6.3.1

$u = (1, -1, 0)$
$v = (1, 2, 0)$

$|u| = \sqrt{1+1} = \sqrt{2}$

$|v| = \sqrt{1+4} = \sqrt{5}$

$$u \times v = \begin{array}{c} \begin{matrix} i & j & k \end{matrix} \\ \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \end{pmatrix} \end{array}$$

$(-1 \cdot 0 - 0 \cdot 2), (0 \cdot 1 - 0 \cdot 1), (1 \cdot 2 - (-1) \cdot 1)$

$u \times v (0, 0, 3)$

$|u \times v| = \sqrt{0+0+9} = \sqrt{9} = 3$

$u \times v = |u| \cdot |v| \cos\theta \leftarrow$ parallelogram Area

$= 1 \cdot 1 + (-1) \cdot 2 + 0 \cdot 0$

$= 1 - 2 + 0$

$\sqrt{2}\ \sqrt{5} = -1 \quad \cos\theta$

$\sqrt{10} = -\cos\theta$

$\cos\theta = \frac{-1}{\sqrt{10}}$

$\sin\theta = \sqrt{1 - \cos^2\theta}$

$\sin\theta = \sqrt{1 - \left(\frac{-1}{\sqrt{10}}\right)^2}$

$\sin\theta = \frac{3}{\sqrt{10}}$

3.6.2

$A=(1,-1,2)$, $B=(0,-1,3)$, $C=(3,0,2)$

$N = AB \times AC$

$A \times B = B - A$

$A \times C = C - A$

$$A \times B = \begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 3 \end{bmatrix} \boxed{= (-1,0,1)}$$

$$A \times C = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \end{bmatrix} \boxed{= (2,1,0)}$$

$$N = A \times B \begin{array}{c} \begin{matrix} i & j & k \end{matrix} \\ \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} \end{array} =$$

$$N = i \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - j \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix} + k \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$$

$= 0-1$ $\quad$ $= 0-2$ $\quad$ $= -1-0$

$= -1$ $\quad$ $= -2$ $\quad$ $= -1$

$\boxed{N = (-1,-2,-1)}$ $\quad$ $\boxed{A = (1,-1,2)}$

$N_1(X - X_a) + N_2(y - y_a) + N_3(z - z_a)$

$-1(X-1) + (-2(y-(-1)) + (-1(z-2)$

$-X - 1 - -2y - \cancel{2} - z + \cancel{2} = 0$

$-X - 2y + z = 1$ so the plan is (1)

6.3.3

$u=(1,-1,0)$, $v=(1,2,0)$, $w=(1,0,3)$

$$u \times v = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \end{bmatrix} (0,0,3)$$

$$v \times u = \begin{bmatrix} 1 & 2 & 0 \\ 1 & -1 & 0 \end{bmatrix} = (0,0,-3)$$

$$v \times w = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 3 \end{bmatrix} = (-3,-3,1)$$

* 1 - symmetry $u \times v = -(v \times w)$

$(0,0,3) = -(0,0,-3)$ Approved

* 2 - Homogenity $2\,u \times v = 2(u \times v)$

$2(0,0,3) = 2(0,0,3)$

$((0,0,6) = (0,0,6)$ Approved

* 3 - linearity $u \times (v+w) = (v \times w) + (v \times w)$

$u \times (2,2,3)$

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \end{bmatrix} = (-3,3,4)$$

$(u \times v) + (v \times w)$

$(0,0,3) + (-3,-3,1) = (-3,-3,4)$

So Approved

6.3.4

$$V \times U = -(U \times V)$$

Theorem 6.3.1 $= \cancel{\quad}$

$$U \times V = \begin{array}{c} \begin{array}{ccc} i & j & k \end{array} \\ \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{bmatrix} \end{array}$$

(1) ... $U \times V = i(u_2 v_3 - u_3 v_2) - j(u_3 v_1 - u_1 v_3) + k(u_1 v_2 - u_2 v_1)$

$$V \times U = \begin{array}{c} \begin{array}{ccc} i & j & k \end{array} \\ \begin{bmatrix} v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \end{bmatrix} \end{array}$$

(2) ... $V \times U = i(V_2 U_3 - V_3 U_2) - j(V_3 U_1 - V_1 U_3) + k(V_1 U_2 - V_2 U_1)$

So from 1 and 2 we can approve $V \times W = -(U \times V)$

6.3.5

$u \cdot (v \times w) = |u \cdot (u \times w)|$ volum of parallelepiped

volum = Area x Height

1- basic Area $= u \times w$

$= |u||w| \sin\theta$

2- Height $= u \cdot \dfrac{v \times w}{|v \times w|}$

$$\frac{|u \cdot (u \times w)|}{v \times w}$$

$$\text{volum} = u \times w \times \frac{|u \cdot (u \times w)|}{|u \times w|}$$

$$= u (v \times w)$$