EC 504
Spring, 2021
Quiz 5

Solutions

1. Which of the following statements are true?

   (a) The following three algorithms are examples of greedy algorithms: Dijkstra's shortest path algorithm, Kruskal's minimum spanning tree algorithm, Prim's minimum spanning tree algorithm.

   Solution: True. They are greedy algorithms, because they select candidates based on a local metric, and once a candidate is selected, there is no backtracking.

   (b) Bellman-Ford algorithm can be used for finding the longest path in an acyclic graph.

   Solution: This is true. Basically, take the negative of all the distances and try to find the shortest path with negative distances. Since the graph has no cycles, it has no negative weight cycles and hence Bellman Ford will find the shortest path.

2. Which of the following statements are true?

   (a) In Floyd-Warshall's algorithm for finding all-pairs shortest paths, after each major outer loop k, upper bounds are computed on shortest distances $D(i, j)$ between each pair of vertices i, j. These upper bounds correspond to the shortest distance among all paths between vertices i and j which use only intermediate vertices in $1, ..., , k$.

   Solution: True. That is the proof of Floyd's algorithm.

   (b) Consider a directed, weighted graph where every arc in the graph has weight 100. For this graph, executing Dijkstra's algorithm to find a shortest path tree starting from a given vertex is equivalent to performing depth first search.

   Solution: True. When the distances are the same on every arc, Dijkstra's algorithm scans the nodes in breadth first search order, since distance and number of hops are equivalent.

3. Which of the following statements are true?

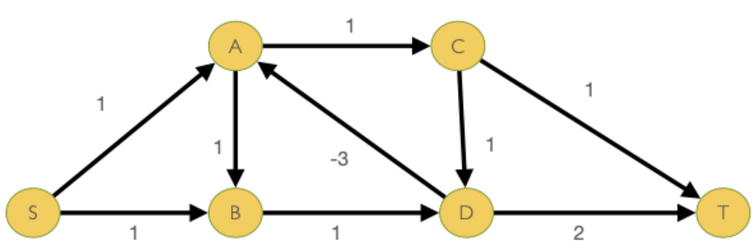
   (a) In Floyd-Warshall's algorithm, the worst case complexity does not depend on the number of edges.

   Solution: This is true. It is $O(n^3)$, where n is the number of vertices.

   (b) Floyd-Warshall's algorithm fails to converge when there are negative weight edges, but no negative weight cycles, in the graph.

   Solution: This is false; the algorithm will converge as long as there are no negative-weight cycles.

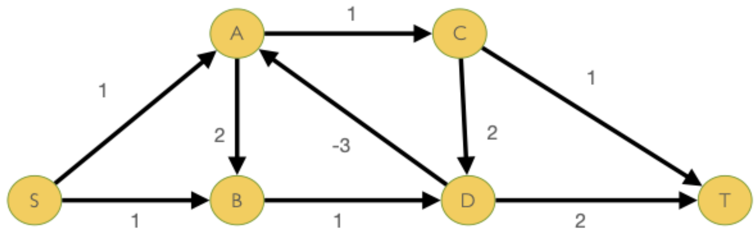
4. Consider the graph below:



What is the length of the shortest path from S to T?

Solution: Since there is a negative length cycle A-B-D, the shortest path does not exist.

5. Consider the graph below:



What is the length of the shortest path from S to T?

Solution: Since there are no negative weight cycles, we can compute this using Bellman-Ford, as there are negative weight edges.

We keep track of the distance labels to the different vertices, and the work queue.

1)

$$D(S) = 0, D(A) = \text{inf}, D(B) = \text{inf}, D(C) = \text{inf}, D(D) = \text{inf}, D(T) = \text{inf}; \quad Q = \{0\}$$

2) Take 0 out of Q, and scan distances to update as:

$$D(S) = 0, D(A) = 1, D(B) = 1, D(C) = \text{inf}, D(D) = \text{inf}, D(T) = \text{inf}; \quad Q = \{A, B\}$$

3) Take A out of Q, and scan distances to update as:

$$D(S) = 0, D(A) = 1, D(B) = 1, D(C) = 2, D(D) = \text{inf}, D(T) = \text{inf}; \quad Q = \{B, C\}$$

4) Take B out of Q and scan distances to update as:

$$D(S) = 0, D(A) = 1, D(B) = 1, D(C) = 2, D(D) = 2, D(T) = \text{inf}; \quad Q = \{C, D\}$$

5) Take C out of Q and scan distances to update as:

$$D(S) = 0, D(A) = 1, D(B) = 1, D(C) = 2, D(D) = 2, D(T) = 3; \quad Q = \{D, T\}$$

6) Take D out of Q and scan distances to update as:

$$D(S) = 0, D(A) = -1, D(B) = 1, D(C) = 2, D(D) = 2, D(T) = 3; \quad Q = \{T, A\}$$

7) Take T out of Q and scan distances to update as:

$$D(S) = 0, D(A) = -1, D(B) = 1, D(C) = 2, D(D) = 2, D(T) = 3; \quad Q = \{A\}$$

8) Take A out of Q and scan distances to update as:

$$D(S) = 0, D(A) = -1, D(B) = 1, D(C) = 0, D(D) = 2, D(T) = 3; \quad Q = \{C\}$$

0) Take C out of Q and scan distances to update as:

$$D(S) = 0, D(A) = -1, D(B) = 1, D(C) = 0, D(D) = 2, D(T) = 1; \quad Q = \emptyset$$

We are done, and the final distance is 1.