9) $p=(2,-3,1)$ $\quad P_{paralel} = Proj_q(p)$

$q=(12,3,4)$ $\quad P_{perpendicular} = p - Proj_q(p)$

$$Proj_q(p) = \frac{p \cdot q}{|q|^2} \cdot q$$

$P_{paralel}$

$$\frac{p \cdot q}{|q|^2} \cdot q = \frac{24+(-9)+4}{(12^2+3^2+4^2)} \cdot (12,3,4) = \frac{19}{169} \cdot (12,3,4)$$

$$= \left(\frac{228}{169}, \frac{57}{169}, \frac{76}{169}\right)$$

$P_{perpendicular}$

$$p - Proj_q(p) = (2,-3,1) - \left(\frac{228}{169}, \frac{57}{169}, \frac{76}{169}\right)$$

$$= \left(\frac{110}{169}, \frac{-564}{169}, \frac{93}{169}\right)$$

$p = P_{paralel} + P_{perpendicular}$ ✓

$$\left(\frac{228}{169}, \frac{57}{169}, \frac{76}{169}\right) + \left(\frac{110}{169}, \frac{-564}{169}, \frac{93}{169}\right) = (2,-3,1)$$

$P_{perpendicular} \cdot q = 0$ ✓

$$\left(\frac{110}{169}, \frac{-564}{169}, \frac{93}{169}\right) \cdot (12,3,4) = \frac{1320 - 1692 + 372}{169} = \frac{0}{169} = 0$$