EC 504
Quiz 2

Solutions

1. Which of the following statements are true?

   (a) In a red-black tree, if a node is red, then one of its children can be red and the other child can be black.
   Solution: False. A red parent cannot have a red child in a red-black tree.

   (b) The following tree can be colored as a red-black search tree with keys ordered alphabetically.

```
           G
         /   \
        E       P
      / \     /  \
     C   F  M     R
    / \    /        \
   A   D  N          T
```

   Solution: False. It is not even a valid search tree, as the key N is on the wrong side of M.

2. Which of the following statements are true?

   (a) If the height of a binary tree is the maximum number of edges in any root to a leaf path, then the maximum number of nodes in a binary tree of height h is $2^{h+1} - 1$.
   Solution: True. The maximum number would be in a perfect binary tree with all levels complete. A perfect binary tree of height h has $2^{h+1} - 1$ nodes.

   (b) In a binary search tree with no repeated keys, deleting the node with key x, followed by deleting the node with key y, will result in the same search tree as deleting the node with key y, then deleting the node with key x.
   Solution: The answer to this is false, because of how we delete nodes with only one child. A simple counter-example is

```
                    Delete 10:              Delete 9:

      10                      11             11
     /  \                    /  \              \
    9    12                 9    12             12
        /
       11

   versus     Delete 9:                   Delete 10:

      10                      10             12
     /  \                       \           /
    9    12                      12        11
        /                       /
       11                      11
```

   Note that delete 9, then 10 results in a different tree than delete 10, then 9.

3. Consider the following statements:

   (a) In a splay tree with n nodes, the maximum height of the tree is O(log(n)) at all times.

   (b) You can insert the numbers 1, 2, ... , n in that order, into an empty splay tree, in O(n)

Solution: (a) is false. If you insert 1, 2, 3, ..., n in order into a splay tree, it results in a a tree of height $n-1$.

(b) is true, because you only do one single rotation each time.

4. Which of the following statements are true?

   (a) There are 12 possible binary search trees that only contain the four keys 1, 2, 3, 4.
   Solution: False. There are 14, resulting from the following insertion orders:
   1234, 1243, 1324 (same as 1342), 1423, 1432.
   2134 (same as 2314, 2341), 2143 (same as 2413, 2431)
   3124 (same as 3412, 3142), 3214 (same as 3412,3241)
   4321, 4312, 4231 (same as 4213), 4123, 4132

   (b) Suppose you have an unsorted array of n elements, where $n = m^2$ for some integer m. One can determine the m smallest elements of the array in $O(n)$ time.
   Solution: This is true; there are several algorithms to do this, but you use the algorithm for finding the k-th smallest element in $O(n)$ using the median of medians techniques. Once you find that element, do a pass through the list again and find all elements smaller than this. Two passes, each $O(n)$.

5. Consider the following statements.

   (a) In B+ trees, the length of a path from a root to a leaf node can differ by 1 from the length of a path from the root to another leaf node.

   (b) In a B+ tree, where the maximum number of keys in an interior node is 5, the minimum number of keys in an interior node is 2.

   Solution: (a) is false; the paths are exactly the same length for all leaves.

   (b) is true, because 5 keys equals 6 children, so minimum number of children is 3, which requires 2 keys.

6. What is the maximum height of the root of a B+ tree of order m=100, with L = m=100 for the leaf nodes, when the tree contains 1,000,000 data elements?

   Solution: The minimum number of keys is 49 in each node. The level below the root will be filled when the root has 100 children and one child saturates with 100 keys. Hence, the minimum number of keys that require three levels is 5049. To get a fourth layer, the root must again have 100 children, each of which have a minimum of 50 children, and one of them has 99. So the minimum number is: 5000 x 49 + 99 which is approximately 250,000. One more layer makes this easily exceed one million, so the maximum number of layers to be accessed is 5. Hence, the maximum height is 4.

7. Consider the following statements:

   (a) The second smallest element in a min-heap must be a child of the root.
   Solution: That is true. If it were not, then the heap property is violated.

   (b) In a min-heap, the largest element is always on a leaf.
   Solution: Yes, because otherwise you violate the min-heap property.