1) $u = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad v = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$

$|u| = \sqrt{(1)^2 + (-1)^2 + 0^2} \quad |v| = \sqrt{(1)^2 + (2)^2 + (0)^2}$

$|u| = \sqrt{2} \quad |v| = \sqrt{5}$

$$u \times v = \begin{pmatrix} (-1)(0) - (0)(2) \\ (0)(1) - (1)(0) \\ (1)(2) - (-1)(1) \end{pmatrix}$$

$u \times v = (0, 0, 3)$

$|u \times v| = \sqrt{0^2 + 0^2 + 3^2} = 3$

$$u \times v = \begin{pmatrix} u_2 v_3 - u_3 v_2 \\ u_3 v_1 - u_1 v_3 \\ u_1 v_2 - u_2 v_1 \end{pmatrix}$$

$u \times v = |u| * |v| * \cos\theta$

$|u \times v| =$

$\cancel{|u \times v|}\ |u| \cdot |v| \cdot \sin\theta$

$\sin\theta = \sqrt{1 - \cos^2\theta}$

$u \times v = |u| \cdot |v| \cdot \cos\theta$

$u \times v = (1) \cdot (1) + (-1) \cdot (2) + (0)(0)$

$u \times v = -1$

$-1 = \sqrt{2} \cdot \sqrt{5} \cdot \cos\theta$

$\frac{-1}{\sqrt{10}} = \cos\theta$

- - - - - - - -

$\sin\theta = \sqrt{1 - \cos^2\theta}$

$\sin\theta = \sqrt{1 - \left(\frac{-1}{\sqrt{10}}\right)^2}$

$\sin\theta = \frac{3}{\sqrt{10}}$

$|u \times v| =$

$\cancel{|u \times v|}\ |u| \cdot |v| \cdot \sin\theta$

$|u \times v| =$

$\cancel{|u \times v|}\ \sqrt{2} \cdot \sqrt{5} \cdot \sin\theta$

$|u \times v| = \sqrt{2} \cdot \sqrt{5} \cdot \frac{3}{\sqrt{10}}$

$|u \times v| = 3$

2)

$N = AB \times AC$

$AB = B - A$

$AC = C - A$

$$N = \begin{vmatrix} i & j & k \\ x_{AB} & y_{AB} & z_{AB} \\ x_{AC} & y_{AC} & z_{AC} \end{vmatrix}$$

$$N = i \begin{vmatrix} y_{AB} & z_{AB} \\ y_{AC} & z_{AC} \end{vmatrix} - j \begin{vmatrix} x_{AB} & z_{AB} \\ x_{AC} & z_{AC} \end{vmatrix} + k \begin{vmatrix} x_{AB} & y_{AB} \\ x_{AC} & y_{AC} \end{vmatrix}$$

$$N_1 (x - x_A) + N_2 (y - y_A) + N_3 (z - z_A) = 0$$

$A = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ $B = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$ $C = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$ $A = (1, -1, 2)$ $B = (0, -1, 3)$ $C = (3, 0, 2)$

$$AB = \begin{pmatrix} (0) - (1) \\ (-1) - (-1) \\ (3) - (2) \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = (-1, 0, 1)$$

$$AC = \begin{pmatrix} (3) - (1) \\ (0) - (-1) \\ (2) - (2) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = (2, 1, 0)$$

$AB = (-1, 0, 1)$ $AC = (2, 1, 0)$

$$N = \begin{vmatrix} i & j & k \\ -1 & 0 & 1 \\ 2 & 1 & 0 \end{vmatrix}$$

$$N = i \begin{vmatrix} 0, & 1 \\ 1, & 0 \end{vmatrix} - j \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} + k \begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix}$$

$= (0)(0) - (1)(1)$ $= (-1)(0) - (1)(2)$ $= (-1)(1) - (0)(2)$

$= -1$ $= -2$ $= -1$

$$N = \begin{vmatrix} i & j & k \\ x_{AB} & y_{AB} & z_{AB} \\ x_{AC} & y_{AC} & z_{AC} \end{vmatrix}$$

$N = (-1, 2, -1)$

$A = (1, -1, 2)$

$$N_1 (x - x_A) + N_2 (y - y_A) + N_3 (z - z_A) = 0$$

$$-1 (x - (1)) + 2 (y - (-1)) + -1 (z - (2)) = 0$$

$$-x + 1 + 2y + 2 - z + 2 = 0$$

$$-x + 2y - z = -5 \dashrightarrow \underline{x - 2y + z = 5}$$

3)

$$u \times v = \begin{vmatrix} i & j & k \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} = i \begin{vmatrix} u_y & u_z \\ v_y & v_z \end{vmatrix} - j \begin{vmatrix} u_x & u_z \\ v_x & v_z \end{vmatrix} + k \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$$

$$= i\left[(u_y)(v_z)-(u_z)(v_y)\right] - j\left[(u_x)(v_z)-(u_z)(v_x)\right] + k\left[(u_x v_y)-(u_y)(v_x)\right]$$

~~$u \times v$~~

$$u = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \qquad v = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \qquad w = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$$

$u = (1, -1, 0)$ $\qquad v = (1, 2, 0)$ $\qquad w = (1, 0, 3)$

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix} = i \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix} - j \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} + k \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}$$

$= (-1)(0) - (0)(2) = 0$ $\qquad = (1)(0) - (0)(1) = 0$ $\qquad = (1)(2) - (-1)(1) = 3$

$u \times v = (0, 0, 3)$ $\qquad 2u \times v = (0, 0, 6)$

$$v \times u = \begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 1 & -1 & 0 \end{vmatrix} = i \begin{vmatrix} 2 & 0 \\ -1 & 0 \end{vmatrix} - j \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}$$

$= (2)(0) - (0)(-1) = 0$ $\qquad = (1)(0) - (0)(1) = 0$ $\qquad = (1)(-1) - (2)(1) = -3$

$v \times u = (0, 0, -3)$

$$u \times w = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & 0 & 3 \end{vmatrix} = i \begin{vmatrix} -1 & 0 \\ 0 & 3 \end{vmatrix} - j \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} + k \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix}$$

$= (-1)(3) - (0)(0) = -3$ $\qquad = (1)(3) - (0)(1) = 3$ $\qquad = (1)(0) - (-1)(1) = 1$

$u \times w = (-3, -3, 1)$

$\boxed{u \times v = -(v \times u)}$ ✓

$(0, 0, 3) = -(0, 0, -3)$ ✓

$\boxed{2u \times v = 2(u \times v)}$ ✓

$(0, 0, 6) = 2(0, 0, 3)$ ✓

$\boxed{u \times (v + w) = (u \times v) + (u \times w)}$ ✓

$$\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 2 & 2 & 3 \end{vmatrix} = i \begin{vmatrix} -1 & 0 \\ 2 & 3 \end{vmatrix} - j \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} + k \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix}$$

$-3 \qquad 3 \qquad 4$

$u \times (v + w) = (-3, -3, 4)$ ✓

$(u \times v) + (u \times w) = (0, 0, 3) + (-3, -3, 1) = (-3, -3, 4)$ ✓

4)

$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = i \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - j \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + k \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

$$i\left[(u_2 v_3 - u_3 v_2)\right] - j\left[(u_1 v_3 - u_3 v_1)\right] + k\left[(u_1 v_2 - u_2 v_1)\right]$$

$$\boxed{u \times v = -(v \times u)} \checkmark$$

$$u \times v = i(u_2 v_3 - u_3 v_2) - j(u_1 v_3 - u_3 v_1) + k(u_1 v_2 - u_2 v_1)$$

$$= (u_2 v_3 - u_3 v_2) - (u_1 v_3 - u_3 v_1) + (u_1 v_2 - u_2 v_1)$$

$$v \times u = \begin{vmatrix} i & j & k \\ v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = i \begin{vmatrix} v_2 & v_3 \\ u_2 & u_3 \end{vmatrix} - j \begin{vmatrix} v_1 & v_3 \\ u_1 & u_3 \end{vmatrix} + k \begin{vmatrix} v_1 & v_2 \\ u_1 & u_2 \end{vmatrix} \checkmark$$

$$i\left[(v_2 u_3 - v_3 u_2)\right] - j\left[(v_1 u_3 - v_3 u_1)\right] + k\left[(v_1 u_2) - (v_2 u_1)\right]$$

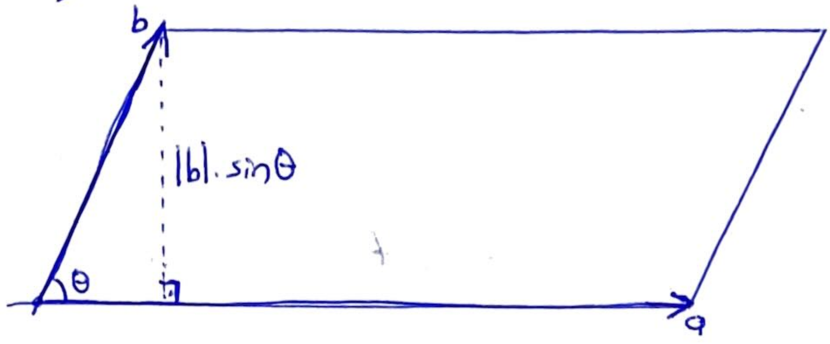
$$v \times u = i(v_2 u_3 - v_3 u_2) - j(v_1 u_3 - v_3 u_1) + k(v_1 u_2 - v_2 u_1)$$

$$= (v_2 u_3 - v_3 u_2) - (v_1 u_3 - v_3 u_1) + (v_1 u_2 - v_2 u_1)$$

$$\boxed{u \times v = -(v \times u)} \checkmark$$

$$(u_2 v_3 - u_3 v_2) - (u_1 v_3 - u_3 v_1) + (u_1 v_2 - u_2 v_1) = -\left[(u_3 v_2 - u_2 v_3) - (u_3 v_1 - u_1 v_3) + (u_2 v_1 - u_1 v_2)\right]$$

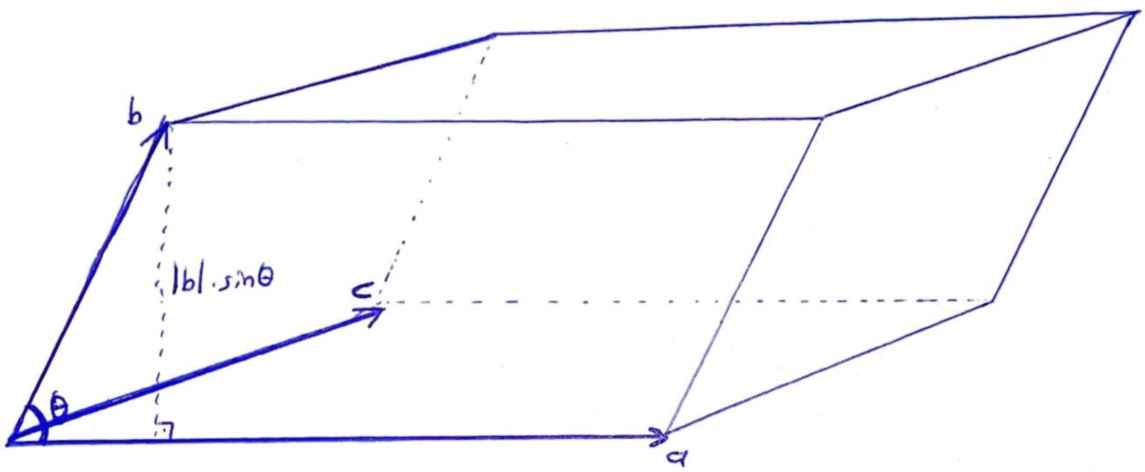
5)



$\text{Area} = |a|.|b|.\sin\theta$

$= (\vec{a} \times \vec{b})$

Parallelogram



Parallelepiped

$\text{Volume} = \text{Height} \times \text{Area}$

$\text{Volume} = |a|.|b|.\sin\theta.|c|$

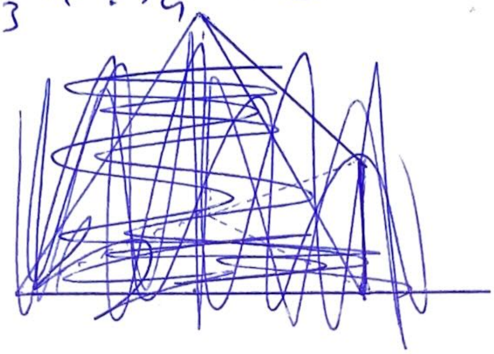
$= |(\vec{a} \times \vec{b}).\vec{c}|$

6)

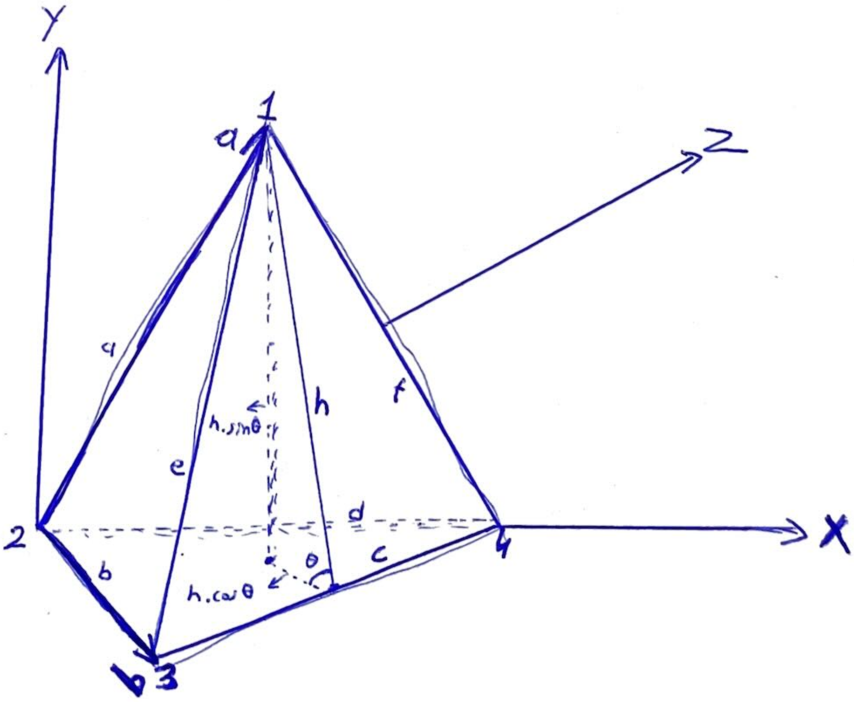
$$n_i = \frac{1}{2}(\vec{a} \times \vec{b})$$

$n_1 = \frac{1}{2} \cdot (a \times b)$ $\quad n_2 = \frac{1}{2} \cdot (b \times c)$ $\quad n_3 = \frac{1}{2} \cdot (c \times f)$ $\quad n_4 = \frac{1}{2} \cdot (d \times f)$

$$n_1 + n_2 + n_3 + n_4 = 0$$



Tetrahedron



7)

$$(a \times b)(c \times d) = (a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c)$$

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - j \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + k \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + k(a_1 b_2 - a_2 b_1)$$

$$a \times b = \begin{vmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{vmatrix}$$

$$c \times d = \begin{vmatrix} i & j & k \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{vmatrix} = i \begin{vmatrix} c_2 & c_3 \\ d_2 & d_3 \end{vmatrix} - j \begin{vmatrix} c_1 & c_3 \\ d_1 & d_3 \end{vmatrix} + k \begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$i(c_2 d_3 - c_3 d_2) - j(c_1 d_3 - c_3 d_1) + k(c_1 d_2 - c_2 d_1)$$

$$c \times d = \begin{vmatrix} c_2 d_3 - c_3 d_2 \\ c_3 d_1 - c_1 d_3 \\ c_1 d_2 - c_2 d_1 \end{vmatrix}$$

$$(a \times b)(c \times d) = \begin{vmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{vmatrix} \cdot \begin{vmatrix} c_2 d_3 - c_3 d_2 \\ c_3 d_1 - c_1 d_3 \\ c_1 d_2 - c_2 d_1 \end{vmatrix}$$

$$(a_2 b_3 - a_3 b_2)(c_2 d_3 - c_3 d_2) = (a_2 b_3 c_2 d_3 - a_2 b_3 c_3 d_2 - a_3 b_2 c_2 d_3 + a_3 b_2 c_3 d_2) \; +$$

$$(a_3 b_1 - a_1 b_3)(c_3 d_1 - c_1 d_3) = (a_3 b_1 c_3 d_1 - a_3 b_1 c_1 d_3 - a_1 b_3 c_3 d_1 + a_1 b_3 c_1 d_3) \; +$$

$$(a_1 b_2 - a_2 b_1)(c_1 d_2 - c_2 d_1) = (a_1 b_2 c_1 d_2 - a_1 b_2 c_2 d_1 - a_2 b_1 c_1 d_2 + a_2 b_1 c_2 d_1)$$

$$(a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c) \qquad a = (a_1, a_2, a_3) \quad b = (b_1, b_2, b_3) \quad c = (c_1, c_2, c_3) \quad d = (d_1, d_2, d_3)$$

$$a \cdot c = a_1 c_1,\ a_2 c_2,\ a_3 c_3$$
$$b \cdot d = b_1 d_1,\ b_2 d_2,\ b_3 d_3$$
$$a \cdot d = a_1 d_1,\ a_2 d_2,\ a_3 d_3$$
$$b \cdot c = b_1 c_1,\ b_2 c_2,\ b_3 c_3$$

$$\dashrightarrow \begin{array}{l} \cancel{a_1 b_1 c_1 d_1} + a_1 b_2 c_1 d_2 + a_1 b_3 c_1 d_3 \\ a_2 b_1 c_2 d_1 + \cancel{a_2 b_2 c_2 d_2} + a_2 b_3 c_2 d_3 \\ a_3 b_1 c_3 d_1 + a_3 b_2 c_3 d_2 + \cancel{a_3 b_3 c_3 d_3} \\ \hline \cancel{a_1 b_1 c_1 d_1} + a_1 b_2 c_2 d_1 + a_1 b_3 c_3 d_1 \\ a_2 b_1 c_1 d_2 + \cancel{a_2 b_2 c_2 d_2} + a_2 b_3 c_3 d_2 \\ a_3 b_1 c_1 d_3 + a_3 b_2 c_2 d_3 + \cancel{a_3 b_3 c_3 d_3} \end{array} \dashrightarrow \begin{array}{l} a_1 b_2 c_1 d_2 + a_1 b_3 c_1 d_3 \\ a_2 b_1 c_2 d_1 + a_2 b_3 c_2 d_3 \\ a_3 b_1 c_3 d_1 + a_3 b_2 c_3 d_2 \\ \\ -a_1 b_2 c_2 d_1 - a_1 b_3 c_3 d_1 \\ -a_2 b_1 c_1 d_2 - a_2 b_3 c_3 d_2 \\ -a_3 b_1 c_1 d_3 - a_3 b_2 c_2 d_3 \end{array}$$

8)

$$\det(a,b,c) = (a \times b) \cdot c$$

$$X \times (Y \times Z) = (X.Z).Y - (X.Y).Z$$

$$(a \times b) \times (c \times d) = [\det(a,c,d)].b - [\det(b,c,d)].a$$

$$(a \times b) \times (c \times d) = [\det(a,b,d)].c - [\det(a,b,c)].d$$

$$X \times (Y \times Z) = (X.Z).Y - (X.Y).Z$$

$$(a \times b) \times (c \times d) = \underbrace{[(a \times b).d]}.c - \underbrace{[(a \times b).c]}.d$$

$$= [\det(a,b,d)].c - [\det(a,b,c)].d$$

$$X \times (Y \times Z) = (X.Z).Y - (X.Y).Z$$

$$(a \times b) \times (c \times d) = \underbrace{[(c \times d).a]}.b - [(c \times d).b].a$$

$$= [\det(a,c,d)].b - [\det(b,c,d)].a$$

9) $T: R^3 \to R^3 \quad T(X) = a \times X \quad x, y \in R^3$

ⓐ Addivity

$T(X+y) = T(x) + T(y) \qquad T(X+y) = a \times (X+y) = a \times X + a \times y$

Homogeneity

$T(cX) = a \times (cX) = c(a \times X) = cT(X)$

ⓑ

$R^3 \{e_1, e_2, e_3\} \quad e_1 = (1,0,0) \quad e_2 = (0,1,0) \quad e_3 = (0,0,1)$

$$T(e_1) = a \times e_1 = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{vmatrix} = i \underbrace{\begin{vmatrix} a_2 & a_3 \\ 0 & 0 \end{vmatrix}}_{0} - j \underbrace{\begin{vmatrix} a_1 & a_3 \\ 1 & 0 \end{vmatrix}}_{-a_3} + k \underbrace{\begin{vmatrix} a_1 & a_2 \\ 1 & 0 \end{vmatrix}}_{-a_2}$$

$$= (0, a_3, -a_2)$$

$$T(e_2) = a \times e_2 = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ 0 & 1 & 0 \end{vmatrix} = i \underbrace{\begin{vmatrix} a_2 & a_3 \\ 1 & 0 \end{vmatrix}}_{-a_3} - j \underbrace{\begin{vmatrix} a_1 & a_3 \\ 0 & 0 \end{vmatrix}}_{0} + k \underbrace{\begin{vmatrix} a_1 & a_2 \\ 0 & 1 \end{vmatrix}}_{a_1}$$

$$= (-a_3, 0, a_1)$$

$$T(e_3) = a \times e_3 = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ 0 & 0 & 1 \end{vmatrix} = i \underbrace{\begin{vmatrix} a_2 & a_3 \\ 0 & 1 \end{vmatrix}}_{a_2} - j \underbrace{\begin{vmatrix} a_1 & a_3 \\ 0 & 1 \end{vmatrix}}_{a_1} + k \underbrace{\begin{vmatrix} a_1 & a_2 \\ 0 & 0 \end{vmatrix}}_{0}$$

$$= (a_2, -a_1, 0)$$

$$[T] = [T(e_1) \quad T(e_2) \quad T(e_3)] = \begin{bmatrix} 0 & -a_3 & +a_2 \\ +a_3 & 0 & -a_1 \\ -a_2 & +a_1 & 0 \end{bmatrix}$$

(C) $X = (x_1, x_2, x_3)$ $\quad T(x) = a \times X = 0$

$a = (a_1, a_2, a_3)$ $\quad a \times X = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ x_1 & x_2 & x_3 \end{vmatrix} = i\begin{vmatrix} a_2 & a_3 \\ x_2 & x_3 \end{vmatrix} - j\begin{vmatrix} a_1 & a_3 \\ x_1 & x_3 \end{vmatrix} + k\begin{vmatrix} a_1 & a_2 \\ x_1 & x_2 \end{vmatrix}$

$\downarrow (a_2x_3 - a_3x_2) \quad \downarrow (a_1x_3 - a_3x_1) \quad \downarrow (a_1x_2 - a_2x_1)$

$= \left((a_2x_3 - a_3x_2), (a_3x_1 - a_1x_3), (a_1x_2 - a_2x_1)\right)$

$\downarrow 0 \quad \downarrow 0 \quad \downarrow 0$

$$[T]x = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -a_3x_2 + a_2x_3 \\ a_3x_1 - a_1x_3 \\ -a_2x_1 + a_1x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(D) Dimension = Rank + Nullity

$\text{Dimension}(R^3) = 3$

Nullity $\rightarrow$ Dimension of null space = 1

$$\underset{3}{\text{Dimension}} = \underset{2}{\text{Rank}} + \underset{1}{\text{Nullity}}$$

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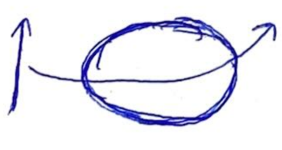
10) Earth Rotation(w) | Wind Velocity(v) | Coriolis Effect



$w = (0, 0, w_z)$

Eastward(v) $= (v_x, 0, 0)$

Northward(v) $= (0, v_y, 0)$

$-2w \times v$

$-2[(0,0,w_z) \times (v_x, v_y, 0)]$

Northern Hemisphere (counterclockwise)

$w_z > 0$ Upward ↑

$-2w_zv_x < 0$ Down ↓

$2w_z v_y > 0$ East →

Eastward Wind

$v = (v_x, 0, 0)$

$-2(w \times v) = -2(0,0,w_z) \times (v_x, 0, 0)$

$= -2(0, w_z v_x, 0)$

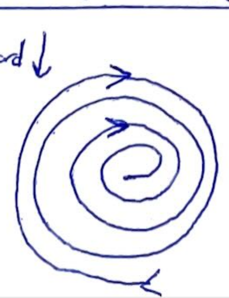
$= (0, -2w_zv_x, 0)$

Southern Hemisphere (clockwise)

$w_z < 0$ Downward ↓

$2w_zv_x > 0$ Up ↑

$2w_zv_y < 0$ West ←



Northward Wind

$v = (0, v_y, 0)$

$-2(w \times v) = -2(0,0,w_z) \times (0, v_y, 0)$

$= -2(-w_z v_y, 0, 0)$

$= (2w_z v_y, 0, 0)$

$= (w_x, w_y, w_z) \times (v_x, v_y, v_z)$

$= \begin{vmatrix} i & j & k \\ w_x & w_y & w_z \\ v_x & v_y & v_z \end{vmatrix}$

$= i\begin{vmatrix} w_y & w_z \\ v_y & v_z \end{vmatrix} - j\begin{vmatrix} w_x & w_z \\ v_x & v_z \end{vmatrix} + k\begin{vmatrix} w_x & w_y \\ v_x & v_y \end{vmatrix}$

$= (w_y v_z - w_z v_y) - (w_x v_z - w_z v_x) + (w_x v_y - w_y v_x)$