

$$9) T: \mathbb{R}^3 \rightarrow \mathbb{R}^3 \quad T(X) = a \times X \quad x, y \in \mathbb{R}^3$$

(a) Additivity  $T(x+y) = \underline{T(x) + T(y)} \quad T(x+y) = a \times (x+y) = \underline{a \times x + a \times y}$

Homogeneity

$$T(cx) = a \times (cx) = c(a \times x) = cT(x)$$

(b)

$$\mathbb{R}^3 \leftarrow \{e_1, e_2, e_3\} \quad e_1 = (1, 0, 0) \quad e_2 = (0, 1, 0) \quad e_3 = (0, 0, 1)$$

$$T(e_1) = a \times e_1 = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{vmatrix} = i \begin{vmatrix} a_2 & a_3 \\ 0 & 0 \end{vmatrix} - j \begin{vmatrix} a_1 & a_3 \\ 1 & 0 \end{vmatrix} + k \begin{vmatrix} a_1 & a_2 \\ 1 & 0 \end{vmatrix}$$

$$= (0, a_3, -a_2)$$

$$T(e_2) = a \times e_2 = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ 0 & 1 & 0 \end{vmatrix} = i \begin{vmatrix} a_2 & a_3 \\ 1 & 0 \end{vmatrix} - j \begin{vmatrix} a_1 & a_3 \\ 0 & 0 \end{vmatrix} + k \begin{vmatrix} a_1 & a_2 \\ 0 & 1 \end{vmatrix}$$

$$= (-a_3, 0, a_1)$$

$$T(e_3) = a \times e_3 = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ 0 & 0 & 1 \end{vmatrix} = i \begin{vmatrix} a_2 & a_3 \\ 0 & 1 \end{vmatrix} - j \begin{vmatrix} a_1 & a_3 \\ 0 & 1 \end{vmatrix} + k \begin{vmatrix} a_1 & a_2 \\ 0 & 0 \end{vmatrix}$$

$$= (a_2, -a_1, 0)$$

$$[T] = [T(e_1) \quad T(e_2) \quad T(e_3)] = \begin{bmatrix} 0 & -a_3 & +a_2 \\ +a_3 & 0 & -a_1 \\ -a_2 & +a_1 & 0 \end{bmatrix}$$