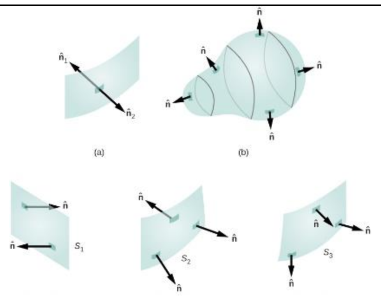
1. Dot Products (scalar product)

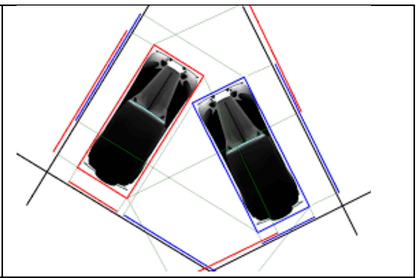
The dot product (also called the scalar product) is a way to multiply two vectors to produce a single scalar (number), not a vector. It's widely used in physics, machine learning, computer graphics, and linear algebra. The dot product serves a wide range of applications across various scientific and engineering disciplines, and the scalar values it produces admit diverse interpretations.

1.1 Dot Product Applications

Usually, several fields use the dot product to solve many issues during working life, such as collision detection, which can help to maintain people's lives as their assets, as shown in Table 1.

Table 1: Dot Product Application

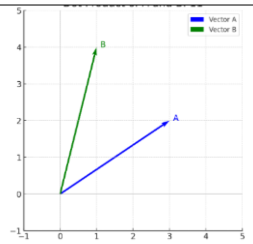
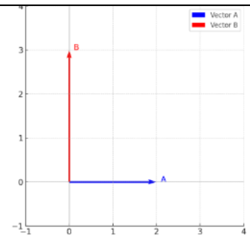
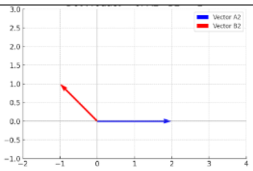
NO	Field	Example	Shows
1	Physics & Engineering	Lifting or pushing an object along a path	
2	Electric and Magnetic Fields	Calculating the electric flux through a surface	
3	Computer Graphics	Realistic shadows, highlights, and reflections in 3D games and movies.	
4	Machine Learning	Recommender systems, search engines, and sentiment analysis	

5	Robotics & Navigation	Collision Detection	

1.2 Interpretation of Dot Product Results

Based on the result obtained from the product, three different meanings, such as vectors point roughly in the same direction, vectors are perpendicular, and vectors point in opposite directions, as shown in Table 2.

Table 2: Dot product Results

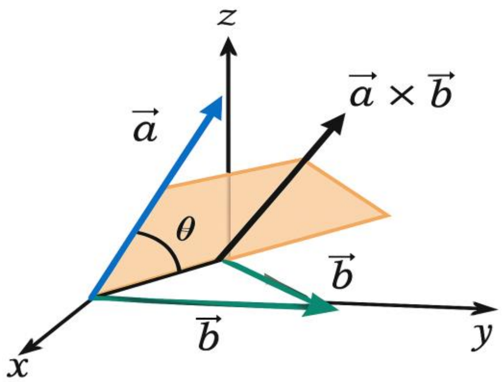
No	Vectors	Result	Interpretation	Drawing
1	**A**= (3,2), B= (1,4)	A. B=11>0	Vectors Point Roughly In The Same Direction	
2	**A**= (2,0), B= (0,4)	A. B=0	Vectors Are Perpendicular	
3	**A**= (2,0), B= (-1,1)	A. B=-2	Vectors Point In Opposite Directions	

2. **Cross Product (vector product)**

The cross product is a mathematical operation between two vectors in 3D space. Unlike the dot product, which gives a scalar, the cross-product results in a new vector that is perpendicular to both input vectors and has a magnitude related to the angle between them.

$$i \quad j \quad k$$
$$A_x \; A_y A_z$$
$$B_x \; B_y B_z$$
$$A * B = (A_y \,.\, B_y - A_z \,.\, B_y), (A_z \; B_x \; A_x \; B_z), (\; A_x B_y A_y B_x)$$

- A = (1,0,0)
- B = (0,1,0)
- A*B = (0,0,1)



2.1 Cross Product Applications

The cross product is a fundamental mathematical technique that has been used in many real-world applications across physics, engineering, and computer science. The cross product, as a vector used to produce a vector perpendicular to two others, makes it essential for understanding and calculating rotational effects, such as torque and angular momentum. In mechanical engineering, the cross product helps determine how forces cause objects to rotate, which is vital in designing engines, tools, and machines, as shown in Table No. 3.

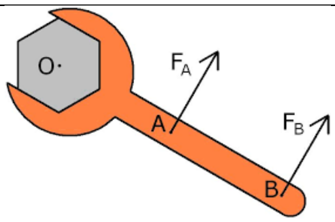
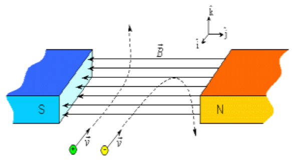
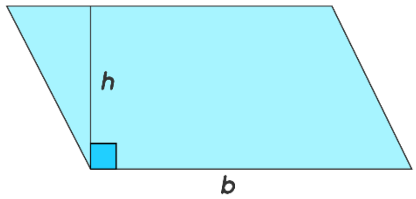
NO	Field	Example	Shows
1	Torque	Torque is the rotational effect of a force	
2	Magnetic Force	The magnetic force on a charged particle	
3	Normal Vector	A vector perpendicular to a surface	
4	Area of Parallelogram	Find the Area	
5	Computer Graphics	Realistic lighting on 3D objects	

Table 3: Cross Product Applications

3. Triple Product

The triple product is the product of three 3-dimensional vectors, usually used in geometry and algebra sciences. The name "triple product" is used for two different products, the scalar triple product and, less often, the vector triple product, as illustrated in Table 4.

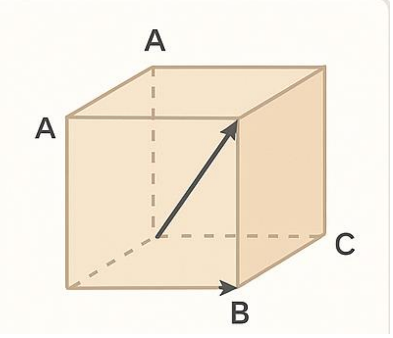
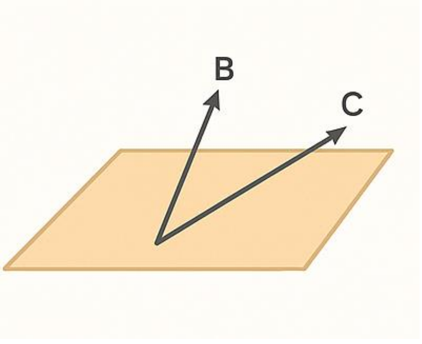
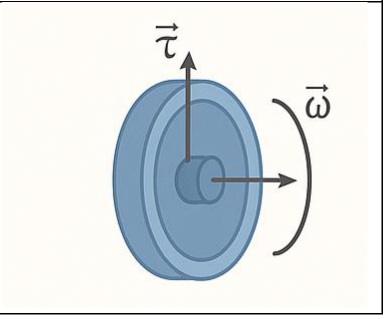
Table 4: Triple Products

NO	Triple Product	Mathematical Formula
1	Scalar Triple Product	$A\cdot(B\times C)$
2	Vector Triple Product	$A\times(B\times C)$

3.1 Triple Product Applications

A wide range of application work based on the triple products, which is primarily used to calculate the volume of a parallelepiped, making it essential in 3D geometry, architecture, and design, where accurate volume measurements are needed. While the vector triple product is used to simplify complex vector expressions, particularly in physics and mechanics, as shown in Table 5.

Table 5: **Triple Products Application**

NO	Field	Example	Shows
1	Volume of a Parallelepiped (3D shape)	Show the size of 3D shapes	
2	Checking Coplanarity	Validate flat surfaces or movements in a single plane.	
3	**Mechanics and Dynamics**	joint rotations and force directions	

4	Computer Graphics & 3D Game Engines	VR	

4. Leaning Out comes

Throughout this course, I have gained a solid understanding of geometry and its significance in the theory part. Studying the dot product has taught me how to determine whether two vectors are parallel, antiparallel, or perpendicular, and to apply this knowledge in real-world scenarios, such as in manual-handling operations. Exploring the cross product revealed how to generate a vector orthogonal to two given vectors, a concept that underpins analyses of rotational forces in mechanical engineering. Furthermore, the scalar triple product offers a straightforward criterion for testing whether three vectors are coplanar, which is invaluable for computing volumes and verifying spatial configurations. In conclusion, although the material initially appeared abstract, I now appreciate the depth and practical power of geometric methods.

5. Exercises

Last section presents the answers of the most important exercises which have been uploaded on the GitHub as different solutions for a further generation of students.