Student Information

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Answer 1

a)

Density function for uniform distribution:

$$f(x) = \frac{1}{b-a}, \quad a < x < b$$

Question says that the A and B events are independent. We can directly apply product rule:

$$f(T_a, T_b) = \frac{1}{100}.\frac{1}{100} = \frac{1}{10000}, \quad 0 \leq T_a \leq 100, \quad 0 \leq T_b \leq 100$$

We know pdf is derivative of cdf. So, if we integrate the pdf, we can reach to the cdf function:

$$F(T_a, T_b) = \int_0^{T_a} \int_0^{T_b} \frac{1}{10000} dxdy = (T_a)(T_b)\frac{1}{10000}, \quad 0 \leq T_a \leq 100, \quad 0 \leq T_b \leq 100$$

b)

Question is asking:

$$P\{0 < T_a < 30\} \cap P\{40 < T_b < 60\}$$

We can integrate density function to compute probabilities:

$$P\{0 < T_a < 30\} = \int_0^{30} \frac{1}{100} dt = 0.3$$

$$P\{40 < T_b < 60\} = \int_{40}^{60} \frac{1}{100} dt = 0.2$$

We know events are independent and question is asking about intersection, so we can multiply them:

$$P\{0 < T_a < 30\} \cap P\{40 < T_b < 60\} = (0.3)(0.2) = \underline{0.06}$$

c)

Our condition is:

$$T_A <= T_B + 10$$

I will show intervals as points, i.e. each point representation is actually an interval. Because uniform distribution is continious.

We know that if T_A is less than or equal to T_B this inequality is valid. If we think the square with 10000 area, there is 5050 cases which satisfies this condition. Also there are cases where T_A is bigger than T_B.

$$(1:10,0),(2:11,1),(3:12,2)....$$

As you can see there is 10 different case for each T_B value up to 90. So we have 910 different cases from here. From 91 to 100, cases are decreasing by one. So there is 55 different cases, but we already counted equality cases, so we should subtract 10 of them. 45.

$$5050+910+45=6005$$

$$6005/10000=0.6005$$

d)

Our condition is:

$$T_A <= T_B+20 \quad \wedge \quad T_B <= T_A+20$$

Logic from the previous question:

$$(0,0:20),(1,1:21),(2,2:22),(3,3:23)....$$

There is 21 different cases for each T_A up to 80:

$$21.81=1701$$

From 81 to 100 cases are decreasing by 1 each step.

$$\frac{20.21}{2}=210$$

$$1701+210=1911$$

Same cases are valid for T_B, but we are double counting the equaltiy situations, so we should subtract 101.

$$1911.2-101=3721$$

$$3721/10000=\underline{0.3721}$$

Answer 2

There is Binomial distribution in the event and Binomial distribution is sum of Bernoulli's, so Central Limit Theorem is appliable.

a)

First, let's calculate μ and σ.

$$\mu = np = 150(0.60) = 90 \qquad \sigma = \sqrt{npq} = \sqrt{150(0.60)(0.40)} = 6$$

Now let's look what question is asking:

$$P\{X \geq 150(0.65)\} = P\{X \geq 97.5\} = P\{X > 97.5\}$$

Binomial distribution is a discrete distribution, so we need to do continuity correction, but it's already corrected. We can do standardization directly:

$$P\{X > 97.5\} = P\{\frac{X - 90}{6} > \frac{97.5 - 90}{6}\} = P\{Z > 1.25\}$$

From the Table A4:

$$P\{Z > 1.25\} = P\{Z < -1.25\} = \underline{0.1056}$$

b)

First, let's calculate μ and σ:

$$\mu = np = 150(0.10) = 15 \qquad \sigma = \sqrt{npq} = \sqrt{150(0.10)(0.9)} = 3.67$$

Now let's look what question is asking:

$$P\{X \leq 150(0.15)\} = P\{X \leq 22.5\} = P\{X < 22.5\}$$

Again we don't need correction for this part as well. We can also ignore equality since we are working on discrete distribution. Standardization:

$$P\{X < 22.5\} = P\{\frac{X - 15}{3.67} < \frac{22.5 - 15}{3.67}\} = P\{Z < 2.04\}$$

From Table A4:

$$P\{Z < 2.04\} = \underline{0.9793}$$

Answer 3

Problem can be fit into standard normal distribution, and it's a neccesity since there is no cdf table for other distributions. To apply the properties variables must be standardized. Formula for this is following:

$$Z = \frac{X - \mu}{\sigma}$$

According to question μ is 175 cm and, σ is 7. Now variables can be standardized:

$$Z_1 = \frac{170 - 175}{7} = -0.71$$

$$Z_2 = \frac{180 - 175}{7} = 0.71$$

Let's get back to the what question is asking about:

$$P\{170 < X < 180\} = P\{-0.71 < Z < 0.71\}$$

From the cdf table (A4) $P\{Z < -0.71\}$ is 0.2389. We know standard normal distribution is symmetric. So, $P\{Z > 0.71\}$ is 0.2389 as well. Also all graph's area is 1, and question is asking about area between those values:

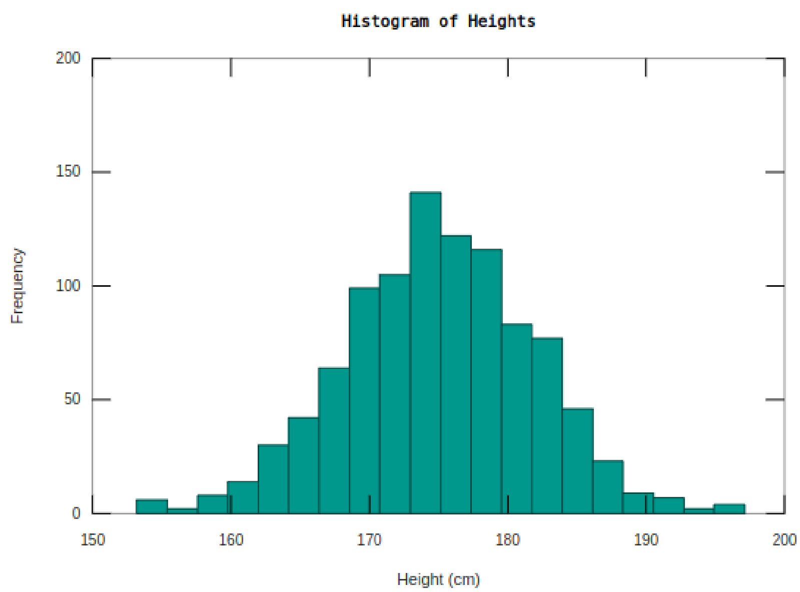
$$P\{-0.71 < Z < 0.71\} = 1 - 2(0.2389) = \underline{0.5222}$$

Answer 4

You can also find the codes in the text file.

a)

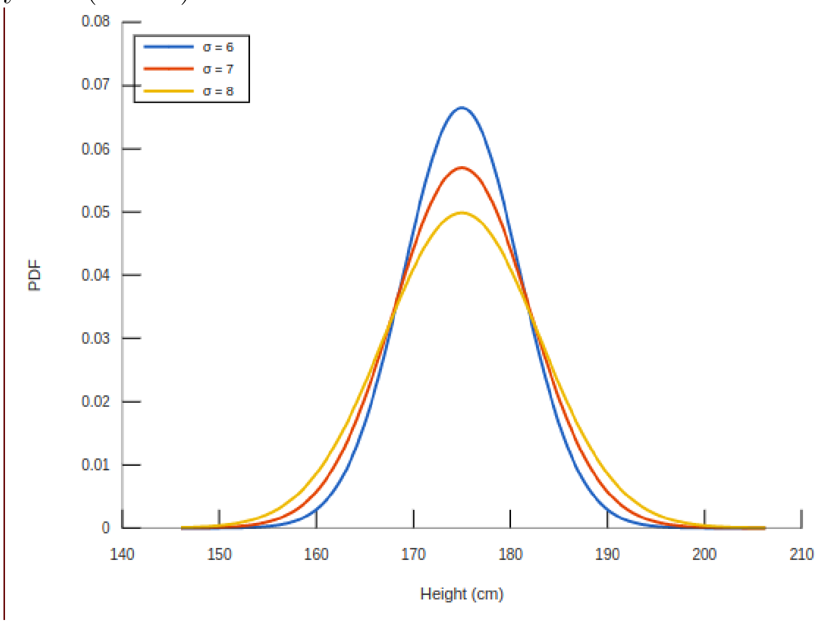
```
x = normrnd(175, 7, [1, 1000]);
hist(x, 20)
xlabel('Height (cm)')
ylabel('Frequency')
title('Histogram of Heights')
```



It's in the form of normal distribution as expected. It has more elements around mean and seperating according to standard deviation. We can see that because of number of bins which is 20. It's important to select suitable bin number. Otherwise, we couldn't see that distribution is normal.

b)

```
sigmas = [6, 7, 8];
sample = normrnd(175, sigmas(end), [1, 1000]);
x_min = min(sample);
x_max = max(sample);
x = linspace(x_min, x_max, 1000);
hold on
for sigma = sigmas
y = normpdf(x, 175, sigma);
plot(x, y, 'LineWidth', 2, 'DisplayName', sprintf('
sigma = end
hold off
legend('Location', 'Northwest')
xlabel('Height (cm)')
ylabel('PDF')
```



For higher deviations, values are differing much more. We can see that here. There are 3 normal distributions and the one with higher deviation has less probability for mean value, because values are more seperated. For sigma = 6, we can see there is higher probability for μ.

c)

```
count45 = 0;
count50 = 0;
count55 = 0;

    for i = 1:1000
count = 0;
```

```
x = normrnd(175, 7, [1, 150]);
for j = 1:150
if(x(j) >= 170 && x(j) <= 180)
count = count + 1;
end
end
if count >= 150*0.45
count45 = count45 + 1;
end
if count >= 150*0.5
count50 = count50 + 1;
end
if count >= 150*0.55
count55 = count55 + 1;
end
end

prob45 = count45 / 1000 * 100;
prob50 = count50 / 1000 * 100;
prob55 = count55 / 1000 * 100;

fprintf('Probability of at least 45%% of adults with heights between 170 cm and 180 cm: %.2f%%\n', prob45)
fprintf('Probability of at least 50%% of adults with heights between 170 cm and 180 cm: %.2f%%\n', prob50)
fprintf('Probability of at least 55%% of adults with heights between 170 cm and 180 cm: %.2f%%\n', prob55)
```

```
Probability of at least 45% of adults with heights between
170 cm and 180 cm: 96.80%
Probability of at least 50% of adults with heights between
170 cm and 180 cm: 75.40%
Probability of at least 55% of adults with heights between
170 cm and 180 cm: 26.30%
```

Since μ is 175, we are expecting more values around here. Also, it depends on standard deviation. Interval's average is also 175, so we can expect nearly half of the values in here, since deviation is 7. As you can see from results there is a high chance for that, but for higher percentages this value is decreasing. (55%)