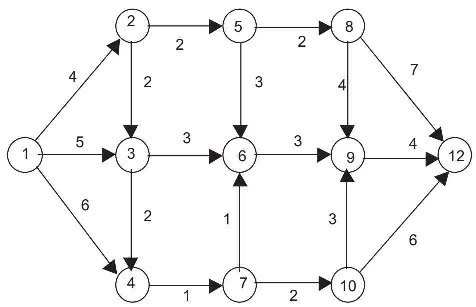
EC 504
Spring, 2021
HW 6

Due Wednesday, April 7, 8PM on Gradescope.



1. (15 pts) Consider the graph in the figure above as a directed, capacitated graph, where the numbers indicate an arc's capacity to carry flow from node 1 to node 12. In the max-flow algorithm of Ford and Fulkerson, the key step is, once a path has been found, to augment the flow and construct the residual graph for the next iteration.

   (a) Consider the path $1 \to 2 \to 3 \to 6 \to 9 \to 12$. What is the capacity of this path?

   (b) Suppose we send two units of the flow along path $1 \to 2 \to 5 \to 8 \to 12$. Construct the residual graph which remains after this flow has been sent. Draw this graph.

   (c) Find the maximum flow from node 1 to node 12 in this graph.

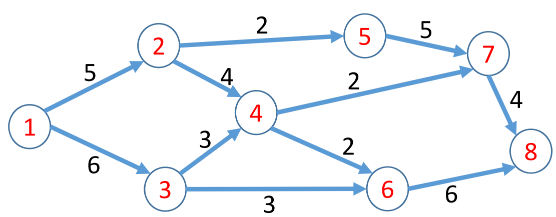


Figure 1: Figure for Problems 2, 3.

2. (15 pts) Consider the weighted, directed graph in Figure 1. Illustrate the steps of the Ford-Fulkerson algorithm for finding the max flow from node 1 to node 8. Show the residual network at each step.

3. (15 pts) Consider the weighted, directed graph in Figure 2. Illustrate the steps of the preflow-push algorithm for finding the max flow from node 1 to node 8. Show the distance labels of each node at each step, and the excesses.

4. (15 pts) Let $G = (V, E)$ be a directed, capacitated graph with source s, t and integer capacities. Suppose that we are given a maximum flow $f(u, v), (u, v) \in E$, in the network G.

   (a) Suppose that we increase the capacity of a single edge $(u', v') \in E$ by one unit. Describe an algorithm of $O(|V| + |E|)$ to modify the previous max flow $f(u, v), (u, v) \in E$, to a new max flow in the new network where the edge (u', v') has increased capacity by one.

   (b) Suppose that we decrease the capacity of a single edge $(u', v') \in E$ by one unit. Describe an algorithm of $O(|V| + |E|)$ to modify the previous max flow $f(u, v), (u, v) \in E$, to a new max flow in the new network where the edge (u', v') has decreased capacity by one.

5. (10 pts) Assume you have a scheduling problem with 10 customers. Customer i has value V_i, and requires processing time T_i. The values of V_i and T_i are listed below as arrays:

```
V = {3,4,7,5,2,3,5,8,9,6}
T = {2,3,4,3,1,3,3,5,6,4}
```

Assume that jobs can be scheduled partially, so that a job of value V_i which requires time T_i will receive value $V_i t / T_i$ if processed only for time T.

Find the maximum value which can be scheduled with total processing time 10 units.

6. (10 pts) Assume you have a scheduling problem with 10 customers. Customer i has value V_i, and requires processing time T_i. The values of V_i and T_i are listed below as arrays:

```
V = {3,4,7,5,2,3,5,8,9,6}
T = {2,3,4,3,1,3,3,5,6,4}
```

Assume that jobs must be scheduled fully, with no partial credit.

Use dynamic programming to find the optimal value which can be scheduled in a total of 9, 10, 11, 12 time units. Find the maximum value which can be scheduled with total processing time 10 units. (Note: this may be easier to do in MATLAB or Python than by hand.)