Middle East Technical University Department of Computer Engineering

CENG 280

Formal Languages and Abstract Machines

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Homework 3

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Answer for Q1

1. We can start off with split the states into two groups according to they are final state or not.

$\equiv_0$ $\{q_0, q_1, q_3, q_4\}$ $\{q_2, q_5\}$

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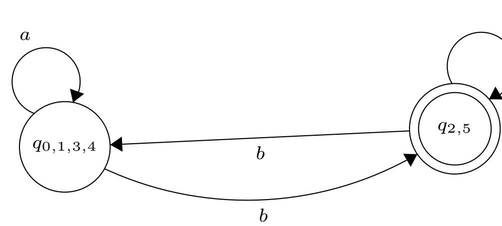
There is no change for second step as well. If we consider transitions, first group's elements can reach second group with consuming 'b', and also second group's elements can reach first group with consuming 'b'. Also both groups can turn same set with consuming 'a'. Therefore we have two states.



2. We know that number of equivalent classes is equal to number of states. Since we have two states there will be 2 equivalent classes. One of them can be expressed as:

$$[e] = L$$

We know that if two equivalent classes are same, when we concatenate a symbol from alphabet they both in the language or they both not in the language. Let's try a and b.

$$[a] \rightarrow ea, aa \notin L \quad \wedge \quad eb, ab \in L \quad \rightarrow [e] = [a]$$

$$[b] \rightarrow ea \notin L, ba \in L \quad \wedge \quad eb \in L, bb \notin L \quad \rightarrow [e] \neq [b]$$

Therefore we can consider $[b]$ as an equivalent class.

$$[e] = L \quad and \quad [b] = Lba^*b$$

3. According to MyHill-Nerode theorem number of states in the FA is equal to number of equivalent classes of the language, and it also states that if a language is regular, then it should has finite number of states(equivalent classes). So if we can show that it has infinite number of equivalent classes, language is not regular.

$$[e] \not\approx_L [a] \not\approx_L [d] \not\approx_L [ab] \not\approx_L [ac]...$$

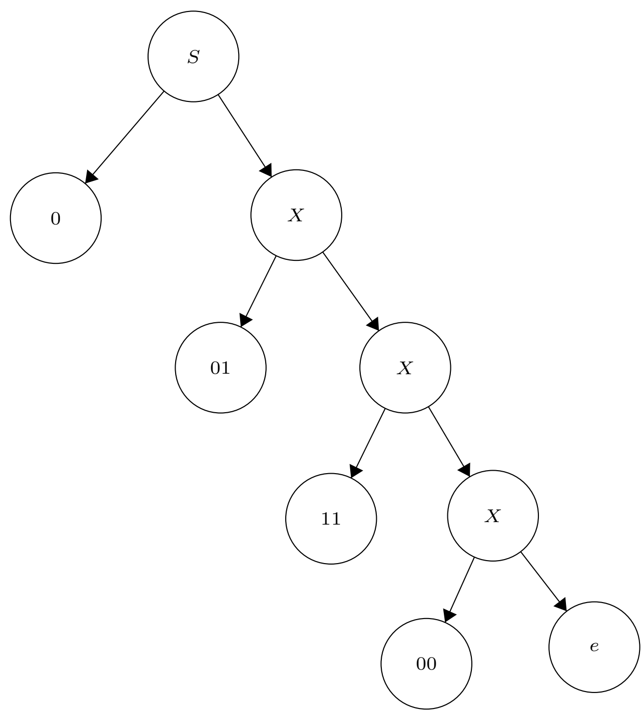
This list can go further to infinity. We can find a new equivalent class every step. Since regular languages can be repsresented with DFA's and DFA's have finite number of states, this language is not regular.

Answer for Q2

1. G = {V, $\sum$, R, S} where V = {S} $\cup$ $\sum$, $\sum$ = {a,b} and
R = {S $\rightarrow$ SS | aSb | bSa | bS | b}

2. G = {V, $\sum$, R, S} where V = {S,X} $\cup$ $\sum$, $\sum$ = {0,1,2} and
R = {S $\rightarrow$ 0S1X | e , X $\rightarrow$ 1X2 | e}

3. G = {V, $\sum$, R, S} where V = {S,X} $\cup$ $\sum$, $\sum$ = {0,1} and
R = {S $\rightarrow$ 0X | 1X, X $\rightarrow$ 00X | 01X | 10X | 11X | e }



Answer for Q3

1. G = {w | w begins and ends with same symbol from the alphabet and w $\in (0, 1)^*$}

2. G = {w | w contains at least two 1's and w $\in (0, 1)^*$.}