

# Deriving the Quadratic equation

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## 1 The Quadratic Equation

Given a quadratic polynomial  $ax^2 + bx + c = 0$  The two solutions of  $x$  that satisfy this equation are given by the quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (1)$$

## 2 Algebraic Proof

The most straight-forward proof is given by a technique referred to as 'Completing the Square'.

$$ax^2 + bx + c = 0$$

Dividing both sides by  $a$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Adding and subtracting  $\frac{b^2}{(2a)^2}$

$$x^2 + \frac{b}{a}x + \frac{b^2}{(2a)^2} + \frac{c}{a} - \frac{b^2}{(2a)^2} = 0$$

Recognizing the 'square'

$$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{(2a)^2} = 0$$

Rearranging and solving

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$