(C)

$X = (x_1, x_2, x_3)$ $\quad T(x) = a \times X = 0$

$a = (a_1, a_2, a_3)$ $\quad a \times X = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ x_1 & x_2 & x_3 \end{vmatrix} = i\begin{vmatrix} a_2 & a_3 \\ x_2 & x_3 \end{vmatrix} - j\begin{vmatrix} a_1 & a_3 \\ x_1 & x_3 \end{vmatrix} + k\begin{vmatrix} a_1 & a_2 \\ x_1 & x_2 \end{vmatrix}$

$(a_2x_3 - a_3x_2) \quad (a_1x_3 - a_3x_1) \quad (a_1x_2 - a_2x_1)$

$= \left((a_2x_3 - a_3x_2), (a_3x_1 - a_1x_3), (a_1x_2 - a_2x_1)\right)$

$0 \quad 0 \quad 0$

$$[T]x = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -a_3x_2 + a_2x_3 \\ a_3x_1 - a_1x_3 \\ -a_2x_1 + a_1x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(D)

Dimension = Rank + Nullity

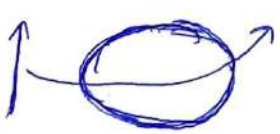
Dimension $(R^3) = 3$

Nullity → Dimension of null space = 1

Dimension = Rank + Nullity

$3 \quad 2 \quad 1$

10) Earth Rotation(w)



$w = (0, 0, w_z)$

Wind Velocity(v)

Eastward(v) $= (v_x, 0, 0)$

Northward(v) $= (0, v_y, 0)$

Coriolis Effect

$-2w \times v$

$-2[(0, 0, w_z) \times (v_x, v_y, 0)]$

$= (w_x, w_y, w_z) \times (v_x, v_y, v_z)$

$= \begin{vmatrix} i & j & k \\ w_x & w_y & w_z \\ v_x & v_y & v_z \end{vmatrix}$

$= i\begin{vmatrix} w_y & w_z \\ v_y & v_z \end{vmatrix} - j\begin{vmatrix} w_x & w_z \\ v_x & v_z \end{vmatrix} + k\begin{vmatrix} w_x & w_y \\ v_x & v_y \end{vmatrix}$

$= (w_yv_z - w_zv_y) - (w_xv_z - w_zv_x) + (w_xv_y - w_yv_x)$

Northern Hemisphere (counterclockwise)

$w_z > 0$ Upward ↑

$-2w_zv_x < 0$ Down ↓

$2w_zv_y > 0$ East →

Southern Hemisphere (clockwise)

$w_z < 0$ Downward ↓

$-2w_zv_x > 0$ Up ↑

$2w_zv_y < 0$ West ←

Eastward Wind

$v = (v_x, 0, 0)$

$-2(w \times v) = -2[(0, 0, w_z) \times (v_x, 0, 0)]$

$= -2(0, w_zv_x, 0)$

$= (0, -2w_zv_x, 0)$

Northward Wind

$v = (0, v_y, 0)$

$-2(w \times v) = -2[(0, 0, w_z) \times (0, v_y, 0)]$

$= -2(-w_zv_y, 0, 0)$

$= (2w_zv_y, 0, 0)$