Student Information

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Answer 1

a)

We can start calculating sample mean and standard deviation.

$$\bar{X} = \frac{8.4 + 7.8 + 6.4 + 6.7 + 6.6 + 6.6 + 7.2 + 4.1 + 5.4 + 6.9 + 7.0 + 6.9 + 7.4 + 6.5 + 6.5 + 8.5}{16} = 6.81$$

$$std = \frac{1}{15}\sqrt{(8.4 - 6.81)^2 + (7.8 - 6.81)^2 + ... + (8.5 - 6.81)^2} = 1.06$$

Since our sample size is small(n < 30), we should use t-distribution. α is 0.02. So we should compute $t_{0.01}$ with (16-1) = 15 degrees of freedom from table.

$$t_{0.01} = 2.602$$

Now we can use the confidence interval formula:

$$\bar{X} \pm t_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 6.81 \pm (2.602)\frac{1.06}{\sqrt{16}} = 6.81 \pm 0.69 = [6.12, 7.5]$$

b)

First, let's determine the null and alternative hypothesis:

$$H_0 \to \mu = 7.5 \qquad H_A \to \mu < 7.5$$

The test is one-sided and left-tail test. α is 0.05. Let's compute t-statistics:

$$t = \frac{\mu - \mu_0}{\sigma/\sqrt{n}} = \frac{6.81 - 7.5}{1.06/4} = -2.60$$

Let's look at -$t_{0.05}$(test is left-sided) with 15 degrees of freedom:

$$-t_{0.05} = -1.753$$

We can clearly see our t-statistics is on the rejection region. So, we can reject null hypothesis. Improvement is succesful.
To double-check we can compute P-value:

$$P\{Z < Z_{obs}\} = P\{Z < -2.60\} = 0.0047$$

P-Value is smaller than critical point 0.01, so we can reject null hypothesis.

c)

No. Wtih this change, our t-statistic will change as well. We need to compute t-statistic to answer this question. Also we need to compute test statistic to compare. We cannot say anything without this calculations.

Answer 2

a)

We know null hypothesis must be an equality. So Ali's claim should be null hypothesis, and Ahmet's claim should be alternative hypothesis.

$$H_0 \rightarrow \mu = \mu_0 \qquad H_A \rightarrow \mu > \mu_0$$

b)

Since our data is big(n > 30), we should use Z-statistics. From the alternative hypothesis we can understand the test should be one-sided right-tail test. So, α is 0.05.

$$Z_{0.05} = \Phi^{-1}(0.95) = 1.645$$

This number will be the limit of acception region. Now, let's compute Z:

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{5500 - 5000}{2000/\sqrt{100}} = 2.5$$

Since Z > 1.645 our test statistic belongs to rejection region. So we should reject null hypothesis. Ahmet can claim there is an increase in prices compared to last year with 95% confidence.

c)

$$P - value = P\{Z \geq Z_{obs}\} = P\{Z \geq Z_{2.5}\} = 1 - \phi^{-1}(2.5) = 1 - 0.9938 = 0.0062$$

Since P-value is smaller than critical point 0.01 we can say that prices are increased i.e. reject null hypothesis with higher confidence.

d)

First, let's determine null and alternative hypothesis.

$$H_0 \rightarrow \mu_A = \mu_I \qquad H_A \rightarrow \mu_A < \mu_I$$

Our data is big again, so we should use Z-statistics. The test is one-sided, left-tail. So α is 0.01.

$$-Z_{0.01} = -\phi^{-1}(0.99) = -2.33$$

Since we dealing with left-tail test, negative Z needed. Now, let's find Z-statistic with the formula for two-samples. Since population means are equal D is 0:

$$\frac{\bar{X}_A - \bar{X}_I}{\sqrt{\frac{\sigma_A^2}{n} + \frac{\sigma_I^2}{m}}} = \frac{5500 - 6500}{\sqrt{\frac{2000^2}{100} + \frac{3000^2}{60}}} = -2.29$$

Since our Z is in acception region, we can accept null hypothesis i.e. we cannot say prices are higher in Istanbul.

Answer 3

The question is asking dependency, so we can use chi-square test. Let's determine the null and alternative hypothesis:
$H_0 \rightarrow$ number of rainy days are independent on the seasons.
$H_A \rightarrow$ number of rainy days are dependent on the seasons.
Now, let's compute expected values. Since every season lasts 90 days, all expected values will be same for rainy and non-rainy days:

Expected value of rainy days $\rightarrow \frac{90.100}{360} = 25$

Expected value of non-rainy days $\rightarrow \frac{90.260}{360} = 65$

Now let's compute χ^2_{obs}:

$$\chi^2_{obs} = \frac{(34-25)^2}{25} + \frac{(32-25)^2}{25} + \frac{(15-25)^2}{25} + \frac{(19-25)^2}{25} + \frac{(56-65)^2}{65} + \frac{(58-65)^2}{65} + \frac{(75-65)^2}{65} + \frac{(71-65)^2}{65}$$

$$\chi^2_{obs} = 14.73$$

To use the table, also we need to know degrees of freedom which can be calculated as follows:

$$(n-1)(m-1) = (4-1)(2-1) = 3$$

From the table:

$$0.001 < P - value < 0.005$$

P-Value is smaller than 0.01, so we can reject the null hypothesis. The rainy days are dependent on seasons.

Answer 4

data = [34 32 15 19; 56 58 75 71];
([pValue,Xobs,df]) = chisquare_test_independence (data);
pValue
Xobs

df
Also you can find the code as a text file.

Name	Class	Dimension	Value
Xobs	double	1x1	14.732
data	double	2x4	[34, 32, 15, 19; 56, 58, 75, 71]
df	double	1x1	3
pValue	double	1x1	0.0020603

You can test different situations with changing data variable.