal (i.e. a Free Choice-like inference), cf (92a).

(92) $[\![\alpha]\!] = \lambda X.\ \textsc{Exh}^2(\exists y \prec \iota\mathsf{oysters}, \exists x \prec X,\ \mathsf{opened}(y)(x))$
alts: $\exists y \prec \mathsf{oyster}_1 + \mathsf{oyster}_2, \exists x \prec X,\ \mathsf{opened}(y)(x)), \ldots$
$= \forall y \prec \iota\mathsf{oysters}, \exists x \prec X,\ \mathsf{opened}(y)(x)$
$\leftrightarrow$ *every oyster was opened by one of X*

In the root position β, $\textsc{Exh}^2$ operates over the sub-domain alternatives of *the cooks*[22]. Here however, the existential represented by *the cooks* finds itself in the scope of the universal corresponding to *the oysters* which was created by the first strengthening. The situation is entirely parallel to the case of cumulative readings of *every* ; $\textsc{Exh}^2$ will generate a distributive-like implicature. Together with the prejacent, this will create the cumulative reading.

(93) a. $[\![\beta]\!] = \textsc{Exh}^2(\forall y \prec \iota\mathsf{oysters}, \exists x \prec \iota\mathsf{cooks}\ \mathsf{opened}(y)(x))$
alts: $\exists y \in [\![\text{oyster}]\!], \exists x \prec \mathsf{Joana} + \mathsf{Marius},\ \mathsf{opened}(y)(x)), \ldots$
$= \forall y \prec \iota\mathsf{oysters}, \exists x \prec \iota\mathsf{cooks},\ \mathsf{opened}(y)(x)$
$\wedge \forall x \prec \iota\mathsf{cooks}, \exists y \prec \iota\mathsf{oysters},\ \mathsf{opened}(y)(x)$
$\leftrightarrow$ *cumulative reading*

All in all, the computation raises no particular issue. The object is first strengthened to a universal meaning ; from then on, the situation is entirely parallel to the case of cumulative readings of *every*.

5.4 Summary

In this section, three extensions to the basic theory of cumulative readings of *every* were studied. We have seen that cumulative readings of other quantifiers than *every* and their participation inferences can be captured by the same process of recursive exhaustification. We have seen how asymmetries follow from assuming that exhaustification happens wherever it can.

[22] In addition to the sub-domain alternatives of *the oysters*.

We have seen how ordinary cumulative sentences follow from applying recursive exhaustification twice. At the end of this section, we reach the following set of assumptions about exhaustification and alternatives:

Assumptions

1. **assumptions about composition**
   - verbs have existential meanings (e.g. $\exists$-*opened*)
   - recursive Exh in positive environments *in all positions.*.
2. **assumptions about alternatives**
   - $[\![$the NP$]\!]$ has as alternatives all pluralities X, such that $X \prec [\![$the NP$]\!]$
   - There is no $\forall$-VP alternative to $\exists$-VP.
   - "*every*", *many*, *most* have *some* as an alternative.
   - quantifiers' alternatives must be of the same monotonicity.

Together, these assumptions constitute the solution proposed to the problem of cumulative readings of *every*. In the next section, we move to a broader discussion by comparing this approach to alternatives from the literature and issues and extensions are discussed.

6 Comparison to previous literature and broader issues

This section opens a broader discussion on cumulativity. In the first section, I discuss approaches from previous literature and compare them to the current system. In the second section, I highlight some differences with the homogeneity system of Bar-Lev (2018b), which the analysis draws much from. In the third section, I discuss how collective action can be incorporated in the current analysis.

6.1 Previous literature

In the next subsection, I review several alternative solutions to the problem raised by cumulative readings of *every*.

6.1.1 Event semantics

Starting with Schein (1993), cumulative readings of *every* have often been accounted for in terms of Neo-Davidsonian event semantics (Champollion, 2016b; Ferreira, 2005; Ivlieva, 2013; Kratzer, 2000; Schein, 1993). In such accounts, the denotation of *every* is typically changed to an event-sensitive denotation which allows to interact with event composition.

Setting variation in technical details aside, (94) illustrates a prototypical derivation, loosely following Champollion (2016b): *every oyster* combines with the VP to form a predicate true of events which are sums of oyster-openings, in which all oysters were opened. The subject "*the cooks*" is asserted to be the agent of one such event.

(94) a. The cooks opened every oyster.

b. **LF:** $\exists e.$ [the cooks AGENT] [every oyster] $\lambda x.$ opened [x THEME]

c. $[\![\text{every oyster}]\!] = \lambda p.\lambda e.\ (e, \bigoplus \text{oyster}) \in {}^{*} \left[\lambda x.\lambda e.\ p(e) \wedge \textsc{Theme}(e) = x \wedge \mathsf{oyster}(x)\right]$
$[\![\text{VP}]\!] = \lambda e.\ (e, \bigoplus \text{oyster}) \in {}^{*} \left[\lambda x.\lambda e.\ \mathsf{open}(e) \wedge \textsc{Theme}(e) = x \wedge \mathsf{oyster}(x)\right]$
$=$ *e is a sum of oyster openings in which all oysters were opened*

d. $[\![(94a)]\!] = \exists e, \textsc{Agent}(e) = \iota\mathsf{cooks} \wedge (e, \bigoplus \text{oyster}) \in {}^{*} \left[\lambda x.\lambda e.\ \mathsf{open}(e) \wedge \textsc{Theme}(e) = x \wedge \mathsf{oyster}(x)\right]$

The resulting truth-conditions correctly assert that in one or more openings, the cooks did open the oysters, which is the cumulative reading.

For the event analysis to work, the denotation of *every* need to be adapted to event semantics in order to deliver the correct reading. The semantics of other quantifiers which partake in cumulative readings, like non-partitive *most*, would similarly need to be adapted.

The need for adapting quantifier denotations, I argue, is a downside of the event solution. Let me explain why. First, independently from any considerations of cumulativity, the theory of quantification in event semantics has problems that do not arise in a more traditional generalized quantifier theory. Downward-entailing quantifiers, for instance, "*require particular care*", as Kratzer (2003) puts is, because naive adaptations of quantifiers into event semantics generate unattested reading.

To be sure, several solutions to the problem of quantification in event semantics have been propose(Champollion, 2014a; de Groote and Winter, 2015; Krifka, 1989; Winter and Zwarts, 2011).

For instance, Champollion (2014a); de Groote and Winter (2015); Winter and Zwarts (2011) each propose a system in which sentences are composed in such a way that quantifiers effectively always take high scope with respect to the event existential.

Although this may seem orthogonal to the problem of cumulative readings of *every*, it isn't: solutions to one problem may interact poorly with solutions to the other problem. For instance, Champollion (2014b) notes that, at least provisionally, the theory of quantification in Champollion (2014a) cannot be reconciled with the theory of *every* in Champollion (2016b), which is used for cumulative readings.

By contrast, the present account latches on to a standard theory of quantification and retains standard denotations for quantifiers.

6.1.2 Plural projection framework

In Haslinger and Schmitt (2018), the plural projection framework of Schmitt (2013) is applied to cumulative readings of *every*. The plural projection framework relies on two premises: 1) pluralities are cross-categorial: in addition to pluralities of individuals, there are pluralities of propositions, predicates, etc., 2) pluralities combine cumulatively.

To account for cumulative readings of *every*, Haslinger and Schmitt (2018) assume that *every oyster* combines with the relation denoted by the verb to form a plural predicate, as in (95a). This plural predicate may compose *cumulatively* with the plural subject to form the cumulative reading of the sentence.

(95) a. The cooks opened every oyster.

b. ⟦opened every oyster⟧ = $\mathsf{open}(\mathsf{oyster}_1) \oplus \ldots \oplus \mathsf{open}(\mathsf{oyster}_n)$

The composition of the VP in an ordinary cumulative sentence is different but yields the same result.

(96) a. The cooks opened every oyster.

b. ⟦opened the oysters⟧ = $\mathsf{open}\left(\mathsf{oyster}_1 \oplus \ldots \oplus \mathsf{oyster}_n\right)$
$= \text{open}(\mathsf{oyster}_1) \oplus \ldots \oplus \text{open}(\mathsf{oyster}_n)$

This parallel between ordinary cumulative sentences and cumulative sentences with *every* makes accounting for the difference in their homogeneity properties challenging. Indeed, as discussed in Chatain (2021), a natural implementation of homogeneity in the plural projection framework (as in e.g. Schmitt (2017)) will make the two sentences exactly equivalent under negation, contrary to what we observe:

(97) a. The cooks didn't open every oyster.

b. The cooks didn't open the oysters.

6.2 Similarity and differences with Bar-Lev (2018b)

In this section, I return to some of the differences between the presentation of homogeneity in this account and the account of Bar-Lev (2018b). The system for homogeneity used in this work departs in three ways from Bar-Lev (2018b).

The first departure concerns the origin of the underlying existential meaning. In Bar-Lev (2018b), the underlying existential meaning is the result of applying a covert operator, the distributivity operator[23] ; in this system, it directly stems from the semantics of the verb.

The second departure is the semantics of the exhaustivity operator. In Bar-Lev (2018b), the exhaustivity operator is applied only once but its semantics is modified so that it can both negate alternatives (exclusion) and assert alternatives (inclusion). In this work, the effect of "*inclusion*" is achieved by double exclusion.

The third departure concerns the source of the alternatives needed for exhaustification. In the present account, these are alternatives obtained by replacing pluralities with sub-pluralities. In Bar-Lev (2018b), these alternatives are sub-domain alternatives, obtained by varying the restriction of the distributivity operator.

(98) a. **Bar-Lev (2018b):** Exh $_{\text{II+IE}}$ the dancers ∃-Dist smiled.

b. **This work:** Exh $_{\text{IE}}$ Exh $_{\text{IE}}$ the dancers ∃-smiled

The chart below summarizes the differences:

	This work	**Bar-Lev (2018b)**
Existential	∃-open	$\exists_C$-Dist
Alternatives	$\text{Alt}(\iota\mathsf{cooks}) = \{\iota\mathsf{fr.\text{-}cooks}, \ldots\}$	$\text{Alt}(C) = \{C_1, C_2, \ldots\}$
Exhaustification	Recursive exhaustification	Innocent exclusion + inclusion

[23] And then, more generally, the cumulativity operator.

Let me comment on each departure in turn.

As already mentioned, recursive exhaustification was chosen because it accounts for distributive implicatures. In the analysis proposed, participation implicatures of cumulative sentences were modeled as distributive implicatures. Innocent Inclusion, as it currently stands, does not seem able to derive these implicatures correctly. (99) illustrates what innocent inclusion predicts for distributive implicatures[24].

(99) a. Every ambassador speaks English or Mandarin.

b. **Alternatives:**

- Some ambassador speaks Mandarin
- Some ambassador speaks English
- Every ambassador speaks English
- Every ambassador speaks Mandarin
- Some ambassador speaks English and Mandarin
- Every ambassador speaks English and Mandarin

c. **Innocently Excludable:**

- Some ambassador speaks English and Mandarin
- Every ambassador speaks English and Mandarin

d. **Innocently Includable:** none

More research is needed to make Innocent Inclusion compatible with distributive implicatures and, concomitantly participation inferences in cumulative sentences with *every*.

Let us now turn to the question whether the underlying existential meaning should be associated with the verb or with a covert distributivity operator. First, note that all readings predicted by the current account can be predicted by a system *à la* Bar-Lev (2018b): it suffices to replace instances of the verb ∃-*open* with [∃-Dist *open*] (or an appropriately polymorphic version thereof, cf Bar-Lev (2018a)). The system of Bar-Lev (2018b) is therefore strictly more powerful.

(100) $\text{Exh}_{\text{IE}}\text{Exh}_{\text{IE}}$ the cooks [every oyster] λx $\exists_C$-Dist opened x

The problem is that the distributivity operator is credited with properties that don't hold of cumulativity. For instance, Champollion (2016a) notes that the distributivity operator is typically a quantifier over *atoms*. With a contextually supplied partition (or cover) of the object at hand, non-atomic readings are possible (Schwarzschild, 1996), but the presence of a rich context is paramount.

[24] This reasoning should also consider what happens if some of the alternatives are pruned. To avoid overgeneration with inclusion, Bar-Lev (2018b) proposes a system in which pruning only affects which alternatives are negated or asserted in the end, but does not affect which alternatives are innocently excludable/includable. This means that the result of pruning is necessarily a weaker statement ; since the statement derived without pruning does not entail the distributive implicature, neither will any statement obtained by pruning some alternatives.

By contrast, cumulative readings, as we will see in the next section, do not require atomicity, regardless of context. This discrepancy between cumulativity and distributivity is challenging to explain if both readings originate from the same operator. This argument is not iron-clad as it depends on implementation specifics but it provides motivation to pursue the alternative presented in this work.

The final difference concerns the source of the alternatives used by Exh: up till now in this work, the alternatives have been sub-pluralities ; in Bar-Lev (2018b), they are obtained by varying covert restrictions on elements of the sentences. Nothing said so far can help decide between these two options, apart from convenience of presentation. In the next section, we will see that adopting covert restriction can help us deal with the cases of collective-cumulativity left aside in this work.

6.3 Collective interpretations

So far, the cumulative examples were given a paraphrase as in (101b). This paraphrase is incorrect if we allow for the possibility that two or more cooks could open an oyster as a group (e.g. one holds a part of the shell while the other does the knife work).

(101) a. The ten cooks opened every oyster.
b. **Truth-conditions:**
Every cook opened an oyster.
Every oyster was opened by a cook.

Switching to predicates with a more plausible collective interpretation, as in (102), a paraphrase corresponding to (102b) comes out too strong: it implies that every jigsaw puzzle was completed *individually*. A more adequate paraphrase would be (102c)[25] where it is merely implied that every player participated in the completion of a jigsaw puzzle.

(103) a. The ten players completed every jigsaw puzzle.
b. **Incorrect truth-conditions:**
Every jigsaw puzzle was completed by a player.
Every player completed a jigsaw puzzle.
c. **Correct truth-conditions:**
Every jigsaw puzzle was completed by a group of players.
Every player was part of a group of players that completed a jigsaw puzzle.

To explain the truth-conditions of (103c), some amendments to the theory are needed. First, we'll need to switch to a restriction-based view on the source of alternatives. The reasons for this will become apparent later on. Second, following Bar-Lev (2018a), we modify the underlying denotation of verbs (or verb+operator) so that it allows for group action.

[25] This is not case of ambiguity between a *collective* and *non-collective* reading. If the non-collective reading in (102b) was available, it could be targeted by denials (Bar-Lev, 2020), but it does not seem to:

(102) _ The ten players completed every jigsaw puzzle _ That's not true: this jigsaw puzzle was completed by the players together! # Oh, you meant *together*…

For a more general argument against a collective/non-collective ambiguity, see Glass (2018).

(104) $[\![\exists_{C,C'} \text{ completed}]\!](X)(Y) = 1 \text{ iff } \begin{array}{l} \exists x \prec X, x \in C \wedge \exists X',\ x \prec X' \prec X \\ \exists y \prec Y, y \in C' \wedge \exists Y',\ y \prec Y' \prec Y \wedge [\![\text{completed}]\!](X)(Y) \end{array}$

The new denotation can be informally paraphrased: X completed Y just in case one of X "*participated*", with others among X, to completing one or more jigsaw puzzles. Without the covert restrictions C or C', this denotation is equivalent to: some among X completed some among Y. By comparison, the original existential stipulation would simply have been: some x among X completed some y among Y.

With the original analysis, a sentence like (105a) would be given the meaning in (105b). But (105b) is too weak: it is compatible with all jigsaw puzzles having been completed in teams.

(105) a. The ten players didn't complete every jigsaw puzzle.
 b. **Truth-conditions under original existential denotation:**
 Not every jigsaw puzzle was completed by one individual player.

With the new collective-friendly denotation in (104), the truth-conditions are as in (106a), which one can informally paraphrase as in (106b). This is more correct[26] ; it correctly comes out false when all puzzles are completed in teams.

(106) a. **Raw truth-conditions:**
 $\neg\forall y \in \text{jigsaw},$
 $\quad \exists x \prec \iota\text{players}, x \in C \wedge \exists X',\ x \prec X' \prec \iota\text{players} \wedge$
 $\quad \exists y' \prec y, y \in D_e \wedge \exists Y',\ y' \prec Y' \prec y \wedge [\![\text{completed}]\!](X)(Y')$
 $\leftrightsquigarrow$
 $\neg\forall y \in \text{jigsaw},$
 $\quad \exists x \prec \iota\text{players}, x \in C \wedge \exists X',\ x \prec X' \prec \iota\text{players} \wedge [\![\text{completed}]\!](X)(y)$
 b. **Truth-conditions:**
 Not every jigsaw puzzle was completed by one or more players.

In positive sentences, participation inferences can be derived just as in the original theory: by recursively exhaustifying the underlying meaning, as in (107a). To see through the computation, note that the configuration highlighted in red in (107b) is the same configuration in which distributive implicatures arise: a universal scoping over an existential with subdomain alternatives ("*every jigsaw puzzle is such that some player ...*"). Exh^2 thus generates a distributive implicature (in gray) that every player is such that for some jigsaw puzzle, they are part of a group that completed that puzzle.

(107) a. Exh Exh The players completed every jigsaw puzzle.
 b. $\text{Exh}^2 \big(\forall y \in \text{jigsaw}, \exists x \prec \iota\text{players}, x \in C \wedge \exists X',\ x \prec X' \prec \iota\text{players} \wedge [\![\text{completed}]\!](y)(X)\big)$
 $= \forall y \in \text{jigsaw}, \exists x \prec \iota\text{players}, x \in C \wedge \exists X',\ x \prec X' \prec \iota\text{players} \wedge [\![\text{completed}]\!](y)(X)$
 $\wedge\ \forall x \prec \iota\text{players}, x \in C \rightarrow \forall y \in \text{jigsaw}, \wedge \exists X',\ x \prec X' \prec \iota\text{players} \wedge [\![\text{completed}]\!](y)(X)$
 = *every jigsaw puzzle was completed by some players*
 and every player is part of a group of players who completed a jigsaw puzzle.

[26]Not entirely, as they ignore what Kriz (2015) calls Sidewards Homogeneity: some players could team up with non-players to complete jigsaw puzzles. I leave the question of how to deal with this form of homogeneity open.

To derive this result, it is critical that one existential over players ($\exists x \prec \iota\mathsf{players},$) is strengthened and not the other ($\exists X'$). This is achieved because only the former existential has subdomain alternatives. This in turn follows from the assumption made earlier that covert domain restriction are the source of alternatives, not the plurality itself.

All in all, the theory can be adjusted to accommodate collective action. These changes do not alter but obscure the main ideas of the theory, which is I have avoided them in the main presentation.

Conclusion

This paper proposed a new theory of cumulative readings of *every* and quantifiers in object position. This theory of cumulativity is special in that it does justice to the homogeneity properties of cumulative readings and gives an account of the truth-conditions of negative sentences, not frequently addressed in previous approaches.

The theory, whose assumptions are repeated in the box below, builds at its core existential quantification in the meaning of the verb. With this assumption alone, the negative versions of all sentences in the data set are all covered. This assumption does not explain the participation inferences of positive sentences.

I showed an analogy between these inferences and the implicatures of Free Choice/distributive implicatures. I showed that an account in terms of recursive exhaustification delivers the correct inferences for both Free Choice/distributive implicatures and participation inferences. The account can be extended to account for the cumulative readings of other quantifiers, and for asymmetries in cumulative readings.

Assumptions

1. verbs have existential meanings (e.g. ∃-*open*).
2. recursive Exh in positive environment *in all positions.*.
3. each sub-plurality is an alternative to a plural-referring expression.
4. existential alternatives to quantifiers.

One limitation of the theory is that it does not do justice to all cumulative readings of quantifiers. In particular, cumulative readings of modified numerals (Brasoveanu, 2013; Buccola and Spector, 2016; Landman, 2000; Schein, 1993) are not dealt with.

(108) More than 15 children ate less than 8 ice-creams.

It is interesting to note that two theories of such readings start of with an underlying meaning akin to "*some of the children ate some of the ice-creams*". In Brasoveanu (2013), the assertive component of the modified numeral is existential (its post-suppositional component contains the numeric test). In Landman (2000), this corresponds to the reading . It would be interesting to see if and how such existential readings can be connected to the existential readings posited in the current theory. This endeavor is left to future research.

A The UE guarantee

We set out to prove the following fact:

> **The "*UE entails participation*" guarantee.**
> If a quantifier $\mathcal{Q}$ is non-trivial, permutation-invariant, conservative and upward-entailing (i.e. $\mathcal{Q}x, P(x)$ entails $\exists x, P'(x)$ whenever $P \subset P'$) and $\mathcal{Q}$ has *some* as its only alternative, then $\text{Exh}\text{Exh}(\mathcal{Q}x, \exists y \in D, R(x,y))$ (where D has sub-domain alternatives) will entail the exhaustive participation inference that $\forall y \in D, \exists x, R(x,y)$

Let's start by unpacking the assumptions. Because of the assumption on Q, we know that the truth of a statement like $\mathcal{Q}x \in D, P(x)$ will only depend on the cardinality of $P \cap D$. Call quant the set of cardinalities $P \cap D$ where $\mathcal{Q}x \in D, P(x)$ is true. Because $\mathcal{Q}$ is non-trivial, quant is non-empty. Because it is upward-entailing, it contains one number between 1 and the cardinality of D.

For clarity, I write D for the full domain of the existential quantifier, D_Q the domain of $\mathcal{Q}$, C for any strict non-empty sub-domain of D, $\bar{C}$ for the complementary of C in D (which is also non-empty because C is a strict subset). I write Exh_{h} for the higher exhaustification operator and Exh_{l} for the lower exhaustification operator.

By our assumptions, the alternatives that Exh_{h} operates on are of one of two types:

(109) a. $\text{Exh}_l\left(\mathcal{Q}x, \exists y \in C, R(x,y)\right)$
b. $\text{Exh}_l\left(\exists x, \exists y \in C, R(x,y)\right)$

Call these alternatives the *higher* alternatives.

Let's gain some insight into what the *higher* alternatives mean. Each of them is the exhaustification of a prejacent against the set of alternatives S. S is the same for all formulas in (109), let's call this the set of *lower* alternatives. It is composed of formulas of the following form:

(110) a. $\mathcal{Q}x, \exists y \in C, R(x,y)$
b. $\exists x, \exists y \in C, R(x,y)$

We can completely determine the meaning of one type of *higher* alternative:

Fact (a). $\text{Exh}(\exists x, \exists_C y, R(x,y))$ is equivalent to $\exists x, \exists_C y, R(x,y) \wedge \neg\exists x, \exists_{\bar{C}} y, R(x,y)$

To see this, let us distinguish two cases. To be brief

- $\mathcal{Q}$ is a pure existential. In other words, $\mathsf{quant} = \{1, \dots, \#D\}$. In that case, all lower alternatives of the form $\mathcal{Q}x, \exists y \in C_0, R(x,y)$ are equivalent to $\exists x \in D_Q, \exists y \in C_0, R(x,y)$ and we can disregard them since they are equivalent to the other type of lower alternatives. Below, I give:
    1. the maximal sets of negatable alternatives,
    2. a world in which the prejacent is true and they are false,

3. an explanation for why they are maximal,

4. the set of innocently excludable alternatives, which is simply the intersection of all these sets

Maximal consistent set:	for any z in C, $\left\{ \exists x, \exists y \in C_0, R(x,y) \middle| z \notin C_0 \right\}$ $\rightsquigarrow$ true in worlds where $R = D \times \{z\}$ $\rightsquigarrow$maximal because $\neg\exists x, \exists y \in C_0, R(x,y)$ and $\neg\exists x, \exists y \in C_1, R(x,y)$ contradict the prejacent when $C_0 \cup C_1 = C$
IE alternatives:	$\left\{ \exists x, \exists y \in C_0, R(x,y) \middle| C_0 \cap C = \varnothing \right\}$

- $\mathcal{Q}$ is not equivalent to $\exists$. Then, there must be some cardinality n greater than 0 that is not in quant ; call S any subset of D_1 of that cardinality. In that case:

Maximal consistent set:	for some z in C, $\left\{ \exists x, \exists y \in C_0, R(x,y) \middle| z \notin C_0 \right\} \cup \left\{ \mathcal{Q}x, \exists y \in C_0, R(x,y) \mid \forall C_0 \right\}$ consistent for worlds where $R = S \times \{z\}$ maximal because $\neg\exists x, \exists y \in C_0, R(x,y)$ and $\neg\exists x, \exists y \in C_1, R(x,y)$ contradict the prejacent when $C_0 \cup C_1 = C$
IE alternatives:	$\left\{ \exists x, \exists y \in C_0, R(x,y) \middle| C_0 \cap C = \varnothing \right\} \cup \left\{ \mathcal{Q}x, \exists y \in C_0, R(x,y) \mid \forall C_0 \right\}$

This gives the truth-conditions of one type of *higher* alternatives: the higher alternatives with only existential quantifiers. We will now give an incomplete characterization of the other type, which will be sufficient for our purpose:

Fact (b).
The higher alternative $\text{Exh}_l(\mathcal{Q}x, \exists y \in C, R(x,y))$ entails:

- $\mathcal{Q}x, \exists y \in C, R(x,y)$
- $\neg\exists x \in D, \exists y \in \bar{C}, R(x,y)$

The first entailment is a simple consequence of the fact that $\text{Exh}(F)$ entails F, as per the definition of Exh. The second entailment rests on the observation that the truth of the prejacent only depends on the value of the relational predicate R on the set $D_{\mathcal{Q}} \times C$ and is completely independent from what values R takes on $D_{\mathcal{Q}} \times \bar{C}$.

For the second entailment, let's take a negatable set of alternatives $\{a_i\}$. By definition, this means that $\mathcal{Q}x, \exists y \in C, R(x,y) \wedge \neg a_1 \wedge \neg a_2 \ldots$ is consistent. Consider the conjunction $\neg a_1 \wedge \neg a_2 \ldots$. Because of the monotonicity of the alternatives, this conjunction is downward-entailing in R: if it is true in a world where R is a certain set, it will be true in any world where R is a smaller set. Now consider the prejacent $\mathcal{Q}x, \exists y \in C, R(x,y)$. It is not downward-entailing in R but it is logically independent from the values taken by R on $D_{\mathcal{Q}} \times C$. This means that if it is true in a world where R is a certain set of pairs R_w; it will also be true in a world where R is the set of pairs $R_w \backslash D_{\mathcal{Q}} \times C$.

Taken together, these observations mean that if there is a world w where $\mathcal{Q}x, \exists y \in C, R(x,y) \wedge$

$\neg a_1 \wedge \neg a_2 \ldots$ and R takes values R_w in w, then there is another world w' where R takes values $R_w \backslash D_{\mathcal{Q}} \times C$ where this is still true. In one such world, the alternative $\neg\exists x \in D, \exists y \in \bar{C}, R(x, y)$ is false. It can be added to the a_i's without incurring contradictions.

Since the a_i's were taken to be arbitrary, this means that this alternative can be added to any set of alternatives without creating contradictions. Therefore, it is innocently excludable. This means in particular that Exh will negate ; in other words, $\text{Exh}_l(\mathcal{Q}x, \exists y \in C, R(x, y))$ entails $\neg\exists x \in D, \exists y \in \bar{C}, R(x, y)$.

Fact (b) is proved. Note that with fact (a), we have shown that higher alternatives of the form $\text{Exh}_l(\mathcal{Q}x, \exists y \in C, R(x, y))$ are stronger than higher alternatives of the form $\text{Exh}(\exists x, \exists_C y, R(x, y))$

At this stage, we have an approximate characterization of all the *higher*alternatives. We are almost ready to compute the meaning of Exh_h. We simply need to ascertain the meaning of its prejacent. Fact (c) establishes it:

Fact (c).
If $\mathcal{Q}$ is UE, $\text{Exh}(\mathcal{Q}x, \exists y \in D, R(x, y))$ is equivalent to $(\mathcal{Q}x, \exists y \in D, R(x, y))$. (In other words, the *higher* prejacent is equivalent to the *lower* prejacent.)

We just need to exhibit maximally consistent set of alternatives which have no alternatives in common. For z in the domain D, let's construct the set of alternatives Alt_z which contains $\exists x, \exists y \in C, R(x, y)$ if z does not belong to C and $\mathcal{Q}x, \exists y \in C, R(x, y)$ if z does not belong to C. I claim that the negation of all alternatives in Alt_z is consistent with the prejacent for all z.

First off, note that any world where $\exists x, \exists y \in C, R(x, y)$ is false for all C which do not include z is a world where the relation R (restricted to $D_1 \times D$) is included in $D_1 \times \{z\}$. Reciprocally, note that whenever R is included in $D_1 \times \{z\}$, all the alternatives in Alt_z are automatically false. Indeed, in such a world, a statement like $\exists y \in C, R(x, y)$ is equivalent to $R(x, z)$ if z is in C and false otherwise. With this observation, the members of Alt_z simplify to $\exists x, \bot$ and $Qx, \bot$, which are all false ($\mathcal{Q}$ has existential entailments).

To prove consistency then, I just need to exhibit one world where the *lower* prejacent is true and R is included in $D_1 \times \{z\}$. This is easily done: take S a subset of size quant of D_1 and take a world where R is equal to $S \times \{z\}$ (and is thus included in $D_1 \times \{z\}$). In this world, $\mathcal{Q}x, \exists y \in D, R(x, y)$ is equivalent to $\mathcal{Q}x, x \in S$ and is true, since S's cardinality is in quant

To prove maximality, let us the previous observation that whenever the alternatives of Alt_z are false and z is in C, $\exists y \in C, R(x, y)$ is equivalent to $R(x, z)$. The alternatives which are not Alt_z are all equivalent to either $\exists x, R(x, z)$ and $\mathcal{Q}x, R(x, z)$, which are equivalent to $\exists x, x \in S$ and the *lower* prejacent respectively. Both of the latter are entailed by the prejacent so cannot be negated.

So all sets of alternatives Alt_z are maximal and consistent. But their intersection is empty: for any alternative $\mathcal{Q}x, \exists y \in C, R(x, y)$, one just need to find a z in D but not in C and this alternative will not be in Alt_z, and similarly for the other alternatives. No alternative is IE and Exh_l is vacuous.

With fact (c) proved, we are ready to show that exhaustive participation is entailed:

Fact (d).
To the extent that the exhaustive participation inference is consistent with the higher prejacent, all *higher* alternatives are negatable and their negation, when conjoined to the prejacent, is equivalent to an exhaustive participation inference.

To show this, we simply need to show that all *higher*alternatives can be negated consistently with the lower prejacent. As seen above, all alternatives of the form $\textsc{Exh}(\mathcal{Q}x, \exists_C y, R(x,y))$ are stronger than $\textsc{Exh}(\exists x, \exists_C y, R(x,y))$. This means that to prove consistence, we only need to look at the latter, since weaker statements makes for stronger negations.

Let us conjoin the negation of all alternatives of the form $\textsc{Exh}(\exists x, \exists_C y, R(x,y))$. Given fact (a), this conjunction can be written as (111). I have grouped together alternatives which have complementary domains.

(111)
$$\begin{aligned}&\neg\left(\exists x, \exists_C y, R(x,y) \wedge \neg\exists x, \exists_{\bar{C}} y, R(x,y)\right)\\ &\wedge\neg\left(\exists x, \exists_{\bar{C}} y, R(x,y) \wedge \neg\exists x, \exists_C y, R(x,y)\right)\\ &\wedge\neg\left(\exists x, \exists_{C_1} y, R(x,y) \wedge \neg\exists x, \exists_{\bar{C}_1} y, R(x,y)\right)\\ &\wedge\neg\left(\exists x, \exists_{\bar{C}_1} y, R(x,y) \wedge \neg\exists x, \exists_{C_1} y, R(x,y)\right)\\ &\ldots\end{aligned}$$

Focusing on one grouping (e.g. the first two lines of (111)), we can make drastic simplifications: The crucial observation is that we can simplify these groupings, as follows[27]:

(112)
$$\begin{aligned}&\neg\left(\exists x, \exists_C y, R(x,y) \wedge \neg\exists x, \exists_{\bar{C}} y, R(x,y)\right)\\ &\wedge\neg\left(\exists x, \exists_{\bar{C}} y, R(x,y) \wedge \neg\exists x, \exists_C y, R(x,y)\right)\\ &\leftrightsquigarrow\\ &\left(\exists x, \exists_C y, R(x,y) \wedge \exists x, \exists_{\bar{C}} y, R(x,y)\right)\\ &\vee\left(\neg\exists x, \exists_{\bar{C}} y, R(x,y) \wedge \neg\exists x, \exists_C y, R(x,y)\right)\\ &\leftrightsquigarrow\\ &\left(\exists x, \exists_C y, R(x,y) \wedge \exists x, \exists_{\bar{C}} y, R(x,y)\right)\\ &\vee\left(\neg\exists x, \exists_D y, R(x,y)\right)\end{aligned}$$

Note that the second disjunct of (112) must be false if the prejacent is true. So together with the prejacent, the negation of the *higher* alternatives contributes the following inference:

(113)
$$\begin{aligned}&\textsc{Exh}(\mathcal{Q}x, \exists y \in D, R(x,y))\\ &\wedge\left(\exists x, \exists_C y, R(x,y) \wedge \exists x, \exists_{\bar{C}} y, R(x,y)\right)\\ &\wedge\left(\exists x, \exists_{C_1} y, R(x,y) \wedge \exists x, \exists_{\bar{C}_1} y, R(x,y)\right)\\ &\ldots\end{aligned}$$

This is exactly the conjunction of the prejacent with the exhaustive participation inferences. If this is consistent, then it is the meaning of the whole.

(114)
$$\begin{aligned}&\textsc{Exh}(\mathcal{Q}x, \exists y \in D, R(x,y))\\ &\wedge\forall y, \exists x, R(x,y)\\ &\ldots\end{aligned}$$

[27] The steps in this calculus: first, De Morgan's law; then, use or/and distributivity ; then simplify contradictory disjuncts

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