

7)

$$(a \times b) (c \times d) = (a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c)$$

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - j \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + k \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$\downarrow \quad \downarrow \quad \downarrow$

$$i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1)$$

$$a \times b = \begin{vmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{vmatrix}$$

$$c \times d = \begin{vmatrix} i & j & k \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{vmatrix} = i \begin{vmatrix} c_2 & c_3 \\ d_2 & d_3 \end{vmatrix} - j \begin{vmatrix} c_1 & c_3 \\ d_1 & d_3 \end{vmatrix} + k \begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}$$

$\downarrow \quad \downarrow \quad \downarrow$

$$i(c_2 d_3 - c_3 d_2) - j(c_1 d_3 - c_3 d_1) + k(c_1 d_2 - c_2 d_1)$$

$$c \times d = \begin{vmatrix} c_2 d_3 - c_3 d_2 \\ c_3 d_1 - c_1 d_3 \\ c_1 d_2 - c_2 d_1 \end{vmatrix}$$

$$(a \times b) (c \times d) = \begin{vmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{vmatrix} \cdot \begin{vmatrix} c_2 d_3 - c_3 d_2 \\ c_3 d_1 - c_1 d_3 \\ c_1 d_2 - c_2 d_1 \end{vmatrix}$$

$$(a_2 b_3 - a_3 b_2)(c_2 d_3 - c_3 d_2) = (a_2 b_3 c_2 d_3 - a_2 b_3 c_3 d_2 - a_3 b_2 c_2 d_3 + a_3 b_2 c_3 d_2)$$

$$(a_3 b_1 - a_1 b_3)(c_3 d_1 - c_1 d_3) = (a_3 b_1 c_3 d_1 - a_3 b_1 c_1 d_3 - a_1 b_3 c_3 d_1 + a_1 b_3 c_1 d_3) +$$

$$(a_1 b_2 - a_2 b_1)(c_1 d_2 - c_2 d_1) = (a_1 b_2 c_1 d_2 - a_1 b_2 c_2 d_1 - a_2 b_1 c_1 d_2 + a_2 b_1 c_2 d_1) +$$

$$(a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c)$$

$$a = (a_1, a_2, a_3) \quad b = (b_1, b_2, b_3) \quad c = (c_1, c_2, c_3) \quad d = (d_1, d_2, d_3)$$

$$a \cdot c = a_1 c_1, a_2 c_2, a_3 c_3$$

$$a_1 b_1 c_1 d_1 + a_1 b_2 c_1 d_2 + a_1 b_3 c_1 d_3$$

$$a_1 b_2 c_1 d_2 + a_1 b_3 c_1 d_3$$

$$b \cdot d = b_1 d_1, b_2 d_2, b_3 d_3$$

$$a_2 b_1 c_2 d_1 + a_2 b_2 c_2 d_2 + a_2 b_3 c_2 d_3$$

$$a_2 b_1 c_3 d_1 + a_2 b_2 c_3 d_2$$

$$a \cdot d = a_1 d_1, a_2 d_2, a_3 d_3$$

$$a_3 b_1 c_3 d_1 + a_3 b_2 c_3 d_2 + a_3 b_3 c_3 d_3$$

$$a_3 b_1 c_2 d_1 + a_3 b_2 c_2 d_2$$

$$b \cdot c = b_1 c_1, b_2 c_2, b_3 c_3$$

$$a_1 b_1 c_1 d_1 + a_1 b_2 c_1 d_2 + a_1 b_3 c_1 d_3$$

$$a_1 b_1 c_2 d_1 + a_1 b_2 c_2 d_2$$