EC 504
Quiz 3

Solutions

1. Which of the following statements are true?

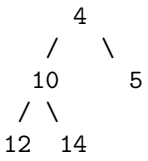
   (a) A binary heap of n elements is a full binary tree for all possible values of n.

   (b) A heapsort algorithm for a given list first forms a min-heap with the elements in that list, then extracts the elements of the heap one by one from the top. This algorithm will sort a list of n elements in time of O(n).

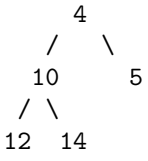
   Solution: Both are false. A binary heap is a complete binary tree, not a full binary tree. A heapsort algorithm will be $\Theta(n \log(n))$. Forming the heap is $\Theta(n)$, and extracting each element is $O(\log(n))$.

2. Which of the following statements are true?

   (a) The following tree is a valid binary min-heap.



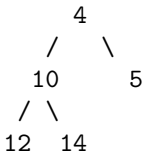
   (b) The following tree can appear in a binomial min-heap. .



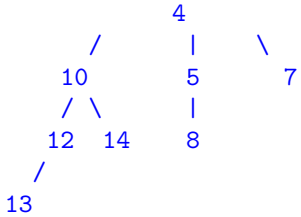
   Solution: (a) is True, because t is a complete binary tree satisfying the min heap property. (b) is False, because it is not a binomial tree, as the number of elements is not a power of 2.

3. Which of the following statements are true?

   (a) The following tree can appear in a Fibonacci heap.



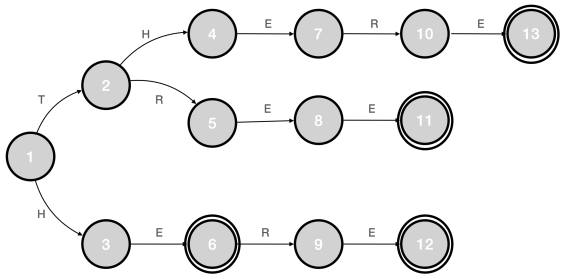
   Solution: True. Suppose originally the tree originally had rank 3, as:



   which can arise from merging two rank 1 trees, then two rank two trees. Now, suppose we reduce the keys in 13, 8 and 7, which cuts them and moves them as roots of new trees. This results in the tree in the question.

(b) The prefix function for the following pattern: "ababab" is [0,0,1,2,1,2].
Solution: This is false, as the correct prefix function is [0,0,1,2,3,4].

4. Consider the trie below, constructed from the patterns TREE, THERE, HERE, HE, where the nodes have been numbered for identification.



Determine whether the following are true: when we construct the suffix links in the Aho-Corasick algorithm,

(a) The suffix link from node 8 will go to node 3.

(b) The suffix link from node 10 will go to node 9.

Solution: (a) is false, because the string that matches to node 8 is "TRE", and neither "RE" nor "E" are valid prefixes in the trie, so the suffix link goes to node 1.

(b) is true, because the string that matches to node 10 is "THER", and the longest suffix that matches a prefix is "HER", which is the string to node 9.