NORMAL FORMS

~~Defin~~ _Basic conjunction_: A statement form containing only connectives as negation ($\sim$) or conjunction ($\wedge$) or both is called a basic conjunction.

<u>NOTE</u>: Basic conjunctions have the truth value true (T) only for a particular combination of truth values that can be assigned to the statement variables appearing in it.

<u>example</u>: ($\sim p \wedge q \wedge r$) is a basic conjunction and it has the ~~o~~ truth value true for the combination of truth values FTT that can be assigned to the statement variables p, q, r respectively. ~~out~~ ~~this~~ Moreover, there is no other combination of truth values for which ($\sim p \wedge q \wedge r$) is true.

<u>NOTE</u>: Basic conjunctions are restricted statement ~~formulas~~. forms.

Principle Conjunctive Normal Form (PCNF):

Every statement form which is not a tautology is logically equivalent to a restricted statement form, of the form $\left(\bigwedge\limits_{i=1}^{m} \left(\bigvee\limits_{j=1}^{n} Q_{ij} \right) \right)$ where each Q_{ij}'s is a statement variables or negation of a statement variable. This form is called principle conjunctive normal form.

<u>example</u>: Principle conjunctive normal form of $((\sim p \vee q) \to r)$ is

$(\sim p \vee \sim q \vee r) \wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee r).$

Principle Disjunctive Normal Form (PDNF):

Every statement form which is not a contradiction is logically equivalent to a restricted statement form, of the form $\left(\overset{m}{\underset{i=1}{V}} \left(\overset{n}{\underset{j=1}{\wedge}} Q_{ij} \right) \right)$; where each Q_{ij} is a statement variable or negation of a statement variable. This form is called principle disjunctive normal form.

example: Principle disjunctive normal form of $(p \leftrightarrow q)$ is

$$(p \wedge q) \vee (\sim p \wedge \sim q).$$

Process of Finding PCNF:

Suppose R is a statement form consisting of some statement variables and we need to find PCNF of R.

Step1: Find the truth table for $(\sim R)$.

Step2: Find the combination of truth values for which $(\sim R)$ have the truth value true (T).

Step3: Find all the basic conjunctions for each combination of truth values obtained in step2.

Step4: Find the disjunctions of all the basic conjunctions obtained in step3.

This is the PDNF which is logically equivalent to $(\sim R)$.

Step5: Find the negation of the PDNF that has been obtained in step4 and apply DeMorgan's laws. Then the resulting restricted statement form will be the PCNF which is logically equivalent to R.

Process of finding PDNF:

Suppose R is a statement form consisting of statement variables and connectives. We need to find PDNF of R.

Step 1: Find the truth table for R.

Step 2: Find the combination of truth values for which R have the truth value true (T).

Step 3: Find all the basic conjunctions for each combination of truth values that has been obtained in step 2.

Step 4: Find disjunction of all the basic conjunctions that has been obtained in step 3.

The resulting restricted statement form is the ~~basic conjuncti~~ PDNF of R.

Problem: Find the principle conjunctive normal form and principle disjunctive normal form of $((\sim p \vee q) \to r)$.

Sol^n: Let, $R : ((\sim p \vee q) \to r)$.

p	q	r	(~p)	(~p∨q)	R:(~p∨q)→r	(~R)
T	T	T	F	T	T	F
T	T	F	F	T	F	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	F
F	T	F	T	T	F	T
F	F	T	T	T	T	F
F	F	F	T	T	F	T

PCNF: The combination of truth values for which (~R) has the truth value true (T) are:

TTF, FTF, FFF.

Basic conjunctions corresponding to above combination of truth values are respectively are:

(p ∧ q ∧ ~r), (~p ∧ q ∧ ~r), (~p ∧ ~q ∧ ~r).

Therefore, the PDNF which is logically equivalent to (~R) is

$$(p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r).$$

i.e. $\sim(\sim R) \Leftrightarrow \sim((p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r)).$

i.e. $R \Leftrightarrow ((\sim p \vee \sim q \vee r) \wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee r)).$

since, $\sim(\sim p) \Leftrightarrow p.$

Thus, $(\sim p \vee \sim q \vee r) \wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee r)$ is the PCNF which is logically equivalent to R.

PDNF :

The combination of truth values for ~~which~~ which R has the truth value true (T) are:

TTT, TFT, TFF, FTT, FFT.

Basic conjunctions corresponding to above combination of truth values respectively are:

(p ∧ q ∧ r), (p ∧ ~q ∧ r), (p ∧ ~q ∧ ~r), (~p ∧ q ∧ r), (~p ∧ ~q ∧ r).

Therefore, the PDNF which is logically equivalent to R is,

$(p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge r).$

Problem: Find the PCNF and PDNF which is logically equivalent to following statement forms:

1. $((p \wedge q) \vee (\sim q \leftrightarrow r)).$
2. $(p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge r).$
3. $\sim(p \rightarrow \sim q \rightarrow r).$