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## 1 Miscellaneous

### 1.1 Day of Date

```
// 0-based
const vector<int> T = {
    0, 3, 2, 5, 0, 3,
    5, 1, 4, 6, 2, 4
}

int day(int d, int m, int y) {
    y -= (m < 3);
    return (y + y / 4 - y / 100 + y / 400 + T[m - 1] + d) % 7;
}
```

### 1.2 Number of Days since 1-1-1

```
int rdn(int d, int m, int y) {
    if(m < 3)
        --y, m += 12;
    return 365 * y + y / 4 - y / 100 + y / 400
        + (153 * m - 457) / 5 + d - 306;
}
```

### 1.3 Enumerate Subsets of a Bitmask

```
int x = 0;
do {
    // do stuff with the bitmask here
    x = (x + 1 + ~m) & m;
} while(x != 0);
```

### 1.4 Fast IO

```
int read() {
    char c;
    do {
        c = getchar_unlocked();
    } while(c < 33);
    int res = 0;
    int mul = 1;
    if(c == '-') {
        mul = -1;
        c = getchar_unlocked();
    }
    while('0' <= c && c <= '9') {
        res = res * 10 + c - '0';
        c = getchar_unlocked();
    }
    return res * mul;
}

void write(int x) {
    static char wbuf[10];
    if(x < 0) {
        putchar_unlocked('-');
        x = -x;
    }
    int idx = 0;
    while(x) {
        wbuf[idx++] = x % 10;
        x /= 10;
    }
    if(idx == 0)
        putchar_unlocked('0');
    for(int i = idx - 1; i >= 0; --i)
        putchar_unlocked(wbuf[i] + '0');
```

```
}
void write(const char* s) {
    while(*s) {
        putchar_unlocked(*s);
        ++s;
    }
}
```

### 1.5 Int to Roman

```
const string R[] = {
    "M", "CM", "D", "CD", "C", "XC", "L",
    "XL", "X", "IX", "V", "IV", "I"
};

const int N[] = {
    1000, 900, 500, 400, 100, 90,
    50, 40, 10, 9, 5, 4, 1
};

string to_roman(int x) {
    if(x == 0) {
        return "0"; // Not decimal 0!
    }
    string res = "";
    for(int i = 0; i < 13; ++i)
        while(x >= N[i])
            x -= N[i], res += R[i];
    return res;
}
```

### 1.6 Josephus Problem

```
ll josephus(ll n, ll k) { // O(k log n)
    if(n == 1)
        return 0;
    if(k == 1)
        return n - 1;
    if(k > n)
        return (josephus(n - 1, k) + k) % n;
    ll cnt = n / k;
    ll res = josephus(n - cnt, k);
    res -= n % k;
    if(res < 0)
        res += n;
    else
        res += res / (k - 1);
    return res;
}

int josephus(int n, int k) { // O(n)
    int res = 0;
    for(int i = 1; i <= n; ++i)
        res = (res + k) % i;
    return res + 1;
}
```

### 1.7 Random Primes

36671 74101 724729 825827 924997 1500005681 2010408371 2010405347

### 1.8 RNG

```
// RNG - rand_int(min, max), inclusive
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());
```

```
template<class T>
T rand_int(T mn, T mx) {
    return uniform_int_distribution<T>(mn, mx)(rng);
}
```

## 2 Data Structures

### 2.1 2D Segment Tree

```
struct Segtree2D {
    struct Segtree {
        struct node {
            int l, r, val;
            node* lc, *rc;
            node(int _l, int _r, int _val = INF) : l(_l), r(_r), val(_val),
                lc(NULL), rc(NULL) {}
        };
        typedef node* pnode;

        pnode root;

        Segtree(int l, int r) {
            root = new node(l, r);
        }

        void update(pnode& nw, int x, int val) {
            int l = nw->l, r = nw->r, mid = (l + r) / 2;
            if(l == r)
                nw->val = val;
            else {
                assert(l <= x && x <= r);
                pnode& child = x <= mid ? nw->lc : nw->rc;
                if(!child)
                    child = new node(x, x, val);
                else if(child->l <= x && x <= child->r)
                    update(child, x, val);
                else {
                    do {
                        if(x <= mid)
                            r = mid;
                        else
                            l = mid + 1;
                        mid = (l + r) / 2;
                    } while((x <= mid) == (child->l <= mid));
                    pnode nxt = new node(l, r);
                    if(child->l <= mid)
                        nxt->lc = child;
                    else
                        nxt->rc = child;
                    child = nxt;
                    update(nxt, x, val);
                }
                nw->val = min(nw->lc ? nw->lc->val : INF,
                    nw->rc ? nw->rc->val : INF);
            }
        }

        int query(pnode& nw, int x1, int x2) {
            if(!nw)
                return INF;
            int& l = nw->l, &r = nw->r;
            if(r < x1 || x2 < l)
                return INF;
            if(x1 <= l && r <= x2)
                return nw->val;
            int ret = min(query(nw->lc, x1, x2),
                query(nw->rc, x1, x2));
            return ret;
        }
    };
}
```

```

}

void update(int x, int val) {
    assert(root->l <= x && x <= root->r);
    update(root, x, val);
}

int query(int l, int r) {
    return query(root, l, r);
}
};

struct node {
    int l, r;
    Segtree y;
    node* lc, *rc;
    node(int _l, int _r) : l(_l), r(_r), y(0, MAX),
        lc(NULL), rc(NULL) {}
};
typedef node* pnode;

pnode root;

Segtree2D(int l, int r) {
    root = new node(l, r);
}

void update(pnode& nw, int x, int y, int val) {
    int& l = nw->l, &r = nw->r, mid = (l + r) / 2;
    if(l == r)
        nw->y.update(y, val);
    else {
        if(x <= mid) {
            if(!nw->lc)
                nw->lc = new node(l, mid);
            update(nw->lc, x, y, val);
        } else {
            if(!nw->rc)
                nw->rc = new node(mid + 1, r);
            update(nw->rc, x, y, val);
        }
        val = min(nw->lc ? nw->lc->y.query(y, y) : INF,
            nw->rc ? nw->rc->y.query(y, y) : INF);
        nw->y.update(y, val);
    }
}

int query(pnode& nw, int x1, int x2, int y1, int y2) {
    if(!nw)
        return INF;
    int& l = nw->l, &r = nw->r;
    if(r < x1 || x2 < l)
        return INF;
    if(x1 <= l && r <= x2)
        return nw->y.query(y1, y2);
    int ret = min(query(nw->lc, x1, x2, y1, y2),
        query(nw->rc, x1, x2, y1, y2));
    return ret;
}

void update(int x, int y, int val) {
    assert(root->l <= x && x <= root->r);
    update(root, x, y, val);
}

int query(int x1, int x2, int y1, int y2) {
    return query(root, x1, x2, y1, y2);
}
};

```

## 2.2 Fenwick RU-RQ

```

void updtRL(int l, int r, ll val) {
    updt(BIT1, l, val), updt(BIT1, r + 1, -val);
    updt(BIT2, l, val * (l - 1)), updt(BIT2, r + 1, -val * r);
}

ll query(int k) {
    return que(BIT1, k) * k - que(BIT2, k);
}

```

## 2.3 Heavy-Light Decomposition

```

struct HLD {
    int n;
    vector<int> id, size, idx, up, root, st;
    vector<vector<int>> adj, chain;
    SegTree seg;

    HLD(const vector<vector<int>>& edges) :
        n(edges.size()), id(n, -1), size(n, -1), idx(n, -1),
        up(n, -1), adj(edges), seg(n) {
        precompute(0, -1);
        decompose(0, -1);
        int cnt = 0;
        st.resize(chain.size());
        for(int i = 0; i < (int) chain.size(); ++i) {
            st[i] = cnt;
            cnt += chain[i].size();
        }
    }

    void precompute(int pos, int dad) {
        size[pos] = 1;
        up[pos] = dad;
        for(auto& i : adj[pos]) {
            if(i != dad) {
                precompute(i, pos);
                size[pos] += size[i];
            }
        }
    }

    void decompose(int pos, int dad) {
        if(id[pos] == -1) {
            id[pos] = chain.size();
            root.push_back(pos);
            chain.emplace_back();
        }
        idx[pos] = chain[id[pos]].size();
        chain[id[pos]].push_back(pos);
        int mx = 0, heavy = -1;
        for(auto& i : adj[pos]) {
            if(i != dad && size[i] > mx) {
                mx = size[i];
                heavy = i;
            }
        }
        if(heavy != -1)
            id[heavy] = id[pos];
        for(auto& i : adj[pos]) {
            if(i != dad)
                decompose(i, pos);
        }
    }

    void update(int ch, int l, int r, int val) {
        seg.update(st[ch] + l, st[ch] + r, val);
    }

    int query(int ch, int l, int r, int val) {

```

```
};
vector<P> node;
vector<int> root;

int newNode() {
    return ptr++;
}

int copyNode(int idx) {
    node[ptr] = node[idx];
    return ptr++;
}

int build(int l, int r) {
    int idx = newNode();
    if(l == r)
        return idx;
    node[idx].l = build(l, (l + r) / 2);
    node[idx].r = build((l + r) / 2 + 1, r);
    return idx;
}

int update(int idx, int l, int r, int x, int val) {
    idx = copyNode(idx);
    if(l == r) {
        node[idx].val += val;
        return idx;
    }
    int mid = (l + r) / 2;
    if(x <= mid)
        node[idx].l = update(node[idx].l, l, mid, x, val);
    else
        node[idx].r = update(node[idx].r, mid + 1, r, x, val);
    node[idx].val = node[node[idx].l].val + node[node[idx].r].val;
    return idx;
}

int query(int idxl, int idxr, int l, int r, int x, int y) {
    if(y < l || r < x)
        return 0;
    if(x <= l && r <= y)
        return node[idxr].val - node[idxl].val;
    int mid = (l + r) / 2;
    return query(node[idxl].l, node[idxr].l, l, mid, x, y)
        + query(node[idxl].r, node[idxr].r, mid + 1, r, x, y);
}

public:
PersistentSegtree(int _n) : n(_n), ptr(0) {
    sz = 30 * n;
    node.resize(sz);
    root.push_back(build(1, n));
}

void update(int x, int val) {
    root.push_back(update(root.back(), 1, n, x, val));
}

int query(int l, int r, int x, int y) {
    return query(root[l - 1], root[r], 1, n, x, y);
}
};
```

## 2.6 STL PBDS

```
// ost = ordered set
// omp = ordered map
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
```

```
using namespace __gnu_pbds;
```

```
template<class T>
using ost = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
template<class T, class U>
```

```
    return seg.query(st[ch] + l, st[ch] + r, val);
}
};

// how to move from u to v
while(1) {
    if(hld.id[u] == hld.id[v]) {
        if(hld.idx[u] > hld.idx[v])
            swap(u, v);
        hld.update(hld.id[u], hld.idx[u], hld.idx[v], w);
        // or hld.query(hld.id[u], hld.idx[u], hld.idx[v]);
        break;
    }
    if(hld.id[u] < hld.id[v])
        swap(u, v);
    hld.update(hld.id[u], 0, hld.idx[u], w);
    // or hld.query(hld.id[u], 0, hld.idx[v]);
    u = hld.up[hld.root[hld.id[u]]];
}
}
```

## 2.4 Li Chao Tree

```
typedef long long int TD;
const TD INF = 100000000000000;
namespace LICHAO {
struct Node {
    TD m, c;
    Node* l, *r;
};
Node* newNode(Node* x = NULL) {
    Node* ret = (Node*)malloc(sizeof(Node));
    if(x)
        ret->m = x->m, ret->c = x->c;
    ret->l = ret->r = NULL;
    return ret;
}

void update(Node* k, TD l, TD r, TD m, TD c) {
    TD mid = l + r >> 1;
    bool le = m * l + c < k->m * l + k->c;
    bool ri = m * mid + c < k->m * mid + k->c;
    if(ri)
        swap(k->m, m), swap(k->c, c);
    if(r - l <= 1)
        return;
    else if(le != ri)
        update((k->l) ? (k->l) : (k->l = newNode(k)), l, mid, m, c);
    else
        update((k->r) ? (k->r) : (k->r = newNode(k)), mid, r, m, c);
}

TD query(Node* k, TD l, TD r, TD p) {
    if(!k)
        return INF;
    if(r - l <= 1)
        return p * k->m + k->c;
    if(p < (l + r >> 1))
        return min(p * k->m + k->c, query(k->l, l, l + r >> 1, p));
    else
        return min(p * k->m + k->c, query(k->r, l + r >> 1, r, p));
}
}
```

## 2.5 Persistent Segment Tree

```
class PersistentSegtree {
private:
    int n, ptr, sz;
    struct P {
        int val = 0, l, r;
```

```
using omp = tree<T, U, less<T>, rb_tree_tag,  
    tree_order_statistics_node_update>;
```

2.7 Treap

```
// Complexity: O(log N) for split and merge  
//  
// empty treap: Treap* tr = nullptr;  
// insert v at x: [l, r] = split(tr, x), m = Treap(v), merge lmr  
// delete at x: [l, r] = split(tr, x), [m, r] = split(r, l), merge lr  
// lazy prop: propagate every time a node is accessed  
  
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());  
  
using Key = int;  
  
struct Treap {  
    Key val;  
    Treap* left;  
    Treap* right;  
    int prio, sz;  
    Treap() {}  
    Treap(int _val);  
};  
  
int size(Treap* tr) {  
    return tr ? tr->sz : 0;  
}  
  
void update(Treap* tr) {  
    tr->sz = 1 + size(tr->left) + size(tr->right);  
}  
  
Treap::Treap(Key _val) :  
    val(_val), left(nullptr), right(nullptr), prio(rng()) {  
    update(this);  
}  
  
pair<Treap*, Treap*> split(Treap* tr, int sz) {  
    if(!tr) return {nullptr, nullptr};  
    int left_sz = size(tr->left);  
    if(sz <= left_sz) {  
        auto [left, mid] = split(tr->left, sz);  
        tr->left = mid;  
        update(tr);  
        return {left, tr};  
    } else {  
        auto [mid, right] = split(tr->right, sz - left_sz - 1);  
        tr->right = mid;  
        update(tr);  
        return {tr, right};  
    }  
}  
  
Treap* merge(Treap* l, Treap* r) {  
    if(!l)  
        return r;  
    if(!r)  
        return l;  
    if(l->prio < r->prio) {  
        l->right = merge(l->right, r);  
        update(l);  
        return l;  
    } else {  
        r->left = merge(l, r->left);  
        update(r);  
        return r;  
    }  
}
```

2.8 Unordered Map Custom Hash

```
struct custom_hash {  
    static uint64_t splitmix64(uint64_t x) {  
        x += 0x9e3779b97f4a7c15;  
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;  
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;  
        return x ^ (x >> 31);  
    }  
    size_t operator()(uint64_t x) const {  
        static const uint64_t FIXED_RANDOM =  
            chrono::steady_clock::now().time_since_epoch().count();  
        return splitmix64(x + FIXED_RANDOM);  
    }  
};  
  
unordered_map<int, int, custom_hash> umap;
```

2.9 Mo's on Tree

$ST(u) \leq ST(v)$   
 $P = LCA(u, v)$   
If  $P = u$ , query  $[ST(u), ST(v)]$   
Else query  $[EN(u), ST(v)] + [ST(P), ST(P)]$

3 Dynamic Programming

3.1 DP Convex Hull

```
/* dp[i] = min k<i {dp[k] + x[i]*m[k]}  
    Make sure gradient (m[i]) is either non-increasing if min,  
    or non-decreasing if max. x[i] must be non-decreasing. just sort */  
int y[N], m[N];  
// while this is true, pop back from dq. a=new line, b=last, c=2nd last  
bool cekx(int a, int b, int c) {  
    // if not enough, change to cross mul  
    // if cross mul, beware of negative denominator, and overflow  
    return (double)(y[b] - y[a]) / (m[a] - m[b]) <= (double)(y[c] - y[b]) /  
        (m[b] - m[c]);  
}
```

3.2 DP DNC

```
void f(int rem, int l, int r, int optl, int optr) {  
    if(l > r)  
        return;  
    int mid = l + r >> 1;  
    int opt = MOD, optid = mid;  
    for(int i = optl; i <= mid && i <= optr; ++i) {  
        if(dp[rem - 1][i] + c[i][mid] < opt) {  
            opt = dp[rem - 1][i] + c[i][mid];  
            optid = i;  
        }  
    }  
    dp[rem][mid] = opt;  
    f(rem, l, mid - 1, optl, optid);  
    f(rem, mid + 1, r, optid, optr);  
    return;  
}  
  
rep(i, 1, n)dp[1][i] = c[0][i];  
rep(i, 2, k)f(i, i, n, i, n);
```

3.3 DP Knuth-Yao

```
// opt[i+1][j] <= opt[i][j] <= opt[i][j+1]  
// dp[i][j] = min{k} dp[i][k]+dp[k][j]+cost[i][j]
```

```
const double PI = acos(-1.0);
const double INF9 = 1E9;
double between_d(double x, double l, double r) {
    return (min(l, r) <= x + EPS && x <= max(l, r) + EPS);
}
double same_d(double x, double y) {
    return between_d(x, y, y);
}
double dabs(double x) {
    if(x < EPS)
        return -x;
    return x;
}
/*Point*/
struct point {
    double x, y;
    point() {
        x = y = 0.0;
    }
    point(double _x, double _y) {
        x = _x;
        y = _y;
    }
    bool operator< (point other) {
        if(x < other.x + EPS)
            return true;
        if(x + EPS > other.x)
            return false;
        return y < other.y + EPS;
    }
    bool operator== (point other) {
        return same_d(x, other.x) && same_d(y, other.y);
    }
};
double e_dist(point P1, point P2) {
    return hypot(P1.x - P2.x, P1.y - P2.y);
}
double m_dist(point P1, point P2) {
    return dabs(P1.x - P2.x) + dabs(P1.y - P2.y);
}
double pointBetween(point P, point L, point R) {
    return (e_dist(L, P) + e_dist(P, R) == e_dist(L, R));
}
bool collinear(point P, point L,
               point R) { //newly added(luis), cek 3 poin segaris
    return P.x * (L.y - R.y) + L.x * (R.y - P.y) + R.x * (P.y - L.y) ==
           0; // bole gnti "dabs(x)<"EPS
}

/*Vector*/
struct vec {
    double x, y;
    vec() {
        x = y = 0.0;
    }
    vec(double _x, double _y) {
        x = _x;
        y = _y;
    }
    vec(point A) {
        x = A.x;
        y = A.y;
    }
    vec(point A, point B) {
        x = B.x - A.x;
        y = B.y - A.y;
    }
};
vec scale(vec v, double s) {
    return vec(v.x * s, v.y * s);
}
```

```
for(int k = 0; k <= n; k++) {
    for(int i = 0; i + k <= n; i++) {
        if(k < 2)
            dp[i][i + k] = 0, opt[i][i + k] = i;
        else {
            int sta = opt[i][i + k - 1];
            int end = opt[i + 1][i + k];
            for(int j = sta; j <= end; j++) {
                if(dp[i][j] + dp[j][i + k] + cost[i][i + k] < dp[i][i + k]) {
                    dp[i][i + k] = dp[i][j] + dp[j][i + k] + cost[i][i + k];
                    opt[i][i + k] = j;
                }
            }
        }
    }
}
```

## 4 Geometry

### 4.1 Geometry Template

```
/*
TABLE OF CONTENT
0. Basic Rule
    0.1. Everything is in double
    0.2. Every comparison use EPS
    0.3. Every degree in rad
1. General Double Operation
    1.1. const double EPS=1E-9
    1.2. const double PI=acos(-1.0)
    1.3. const double INF9=1E9
    1.3. between_d(double x,double l,double r)
        check whether x is between l and r inclusive with EPS
    1.4. same_d(double x,double y)
        check whether x=y with EPS
    1.5. dabs(double x)
        absolute value of x
2. Point
    2.1. struct point
        2.1.1. double x,y
            cartesian coordinate of the point
        2.1.2. point()
            default constructor
        2.1.3. point(double _x,double _y)
            constructor, set the point to (_x,_y)
        2.1.4. bool operator< (point other)
            regular pair<double,double> operator < with EPS
        2.1.5. bool operator== (point other)
            regular pair<double,double> operator == with EPS
    2.2. hypot(point P)
        length of hypotenuse of point P to (0,0)
    2.3. e_dist(point P1,point P2)
        euclidean distance from P1 to P2
    2.4. m_dist(point P1,point P2)
        manhattan distance from P1 to P2
    2.5. point rotate(point P,point O,double angle)
        rotate point P from the origin O by angle ccw
3. Vector
    3.1. struct vec
        3.1.1. double x,y
            x and y magnitude of the vector
        3.1.2. vec()
            default constructor
        3.1.3. vec(double _x,double _y)
            constructor, set the vector to (_x,_y)
        3.1.4. vec(point A,point B)
            constructor, set the vector to vector AB (A->B)
*/
/*General Double Operation*/
```

```

}
vec flip(vec v) {
    return vec(-v.x, -v.y);
}
double dot(vec u, vec v) {
    return (u.x * v.x + u.y * v.y);
}
double cross(vec u, vec v) {
    return (u.x * v.y - u.y * v.x);
}
double norm_sq(vec v) {
    return (v.x * v.x + v.y * v.y);
}
point translate(point P, vec v) {
    return point(P.x + v.x, P.y + v.y);
}
point rotate(point P, point O, double angle) {
    vec v(O);
    P = translate(P, flip(v));
    return translate(point(P.x * cos(angle) - P.y * sin(angle),
        P.x * sin(angle) + P.y * cos(angle)), v);
}
point mid(point P, point Q) {
    return point((P.x + Q.x) / 2, (P.y + Q.y) / 2);
}
double angle(point A, point O, point B) {
    vec OA(O, A), OB(O, B);
    return acos(dot(OA, OB) / sqrt(norm_sq(OA) * norm_sq(OB)));
}
int orientation(point P, point Q, point R) {
    vec PQ(P, Q), PR(P, R);
    double c = cross(PQ, PR);
    if(c < -EPS)
        return -1;
    if(c > EPS)
        return 1;
    return 0;
}
/*Line*/
struct line {
    double a, b, c;
    line() {
        a = b = c = 0.0;
    }
    line(double _a, double _b, double _c) {
        a = _a;
        b = _b;
        c = _c;
    }
    line(point P1, point P2) {
        if(P1 < P2)
            swap(P1, P2);
        if(same_d(P1.x, P2.x))
            a = 1.0, b = 0.0, c = -P1.x;
        else
            a = -(P1.y - P2.y) / (P1.x - P2.x), b = 1.0, c = -(a * P1.x) - P1.y;
    }
    line(point P, double slope) {
        if(same_d(slope, INFD))
            a = 1.0, b = 0.0, c = -P.x;
        else
            a = -slope, b = 1.0, c = -(a * P.x) - P.y;
    }
    bool operator==(line other) {
        return same_d(a, other.a) && same_d(b, other.b) && same_d(c, other.c);
    }
    double slope() {
        if(same_d(b, 0.0))
            return INFD;
        return -(a / b);
    }
}

```

```

};
bool paralel(line L1, line L2) {
    return same_d(L1.a, L2.a) && same_d(L1.b, L2.b);
}
bool intersection(line L1, line L2, point& P) {
    if(paralel(L1, L2))
        return false;
    P.x = (L2.b * L1.c - L1.b * L2.c) / (L2.a * L1.b - L1.a * L2.b);
    if(same_d(L1.b, 0.0))
        P.y = -(L2.a * P.x + L2.c);
    else
        P.y = -(L1.a * P.x + L1.c);
    return true;
}
double pointToLine(point P, point A, point B, point& C) {
    vec AP(A, P), AB(A, B);
    double u = dot(AP, AB) / norm_sq(AB);
    C = translate(A, scale(AB, u));
    return e_dist(P, C);
}
double lineToLine(line L1, line L2) {
    if(!paralel(L1, L2))
        return 0.0;
    return dabs(L2.c - L1.c) / sqrt(L1.a * L1.a + L1.b * L1.b);
}
/*Line Segment*/
struct segment {
    point P, Q;
    line L;
    segment() {
        point T1;
        P = Q = T1;
        line T2;
        L = T2;
    }
    segment(point _P, point _Q) {
        P = _P;
        Q = _Q;
        if(Q < P)
            swap(P, Q);
        line T(P, Q);
        L = T;
    }
    bool operator==(segment other) {
        return P == other.P && Q == other.Q;
    }
};
bool onSegment(point P, segment S) {
    if(orientation(S.P, S.Q, P) != 0)
        return false;
    return between_d(P.x, S.P.x, S.Q.x) && between_d(P.y, S.P.y, S.Q.y);
}
bool s_intersection(segment S1, segment S2) {
    double o1 = orientation(S1.P, S1.Q, S2.P);
    double o2 = orientation(S1.P, S1.Q, S2.Q);
    double o3 = orientation(S2.P, S2.Q, S1.P);
    double o4 = orientation(S2.P, S2.Q, S1.Q);
    if(o1 != o2 && o3 != o4)
        return true;
    if(o1 == 0 && onSegment(S2.P, S1))
        return true;
    if(o2 == 0 && onSegment(S2.Q, S1))
        return true;
    if(o3 == 0 && onSegment(S1.P, S2))
        return true;
    if(o4 == 0 && onSegment(S1.Q, S2))
        return true;
    return false;
}
double pointToSegment(point P, point A, point B, point& C) {
    vec AP(A, P), AB(A, B);
}

```

```
double u = dot(AP, AB) / norm_sq(AB);
if(u < EPS) {
    C = A;
    return e_dist(P, A);
}
if(u + EPS > 1.0) {
    C = B;
    return e_dist(P, B);
}
return pointToLine(P, A, B, C);
}
double segmentToSegment(segment S1, segment S2) {
    if(s_intersection(S1, S2))
        return 0.0;
    double ret = INFD;
    point dummy;
    ret = min(ret, pointToSegment(S1.P, S2.P, S2.Q, dummy));
    ret = min(ret, pointToSegment(S1.Q, S2.P, S2.Q, dummy));
    ret = min(ret, pointToSegment(S2.P, S1.P, S1.Q, dummy));
    ret = min(ret, pointToSegment(S2.Q, S1.P, S1.Q, dummy));
    return ret;
}
/*Circle*/
struct circle {
    point P;
    double r;
    circle() {
        point P1;
        P = P1;
        r = 0.0;
    }
    circle(point _P, double _r) {
        P = _P;
        r = _r;
    }
    circle(point P1, point P2) {
        P = mid(P1, P2);
        r = e_dist(P, P1);
    }
    circle(point P1, point P2, point P3) {
        vector<point> T;
        T.clear();
        T.pb(P1);
        T.pb(P2);
        T.pb(P3);
        sort(T.begin(), T.end());
        P1 = T[0];
        P2 = T[1];
        P3 = T[2];
        point M1, M2;
        M1 = mid(P1, P2);
        M2 = mid(P2, P3);
        point Q2, Q3;
        Q2 = rotate(P2, P1, PI / 2);
        Q3 = rotate(P3, P2, PI / 2);
        vec P1Q2(P1, Q2), P2Q3(P2, Q3);
        point M3, M4;
        M3 = translate(M1, P1Q2);
        M4 = translate(M2, P2Q3);
        line L1(M1, M3), L2(M2, M4);
        intersection(L1, L2, P);
        r = e_dist(P, P1);
    }
    bool operator==(circle other) {
        return (P == other.P && same_d(r, other.r));
    }
};
bool insideCircle(point P, circle C) {
    return e_dist(P, C.P) <= C.r + EPS;
}
bool c_intersection(circle C1, circle C2, point& P1, point& P2) {
```

```
double d = e_dist(C1.P, C2.P);
if(d > C1.r + C2.r) {
    return false; //d+EPS kalo butuh
}
if(d < dabs(C1.r - C2.r) + EPS)
    return false;
double x1 = C1.P.x, y1 = C1.P.y, r1 = C1.r, x2 = C2.P.x, y2 = C2.P.y, r2 = C2.r;
double a = (r1 * r1 - r2 * r2 + d * d) / (2 * d), h = sqrt(r1 * r1 - a * a);
point T(x1 + a * (x2 - x1) / d, y1 + a * (y2 - y1) / d);
P1 = point(T.x - h * (y2 - y1) / d, T.y + h * (x2 - x1) / d);
P2 = point(T.x + h * (y2 - y1) / d, T.y - h * (x2 - x1) / d);
return true;
}
bool lc_intersection(line L, circle C, point& P1, point& P2) {
    double a = L.a, b = L.b, c = L.c, x = 0.P.x, y = 0.P.y, r = 0.r;
    double A = a * a + b * b, B = 2 * a * b * y - 2 * a * c - 2 * b * b * x,
        C = b * b * x * x + b * b * y * y - 2 * b * c * y + c * c - b * b * r * r;
    double D = B * B - 4 * A * C;
    point T1, T2;
    if(same_d(b, 0.0)) {
        T1.x = c / a;
        if(dabs(x - T1.x) + EPS > r)
            return false;
        if(same_d(T1.x - r - x, 0.0) || same_d(T1.x + r - x, 0.0)) {
            P1 = P2 = point(T1.x, y);
            return true;
        }
        double dx = dabs(T1.x - x), dy = sqrt(r * r - dx * dx);
        P1 = point(T1.x, y - dy);
        P2 = point(T1.x, y + dy);
        return true;
    }
    if(same_d(D, 0.0)) {
        T1.x = -B / (2 * A);
        T1.y = (c - a * T1.x) / b;
        P1 = P2 = T1;
        return true;
    }
    if(D < EPS)
        return false;
    D = sqrt(D);
    T1.x = (-B - D) / (2 * A);
    T1.y = (c - a * T1.x) / b;
    P1 = T1;
    T2.x = (-B + D) / (2 * A);
    T2.y = (c - a * T2.x) / b;
    P2 = T2;
    return true;
}
bool sc_intersection(segment S, circle C, point& P1, point& P2) {
    bool cek = lc_intersection(S.L, C, P1, P2);
    if(!cek)
        return false;
    double x1 = S.P.x, y1 = S.P.y, x2 = S.Q.x, y2 = S.Q.y;
    bool b1 = between_d(P1.x, x1, x2) && between_d(P1.y, y1, y2);
    bool b2 = between_d(P2.x, x1, x2) && between_d(P2.y, y1, y2);
    if(P1 == P2)
        return b1;
    if(b1 || b2) {
        if(!b1)
            P1 = P2;
        if(!b2)
            P2 = P1;
        return true;
    }
    return false;
}
/*Triangle*/
double t_perimeter(point A, point B, point C) {
    return e_dist(A, B) + e_dist(B, C) + e_dist(C, A);
}
```



```
double t_area(point A, point B, point C) {
    double s = t_perimeter(A, B, C) / 2;
    double ab = e_dist(A, B), bc = e_dist(B, C), ac = e_dist(C, A);
    return sqrt(s * (s - ab) * (s - bc) * (s - ac));
}

circle t_inCircle(point A, point B, point C) {
    vector<point> T;
    T.clear();
    T.pb(A);
    T.pb(B);
    T.pb(C);
    sort(T.begin(), T.end());
    A = T[0];
    B = T[1];
    C = T[2];
    double r = t_area(A, B, C) / (t_perimeter(A, B, C) / 2);
    double ratio = e_dist(A, B) / e_dist(A, C);
    vec BC(B, C);
    BC = scale(BC, ratio / (1 + ratio));
    point P;
    P = translate(B, BC);
    line AP1(A, P);
    ratio = e_dist(B, A) / e_dist(B, C);
    vec AC(A, C);
    AC = scale(AC, ratio / (1 + ratio));
    P = translate(A, AC);
    line BP2(B, P);
    intersection(AP1, BP2, P);
    return circle(P, r);
}

circle t_outCircle(point A, point B, point C) {
    return circle(A, B, C);
}

/*Polygon*/
struct polygon {
    vector<point> P;
    polygon() {
        P.clear();
    }
    polygon(vector<point>& _P) {
        P = _P;
    }
};

bool rayCast(point P, polygon& A) {
    point Q(P.x, 10000);
    line cast(P, Q);
    int cnt = 0;
    FOR(i, (int)(A.P.size()) - 1) {
        line temp(A.P[i], A.P[i + 1]);
        point I;
        bool B = intersection(cast, temp, I);
        if(!B)
            continue;
        else if(I == A.P[i] || I == A.P[i + 1])
            continue;
        else if(pointBetween(I, A.P[i], A.P[i + 1]) && pointBetween(I, P, Q))
            cnt++;
    }
    return cnt % 2 == 1;
}

// line segment p-q intersect with line A-B.
point lineIntersectSeg(point p, point q, point A, point B) {
    double a = B.y - A.y;
    double b = A.x - B.x;
    double c = B.x * A.y - A.x * B.y;
    double u = fabs(a * p.x + b * p.y + c);
    double v = fabs(a * q.x + b * q.y + c);
    return point((p.x * v + q.x * u) / (u + v), (p.y * v + q.y * u) / (u + v));
}

// cuts polygon Q along the line formed by point a -> point b
// (note: the last point must be the same as the first point)
```

```
vector<point> cutPolygon(point a, point b, const vector<point>& Q) {
    vector<point> P;
    for(int i = 0; i < (int)Q.size(); i++) {
        double left1 = cross(toVec(a, b), toVec(a, Q[i]));
        double left2 = 0;
        if(i != (int)Q.size() - 1)
            left2 = cross(toVec(a, b), toVec(a, Q[i + 1]));
        if(left1 > -EPS)
            P.push_back(Q[i]);
        if(left1 * left2 < -EPS)
            P.push_back(lineIntersectSeg(Q[i], Q[i + 1], a, b));
    }
    if(!P.empty() && !(P.back() == P.front()))
        P.push_back(P.front());
    return P;
}

circle minCoverCircle(polygon& A) {
    vector<point> p = A.P;
    point c;
    circle ret;
    double cr = 0.0;
    int i, j, k;
    c = p[0];
    for(i = 1; i < p.size(); i++) {
        if(e_dist(p[i], c) >= cr + EPS) {
            c = p[i], cr = 0;
            ret = circle(c, cr);
            for(j = 0; j < i; j++) {
                if(e_dist(p[j], c) >= cr + EPS) {
                    c = mid(p[i], p[j]);
                    cr = e_dist(p[i], c);
                    ret = circle(c, cr);
                    for(k = 0; k < j; k++) {
                        if(e_dist(p[k], c) >= cr + EPS) {
                            ret = circle(p[i], p[j], p[k]);
                            c = ret.P;
                            cr = ret.r;
                        }
                    }
                }
            }
        }
    }
    return ret;
}

/*Geometry Algorithm*/
double DP[110][110];
double minCostPolygonTriangulation(polygon& A) {
    if(A.P.size() < 3)
        return 0;
    FOR(i, A.P.size()) {
        for(int j = 0, k = i; k < A.P.size(); j++, k++) {
            if(k < j + 2)
                DP[j][k] = 0.0;
            else {
                DP[j][k] = INF;
                REP(l, j + 1, k - 1) {
                    double cost = e_dist(A.P[j], A.P[k]) + e_dist(A.P[k], A.P[l]) + e_dist(A.P[l], A.P[j]);
                    DP[j][k] = min(DP[j][k], DP[j][l] + DP[l][k] + cost);
                }
            }
        }
    }
    return DP[0][A.P.size() - 1];
}
```

## 4.2 Convex Hull

```
typedef double TD;           // for precision shifts
namespace GEOM {
    typedef pair<TD, TD> Pt;   // vector and points
    const TD EPS = 1e-9;
    const TD maxD = 1e9;
    TD cross(Pt a, Pt b, Pt c) { // right hand rule
        TD v1 = a.first - c.first; // (a-c) X (b-c)
        TD v2 = a.second - c.second;
        TD u1 = b.first - c.first;
        TD u2 = b.second - c.second;
        return v1 * u2 - v2 * u1;
    }
    TD cross(Pt a, Pt b) { // a X b
        return a.first * b.second - a.second * b.first;
    }
    TD dot(Pt a, Pt b, Pt c) { // (a-c) . (b-c)
        TD v1 = a.first - c.first;
        TD v2 = a.second - c.second;
        TD u1 = b.first - c.first;
        TD u2 = b.second - c.second;
        return v1 * u1 + v2 * u2;
    }
    TD dot(Pt a, Pt b) { // a . b
        return a.first * b.first + a.second * b.second;
    }
    TD dist(Pt a, Pt b) {
        return sqrt((a.first - b.first) * (a.first - b.first) +
            (a.second - b.second) * (a.second - b.second));
    }
    TD shoelaceX2(vector<Pt>& convHull) {
        TD ret = 0;
        for(int i = 0, n = convHull.size(); i < n; i++)
            ret += cross(convHull[i], convHull[(i + 1) % n]);
        return ret;
    }
    vector<Pt> createConvexHull(vector<Pt>& points) {
        sort(points.begin(), points.end());
        vector<Pt> ret;
        for(int i = 0; i < points.size(); i++) {
            while(ret.size() > 1 &&
                cross(points[i], ret[ret.size() - 1], ret[ret.size() - 2]) < -EPS)
                ret.pop_back();
            ret.push_back(points[i]);
        }
        for(int i = points.size() - 2, sz = ret.size(); i >= 0; i--) {
            while(ret.size() > sz &&
                cross(points[i], ret[ret.size() - 1], ret[ret.size() - 2]) < -EPS)
                ret.pop_back();
            if(i == 0)
                break;
            ret.push_back(points[i]);
        }
        return ret;
    }
    bool isInside(Pt pv, vector<Pt>& x) { //using winding number
        int n = x.size(), wn = 0;
        x.push_back(x[0]);
        for(int i = 0; i < n; ++i) {
            if(((x[i + 1].first <= pv.first && x[i].first >= pv.first) ||
                (x[i + 1].first >= pv.first && x[i].first <= pv.first)) &&
                ((x[i + 1].second <= pv.second && x[i].second >= pv.second) ||
                (x[i + 1].second >= pv.second && x[i].second <= pv.second))) {
                if(cross(x[i], x[i + 1], pv) == 0) {
                    x.pop_back();
                    return true;
                }
            }
        }
        for(int i = 0; i < n; ++i) {
            if(x[i].second <= pv.second) {
                if(x[i + 1].second > pv.second && cross(x[i], x[i + 1], pv) > 0)
                    ++wn;
            } else if(x[i + 1].second <= pv.second && cross(x[i], x[i + 1], pv) < 0)
                --wn;
        }
        x.pop_back();
        return wn != 0;
    }
}

vector<Pt> createConvexHull(vector<Pt>& points) {
    sort(points.begin(), points.end());
    vector<Pt> ret;
    for(int i = 0; i < points.size(); i++) {
        while(ret.size() > 1 &&
            cross(points[i], ret[ret.size() - 1], ret[ret.size() - 2]) < -EPS)
            ret.pop_back();
        ret.push_back(points[i]);
    }
    for(int i = points.size() - 2, sz = ret.size(); i >= 0; i--) {
        while(ret.size() > sz &&
            cross(points[i], ret[ret.size() - 1], ret[ret.size() - 2]) < -EPS)
            ret.pop_back();
        if(i == 0)
            break;
        ret.push_back(points[i]);
    }
    return ret;
}
bool isInside(Pt pv, vector<Pt>& x) { //using winding number
    int n = x.size(), wn = 0;
    x.push_back(x[0]);
    for(int i = 0; i < n; ++i) {
        if(((x[i + 1].first <= pv.first && x[i].first >= pv.first) ||
            (x[i + 1].first >= pv.first && x[i].first <= pv.first)) &&
            ((x[i + 1].second <= pv.second && x[i].second >= pv.second) ||
            (x[i + 1].second >= pv.second && x[i].second <= pv.second))) {
            if(cross(x[i], x[i + 1], pv) == 0) {
                x.pop_back();
                return true;
            }
        }
    }
    for(int i = 0; i < n; ++i) {
        if(x[i].second <= pv.second) {
            if(x[i + 1].second > pv.second && cross(x[i], x[i + 1], pv) > 0)
                ++wn;
        } else if(x[i + 1].second <= pv.second && cross(x[i], x[i + 1], pv) < 0)
            --wn;
    }
    x.pop_back();
    return wn != 0;
}
}
```

```
++wn;
    } else if(x[i + 1].second <= pv.second && cross(x[i], x[i + 1], pv) < 0)
        --wn;
    }
    x.pop_back();
    return wn != 0;
}
}
bool isInside(Pt pv, vector<Pt>& x) { //using winding number
    int n = x.size(), wn = 0;
    x.push_back(x[0]);
    for(int i = 0; i < n; ++i) {
        if(((x[i + 1].first <= pv.first && x[i].first >= pv.first) ||
            (x[i + 1].first >= pv.first && x[i].first <= pv.first)) &&
            ((x[i + 1].second <= pv.second && x[i].second >= pv.second) ||
            (x[i + 1].second >= pv.second && x[i].second <= pv.second))) {
            if(cross(x[i], x[i + 1], pv) == 0) {
                x.pop_back();
                return true;
            }
        }
    }
    for(int i = 0; i < n; ++i) {
        if(x[i].second <= pv.second) {
            if(x[i + 1].second > pv.second && cross(x[i], x[i + 1], pv) > 0)
                ++wn;
        } else if(x[i + 1].second <= pv.second && cross(x[i], x[i + 1], pv) < 0)
            --wn;
    }
    x.pop_back();
    return wn != 0;
}
}
```

### 4.3 Closest Pair of Points

```
#define fi first
#define se second
typedef pair<int, int> pii;
struct Point {
    int x, y, id;
};
int compareX(const void* a, const void* b) {
    Point* p1 = (Point*)a, *p2 = (Point*)b;
    return (p1->x - p2->x);
}
int compareY(const void* a, const void* b) {
    Point* p1 = (Point*)a, *p2 = (Point*)b;
    return (p1->y - p2->y);
}
double dist(Point p1, Point p2) {
    return sqrt((double)(p1.x - p2.x) * (p1.x - p2.x) +
        (double)(p1.y - p2.y) * (p1.y - p2.y)
        );
}
pair<pii, double> bruteForce(Point P[], int n) {
    double min = 1e8;
    pii ret = pii(-1, -1);
    for(int i = 0; i < n; ++i)
        for(int j = i + 1; j < n; ++j)
            if(dist(P[i], P[j]) < min) {
                ret = pii(P[i].id, P[j].id);
                min = dist(P[i], P[j]);
            }
    return pair<pii, double> (ret, min);
}
pair<pii, double> getmin(pair<pii, double> x, pair<pii, double> y) {
    if(x.fi.fi == -1 && x.fi.se == -1)
        return y;
    if(y.fi.fi == -1 && y.fi.se == -1)
```

```

    return x;
    return (x.se < y.se) ? x : y;
}
pair<pii, double> stripClosest(Point strip[], int size, double d) {
    double min = d;
    pii ret = pii(-1, -1);
    qsort(strip, size, sizeof(Point), compareY);
    for(int i = 0; i < size; ++i)
        for(int j = i + 1; j < size && (strip[j].y - strip[i].y) < min; ++j)
            if(dist(strip[i], strip[j]) < min) {
                ret = pii(strip[i].id, strip[j].id);
                min = dist(strip[i], strip[j]);
            }
    return pair<pii, double>(ret, min);
}
pair<pii, double> closestUtil(Point P[], int n) {
    if(n <= 3)
        return bruteForce(P, n);
    int mid = n / 2;
    Point midPoint = P[mid];
    pair<pii, double> dl = closestUtil(P, mid);
    pair<pii, double> dr = closestUtil(P + mid, n - mid);
    pair<pii, double> d = getmin(dl, dr);
    Point strip[n];
    int j = 0;
    for(int i = 0; i < n; i++)
        if(abs(P[i].x - midPoint.x) < d.second)
            strip[j] = P[i], j++;
    return getmin(d, stripClosest(strip, j, d.second));
}
pair<pii, double> closest(Point P[], int n) {
    qsort(P, n, sizeof(Point), compareX);
    return closestUtil(P, n);
}
Point P[50005];
int main() {
    int n;
    scanf("%d", &n);
    for(int a = 0; a < n; a++) {
        scanf("%d%d", &P[a].x, &P[a].y);
        P[a].id = a;
    }
    pair<pii, double> hasil = closest(P, n);
    if(hasil.fi.fi > hasil.fi.se)
        swap(hasil.fi.fi, hasil.fi.se);
    printf("%d %d %.6lf\n", hasil.fi.fi, hasil.fi.se, hasil.se);
    return 0;
}

```

#### 4.4 Smallest Enclosing Circle

```

// welzl's algo to find the 2d minimum enclosing circle of a set of points
// expected O(N)
// directions: remove duplicates and shuffle points, then call welzl(points)

struct Point {
    double x;
    double y;
};

struct Circle {
    double x, y, r;
    Circle() {}
    Circle(double _x, double _y, double _r): x(_x), y(_y), r(_r) {}
};

Circle trivial(const vector<Point>& r) {
    if(r.size() == 0)
        return Circle(0, 0, -1);
    else if(r.size() == 1)

```

```

        return Circle(r[0].x, r[0].y, 0);
    else if(r.size() == 2) {
        double cx = (r[0].x + r[1].x) / 2.0, cy = (r[0].y + r[1].y) / 2.0;
        double rad = hypot(r[0].x - r[1].x, r[0].y - r[1].y) / 2.0;
        return Circle(cx, cy, rad);
    } else {
        double x0 = r[0].x, x1 = r[1].x, x2 = r[2].x;
        double y0 = r[0].y, y1 = r[1].y, y2 = r[2].y;
        double d = (x0 - x2) * (y1 - y2) - (x1 - x2) * (y0 - y2);
        double cx = (((x0 - x2) * (x0 + x2) + (y0 - y2) * (y0 + y2)) / 2 *
            (y1 - y2) - ((x1 - x2) * (x1 + x2) + (y1 - y2) * (y1 + y2)) / 2 *
            (y0 - y2)) / d;
        double cy = (((x1 - x2) * (x1 + x2) + (y1 - y2) * (y1 + y2)) / 2 *
            (x0 - x2) - ((x0 - x2) * (x0 + x2) + (y0 - y2) * (y0 + y2)) / 2 *
            (x1 - x2)) / d;
        return Circle(cx, cy, hypot(x0 - cx, y0 - cy));
    }
}

Circle welzl(const vector<Point>& p, int idx = 0, vector<Point> r = {}) {
    if(idx == (int) p.size() || r.size() == 3)
        return trivial(r);
    Circle d = welzl(p, idx + 1, r);
    if(hypot(p[idx].x - d.x, p[idx].y - d.y) > d.r) {
        r.push_back(p[idx]);
        d = welzl(p, idx + 1, r);
    }
    return d;
}

```

#### 4.5 Sutherland-Hodgman Algorithm

```

// Complexity: linear time
// Ada 2 poligon, cari poligon intersectionnya
// poly_point = hasilnya, clipper = pemotongnya
#include<bits/stdc++.h>
using namespace std;

const double EPS = 1e-9;

struct point {
    double x, y;
    point(double _x, double _y): x(_x), y(_y) {}
};

struct vec {
    double x, y;
    vec(double _x, double _y): x(_x), y(_y) {}
};

point pivot(0, 0);
vec toVec(point a, point b) {
    return vec(b.x - a.x, b.y - a.y);
}

double dist(point a, point b) {
    return hypot(a.x - b.x, a.y - b.y);
}

double cross(vec a, vec b) {
    return a.x * b.y - a.y * b.x;
}

bool ccw(point p, point q, point r) {
    return cross(toVec(p, q), toVec(p, r)) > 0;
}

bool collinear(point p, point q, point r) {
    return fabs(cross(toVec(p, q), toVec(p, r))) < EPS;
}

bool lies(point a, point b, point c) {
    if((c.x >= min(a.x, b.x) && c.x <= max(a.x, b.x)) &&
        (c.y >= min(a.y, b.y) && c.y <= max(a.y, b.y)))
        return true;
    else

```

```

        return false;
    }
    bool anglecmp(point a, point b) {
        if(collinear(pivot, a, b))
            return dist(pivot, a) < dist(pivot, b);
        double d1x = a.x - pivot.x, d1y = a.y - pivot.y;
        double d2x = b.x - pivot.x, d2y = b.y - pivot.y;
        return (atan2(d1y, d1x) - atan2(d2y, d2x)) < 0;
    }

    point intersect(point s1, point e1, point s2, point e2) {
        double x1, x2, x3, x4, y1, y2, y3, y4;
        x1 = s1.x;
        y1 = s1.y;
        x2 = e1.x;
        y2 = e1.y;
        x3 = s2.x;
        y3 = s2.y;
        x4 = e2.x;
        y4 = e2.y;
        double num1 = (x1 * y2 - y1 * x2) * (x3 - x4) - (x1 - x2) * (x3 * y4 - y3 * x4);
        double num2 = (x1 * y2 - y1 * x2) * (y3 - y4) - (y1 - y2) * (x3 * y4 - y3 * x4);
        double den = (x1 - x2) * (y3 - y4) - (y1 - y2) * (x3 - x4);
        double new_x = num1 / den;
        double new_y = num2 / den;
        return point(new_x, new_y);
    }

    void clip(vector<point>& poly_points, point point1, point point2) {
        vector<point> new_points;
        new_points.clear();
        for(int i = 0; i < poly_points.size(); i++) {
            int k = (i + 1) % poly_points.size();
            double i_pos = ccw(point1, point2, poly_points[i]);
            double k_pos = ccw(point1, point2, poly_points[k]);
            //in in
            if(i_pos <= 0 && k_pos <= 0)
                new_points.push_back(poly_points[k]);
            //out in
            else if(i_pos > 0 && k_pos <= 0) {
                new_points.push_back(intersect(point1, point2, poly_points[i],
                                                poly_points[k]));
                new_points.push_back(poly_points[k]);
            }
            // in out
            else if(i_pos <= 0 && k_pos > 0) {
                new_points.push_back(intersect(point1, point2, poly_points[i],
                                                poly_points[k]));
            }
            //out out
            else {
            }
        }
        poly_points.clear();
        for(int i = 0; i < new_points.size(); i++)
            poly_points.push_back(new_points[i]);
    }

    double area(const vector<point>& P) {
        double result = 0.0;
        double x1, y1, x2, y2;
        for(int i = 0; i < P.size() - 1; i++) {
            x1 = P[i].x;
            y1 = P[i].y;
            x2 = P[i + 1].x;
            y2 = P[i + 1].y;
            result += (x1 * y2 - x2 * y1);
        }
        return fabs(result) / 2;
    }

    void suthHodgClip(vector<point>& poly_points, vector<point> clipper_points) {
        for(int i = 0; i < clipper_points.size(); i++) {

```

```

            int k = (i + 1) % clipper_points.size();
            clip(poly_points, clipper_points[i], clipper_points[k]);
        }
    }

    vector<point> sortku(vector<point> P) {
        int P0 = 0;
        int i;
        for(i = 1; i < 3; i++) {
            if(P[i].y < P[P0].y || (P[i].y == P[P0].y && P[i].x > P[P0].x))
                P0 = i;
        }
        point temp = P[0];
        P[0] = P[P0];
        P[P0] = temp;
        pivot = P[0];
        sort(++P.begin(), P.end(), anglecmp);
        reverse(++P.begin(), P.end());
        return P;
    }

    int main {
        clipper_points = sortku(clipper_points);
        suthHodgClip(poly_points, clipper_points);
    }

```

## 4.6 Centroid of Polygon

$$C_x = \frac{1}{6A} \sum_{i=0}^{n-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$

$$C_y = \frac{1}{6A} \sum_{i=0}^{n-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$

## 4.7 Pick Theorem

A: Area of a simply closed lattice polygon  
 B: Number of lattice points on the edges  
 I: Number of points in the interior  
 $A = I + \frac{B}{2} - 1$

## 5 Graphs

### 5.1 Articulation Point and Bridge

```

// gr -> adj list
// vector vis, low -> initialize to -1
// int timer -> initialize to 0
void dfs(int pos, int dad = -1) {
    vis[pos] = low[pos] = timer++;
    int kids = 0;
    for(auto& i : gr[pos]) {
        if(i == dad)
            continue;
        if(vis[i] >= 0)
            low[pos] = min(low[pos], vis[i]);
        else {
            dfs(i, pos);
            low[pos] = min(low[pos], low[i]);
            if(low[i] > vis[pos])
                is_bridge(pos, i)
                if(low[i] >= vis[pos] && dad >= 0)
                    is_articulation_point(pos)
                    ++kids;
        }
    }
    if(dad == -1 && kids > 1)
        is_articulation_point(pos)
}

```

## 5.2 SCC and Strong Orientation

```
#define N 10020
vector<int> adj[N];
bool vis[N], ins[N];
int disc[N], low[N], gr[N];
stack<int> st;
int id, grid;
void scc(int cur, int par) {
    disc[cur] = low[cur] = ++id;
    vis[cur] = ins[cur] = 1;
    st.push(cur);
    for(int to : adj[cur]) {
        //if (to==par) continue; // ini untuk S0(scc undirected)
        if(!vis[to])
            scc(to, cur);
        if(ins[to])
            low[cur] = min(low[cur], low[to]);
    }
    if(low[cur] == disc[cur]) {
        grid++; // group id
        while(ins[cur]) {
            gr[st.tp] = grid;
            ins[st.tp] = 0;
            st.pop();
        }
    }
}
```

## 5.3 Centroid Decomposition

```
int build_cen(int nw) {
    com_cen(nw, 0); //fungsi untuk itung size subtree
    int siz = sz[nw] / 2;
    bool found = false;
    while(!found) {
        found = true;
        for(int i : v[nw]) {
            if(!rem[i] && sz[i] < sz[nw]) {
                if(sz[i] > siz) {
                    found = false;
                    nw = i;
                    break;
                }
            }
        }
    }
    big
    rem[nw] = true;
    for(int i : v[nw]) if(!rem[i])
        par_cen[build_cen(i)] = nw;
    return nw;
}
```

## 5.4 Dinic's Maximum Flow

```
// O(VE log(max_flow)) if scaling == 1
// O((V + E) sqrt(E)) if unit graph (turn scaling off)
// O((V + E) sqrt(V)) if bipartite matching (turn scaling off)
// indices are 0-based
const ll INF = 1e18;
```

```
struct Dinic {
    struct Edge {
        int v;
        ll cap, flow;
        Edge(int _v, ll _cap): v(_v), cap(_cap), flow(0) {}
    };
};
```

```
int n;
ll lim;
vector<vector<int>> gr;
vector<Edge> e;
vector<int> idx, lv;
```

```
bool has_path(int s, int t) {
    queue<int> q;
    q.push(s);
    lv.assign(n, -1);
    lv[s] = 0;
    while(!q.empty()) {
        int c = q.front();
        q.pop();
        if(c == t)
            break;
        for(auto& i : gr[c]) {
            ll cur_flow = e[i].cap - e[i].flow;
            if(lv[e[i].v] == -1 && cur_flow >= lim) {
                lv[e[i].v] = lv[c] + 1;
                q.push(e[i].v);
            }
        }
    }
    return lv[t] != -1;
}

ll get_flow(int s, int t, ll left) {
    if(!left || s == t)
        return left;
    while(idx[s] < (int) gr[s].size()) {
        int i = gr[s][idx[s]];
        if(lv[e[i].v] == lv[s] + 1) {
            ll add = get_flow(e[i].v, t, min(left, e[i].cap - e[i].flow));
            if(add) {
                e[i].flow += add;
                e[i ^ 1].flow -= add;
                return add;
            }
        }
        ++idx[s];
    }
    return 0;
}

Dinic(int vertices, bool scaling = 1) : // toggle scaling here
    n(vertices), lim(scaling ? 1 << 30 : 1), gr(n) {}

void add_edge(int from, int to, ll cap, bool directed = 1) {
    gr[from].push_back(e.size());
    e.emplace_back(to, cap);
    gr[to].push_back(e.size());
    e.emplace_back(from, directed ? 0 : cap);
}

ll get_max_flow(int s, int t) { // call this
    ll res = 0;
    while(lim) { // scaling
        while(has_path(s, t)) {
            idx.assign(n, 0);
            while(ll add = get_flow(s, t, INF))
                res += add;
        }
        lim >>= 1;
    }
    return res;
}
};
```

## 5.5 Minimum Cost Maximum Flow

```
// 1-based index
template<class T>
using rpq = priority_queue<T, vector<T>, greater<T>>;

const ll INF = 1e18;

struct MCMF {
    struct Edge {
        int v;
        ll cap, cost;
        int rev;
        Edge(int _v, ll _cap, ll _cost, int _rev) :
            v(_v), cap(_cap), cost(_cost), rev(_rev) {}
    };

    ll flow, cost;
    int st, ed, n;
    vector<ll> dist, H;
    vector<int> pv, pe;
    vector<vector<Edge>> adj;

    bool dijkstra() {
        rpq<pair<ll, int>> pq;
        dist.assign(n + 1, INF);
        dist[st] = 0;
        pq.emplace(0, st);
        while(!pq.empty()) {
            auto [cst, pos] = pq.top();
            pq.pop();
            if(dist[pos] < cst)
                continue;
            for(int i = 0; i < (int) adj[pos].size(); ++i) {
                auto& e = adj[pos][i];
                int nxt = e.v;
                ll nxt_cst = dist[pos] + e.cost + H[pos] - H[nxt];
                if(e.cap > 0 && nxt_cst < dist[nxt]) {
                    dist[nxt] = nxt_cst;
                    pe[nxt] = i;
                    pv[nxt] = pos;
                    pq.emplace(nxt_cst, nxt);
                }
            }
        }
        return dist[ed] != INF;
    }

    MCMF(int _n) : n(_n), pv(n + 1), pe(n + 1), adj(n + 1) {}

    void add_edge(int u, int v, ll cap, ll cst) {
        adj[u].emplace_back(v, cap, cst, adj[v].size());
        adj[v].emplace_back(u, 0, -cst, adj[u].size() - 1);
    }

    pair<ll, ll> solve(int _st, int _ed) {
        st = _st, ed = _ed;
        flow = 0, cost = 0;
        H.assign(n + 1, 0);
        while(dijkstra()) {
            for(int i = 0; i <= n; ++i)
                H[i] += dist[i];
            ll f = INF;
            for(int i = ed; i != st; i = pv[i])
                f = min(f, adj[pv[i]][pe[i]].cap);
            flow += f;
            cost += f * H[ed];
            for(int i = ed; i != st; i = pv[i]) {
                auto& e = adj[pv[i]][pe[i]];
                e.cap -= f;
                adj[i][e.rev].cap += f;
            }
        }
    }
};
```

```
    }
    return {flow, cost};
}
};
```

## 5.6 Flows with Demands

let  $S_0$  be the source and  $T_0$  be the original sink

1. add 2 additional nodes, call them  $S_1$  and  $T_1$
2. connect  $S_0$  to nodes normally
3. connect nodes to  $T_0$  normally
4. for each edge  $(U, V)$ ,  $\text{cap} = \text{original cap} - \text{demand}$
5. for each node  $N$  :
  1. add an edge  $(S_1, N)$ ,  $\text{cap} = \text{sum of inward demand to } N$
  2. add an edge  $(N, T_1)$ ,  $\text{cap} = \text{sum of outward demand from } N$
6. add an edge  $(T_0, S_0)$ ,  $\text{cap} = \text{INF}$
7. the above is not a typo!
8. run max flow normally
9. for each edge  $(S_1, V)$  and  $(U, T_1)$ , check if  $\text{flow} == \text{cap}$

if step #9 fails, then it is not possible to satisfy the given demand

Mathematically, let  $d(e)$  be the demand of edge  $e$ . Let  $V$  be the set of every vertex in the graph.

- $c'(S_1, v) = \sum_{u \in V} d(u, v)$  for each edge  $(s', v)$ .
- $c'(v, T_1) = \sum_{w \in V} d(v, w)$  for each edge  $(v, t')$ .
- $c'(u, v) = c(u, v) - d(u, v)$  for each edge  $(u, v)$  in the old network.
- $c'(T_0, S_0) = \infty$

## 5.7 Hungarian

```
template <typename TD> struct Hungarian {
    TD INF = 1e9; //max_inf
    int n;
    vector<vector<TD> > adj; // cost[left][right]
    vector<TD> hl, hr, slk;
    vector<int> fl, fr, vl, vr, pre;
    deque<int> q;
    Hungarian(int _n) {
        n = _n;
        adj = vector<vector<TD> >(n, vector<TD>(n, 0));
    }
    int check(int i) {
        if(vl[i] == 1, fl[i] != -1)
            return q.push_back(fl[i]), vr[fl[i]] = 1;
        while(i != -1)
            swap(i, fr[fl[i] = pre[i]]);
        return 0;
    }
    void bfs(int s) {
        slk.assign(n, INF);
        vl.assign(n, 0);
        vr = vl;
        q.assign(vr[s] = 1, s);
        for(TD d; ) {
            for(!q.empty(); q.pop_front()) {
                for(int i = 0, j = q.front(); i < n; i++) {
                    if(d = hl[i] + hr[j] - adj[i][j], !vl[i] && d <= slk[i]) {
                        if(pre[i] == j, d)
                            slk[i] = d;
                        else if(!check(i))
                            return;
                    }
                }
            }
            d = INF;
        }
    }
};
```

```

    for(int i = 0; i < n; i++) if(!vl[i] && d > slk[i])
        d = slk[i];
    for(int i = 0; i < n; i++) {
        if(vl[i])
            hl[i] += d;
        else
            slk[i] -= d;
        if(vr[i])
            hr[i] -= d;
    }
    for(int i = 0; i < n; i++) if(!vl[i] && !slk[i] && !check(i))
        return;
}
}
TD solve() {
    fl.assign(n, -1);
    fr = fl;
    hl.assign(n, 0);
    hr = hl;
    pre.assign(n, 0);
    for(int i = 0; i < n; i++)
        hl[i] = *max_element(adj[i].begin(), adj[i].begin() + n);
    for(int i = 0; i < n; i++)
        bfs(i);
    TD ret = 0;
    for(int i = 0; i < n; i++) if(adj[i][fl[i]])
        ret += adj[i][fl[i]];
    return ret;
}
}; //i wiLL be matched with fl[i]

```

## 5.8 Edmonds' Blossom

// Maximum matching on general graphs in  $O(V^2 E)$   
 // Indices are 1-based  
 // Stolen from ko\_osaga's cheatsheet

```

struct Blossom {
    vector<int> vis, dad, orig, match, aux;
    vector<vector<int>> conn;
    int t, N;
    queue<int> Q;

    void augment(int u, int v) {
        int pv = v;
        do {
            pv = dad[pv];
            int nv = match[pv];
            match[pv] = pv;
            match[pv] = v;
            v = nv;
        } while(u != pv);
    }

    int lca(int v, int w) {
        ++t;
        while(true) {
            if(v) {
                if(aux[v] == t)
                    return v;
                aux[v] = t;
                v = orig[dad[match[v]]];
            }
            swap(v, w);
        }
    }

    void blossom(int v, int w, int a) {
        while(orig[v] != a) {
            dad[v] = w;
            w = match[v];
        }
    }
}

```

```

    if(vis[w] == 1) {
        Q.push(w);
        vis[w] = 0;
    }
    orig[v] = orig[w] = a;
    v = dad[w];
}

bool bfs(int u) {
    fill(vis.begin(), vis.end(), -1);
    iota(orig.begin(), orig.end(), 0);
    Q = queue<int>();
    Q.push(u);
    vis[u] = 0;
    while(!Q.empty()) {
        int v = Q.front();
        Q.pop();
        for(int x : conn[v]) {
            if(vis[x] == -1) {
                dad[x] = v;
                vis[x] = 1;
                if(!match[x]) {
                    augment(u, x);
                    return 1;
                }
                Q.push(match[x]);
                vis[match[x]] = 0;
            } else if(vis[x] == 0 && orig[v] != orig[x]) {
                int a = lca(orig[v], orig[x]);
                blossom(x, v, a);
                blossom(v, x, a);
            }
        }
    }
    return false;
}

Blossom(int n) : // n = vertices
    vis(n + 1), dad(n + 1), orig(n + 1), match(n + 1),
    aux(n + 1), conn(n + 1), t(0), N(n) {
    for(int i = 0; i <= n; ++i) {
        conn[i].clear();
        match[i] = aux[i] = dad[i] = 0;
    }
}

void add_edge(int u, int v) {
    conn[u].push_back(v);
    conn[v].push_back(u);
}

int solve() { // call this for answer
    int ans = 0;
    vector<int> V(N - 1);
    iota(V.begin(), V.end(), 1);
    shuffle(V.begin(), V.end(), mt19937(0x94949));
    for(auto x : V) {
        if(!match[x]) {
            for(auto y : conn[x]) {
                if(!match[y]) {
                    match[x] = y, match[y] = x;
                    ++ans;
                    break;
                }
            }
        }
    }
    for(int i = 1; i <= N; ++i) {
        if(!match[i] && bfs(i))
            ++ans;
    }
}

```



```
    }
    return ans;
}
};
```

## 5.9 Eulerian Path or Cycle

```
// finds a eulerian path / cycle
// visits each edge only once
// properties:
// - cycle: degrees are even
// - path: degrees are even OR degrees are even except for 2 vertices
// how to use: g = adjacency list g[n] = connected to n, undirected
// if there is a vertex u with an odd degree, call dfs(u)
// else call on any vertex
// ans = path result
```

```
vector<set<int>> g;
vector<int> ans;
```

```
void dfs(int u) {
    while(g[u].size()) {
        int v = *g[u].begin();
        g[u].erase(v);
        g[v].erase(u);
        dfs(v);
    }
    ans.push_back(u);
}
```

## 5.10 Hierholzer's Algorithm

```
// Eulerian on Directed Graph
stack<int> path;
vector<int> euler;
inline void hierholzer() {
    path.push(0);
    int cur = 0;
    while(!path.empty()) {
        if(!adj[cur].empty()) {
            path.push(cur);
            int next = adj[cur].back();
            adj[cur].pop();
            cur = next;
        } else {
            euler.pb(cur);
            cur = path.top();
            path.pop();
        }
    }
    reverse(euler.begin(), euler.end());
}
```

## 5.11 2-SAT

```
struct TwoSAT {
    int n;
    vector<vector<int>> g, gr;
    vector<int> comp, topological_order, answer;
    vector<bool> vis;

    TwoSAT() {}
    TwoSAT(int _n) :
        n(_n), g(2 * n), gr(2 * n), comp(2 * n), answer(2 * n), vis(2 * n) {}

    void add_edge(int u, int v) {
        g[u].push_back(v);
        gr[v].push_back(u);
    }
};
```

```

}

// For the following three functions
// int x, bool val: if 'val' is true, we take the variable to be x.
// Otherwise we take it to be x's complement.

// At least one of them is true
void add_clause_or(int i, bool f, int j, bool p) {
    add_edge(i + (f ? n : 0), j + (p ? 0 : n));
    add_edge(j + (p ? n : 0), i + (f ? 0 : n));
}

// Only one of them is true
void add_clause_xor(int i, bool f, int j, bool p) {
    add_clause_or(i, f, j, p);
    add_clause_or(i, !f, j, !p);
}

// Both of them have the same value
void add_clause_and(int i, bool f, int j, bool p) {
    add_clause_xor(i, !f, j, p);
}

// Topological sort
void dfs(int u) {
    vis[u] = true;
    for(const auto& v : g[u])
        if(!vis[v])
            dfs(v);
    topological_order.push_back(u);
}

// Extracting strongly connected components
void scc(int u, int id) {
    vis[u] = true;
    comp[u] = id;
    for(const auto& v : gr[u])
        if(!vis[v])
            scc(v, id);
}

bool satisfiable() {
    fill(vis.begin(), vis.end(), false);
    for(int i = 0; i < 2 * n; i++)
        if(!vis[i])
            dfs(i);
    fill(vis.begin(), vis.end(), false);
    reverse(topological_order.begin(), topological_order.end());
    int id = 0;
    for(const auto& v : topological_order)
        if(!vis[v])
            scc(v, id++);
    // Constructing the answer
    for(int i = 0; i < n; i++) {
        if(comp[i] == comp[i + n])
            return false;
        answer[i] = (comp[i] > comp[i + n] ? 1 : 0);
    }
    return true;
}
};
```

## 6 Math

### 6.1 Extended Euclidean GCD

```
// computes x and y such that ax + by = gcd(a, b) in O(log (min(a, b)))
// returns {gcd(a, b), x, y}
tuple<int, int, int> gcd(int a, int b) {
```



```
if(b == 0) return {a, 1, 0};
auto [d, x1, y1] = gcd(b, a % b);
return {d, y1, x1 - y1* (a / b)};
}
```

## 6.2 Generalized CRT

```
template<typename T>
T extended_euclid(T a, T b, T& x, T& y) {
    if(b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    T xx, yy, gcd;
    gcd = extended_euclid(b, a % b, xx, yy);
    x = yy;
    y = xx - (yy * (a / b));
    return gcd;
}
template<typename T>
T MOD(T a, T b) {
    return (a % b + b) % b;
}
// return x, lcm. x = a % n && x = b % m
template<typename T>
pair<T, T> CRT(T a, T n, T b, T m) {
    T _n, _m;
    T gcd = extended_euclid(n, m, _n, _m);
    if(n == m) {
        if(a == b)
            return pair<T, T>(a, n);
        else
            return pair<T, T>(-1, -1);
    } else if(abs(a - b) % gcd != 0)
        return pair<T, T>(-1, -1);
    else {
        T lcm = m * n / gcd;
        T x = MOD(a + MOD(n * MOD(_n * ((b - a) / gcd), m / gcd), lcm), lcm);
        return pair<T, T>(x, lcm);
    }
}
```

## 6.3 Generalized Lucas Theorem

```
/*Special Lucas : (n,k) % p^x
fctp[n] = Product of the integers less than or equal
to n that are not divisible by p
Precompute fctp*/
LL p
LL E(LL n, int m) {
    LL tot = 0;
    while(n != 0)
        tot += n / m, n /= m;
    return tot;
}
LL funct(LL n, LL base) {
    LL ans = fast(fctp[base], n / base, base) * fctp[n % base] % base;
    return ans;
}
LL F(LL n, LL base) {
    LL ans = 1;
    while(n != 0) {
        ans = (ans * funct(n, base)) % base;
        n /= p;
    }
    return ans;
}
```

```
LL special_lucas(LL n, LL r, LL base) {
    p = fprime(base);
    LL pow = E(n, p) - E(n - r, p) - E(r, p);
    LL TOP = fast(p, pow, base) * F(n, base) % base;
    LL BOT = F(r, base) * F(n - r, base) % base;
    return (TOP * fast(BOT, totien(base) - 1, base)) % base;
}
//End of Special Lucas
```

## 6.4 Linear Diophantine

```
//FOR SOLVING MINIMUM ABS(X) + ABS(Y)
ll x, y, newX, newY, target = 0;
ll extGcd(ll a, ll b) {
    if(b == 0) {
        x = 1, y = 0;
        return a;
    }
    ll ret = extGcd(b, a % b);
    newX = y;
    newY = x - y * (a / b);
    x = newX;
    y = newY;
    return ret;
}
ll fix(ll sol, ll rt) {
    ll ret = 0;
    //CASE SOLUTION(X/Y) < TARGET
    if(sol < target)
        ret = -floor(abs(sol + target) / (double)rt);
    //CASE SOLUTION(X/Y) > TARGET
    if(sol > target)
        ret = ceil(abs(sol - target) / (double)rt);
    return ret;
}
ll work(ll a, ll b, ll c) {
    ll gcd = extGcd(a, b);
    ll solX = x * (c / gcd);
    ll solY = y * (c / gcd);
    a /= gcd;
    b /= gcd;
    ll fi = abs(fix(solX, b));
    ll se = abs(fix(solY, a));
    ll lo = min(fi, se);
    ll hi = max(fi, se);
    ll ans = abs(solX) + abs(solY);
    for(ll i = lo; i <= hi; i++) {
        ans = min(ans, abs(solX + i * b) + abs(solY - i * a));
        ans = min(ans, abs(solX - i * b) + abs(solY + i * a));
    }
    return ans;
}
```

## 6.5 Modular Linear Equation

```
// finds all solutions to ax = b (mod n)
vi modular_linear_equation_solver(int a, int b, int n) {
    int x, y;
    vi ret;
    int g = extended_euclid(a, n, x, y);
    if(!(b % g)) {
        x = mod(x * (b / g), n);
        for(int i = 0; i < g; i++)
            ret.push_back(mod(x + i * (n / g), n));
    }
    return ret;
}
```

## 6.6 Miller-Rabin and Pollard's Rho

```
namespace MillerRabin {
const vector<ll> primes = { // deterministic up to 2^64 - 1
    2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37
};
ll gcd(ll a, ll b) {
    return b ? gcd(b, a % b) : a;
}
ll powa(ll x, ll y, ll p) { // (x ^ y) % p
    if(!y)
        return 1;
    if(y & 1)
        return ((__int128) x * powa(x, y - 1, p)) % p;
    ll temp = powa(x, y >> 1, p);
    return ((__int128) temp * temp) % p;
}
bool miller_rabin(ll n, ll a, ll d, int s) {
    ll x = powa(a, d, n);
    if(x == 1 || x == n - 1)
        return 0;
    for(int i = 0; i < s; ++i) {
        x = ((__int128) x * x) % n;
        if(x == n - 1)
            return 0;
    }
    return 1;
}
bool is_prime(ll x) { // use this
    if(x < 2)
        return 0;
    int r = 0;
    ll d = x - 1;
    while((d & 1) == 0) {
        d >>= 1;
        ++r;
    }
    for(auto& i : primes) {
        if(x == i)
            return 1;
        if(miller_rabin(x, i, d, r))
            return 0;
    }
    return 1;
}
}

namespace PollardRho {
mt19937_64 generator(chrono::steady_clock::now()
    .time_since_epoch().count());
uniform_int_distribution<ll> rand_ll(0, LLONG_MAX);
ll f(ll x, ll b, ll n) { // (x^2 + b) % n
    return (((__int128) x * x) % n + b) % n;
}
ll rho(ll n) {
    if(n % 2 == 0)
        return 2;
    ll b = rand_ll(generator);
    ll x = rand_ll(generator);
    ll y = x;
    while(1) {
        x = f(x, b, n);
        y = f(f(y, b, n), b, n);
        ll d = MillerRabin::gcd(abs(x - y), n);
        if(d != 1)
            return d;
    }
}
void pollard_rho(ll n, vector<ll>& res) {
    if(n == 1)
        return;
    if(MillerRabin::is_prime(n)) {
        res.push_back(n);
        return;
    }
    ll d = rho(n);
    pollard_rho(d, res);
    pollard_rho(n / d, res);
}
vector<ll> factorize(ll n, bool sorted = 1) { // use this
    vector<ll> res;
    pollard_rho(n, res);
    if(sorted)
        sort(res.begin(), res.end());
    return res;
}
}
```

```
if(MillerRabin::is_prime(n)) {
    res.push_back(n);
    return;
}
ll d = rho(n);
pollard_rho(d, res);
pollard_rho(n / d, res);
}
vector<ll> factorize(ll n, bool sorted = 1) { // use this
    vector<ll> res;
    pollard_rho(n, res);
    if(sorted)
        sort(res.begin(), res.end());
    return res;
}
}
```

## 6.7 Berlekamp-Massey

```
#include <bits/stdc++.h>
using namespace std;
#define pb push_back
typedef long long ll;
#define SZ 233333
const int MOD = 1e9 + 7; //or any prime
ll qp(ll a, ll b) {
    ll x = 1;
    a %= MOD;
    while(b) {
        if(b & 1)
            x = x * a % MOD;
        a = a * a % MOD;
        b >>= 1;
    }
    return x;
}
namespace linear_seq {
vector<int> BM(vector<int> x) {
    //ls: (shortest) relation sequence (after filling zeroes) so far
    //cur: current relation sequence
    vector<int> ls, cur;
    //lf: the position of ls (t')
    //ld: delta of ls (v')
    int lf = -1, ld = -1;
    for(int i = 0; i < int(x.size()); ++i) {
        ll t = 0;
        //evaluate at position i
        for(int j = 0; j < int(cur.size()); ++j)
            t = (t + x[i - j - 1] * (ll)cur[j]) % MOD;
        if((t - x[i]) % MOD == 0) {
            continue; //good so far
        }
        //first non-zero position
        if(!cur.size()) {
            cur.resize(i + 1);
            lf = i;
            ld = (t - x[i]) % MOD;
            continue;
        }
        //cur=cur-c/ld*(x[i]-t)
        ll k = -(x[i] - t) * qp(ld, MOD - 2) % MOD/*1/ld*/;
        vector<int> c(i - lf - 1); //add zeroes in front
        c.pb(k);
        for(int j = 0; j < int(ls.size()); ++j)
            c.pb(-ls[j]*k % MOD);
        if(c.size() < cur.size())
            c.resize(cur.size());
        for(int j = 0; j < int(cur.size()); ++j)
            c[j] = (c[j] + cur[j]) % MOD;
        //if cur is better than ls, change ls to cur
    }
}
```

```

    if(i - lf + (int)ls.size() >= (int)cur.size())
        ls = cur, lf = i, ld = (t - x[i]) % MOD;
    cur = c;
}
for(int i = 0; i < int(cur.size()); ++i)
    cur[i] = (cur[i] % MOD + MOD) % MOD;
return cur;
}
int m; //length of recurrence
//a: first terms
//h: relation
ll a[SZ], h[SZ], t_[SZ], s[SZ], t[SZ];
//calculate p*q mod f
void mull(ll* p, ll* q) {
    for(int i = 0; i < m + m; ++i)
        t_[i] = 0;
    for(int i = 0; i < m; ++i) if(p[i])
        for(int j = 0; j < m; ++j)
            t_[i + j] = (t_[i + j] + p[i] * q[j]) % MOD;
    for(int i = m + m - 1; i >= m; --i) if(t_[i])
        //miuns t_[i]x^{i-m}(x^m-\sum_{j=0}^{m-1} x^{m-j-1}h_j)
        for(int j = m - 1; ~j; --j)
            t_[i - j - 1] = (t_[i - j - 1] + t_[i] * h[j]) % MOD;
    for(int i = 0; i < m; ++i)
        p[i] = t_[i];
}
ll calc(ll K) {
    for(int i = m; ~i; --i)
        s[i] = t[i] = 0;
    //init
    s[0] = 1;
    if(m != 1)
        t[1] = 1;
    else
        t[0] = h[0];
    //binary-exponentiation
    while(K) {
        if(K & 1)
            mull(s, t);
        mull(t, t);
        K >>= 1;
    }
    ll su = 0;
    for(int i = 0; i < m; ++i)
        su = (su + s[i] * a[i]) % MOD;
    return (su % MOD + MOD) % MOD;
}
int work(vector<int> x, ll n) {
    if(n < int(x.size()))
        return x[n];
    vector<int> v = BM(x);
    m = v.size();
    if(!m)
        return 0;
    for(int i = 0; i < m; ++i)
        h[i] = v[i], a[i] = x[i];
    return calc(n);
}
}
using linear_seq::work;

const vector<int> sequence = {
    0, 2, 2, 28, 60, 836, 2766
};
int main() {
    cout << work(sequence, 7) << '\n';
}

```

## 6.8 Fast Fourier Transform

```

using ld = double; // change to long double if reach 10^18
using cd = complex<ld>;
const ld PI = acos(-(ld)1);

void fft(vector<cd>& a, int sign = 1) {
    int n = a.size();
    ld theta = sign * 2 * PI / n;
    for(int i = 0, j = 1; j < n - 1; j++) {
        for(int k = n >> 1; k > (i ^= k); k >>= 1);
        if(j < i)
            swap(a[i], a[j]);
    }
    for(int m, mh = 1; (m = mh << 1) <= n; mh = m) {
        int irev = 0;
        for(int i = 0; i < n; i += m) {
            cd w = exp(cd(0, theta * irev));
            for(int k = n >> 2; k > (irev ^= k); k >>= 1);
            for(int j = i; j < mh + i; j++) {
                int k = j + mh;
                cd x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
    }
    if(sign == -1) for(cd& i : a)
        i /= n;
}

vector<ll> multiply(vector<ll> const& a, vector<ll> const& b) {
    vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 1;
    while(n < a.size() + b.size())
        n <<= 1;
    fa.resize(n);
    fb.resize(n);
    fft(fa);
    fft(fb);
    for(int i = 0; i < n; i++)
        fa[i] *= fb[i];
    fft(fa, -1);
    vector<ll> res(n);
    for(int i = 0; i < n; i++)
        res[i] = round(fa[i].real());
    return res;
}

```

## 6.9 Number Theoretic Transform

```

namespace FFT {
/* ----- Adjust the constants here ----- */
const int LN = 24; //23
const int N = 1 << LN;
typedef long long LL; // 2**23 * 119 + 1. 998244353
// `MOD` must be of the form 2**`LN` * k + 1, where k odd.
const LL MOD = 9223372036737335297; // 2**24 * 54975513881 + 1.
const LL PRIMITIVE_ROOT = 3; // Primitive root modulo `MOD`.
/* ----- End of constants ----- */
LL root[N];
inline LL power(LL x, LL y) {
    LL ret = 1;
    for(; y; y >>= 1) {
        if(y & 1)
            ret = (__int128) ret * x % MOD;
        x = (__int128) x * x % MOD;
    }
    return ret;
}
inline void init_fft() {

```

```
const LL UNITY = power(PRIMITIVE_ROOT, MOD - 1 >> LN);
root[0] = 1;
for(int i = 1; i < N; i++)
    root[i] = (__int128) UNITY * root[i - 1] % MOD;
return;
}
// n = 2^k is the length of polynom
inline void fft(vector<LL>& a, bool invert) {
    for(int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for(; j >= bit; bit >>= 1)
            j -= bit;
        j += bit;
        if(i < j)
            swap(a[i], a[j]);
    }
    for(int len = 2; len <= n; len <<= 1) {
        LL wlen = (invert ? root[N - N / len] : root[N / len]);
        for(int i = 0; i < n; i += len) {
            LL w = 1;
            for(int j = 0; j < len >> 1; j++) {
                LL u = a[i + j];
                LL v = (__int128) a[i + j + len / 2] * w % MOD;
                a[i + j] = ((__int128) u + v) % MOD;
                a[i + j + len / 2] = ((__int128) u - v + MOD) % MOD;
                w = (__int128) w * wlen % MOD;
            }
        }
    }
    if(invert) {
        LL inv = power(n, MOD - 2);
        for(int i = 0; i < n; i++)
            a[i] = (__int128) a[i] * inv % MOD;
    }
    return;
}
inline vector<LL> multiply(vector<LL> a, vector<LL> b) {
    vector<LL> c;
    int len = 1 << 32 - __builtin_clz(a.size() + b.size() - 2);
    a.resize(len, 0);
    b.resize(len, 0);
    fft(len, a, false);
    fft(len, b, false);
    c.resize(len);
    for(int i = 0; i < len; i++)
        c[i] = (__int128) a[i] * b[i] % MOD;
    fft(len, c, true);
    return c;
}
//FFT::init_fft(); wajib di panggil init di awal
}
```

## 6.10 Gauss-Jordan

```
// Gauss-Jordan elimination with full pivoting.
//
// Uses:
// (1) solving systems of linear equations (AX=B)
// (2) inverting matrices (AX=I)
// (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT:    a[][] = an nxn matrix
//           b[][] = an nxm matrix
//
// OUTPUT:   X      = an nxm matrix (stored in b[][])
//           A^{-1} = an nxn matrix (stored in a[][])
//           returns determinant of a[][]
const double EPS = 1e-10;
```

```
typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
T GaussJordan(VVT& a, VVT& b) {
    const int n = a.size();
    const int m = b[0].size();
    VI irow(n), icol(n), ipiv(n);
    T det = 1;
    for(int i = 0; i < n; i++) {
        int pj = -1, pk = -1;
        for(int j = 0; j < n; j++) if(!ipiv[j])
            for(int k = 0; k < n; k++) if(!ipiv[k])
                if(pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) {
                    pj = j;
                    pk = k;
                }
        if(fabs(a[pj][pk]) < EPS) {
            cerr << "Matrix is singular." << endl;
            exit(0);
        }
        ipiv[pk]++;
        swap(a[pj], a[pk]);
        swap(b[pj], b[pk]);
        if(pj != pk)
            det *= -1;
        irow[i] = pj;
        icol[i] = pk;
        T c = 1.0 / a[pk][pk];
        det *= a[pk][pk];
        a[pk][pk] = 1.0;
        for(int p = 0; p < n; p++)
            a[pk][p] *= c;
        for(int p = 0; p < m; p++)
            b[pk][p] *= c;
        for(int p = 0; p < n; p++) if(p != pk) {
            c = a[p][pk];
            a[p][pk] = 0;
            for(int q = 0; q < n; q++)
                a[p][q] -= a[pk][q] * c;
            for(int q = 0; q < m; q++)
                b[p][q] -= b[pk][q] * c;
        }
    }
    for(int p = n - 1; p >= 0; p--) if(irow[p] != icol[p]) {
        for(int k = 0; k < n; k++)
            swap(a[k][irow[p]], a[k][icol[p]]);
    }
    return det;
}
int main() {
    const int n = 4;
    const int m = 2;
    double A[n][n] = { {1, 2, 3, 4}, {1, 0, 1, 0}, {5, 3, 2, 4}, {6, 1, 4, 6} };
    double B[n][m] = { {1, 2}, {4, 3}, {5, 6}, {8, 7} };
    VVT a(n), b(n);
    for(int i = 0; i < n; i++) {
        a[i] = VT(A[i], A[i] + n);
        b[i] = VT(B[i], B[i] + m);
    }
    double det = GaussJordan(a, b);
    // expected: 60
    cout << "Determinant: " << det << endl;
    // expected: -0.233333 0.166667 0.133333 0.066667
    //           0.166667 0.166667 0.333333 -0.333333
    //           0.233333 0.833333 -0.133333 -0.066667
    //           0.05 -0.75 -0.1 0.2
    cout << "Inverse: " << endl;
    for(int i = 0; i < n; i++) {
        for(int j = 0; j < n; j++)
```

```
        cout << a[i][j] << ' ';
        cout << endl;
    }
    // expected: 1.63333 1.3
    //            -0.166667 0.5
    //            2.36667 1.7
    //            -1.85 -1.35
    cout << "Solution: " << endl;
    for(int i = 0; i < n; i++) {
        for(int j = 0; j < m; j++)
            cout << b[i][j] << ' ';
        cout << endl;
    }
}
```

### 6.11 Fibonacci Check

```
bool is_fibonacci(int n) {
    return is_perfect_square(5 * n * n + 4)
        || is_perfect_square(5 * n * n - 4);
}
```

### 6.12 Derangement

```
der[0] = 1;
der[1] = 0;
for(int i = 2; i <= 10; ++i)
    der[i] = (11)(i - 1) * (der[i - 1] + der[i - 2]);
```

### 6.13 Bernoulli Number

$$\sum_{k=1}^n k^p = \frac{1}{p+1} \sum_{i=0}^p (-1)^i \binom{p+1}{i} B_i n^{p+1-i} \quad B_m^+ = 1 - \sum_{k=0}^{m-1} \binom{m}{k} \frac{B_k^+}{m-k+1}$$

### 6.14 Forbenius Number

$(X * Y) - (X + Y)$  and total count is  $(X - 1) * (Y - 1) / 2$

### 6.15 Stars and Bars with Upper Bound

$$P = (1 - X^{r_1+1}) \dots (1 - X^{r_n+1}) = \sum_i c_i X^{e_i}$$
$$Ans = \sum_i c_i \binom{N - e_i + n - 1}{n - 1}$$

## 7 Strings

### 7.1 Aho-Corasick

```
const int K = 26;
struct Vertex {
    int next[K];
    bool leaf = 0;
    int p = -1, ans = 0;
    char pch;
    int link = -1,mlink = -1;
    //magic link, is the link to find the nearest leaf
    int go[K];
    Vertex(int p = -1, char ch = '$') : p(p), pch(ch) {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};
vector<Vertex> t;
int add_string(string const& s) {
    int v = 0;
    for(char ch : s) {
```

```
        int c = ch - 'a';
        if(t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.emplace_back(v, ch);
        }
        v = t[v].next[c];
    }
    t[v].leaf = 1;
    return v;
}
int go(int v, char ch);
int get_link(int v) {
    if(t[v].link == -1) {
        if(v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].pch);
    }
    return t[v].link;
}
int get_mlink(int v) {
    if(t[v].mlink == -1) {
        if(v == 0 || t[v].p == 0)
            t[v].mlink = 0;
        else {
            t[v].mlink = go(get_link(t[v].p), t[v].pch);
            if(t[v].mlink && !t[t[v].mlink].leaf) {
                if(t[t[v].mlink].mlink == -1)
                    get_mlink(t[v].mlink);
                t[v].mlink = t[t[v].mlink].mlink;
            }
        }
    }
    return t[v].mlink;
}
int go(int v, char ch) {
    int c = ch - 'a';
    if(t[v].go[c] == -1) {
        if(t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
    }
    return t[v].go[c];
}
//t.pb(Vertex());
```

### 7.2 Eertree

```
/*
    Eertree - keep track of all palindromes and its occurrences
    This code refers to problem Longest Palindromic Substring
    https://www.spoj.com/problems/LPS/
*/
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;

struct node {
    int next[26];
    int sufflink;
    int len, cnt;
};

const int N = 1e5 + 69;
int n;
string s;
node tree[N];
int idx, suff;
int ans = 0;
```

```
while(lps[j] - j < lps[i] - i)
    lps[k++] = lps[j--];
lps[k] = lps[i] - (i - j); // set lps[k] to edge of lps[i]
i = k - 1; // jump to mirror, which is k
}
```

## 7.4 Suffix Array

```
// stores result in sa and lcp
// if lcp is needed, call SuffixArray(str, 1)
struct SuffixArray {
    int n;
    vector<int> sa, lcp, rnk, cnt;
    vector<pair<int, int>> p;
    SuffixArray(const string& s, bool calc_lcp = 0) :
        n(s.length()), sa(n), lcp(calc_lcp ? n : 0), rnk(n),
        cnt(max(n, 256)), p(n) {
        for(int i = 0; i < n; ++i)
            rnk[i] = s[i];
        iota(sa.begin(), sa.end(), 0);
        for(int i = 1; i < n; i <= 1)
            update_sa(i);
        if(!calc_lcp)
            return;
        vector<int> phi(n), plcp(n);
        phi[sa[0]] = -1;
        for(int i = 1; i < n; ++i)
            phi[sa[i]] = sa[i - 1];
        int l = 0;
        for(int i = 0; i < n; ++i) {
            if(phi[i] == -1)
                plcp[i] = 0;
            else {
                while((i + l < n) && (phi[i] + l < n)
                    && (s[i + l] == s[phi[i] + l]))
                    ++l;
                plcp[i] = l;
                l = max(l - 1, 0);
            }
        }
        for(int i = 0; i < n; ++i)
            lcp[i] = plcp[sa[i]];
    }

    void update_sa(int len) {
        sort_sa(len);
        sort_sa(0);
        for(int i = 0; i < n; ++i) p[i] = {rnk[i], rnk[(i + len) % n]};
        auto lst = p[sa[0]];
        rnk[sa[0]] = 0;
        int cur = 0;
        for(int i = 1; i < n; ++i) {
            if(lst != p[sa[i]]) {
                lst = p[sa[i]];
                ++cur;
            }
            rnk[sa[i]] = cur;
        }
    }

    void sort_sa(int offset) {
        fill(cnt.begin(), cnt.end(), 0);
        for(int i = 0; i < n; ++i)
            ++cnt[rnk[(i + offset) % n]];
        int sum = 0;
        for(int i = 0; i < (int) cnt.size(); ++i) {
            int temp = cnt[i];
            cnt[i] = sum;
            sum += temp;
        }
        vector<int> temp(n);
```

```
void init_eertree() {
    idx = suff = 2;
    tree[1].len = -1, tree[1].sufflink = 1;
    tree[2].len = 0, tree[2].sufflink = 1;
}

bool add_letter(int x) {
    int cur = suff, curlen = 0;
    int nw = s[x] - 'a';
    while(1) {
        curlen = tree[cur].len;
        if(x - curlen - 1 >= 0 && s[x - curlen - 1] == s[x])
            break;
        cur = tree[cur].sufflink;
    }
    if(tree[cur].next[nw]) {
        suff = tree[cur].next[nw];
        return 0;
    }
    tree[cur].next[nw] = suff = ++idx;
    tree[idx].len = tree[cur].len + 2;
    ans = max(ans, tree[idx].len);
    if(tree[idx].len == 1) {
        tree[idx].sufflink = 2;
        tree[idx].cnt = 1;
        return 1;
    }
    while(1) {
        cur = tree[cur].sufflink;
        curlen = tree[cur].len;
        if(x - curlen - 1 >= 0 && s[x - curlen - 1] == s[x]) {
            tree[idx].sufflink = tree[cur].next[nw];
            break;
        }
    }
    tree[idx].cnt = tree[tree[idx].sufflink].cnt + 1;
    return 1;
}

int main() {
    ios::sync_with_stdio(0);
    cin.tie(0);
    cin >> n >> s;
    init_eertree();
    for(int i = 0; i < n; i++)
        add_letter(i);
    cout << ans << '\n';
    return 0;
}
```

## 7.3 Manacher's Algorithm

```
// Computes lps array. lps[i] means the longest palindromic substring centered at i (↔
    when i is even, it is between characters. when it is odd, it is on characters)lps↔
[0] = 0; lps[1] = 1;
REP(i, 2, 2 * str.size()) {
    int l = i / 2 - lps[i] / 2;
    int r = (i - 1) / 2 + lps[i] / 2;
    while(1) { // widen
        if(l == 0 || r + 1 == str.size())
            break;
        if(str[l - 1] != str[r + 1])
            break;
        l--, r++;
    }
    lps[i] = r - l + 1;
    // jump
    if(lps[i] > 2) {
        int j = i - 1, k = i + 1; // while lps[j] inside lps[i]
```

```
for(int i = 0; i < n; ++i) {
    int cur = cnt[rnk[(sa[i] + offset) % n]]++;
    temp[cur] = sa[i];
}
sa = move(temp);
};
```

## 7.5 Suffix Automaton

```
struct state {
    int len, link;
    map<char, int>next; //use array if TLE
};

const int MAXLEN = 100005;
state st[MAXLEN * 2];
int sz, last;

void sa_init() {
    sz = last = 0;
    st[0].len = 0;
    st[0].link = -1;
    st[0].next.clear();
    ++sz;
}

void sa_extend(char c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    st[cur].next.clear();
    int p;
    for(p = last; p != -1 && !st[p].next.count(c); p = st[p].link)
        st[p].next[c] = cur;
    if(p == -1)
        st[cur].link = 0;
    else {
        int q = st[p].next[c];
        if(st[p].len + 1 == st[q].len)
            st[cur].link = q;
        else {
            int clone = sz++;
            st[clone].len = st[p].len + 1;
            st[clone].next = st[q].next;
            st[clone].link = st[q].link;
            for(; p != -1 && st[p].next[c] == q; p = st[p].link)
                st[p].next[c] = clone;
            st[q].link = st[cur].link = clone;
        }
    }
    last = cur;
}

// forwarding
for(int i = 0; i < m; i++) {
    while(cur >= 0 && st[cur].next.count(pa[i]) == 0) {
        cur = st[cur].link;
        if(cur != -1)
            len = st[cur].len;
    }
    if(st[cur].next.count(pa[i])) {
        len++;
        cur = st[cur].next[pa[i]];
    } else
        len = cur = 0;
}

// shortening abc -> bc
if(l == m) {
    l--;
```

```
if(l <= st[st[cur].link].len)
    cur = st[cur].link;
}

// finding lowest and highest length
int lo = st[st[cur].link].len + 1;
int hi = st[cur].len;

//Finding number of distinct substrings
//answer = distsub(0)
LL d[MAXLEN * 2];
LL distsub(int ver) {
    LL tp = 1;
    if(d[ver])
        return d[ver];
    for(map<char, int>::iterator it = st[ver].next.begin();
        it != st[ver].next.end(); it++)
        tp += distsub(it->second);
    d[ver] = tp;
    return d[ver];
}

//Total Length of all distinct substrings
//call distsub first before call lesub
LL ans[MAXLEN * 2];
LL lesub(int ver) {
    LL tp = 0;
    if(ans[ver])
        return ans[ver];
    for(map<char, int>::iterator it = st[ver].next.begin();
        it != st[ver].next.end(); it++)
        tp += lesub(it->second) + d[it->second];
    ans[ver] = tp;
    return ans[ver];
}

//find the k-th lexicographical substring
void kthsub(int ver, int K, string& ret) {
    for(map<char, int>::iterator it = st[ver].next.begin();
        it != st[ver].next.end(); it++) {
        int v = it->second;
        if(K <= d[v]) {
            K--;
            if(K == 0) {
                ret.push_back(it->first);
                return;
            } else {
                ret.push_back(it->first);
                kthsub(v, K, ret);
                return;
            }
        } else
            K -= d[v];
    }
}

// Smallest Cyclic Shift to obtain lexicographical smallest of All possible
//in int main do this
int main() {
    string S;
    sa_init();
    cin >> S; //input
    tp = 0;
    t = S.length();
    S += S;
    for(int a = 0; a < S.size(); a++)
        sa_extend(S[a]);
    minshift(0);
}

//the function
int tp, t;
void minshift(int ver) {
    for(map<char, int>::iterator it = st[ver].next.begin();
```

```
        it != st[ver].next.end(); it++) {
            tp++;
            if(tp == t) {
                cout << st[ver].len - t + 1 << endl;
                break;
            }
            minshift(it->second);
            break;
        }
    }
}
//end of function

// LONGEST COMMON SUBSTRING OF TWO STRINGS
string lcs(string s, string t) {
    sa_init();
    for(int i = 0; i < (int)s.length(); ++i)
        sa_extend(s[i]);
    int v = 0, l = 0,
        best = 0, bestpos = 0;
    for(int i = 0; i < (int)t.length(); ++i) {
        while(v && ! st[v].next.count(t[i])) {
            v = st[v].link;
            l = st[v].length;
        }
        if(st[v].next.count(t[i])) {
            v = st[v].next[t[i]];
            ++l;
        }
        if(l > best)
            best = l, bestpos = i;
    }
    return t.substr(bestpos - best + 1, best);
}
```

8 OEIS

8.1 A000108 (Catalan)

Catalan numbers  
f(n) = nCk(2n,n) / (n+1) = nCk(2n,n) - nCk(2n,n+1) = f(n-1) \* 2\*(2\*n-1) / (n+1)  
1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900,  
2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420,  
24466267020, 91482563640, 343059613650, 1289904147324, 4861946401452,  
18367353072152, 69533550916004, 263747951750360, 1002242216651368,  
3814986502092304

8.2 A000127

Maximal number of regions obtained by joining n points around a circle by  
straight lines  
f(n) = (n^4 - 6\*n^3 + 23\*n^2 - 18\*n + 24) / 24  
1, 2, 4, 8, 16, 31, 57, 99, 163, 256, 386, 562, 794, 1093, 1471, 1941, 2517,  
3214, 4048, 5036, 6196, 7547, 9109, 10903, 12951, 15276, 17902, 20854, 24158,  
27841, 31931, 36457, 41449, 46938, 52956, 59536, 66712, 74519, 82993, 92171,  
102091, 112792, 124314

8.3 A000668 (Mersene Primes)

Mersenne primes (of form 2^p - 1 where p is a prime)  
3, 7, 31, 127, 8191, 131071, 524287, 2147483647, 2305843009213693951,  
618970019642690137449562111, 162259276829213363391578010288127,  
170141183460469231731687303715884105727

8.4 A001434

Number of graphs with n nodes and n edges.  
0, 0, 1, 2, 6, 21, 65, 221, 771, 2769, 10250, 39243, 154658, 628635, 2632420,  
11353457, 50411413, 230341716, 1082481189, 5228952960, 25945377057,  
132140242356, 690238318754

8.5 A018819

Binary partition function: number of partitions of n into powers of 2  
f(2m+1) = f(2m); f(2m) = f(2m-1) + f(m)  
1, 1, 2, 2, 4, 4, 6, 6, 10, 10, 14, 14, 20, 20, 26, 26, 36, 36, 46, 46, 60,  
60, 74, 74, 94, 94, 114, 114, 140, 140, 166, 166, 202, 202, 238, 238, 284,  
284, 330, 330, 390, 390, 450, 450, 524, 524, 598, 598, 692, 692, 786, 786,  
900, 900, 1014, 1014, 1154, 1154, 1294, 1294

8.6 A092098

3-Portolan numbers: number of regions formed by n-secting the angles of  
an equilateral triangle.  
long long solve(long long n) {  
 long long res = (n % 2 == 1 ? 3\*n\*n - 3\*n + 1 : 3\*n\*n - 6\*n + 6);  
 const int bats = n/2 - 1;  
 for (long long i=1; i<=bats; i++) for (long long j=1; j<=bats; j++) {  
 long long num = i \* (n-j) \* n;  
 long long denum = (n-i) \* j + i \* (n-j);  
 res -= 6 \* (num % denum == 0 && num / denum <= bats);  
 } return res;  
}  
1, 6, 19, 30, 61, 78, 127, 150, 217, 246, 331, 366, 469, 510, 625, 678, 817,  
870, 1027, 1080, 1261, 1326, 1519, 1566, 1801, 1878, 2107, 2190, 2437, 2520,  
2791, 2886, 3169, 3270, 3559, 3678, 3997, 4110, 4447, 4548, 4921, 5034, 5419,  
5550, 5899, 6078, 6487

8.7 A277402

3-Portolan numbers: number of regions formed by n-secting the angles of  
an equilateral triangle.  
a(n) = 6n + 6(n-1 - n%2) + a(n-2); f(n) = a(n) if n % 10 != 0 else a(n) - 12  
1, 6, 19, 30, 61, 78, 127, 150, 217, 234, 331, 366, 469, 510, 631, 678, 817,  
870, 1027, 1074, 1261, 1326, 1519, 1590, 1801, 1878, 2107, 2190, 2437, 2514,  
2791, 2886, 3169, 3270, 3571, 3678, 3997, 4110, 4447, 4554, 4921, 5046, 5419,  
5550, 5941, 6078, 6487, 6630, 7057, 7194