EC 504
Spring, 2021
Quiz 6

Solutions

1. Which of the following statements are true?

   (a) In the Ford-Fulkerson algorithm with integer capacities, there is a sequence of augmenting paths that carry flow from the origin s to the destination t. The amount of flow carried by each augmenting path can be no greater than the amount of flow carried by the preceding augmenting path.

   Solution: This is false. There is no guarantee that we find the augmenting path with the maximum capacity in the first augmenting path.

   (b) The Edmonds-Karp variation of the Ford-Fulkerson algorithm has complexity which is polynomial in the number of edges in the graph.

   Solution: This is true. Finding each augmenting path is of order number of edges, and the complexity of the Edmonds-Karp algorithm is $O(|V||E|^2)$

2. Which of the following statements are true?

   (a) Consider a directed, capacitated graph, with origin vertex s and destination vertex t. The maximum flow which can be sent from s to t is less than or equal to the minimum of the following two quantities: the sum of the capacities of the edges leaving s, and the sum of the capacities of the edges entering t.

   Solution: This is True. Each of those numbers is the capacity of a cut in the network separating s and t, and the maximum flow must be no greater than the capacity of the minimum cut.

   (b) Consider a capacitated, directed graph, with an origin vertex s and a destination vertex t. Then, the maximum flow from vertex s to vertex t must be greater than or equal to the smallest capacity of any edge going into vertex t.

   Solution: This is false. You can have only one edge leaving vertex s with capacity 1, and only one edge entering t with capacity 10. The max flow will be 1.

3. Which of the following statements are true?

   (a) There are instances of the integer knapsack problem which can be solved in polynomial time by a greedy algorithm.

   Solution: This is true. Consider any instance where the capacity of the knapsack is such that the fractional knapsack algorithm assigns only whole objects.

   (b) Dynamic programming can be used to solve the integer knapsack problem in time that is polynomial in the number of objects and the capacity of the knapsack.

   Solution: This is true; the complexity is $O(nC)$, where n is the number of objects and C is the capacity of the knapsack.

4. Which of the following statements are true?

   (a) In a preflow, the excess at a vertex can be negative.

   Solution: This is False. The excesses at vertices must be non-negative for preflows.

   (b) In the Preflow-push algorithm, assume a vertex with excess flow has an outgoing edge in the reduced graph to a neighbor. However, its distance label is equal to the distance label of that neighbor. In this case, the vertex will be able to push part or all of its excess to that neighbor.

   Solution: This is False; flow pushes are allowed only when the distance label is one higher than the neighbor's distance label.

5. Which of the following statements are true?

   (a) Finding the optimal alignment of two sequences of symbols of length n can be done exactly with an algorithm of complexity $O(n^2)$.

   Solution: This is True; that is the complexity of the dynamic programming algorithm in this case.

   (b) Consider the maximum subarray sum problem. The maximum subarray must start and end with a non-negative number.

   Solution: This is True. If all the numbers were negative in the array, then the maximum subarray would be empty. Assume there are positive numbers in the array. Then, if the maximum subarray sum starts with a negative number, we can get a larger sum by removing that start from the subarray, which is a contradiction. The same argument applies to the end of the subarray if it is a negative number.