3)

$$U \times V = \begin{vmatrix} i & j & k \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} = i \begin{vmatrix} u_y & u_z \\ v_y & v_z \end{vmatrix} - j \begin{vmatrix} u_x & u_z \\ v_x & v_z \end{vmatrix} + k \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$$

$$= i\left[(u_y)(v_z) - (u_z)(v_y)\right] - j\left[(u_x)(v_z) - (u_z)(v_x)\right] + k\left[(u_x v_y) - (u_y)(v_x)\right]$$

~~U x V~~

$$U = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \qquad V = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \qquad W = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$$

$$U = (1, -1, 0) \qquad V = (1, 2, 0) \qquad W = (1, 0, 3)$$

$$U \times V = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix} = i \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix} - j \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} + k \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}$$

$$= (-1)(0) - (0)(2) \qquad = (1)(0) - (0)(1) \qquad = (1)(2) - (-1)(1)$$
$$= 0 \qquad\qquad = 0 \qquad\qquad = 3$$

$$U \times V = (0, 0, 3) \qquad\qquad\qquad 2U \times V = (0, 0, 6)$$

$$V \times U = \begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 1 & -1 & 0 \end{vmatrix} = i \begin{vmatrix} 2 & 0 \\ -1 & 0 \end{vmatrix} - j \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= (2)(0) - (0)(-1) \qquad = (1)(0) - (0)(1) \qquad = (1)(-1) - (2)(1)$$
$$= 0 \qquad\qquad = 0 \qquad\qquad = -3$$

$$V \times U = (0, 0, -3)$$

$$U \times W = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & 0 & 3 \end{vmatrix} = i \begin{vmatrix} -1 & 0 \\ 0 & 3 \end{vmatrix} - j \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} + k \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix}$$

$$= (-1)(3) - (0)(0) \qquad = (1)(3) - (0)(1) \qquad = (1)(0) - (-1)(1)$$
$$= -3 \qquad\qquad = 3 \qquad\qquad = 1$$

$$U \times W = (-3, -3, 1)$$

$$\boxed{U \times V = -(V \times U)} \checkmark \qquad \boxed{U \times (V + W) = (U \times V) + (U \times W)} \checkmark$$

$$(0, 0, 3) = -(0, 0, -3) \checkmark \qquad \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 2 & 2 & 3 \end{vmatrix} = i \begin{vmatrix} -1 & 0 \\ 2 & 3 \end{vmatrix} - j \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} + k \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix}$$

$$\boxed{2U \times V = 2(U \times V)} \checkmark \qquad\qquad\qquad\qquad -3 \qquad\qquad 3 \qquad\qquad 4$$

$$(0, 0, 6) = 2(0, 0, 3) \checkmark \qquad U \times (V + W) = (-3, -3, 4) \checkmark$$
$$(U \times V) + (U \times W) = (0, 0, 3) + (-3, -3, 1) = (-3, -3, 4) \checkmark$$