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Answer 1

$$\sum_{n=2}^{\infty} a_n x^n = 3\sum_{n=2}^{\infty} a_{n-1}x^n + 4\sum_{n=2}^{\infty} a_{n-2}x^n$$

$$\sum_{n=2}^{\infty} a_n x^n = 3x\sum_{n=2}^{\infty} a_{n-1}x^{n-1} + 4x^2\sum_{n=2}^{\infty} a_{n-2}x^{n-2}$$

Let me define a function:

$$A(x) = \sum_{n=0}^{\infty} a_n x^n$$

Now, we can rearrange the equation in terms of $A(x)$.

$$A(x) - a_0 - xa_1 = 3x(A(x) - a_0) + 4x^2 A(x)$$

$$A(x) - 1 - x = 3xA(x) - 3x + 4x^2 A(x)$$

$$2x - 1 = (4x^2 + 3x - 1)A(x)$$

$$A(x) = \frac{2x-1}{(4x-1)(x+1)}$$

$$A(x) = \frac{X}{4x-1} + \frac{Y}{x+1}$$

$$X + 4Y = 2, X - Y = -1$$

$$X = -2/5, Y = 3/5$$

$$A(x) = \frac{2}{5}(\frac{1}{1-4x}) + \frac{3}{5}(\frac{1}{1+x})$$

We know that $< 1, 1, 1, 1, 1... >= \frac{1}{1-x}$:

$$\frac{1}{1-4x} =< 1, 4, 16, 64...4^n... >$$

$$\frac{1}{1+x} < 1, -1, 1, -1...(-1)^n... >$$

Final solution:

$$A(x) = \frac{2.4^n}{5} + \frac{(-1)^n.3}{5}$$

Answer 2

a)

We can clearly see that generating function is:

$$< 2, 5, 11, 29, 83, 245.....(3^n + 2)... >$$

$$< 1, 3, 9, 27, 81, 243...3^n... > + < 2, 2, 2, 2, 2, 2...2... > - < 1, 0, 0, 0, 0...0... >$$
$$< 1, 3, 9, 27, 81, 243...3^n... > +2 < 1, 1, 1, 1, 1, 1...1... > - < 1, 0, 0, 0, 0...0... >$$

We know that $< 1, 1, 1, 1, 1... >= \frac{1}{1-x}$.(geometric series)

We can mulitply it by a constant(Theorem 1):

$$2\frac{1}{1-x} = \frac{2}{1-x}$$

If we substitute x with 3x we will have $< 1, 3, 9, 27, 81, 243...3^n... >= \frac{1}{1-3x}$

$$< 1, 0, 0, 0...0... >= 1 + 0.x^1 + 0.x^2... = 1$$

Final result:

$$\frac{2}{1-x} + \frac{1}{1-3x} - 1$$

b)

$$1 - 3x + 2x^2 = (2x - 1)(x - 1)$$
$$\frac{7 - 9x}{1 - 3x + 2x^2} = \frac{A}{2x - 1} + \frac{B}{x - 1}$$

$$A + 2B = -9 \quad and \quad - A - B = 7$$
$$A = -5 \quad and \quad B = -2$$
$$\frac{7 - 9x}{1 - 3x + 2x^2} = \frac{-5}{2x - 1} + \frac{-2}{x - 1} = \frac{5}{1 - 2x} + \frac{2}{1 - x}$$

$$\frac{5}{1 - 2x} = 5(\frac{1}{1 - (2x)})$$

We know that $\frac{1}{1-x} =< 1, 1, 1, 1, 1... >$.(geometric series)
If we susbstitute x with 2x and multiply expression by 5:

$$\frac{1}{1 - 2x} =< 1, 2, 4, 8, 16...2^n... >$$

$$\frac{5}{1-2x} =< 5, 10, 20, 40, 80...5.2^n... >$$

We can use $\frac{1}{1-x} =< 1, 1, 1, 1, 1... >$ again.

$$\frac{2}{1-x} = 2 < 1, 1, 1, 1...1... >=< 2, 2, 2, 2...2... >$$

Final result:

$$< 5, 10, 20, 40, 80...5.2^n... > + < 2, 2, 2, 2...2... >=< 7, 12, 22, 42, 82...5.2^n + 2 >$$

Answer 3

a)

A relation is equivalence iff it has refflexivity, symmetricity, transitvity properties.

1)Reflexive: We should show that this relation has (a,a) as an element. For (a,a) our n will be $a\sqrt{2}$, but relation defined in Z and n $\in Z$ as well. Therefore $a\sqrt{2}$ cannot be a integer. Relation is not reflexive.

2)Symmetric: We should show that if relation has (a,b), then it must contain (b,a).

$$a^2 + b^2 = b^2 + a^2 = n$$

Therefore, the relation is symmetric.

3)Transitive: We should show that if this relation has (a,b) and (b,c), then it must contain (a,c). We can give a counter example: 5R12 (where n is 13) and 12R16 (where n is 20). By property we must have 5R16 (where n is $\sqrt{281}$ which is not a integer). Therefore the relation is not transitive.

The relation symmetric but not reflexive and not transitive, so this relation is not equivalence relation.

b)

A relation is equivalence iff it has refflexivity, symmetricity, transitvity properties.
1)Reflexive: We should show that this relation has $((x_1, y_1), (x_1, y_1))$ as an element.

$$2x_1 + y_1 = 2x_1 + y_1$$

2)Symmetric: We should show that if this relation has $((x_1, y_1), (x_2, y_2))$ then, it must contain $((x_2, y_2), (x_1, y_1))$ as well.

$$2x_1 + y_1 = 2x_2 + y_2$$

$$2x_2 + y_2 = 2x_1 + y_1$$

3)Transitive: We should show that if this relation has $((x_1, y_1), (x_2, y_2))$ and $((x_2, y_2), (x_3, y_3))$, then it must contain $((x_1, y_1), (x_3, y_3))$.

$$2x_1 + y_1 = 2x_2 + y_2 = A$$

$$2x_2 + y_2 = 2x_3 + y_3 = A$$

where A $\in R$

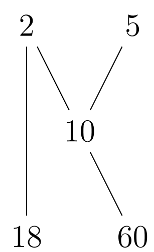
$$2x_1 + y_1 = 2x_3 + y_3 = A$$

Therefore this relation is a equivalence relation.
Equivalence class [(1,-2)] has all the points which has (A,-2A) form and A $\in R$. Since our relation is defined in R, this represents the $y = -2x$ line in Cartesian coordinate system.

Answer 4

Since "divides" relation is antisymmetric, reflexive, transitive it is partial ordering relation. Its POSET is (2,5,10,18,60, |)

a)



Since Hasse diagram does not contain loops and transivity reduncuncies, we don't need to 5 – 60, 2–60 and all the loops.

b)

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$ If there is a relation between two elements it represented with 1.

c)

We need to show $R \cup R^{-1}.R^{-1} is :$ $$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$M_{R\cup R^{-1}} = M_R \vee M_{R^{-1}} = \begin{bmatrix}1&0&1&1&1\\0&1&1&0&1\\0&0&1&0&1\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix} \vee \begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\1&1&1&0&0\\1&0&0&1&0\\1&1&1&0&1\end{bmatrix} = \begin{bmatrix}1&0&1&1&1\\0&1&1&0&1\\1&1&1&0&1\\1&0&0&1&0\\1&1&1&0&1\end{bmatrix}$$

d)

We need to see a chain Hasse diagram to determine it's total ordering or not. It's not possible with removing 1 element. If we remove 5 to fix 2R5, we still have 10R18 and 18R60 relation which is invalid, so we need to remove 5 and 18. If we put 1 and 20, the Hasse diagram will look like this:

1 —— 2 —— 10 —— 20 —— 60

Therefore we have a chain. Now we have a total order relation on this set.