

$$19) \quad p \cdot q = 2p_1q_1 + p_2q_2$$

 \mathbb{R}^2 $a, b \quad \mathbb{R}$ $p, r \quad \mathbb{R}^2$

$$\begin{aligned}(ap + br) \cdot q &= 2(a p_1 + b r_1) q_1 + (a p_2 + b r_2) q_2 \\&= 2a p_1 q_1 + 2b r_1 q_1 + a p_2 q_2 + b r_2 q_2 \\&= a(2 p_1 q_1 + p_2 q_2) + b(2 r_1 q_1 + r_2 q_2) \\&= a(p \cdot q) + b(r \cdot q)\end{aligned}$$

$$p \cdot p \geq 0$$

$$p \cdot p = 2p_1 p_1 + p_2 p_2 = 2p_1^2 + p_2^2$$

$$\underline{2p_1^2 + p_2^2 \geq 0}$$