Sequences

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1 Arithmetic Sequence

Definition 1.1 *Given a starting value of a_0 and a constant difference value of d the arithmetic sequence is defined as follows.*

$$\boxed{a_n = a_0 + n \cdot d}$$

1.1 Sum of Arithmetic sequence

Theorem 1.1

$$\sum_{k=0}^{n} a_k = \frac{(n+1)}{2}(2a_0 + n \cdot d) \tag{1}$$

Proof

Define S_n such that:

$$S_n = \sum_{k=0}^{n} a_k$$

Note that

$$2S_n = \sum_{k=0}^{n} a_k + \sum_{k=0}^{n} a_{n-k} = \sum_{k=0}^{n} \Big(a_k + a_{n-k}\Big)$$

Using the definition we see $a_k + a_{n-k} = \big(a_0 + k \cdot d\big) + \big(a_0 + (n-k) \cdot d\big) = 2a_0 + n \cdot d$ which does not depend on k

$$2S_n = \sum_{k=0}^{n} \Big(2a_0 + n \cdot d\Big) = (n+1)\Big(2a_0 + n \cdot d\Big)$$

$$S_n = \frac{(n+1)}{2}(2a_0 + n \cdot d)$$

1.2 Example: Sum of Natural Numbers

Prove that

$$\sum_{i=0}^{n} i = 0 + 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$

Proof: Set $a_0 = 0$ and $d = 1$

$$\sum_{i=0}^{n} i = \frac{(n+1)}{2}(n)$$