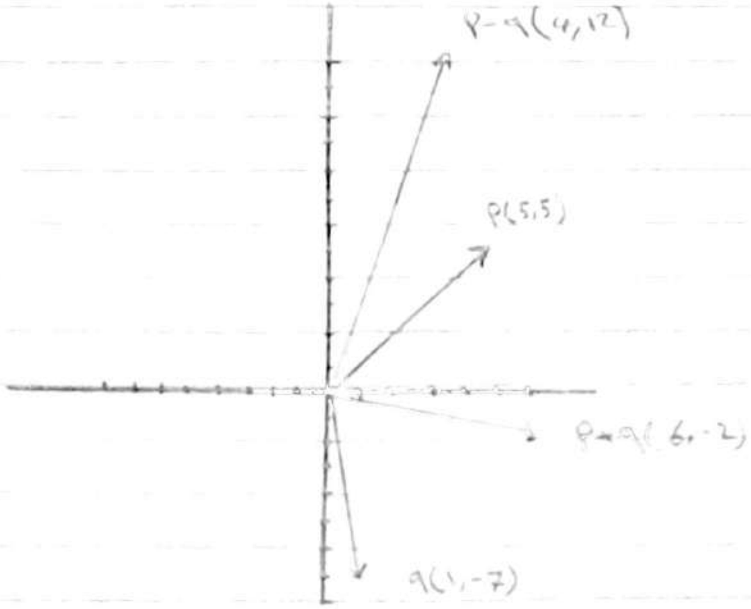
1.2.1

$p = (5,5)$, $q = (1,-7)$

A- $p + q = (6,-2)$, $p - q = (4,12)$

B-



C- $|p| = \sqrt{5^2+5^2} = \sqrt{50}$

$|q| = \sqrt{1^2+(-7)^2} = \sqrt{50}$

$|p+q| = \sqrt{6^2+(-2)^2} = \sqrt{40}$

$|p-q| = \sqrt{4^2+12^2} = \sqrt{160}$

D- $|p+q|^2 = (\sqrt{40})^2 = 40$

$|p|^2 = (\sqrt{50})^2 = 50$

$|q|^2 = (\sqrt{50})^2 = 50$

so $|p+q|^2 \neq |p|^2 + |q|^2$

since $40 \neq 50 + 50$

1.2.2

$p = (2, -2, 1)$, $q = (2, 3, 2)$

$p + q = (4, 1, 3)$, $p - q = (0, -5, -1)$

$|p| = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{9}$

$|q| = \sqrt{2^2 + 3^2 + 2^2} = \sqrt{17}$

$|p+q| = \sqrt{4^2 + 1^2 + 3^2} = \sqrt{26}$

$|p-q| = \sqrt{0^2 + (-5)^2 + (-1)^2} = \sqrt{26}$

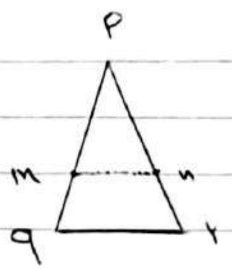
$|p+q|^2 = |p|^2 + |q|^2$

part 1) $(\sqrt{26})^2 = (\sqrt{9})^2 + (\sqrt{17})^2 = 26 = 9 + 17$ so it is correct equal

$|p-q|^2 = |p|^2 + |q|^2$

part 2) $(\sqrt{26})^2 = (\sqrt{9})^2 + (\sqrt{17})^2 = 26 = 9 + 17$ so equal correct

1.2.3



let $m = \frac{p+q}{2}$, $n = \frac{p+r}{2}$

$\overrightarrow{mn} = n - m$

$\overrightarrow{mn} = \frac{p+r}{2} - \frac{p+q}{2}$

$\overrightarrow{mn} = \frac{p+r-p-q}{2}$

$\overrightarrow{mn} = \frac{r-q}{2}$ So $\overrightarrow{mn} = \underline{1}\, qr$ Half is approved

$\overrightarrow{qr} = r - q$

$\overrightarrow{mn} = \frac{1}{2}\, \overrightarrow{qr}$ So theyare Parallel

1.2.4

Angle $\cos\theta \frac{p \cdot q}{|p| \cdot |q|}$ $\quad p = (-2, 4), q = (3, -5)$

$|p| = \sqrt{(-2)^2 + 4^2} = \sqrt{20}$

$|q| = \sqrt{3^2 + (-5)^2} = \sqrt{34}$

$p \cdot q = -6 + (-20) = -26$

$\text{Angle}(p, q) = \cos\theta \frac{-26}{\sqrt{20}\sqrt{34}}$

$= \cos\theta \cdot \frac{-26}{26.07}$

$\simeq 176.5°$

1.2.5

$p = (1, -2, 4)$, $q = (3, 5, 2)$, $p \cdot q = 3 + (-10) + 8 = 1$

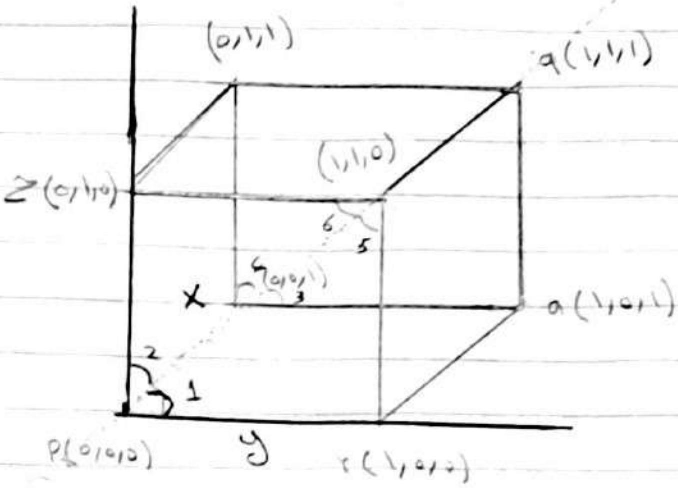
$|p| = \sqrt{1^2 + (-2)^2 + 4^2} = \sqrt{21}$

$|q| = \sqrt{3^2 + 5^2 + 2^2} = \sqrt{38}$

Angle $= \cos\theta \quad \dfrac{p \cdot q}{|p||q|} = \cos\theta \quad \dfrac{1}{\sqrt{21}\ \sqrt{38}}$

Angle $\simeq 87.97^\circ$

1.2.6



$x = \vec{pq} = q - p = (1,1,1)$

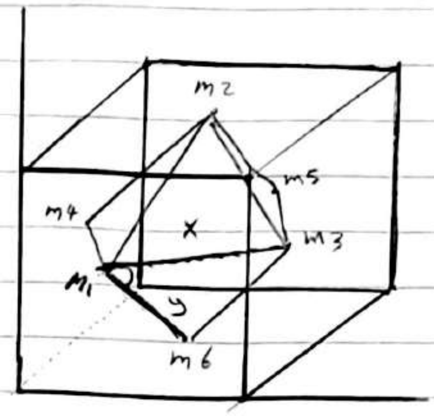
$y = \vec{pr} = r - p = (1,0,0)$

$$\text{Angle}_1(pq \,\&\, pr) = \frac{x \cdot y}{|x||y|} \cdot \cos\theta$$

$$= \frac{1}{\sqrt{3}\,\sqrt{1}} \cdot \cos\theta = \frac{1}{\sqrt{3}} \cdot \cos\theta = 0.577 \cdot \cos\theta$$

First Angle $(x,y) = 54.73°$

1.2.7



$m_1 = (\frac{1}{2}, \frac{1}{2}, 0)$ Front

$m_2 = (\frac{1}{2}, 1, \frac{1}{2})$ Top

$m_3 = (1, \frac{1}{2}, \frac{1}{2})$ Right

$m_4 = (0, \frac{1}{2}, \frac{1}{2})$ left

$m_5 = (\frac{1}{2}, \frac{1}{2}, 1)$ Back

$m_6 = (\frac{1}{2}, 0, \frac{1}{2})$ Bottom

$\vec{x} = \overrightarrow{m_1 m_3} = m_3 - m_1 = (1, \frac{1}{2}, \frac{1}{2}) - (\frac{1}{2}, \frac{1}{2}, 0) = (\frac{1}{2}, 0, \frac{1}{2})$

$x = |m_1 m_3| = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2} = \sqrt{\frac{1}{2}}$

$\vec{y} = \overrightarrow{m_1 m_6} = m_6 - m_1 = (\frac{1}{2}, 0, \frac{1}{2}) - (\frac{1}{2}, \frac{1}{2}, 0) = (0, -\frac{1}{2}, \frac{1}{2})$

$y = |m_1 m_6| = \sqrt{(-\frac{1}{2})^2 + (\frac{1}{2})^2} = \sqrt{\frac{1}{2}}$

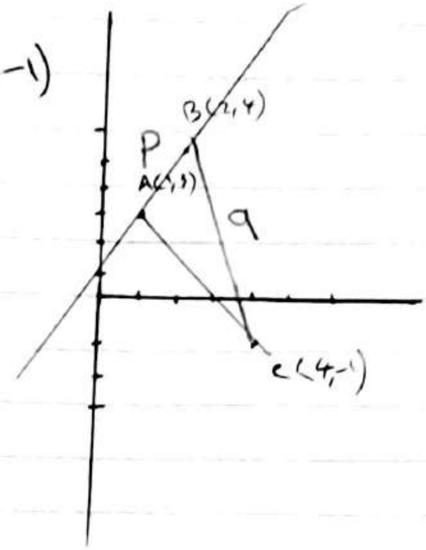
$$\text{Angle} = \cos\theta \; \frac{x \cdot y}{|x||y|} = \frac{\sqrt{\frac{1}{2}} \; \sqrt{\frac{1}{2}}}{\frac{1}{2} \cdot \frac{1}{2}} = \frac{0.5}{0.25}$$

$\cos\theta \, . \, 0.5$

$\theta \approx 60^\circ$

1.2.8

A- $A=(1,3)$, $B=(2,4)$, $C=(4,-1)$



$$Proj(P) = \frac{P \cdot q}{|q|^2} \cdot q$$

$$P = \vec{B} - \vec{A} = (1,1), \quad |P| = \sqrt{2}$$
$$q = \vec{C} - \vec{B} = (2,-5), \quad |q| = \sqrt{29}$$

$$Proj(P) = \frac{P \cdot q}{|q|^2} \cdot q =$$

$$= \frac{(1 \cdot 2)+(1 \times -5)}{(\sqrt{29})^2} = \frac{2-5}{29} = \frac{3}{29} \cdot (2,-5)$$

$$Proj(P) = \left(\frac{6}{29}, \frac{-15}{29}\right)$$

B- Distance of A to BC

$$D = \vec{BA} - Proj(\vec{BA}), \quad \vec{BA} = A - B = (-1,-1)$$

$$D = (-1,-1) - \left(\frac{6}{29}, \frac{-15}{29}\right)$$

$$D = \frac{6}{29}, \frac{-14}{29}$$

$$|D| = \sqrt{\left(\frac{6}{29}\right)^2 + \left(\frac{-14}{29}\right)^2} = \sqrt{0.04 + 0.230} = \sqrt{0.27}$$

Dist $= 0.51$

C- Area = Distance $\times$ $|BC| \times \frac{1}{2}$

$$= 0.51 \cdot \sqrt{29} \cdot \frac{1}{2}$$

$$= 0.25 \sqrt{29}$$

$$\approx 1.3$$

1.2.9

$p = (2, -3, 1)$, $q = (12, 3, 4)$

$|p| = \sqrt{4+9+1} = \sqrt{14}$

$|q| = \sqrt{144+9+16} = \sqrt{169}$

$p \cdot q = 24 + (-9) + 4 = 19$

$\text{proj}(p) = \dfrac{p \cdot q}{|q|^2} \cdot q = \dfrac{19}{\sqrt{169}^{\,2}} \cdot (12, 3, 4)$

$= \left(\dfrac{228}{169} , \dfrac{57}{169} , \dfrac{76}{169} \right)$

$\text{perpendicular}(p) = p - \text{proj}(p)$

$= (2, -3, 1) - \left(\dfrac{228}{169} , \dfrac{57}{169} , \dfrac{76}{169} \right)$

$= \left(\dfrac{116}{169} , \dfrac{-564}{169} , \dfrac{93}{169} \right)$

1.2.10

$$|p+q|^2 + |p-q|^2 = 2|p|^2 + 2|q|^2$$

$$(|p+q| \cdot |p+q|) + (|p-q||p-q|) =$$

$$(|p|^2 + 2\cancel{|pq|} + |q|^2) + (|p|^2 - 2\cancel{|pq|} + |q|^2)$$

$$|p|^2 + |q|^2 + |p|^2 + |q|^2$$

$$2|p|^2 + 2|q|^2 = 2|p|^2 + 2|q|^2$$

let $p = (1,0)$ $\quad q = (0,1)$

$|p| = \sqrt{2}$ $\quad |q| = \sqrt{2}$

$p+q = (1,1)$ $\quad p-q = (1,-1)$

now prove the parallelogram

$$|p+q|^2 + |p-q|^2 = 2|p|^2 + 2|q|^2$$

$$(\sqrt{2})^2 + (\sqrt{2})^2 = 2(\sqrt{1})^2 + 2(\sqrt{1})^2$$

$$2+2 = 2+2 \quad S\cancel{o} \quad 4=4$$

now prov the nonparalleogram

let $p = (1,0)$ $\quad q = (1,1)$

$|p| = \sqrt{1}$ $\quad |q| = \sqrt{2}$

$p+q = (2,1)$ $\quad p-q = (0,1)$

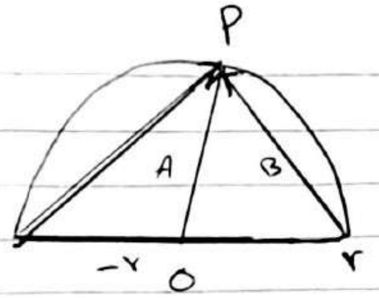
now $|p+q|^2 + |p-q|^2 = 2|p|^2 + 2|q|^2$

$$(\sqrt{5})^2 + (\sqrt{1})^2 = 2(\sqrt{1})^2 + 2(\sqrt{2})^2$$

$$5+1 = 2+4$$

$$5 \neq 6$$

1.2.11



$\vec{A} = P - (-r) = P + r$

$\vec{B} = P - r$

$A \cdot B = (P + r)(P - r)$

$= \vec{P}\vec{P} - Pr + Pr - \vec{r}\vec{r}$

$= \vec{P}\vec{P} - \vec{r}\vec{r}$ since $\vec{P}$ & $\vec{r}$ lies on same circle connected to same center

$= \vec{P}\vec{P} - \vec{P}\vec{P}$ so $P = r$

$= 0$ because the result is zero the Right

1.2.12

$$(p - \lambda q)(p - \lambda q) \geq 0$$ Apply Square

$$|p|^2 - 2\lambda pq + \lambda^2 |q|^2 \geq 0$$ Apply ~~&~~ λ value $\frac{p \cdot q}{|q|^2}$

$$|p|^2 - 2\frac{p \cdot q}{|q|^2} \cdot pq + \left(\frac{p \cdot q}{|q|^2}\right)^2 \cdot |q|^2 \geq 0$$

$$|p|^2 - 2\frac{(p \cdot q)^2}{|q|^2} + \frac{(p \cdot q)^2}{|q|^2} \geq 0$$

$$|p|^2 - \frac{(p \cdot q)^2}{|q|^2} \geq 0$$

$$|p|^2 \cdot |q|^2 \geq 0$$

$$|p| \cdot |q| \geq p \cdot q$$

Since the dot product grater than zero
They are in same direction and not aligned

1.2.13

inequal triangle = $|p|+|q| \geq |p+q|$

$$(|p|+|q|)^2 \geq |p+q|^2$$

$$|p|^2 + 2|pq| + |q|^2 \geq |p+q|^2$$

$$|p|^2 + 2|pq| + |q|^2 \geq |p|^2 + 2pq + |q|^2$$

$$2|pq| \geq 2pq$$

since $|p \cdot q| \geq 0$ so they are in same direction cannot be equal triangle which approve as not equal triangle

1.2.14

$||p|-|q||^2 \leq |p-q|$ apply the squaring

$||p|-|q||^2 \leq |p-q|^2$

$\cancel{|p|^2} - 2|pq| + \cancel{|q|^2} \leq \cancel{|p|^2} - 2pq + \cancel{|q|^2}$

$-\cancel{2}|pq| \leq -\cancel{2}pq$

$|pq| \leq pq$

so all vectors in $\mathbb{R}^n$ is inequality

1.2.15

$P = |P| (\cos\theta, \sin\theta)$ Pythaeus

$P = (P_x, P_y)$, $P_x = |P| \cdot \cos\theta$, $P_y = |P| \cdot \sin\theta$

$|P| = \sqrt{P_x^2 + P_y^2}$

$P_x, P_y = |P| (\cos\theta, \sin\theta)$

$|P| \cdot \cos\theta, |P| \cdot \sin\theta = |P| (\cos\theta, \sin\theta)$

Approved $|P| (\cos\theta, \sin\theta) = |P| (\cos\theta, \sin\theta)$

1.2.16

$P = (P_x, P_y, P_z)$

$u_P = \frac{P}{|P|}$ From 15

$x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3)$

$x \cdot y = x_1 y_1 + x_2 y_2 + x_3 y_3$

$u_P \cdot i = \left(\frac{P_1}{|P|}, \frac{P_2}{|P|}, \frac{P_3}{|P|}\right) \cdot (1, 0, 0) = \frac{P_1}{|P|}$

$u_P \cdot j = \left(\frac{P_1}{|P|}, \frac{P_2}{|P|}, \frac{P_3}{|P|}\right) \cdot (0, 1, 0) = \frac{P_2}{|P|}$

$u_P \cdot k = \left(\frac{P_1}{|P|}, \frac{P_2}{|P|}, \frac{P_3}{|P|}\right) \cdot (0, 0, 1) = \frac{P_3}{|P|}$

$P = |P| \cdot \cos\theta$

$\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3 = 1$

replace $P = u_P$ 2 anations lead to 1

$\cos^2\theta_1 = \frac{P_1}{|P|}$, $\cos^2\theta_2 = \frac{P_2}{|P|}$, $\cos^2\theta_3 = \frac{P_3}{|P|}$

$P = (P_x, P_y, P_z) = |P|(\cos\theta_1, \cos\theta_2, \cos\theta_3)$

CamScanner

1.2.17

$\cos\Theta_1 = \frac{P_1}{|P|}$ From Book

$P = (3, -4, 12)$

$|P| = \sqrt{9+16+144} = \sqrt{169} = 13$

$\cos\Theta_1 = \frac{P_1}{|P|} = \frac{3}{13} = 76.7^\circ$

$\cos\Theta_2 = \frac{P_2}{|P|} = \frac{-4}{13} = 107.1^\circ$

$\cos\Theta_3 = \frac{P_3}{|P|} \quad \frac{12}{13} = 22.7^\circ$

1.2.18

$\cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b$

$e_b = (\cos b, \sin b)$, $e_a = (\cos a, \sin a)$

$b = (b_1, b_2)$ $\quad a = (a_1, a_2)$

$e_a \cdot e_b = \cos a \cdot \cos b + \sin a \sin b$

$* \; e_a \cdot e_b = |e_a||e_b| \cdot \cos\theta$

$\underline{|e_a||e_b|} \cdot \cos\theta = \cos a \cdot \cos b + \sin a \cdot \sin b$

$1 \cdot 1 \cos(a-b) = \cos a \cdot \cos b + \sin a \sin b$