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Answer 1

a)

p	q	$p \wedge q$	$\neg p \vee \neg q$	$(p \wedge q) \iff (\neg p \vee \neg q)$
F	F	F	T	F
F	T	F	T	F
T	F	F	T	F
T	T	T	F	F

Since every value is False, this is a contradiction.

b)

1. $p \implies ((q \vee \neg q) \implies (p \wedge q))$ given
2. $p \implies (T \implies (p \wedge q))$ Negation Law
3. $p \implies (F \vee (p \wedge q))$ Imp. Elimination(Lemma)
4. $p \implies (p \wedge q)$ Identity Law
5. $\neg p \vee (p \wedge q)$ Imp. Elimination(Lemma)
6. $(\neg p \vee p) \wedge (\neg p \vee q)$ Distrubtive Law
7. $T \wedge (\neg p \vee q)$ Negation Law
8. $\neg p \vee q$ Identity Law

Answer 2

1. $\forall x \exists y (W(x, y))$
2. $\forall x \exists y (\neg F(x, y))$
3. $\forall x (W(x, P) \implies A(Ali, x))$
4. $\exists y (W(Busra, y) \wedge F(TUBITAK, y))$
5. $\exists x \exists y \exists z (S(x, y) \wedge S(x, z))$
6. $\neg(\exists x \exists y \exists z (W(x, z) \wedge W(y, z)))$
7. $\exists x \exists y \exists z \forall t (((W(x, z) \wedge W(y, z)) \implies \neg w(t, z)) \wedge (t \neq x \wedge t \neq y))$

Answer 3

1	$p \implies q$	
2	$(q \land \neg r) \implies s$	
3	$\neg s$	
4	$\quad (q \land \neg r) \land \neg s$	
5	$\quad q \land \neg r$	$\land$E, 4
6	$\quad s$	$\Rightarrow$E, 2, 5
7	$\quad \bot$	$\neg$E, 3, 6
8	$\neg((q \land \neg r) \land \neg s)$	$\neg$I, 4–7
9	$\quad p$	
10	$\quad q$	$\Rightarrow$E, 1, 9
11	$\quad\quad \neg r$	
12	$\quad\quad q \land \neg r$	$\land$I, 10, 11
13	$\quad\quad (q \land \neg r) \land \neg s$	$\land$I, 12, 3
14	$\quad\quad \bot$	$\neg$E, 8, 13
15	$\quad \neg\neg r$	$\neg$I, 11–14
16	$\quad r$	$\neg\neg$E, 15
17	$p \implies r$	$\Rightarrow$I, 9–16

Answer 4

Ayşe : p
Barış : $s \implies \neg q$
Can : $p \implies (q \land r)$
Duygu : $r \implies s$
According to Ekim, Barış is lying, so we need to prove $\neg(s \implies \neg q)$ with using Ayse, Can and Duygu's statements.

1	p	
2	$p \implies (q \wedge r)$	
3	$r \implies s$	
4	$\quad s \implies \neg q$	
5	$\quad q \wedge r$	$\Rightarrow$E, 1, 2
6	$\quad r$	$\wedge$E, 5
7	$\quad s$	$\Rightarrow$E, 3
8	$\quad \neg q$	$\Rightarrow$E, 4
9	$\quad q$	$\wedge$E, 5
10	$\quad \bot$	$\neg$E, 8, 9
11	$\neg(s \implies \neg q)$	$\neg$I, 4–10

Answer 5

I am going to use Modus Tollens Lemma in my proof so here is the MT's ND proof:

1	$p \implies q$	
2	$\neg q$	
3	$\quad p$	
4	$\quad q$	$\Rightarrow$E, 1, 3
5	$\quad \bot$	$\neg$E, 2, 4
6	$\neg p$	$\neg$I, 3–5

Here is my proof:

1	$\forall x(P(x) \implies (Q(x) \implies R(x)))$	
2	$\exists x(P(x))$	
3	$\forall x(\neg R(x))$	
4	$a \quad p(a)$	
5	$p(a) \implies (Q(a) \implies R(a))$	$\forall$E, 1
6	$\neg R(a)$	$\forall$E, 3
7	$Q(a) \implies R(a)$	$\Rightarrow$E, 4, 5
8	$\neg Q(a)$	

I used Modus Tollens Lemma (6,7) here.

9	$\exists x(\neg Q(x))$	$\exists$I, 8
10	$\exists x(\neg Q(x))$	$\exists$E, 2, 4–9