EC 504
Spring, 2021
Exam 1

Monday, March 8, 2021

Instructions

- Put your name on each page; and both your name and BU ID on the first page.
- This exam is open notes, but no consultation with anyone other than the instructors are permitted during the exam.
- You have one hour and 45 minutes to complete this exam. It is designed to require less than one hour and fifteen minutes of work, so there should not be significant time pressure. Nevertheless, there are many problems, so do not spend too much time on any one question; you can always return to it.
- At the end of the exam, you will have 15 minutes to scan your work and upload it to Gradescope under Exam 1 submission. You don't have to assign pages if you don't want to.

1. (12 pts) Answer True or False to each of the questions below. Each question is worth 2 points. Answer true only if it is true as stated, with no additional assumptions. No explanation is needed, but any explanation is likely to earn partial credit, and no explanation will not earn any credit if the answer is wrong.

   (a) $n^{1.1} \in O((0.1)^n)$

   (b) Inserting numbers $1, \ldots, n$ into a binary min-heap in that order will take $\Theta(n)$ time.

   (c) If X is the inorder successor of Y, then X has no right child.

   (d) Consider a B+-tree in which the maximum number of keys in a non-leaf node is 5. Then, the minimum number of keys in any non-root, non-leaf node is 3.

   (e) In the KMP algorithm for string matching, the prefix function for the string "ABABCAB" is [0,0,1,2,1,1,2].

   (f) A perfect binary tree is a full binary tree where all the leaves are at the same level (the last level). A Red-Black Tree which is also a perfect binary tree can have all black nodes

2. (16 pts) Quick questions

   (a) A B+-tree of order 4 is built from scratch by 10 successive insertions. What is the minimum number of node splitting operations that must take place?

   (b) The height of a tree is the length of the longest root-to-leaf path in it, in terms of number of edges in the path. What are the minimum and maximum number of nodes in a binary tree of height 5?

   (c) Consider a complete binary tree with n elements, where the left and the right subtrees of the root are max-heaps. What is the worst case complexity for an efficient algorithm that converts this tree to a max-heap?

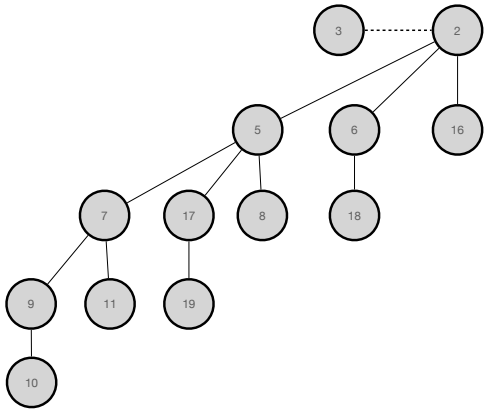
   (d) Consider two binomial heaps H_1, H_2, where H_1 has binomial trees of rank 0, 2,3 and 4, and H_2 has binomial trees of ranks 0, 3, and 4. What are the ranks of the binomial trees that result when we join (merge, not lazy merge) H_1 and H_2?

3. (6 pts) Given a binary tree (not a binary search tree), post-order traversal of this tree yields the following sequence: 1,6,5,9,10,7. Draw a complete binary tree of height 2 consistent with this traversal.

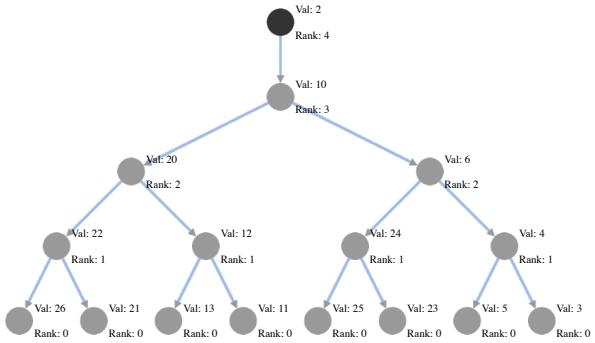
4. (6 pts) Consider a hash table of size seven, with starting index zero, and a hash function (3x + 4)mod 7. Assuming the hash table is initially empty, which of the following is the contents of the table when the sequence 1, 3, 8, 10 is inserted into the table using open hashing with linear probing for collision resolution?

5. (6 pts) A binary max-heap implemented using an array is given as $[25, 14, 16, 13, 10, 8, 12]$. What is the content of the array after two delete max operations?

6. (6 pts) Consider the Fibonacci min-heap illustrated on the right. Show the Fibonacci heap that results after the following operation: Reduce key 17 to 14. Then, show the Fibonacci heap that results when you delete minimum in the resulting heap from the first part.

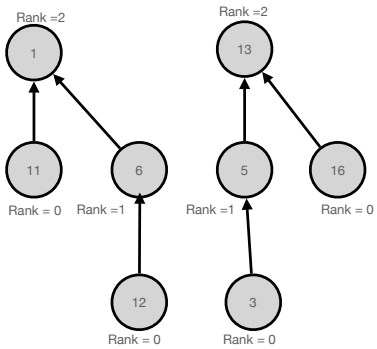


7. (6 pts) Consider the rank-pairing min-heap illustrated on the right. Show the rank-pairing heap, including ranks, that results after the following operation: Reduce key 12 to 9. Then, show the rank-pairing heap that results when you delete minimum in the resulting heap from the first part, using recursive merging, showing the resulting ranks.

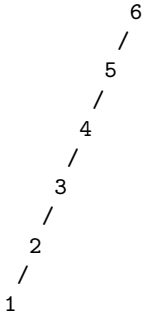


8. (6 pts) Draw the Aho-Corasick trie corresponding to the following search patterns: TEAM, STEAM, STEM, EAR. Show the suffix links, except for the suffix links that revert back to the root. Don't bother showing the output links.

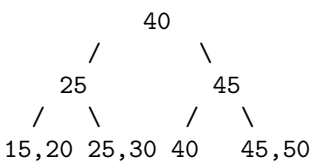
9. (6 pts) Consider the two trees, shown on the right, that are part of a disjoint set forest. Suppose we find a relation between keys 12 and 3. Show the disjoint set tree that results from the union operation using merge by rank and path compression, where, if two roots have the same rank, merge the larger value root under the smaller value root.



10. (6 pts) Consider the splay tree shown on the right, arising from inserting the numbers 1,2,3,4,5,6 in sequence.
Show the splay tree that results when you find the key 2 (and splay it to the top.)



11. (6 pts) Consider the B+ tree of order 3, shown on the right. Show the two B+ trees that result when you delete 40 first, then when you delete 45 subsequently.



12. (6 pts) Consider inserting the keys 2, 6, 5, 3, 8, 9 in this order into the initially empty data structures below. Identify for which of the following structures is the node with the key 6 a child of the node with the key 2.

    (a) A binary search tree.

    (b) A red-black tree.

    (c) A B tree of order 3 (maximum of 3 children).

    (d) A binomial min-heap.

    (e) A splay tree.

    (f) A Fibonacci min-heap.

13. (6 pts) For each of these recursions, please give the tightest upper bound for the recursion. You can write your answer as $T(n) \in O(g(n))$ for your best choice of function $g(n)$.

    (a) $T(n) = 5T(n/2) + (n \log n)^2$

    (b) $T(n+1) = 2n + T(n)$, with $T(1) = 1$.

    (c) $T(n) = 2T(n^{1/2}) + 1; T(1) = 1$.

14. (6 pts) Given two Red-Black search trees, each having n, describe and estimate the asymptotic complexity of an algorithm for merging these trees to form another Red-Black binary tree containing $2n$ elements. For full credit, describe an $O(n)$ algorithm. Any slower algorithm will only receive at most half credit.