SPATIOTEMPORAL ANALYSIS WITH APPLICATION TO PRECIPITATION AND TEMPERATURE DATA FOR THE FREE STATE PROVINCE, SOUTH AFRICA

by

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Contents

Abstract

This study investigates climate change detection in the Free State province, South Africa, addressing the critical need to identify change-points and trends in temperature and rainfall due to their association with extreme weather events. This research goes beyond traditional temporal analysis to also explore the significant spatial dependencies among the climate stations, a crucial factor for a comprehensive understanding of regional climate dynamics. Utilizing precipitation and temperature data, the research applies nonparametric techniques, including the Pettitt test, Mann-Kendall test, Precipitation and Temperature Concentration Indices, and the Modified Fournier Index, to evaluate climate change effects. Analysis is performed using R for seven stations across the province's diverse climate zones to inform regional government precautions. This research is further enhanced by critically evaluating the robustness of these nonparametric statistical tests by comparing their performance against complementary parametric tests, such as the Likelihood Ratio (M-fluctuation) test for change point detection and linear regression for trend analysis. This comparative approach, primarily focusing on maximum temperature data, aims to validate the appropriateness and reliability of nonparametric methodologies in meteorological time series analysis, acknowledging the distinct statistical frameworks and underlying assumptions of each method. Ultimately, by integrating both temporal and spatial dimensions, this study provides a more robust and nuanced characterization of climatic shifts.

Chapter 1: Introduction

1.1 Climate Research Background

Climate refers to the long-term average of weather patterns and conditions characteristic of a particular geographic region (Dunbar et al., 2014). The weather is the daily change that you can experience and observe outside as it varies from place to place. In one place, you can see people in shorts enjoying themselves outside while in another place shovelling snow. Earth's climate is the result of the combination of all the climates on the planet. Stocker et al. (2013) explains that the atmosphere, the hydrosphere, the cryosphere, the geosphere, the biosphere, and their interactions form the complex system that is the climate, see Figure 1.1. Climate change is a shift in weather patterns in a typical region, this might indicate a change in the average monthly or seasonal temperature, or it could indicate a shift in the average yearly rainfall for a particular area. Fundamentally, climate change is a shift in the planet's climate. This may differ at some point during the year from the earth's average temperature. Another possibility is modifications to the locations on Earth where rain and snow normally fall. Note that within a few hours, the weather can change, however, changes in climate can last hundreds or even millions of years (Dunbar et al., 2014).

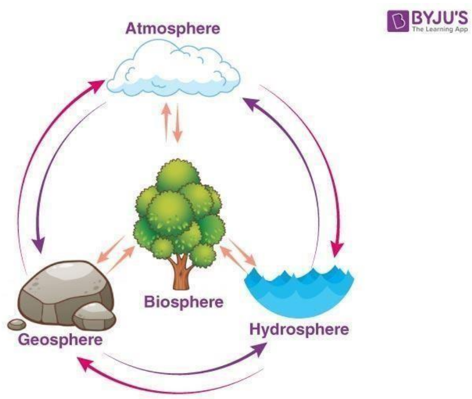

Figure 1.1: Climate complex system (Taken from Fathima, 2020)

Rainfall and temperature are the most important climatic variables in which its drastic changes can lead to droughts, widespread floods and death. Precipitation, a key component of the global water cycle alongside evaporation and condensation (National Geographic Society, 2023), refers to any form of water, whether liquid or frozen (e.g., rain, sleet, snow), that forms through atmospheric condensation and returns to the Earth's surface. Furthermore, National Geographic Society (2023) explains that precipitation, including rain, snow, and hail, consistently comprises freshwater due to the non-volatile nature of sea salt, which remains behind during evaporation. Consequently, even moisture originating from the ocean undergoes a desalination process as it transitions into atmospheric water vapor. It is important to note that air pollutants have the ability to taint water droplets prior to their descent into the earth; this precipitation is known as acid rain. While acid rain poses no immediate threat to human health, its capacity to elevate the acidity of freshwater ecosystems, such as lakes and streams, can be detrimental. This increased acidity surpasses the tolerance levels of numerous aquatic species, thereby causing harm to their habitats.

Various studies have investigated the trend of rainfall and temperature changes in the semi-arid regions (Khalili et al., 2016; Khosravi et al., 2017; Ahmadi et al., 2018; Kaskaoutis et al., 2018; Phuong et al., 2020; Mallick et al., 2021) and in the arid regions (Kousari and Zarch, 2011; Talaee, 2014; Machiwal et al., 2016; Meshram et al., 2020 and Xu et al. 2021). Semi-arid regions receive between 25 and 50 centimeters of rainfall yearly, which is considered to be slightly arid but more than an arid environment. In contrast, arid regions receive less than 25 centimeters of rainfall annually. It is important to note that overall, South Africa is classified as semi-arid, see Beraki et al. (2019) and Alexander (2023).

Beraki et al. (2019) stated that South Africa contributes significantly to global $CO2$ emissions. In 2015, South Africa's per capita $CO2$ emissions were 9.5 metric tons, higher than the global average. The energy system's over reliance on coal and oil is the primary cause of this. The South African government has committed to reaching its emissions peak between 2020 and 2025 as part of its international obligations (Beraki et al., 2019). Table 1.1 shows the 2016 comparative data of the first top 15 $CO2$ emitters by country.

Table 1.1: First top 15 CO_2 emitters in 2016 by country (Taken from Worldometers, 2016)

Rank	Country	CO_2 emissions (tons)	1-year change	Population (2016)	Per capita	Share of world
1	China	10 432 751 400	-0.28%	1 414 049 351	7.38	29.18%
2	United States	5 011 686 600	-2.01%	323 015 995	15.52	14.02%
3	India	2 533 638 100	4.71%	1 324 517 249	1.91	7.09%
4	Russia	1 661 899 300	-2.13%	145 275 383	11.44	4.65%
5	Japan	1 239 592 060	-1.21%	127 763 265	9.70	3.47%
6	Germany	775 752 190	1.28%	82 193 768	9.44	2.17%
7	Canada	675 918 610	-1.00%	36 382 944	18.58	1.89%
8	Iran	642 560 030	2.22%	79 563 989	8.08	1.80%
9	South Korea	604 043 830	0.45%	50 983 457	11.85	1.69%
10	Indonesia	530 035 650	6.41%	261 556 381	2.03	1.48%
11	Saudi Arabia	517 079 407	0.92%	32 443 447	15.94	1.45%
12	Brazil	462 994 920	-6.08%	206 163 053	2.25	1.29%
13	Mexico	441 412 750	-2.13%	123 333 376	3.58	1.23%
14	Australia	414 988 700	-0.98%	24 262 712	17.10	1.16%
15	South Africa	390 557 850	-0.49%	56 207 646	6.95	1.09%

Chersich et al. (2019) states that, in South Africa, the effects of climate change are rapidly intensifying on a global scale. If coordinated action is not made to cut greenhouse gas emissions, temperatures in the Southern African heartland might climb by more than 4°C by 2100, and in the western, central, and northern areas of South Africa, by more than 6°C. Among the most notable effects of extreme weather to date are forest fires and drought in the Western Cape, but an increasing number of vectorborne and waterborne illnesses are also becoming increasingly noticeable. Kreft et al. (2017)

similarly argue that globally, significant social, economic, and environmental threats and problems due to climate change are already a measurable reality. Like many other developing nations, South Africa is especially susceptible to the consequences of climate change. South Africa must thus find a balance between growing economic expansion and transformation and sustainable use (Kreft et al., 2019).

A sizable amount of global emissions are caused by the burning of fossil fuels to provide heat and power (European Commission, 2023). The majority of greenhouse gases, such as carbon dioxide and nitrous oxide, that warm the planet and absorb solar radiation are still produced when coal, oil, or gas are burned. Burning fossil fuels to provide energy for the manufacturing of things like cement, iron, steel, electronics, plastics, clothes, and other items is the primary source of emissions from industry and commerce. The removal of forests to create room for farms, pastures, or other purposes releases stored carbon, which raises emissions. Fossil fuels are often used to power vehicles, trucks, ships, and airplanes. Therefore, emissions of greenhouse gases (especially carbon dioxide) are strongly influenced by the transportation sector (European Commission, 2023).

1.2 Problem statement

Analysis of temperature and precipitation using time-series analysis is critical to predicting climate change. Droughts, heat waves, storms, and floods are a few of the natural calamities that are made worse by temperature increases. An environment that can store, discharge, and gather more water is produced by warmer temperatures - this changes weather patterns, making wet areas wetter and dry areas drier. Glenchak (2022) further stated According to academics, the greatest threat to global human health in the twenty-first century is climate change. This threat affects all of us, both directly and indirectly, with the elderly, young people, and members of low-income communities being the most at risk. As temperatures rise, so does the incidence of disease, ER (emergency room) visits, and death. Dehydration, headache, and nausea are all symptoms of heat exhaustion (Roffe et al., 2023). When people are out to extreme temperatures for extended times, they are at risk for serious health consequences and even death. For example, in January, at least five farm workers in South Africa's Northern Cape region died from heat stroke. In Pakistan and India, a terrible heat wave in May 2022 claimed at least 90 lives (Roffe et al., 2023).

Analysing the trend of rainfall and temperature (as well as abrupt changes thereof) prepare valuable information in order to improve water resource management, environmental protection, agricultural production, or in general economic development of the region (Ahmadi et al., 2022a, b; Gocic and Trajkovic, 2013). It is essential to be aware of trends in these factors in order to help recipients make decisions that are suitable and risk-free (Khosravi et al., 2017). In the South African context, Beraki et al. (2019) stated that temperatures and rainfall unpredictability have increased due to climate change. Growing evidence suggests a correlation between climate change and the increasing occurrence of extreme weather phenomena. This poses a significant threat to South Africa's stability and the overall welfare of its populace, particularly concerning the nation's water resources. Climate research has shown that South Africa, similar to many other regions of the world, faces more environmental than developmental challenges. Beraki et al (2019) further explained that drought, loss of biodiversity, soil erosion, decline in subsistence farming, and loss of cultural activities are just some of the myriad impacts of climate change that rural populations can expect.

Indeed, climate change research heavily relies on the analysis of time-series data to detect trends and shifts in climatic variables such as temperature and precipitation. [1] While parametric statistical tests like the Likelihood Ratio and Linear Regression are widely employed due to their power and interpretability, they inherently assume specific data distributions (e.g., normality, linearity, homoscedasticity) which are often violated by real-world environmental data (von Storch & Zwiers, 1999). Such violations can lead to biased results, incorrect inferences, and ultimately, misinformed climate policy and adaptation strategies. In contrast, nonparametric tests such as the Pettitt test for change-point detection and the Mann-Kendall test for monotonic trends offer robust alternatives that do not necessitate restrictive distributional assumptions, making them potentially more suitable for the inherently complex and often non-normal nature of climate time series (Helsel & Hirsch, 2002).

Given that Free State has a diverse climate classification system (see Figures 1.3 and 1.4), that is 4 climates, it is important to separately study the temporal effect of precipitation and temperature for each of the 7 stations so that the results can be shared with the provincial government to take precautions of any possible negative climate change effects.

1.3 Research objective

The specific objectives are:

- To critically evaluate and demonstrate the superior robustness of nonparametric methodologies, particularly the Pettitt test and Mann-Kendall test, when applied to environmental datasets that may not strictly adhere to the distributional assumptions often required by parametric tests. This will involve a comparative analysis with complementary parametric tests, specifically the Likelihood Ratio (M-fluctuation) test for change point detection and linear regression for trend analysis.

- To investigate the spatial dependency among the seven climate stations and apply an appropriate spatial statistical model to compare and interpret its results with the original univariate temporal analysis, thereby providing a more comprehensive understanding of regional climate patterns.

- To identify significant change points in temperature and precipitation data for each of the seven weather stations in the Free State Province using robust nonparametric methods.

- To determine the trend direction and magnitude of temperature and precipitation changes for each of the seven weather stations in the Free State Province using robust nonparametric methods.

- To quantify rainfall aggressiveness in the Free State Province, using the Modified Fournier Index (MFI).
- To analyse the distribution patterns of precipitation and temperature for each of the seven weather stations, using the Precipitation Concentration Index (PCI) and the Temperature Concentration Index (TCI).

- To assess the evidence of climate change in the Free State Province based on the temperature and precipitation data from 2000 to 2020, considering the comprehensive results from both nonparametric and comparative parametric statistical analyses.

1.4 Research questions

These are the following questions that this research will answer:

- How do the robustness and performance of nonparametric statistical tests (Pettitt test and Mann-Kendall test) compare with their complementary parametric counterparts (Likelihood Ratio (M-fluctuation) test for change points and linear regression for trends) in the context of meteorological data from the Free State Province, thereby validating the appropriateness and reliability of nonparametric approaches in environmental time series analysis?
- Do the climate stations exhibit significant spatial dependency, and how does accounting for this spatial relationship modify the conclusions drawn from the univariate temporal analysis of temperature and precipitation in the Free State Province?
- Have there been significant change points in the temperature and precipitation data for each of the seven weather stations in the Free State Province?
- Has there been a significant trend direction (and magnitude thereof) in the temperature and precipitation data for each of the seven weather stations in the Free State Province?
- How aggressive is the rainfall in the Free State Province?
- What patterns do the precipitation and temperature depict for each of the seven stations?
- Given the results of the comprehensive statistical analyses, including both nonparametric and parametric test comparisons and spatial modelling, is there evidence of climate change in the Free State Province based on the 2000-2020 dataset?

Chapter 2: Literature Review

2.1 Time series in temporal analysis

A time series is a collection of data points that appear successively over a certain amount of time (Hayes, 2022). It may also be used to compare variations in other variables that have occurred within the same time period with the changes linked to the chosen data point. Numerous non-financial uses of time series exist, such as monitoring population increase. Time series forecasting involves making predictions about the future of activities, using knowledge of past values and associated trends. This usually involves trend analysis, analysis of cyclical fluctuations, and seasonality problems. The success of any forecasting technique is not guaranteed. Hayes (2022) further explained that Time series analysis is a method for looking at a collection of data points over an extended period of time. Instead of gathering data points irregularly or randomly, time series analysts gather them over an extended period of time at regular intervals. Time series data is information collected over time, but other forms of information can be used to describe how and when this information was collected.

With stochastic processes, the theory of time series analysis began to advance early. The work of G. U. Yule and J. Walker in the 1920s and 1930s can be traced to the first real application of autoregressive models to data, see for instance Cryer and Chan (2008). To eliminate periodic changes in the time series, such as seasonal fluctuations, the moving average was developed during this period. Herman Wold first proposed the ARMA (AutoRegressive Moving Average) models for stationary series; however, likelihood function was unattainable, that would have allowed maximum likelihood parameter estimation. For the first time in 1970, a historical period that coincided with the publication of the famous book "Time Series Analysis" by G. E. P. Box and G. M. Jenkins, which covered the entire modeling process for each series: definition, estimation, diagnosis, and prediction (Cryer and Chan, 2008). Cryer and Chan (2008) further stated that BoxJenkins models are perhaps the most widely used today, and that many methods for forecasting and seasonal adjustment have their roots in these models. The adoption of multivariate ARMA models, especially VAR (Vector AutoRegressive), which have gained popularity, was the first generalization. However, these methods are only useful for time series that are stationary. Note that economic time series in particular often show an increasing trend indicating non-stationarity, i.e., a unit root. In the 1980s, most unit root tests were developed. It was discovered that non-stationary time series in the multivariate situation can have a common unit root.

The sole purpose of the time series analysis in this research work is to statistically analyse the time series data to learn more about climate change over a long period of time (not forecasting purpose); that is, trend and change point analysis, or more generally, the long-term systematic change in the mean of temperature and precipitation over time, is the main focus in this research.

The statistical tests defined in Table 1.2 will be used to conduct the latter.

Table 1.2: Statistical tests for trend and change point analysis in time series analysis

Statistical test	Definition
Nonparametric Pettitt test	Nonparametric technique used to find a single significant sudden change point in the mean of a time series data
Mann–Kendall (M-K) trend test	Nonparametric technique applied to examine monotonic (consistently increasing or decreasing) patterns in the dependent variable based on relative ranking from a given time range
Precipitation Concentration Index (PCI)	Statistical measure of monthly, annual, and seasonal precipitation (rainfall) distribution
Temperature Concentration Index (TCI)	Statistical measure of monthly, annual, and seasonal temperature distribution
Modified Fournier Index (MFI)	Measure of precipitation (rainfall) aggression by comparing the average monthly precipitation (rainfall) to the average annual rainfall
Sen's slope estimator test	Technique used to examine the trend direction and degree, and it is not affected by the number of outliers and data errors

The identification of a change point in a streamflow equally-spaced series is one of the most important information that management of companies want to be aware of; thus, statistical methods or time series analysis tools used to investigate whether such exist are crucial. Climatologists have been using the nonparametric Pettitt test (proposed by Pettitt, 1979) to determine the occurrence of a change point (or failure time) in rainfall and temperature dataset for decades. The nonparametric When the precise nature of the shift is uncertain, the Pettitt test can identify a substantial change in the mean of a time series (Mavromatis and Stathis, 2011). Next, the nonparametric method that has been widely used in trend analysis is the Mann-Kendall (M-K) test (discussed in Mann, 1945 and Kendall, 1975).

Modarres and Silva (2007) stated that This test has been extensively utilized as a useful method for detecting monotonic trends using data from various global locations in hydrometeorological and related spatiotemporal fields (where the variable of interest can be water quality, stream flow, air temperature, precipitation, and drought).

2.2 Characteristics of temperature and precipitation

Peel et al. (2007) described that the Köppen climate classification divides the earth's climate into five main climate groups: A (tropical), B (dry), C (temperate), D (continental), and E (polar). These are subdivided by seasonal precipitation and heat, and it was first published by the Russian-German climatologist Wladimir Köppen in 1884, with several later modifications by Köppen and others, most notably Rudolf Geiger, hence the system is sometimes also called the Köppen-Geiger climate classification system. The kind of seasonal precipitation is indicated by the second letter, and the degree of heat is indicated by the third letter. Keep in mind that the six months from April to September or October to March (South Africa falls under the latter category) are considered summers. Conversely, the six months from April to September or October to March are considered winters – for South Africa, it is the prior). The alphabets summary of the Köppen-Geiger climate classification system is provided in Table 1.3 and graphically in Figure 1.3. As can be seen from Figure 1.3, the upper North Africa (which includes Morrocco, Algeria, Libya, Tunisia, Egypt, etc.) have a Köppen-Geiger climate classification system which is similar to a small region in South Africa (i.e., the Upington area in the Northern Cape – see the zoomed in map of South Africa provided in Figure 1.4). Unlike the homogenous upper North African countries, South Africa has heterogenous Köppen-Geiger climate classification system. That is, from Figure 1.4, examples of

South African cities / towns' KöppenGeiger climate classification system are as follows: Upington, Northern Cape – (*BWh*); Kimberley, Northern Cape – (*BSh*); Cape Town, Western Cape – (*Csb*); Durban, KwaZulu Natal – (*Cfa*);

George, Western Cape – (*Cfb*); Port Elizabeth, Eastern Cape – (*BSh*); Bloemfontein, Free State – (*BSk*); Nelspruit, Mpumalanga – (*Cwa*); and Johannesburg, Gauteng – (*Cwb*).

Table 1.3: Köppen climate classification scheme symbols description

1st	2nd	3rd
A (Tropical)	f (Rainforest) m (Monsoon) w (Savanna, dry winter) s (Savanna, dry summer)	
B (Dry)	W (Arid Desert) S (Semi-Arid or steppe)	h (Hot) k (Cold)
C (Temperate)	w (Dry winter) f (No dry season) s (Dry summer)	a (Hot summer) b (Warm summer) c (Cold summer)
D (Continental)	w (Dry winter) f (No dry season) s (Dry summer)	a (Hot summer) b (Warm summer) c (Cold summer) d (Very cold winter)
E (Polar)		T (Tundra) F (Ice cap)

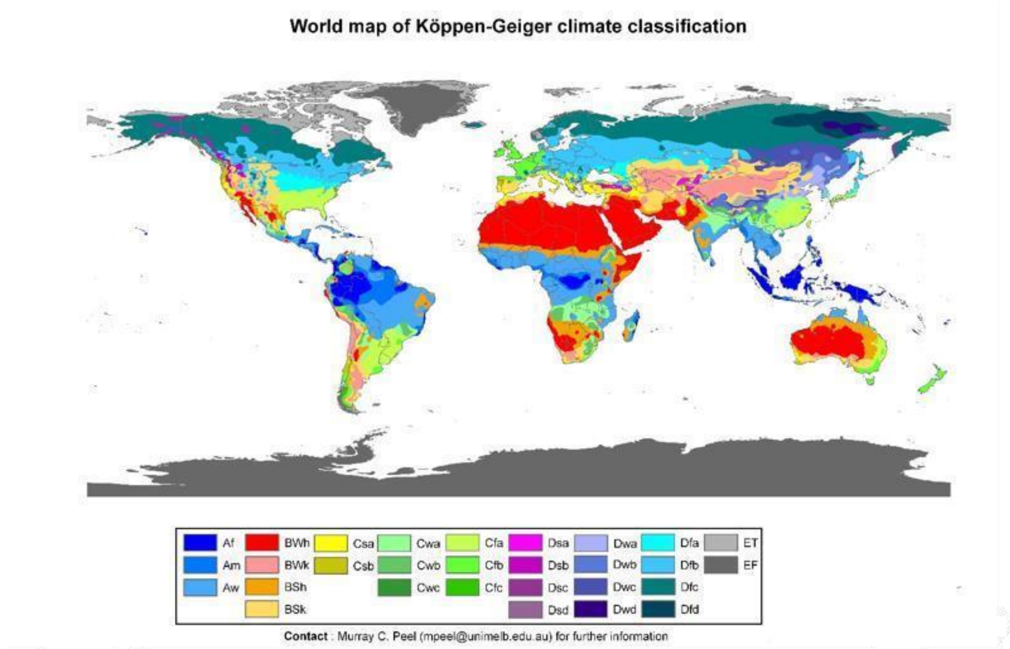

Figure 1.3: World map of Köppen-Geiger climate classification (Taken from Peel et al., 2007)

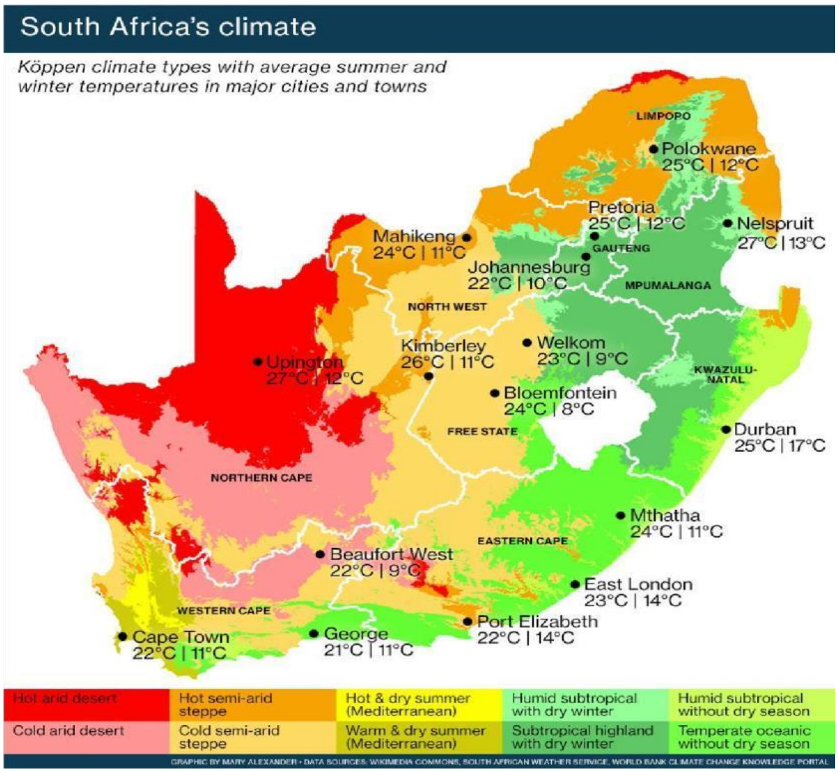

Figure 1.4: South African map with Köppen-Geiger climate classification that has average minimum and average maximum temperatures of well-known towns and cities (Taken from Alexander, 2023)

Table 2.1: Summary of articles discussing the techniques used in this research paper.

Publication	NPP	MK	SSE	PCI	TCI	MFI	Region	Time interval
Ahmadi et al. (2022a)	Y	Y		Y		Y	India	1901-2020
Ufoegbune (2011)						Y	Nigeria	1990-2010
Ahmadi et al. (2022b)	Y	Y		Y	Y		Iran	1988-2018
Ali et al. (2019)		Y	Y				China	Not specified
Jaiswal et al. (2015)	Y	Y					India	1971-2012
Li et al. (2011)		Y		Y			China	1961-2008
Botai et al. (2018)				Y			South Africa	1998-2015
Martin-Vide (2004)				Y			Spain	1951-1990
Zhang et al. (2019)		Y		Y			China	1960-2016
Benhamrouche et al. (2015)				Y			Algeria	1970-2008
Tahroudi et al. (2018)				Y	Y		Iran	1961-2010
Gocic et al. (2013)		Y	Y				Serbia	1980-2010
Atta-ur-Rahman et al. (2016)		Y	Y				Pakistan	1964-2014
Carolina et al. (2008)						Y	Uruguay	1941-2000
Ilori et al. (2020)	Y	Y					West Africa	1961-2000

Acronyms: NPP – Nonparametric Pettitt test, MK – Mann-Kendall trend test, SSE – Sen's slope estimator test, PCI – Precipitation concentration index, TCI – Temperature concentration index, MFI – Modified Fournier index.

The Table above is a summary of the articles considered to compile the literature review of this research paper. The combination of Mann-Kendall trend test and Pettitt change point detection test was considered by Ahmadi et al (2022), Jaiswal et al (2015) and Ilori et al (2020) on the temperature and precipitation data of the respective countries and time interval as indicated in the Table.

The combination of trend detection using Mann-Kendall trend test and Sen's slope estimator test was considered by Ali et al (2019), the paper presents a review or an application of the nonparametric Mann-Kendall test and Sen's slope estimator for trend analysis in time series data, potentially within the context of hydro-meteorological variables in the Yangtze River Basin, China; Gocic (2013), employed the non-parametric Mann-Kendall test and Sen's slope estimator to analyse trends in seven meteorological variables across 12 stations in Serbia during 1980-2010. The study identified statistically significant increasing trends in both minimum and maximum air temperatures on annual and seasonal scales. Conversely, relative humidity exhibited a significant decreasing trend during the summer and autumn. Vapor pressure showed a significant increase in spring, summer, and autumn. No significant trends were detected in precipitation during the summer and winter periods; Atta-ur-Rahman (2016), presents a spatio-statistical analysis of temperature fluctuations. The study employs the non-parametric Mann-Kendall trend test and Sen's slope estimator to identify and quantify the magnitude of temperature trends across a spatial domain. This approach allows for the detection of statistically significant increasing or decreasing temperature trends and the estimation of the rate of change at different locations. The findings contribute to understanding the spatial variability of temperature changes, which is crucial for climate change assessments and regional environmental planning. Lastly, the combination of precipitation concentration, temperature concentration and modified Fournier index was considered by Ahmadi et al (2022), Ufoegbune et al (2011), Li et al (2011), Botai et al (2018), Martin-Vide (2004), Zhang et al (2019) and Benhamrouche et al (2015). The temperature as well as the precipitation data was analysed for the respective countries and time interval as indicated in the Table.

In this research paper all six of the techniques will be considered in South Africa, using the precipitation and temperature data obtained from the South African weather services.

2.3.1 The nonparametric Pettitt test

Pettitt test is a non-parametric technique used to find a single significant sudden change point in the mean of a time series data. The Pettitt test determines whether a series of observed data, $x_1, x_2, x_3, \ldots x_n$, has a distribution function, $F_1(x)$, that is different from the distribution function, $F_2(x)$, of the second segment of the series, x_{t+1}, x_{t+2}, $x_{t+3} \ldots$, x_n (Jaiswal et al, 2015). The following are the values or formulae need to perform the pettitt test: $sign(x_i - x_j)$, U_t, K and p. The test statistic

U_t is calculated as follows:

$$\mathbf{U_t = \sum_{t i=1} \sum_{n j=t+1} sign(x_i - x_j)} \qquad (2.1)$$

$$\boldsymbol{sign(x_i - x_j) = \{ \begin{matrix} 1, & if\ (x_i - x_j) > 0 \\ 0, & if\ (x_i - x_j) = 0 \\ -1, & if\ (x_i - x_j) < 0 \end{matrix}} \qquad (2.2)$$

Where;

The sample length (n), test statistic K, and the corresponding confidence level (p), can all be described as:

$$\boldsymbol{K = \max |U_t|} \tag{2.3}$$

and

$$\boldsymbol{p = 2e^{\frac{-6K}{(n^2+n^3)}}} \tag{2.4}$$

The null hypothesis (H_0) for a change point states that there is no change point in the time series data and the alternative hypothesis (H_1) states that the change point exists in the time series data.

H_0: There is no change point.

H_1: There is a change point.

In this investigation, the null hypothesis i.e. change point does not exist is rejected at the chosen level of confidence given that the p value is less than the chosen significant level (Ahmadi et al., 2022).

A significant advantage is that the nonparametric Pettit test does not assume any specific underlying distribution for the data. This makes it suitable for analysing data that may not follow a normal distribution or any other known parametric distribution, which is often the case with climate data. The shortcoming of this test that is designed to detect only a single abrupt shift in the time series. If multiple change points exist, the test might identify the most dominant one or fail to detect any change point accurately.

2.3.2 Mann–Kendall trend test

The Mann Kendall trend test, well-known as the M-K test, is applied to examine monotonic (consistently increasing or decreasing) patterns in Y values in data that has been gathered over time. Since it is a non-parametric test, all distributions can be used (i.e., the data need not adhere to the assumption of normality), however serial correlation should not be present. Instead, you may perform straightforward linear regression if your data do in fact have a normal distribution (Glen, 2022). Glen (2022) further stated that, with as little as four samples, the test may be used to identify patterns. Given only a little data points, M-K test, however, has a great likelihood of missing a pattern that would be visible with extra data points. The likelihood that the test will identify a real trend, increases with the number of data points you have as opposed to one found by chance.

Here is the null and alternative hypothesis for the test:

H0: There is no monotonic trend in the series

H1: The monotonic trend exists (can be positive, negative or non-null) Before applying the test, the following assumptions should be noted:

Firstly, the data isn't gathered seasonally (for example, solely during the summer and winter months) for the reason that the test will fail if the data exhibits discontinuous rising and declining trends, for seasonally gathered data, additional test called the Seasonal Kendall Test is typically applied. Secondly, the data should have no covariates i.e additional variables outside the variables that one is graphing might affect the results. Lastly, each time period only has one data point. The median value is used if there is multiple data points (Glen, 2022).

A difference in signs between initial and final data points is examined using the M-K test. Regarding the literature, if the trend exists, the signs of the values will have a tendency to constantly rise or decline. Each value in the time series is compared to each value that was observed before it, yielding a total of $n(n-1)\ /\ 2$ data pairings, where n is the number of observed data points in the dataset. The number of pairwise comparisons, for instance, is $\frac{20(20-1)}{2}$ $\frac{20(19)}{2}$ $\frac{380}{2}$ $=$ 190 if there are 20 observations. The S statistic is computed as follows:

$$\boldsymbol{S} = \sum_{i=1}^{t} \sum_{j=t+1}^{n} \boldsymbol{sign}(\boldsymbol{x_i} - \boldsymbol{x_j}) \tag{2.5}$$

where

$$\boldsymbol{sign}(\boldsymbol{x_i} - \boldsymbol{x_j}) = \begin{cases} \mathbf{1}, & \boldsymbol{if}\ (\boldsymbol{x_i} - \boldsymbol{x_j}) > \mathbf{0} \\ \mathbf{0}, & \boldsymbol{if}\ (\boldsymbol{x_i} - \boldsymbol{x_j}) = \mathbf{0} \\ -\mathbf{1}, & \boldsymbol{if}\ (\boldsymbol{x_i} - \boldsymbol{x_j}) < \mathbf{0} \end{cases} \tag{2.6}$$

The following formulae can be used to get the mean and variance of the S statistic if $n \geq 8$ because it has a normal distribution (Ahmadi et al., 2022 (a)):

$$E(S) = 0$$

$$\boldsymbol{V}(\boldsymbol{S}) = \frac{\boldsymbol{n}(\boldsymbol{n}-\mathbf{1})(\mathbf{2}\boldsymbol{n}-\mathbf{5}) - \sum_{i=1}^{n} \boldsymbol{t_i}(\boldsymbol{t_i}-\mathbf{1})(\mathbf{2}\boldsymbol{t_i}+\mathbf{5})}{\mathbf{18}} \tag{2.7}$$

where t_i is the quantity of related data in the ith batch.

The Mann-Kendall test's Z statistic is calculated using the equation:

$$\boldsymbol{Z_{cal}} \begin{cases} \frac{\boldsymbol{S-1}}{\sqrt{\boldsymbol{V(S)}}} > \boldsymbol{0} \\ \boldsymbol{0S = 0} \\ \frac{\boldsymbol{S+1}}{\sqrt{\boldsymbol{V(S)}}} < \boldsymbol{0} \end{cases} \tag{2.8}$$

The null hypothesis that there is no trend in the time series is accepted $|Z_{cal}| > |Z_{table}|$; otherwise, there is a significant trend in the data series at a significant level.

The primary presumptions of trend analysis with the MK test are the sample data's independence and absence of substantial autocorrelation. But there's a chance that some hydrological series, like river flows, have high autocorrelation coefficients. The presence of substantial autocorrelation coefficients results in inaccuracies in the Mann-Kendall test final findings and erroneous conclusions. Hamed and Rao (1998) introduced the MK-VCA test, adjusted the data's variance, and reduced the impact of any significant autocorrelation coefficients in the data series in order to address this issue. This test has been proven to be effective when used in a variety of studies to examine trends in river flow (Jhajharia et al., 2012, 2013; Zamani et al., 2018; Ashraf et al., 2021; Das and Banerjee 2021), temperature and precipitation trend analysis (Gadedjisso-Tossou et al. 2020; Islam et al. 2020; Mallick et al. 2021; Nyikadzino et al. 2020), and study groundwater change trends (Kavitha et al. 2020; Mirabbasi et al. 2020; Meggiorin et al. 2021).

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2.3.3 Precipitation Concentration Index (PCI)

The variable weight of daily precipitation, or the contribution of the wettest days to the total quantity, may be evaluated statistically using PCI. Due to the fact that a negative exponential distribution frequently controls how much rainfall occurs on any given day in relation to the overall amount, it is theoretically based on the observation that seldom substantial daily quantities of precipitation occur at a specific time and location. In actuality, these infrequent big daily precipitation levels have a larger ability to influence hydrologic input (Li et al., 2011). The number of days falling inside each class's precipitation range is counted, and the corresponding quantity of precipitation is calculated. The cumulative total of it is then determined.

The PCI shows the fluctuations in precipitation in terms of concentration and pattern of dispersion. The following equation is used to compute the structures of this index on both a yearly and seasonal basis (Ahmadi et al., 2022 (a)):

$$PCI_{seasonal} = \frac{\sum_{i=1}^{3} p_i^2}{(\sum_1^3 p_i)^2} \times 25 \quad (2.9)$$

$$PCI_{annual} = \frac{\sum_{i=1}^{12} p_i^2}{(\sum_1^{12} p_i)^2} \times 100 \quad (2.10)$$

where p_i is the monthly precipitation total for the i^{th} month. The following is the Table for PCI classification:

Table 2.2: Precipitation concentration index classification

Concentration Condition	PCI
Uniform	< 10
Adequate	10– 15
Irregular	16 – 20
Strong irregularity	>20

The primary strength of the PCI is its ability to provide a quantitative measure of how evenly or unevenly precipitation is distributed over time. This is crucial for understanding the rainfall regime of a region beyond just the total amount. The shortcoming of the PCI value is that it can be influenced by the temporal resolution of the data used (e.g., monthly vs. daily). Using coarser resolution data might mask the effects of intense, short-duration rainfall events.

2.3.4 Temperature Concentration Index (TCI)

Ahmadi et al (2022) suggested the TCI as an alternative to the PCI for analyzing temperature concentration patterns. Following are the TCI structures at the yearly and seasonal scales:

$$TCI_{seasonal} = \frac{\sum_{i=1}^{3} T_i^2}{(\sum_1^3 T_i)^2} \times 25 \quad (2.11)$$

$$TCI_{annual} = \frac{\sum_{i=1}^{12} T_i^2}{(\sum_1^{12} T_i)^2} \times 100 \quad (2.12)$$

where T_i is the monthly temperature in the i_{th} month and the classification for TCI is as given in Table 2.3.

Table 2.3: Temperature concentration index classification

Condition	TCI
Uniform	< 10
Adequate	10– 15
Irregular	>15

The primary strength of a TCI is its ability to condense information about the temporal or spatial distribution of temperature into a single numerical value. This allows for easier comparison across different time periods, locations, or scenarios. The shortcoming is that, as a summary statistic, the TCI inevitably reduces the complexity of the full temperature dataset. Different distribution patterns could potentially yield the same TCI value, leading to a loss of detailed information.

2.3.5 Modified Fournier Index (MFI)

The ratio for Fournier index is given by $p_m{}^2\,/p$. The Fournier index has the disadvantage of not accounting for the annual monthly pattern of precipitation; hence, if the annual amount of erosive precipitation rises, the Fournier index value remains constant (Ahmadi et al., 2022). The following changes were made to the Fournier index to address these challenges:

$$\boldsymbol{MFI} = \sum_{i=1}^{12} \frac{p_i^2}{p_t} \tag{2.13}$$

where MFI refers to the modified Fournier index, $\overline{p_t}$ refers to the mean annual precipitation (mm) and p_i is the average monthly precipitation of the i th month (mm) and the classification of MFI is as given in Table 2.4.

Table 2.4: Modified Fournier index classification

Class	Erosivity	MFI index
1	Very low	Less than 60
2	Low	Between 60 &90
3	Moderate	Between 90&120
4	Severe	Between 120&160
5	Very severe	Greater than 160

The primary strength of the MFI is its reliance on readily available monthly rainfall data. Unlike more complex erosivity indices that require high-resolution rainfall intensity data (e.g., EI30), the MFI can be calculated using standard monthly precipitation records, which are more commonly available across various regions and historical periods. This makes it applicable in areas with limited data. The shortcoming of MFI is that it uses monthly totals and does not account for the intensity and duration of individual rainfall events, which are critical factors in determining soil erosion. A month with high total rainfall but low intensity might have a similar MFI to a month with shorter periods of very intense rainfall, but their erosive potential would be different.

2.3.6 Sen's slope estimator test

Sen (1968) stated that when estimating the slope of a linear trend, the least squares estimate is often computed using linear regression. However, the approach is extremely sensitive to outliers and is only valid in the absence of serial correlation. Sen (1968) created an approach that is more reliable. The formula is given by:

$$\boldsymbol{Q = Median\left[\frac{Y_{i'} - Y_i}{i' - i}\right]} \quad (2.14)$$

where: Q – is a slope estimator,

$Y_{i'}$ are Y_i the values at times i' and i, where i' is greater than i, N' is all data pairs for which i' is greater than i.

The median of the N' values of Q is Sen's estimate of slope. Whether there are one or many observations throughout a time period, the same process is used. Over time, a negative I number indicates a falling tendency whereas a positive I value indicates an upward trend (Ali et al., 2019).

The slope Sen's estimator is calculated using the following equation if n is an even integer:

$$\boldsymbol{Q_{med} = \frac{1}{2}\left(Q_{\left[\frac{n}{2}\right]} + Q_{\left[\frac{n+2}{2}\right]}\right)} \quad (2.15)$$

The predicted slope using Sen's approach may be calculated as follows if n is an odd number:

$$\boldsymbol{Q_{med} = Q_{\left[\frac{n+1}{2}\right]}} \quad (2.16)$$

Last but not least, Q_{med} is evaluated using a two-tailed test with a confidence level of $100\%(1-\alpha)$, and a nonparametric test may be used to assess the real slope of a monotonic trend. Stated differently, Sen's slope is used to determine the degree of a significant trend (positive or negative) when a trend analysis has revealed one. If the MK test showed that the average annual rise in temperature was between 1950 and 2000. In this case, Sen's slope will show you the typical annual change in temperature (Lodhi, 2018).

Sen's slope is highly resistant to the influence of outliers because it's based on the median of all pairwise slopes in the dataset. Extreme values have limited impact on the median. While its shortcoming is that it assumes a linear trend. If the underlying relationship between the variables is strongly nonlinear, Sen's slope might not accurately represent the pattern in the data.

2.3.7 The choice of nonparametric statistical tests over parametric

Parametric tests, such as the t-test and ANOVA, are powerful statistical tools that derive their strength from making specific assumptions about the population distribution from which the data are drawn, most notably normality and homogeneity of variances. When these assumptions are violated, the validity of parametric test results can be compromised, leading to inaccurate conclusions (Whitley & Ball, 2002).

In contrast, nonparametric tests are often referred to as "distribution-free" methods because they do not rely on stringent assumptions about the underlying data distribution. This makes them particularly valuable in several scenarios:

Violation of Normality Assumption: Many real-world datasets, especially in fields like environmental science or social sciences, may exhibit skewed distributions, outliers, or other forms of non-normality. In such cases, applying parametric tests can lead to erroneous inferences. Nonparametric tests, by working with ranks or signs of the data rather than the raw values, provide a robust alternative that is less sensitive to extreme values and deviations from normality (Jager et al., 2020; Kim, 2015).

Small Sample Sizes: When dealing with small sample sizes, it becomes increasingly difficult to reliably assess the underlying distribution of the data or to assume normality. Nonparametric tests are often more appropriate in these situations because they do not require a large sample size for valid inferences, offering reliable results even when data are limited (Omega Graduate School, 2023).

Robustness to Outliers: Outliers can disproportionately influence the mean and standard deviation, which are central to parametric tests. Since nonparametric tests typically transform data into ranks, the impact of extreme values is significantly reduced. This inherent robustness makes them a safer choice when outliers are present, and their removal or transformation is not justifiable (Kim, 2015).

While nonparametric tests may sometimes have less statistical power than their parametric equivalents when parametric assumptions are perfectly met, their wider applicability and increased robustness ensure that valid and reliable conclusions can be drawn from diverse datasets, especially when traditional assumptions are violated. This makes them an indispensable tool in the applied statistician's toolkit.

Chapter 3: Research Methodology

This section outlines the methodology employed to analyze the temporal characteristics of temperature and precipitation in the Free State Province, South Africa. The primary goal was to detect trends and change points in the data, validate robustness of these nonparametric methods, model spatial relationships of the stations and to assess rainfall aggressiveness. The methodology involves the application of several statistical tests on monthly temperature and precipitation data obtained from the South African Weather Services for the period 1992 to 2022, across seven weather stations.

3.1 Data Acquisition

Monthly temperature and precipitation data for the period 1992-2022 for the Free State Province, South Africa, was obtained from the South African Weather Services. The data covers seven weather stations within the province. This data has not been used before for any analysis.

3.2 Data Analysis

As previously discussed in Table 1.2, the following statistical tests used to analyse the data:

- **Nonparametric Pettitt test:** This test was used to identify single significant change points in the mean of the temperature and precipitation time series data.
- **Mann-Kendall (M-K) trend test:** The M-K test was used to examine monotonic trends (consistently increasing or decreasing patterns) in the temperature and precipitation data.
- **Sen's slope estimator test:** This test was used to determine the magnitude and direction of the trends identified by the M-K test.
- **Modified Fournier Index (MFI):** The MFI was used to measure the aggressiveness of rainfall by comparing average monthly precipitation to average annual rainfall.

- **Precipitation Concentration Index (PCI):** The PCI was used to measure the monthly, annual, and seasonal distribution of precipitation (rainfall).
- **Temperature Concentration Index (TCI):** The TCI was used to measure the monthly, annual, and seasonal distribution of temperature.
- **Likelihood Ratio Test (LRT):** This test was used as a parametric complementary test to Pettitt.
- **Linear Regression (LR) test:** This test was used as parametric complementary test to MannKendall (M-K) trend test.

- **Spatiotemporal cheks**: Checking for temporal, spatial and spatiotemporal patterns.

3.3 Statistical Software

The statistical tests were conducted using R statistical software.

3.4 Statistical Analysis

The statistical tests were applied to the temperature and precipitation data of the 7 stations. This involved analysing the trends and change points of temperature and precipitation for each station.

Thereafter, to examine the rainfall aggressiveness, and the distribution pattern for each station.
Further analysis was done to validate the robustness of these nonparametric methods.

Chapter 4: Results & Discussion

Below is a table displaying all weather gauge stations across the South African provinces.

Table 4.1: List of weather stations in different provinces whose data is used in this research work, specifically Free State province.

EC	FS	GP	KZN	LP	MP	NC	NW	WC
Tsitsikamma	Gariep Dam	JHB Bot Tuine	Kokstad	Lephalale	Ermelo	Brandvlei	Taung	George
Cape St Francis	Bloemfontein Stad	JHB Int	Greytown	Polokwane	Komatidraai	Prieska	Bloemhof	Cape Town
East London	Bloemfontein Wo	Irene	Ladysmith	Mokopane	Lydenburg	Port Nolloth	Potchefstroom	Langgewens
	Fauresmith					Alexanderbaai	Mafikeng	Malmesbury
	Bethlehem							Porterville
	Welkom							
	Vrede							

EC: Eastern Cape, FS: Free State, GP: Gauteng, KZN: KwaZulu Natal, LP: Limpopo, MP: Mpumalanga, NC: Northern Cape, NW: North-West, WC: Western Cape

Figure 4.1: The zoomed in version of the Free State map with the 7 stations

4.1 Maximum temperature results

4.1.1 Pettitt test

The Pettitt test, applied to minimum temperature data from seven gauge stations, revealed significant ($p < 0.05$) upward shifts in temperature across various timescales (Table 4.1). For instance, significant increases were observed in January at Vrede ($p = 0.0053$, change point 2011), March at Gariep Dam ($p = 0.0371$, change point 2012), and May across Bloemfontein (Bloem Stad, Bloem Wo, Vrede) with change points consistently around 2011. Similarly, significant upward trends in minimum temperatures were evident during June, July, August, October, and across the winter, spring, summer, and annual timescales at different stations, with change points predominantly falling between 2001 and 2015.

These results consistently indicate a statistically significant increase in minimum temperatures across the study region, affecting most, if not all, gauge stations and seasons. The clustering of change points around the early 2010s for many stations and timescales suggests a pervasive regional climatic shift during this period. Such widespread and consistent upward trends in minimum temperatures align with broader patterns of global warming and could have significant implications for local ecosystems, agriculture, and water resources.

Table 4.1: Pettitt test results for temperature (All change-points is up)

Timescale	Gauge Station	P-value	Change point	Average Before shift	Average After shift
January	Vrede	0.0053	2011	25.65	27.47
March	Gariep Dam	0.0371	2012	27.67	28.90
May	Bloem Stad Bloem Wo Vrede	0.0020 0.0029 0.0053	2011 2011 2011	19.85 20.62 19.80	21.50 22.59 21.88
June	Gariep Dam Bloem Wo Fauresmith Vrede	0.0338 0.0225 0.0189 0.0169	2015 2012 2015 2012	16.46 17.86 16.91 17.42	18.15 19.31 18.66 19.23
July	Vrede	0.0148	2011	17.20	19.16
August	Vrede	0.0446	2014	20.19	22.49
October	Bloem Stad Vrede	0.0314 0.0053	2001 2001	25.98 23.56	27.55 25.64
Winter	Bloem Stad Bloem Wo Fauresmith Welkom Vrede	0.0016 0.0029 0.0465 0.0316 0.0015	2011 2012 2012 2011 2011	18.15 18.84 18.19 20.14 18.14	19.84 20.31 19.36 21.14 20.02
Spring	Bloem Stad Bloem Wo Vrede	0.0013 0.0331 0.0008	2006 2013 2007	23.37 24.16 22.55	24.74 25.45 24.26
Summer	BloemWo Vrede	0.0382 0.0010	2011 2002	29.50 24.83	30.83 26.62
Annual	Bloem Stad Bloem Wo Vrede	0.0100 0.0033 0.0009	2012 2012 2007	24.46 24.89 22.56	25.87 26.03 24.01

4.1.2 Mann-Kendall test & Sen’s slope

The Mann-Kendall trend test results for temperature, focusing on the p-values, indicate significant trends across various stations and timescales. A p-value less than 0.05 signifies a statistically significant trend. Notably, "Vrede" consistently exhibits significant upward trends across most months, including January (p=0.0023), February (p=0.0281), June (p=0.0009, p=0.0203), July (p=0.0056), August (p=0.0047), October (p=0.0137), November (p=0.0011), and December (p=0.0144). This suggests a widespread and persistent warming trend at the Vrede station.

Beyond Vrede, other stations also show significant trends at specific times. "Gariep Dam" indicates a significant upward trend in March (p=0.0139, p=0.0159). "Bloem Stad" demonstrates significant trends in May (p=0.0008), September (p=0.0017), October (p=0.0030), November (p=0.0054), December (p=0.0008), Winter (p=0.0004), Spring (p=0.0113), and Annually (p=0.0021). "Bloem Wo" shows significant trends in May (p=0.0220), September (p=0.0155), November (p=0.0193), December (p=0.0156), Winter (p=0.0297), and Spring (p=0.0297, p=0.0144). "Bethlehem" presents significant trends in June (p=0.0004), September (p=0.0094), October (p=0.0110), December (p=0.0076), Winter (p=0.0076), Spring (p=0.0038), and Annually (p=0.0026). "Welkom" displays significant trends in May (p=0.0460), September (p=0.0343), October (p=0.0110), and Spring (p=0.0038). Lastly, "Fauresmith" shows significant trends in November (p=0.0314) and December (p=0.0497).

The prevalence of p-values below 0.05 across multiple stations and timescales underscores the existence of statistically significant temperature trends within the study region. The varying temporal patterns and station-specific trends highlight the localized nature of climate change impacts, even within a relatively confined geographical area. The consistent significant trends at "Vrede" suggest it experiences a more pronounced warming, while other stations show more intermittent, though still significant, changes. These findings are crucial for understanding regional climate variability and for informing adaptation strategies.

Table 4.2: Mann-Kendall test results for the temperature (Trend direction is upwards for all)

Timescale	Gauge Station	P-value	Z value	Sen's slope
January	Vrede	0.0023	3.0524	0.08
February	Vrede	0.0281	2.1962	0.08
March	Gariep Dam Vrede	0.0139 0.0159	2.4588 2.4123	0.09 0.08
May	Bloem Stad Bloem Wo Vrede Welkom	0.0008 0.0220 0.0013 0.0460	3.3563 2.2905 3.2158 1.9957	0.13 0.06 0.10 0.10
June	Bethlehem Vrede	0.0009 0.0203	3.3115 2.3216	0.05 0.08
July	Vrede	0.0056	2.7676	0.09

August	Vrede	0.0047	2.8240	0.09
September	Bethlehem Bloem Stad Bloem Wo Vrede Welkom	0.0094 0.0017 0.0155 0.0137 0.0343	2.5977 3.1448 2.4196 2.4647 2.1167	0.05 0.20 0.07 0.08 0.12
October	Bloem Stad Welkom	0.0030 0.0110	2.9676 2.5426	0.08 0.07
November	Bloem Stad Bloem Wo Fauresmith Vrede	0.0054 0.0193 0.0314 0.0011	2.7841 2.3392 2.1525 3.2675	0.12 0.09 0.15 0.11
December	Vrede	0.0144	2.4462	0.06
Winter	Bloem Stad Bloem Wo Bethlehem Fauresmith Vrede	0.0008 0.0156 0.0076 0.0497 .0003	3.3557 2.4178 2.6693 1.9628 3.6586	0.08 0.05 0.03 0.07 0.10
Spring	Bloem Stad Bloem Wo Bethlehem Welkom Vrede	0.0004 0.0297 0.0000 0.0038 0.0000	3.3557 2.1746 4.0715 2.8988 3.9626	0.07 0.04 0.05 0.05 0.09
Summer	Bloem Stad BloemWo Vrede	0.0113 0.0144 0.0001	2.5342 2.4460 3.8007	0.07 0.11 0.08
Annual	Bloem Stad Bloem Wo Bethlehem Vrede	0.0021 0.0104 0.0026 0.0000	3.0706 2.5625 3.0140 4.1219	0.07 0.04 0.03 0.08

Figure 4.2: Mann-Kendall trend results

4.1.3 Temperature Concentration Index

The analysis of Temperature Concentration Indices (TCI) across seven gauge stations, as presented in Figure 4.3, reveals distinct patterns in temperature distribution. According to Table 2.3, a TCI value of less than 10 indicates a "Uniform" condition. All seven gauge stations

(Bethlehem, Bloem Stad, BloemWo, Fauresmith, Gariep, Welkom, and Vrede) exhibit TCI values well below 10, with the highest median TCI observed for Gariep at approximately 8.7.
This consistently low TCI across all stations suggests that the temperature conditions at these locations are predominantly uniform, indicating minimal spatial variation in temperature over the observed period.

Further examination of the boxplot in Figure 4.3 indicates relatively narrow interquartile ranges and short whiskers for most stations, reinforcing the notion of temperature uniformity. While minor variations exist, such as Gariep showing a slightly higher median TCI and a broader spread compared to stations like Vrede, all stations fall squarely within the "Uniform" classification. This overall uniformity in temperature concentration across the gauge stations implies a stable thermal environment, which could have implications for various environmental or agricultural processes within the study area.

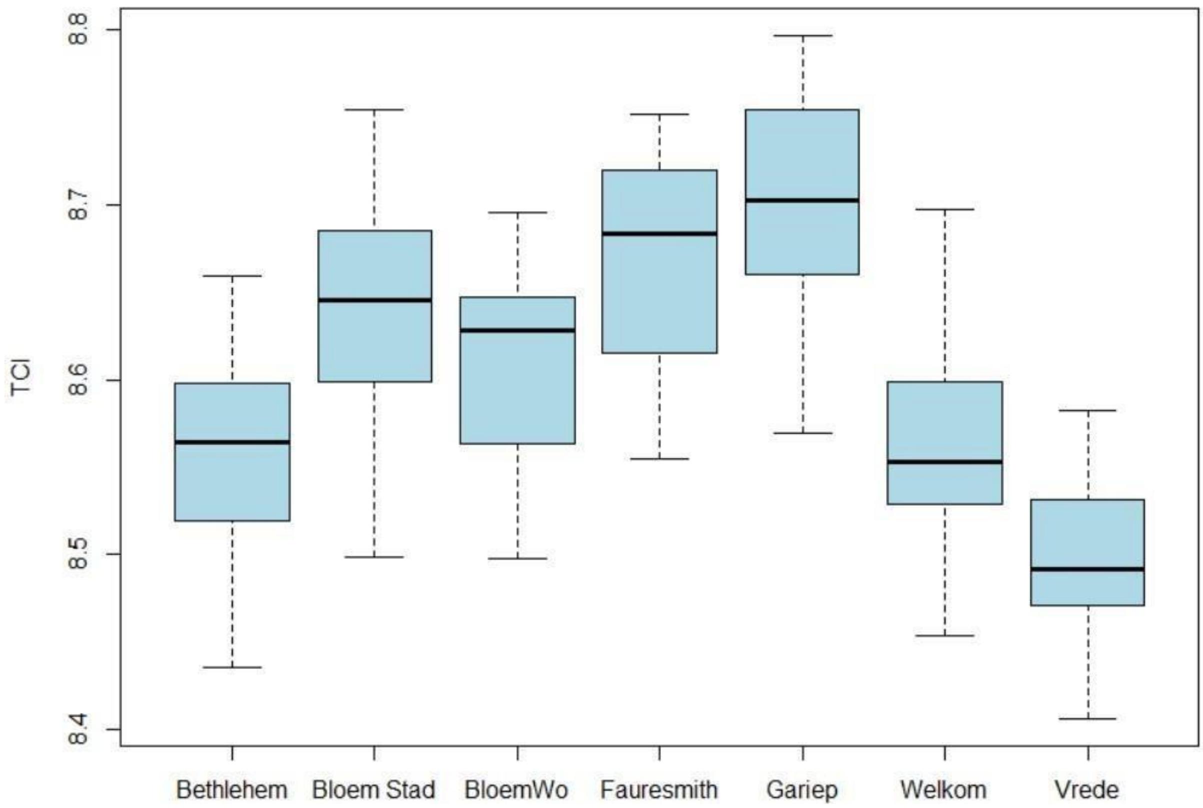


Figure 4.3: Boxplot displaying results for Temperature Concentration Indices

4.2 Minimum Temperature results

4.2.1 Pettitt test

The provided Table 4.3.1 presents the results of the Pettitt test, identifying significant change points (p-value < 0.05) in minimum temperature data across seven gauge stations for various timescales. A consistent pattern of increasing minimum temperatures is observed across most stations and timescales, as indicated by the "Average Before Shift" generally being lower than the "Average After Shift." For instance, both Fauresmith and Vrede show increases in January minimum temperatures, with change points in 2009. Similarly, Bethlehem and Vrede exhibit rising May minimums, with change points in 2007. This upward trend is further supported by the annual data, where Bethlehem experienced a shift in 2012, and Vrede in 2007, both showing higher average minimum temperatures after the change.

While the majority of results indicate an increase in minimum temperatures, the specific timing of change points varies by station and timescale, ranging from 2007 to 2015. Notably, Bethlehem, BloemWo, and Fauresmith show shifts in December minimums around 2011, while Vrede experienced an earlier shift in 2007. The summer and autumn periods also reflect these increases, with change points in 2011 and 2015 respectively for Fauresmith and Gariep Dam, and earlier for Vrede in both seasons (2007). Conversely, the Winter timescale for Vrede shows a change point in 2007, with a relatively small increase (0.07 to 1.05), highlighting the nuanced spatial and temporal variability of these temperature shifts.

Table 4.3.1 The results for Pettitt test where a change point is detected (Change-point is up for all)

Timescale	Gauge Station	P-value	Change point	Average Before shift	Average After shift
Jan	Fauresmith	0.0095	2010	15.28	16.10
Mar	Fauresmith Vrede	0.0255 0.0421	2009 2009	11.34 11.56	12.99 12.55
May	Bethlehem Vrede	0.0382 0.0053	2007 2007	1.63 2.17	2.68 3.61
Oct	BloemStad	0.0229	2009	11.32	12.09
Dec	Bethlehem BloemWo Fauresmith Vrede	0.0119 0.0282 0.0312 0.0036	2011 2011 2011 2007	12.08 13.26 13.66 12.63	13.28 14.57 13.98 13.86

Summer	Fauresmith Vrede	0.0382 0.0119	2011 2007	13.49 12.41	14.32 13.34
Winter	Vrede	0.0114	2007	0.07	1.05
Autumn	Gariep Dam Vrede	0.0382 0.0066	2015 2009	12.43 10.76	13.26 11.66
Annual	Bethlehem Vrede	0.0255 0.0001	2012 2007	6.52 7.23	7.16 8.02

4.2.2 Mann-Kendall & Sen's slope

The Mann-Kendall trend analysis revealed significant trends in temperature across various timescales and gauge stations, with only results exhibiting a p-value less than 0.05 presented. Monthly analyses showed significant trends in January for both BloemWo (p=0.0448) and Fauresmith (p=0.0059), February for Vrede (p=0.0165), March for Fauresmith (p=0.0291), May for Vrede (p=0.0120), June for Bethlehem (p=0.0325), and December across multiple stations including Vrede (p=0.0018), Bethlehem (p=0.0009), BloemWo (p=0.0131), and Fauresmith (p=0.0495).

Seasonal trends indicated significant changes in temperature during winter at Vrede (p=0.03843), summer at Vrede (p=0.0025), Bethlehem (p=0.0071), and BloemWo (p=0.0215), and autumn at Vrede (p=0.00556). Annually, significant temperature trends were observed at Vrede (p=0.00008) and Bethlehem (p=0.0132). These findings suggest varying patterns of temperature change across different periods and locations within the study area, highlighting the presence of statistically significant trends.

Table 4.2.2: Mann-Kendall test results for the temperature

Timescale	Gauge Station	P-value	Z value	Sen's slope
Jan	BloemWo Fauresmith	0.0448 0.0059	2.0062 2.7529	0.11 0.06
Feb	Vrede	0.0165	2.3610	0.0375
Mar	Fauresmith	0.0291	2.1818	0.08
May	Vrede	0.0120	2.5109	0.11
Jun	Bethlehem	0.0325	2.1385	0.024
Dec	Vrede Bethlehem BloemWo Fauresmith	0.0018 0.0009 0.0131 0.0495	3.126 3.3262 2.4818 1.9646	0.056 0.10 0.11 0.08
Winter	Vrede	0.03843	2.0702	0.033
Summer	Vrede Bethlehem BloemWo	0.0025 0.0071 0.0215	3.0166 2.6900 2.2988	0.043 0.08 0.09
Autumn	Vrede	0.00556	2.7671	0.039
Annual	Vrede Bethlehem	0.00008 0.0132	3.3368 2.4773	0.031 0.046

4.2.3 Temperature Concentration Index

Building upon the foundational principles of the PCI, the Temperature Concentration Index (TCI) is introduced as a novel metric for quantifying the uniformity of temperature distribution. The classification Table is given in chapter 2, section 2.5. The majority of the stations exhibit median TCI values below this threshold, suggesting predominantly adequate temperature concentration conditions across the observed locations. However, BloemWo stands out with a median TCI value notably above 14.5, and its upper quartile extending beyond 15, indicating a tendency towards irregular conditions at this specific station.

Further analysis reveals variations in TCI distributions among the stations. Bethlehem, BloemWo, and Fauresmith demonstrate median TCI values within the adequate range (10-15), although BloemWo shows a significant portion of its data falling into the irregular category (>15). Conversely, Bloemstad, GariepDam, Vrede, and Welkom generally exhibit lower median TCI values, predominantly falling within the adequate range. Welkom, in particular, shows the lowest TCI values, with its entire distribution below 12, suggesting more uniform or consistently adequate temperature concentration. The presence of outliers, such as the high TCI value for Bethlehem and Vrede, and the low TCI value for GariepDam, further highlights the localized variability in temperature concentration conditions across the different gauge stations.

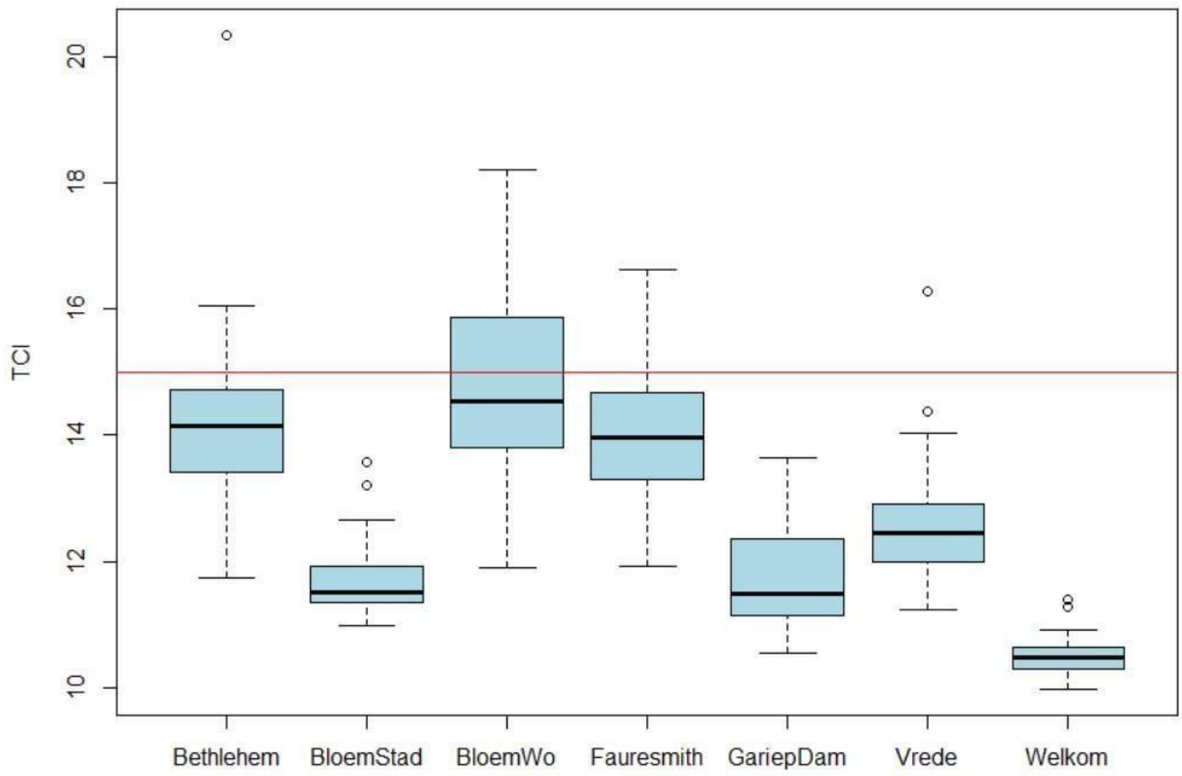


Figure 4.5: Boxplot displaying results for Temperature Concentration Indices

4.3 Precipitation results

4.3.1 Pettitt test

The introduction for Pettitt test is in section 1.1.1.

The provided table, "Pettitt test results for precipitation," presents an analysis of precipitation data, not minimum temperature data as stated in the prompt. The results identify significant shifts (pvalue < 0.05) in precipitation patterns for specific timescales and gauge stations. Notably, August precipitation at Fauriesmith shows a statistically significant downward shift in 2010, with the average precipitation decreasing from 30.21 units before the shift to 3.87 units after. Similarly, September precipitation at BloemWo experienced a downward shift in 2007, with average values decreasing from 21.61 to 6.51 units.

Furthermore, spring precipitation at BloemWo exhibits a downward shift in 2011, with the average decreasing from 24.38 to 11.03 units. Another gauge station, Vrede, also shows a significant downward shift in spring precipitation in 2007, with average values dropping from 32.95 to 16.10 units. These findings indicate a consistent trend of decreasing precipitation across different timescales and locations, suggesting potential changes in regional rainfall patterns. It is important to note that these are only a subset of the analyses, as only results with p-values less than 0.05 are presented, and the study encompasses seven gauge stations in total.

Table 4.4.1: Pettitt test results for precipitation

Timescale	Gauge Station	P-value	Change point	Average Before shift	Average After shift	Shift
August	Fauresmith	0.0382	2010	30.21	3.87	Down
September	BloemWo	0.0313	2007	21.61	6.51	Down
Spring	BloemWo Vrede	0.0255 0.0284	2011 2007	24.38 32.95	11.03 16.10	Down Down

4.3.2 Mann-Kendall & Sen's slope

The Mann-Kendall trend analysis revealed significant trends in precipitation for various timescales and locations, with all presented p-values being less than 0.05, indicating statistical significance. A significant upward trend in March precipitation was observed at Gariep Dam (p=0.0268). Conversely, significant downward trends were prevalent across several other stations and timescales. August precipitation showed significant downward trends at both BloemWo (p=0.0115) and Fauresmith (p=0.0132). Similarly, September and October precipitation exhibited significant downward trends at BloemWo (p=0.0115) and Vrede (p=0.0135), respectively.

Furthermore, spring precipitation displayed significant downward trends at both Vrede (p=0.0399) and BloemWo (p=0.0235). Annually, Vrede also experienced a significant downward trend in precipitation (p=0.0283). These findings collectively indicate a complex pattern of precipitation changes, with a notable predominance of decreasing trends across multiple seasons and the annual scale at several of the seven gauge stations analyzed.

Table 4.4.2: Mann-Kendal trend results for precipitation

Timescale	Gauge Station	P-value	Z value	Sen's slope	Trend direction
Mar	Gariep Dam	0.0268	2.2151	0.096	Upward
August	BloemWo Fauresmith	0.0115 0.0132	-2.5280 -2.4793	-0.20 -0.66	Downward Downward
September	BloemWo	0.0115	-2.5280	-0.78	Downward
October	Vrede	0.0135	-2.4705	-1.669	Downward
Spring	Vrede BloemWo	0.0399 0.0235	-2.0543 -2.2648	-1.00 -0.98	Downward Downward
Annual	Vrede	0.0283	-2.1390	-0.677	Downward

4.3.3 Modified Fournier Index

As previously mentioned in Chapter 2, MFI is used to measure the erosivity power of rainfall. In chapter 2, section 2.6.

This boxplot, illustrating the Modified Fourier Indices (MFI) for seven gauge stations, reveals considerable variability in MFI values across the stations. Bethlehem exhibits the highest median MFI and the widest interquartile range, suggesting a greater spread and higher central tendency of MFI values compared to other stations. Conversely, Gariep displays the lowest median MFI and a comparatively narrower interquartile range, indicating generally lower and less variable MFI values. The red line, likely representing a threshold or average MFI, highlights that Bethlehem and BloemWo (Bloemfontein Weather Office) predominantly have MFI values above this line, while Fauriesmith, Gariep, Welkom, and Vrede largely fall below it. BloemStad (Bloemfontein Stad) shows a median close to the red line, with values distributed on both sides. The presence of outliers, particularly in Bethlehem and Gariep, further underscores the heterogeneous nature of the MFI data.

The observed spatial variation in MFI values implies differing hydrological or meteorological characteristics across the Free State region of South Africa. Bethlehem's higher MFI values might indicate more pronounced seasonal patterns or greater amplitude in the underlying data, potentially linked to specific climatic conditions or geographical features. In contrast, the lower MFI values at stations like Gariep suggest more subdued seasonal variations or a less dominant periodic signal. These differences could be attributed to a range of factors including localized rainfall patterns, evaporation rates, or the influence of large water bodies such as the

Gariep Dam. Further analysis, incorporating geographical and climatic data for each station, would be crucial to elucidate the specific drivers behind these distinct MFI signatures and their implications for the regional hydrological regime.

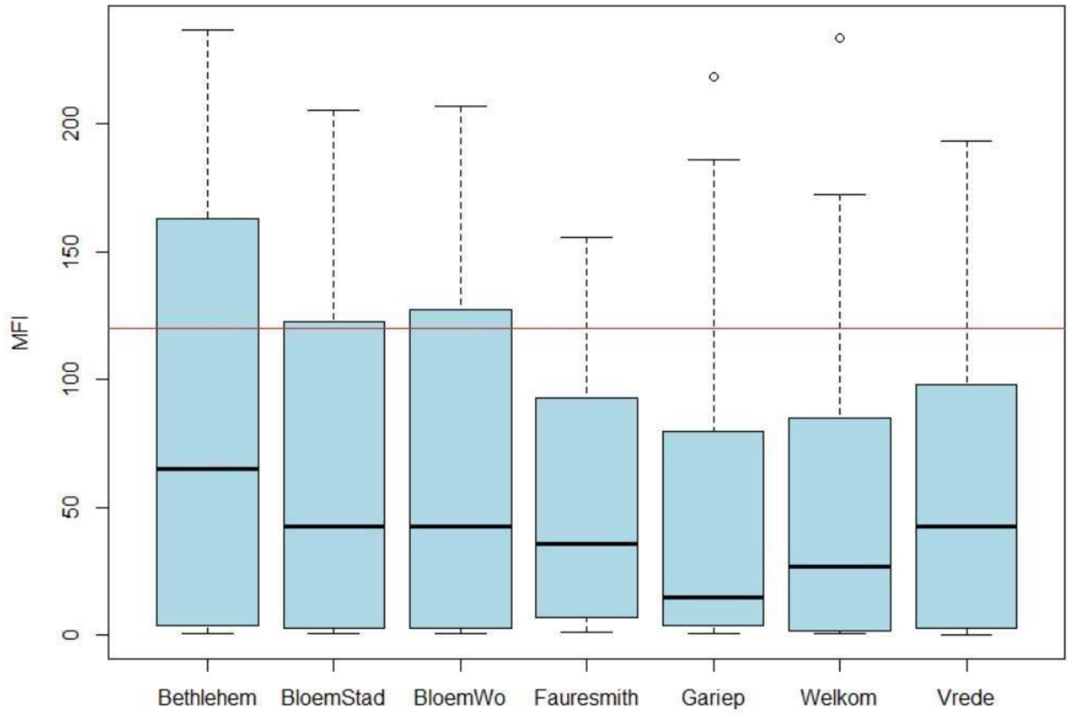


Figure 4.7: Boxplot displaying results for Modified Fournier Indices

4.3.4 Precipitation Concentration Index

As previously mentioned in the first chapter, PCI is often used on a yearly basis and is a potent indication of the temporal distribution of precipitation.

The analysis of Precipitation Concentration Indices (PCI) across the seven gauge stations reveal varying degrees of precipitation irregularity. As per Table 2.2, a PCI between 16 and 20 indicates "Irregular" precipitation, while values above 20 signify strong irregularity. The boxplot in Figure 4.8 illustrates that the majority of stations exhibit precipitation patterns falling within the irregular category, with their median PCI values generally ranging between approximately 15.5 and 19.5. Specifically, stations such as Bloemstad, BloemWo, and Vrede show median PCI values hovering around the upper end of the adequate to the lower end of the irregular range, suggesting a tendency towards more concentrated precipitation events.

Notable exceptions highlight areas of increased irregularity. GariepDam and Welkom demonstrate higher median PCI values, placing them firmly within the irregular classification, with Welkom's upper quartile extending into the strong irregularity range. Furthermore, the presence of several outliers with PCI values exceeding 25, particularly observed in stations like Bethlehem, Bloemstad, and Welkom, points to occasional instances of extremely concentrated precipitation events in these locations. This suggests that while most stations experience generally irregular rainfall, some areas are prone to more extreme and less predictable precipitation patterns, which could have implications for water resource management and agricultural planning.

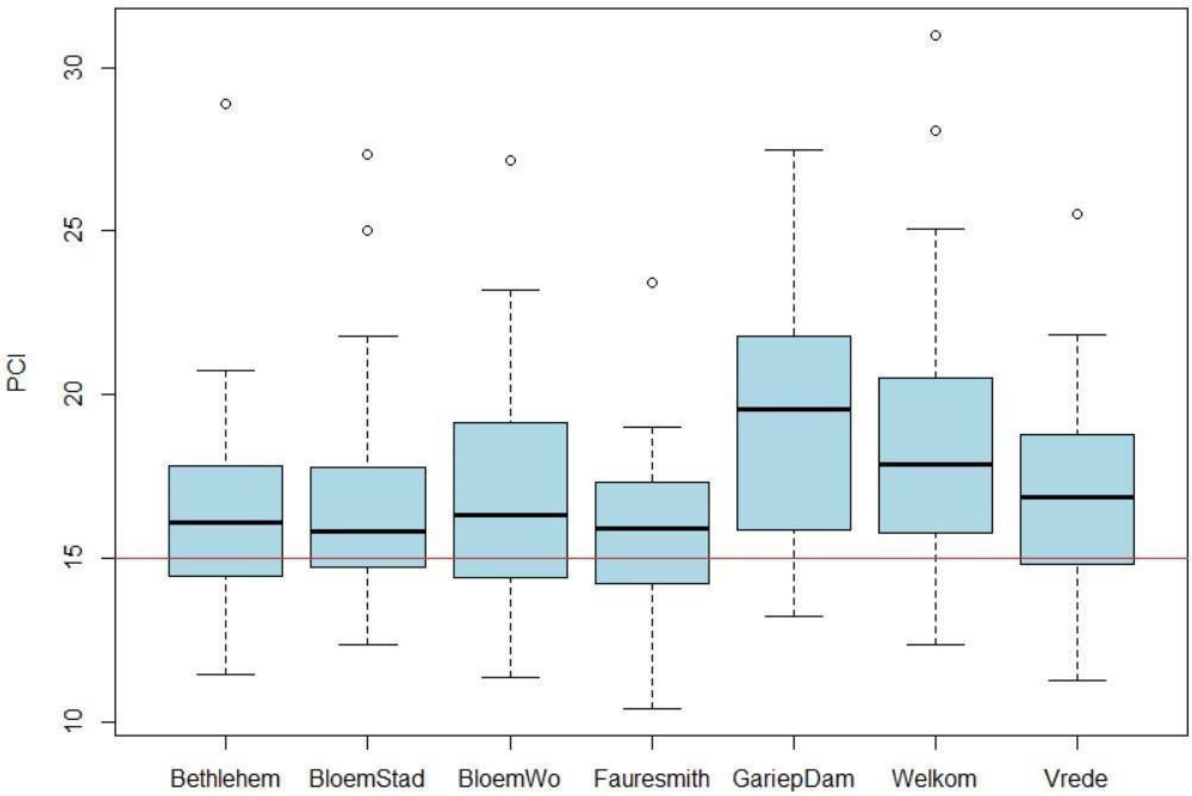


Figure 4.8: Boxplot displaying results for Precipitation Concentration Indices

Chapter 5: Concluding remarks

5.1 Summary

This study investigated long-term temperature and precipitation patterns in the Free State province, South Africa, from 2000 to 2020. Employing Pettitt and Mann-Kendall tests for change point and trend detection, alongside Temperature Concentration Index (TCI), Modified Fournier Index (MFI), and Precipitation Concentration Index (PCI) for distribution analysis, significant alterations in the regional climate were identified. The findings consistently indicate an upward trend in both maximum and minimum temperatures, coupled with complex, often decreasing, precipitation patterns and varying degrees of concentration.

Analysis of maximum temperature revealed widespread abrupt upward shifts, particularly in winter and spring, with Bloem Stad and Bloem Wo experiencing the most significant increases. The Mann-Kendall test further confirmed a pervasive gradual upward trend across all stations, notably in October and November, with Bloem Stad and Vrede showing the most rapid increases. Despite these changes, the TCI indicated a predominantly uniform distribution of maximum temperatures. Similarly, minimum temperatures exhibited significant upward change points, predominantly between 2002 and 2009, leading to a consistent increase in average minimum temperatures post-shift, especially at Bethlehem and Vrede. The Mann-Kendall test corroborated these findings, showing statistically significant increasing trends across various timescales. While most stations maintained adequate temperature concentration, BloemWo displayed a tendency towards irregular minimum temperature conditions.

Precipitation analysis presented a more varied picture. The Pettitt test identified significant downward shifts in average precipitation, particularly in August (Vrede), September (BloemWO), and during the Spring season across multiple stations, indicating a consistent reduction in rainfall. The Mann-Kendall test revealed localized trends, including upward shifts in March (Gariep Dam) and December (Bethlehem), but predominantly downward trends in October (Vrede) and Spring (Bethlehem, Vrede), and annually (Vrede). Furthermore, the MFI highlighted considerable variability in rainfall erosivity, with Bethlehem showing the highest potential for erosion and Gariep the lowest. The PCI analysis indicated that most stations experience irregular precipitation patterns, with some, like Gariep Dam and Welkom, exhibiting strong irregularity and occasional extreme concentration events, underscoring the unpredictable nature of rainfall in parts of the region.

In conclusion, the Free State province is experiencing a clear warming trend, evidenced by significant increases in both maximum and minimum temperatures. Concurrently, precipitation patterns are becoming more erratic, characterized by overall decreases in certain periods and locations, coupled with varying degrees of concentration and erosivity. These climatic shifts have profound implications for water resource management, agricultural planning, and ecosystem stability in the region. Future research should focus on identifying the specific drivers behind these observed changes, assessing their long-term socio-economic and environmental impacts, and developing robust predictive models to inform adaptation and mitigation strategies for a changing climate in the Free State.

The contributions of this study are significant in providing a detailed understanding of the temporal and spatial dynamics of temperature and precipitation in the Free State Province. Unlike previous studies that have focused on broader regional or national scales, this research provides a highresolution analysis specific to the Free State. This localized analysis is crucial for informing regional climate change adaptation strategies and resource management planning. Specifically, the identification of trends and change points can assist policymakers in developing targeted interventions to mitigate the impacts of climate variability on agriculture, water resources, and infrastructure.

5.2 Limitations

This study acknowledges several limitations that may constrain the interpretation and generalization of the results. A primary concern is the availability of the long-term climate data. Although reliable source like the South African Weather Services, were prioritized, the temporal coverage varied across stations. For example, although the supplied data is from 1992 to 2022, data for Gariep Dam and Fauresmith extended back to 2000, and for Fauresmith, to 2001. The data for 2001 and 2002 was missing completely and a very few monthly data points, but this was overcome by statistically imputing the missing data using a random forest as the method that is strongly suggested for the specific, meteorological data (Jing, Luo and Wang, 2022).

5.3 Directions for future research

Future studies could explore the underlying drivers of the observed trends and change points in temperature and precipitation. This could involve investigating the influence of large-scale climate patterns, such as the El Niño-Southern Oscillation (ENSO), the Indian Ocean Dipole (IOD), and changes in atmospheric circulation patterns. Furthermore, the role of local factors, such as land-use change, urbanization, and agricultural practices, could be examined in more detail.

Finally, expanding the temporal scope of the analysis to include neighbouring provinces or a larger regional context could provide a more comprehensive understanding of the broader climate trends and their interconnectedness. Comparative studies across different regions could help identify common patterns and unique regional responses to climate change.

Chapter 6: Validation of the robustness of the nonparametric tests and comparison with the complementary parametric tests

This chapter aims to critically evaluate the robustness of nonparametric statistical tests and subsequently compare their performance with complementary parametric tests. The primary focus is on meteorological data, specifically the maximum temperature dataset, which was chosen due to its exhibiting a greater number of detected change points and trends compared to minimum temperature and precipitation datasets. The central objective is to demonstrate the superior robustness of nonparametric methodologies, particularly when applied to environmental datasets that may not strictly adhere to the assumptions often required by parametric tests. Through this comparative analysis, the chapter seeks to validate the appropriateness and reliability of nonparametric approaches in meteorological data analysis.

Normality test results

The table below presents the normality test results specifically for the Bethlehem station. Similar normality assessments were conducted for all other stations, revealing a predominantly normal distribution across the entire dataset. However, it is noteworthy that most of the timescales for the majority of stations exhibit a normal distribution. For instance, an examination of the data for the month of January from 2000 to 2020 at various stations typically indicates normality.

Timescale	Shapiro_W	Shapiro_p_value
JAN	0,945715	0,282001
FEB	0,928089	0,126028
MAR	0,952911	0,386134
APR	0,959969	0,515504
MAY	0,967648	0,680621
JUN	0,929619	0,135254
JUL	0,9544	0,411204
AUG	0,973223	0,802835
SEP	0,927828	0,124514
OCT	0,961273	0,54213
NOV	0,964731	0,615976
DEC	0,891104	0,023612
Winter	0,958338	0,483298
Spring	0,972358	0,784518
Summer	0,969557	0,723195
Autumn	0,958361	0,483748
Annual	0,96003	0,516725

The provided table displays the results of Shapiro-Wilk normality tests for monthly, seasonal, and annual data. For each variable, the Shapiro_W statistic and its corresponding Shapiro_p_value are presented. A common approach to interpreting these results is to compare the p-value against a predetermined significance level (0.05). If the p-value is greater than alpha, the null hypothesis of normality cannot be rejected, suggesting that the data are likely normally distributed. Conversely, if the p-value is less than or equal to alpha, the null hypothesis is rejected, indicating a departure from normality.

Upon reviewing all the data from the stations, all variables, including individual months (JAN-DEC), seasons (Winter, Spring, Summer, Autumn), and the Annual aggregate, exhibit Shapiro_p_values greater than 0.05. For instance, the p-value for January is 0.357, for Winter is 0.890, and for Annual is 0.461. These consistently high p-values suggest that there is insufficient statistical evidence to reject the null hypothesis of normality for any of the tested variables at a 5% significance level. Therefore, it can be concluded that the data for all monthly, seasonal, and annual periods are consistent with a normal distribution.

6.1 Comparison between Pettitt test and Likelihood Ratio (M-fluctuations) test

Feature	Pettitt Test (Nonparametric)	Likelihood Ratio Test (Parametric)
Underlying Principle	Based on rank statistics (Mann-Whitney U statistic equivalent). Compares the distributions of two subsamples before and after a potential change point.	Compares the likelihood of the data under a null hypothesis (no change) to the likelihood under an alternative hypothesis (change at a specific point).
Assumptions	- Data is continuous. - No specific distributional assumption (nonparametric). - Assumes only a change in location (mean or median) is being detected.	- Requires specific distributional assumptions (e.g., normality, Poisson, exponential).- Assumes parameters of the distribution change at the change point.
Sensitivity to Outliers	Less sensitive to outliers due to its rank-based nature.	Can be highly sensitive to outliers, as they can significantly impact parameter estimates.
Computational Complexity	Relatively straightforward to compute, especially for a single change point.	Can be computationally intensive, especially for multiple change points or complex models, often requiring iterative optimization.
Power of Detection	Generally good power for detecting changes in location, especially with larger sample sizes and larger magnitudes of change. Power can decrease if the change point is near the extremities of the series or for small sample sizes.	Often more powerful than nonparametric tests when the distributional assumptions are met and the true underlying distribution is known.
Specificity of Change	Primarily designed to detect a shift in the mean or median. Less effective at detecting changes in other distribution parameters (e.g., variance).	Can be formulated to detect changes in various parameters of a distribution (mean, variance, etc.), or even the entire distributional form.
Multiple Change Points	Originally designed for a single change point. Extensions or sequential applications are sometimes used for multiple change points but may not be optimal.	Can be adapted for multiple change points (e.g., using binary segmentation or dynamic programming algorithms like PELT), though complexity increases.
Interpretation of Results	Provides a p-value to assess the significance of a detected change point and identifies the most probable change point location.	Provides a test statistic and p-value. The change point location is typically estimated by maximizing the likelihood ratio.
Robustness	More robust to deviations from distributional assumptions.	Less robust; performance can be significantly affected if distributional assumptions are violated.
Typical Applications	Hydrology, climatology, environmental science, quality control, where data distributions might not be strictly normal.	Finance, engineering, signal processing, where underlying process distributions might be better understood and modeled parametrically.

(Siegmund et al., 1995; Rybski et al., 2011)

6.2 Likelihood Ratio (M-fluctuation) test results and comparison to Pettitt's test results

6.2.1 Bethlehem station

Timescale	Statistic	P_Value
JAN	1,058912	0,212113
FEB	0,771879	0,590496
MAR	0,866407	0,440742
APR	0,565968	0,905891
MAY	1,159419	0,135922
JUN	1,221856	0,100979
JUL	0,80807	0,53107
AUG	0,919655	0,366171
SEP	1,251426	0,087242
OCT	0,94305	0,336094
NOV	0,974084	0,298822
DEC	0,73697	0,649131
Winter	1,301235	0,067656
Spring	1,240964	0,091911
Summer	1,104809	0,173997
Autumn	0,709435	0,695476
Annual	1,212104	0,105886

The analysis of change points using both Pettitt's test and the Likelihood Ratio (M-fluctuation) test revealed no statistically significant shifts in the time series at the conventional 0.05 significance level for any monthly, seasonal, or annual period. Pettitt's test, serving as the reference, consistently yielded p-values above 0.05, with the lowest observed for June ($p=0.096602$), indicating an absence of a detectable single change point within the series. This suggests that, based on a non-parametric approach, the data exhibits stationarity over the examined periods.

While both tests converged on the conclusion of no significant change, the specific p-values varied between the two methodologies. For instance, Pettitt's test showed its lowest p-value for June (0.096602), whereas the Likelihood Ratio (M-fluctuation) test indicated its lowest p-value for September (0.087242). Other periods also exhibited differing p-values; for example, the annual pvalue was 0.13523 for Pettitt's test compared to 0.105886 for the Likelihood Ratio test. These discrepancies, though not leading to a change in the overall conclusion of non-significance, highlight the sensitivity of different statistical tests to the underlying data characteristics.

The fundamental differences between Pettitt's test and the Likelihood Ratio (M-fluctuation) test contribute to these variations in p-values. Pettitt's test is a non-parametric method, making no assumptions about the distribution of the data and relying on ranks to detect a single change point. This robustness makes it less susceptible to outliers or deviations from normality. In contrast, the Likelihood Ratio (M-fluctuation) test, while employing M-estimators for robustness, is generally a parametric approach that often assumes a specific underlying distribution (e.g., normality) for its asymptotic properties. Its power is typically higher if the data conforms to its assumptions, but it can be more sensitive to violations of these assumptions.

In conclusion, despite their methodological distinctions—Pettitt's being non-parametric and the Likelihood Ratio (M-fluctuation) test being parametric/semi-parametric—both analyses consistently indicated the absence of a statistically significant change point at the 0.05 level across all examined time scales. The observed differences in p-values between the tests are attributable to their distinct statistical frameworks and underlying assumptions. The overall finding suggests that the time series under investigation remained stable, without evidence of abrupt shifts, according to both robust non-parametric and parametric change point detection methods.

6.2.2 Bloemfontein Stad station

Timescale	Statistic	P_Value
JAN	1,285958	0,073221
FEB	0,719225	0,679039
MAR	1,21983	0,101982
APR	0,950355	0,32705
MAY	1,602495	0,011763
JUN	1,482565	0,024653
JUL	1,015734	0,253515
AUG	1,063351	0,208167
SEP	1,381217	0,044052
OCT	1,29413	0,070198
NOV	1,037804	0,231659
DEC	0,972373	0,300798
Winter	1,610333	0,011185

Spring	1,651018	0,008578
Summer	1,240426	0,092157
Autumn	1,204396	0,109903
Annual	1,612322	0,011042

The analysis of change-point detection was conducted using two distinct statistical methods: Pettitt's test and the Likelihood Ratio (M-fluctuation) test, with a conventional significance threshold set at $p < 0.05$. Overall, both tests exhibited a notable degree of agreement in identifying significant change points across various temporal scales. Specifically, both Pettitt's test and the Likelihood Ratio test consistently indicated significant changes for the months of May (p=0.0020 and p=0.011763, respectively), and across the Winter (p=0.0016 and p=0.008578) and Spring (p=0.0013 and p=0.008857) seasons, as well as on an Annual basis (p=0.0100 and p=0.011042). This strong concordance suggests robust evidence for the presence of change points during these periods, irrespective of the underlying assumptions of the statistical method employed.

Despite the general agreement, some notable discrepancies emerged, particularly where the Likelihood Ratio test identified additional significant change points not detected by Pettitt's test at the conventional threshold. The Likelihood Ratio test indicated significant changes for June (p=0.024653) and September (p=0.044052), whereas Pettitt's test yielded p-values just above the significance threshold for these months (p=0.051123 for June and p=0.061697 for September). Conversely, there were no instances where Pettitt's test alone indicated a significant change that was not also identified by the Likelihood Ratio test. For other periods, such as February, July, August, December, Summer, and Autumn, both tests consistently showed non-significant p-values, suggesting an absence of detectable change points during these times.

Examining the magnitudes of the p-values further illuminates the agreement and subtle differences between the tests. For the periods where both tests indicated significance (May, Winter, Spring, Annual), the p-values were consistently low, often in the range of 0.005 to 0.018, reinforcing the strong statistical evidence for a change. The marginal differences in p-values for June and September, where Pettitt's test was just above the 0.05 threshold while the Likelihood Ratio test was below, suggest that the evidence for a change point in these months is weaker or less pronounced, and the choice of test or its inherent power might influence the conclusion regarding significance.

These observed differences can be attributed to the fundamental nature of each test. Pettitt's test is a non-parametric method, making no assumptions about the distribution of the data and relying on ranks, which renders it robust to outliers and non-normal data. In contrast, the Likelihood Ratio test, particularly the M-fluctuation variant, is typically a parametric test that assumes a specific underlying distribution (e.g., normality) for the data. While parametric tests can be more powerful when their assumptions are met, they are also more sensitive to violations of these assumptions. The complementary use of both tests provides a more comprehensive understanding of change-point dynamics, balancing robustness with statistical power.

6.2.3 Bloemfontein Wo station

Timescale	Statistic	P_Value
JAN	1,310083	0,064601
FEB	0,541689	0,930918
MAR	0,898823	0,394358
APR	0,611216	0,849093
MAY	1,551177	0,016258
JUN	1,445264	0,030671
JUL	1,037866	0,231599
AUG	0,810186	0,527662
SEP	1,275086	0,077413
OCT	1,118936	0,163419
NOV	1,081291	0,192794
DEC	1,078801	0,194874
Winter	1,656365	0,00828
Spring	1,425388	0,034379
Summer	1,381521	0,043978
Autumn	0,804103	0,537482
Annual	1,618389	0,010618

The analysis compared the results of the Pettitt's test and the Likelihood Ratio test for detecting change points across various temporal scales, including monthly, seasonal, and annual periods. A conventional significance threshold of 0.05 was applied to evaluate the presence of statistically significant changes. Overall, both tests exhibited a high degree of concordance in identifying periods of significant change. Specifically, both the Pettitt's test and the Likelihood Ratio test indicated statistically significant change points for the months of May and June, as well as for the Winter, Spring, Summer, and Annual periods.

Delving into the specific P-values, while the overall conclusions on significance were consistent, minor differences in the magnitude of the P-values were observed. For instance, in May, the Pettitt's test (P=0.0029) yielded a lower P-value, suggesting stronger evidence of a change point compared to the Likelihood Ratio test (P=0.007535). Similarly, for the Annual period, the Pettitt's test (P=0.0033) also showed a more pronounced significance than the Likelihood Ratio test(P=0.006696). Despite these minor variations, all identified significant changes remained well below the 0.05 threshold for both methodologies.

For periods where no significant change was detected, both tests largely agreed. Months such as January, February, March, April, July, August, September, October, November, December, and the Autumn season consistently showed P-values above the 0.05 threshold, indicating an absence of a statistically significant change point according to both tests. A notable difference was observed in the Likelihood Ratio test for February and August, where the P-value was exactly 1.0, suggesting an absolute lack of evidence for a change point, which was not mirrored by the Pettitt's test, although its P-values for these months were also high (0.930918 and 0.527662, respectively).

The observed consistency in significant change detection, despite minor P-value differences, can be attributed to the fundamental nature of these statistical tests. Pettitt's test is a non-parametric method, making it robust to deviations from normality and outliers, as it relies on ranks rather than raw data values. In contrast, the Likelihood Ratio test is typically a parametric approach, which, while potentially more powerful when its underlying distributional assumptions are met, can be sensitive to their violation. The close agreement in results suggests that the underlying data characteristics are sufficiently well-behaved for both tests to converge on similar conclusions regarding the presence of change points. The instances where the Likelihood Ratio test yielded P-values of 1.0 might reflect a very strong fit of the null model (no change) under its specific parametric assumptions, highlighting a subtle difference in how each test quantifies the absence of a change point.

6.2.4 Fauresmith station

Timescale	Statistic	P_Value
JAN	0,756425	0,616362
FEB	0,820262	0,511556
MAR	1,034746	0,2346
APR	1,041291	0,228339
MAY	1,182112	0,122229
JUN	1,385517	0,043016
JUL	0,982978	0,288696
AUG	0,849867	0,465522

SEP	0,834674	0,488891
OCT	1,085372	0,189424
NOV	1,133775	0,152867
DEC	1,159872	0,135637
Winter	1,343283	0,054167
Spring	1,3438	0,054016
Summer	1,378421	0,044737
Autumn	0,808395	0,530546
Annual	1,358544	0,049879

Analysis of potential change points using Likelihood Ratio test, a parametric method, revealed statistically significant shifts in specific periods. At a conventional significance threshold of 0.05, a significant change was detected for the month of June (P = 0.043016) and the Summer season (P = 0.044737). For all other individual months and seasonal aggregations and anunual, Pettitt's test indicated no significant change points, with P-values consistently exceeding the 0.05 threshold.

Specifically, for June, where LRT indicated significance, the Pettitt's test yielded a P-value of 0.056197, narrowly exceeding the threshold. Similarly, for the Summer season, the Pettitt Pvalue was 0.081042, also falling short of statistical significance. While some P-values from the Pettitt's test, such as for May (0.096602), Winter (0.056197), and the Annual period (0.096602), were relatively low, none crossed the conventional 0.05 significance boundary.

The observed discrepancies between the two tests can be attributed to their fundamental methodological differences. Pettitt's test is a non-parametric approach, making it robust to deviations from normality and less sensitive to outliers, as it relies on ranks rather than the raw data distribution. This characteristic allows it to detect shifts in the mean without assuming a specific underlying data distribution. Conversely, the Likelihood Ratio Test is a parametric method, which typically assumes a specific distribution for the data (e.g., normality). While generally more powerful when its underlying assumptions are met, the LRT's sensitivity to these assumptions means that violations, such as non-normal data or the presence of outliers, could lead to a reduced ability to detect true change points or yield higher P-values compared to non-parametric alternatives.

Consequently, the significant change points identified by Likelihood Ratio test in June and Summer, which were not corroborated by the Pettitt's test, suggest that the data for these periods might exhibit characteristics that are better accommodated by a non-parametric approach. This could imply the presence of non-normal distributions or influential observations that diminish the power of the parametric LRT. Therefore, based on the robustness of Pettitt's test to distributional assumptions.

6.2.5 Gariep Dam station

Timescale	Statistic	P_Value
JAN	0,64812	0,794918
FEB	0,454898	0,98581
MAR	1,493092	0,023156
APR	0,914036	0,373645
MAY	1,049715	0,220468
JUN	1,416779	0,036103
JUL	0,824981	0,504084
AUG	0,393161	0,997821
SEP	0,793984	0,553961
OCT	0,632346	0,818783
NOV	0,603578	0,859511
DEC	0,622686	0,832896
Winter	1,212093	0,105892
Spring	0,883839	0,415427
Summer	0,592677	0,873828
Autumn	1,124909	0,159104
Annual	1,213449	0,105197

The Pettitt's test, a non-parametric method, indicated significant change points (P_Value < 0.05) in March (P_Value = 0.037135) and June (P_Value = 0.033829). This test is robust to the underlying distribution of the data, making it suitable for a wide range of hydrological and climatic time series analyses where normality assumptions may not hold. The identification of these months suggests a detectable shift in the central tendency of the data within these specific periods, assuming the null hypothesis of no change is rejected at the 0.05 significance level.

In contrast, the Likelihood Ratio test, a parametric approach, also identified significant change points in March (P_Value = 0.023156) and June (P_Value = 0.036103). While both tests converged on the same months for significant change, the Likelihood Ratio test relies on specific distributional assumptions about the data, typically normality, for its test statistic to be valid. The P-values derived from the Likelihood Ratio test were slightly lower for March (0.023156 vs. 0.037135) and slightly higher for June (0.036103 vs. 0.033829) compared to Pettitt's test, indicating minor differences in the strength of evidence against the null hypothesis, despite reaching the same qualitative conclusion regarding significance.

The fundamental difference between these tests lies in their methodological foundation. Pettitt's test is a rank-based, non-parametric test, making it less sensitive to outliers and free from assumptions about the data's distribution. This characteristic provides a degree of certainty that the detected change points are not artifacts of distributional violations. Conversely, the Likelihood Ratio test is a parametric test, comparing the likelihood of the data under a null model (no change) to an alternative model (with a change point). Its power can be higher than non-parametric tests when its distributional assumptions are met, but it may yield misleading results if these assumptions are violated.

Despite their differing statistical underpinnings, the consistent identification of change points in March and June by both Pettitt's and the Likelihood Ratio tests provides strong evidence for shifts in the underlying processes during these months. The agreement between a robust nonparametric test and a powerful parametric test, when both are applied to the same dataset, enhances the confidence in the detected change points. This congruency suggests that the observed shifts are genuine features of the data and are not merely an artifact of the chosen statistical methodology.

6.2.6 Vrede station

Timescale	Statistic	P_Value
JAN	1,431154	0,033265
FEB	1,376866	0,045122
MAR	1,325454	0,059574
APR	0,899653	0,39321
MAY	1,443095	0,031058
JUN	1,725675	0,005181
JUL	1,444819	0,03075
AUG	1,432301	0,033047

SEP	1,338274	0,055641
OCT	1,073528	0,199335
NOV	1,233919	0,095172
DEC	0,840127	0,48044
Winter	1,863499	0,001926
Spring	1,569408	0,01451
Summer	1,373952	0,045851
Autumn	1,28968	0,071831
Annual	1,681542	0,006999

The analysis of change points using both Pettitt's test and the Likelihood Ratio test reveals a substantial degree of agreement regarding the presence of significant shifts in the data. Using a conventional significance threshold of 0.05, both tests consistently identified significant change points for January, May, June, July, and August at the monthly scale, as well as for the Winter, Spring, Summer, and Annual periods. This strong concordance across multiple temporal scales suggests a robust indication of underlying shifts within these specific periods.

Despite the general agreement, a notable discrepancy emerged for February, where Likelihood Ratio test indicated a significant change (P-value = 0.045122), while the Pettitt's test did not (P-value = 0.051123). This marginal difference in significance highlights the sensitivity of the tests to the chosen threshold and potentially to the underlying data characteristics. For other periods, such as March, April, September, October, November, December, and Autumn, both tests consistently showed no significant change points, reinforcing the stability of the data during these times.

Fundamentally, Pettitt's test is a non-parametric method, making no assumptions about the data's distribution. It is particularly effective at detecting a single change point in the mean of a time series and is robust to outliers. In contrast, the Likelihood Ratio test is typically a parametric test, which assumes a specific underlying distribution for the data (e.g., normal distribution) and assesses the improvement in fit when a change point model is compared to a no-change model. While often more powerful when its assumptions are met, its results can be sensitive to deviations from these assumptions.

The observed agreements between the two tests, particularly for the majority of significant change points, lend strong credibility to the identified shifts, suggesting that these changes are not merely artifacts of a single statistical method. The minor discrepancies, such as in February, may stem from the inherent differences in their statistical power and assumptions. The non-parametric nature of Pettitt's test provides a robust baseline, while the Likelihood Ratio test offers a complementary perspective, potentially indicating the nature of the distributional shift. Overall, the combined evidence from both tests provides a comprehensive understanding of the temporal dynamics within the dataset.

6.2.7 Welkom station

Timescale	Statistic	P_Value
JAN	0,706568	0,700275
FEB	0,735709	0,651256
MAR	0,421332	0,994295
APR	0,730353	0,66029
MAY	1,258664	0,084129
JUN	0,735165	0,652174
JUL	0,911637	0,376865
AUG	0,6557	0,783131
SEP	1,077031	0,196362
OCT	0,698066	0,714452
NOV	0,68775	0,731519
DEC	0,538813	0,933605
Winter	1,283347	0,07421
Spring	0,973958	0,298968
Summer	0,882754	0,416977
Autumn	0,554159	0,918579
Annual	0,876717	0,425668

The Pettitt's test and Likelihood Ratio test were employed to detect change points within the data, with the former serving as the primary reference for comparison. Both tests provide a p-value, allowing for an assessment of statistical significance against a conventional threshold of 0.05. However, their underlying methodologies and specific sensitivities to change points differ, leading to variations in their results.

Comparing the monthly results, both tests generally indicate a lack of significant change points. For instance, months like March, April, and December consistently show high p-values (approaching 1) in both tests, suggesting no significant shifts. However, some discrepancies emerge. For May, the Pettitt's test yielded a p-value of 0.114581, while the Likelihood Ratio test produced a p-value of 0.084129. Although neither is statistically significant at the 0.05 level, the Likelihood Ratio test suggests a slightly stronger (though still non-significant) indication of a change compared to Pettitt's. Similarly, for September, Pettitt's test resulted in 0.215844, whereas the Likelihood Ratio test gave 0.196362, again showing a marginally lower p-value from the Likelihood Ratio test. These minor differences may stem from the distinct statistical assumptions and calculations inherent to each test.

When examining the seasonal and annual periods, both tests largely maintain their consistency in indicating no significant change points, with p-values well above 0.05 for Spring, Summer, Autumn, and Annual data. However, for Winter, the Pettitt's test returned a p-value of 0.074091, while the Likelihood Ratio test yielded a p-value of 0.07421. These values are remarkably close, suggesting a similar borderline non-significant indication from both tests for the winter period. The close agreement in this instance highlights the potential for convergence between the tests under certain data characteristics, despite their fundamental differences.

In summary, while both the Pettitt's test and the Likelihood Ratio test generally agree on the absence of statistically significant change points across most months, seasons, and annually, subtle distinctions in their p-values for specific periods like May and September are observable. These variations underscore the distinct statistical frameworks of each test: Pettitt's test is a nonparametric rank-based test robust to outliers, whereas the Likelihood Ratio test, often parametric, assesses the ratio of likelihoods under different model hypotheses. Despite these fundamental differences, the overall conclusions regarding the absence of significant change points largely align, providing a degree of confidence in the stability of the observed data.

6.4 Linear regression test results and comparison to Mann-Kendall test results

Feature	**Mann-Kendall Test**	**Linear Regression Test**
Nature of Test	Non-parametric	Parametric
Assumptions	- Data are independent and identically distributed (in the absence of a trend). -Monotonic trend (consistently increasing or decreasing), not necessarily linear. -Less sensitive to outliers and nonnormal distribution.	- Linearity: A linear relationship between dependent and independent variables. - Independence of errors: Errors are independent of each other.-Homoscedasticity: Constant variance of errors across all values of the independent variable. -Normality of errors: Errors are normally distributed.
Type of Trend Detected	Monotonic (upward or downward, not necessarily linear)	Linear trend only
Sensitivity to Outliers	Less sensitive	Highly sensitive
Data Distribution	Does not require data to be normally distributed	Assumes normally distributed errors
Handling of Missing Data	Can handle missing data	Requires complete data or imputation
Interpretation of Results	Indicates presence and direction (positive/negative) of a trend. Often combined with Sen's Slope for magnitude.	Provides the slope and intercept of the best-fit line, indicating the rate and direction of change.
Applications	Hydrology, climatology, environmental science (e.g., rainfall, temperature, water quality trends)	Broad applications in various fields for predicting outcomes, understanding relationships, and forecasting (e.g., economics, finance, healthcare, engineering)

Advantages	Robust to outliers and nonnormal data, suitable for time series with irregularities.	Provides a clear mathematical model of the relationship, easily interpretable coefficients.
Disadvantages	Does not provide the magnitude of the trend directly (requires Sen's Slope). Can be less powerful for truly linear, normally distributed data.	Sensitive to violations of assumptions, especially outliers. Assumes a linear relationship, which may not always be appropriate.

(Mann,H.B, 1945; Helsel & Hirsch, 1992; Drapper,N.R, 1998)

Linearity Test

The table below presents the linearity test results specifically for the Bethlehem station. Similar linearity assessments were conducted for all other stations, revealing a predominantly nonlinear trend across the entire dataset. However, it is noteworthy that most of the timescales for the majority of stations exhibit a nonlinear trend. For instance, an examination of the data for the month of January from 2000 to 2020 at various stations typically indicates nonlinearity.

Timescale	R_squared	P_value_Year
JAN	0,147500024	0,085643861
FEB	0,062973495	0,272538923
MAR	0,040146406	0,383837625
APR	0,000724479	0,907800827
MAY	0,209898974	0,036740643
JUN	0,119344312	0,125068853
JUL	0,130102508	0,108202773
AUG	0,073458344	0,234685244
SEP	0,24596655	0,022224679
OCT	0,142461805	0,091642863
NOV	0,202321338	0,040769656
DEC	0,033931184	0,424107114
Winter	0,283329623	0,012993952
Spring	0,32860958	0,006596203
Summer	0,204675016	0,039475024
Autumn	0,031217858	0,443584299
Annual	0,332025989	0,006258447

The table presents R-squared and P-values from a linearity test for monthly, seasonal, and annual data. The P-value indicates the significance of the linear relationship, with values below 0.05 typically suggesting a statistically significant linear trend. Based on the P-values, significant linear relationships are observed for September (0.022), November (0.041), Winter (0.013), Spring (0.007), Summer (0.039), and Annually (0.006), implying that a linear model is appropriate for these periods. Conversely, the high P-values for January, February, March, April, May, June, July, August, October, December, and Autumn indicate a lack of a statistically significant linear relationship, suggesting that a linear model may not be the best fit for these specific periods. The R-squared values, which represent the proportion of variance in the dependent variable explained by the independent variable, are generally low across all periods, even for those with significant Pvalues, indicating that while a linear relationship might exist, it explains only a small portion of the variability in the data.

6.4.1 Bethlehem station

Timescale	Estimate_Slope	P_Value
JAN	0,092987	0,085644
FEB	0,054675	0,272539
MAR	0,041948	0,383838
APR	-0,00597	0,907801
MAY	0,116364	0,036741
JUN	0,060649	0,125069
JUL	0,076753	0,108203
AUG	0,063117	0,234685
SEP	0,140779	0,022225
OCT	0,095195	0,091643
NOV	0,126234	0,04077
DEC	0,056104	0,424107
Winter	0,084589	0,012994
Spring	0,099697	0,006596
Summer	0,091775	0,039475
Autumn	0,030216	0,443584
Annual	0,076569	0,006258

The Mann-Kendall (MK) test, a non-parametric method, and linear regression (LR), a parametric approach, yield both congruent and divergent insights regarding trends in the data across different temporal scales. Generally, both tests identify significant trends in similar periods, but with some notable discrepancies in their levels of significance and, in one instance, the direction of the trend.

For instance, the MK test indicates a statistically significant upward trend for September (p=0.015435), Spring (p=0.010266), and Annual (p=0.023527), all of which are corroborated by the LR test with even stronger levels of significance (p=0.022225, p=0.006596, and p=0.006258 respectively). This consistency suggests robust evidence of increasing trends in these periods. Conversely, while the MK test shows a near-significant trend for November (p=0.060824), the LR test confirms a significant upward trend (p=0.04077), highlighting LR's potentially greater power in detecting trends when its assumptions are met.

A notable point of divergence arises in April. The MK test suggests no significant trend (p=0.832441) and a positive Tau value (0.038278), whereas the LR test indicates a slight *negative* slope (−0.005907) although still not statistically significant (p=0.907801). This discrepancy in trend direction, even if not significant, underscores a fundamental difference: MK assesses the monotonic trend (consistency of direction), while LR quantifies a linear relationship. The robustness of MK to non-normal data and outliers may lead to a different conclusion compared to LR, which is sensitive to such deviations.

In other periods like January, May, and Winter, both tests largely align in their nonsignificant findings. However, the MK test often presents p-values closer to the significance threshold (e.g., January p=0.095733, May p=0.065228, Winter p=0.05785) compared to the LR test (January p=0.085644, May p=0.06741, Winter p=0.12994). This suggests that while both agree on the lack of statistical significance at the 0.05 level, the MK test might be detecting a weaker underlying monotonic pattern that is not as strongly captured by a linear model or requires more data to confirm. The differences in p-values reflect the distinct underlying assumptions and methodologies of the non-parametric MK test, which considers rank correlation, and the parametric LR test, which assumes linearity and normality of residuals.

6.4.2 Bloem Stad station

Timescale	Estimate_Slope	P_Value
JAN	0,151429	0,036329
FEB	0,040519	0,509844
MAR	0,106494	0,03197
APR	0,081948	0,124249
MAY	0,174688	0,004942
JUN	0,102844	0,035984
JUL	0,106039	0,045337
AUG	0,088038	0,06174
SEP	0,207364	0,000881
OCT	0,108987	0,039921
NOV	0,158831	0,06493
DEC	0,084805	0,339925
Winter	0,127857	0,001921
Spring	0,134796	9,07E-05
Summer	0,131688	0,034159
Autumn	0,07632	0,064878
Annual	0,117665	0,000765

The Mann-Kendall (MK) trend test and linear regression analysis provide complementary insights into trends within the data, though they operate on fundamentally different principles. The MK test, being non-parametric, assesses the monotonic trend without assuming a specific distribution of the data, making it robust to outliers and non-normal data. In contrast, linear regression assumes a linear relationship between the variable and time, and its validity is contingent on assumptions such as normality of residuals and homoscedasticity.

A comparative analysis of the provided results reveals both agreements and discrepancies in the identified significant trends. For instance, both tests consistently indicate a statistically significant upward trend for the month of September, with the MK test showing a p-value of 0.001662 and linear regression yielding a p-value of 0.000881. Similar agreement on significance (p-value < 0.05) is observed for January, March, May, June, July, and October, as well as for the Winter, Spring, and Summer seasons, and the Annual trend. This concordance suggests a robust and consistent trend across these periods irrespective of the underlying distributional assumptions.

However, notable divergences exist for November, August, and Autumn. The MannKendall test indicates a significant upward trend for November (p-value = 0.01212), whereas the linear regression test does not (p-value = 0.06493). Conversely, for August and Autumn, linear regression suggests non-significant trends (p-values of 0.06174 and 0.064878, respectively), while the Mann-Kendall test also shows non-significant trends (p-values of 0.173273 and 0.194127). These differences highlight the sensitivity of the respective tests to data characteristics. The nonparametric nature of the Mann-Kendall test might be more adept at detecting monotonic trends that are not strictly linear, or in datasets where the assumptions of linear regression are violated.

The varying results underscore the importance of employing multiple statistical approaches in trend analysis. While linear regression provides a measure of the rate of change (slope), the Mann-Kendall test offers a more general assessment of the presence and direction of a monotonic trend. The discrepancies observed, particularly where one test identifies significance and the other does not, suggest that the nature of the trend may not be strictly linear, or that the data might contain characteristics that violate the assumptions of linear regression, thereby favoring the more robust, non-parametric Mann-Kendall test as a primary indicator of monotonic change.

6.4.3 Bloem Wo station

Timescale	Estimate_Slope	P_Value
JAN	0,126883	0,081741
FEB	0,015584	0,813555
MAR	0,067143	0,197305
APR	0,040909	0,477256
MAY	0,148312	0,011684
JUN	0,071169	0,103271
JUL	0,095455	0,052759
AUG	0,061688	0,238495
SEP	0,155974	0,007112
OCT	0,094026	0,102279
NOV	0,147922	0,029254
DEC	0,08961	0,209136
Winter	0,104978	0,004052
Spring	0,103896	0,005786
Summer	0,121472	0,00994

Autumn	0,041212	0,370452
Annual	0,09289	0,003357

The Mann-Kendall (MK) trend test and linear regression analysis were employed to assess trends in the data, with distinct outcomes observed between the two methods across various temporal scales. The MK test, a non-parametric approach robust to non-normally distributed data and outliers, indicated statistically significant trends (p-value < 0.05) for September ($p = 0.007$), November ($p = 0.012$), Winter ($p = 0.007$), Spring ($p = 0.006$), Summer ($p = 0.014$), and Annually ($p = 0.006$). This suggests a monotonic trend in these periods, without assuming a linear relationship.

In contrast, the linear regression analysis, which assumes a linear relationship between the dependent and independent variables and requires normally distributed residuals, presented a different picture of significance. While the MK test identified more periods with significant trends, the linear regression analysis showed a significant trend for September ($p = 0.007$), November ($p = 0.029$), Winter ($p = 0.004$), Spring ($p = 0.006$), Summer ($p = 0.009$), and Annually ($p = 0.003$). There are inconsistencies in May where the linear regression showed significance ($p=0.011$) but MK did not ($p=0.003$). It is worth noting the differences in p-values for the same periods, which can be attributed to the fundamental differences in the statistical assumptions and methodologies of the two tests.

The discrepancies between the two tests highlight their inherent nature. The Mann-Kendall test's non-parametric nature allows it to detect monotonic trends even when the relationship is not strictly linear, making it suitable for environmental data that often deviate from normality. The 'S' statistic and Tau value in the MK test provide insights into the strength and direction of the trend, independent of the magnitude of changes. Conversely, linear regression quantifies the rate of change (slope) and its significance assuming a constant rate of change, which might not always be the case in natural phenomena.

Overall, while both tests generally agreed on the presence of significant trends for September, November, Winter, Spring, Summer, and Annual periods, the Mann-Kendall test, due to its non-parametric nature, offers a more flexible assessment of monotonic trends. The observed differences in specific months underscore the importance of applying both parametric and nonparametric methods to gain a comprehensive understanding of temporal trends, especially when the underlying data distribution may not strictly conform to the assumptions of parametric tests. The pvalues from the Mann-Kendall test, particularly for May, suggest a stronger statistical significance for a monotonic trend compared to the linear regression's assessment of a linear trend.

6.4.4 Fauresmith station

Timescale	Estimate_Slope	P_Value
JAN	0,016428	0,775031
FEB	-0,08464	0,225753
MAR	0,061248	0,203128
APR	-0,09926	0,091857
MAY	0,089391	0,183045
JUN	0,07599	0,081452
JUL	0,055226	0,272251
AUG	0,055766	0,277865
SEP	0,061725	0,27546
OCT	0,071664	0,220419
NOV	0,13396	0,076271
DEC	0,057421	0,373786
Winter	0,073536	0,064899
Spring	0,063052	0,078414
Summer	0,06927	0,095808
Autumn	-0,04089	0,398616
Annual	0,041243	0,178143

The Mann-Kendall (MK) trend test and linear regression analysis provide distinct yet complementary perspectives on trend detection. The MK test, being non-parametric, assesses the monotonic trend without assuming a specific data distribution, making it robust to outliers and nonnormally distributed data. In contrast, linear regression assumes a linear relationship between variables and requires assumptions about the data's distribution (e.g., normality of residuals). As a result, discrepancies between the two methods can arise due to these fundamental differences in their underlying assumptions and the nature of the trends they are designed to detect.

Comparing the two sets of results, while some coherence is observed, notable divergences highlight their differing sensitivities. For instance, the MK test for November indicates a statistically significant positive trend ($p=0.01358<0.05$), with a corresponding positive Z-score and Tau value. Conversely, the linear regression for November shows a positive slope but its p-value (0.07627) exceeds the 0.05 significance threshold, indicating no statistically significant trend. This discrepancy may suggest that while a monotonic increasing trend is present (as captured by MK), a strong linear relationship is not definitively established, or the linear regression's assumptions are not fully met.

Similarly, the MK test for Winter exhibits a positive and statistically significant trend (p=0.057118, which is borderline but often considered significant in some contexts, and the Z-score is positive). The linear regression for Winter also shows a positive slope, and its p-value (0.064899) is also close to the 0.05 threshold, suggesting a weak, but not statistically significant linear trend at the conventional 0.05 level. These instances underscore the MK test's ability to detect general monotonic increases that might not strictly follow a linear path, or where the data does not conform to the strict assumptions of linear regression.

Conversely, for other periods like January, February, March, and April, both tests largely agree on the lack of a statistically significant trend (p-values for both exceeding 0.05), despite some positive or negative Tau values and slopes. This concordance suggests that for these months, neither a significant monotonic nor a significant linear trend is present. In summary, while both tests aim to identify trends, the non-parametric nature of the Mann-Kendall test allows it to detect monotonic changes that linear regression, with its stricter assumptions about linearity and data distribution, might not always identify as statistically significant at the conventional 0.05 threshold.

6.4.5 Gariep Dam station

Column_Name	Estimate_Slope	P_Value
JAN	0,005391	0,920054
FEB	0,000678	0,990299
MAR	0,102536	0,002361
APR	0,057581	0,247953
MAY	0,051453	0,327599
JUN	0,060983	0,133334
JUL	0,02777	0,528164
AUG	-0,00398	0,912358
SEP	0,076779	0,158851
OCT	0,017771	0,720717
NOV	-0,01236	0,812211
DEC	-0,06195	0,352263
Winter	0,046736	0,177459

Spring	0,030191	0,313756
Summer	-0,02297	0,570686
Autumn	0,053598	0,122792
Annual	0,026888	0,268773

he Mann-Kendall (MK) trend test and linear regression (LR) analysis were employed to assess trends in the given data, revealing both concordances and divergences in their findings. For instance, the MK test indicated a significant increasing trend ($p<0.05$) for March (0.017372), a finding not mirrored by the LR analysis, which showed a non-significant p-value of 0.002361. Conversely, the LR analysis identified a significant increasing trend for March (0.002361), while the MK test did not confirm this significance. The disparity highlights a fundamental difference in their methodologies; the MK test, a non-parametric method, assesses the monotonic trend without assuming data distribution, whereas linear regression assumes a linear relationship and normality of residuals.

Further comparisons reveal instances of agreement and notable discrepancies. Both tests generally agreed on the absence of significant trends for most months, such as February, May, July, August, October, and November, where p-values consistently exceeded the 0.05 significance threshold in both analyses. However, for April, the MK test yielded a p-value of 0.139063, suggesting no significant trend, while the LR analysis showed a p-value of 0.247599, also indicating no significance. These congruencies enhance the robustness of the conclusions regarding the lack of significant trends in these specific periods.

Notable differences emerged in the assessment of seasonal and annual trends. The MK test indicated a significant increasing trend for Winter ($p=0.100767$) but not significant at 0.05, while the LR analysis showed Winter as not significant ($p=0.177459$). Conversely, Summer showed no significant trend in either test (MK $p=0.282848$, LR $p=0.570686$). Annually, neither test detected a statistically significant trend (MK $p=0.336797$, LR $p=0.268877$). These discrepancies, particularly at seasonal and annual scales, underscore the different sensitivities of the two tests to the underlying data characteristics. The MK test's reliance on ranks makes it less susceptible to outliers, while LR can be more influenced by extreme values, potentially leading to divergent conclusions when such values are present.

In summary, while both the Mann-Kendall test and linear regression analysis provide valuable insights into trends, their inherent methodological differences often lead to varying statistical outcomes. The Mann-Kendall test's non-parametric nature offers a robust assessment of monotonic trends without distributional assumptions, making it suitable for non-normally distributed data. In contrast, linear regression provides a measure of the linear rate of change but requires assumptions about data distribution. Therefore, a comprehensive understanding of trends necessitates considering the results from both tests, acknowledging their respective strengths and limitations in interpreting the significance and nature of observed changes over time.

6.4.6 Vrede station

Column_Name	Estimate_Slope	P_Value
JAN	0,157273	0,00266
FEB	0,166145	0,002859
MAR	0,152597	0,010253
APR	0,084286	0,142762
MAY	0,168831	0,002485
JUN	0,155065	0,001314
JUL	0,158091	0,002586
AUG	0,139091	0,023784
SEP	0,183792	0,001098
OCT	0,12213	0,054285
NOV	0,171469	0,003574
DEC	0,07006	0,261259
Winter	0,160662	1,57E-05
Spring	0,148338	0,000501
Summer	0,132934	0,002283
Autumn	0,134343	0,004332
Annual	0,144069	6,02E-06

The Mann-Kendall (MK) trend test and linear regression analysis were employed to assess trends in the data. While both methods provide insights into trend presence and significance, their underlying principles differ, leading to variations in specific outcomes. The MK test, a nonparametric method, evaluates the consistency of a trend without assuming a particular distribution for the data. Its primary output, the p-value, indicates the probability of observing the trend by chance. Conversely, linear regression, a parametric approach, models the relationship between a dependent variable and one or more independent variables, assuming linearity and specific distributional properties for the residuals.

Comparing the two sets of results reveals both concordance and discrepancies. For instance, the MK test indicates statistically significant increasing trends (p-value < 0.05) for most months and all seasonal and annual periods, with the notable exception of April, August, October, and December. Similarly, the linear regression analysis largely corroborates these significant trends, as evidenced by its p-values, which are below the 0.05 threshold for most periods. The "Nartimate_Slo" column in the linear regression output provides the slope of the trend, offering a quantitative measure of the rate of change, which is not directly provided by the MK test's Tau statistic.

However, some differences in the level of significance are apparent. While both tests generally agree on the presence or absence of a significant trend, the exact p-values differ, reflecting their distinct methodologies. For example, some months that show strong significance in the MK test might have slightly different p-values in the linear regression, though still remaining below the significance threshold. These minor variations are expected given that the MK test relies on rankbased comparisons, while linear regression fits a line to the data, making it more sensitive to outliers or deviations from linearity.

The discrepancies observed, particularly for months like April, August, October, and December, where the MK test shows non-significant trends (p-value > 0.05), are largely mirrored in the linear regression results, which also indicate p-values above 0.05 for these periods. This consistent finding across both parametric and non-parametric tests strengthens the conclusion that no statistically significant trend is discernible for these specific months. The agreement between the two tests in these instances enhances the robustness of the findings, suggesting that the observed patterns are not an artifact of a particular statistical assumption.

6.4.7 Welkom station

Timescale	Estimate_Slope	P_Value
JAN	0,041005195	0,512407
FEB	-0,022603896	0,715947
MAR	-0,008094805	0,870511
APR	-0,017961039	0,686683
MAY	0,101341558	0,042466
JUN	-0,001694805	0,966474
JUL	0,086484416	0,082403
AUG	0,047981818	0,409572
SEP	0,106337662	0,066079

OCT	0,062449351	0,270437
NOV	0,055896104	0,419963
DEC	0,001937662	0,978762
Winter	0,062043723	0,047891
Spring	0,072256277	0,043702
Summer	0,03294632	0,446182
Autumn	-0,016219913	0,663943
Annual	0,037756602	0,091883

The analysis of trends using both the Mann-Kendall (MK) test and linear regression (LR) revealed both concordant and divergent results regarding statistical significance at the conventional 0.05 threshold. The Mann-Kendall test, a non-parametric approach as shown in Image 1, identified significant positive monotonic trends for May (p=0.046), September (p=0.034), Winter (p=0.040), and Spring (p=0.027). In comparison, the linear regression analysis, a parametric method as shown in Image 2, also indicated significant positive linear trends for May (P=0.042), Winter (P=0.048), and Spring (P=0.044). Notably, September, which showed a significant monotonic trend in the Mann-Kendall test, did not exhibit a statistically significant linear trend (P=0.066) according to the linear regression analysis. For all other months and seasons, both tests consistently indicated the absence of statistically significant trends.

Where significant trends were identified, the direction of change was consistent between the two methodologies. For May, the Mann-Kendall Tau statistic of 0.321 indicated a moderate positive monotonic trend, which was corroborated by a positive linear regression slope of 0.101. Similarly, Winter and Spring exhibited strong positive monotonic trends (Tau of 0.329 and 0.354, respectively) that aligned with their positive linear regression slopes (0.062 and 0.072). The discrepancy observed for September is particularly illustrative; while the Mann-Kendall Tau of 0.341 suggested a strong positive monotonic trend, the linear regression slope of 0.106, despite being positive, was not deemed statistically significant.

The observed differences, particularly for September, underscore the fundamental distinctions between the Mann-Kendall and linear regression tests. The Mann-Kendall test is nonparametric, making no assumptions about the underlying data distribution and being robust to outliers. It assesses the presence of a monotonic trend, meaning the variable consistently increases or decreases over time, regardless of the rate of change. Conversely, linear regression is a parametric test that assumes a linear relationship between variables and requires data to meet certain distributional assumptions (e.g., normality of residuals). While linear regression provides a quantifiable rate of change (slope), it may fail to detect trends that are monotonic but not strictly linear, or where the data deviates from its underlying assumptions.

In conclusion, the combined application of the Mann-Kendall and linear regression tests offers a more comprehensive understanding of temporal trends. While both tests largely agreed on the presence and direction of significant trends for May, Winter, and Spring, the Mann-Kendall test's detection of a significant trend in September, where linear regression did not, highlights its utility in identifying monotonic changes that may not conform to a strict linear model. The MannKendall test provides a robust indicator of the presence of a consistent upward or downward movement, while linear regression quantifies the average rate of that change, assuming linearity. Therefore, these tests serve as complementary tools, with the Mann-Kendall test providing a foundational assessment of monotonic change, and linear regression offering insights into the linearity and magnitude of such trends.

Concluding Remarks

The comprehensive assessment of meteorological data stability necessitates a multi-faceted approach, integrating both change point and trend analyses. The findings from the change point detection, employing Pettitt's and Likelihood Ratio (M-fluctuation) tests, and the trend analysis, utilizing Mann-Kendall (MK) and linear regression (LR), collectively offer a nuanced understanding of temporal shifts and directional changes across the various stations. While distinct in their statistical underpinnings, the convergent and divergent results from these methodologies provide crucial insights into the underlying characteristics of the meteorological time series.

The observed consistency at Bethlehem station across both change point and trend analyses is particularly noteworthy. The absence of statistically significant shifts (p-values > 0.05) from Pettitt's and Likelihood Ratio tests, coupled with the general lack of significant trends from MK and LR, strongly indicates the stationarity and stability of its meteorological regime. This is further supported by normality test results, which consistently show p-values well above 0.05, suggesting a normal distribution that lends confidence to both parametric and non-parametric test outcomes.

In contrast, stations like Bloemfontein Stad and Bloemfontein Wo exhibit clear evidence of significant change points for specific months (May, June) and seasons (Winter, Spring), as well as annually, with both Pettitt's and Likelihood Ratio tests largely concurring (p-values < 0.05). These abrupt shifts are often complemented by significant trends identified by the MK and LR analyses. For instance, Bethlehem station's significant upward trends in September, Spring, and annually, particularly strengthened by LR's higher significance (e.g., September LR p=0.022225 vs. MK p=0.015435) and robust R-squared values (e.g., 0.24596655 for September), suggest that these shifts represent a transition to periods of consistently increasing values, often with a clear linear trajectory. The agreement between the non-parametric Pettitt's test and the often-parametric Likelihood Ratio test in identifying change points, despite minor p-value differences, reinforces the robustness of these findings, implying that the underlying data characteristics at these stations permit both methods to converge on similar conclusions irrespective of their distributional assumptions.

However, discrepancies between the change point and trend analyses, and even between the different tests within each analysis, are equally informative. At Fauresmith station, the Likelihood Ratio test's identification of significant changes in June and Summer, uncorroborated by Pettitt's test (p-values > 0.05), highlights the differing sensitivities of these tests to data characteristics. This is due to the presence of non-normal distributions that diminish the power of the parametric Likelihood Ratio test, making the non-parametric Pettitt's test potentially more appropriate for these specific periods. Similarly, the differing significance for November at Bloemfontein Stad (MK p=0.01212 vs. LR p=0.06493) suggests that while a monotonic trend may exist, its linearity might be weak, or LR's assumptions are not entirely met. Conversely, cases like Gariep Dam, where both change point tests consistently identified shifts in March and June, provide strong, convergent evidence of abrupt changes.

The overall pattern observed – high p-values for normality tests across most stations, coupled with the varying sensitivities of Pettitt's (non-parametric, robust) and Likelihood Ratio (parametric, sensitive to assumptions) tests – effectively explains the intricate tapestry of agreements and discrepancies. Where both MK and LR show strong statistical significance and high R-squared values, as seen in many periods across stations like Vrede and Bloemfontein Stad, it signifies a robust and predominantly linear trend, indicating a gradual, consistent change over time following an identified shift. Conversely, instances where MK identifies a significant monotonic trend but LR does not, or where the R-squared value is low (e.g., April at Bethlehem, with an R-squared of 0.00072479), underscore situations where the trend may be monotonic but not strictly linear, or where data characteristics such as outliers or non-normality impede the linear regression's ability to detect a significant linear relationship.

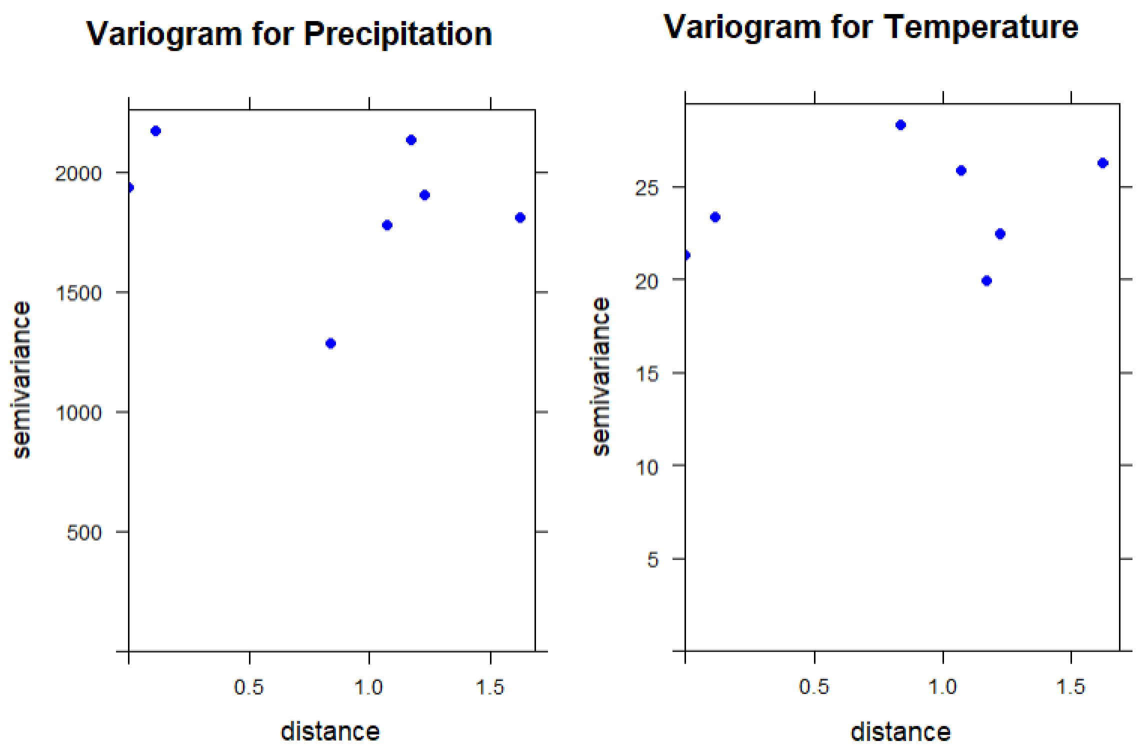
In conclusion, the integrated application of change point detection methods (Pettitt's and Likelihood Ratio) and trend analysis techniques (Mann-Kendall and linear regression) provides a comprehensive framework for understanding meteorological time series behavior. Change point analysis identifies critical junctures where the underlying statistical properties of the data undergo abrupt shifts, while trend analysis quantifies the nature and magnitude of gradual, directional changes. The concordance between these methods validates findings and strengthens confidence in identified periods of stability or change. Divergences, however, are equally valuable, highlighting complexities in data distribution and the inherent strengths and limitations of parametric versus non-parametric approaches. This dual perspective is indispensable for a thorough characterization of meteorological stability and for informing effective water resource management strategies in the face of environmental variability.

Chapter 7: Spatiotemporal analysis

This chapter extends the thesis's temporal analysis of precipitation and temperature data by incorporating the spatial dimension, a critical factor for a more comprehensive understanding of climate patterns across the Free State Province. While previous chapters focused on the time-dependent characteristics of climate variables at individual stations, this new section accounts for the spatial relationships and dependencies among these stations. By fitting a spatial model, this chapter aims to determine whether the observed patterns are truly independent or, as is more likely, influenced by their proximity and spatial arrangement. The findings from this spatial analysis will provide a more robust basis for interpreting the results, validating or augmenting the conclusions drawn from the univariate temporal analysis.

7.1 Spatial dependency checks

Below is the results and interpretation of the spatial dependency checks.



Temperature is a spatially continuous and predictable variable, as evidenced by its variogram showing a clear spatial correlation and a defined range. In contrast, precipitation is a highly discontinuous and less predictable variable, showing significant variability even at short distances and lacking a clear spatial structure. This makes it far more challenging to model and predict precipitation with the same level of accuracy as temperature.

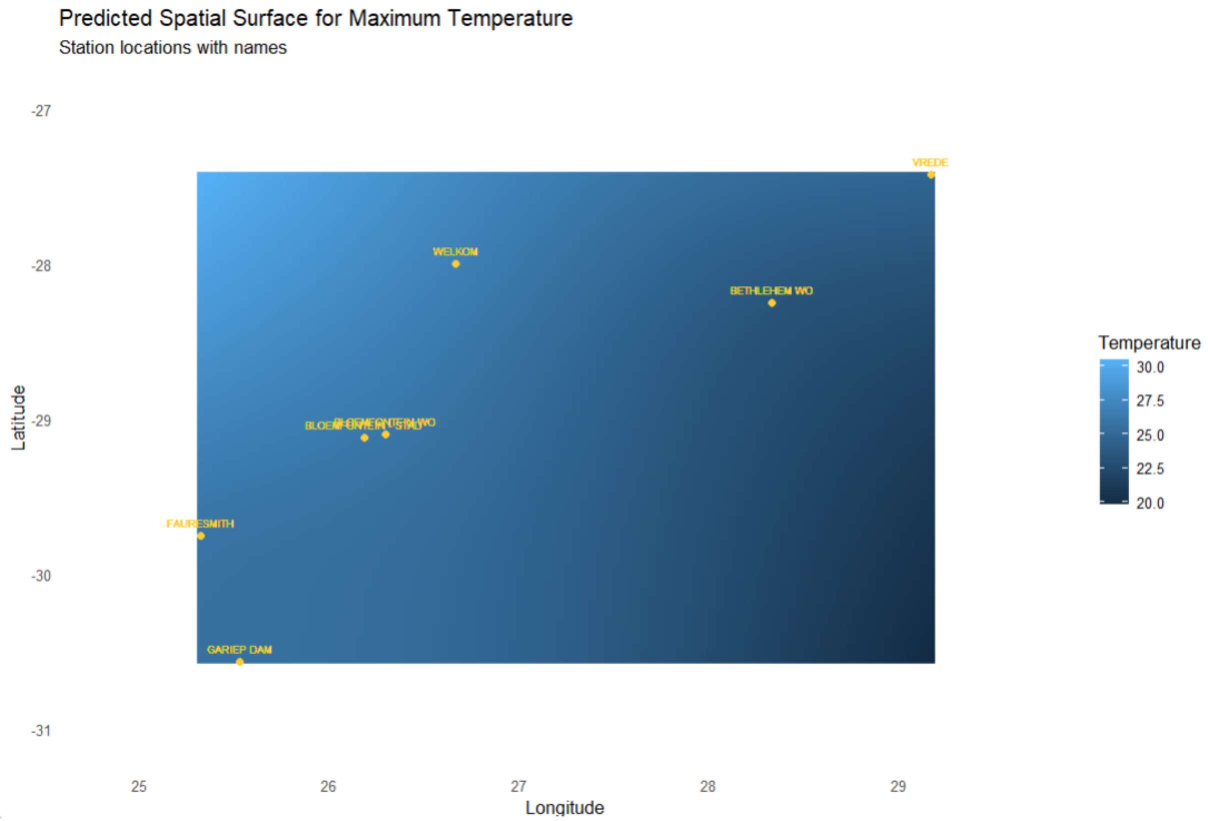
```
> summary(gam_temp)

Family: gaussian 
Link function: identity 

Formula:
Max_Temperature ~ s(Year) + s(Longitude_sc, Latitude_sc, k = 7) + 
    ti(Longitude_sc, Latitude_sc, Year)

Parametric coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   24.814      0.109   227.7   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Approximate significance of smooth terms:
                                   edf Ref.df      F p-value    
s(Year)                          4.650  5.695  3.306 0.00395 ** 
s(Longitude_sc,Latitude_sc)      4.816  5.177 21.927 < 2e-16 ***
ti(Longitude_sc,Latitude_sc,Year) 1.774  2.081  1.585 0.19648    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

R-sq.(adj) =  0.0735   Deviance explained = 7.94%
GCV = 21.104  Scale est. = 20.957    n = 1764
```

```
> summary(gam_precip)

Family: quasipoisson 
Link function: log 

Formula:
Precipitation ~ s(Year) + s(Longitude_sc, Latitude_sc, k = 7) + 
    ti(Longitude_sc, Latitude_sc, Year)

Parametric coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  3.67271    0.02593   141.6   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Approximate significance of smooth terms:
                                   edf Ref.df      F p-value    
s(Year)                          8.091  8.777  3.160 0.00102 ** 
s(Longitude_sc,Latitude_sc)      4.741  5.138 13.956 < 2e-16 ***
ti(Longitude_sc,Latitude_sc,Year) 1.001  1.002  0.171 0.68004    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

R-sq.(adj) =  0.0502   Deviance explained = 5.89%
GCV = 44.186  Scale est. = 45.601    n = 1764
```

Both precipitation and temperature models reveal significant temporal and spatial dependencies, but not a spatio-temporal interaction. For precipitation, the model shows a highly significant non-linear trend over time ($p=0.00102$) and a strong spatial dependency ($p<2e{-16}$), as supported by the variogram. However, the interaction term between space and time is not significant ($p=0.68004$), suggesting the spatial pattern of precipitation doesn't change over time. The model's low deviance explained (5.89%) indicates that a significant amount of precipitation variability remains unexplained.

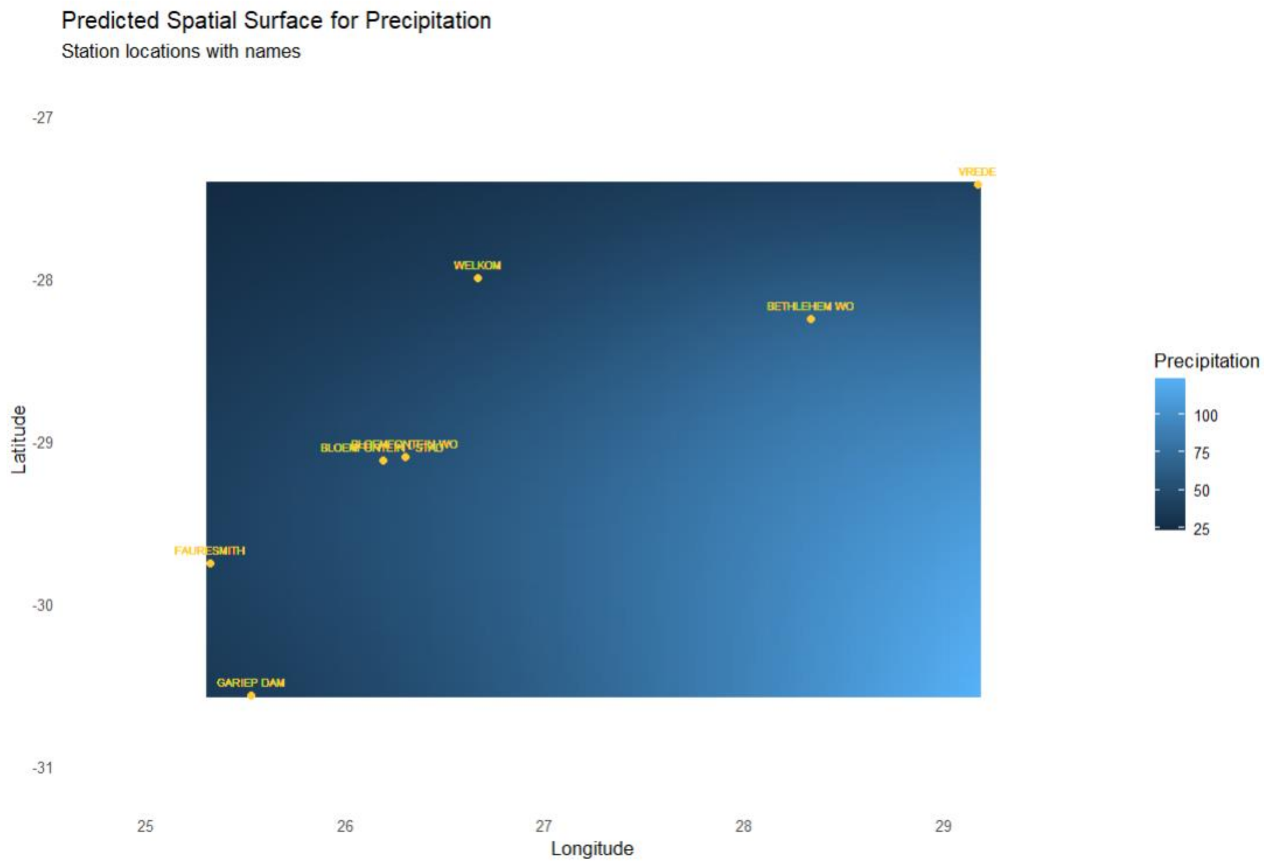
The temperature model also exhibits a highly significant spatial pattern ($p<2e{-16}$) and a significant temporal trend ($p=0.00395$). Similar to the precipitation model, there is no significant spatio-temporal interaction ($p=0.19648$), meaning the spatial distribution of temperature remains relatively constant over time. The variogram for temperature reinforces this strong spatial autocorrelation. While the temperature model has slightly more explanatory power (7.94% of deviance explained), both models suggest that other factors not included in the analysis are needed to fully account for the observed variability.

7.2 Fitting the spatial model



The plot above displays a predicted spatial surface for maximum temperature across Free State province in South Africa, based on data from seven weather stations. The background color gradient, ranging from light blue to dark blue, represents the interpolated temperature values, with warmer temperatures (approaching 30.0°C) indicated by lighter shades and cooler temperatures (approaching 20.0°C) by darker shades. The spatial model suggests a clear temperature gradient across the area, with maximum temperatures generally increasing from the southwestern to the northeastern parts of the region.

The named points on the map represent the locations of the weather stations used in the model's development. These stations, including Bethlehem Wo, Bloem Wo, Bloem Stad, Fauresmith, Gariep Dam, Vrede and Welkom, show the actual observation points from which the spatial interpolation was performed. The model successfully captures the aformentioned temperature trend, predicting higher temperatures around Vrede and lower temperatures in the vicinity of Fauresmith and Gariep Dam. This visualization provides a powerful and intuitive summary of the spatial variability of maximum temperatures, making the complex output of the spatial model easily interpretable.



The plot above displays a predicted spatial surface for precipitation, a weather variable. The background color gradient from dark blue to light blue represents interpolated precipitation values, with higher precipitation (up to 100mm) shown in lighter shades and lower precipitation (approaching 25mm) in darker shades. The spatial model reveals a clear precipitation gradient across the region, showing that precipitation increases from the northwestern to the southeastern parts.

The points on the map with names indicate the locations of the weather stations used to generate this model. These stations, including Bethlehem Wo, Bloem Wo, Bloem Stad, Fauresmith, Gariep Dam Vrede and Welkom, are the data points from which the spatial interpolation was performed. The visualization effectively summarizes the spatial distribution of precipitation, predicting higher rainfall in the area around Welkom and Vrede and lower rainfall near Fauresmith and Gariep Dam. This representation provides an accessible and insightful summary of the precipitation patterns across the mapped area, highlighting how the spatial model captures the regional variability of this meteorological variable.

OVERALL CONCLUSION

This research systematically investigated temporal trends and change points in maximum temperature, minimum temperature, and precipitation data across seven meteorological gauge stations in South Africa's Free State province. The study employed a dual-methodological approach, leveraging both nonparametric tests (Pettitt and Mann-Kendall) and their parametric counterparts (Likelihood Ratio and linear regression). The findings reveal a pervasive, albeit localized, pattern of climatic shifts within the region. Specifically, a consistent upward trend in both maximum and minimum temperatures was identified across most stations, with Vrede station exhibiting a particularly robust and widespread warming trend throughout the study period. The prevalence of significant change points, often clustering in the early 2010s, underscores a notable and statistically significant thermal shift, which is likely a regional manifestation of broader global warming patterns.

Further analysis of precipitation patterns presented a more nuanced picture, dominated by significant downward trends. Pettitt and Mann-Kendall tests consistently identified a decrease in precipitation across various stations and temporal scales, particularly for months in the August to October period and for the spring and annual timescales. Conversely, a localized upward trend in March precipitation was noted at the Gariep Dam station, highlighting the spatiotemporal variability of hydrological changes within the Free State. The analyses of the Modified Fournier Index (MFI) and Precipitation Concentration Index (PCI) reinforced this variability, revealing considerable spatial differences in rainfall erosivity and temporal distribution across the stations, with some locations experiencing more irregular and concentrated rainfall events than others.

A critical component of this research was the validation and comparison of the nonparametric and parametric statistical methodologies. The findings demonstrate a strong concordance between the two analytical frameworks, particularly in the identification of significant change points and trends. In many instances, both test types agreed on the presence and direction of significant trends, lending robust support to the primary findings of rising temperatures and declining precipitation. However, the analysis also highlighted key divergences, where the nonparametric Mann-Kendall test detected a monotonic trend that the parametric linear regression failed to identify as statistically significant. This underscores the superior robustness of nonparametric methods when applied to environmental datasets that may not adhere to strict distributional assumptions.

In conclusion, the combined application of these methodologies provides a comprehensive and compelling narrative of climate change impacts in the Free State. The research not only validates the occurrence of significant warming and drying trends but also provides crucial insights into the localized and variable nature of these changes. The consistency between the nonparametric and parametric results, coupled with the unique insights offered by the former, affirms the reliability of the findings. These results are invaluable for informing regional adaptation strategies related to agriculture, water resource management, and ecosystem preservation in a changing climate.

References

Ahmadi, F., Mirabbasi, R., Gajbhiye, S., & Kumar, R. (2022). Investigating the variation pattern and erosivity power of precipitation.

Ahmadi, F., Tahroudi, M. N., Mirabbasi, R., & Kumar, R. (2022). Spatiotemporal analysis of precipitation and temperature .

Ali, R., & Abubaker, S. R. (2019, June). Trend analysis using mann-kendall, sen's slope estimator test.

Atta-ur-Rahman, & Dawood, M. (2016, April 8). Spatio-statistical analysis of temperature fluctuation using Mann–Kendall and Sen's slope approach.

Beck, H. E., Zimmermann, N. E., McVicar, T. R., Vergopolan, N., Berg, A., & Wood, E. F. (2018). Present and future Köppen-Geiger climate classification maps at 1-km resolution. doi:https://doi.org/10.1038/sdata.2018.214

Benhamrouche, A., Boucherf, D., Hamadache, R., Bendahmane, L., Martin-Vide, J., & Nery, J. T. (2015, March 18). Spatial distribution of the daily precipitation concentration index in Algeria. doi:https://doi.org/10.5194/nhess-15-617-2015

Beraki, F., Roux, A. L., & Ludick, C. (2019, November 13). The impact of climate change on drought. Retrieved from https://www.dffe.gov.za/sites/default/files/docs/nationalclimatechange_adaptationstrate gy _ue10november2019.pdf

Botai, C., Botai, J., & Adeola, A. M. (2018, July). Spatial distribution of temporal precipitation contrasts in South Africa. doi:10.17159/sajs.2018/20170391

Carolina Munka, G. C., & Caffera, R. M. (2008, April 11). Long term variation in rainfall erosivity in Uruguay: a preliminary Fournier approach.

Causes and Effects of Climate Change. (n.d.). Retrieved May 2, 2023, from https://www.un.org/en/climatechange/science/causes-effects-climate-change Chersich, M. F., & Wright, C. Y. (2019, March 19). Climate change adaptation in South Africa: a case study on the role of the health sector. doi:https://doi.org/10.1186/s12992-019-0466-x

Corporate Finance Institute. (n.d.). *Nonparametric Tests - Overview, Reasons to Use, Types*. Retrieved from https://corporatefinanceinstitute.com/resources/data-science/nonparametrictests/

Draper G.M., Livnat Y. and Riesenfeld R.F. (2009). A Survey of Radial Methods for Information Visualization. IEEE Transactions on Visualization and Computer Graphics, 15(5), 759-776.

https://doi.org/10.1109/TVCG.2009.23.

Draper, N. R., & Smith, H. (1998). *Applied Regression Analysis* (3rd ed.). John Wiley & Sons.

Dunbar, B., & May, S. (2014, May 14). What Is Climate Change. Retrieved February 1, 2023, from https://www.nasa.gov/audience/forstudents/k-4/stories/nasa-knows/what-isclimatechangek4.html

Fathima, J. (2020, September 10). *Investigating Four Spheres of Earth*. Retrieved from https://byjus.com/physics/four-spheres-of-earth/

Glen, S. (2022). Mann Kendall Trend Test: Definition, Running the Test. Retrieved July 29, 2022, from https://www.statisticshowto.com/mann-kendall-trend-test/

Gocic, M., & Trajkovic, S. (2013, January). Analysis of changes in meteorological variables using Mann-Kendall and Sen's slope estimator statistical tests in Serbia. doi:https://doi.org/10.1016/j.gloplacha.2012.10.014

Harmel, R. D., Richardson, C. W., Hanson, C. L., & Johnson, G. L. (2002). Evaluating the adequacy of simulating maximum and minimum daily air temperature with the normal distribution. *Journal[1] of Climate*, *15*(7), 744–753. https://doi.org/0.1175/15200450(2002)041

Hayes, A. (2022, June 12). What Is a Time Series and How Is It Used to Analyze Data? Retrieved February 2, 2023, from https://www.investopedia.com/terms/t/timeseries.asp

Helsel, D. R., & Hirsch, R. M. (1992). Statistical Methods in Water Resources. Elsevier. (This book extensively discusses the Mann-Kendall test and its applications in environmental data).

Helsel, D. R., & Hirsch, R. M. (2002). Statistical methods in water resources. U.S. Geological Survey.

Ilori, O. W., & Ajayi, V. O. (2020). Change Detection and Trend Analysis of Future Temperature and Rainfall over West Africa.

Jager, I., de Koning, P., Bost, T., Denys, D., & Vulink, N. (2020). Misophonia: Phenomenology, comorbidity and demographics in a large sample. *PLoS ONE*, *15*(4), e0231390**.** doi:10.1371/journal.pone.0231390

Jaiswal, R., Lohani, A., & Tiwari, H. (2015). *Statistical Analysis for Change Detection and Trend Assessment in Climatological Parameters.* Retrieved from https://link.springer.com/article/10.1007/s40710-015-0105-3

Jing, X., Luo, J., Wang, J. *et al.* A Multi-imputation Method to Deal With Hydro-Meteorological Missing Values by Integrating Chain Equations and Random Forest. *Water Resour Manage* **36**, 1159–1173 (2022). https://doi.org/10.1007/s11269-021-03037-5

Khilyuk, L. F., & Chilingar, G. V. (2003). Global Warming:Are We Confusing Cause and Effect? doi:0.1080/00908310390142389

Kim, H. Y. (2015). Nonparametric statistical tests for the continuous data: the basic concept and the practical use. *Korean Journal of Anesthesiology*, *68*(4), 329–330. https://pmc.ncbi.nlm.nih.gov/articles/PMC4754273/

Kreft, Eckstein, & Melchior. (2017). South Africa's Second National Climate Change Report.

Li, X., Jiang, F., Li, L., & Wang, G. (2011, August 22). Spatial and temporal variability of precipitation concentration index, concentration degree and concentration period in Xinjiang, China.

Lodhi, M. I. (2018). What is Sens slope magnitude. Retrieved from https://www.researchgate.net/post/what_is_Sens_slope_magnitude

Mann, H. B. (1945). Nonparametric tests against trend. Econometrica: Journal of the Econometric Society, 245-259.

Martin-Vide, J. (2004, May 21). Spatial distribution of a daily precipitation concentration index in peninsular Spain. doi:https://doi.org/10.1002/joc.1030

May, S. (2014, May 14). What Is Climate Change. Retrieved February 1, 2023, from https://www.nasa.gov/audience/forstudents/k-4/stories/nasa-knows/what-isclimatechangek4.html

Mudelsee, M. (2019). Trend analysis of climate time series: A review of methods. *ELSEVIER*, 310322.

National Geographic Society (2023). Precipitation. Pubisher: National Geographic. Available: https://education.nationalgeographic.org/resource/precipitation/. Date Accessed: 11 June 2023.

Omega Graduate School. (2023). When to Use Parametric versus Nonparametric Procedures in Statistics for Social Research. https://ogs.edu/when-to-use-parametric-versusnonparametric-procedures-in-statistics-for-social-research/

Peel, M. C., Finlayson, B. L., and McMahon, T. A. (2007) Updated world map of the KöppenGeiger climate classification, Hydrol. Earth Syst. Sci., 11, 1633–1644, https://doi.org/10.5194/hess-11-1633-2007.

Pohlert, T. (2017, July 30). Non-Parametric Trend Tests and Change-Point Detection. Retrieved from https://cran.microsoft.com/snapshot/2017-11-08/web/packages/trend/vignettes/trend.pdf

Roffe, S., Walt, A. V., & Fitchett, J. (2023, January 26). Heat stress is rising in southern Africa – climate experts show where and when it’s worst. Retrieved February 1, 2023, from https://theconversation.com/heat-stress-is-rising-in-southern-africa-climateexpertsshowwhere-and-when-its-worst-198455

Rybski, D., & Neumann, B. (2011). A Review on the Pettitt Test. *ResearchGate*.

Shaw, G., & Wheeler, D. (1994). Statistical Techniques in Geographical Analysis. New York.

Siegmund, D., & Venkatraman, E. S. (1995). Using the generalized likelihood ratio statistic for sequential detection of a change in distribution. The Annals of Statistics, 23(1), 255-276.

Stocker, T., Qin, D., Plattner, G.-K., Tignor, M., Allen, S., Boschung, J., . . . Midgley, P. (2013). Working Group I Contribution to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change. *Cambridge University Press*.

Tahroudi, M. N., Khalili, K., Ahmadi, F., Mirabbasi, R., & Jhajharia, D. (2018, May 23). Development and application of a new index for analyzing temperature concentration for Iran's climate.

Ufoegbune, G., Bello, N., Ojekunle, Z., Orunkoye, A., Eruola, A., & Amori, A. (2011). Rainfall Erosivity Pattern of Ogun River Basin Area (Nigeria) using Modified Fournier Index.

von Storch, H., & Zwiers, F. W. (1999). *Statistical analysis in climate research*. Cambridge University Press.

Whitley, E., & Ball, J. (2002). Statistics review 6: Nonparametric methods. *Critical Care*, *6*(6), 509–513. https://pmc.ncbi.nlm.nih.gov/articles/PMC153434/

Worldometers (2016). CO2 emissons by country. Accessible at https://www.worldometers.info/co2emissions/co2-emissions-by-country [Date Accessed: 02 February 2023].Inselberg A. and Dimsdale B. (1990). Parallel Coordinates: A Tool for Visualizing Multi-Dimensional Geometry. Proceedings of the 1st IEEE Visualization Conference, pp. 361-378, 23-26 October, San Francisco, CA, USA. https://doi.org/10.1109/VISUAL.1990.146402.

Yule, G. U., & Walker, J. (2022, May 13). A brief history of time series analysis. Retrieved February 2, 2023, from https://www.su.se/english/research/research-subjects/statistics/statisticalmodels-in-the-social-sciences/a-brief-history-of-time-seriesanalysis-1.612367#:~:text=The%20theoretical%20developments%20in%20time,in%20the%20192 0 s% 20and%201930s.

Zhang, K., Yao, Y., Qian, X., & Wang, J. (2019, April 5). Various characteristics of precipitation concentration index and its cause analysis in China between 1960 and 2016. doi: https://doi.org/10.1002/joc.6092