

9) $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ $T(X) = \alpha \times X$ $x, y \in \mathbb{R}^3$

a) Additivity
 $T(X+Y) = T(X) + T(Y)$ $T(X+Y) = \alpha \times (X+Y) = \alpha \times X + \alpha \times Y$

Homogeneity

$T(cX) = \alpha \times (cX) = c(\alpha \times X) = cT(X)$

b)

$\mathbb{R}^3 \leftarrow \{e_1, e_2, e_3\}$ $e_1 = (1, 0, 0)$ $e_2 = (0, 1, 0)$ $e_3 = (0, 0, 1)$

$T(e_1) = \alpha \times e_1 = \begin{vmatrix} i & j & k \\ \alpha_1 & \alpha_2 & \alpha_3 \\ 1 & 0 & 0 \end{vmatrix} = i \begin{vmatrix} \alpha_2 & \alpha_3 \\ 0 & 0 \end{vmatrix} - j \begin{vmatrix} \alpha_1 & \alpha_3 \\ 1 & 0 \end{vmatrix} + k \begin{vmatrix} \alpha_1 & \alpha_2 \\ 1 & 0 \end{vmatrix}$

\downarrow \downarrow \downarrow
 0 $-\alpha_3$ $-\alpha_2$

$= (0, \alpha_3, -\alpha_2)$

$T(e_2) = \alpha \times e_2 = \begin{vmatrix} i & j & k \\ \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & 1 & 0 \end{vmatrix} = i \begin{vmatrix} \alpha_2 & \alpha_3 \\ 1 & 0 \end{vmatrix} - j \begin{vmatrix} \alpha_1 & \alpha_3 \\ 0 & 0 \end{vmatrix} + k \begin{vmatrix} \alpha_1 & \alpha_2 \\ 0 & 1 \end{vmatrix}$

\downarrow \downarrow \downarrow
 $-\alpha_3$ 0 α_1

$= (-\alpha_3, 0, \alpha_1)$

$T(e_3) = \alpha \times e_3 = \begin{vmatrix} i & j & k \\ \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & 0 & 1 \end{vmatrix} = i \begin{vmatrix} \alpha_2 & \alpha_3 \\ 0 & 1 \end{vmatrix} - j \begin{vmatrix} \alpha_1 & \alpha_3 \\ 0 & 1 \end{vmatrix} + k \begin{vmatrix} \alpha_1 & \alpha_2 \\ 0 & 0 \end{vmatrix}$

\downarrow \downarrow \downarrow
 α_2 $-\alpha_1$ 0

$= (\alpha_2, -\alpha_1, 0)$

$[T] = [T(e_1) \quad T(e_2) \quad T(e_3)] = \begin{bmatrix} 0 & -\alpha_3 & +\alpha_2 \\ +\alpha_3 & 0 & -\alpha_1 \\ -\alpha_2 & +\alpha_1 & 0 \end{bmatrix}$