THE COOPERATIVE UNIVERSITY OF KENYA

DSTA 1201: FOUNDATIONS OF MATHEMATICS

April 2, 2021

Contents

1 Set Theory

1.1 Introduction

In this lesson we introduce sets and set operations. The concept of set is fundamental to mathematics and computer science. Everything mathematical starts with sets. For example, relationships between two objects are represented as a set of ordered pairs of objects, the natural numbers, which are the basis of other numbers, are also defined using sets, graphs and digraphs consisting of lines and points are described as an ordered pair of sets and so on.

1.2 Objectives

By the end of this topic the learner should be able to:

- Define a set
- Perform set operations
- Apply set theory to counting.

1.3 Introduction to Sets

Definition

A set is a *well defined* list or collection of objects. The different objects that form a set are called *members* or *elements* of a set. The members making up a set are enclosed in braces (curly brackets) i.e. { }. We usually denote sets by capital letters. For example

$A = \{1, 10, 5, 3\}$

A set is finite or infinite according to whether it has a finite or infinite number of members.

For example

i $A = \{a, b, c, d\}$ is a finite set.

ii $N = \{1, 2, 3, \dots\}$ is an infinite set.

There are two ways of describing a set

- By listing down all of its members
- By stating the properties characterizing each one of the members.

For example;

a. $A = \{a, b, c, d, e, f\}$ denotes a set A whose elements are a,b,c,d,e and f.

b. $B = \{x : x > 0 \text{ and x is an integer}\}$. Then the members of B are 1,2,3,...

c. $E = \{x | x^2 - 3x + 2 = 0 \text{ and x is a real number}\}$

Then the members of E are all real numbers satisfying the equation $x^2 - 3x + 2 = 0$, that is $E = \{1, 2\}$

Null or Empty Set

It is a set with no members and it is denoted by $\emptyset$ or $\{\ \}$.

Example 1.1 The set $\{x | x^2 + 1 = 0 \text{ and x is a real number}\}$ is empty.

The set { x:x is an even integer lying between 10 and 11, 10 not included} is empty.

Membership of a Set

The symbol $\in$ means 'a member of' or 'belong to'

Thus if $A = \{1, 4, 3, 5\}$, then $1 \in A$

The symbol $\notin$ means 'is not a member of' or 'does not belong to.'

Thus is $A = \{1, 4, 3, 5\}$, then $2 \notin A$

Equality of Sets

Two sets A and B are said to be equal, written $A = B$, if they have exactly the same numbers.

For example

If A= {a,b,c,d,e,f} and $B = \{e, f, d, c, b, a\}$ then $A = B$

Note

- The order in which the members of a set are listed is immaterial
- It is not allowed to repeat a member of a set more than once,

Subsets

If every member of a set A is also a member of set B, then A is said to be a subset of B or A is contained in B. If A is contained in B, we denote this by $A \subseteq B.$ If A is subset of B, then B is called a super set of A. The notation $A \subset B$ means A is contained in B but $A \neq B.$ In this case, A is called a proper subset of B.

Example 1.2

Find all the subsets of $A = \{1, 2, 3\}$

Solution

a) $\emptyset$ b) $\{1\}$ c) $\{2\}$ d) $\{3\}$ e) $\{1, 2\}$ f) $\{1, 3\}$ g) $\{2, 3\}$ h) $A = \{1, 2, 3\}$

Note

The empty set $\emptyset$ is a subset of any other set. The subsets a to g are proper subsets of A

If A contains an element which is not in B, then A cannot be a subset of B. We denote this by $A \not\subset B$

Example

If $A = \{1, 2, 4, 5\}$ and $B = \{4, 5, 7, 8\}$ then $A \not\subset B$ since $1, 2 \in A$ but $1, 2 \notin B$

If $n(A) = m$, then the number N of all subsets of A is $N = 2^m$

Example 1.5

If $n(A) = 3$, then the number of all subsets of A is $2^3 = 8$

Equality of Sets

A set A is said to be equal to B written $A = B$ if $A \subseteq B$ and $B \subseteq A.$ Therefore in order to prove that $A = B$ we need to show that $x \in A \implies x \in B$ and $x \in B \implies x \in A$

Universal Set

If we have some sets under consideration, a fixed set which contains all these subsets is called the universal set and it is denoted by $\bigcup$

Example 1.6

Let $A = \{1,2,3,4\}$ and $B = \{5,7,9\}$ and $C = \{10,12,13\}$. Then we can take the universal set to be any of these;

{1,2,3,4,5,6,7,8,9,10,11,12,13}

{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15}

$N = \{1,2,3,...\}$ or

$Z = \{...,-2,-1,0,1,2,...\}$