An Affordance Theory of Logical Reasoning: Conditional Inferences as a Way to Test Mental Logic, Mental Models, and the Theory of Affordances

Janet Marie Gibson
Grinnell College

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Drew Johnson and Janet M. Gibson

Grinnell College

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Contact information: drew.johnson@uconn.edu, Dept. of Philosophy, University of Connecticut

Abstract

Two of the main theories of logical reasoning, mental logic theory and mental model theory, make different predictions about how people ought to perform on the four kinds of conditional inferences (modus ponens, modus tollens, the affirming the consequent fallacy, and the denying the antecedent fallacy). These predictions are tested with multiple choice conditional inference logic problems. We tested the possibility that people make use of both mental logic and mental model methods of reasoning by manipulating instructions to influence participants' patterns of response on these multiple-choice problems to fit with mental logic or mental model predictions. We gauged the effects of these instructions with a proof conviction task. We explore the possibility that the theory of affordances can explain which reasoning strategy participants choose by using concrete examples in logic problems to attune people to constraints in logical reasoning. Our data showed that people's performance on conditional inference problems was affected by instruction, but not as predicted. Concrete examples did not appear effective in attuning participants to constraints. We also found a fatigue effect in people's ability to complete modus tollens problems.

An Affordance Theory of Logical Reasoning: Conditional Inferences as a Way to Test Mental Logic, Mental Models, and the Theory of Affordances

Conditional reasoning is replete in everyday life. Whether it occurs as warnings (Couto, Quelhas, & Byrne, 2017) or competes with beliefs (Trippas, Thompson, & Handley, 2017), we often deal with the consequences of faulty reasoning. Although we have the capacity to reason correctly, we still make consistent and predictable errors in logical reasoning. If we are able to reason well, why do we make errors so often? The philosophical study of formal logic has shown us how we ought to reason, but we need is a psychological explanation of how we actually do reason.

Mental logic and mental models

Two major theories of logical reasoning in the literature are mental logic theory (Braine & O'Brien, 1998) and mental model theory (Johnson-Laird, 1994). According to mental model theory, people reason by constructing models of the premises of logical problems. These models represent the truth contingencies of the premises. The truth contingencies of a statement are the conditions under which the statement would be true or not true. The premises are then compared with each other for consistency. When subsequent premises are consistent with the previous premise, they can be combined, and where they are inconsistent, one must be rejected. Two premises are consistent with each other when they can be true at the same time, and they are inconsistent with each other when they cannot both be true at the same time (given their truth contingencies). Combining consistent premises and rejecting inconsistent premises constructs a final model which can then be checked for its consistency with the conclusion; if they are consistent, the conclusion is accepted, and if not, the conclusion is rejected as false (Schroyens, Schaeken, & d'Ydewalle, 2001).

According to mental logic theory, people make logical inferences in roughly the same way we make them in formal logic (Schroyens et al., 2001). The main proponents of mental logic, Braine and O'Brien (1998), suggest that individuals make deductions by applying inference schemas (or rules) to premises to reach conclusions. Each inference schema is a rule that tells us what conclusion can be drawn from premises of a certain form. For example, the modus ponens rule (as it is called in logic) tells us that from premises of the form “if p, then q” and “p” then we can infer “q.” Another example would be the disjunctive syllogism rule which says that from premises of the form “p or q” and “not p” we can infer “q.” These different argument forms (patterns of premises) elicit the different inference schemas.

Although these theories offer different accounts of how we make logical inferences, they are difficult to distinguish between in practice because the fact that a person makes a logical conclusion in response to a logic problem tells us nothing about whether that person used mental models or mental logic. The aim of our studies is to successfully test the predictions of each of these theories.

Any paradigm we might use to test which (if any, or both) of these theories most accurately describes how individuals make logical inferences must provide a way to test varying predictions of these theories. The main paradigm we have observed in the literature involves asking participants to write down each intermediate inference that occurs to them in the process of solving a logic problem. The researchers then code the content and order of the inferences as agreeing with either mental logic or mental models (e.g. Braine, O’Brien, & Yang, 1994; Braine, O’Brien, Noveck, Samuels, Lea, Fisch, & Yang, 1995; Johnson-Laird, Byrne, Schaeken, 1994). The rationale behind using this paradigm is that the primary difference between the mental logic and mental model theories is the sorts of inferences people make on the way to reaching a conclusion.

Braine et al. (1994) made use of this paradigm in order to support mental logic theory. The experimenters created logic problems and then determined how many models would have to be constructed to complete the problem in mental model theory and how many inference schemas would have to be applied in mental logic theory. Braine et al. (1994) cited the mental model prediction that problems requiring more than four models should be very difficult to complete due to restrictions on working memory. In their experiment, participants were asked to complete problems which Braine et al. (1994) calculated to require more than four models to complete, but four or less inference schemas. They expected that if mental models theory accurately described how people make logical inferences, then participants would be unable to solve the problems correctly (because they could not hold all of the required models in working memory). But if mental logic were the more accurate way to describe the way people make logical inferences, then participants would be able to solve the problems easily. Braine et al. (1994) found that participants overwhelmingly avoided making the errors they ought to have made if they were using mental models to complete the problems. Participants' responses were consistent with the predictions of mental logic.

However, Johnson-Laird et al. (1994) responded specifically to this article, proposing an understanding of mental models in which not all truth conditions of a model need to be explicitly considered in working memory. In essence, Johnson-Laird et al. (1994) challenged Braine et al.'s (1994) method of calculating the number of models required to solve a logic problem. According to Johnson-Laird et al., (1994), people are able to select the models of a set of premises that are relevant to the solution of the problem, rather than considering every truth contingency of every premise at once. This allows them to avoid having to construct large numbers of models all at once, instead creating models in order of semantic relatedness.

Evans, Handley, and Buck (1998) explain mental models slightly differently: they say that “owing to limited working memory and processing capacity, however, initial representations tend to be only partially formed (p. 385).” When individuals construct a model of a premise, they begin only by considering only one of the truth conditions of the premise explicitly while understanding that there may be other possible truth conditions. In Evans et al.’s (1998) example, the initial representation of the premise “if p then q” is:

[p] *q* *(1)*

. . .

Where the brackets indicate the understanding that “p is exhausted with respect to q” (i.e., given that “p,” the only possibility for “q” is that it is true), and the “. . .” indicates an *implicit* model. The “. . .” show that there are other possible conditions under which the rule “if p then q” may be true, but they are not considered explicitly (Evans et al., 1998). Essentially, if our mental models reasoner were to speak his or her thoughts out loud, he or she would create a model from “if p then q” thus: “it is possible that both p and q are true, and p’s being true limits the possible truth values of q to “true,” and there may be other possibilities.” Under Braine et al.’s (1994) understanding of mental models, on the other hand, our participant would have to say: “it is possible that both p and q are true, and it is possible that p is false but q is true, and it is possible that p is false and q is false, and those are the only possibilities.” When you begin to add more premises, the combinations of premises that could potentially form models increases exponentially under Braine et al.’s (1994) understanding of mental models.

However, according to Evans et al. (1998), people do not consider the other possible models of the premise unless subsequent premises are inconsistent with the initial explicit model of the first premise. For example, if our reasoner is next presented with the premise “not q,” he or she will see that the possibility of both p and q being true is inconsistent with q being false, so he or she will have to move on and consider more implicit models of the initial premise to see which is consistent with “not q.” The next premise, “p is false, and q is true” is also not consistent with “not q,” but the final premise to be considered, “not p and not q” *is* consistent with “not q,” and this is the model which should be accepted and which would lead to the modus tollens inference.

When we consider Evans et al.’s (1998) theory of implicit and explicit models, it may seem it is not so easy to design a test like Braine et al. (1994) attempted, where mental logic or mental models *clearly* requires more or fewer steps than the other theory and where participants write these steps down in the process of solving the problems. In response to the confusion surrounding coding and counting inferences, we used a different testing paradigm to distinguish between mental logic and mental models. We examined choices made on a multiple-choice test constructed to contain the different predictions of mental logic and mental models.

Predictions for conditional inferences

Conditional inference problems are problems involving "if . . . then" statements (where the "if" clause is called the antecedent and the "then" clause is called the consequent). The philosophical study of formal logic shows that there are four basic forms of conditional inference arguments (Hausman, Kahane, & Tidman, 2010). These are:

- Modus Ponens (MP): This is a valid argument form (which means that if all of its premises are true, the conclusion has been proved to be necessarily true). The form: "If p, then q. p. therefore, q."
- Modus Tollens (MT): This is also a valid argument form. The form: "if p, then q. not q.

Therefore, not p."

- Affirming the Consequent fallacy (AC): This is an invalid argument form (which means that even if all the premises are true, the conclusion can still be false). The form: "if p, then q. q. Therefore, p."
- Denying the Antecedent fallacy (DA): This is also an invalid argument form. The form: "if p, then q. not p. Therefore, not q."

The differences between these forms are that while in modus ponens and denying the antecedent, the second premise gives information about the antecedent, in modus tollens and affirming the consequent, the second premise gives information about the consequent. Modus ponens and affirming the consequent both affirm part of the conditional, while modus tollens and denying the antecedent both negate part of the conditional.

Mental models theory predicts that people are likely to commit the “affirming the consequent” fallacy and are unlikely to commit the “denying the antecedent” fallacy, while mental logic theory predicts that people will make these fallacies with equal frequency (O'Brien et al., 1994; Schroyens et al., 2001). For the affirming the consequent fallacy, Mental models would proceed thus (following Evans et al.’s description of mental models): “if p then q” has the initial model

[p] *q*

. . .

The second premise “q” is consistent with this initial model, and so reasoners may conclude that “p” follows if they do not consider the implicit models of “if p then q” (which includes the model “not p and q,” which is also consistent with the second premise “q”). The denying the antecedent fallacy is easier to avoid under mental models because the second premise (“not p”) is inconsistent with the initial explicit model and so invites reasoners to continue fleshing out models to find one which is consistent. The two implicit models of the truth possibilities for “if p then q,” “not p and not q” and “not p and q” are both consistent with the second premise of denying the antecedent, not q; if people recognize this, they will avoid committing the fallacy and rightly conclude nothing follows, but if participants stop with the model claiming “not p and not q,” they may commit the denying the antecedent fallacy. Mental logic theory predicts that modus tollens inferences are more challenging than modus ponens inferences. Modus pollens require three steps because it is a basic inference rule whereas a proof of modus tollens requires six steps involving an assumed premise, and is proved by the indirect proof method (see Braine et al., 1998; Hausman et al., 2010).

According to Evans et al. (1998), “an interesting difference between the mental logic and mental model theories occurs in their treatment of the so-called fallacies of DA [denying the antecedent] and AC [affirming the consequent]. There are no rules for fallacious inferences in the mental logic systems and such inferences are assumed to reflect the operation of invited inferences (Geiss & Zwicky, 1971) through a separate pragmatic system (see Braine & O’Brien, 1991).” In Braine and O’Brien (1991), invited inferences are essentially the confusion of conditionals with biconditionals. Braine and O’Brien say that the conditional “if p then q” invites the inference “if not p, then not q,” which by the logical inference of *contraposition* (see Hausman et al., 2010) is equivalent to “if q, then p.” Combining “if p, then q” and “if q, then p” creates a biconditional. Under a biconditional, “not p” *does* imply “not q” and “q” *does* imply “p,” although these inferences would commit the denying the antecedent and affirming the consequent fallacies if we were using a conditional rather than a biconditional.

According to Braine and O’Brien (1991), people make the invited inference and treat conditionals as biconditionals based on their pragmatic experience in everyday conversation. For example, if you are told "if you mow the lawn, you will get $5," it is implied that "if you do not mow the lawn, you will not get $5" (creating a biconditional because without this clause, there would be no motivation to mow the lawn when you might still get $5 anyway). However, if we do treat conditionals as biconditionals unless we have some explicit reason not to (as Braine & O'Brien, 1998, say), we ought to commit affirming the consequent and denying the antecedent fallacies with equal frequency, because both of these are valid inferences under biconditionals. So under mental model theory, the affirming the consequent fallacy should be committed more often than the denying the antecedent fallacy, but under mental logic, the two fallacies should occur with equal frequency.

Based on the rationale provide above, predictions of the two models are hypothetically represented in Figure 1. We report first a pilot study[1] using our multiple-choice task that tested these predictions in order to find which model described our observed results.

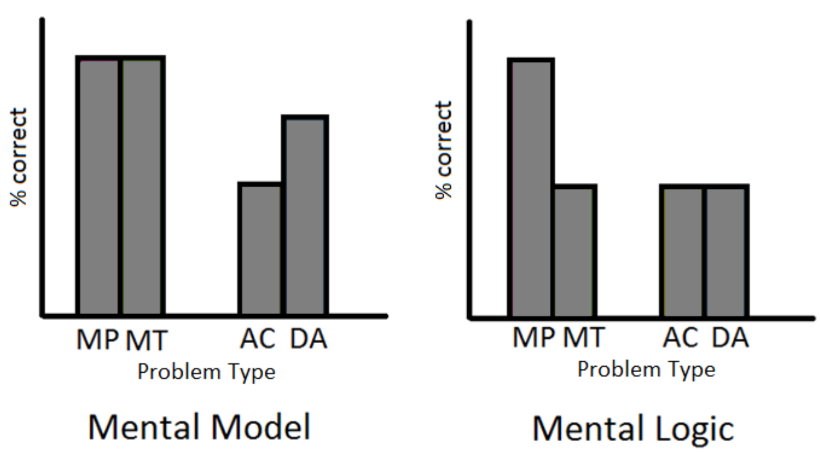

Figure 1. Predictions of mental logic and mental models on conditional inferences

Pilot experiment

We tested the different predictions of mental logic and mental model on people's performance on the types of conditional inference problems. We gave 26 participants sets of multiple choice logic problems (see an example in the Appendix) containing eight problems of each type (modus ponens, modus tollens, affirming the consequent fallacy, and denying the antecedent fallacy), and recorded performance by problem type. We expected that if participants were following mental models, they ought to commit the affirming the consequent fallacy frequently and avoid the denying the antecedent fallacy frequently, while showing no particular difference in performance between modus ponens and modus tollens inferences. If participants were following mental logic, we expected them to perform well on modus ponens inferences and poorly on modus tollens inferences, while showing no real difference between affirming the consequence and denying the antecedent.

The results were interesting and unexpected. As shown in Figure 2, participants performed well on modus ponens inferences, poorly on modus tollens inferences, committed the affirming the consequent fallacy frequently, and committed the denying the antecedent fallacy infrequently. We found significant differences between MP (M = 7.73, SD = .13) and MT (M = 2.88, SD = 1.48), $t(25) = 12.27$, $p < .001$ and between AC (M = 4.00, SD = .91) and DA (M = 6.19, SD = .57), $t(25) = 5.02$, $p < .001$.

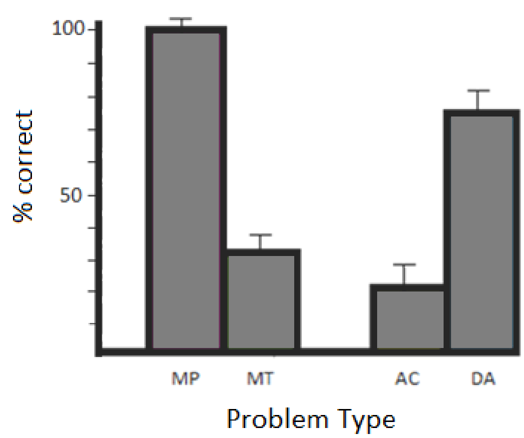

Figure 2. Percentage of problems correct.

Participants' performance on modus ponens and modus tollens fits with the mental logic prediction that participants would have more difficulty with modus tollens inferences than modus ponens inferences. However, their performance on affirming the consequent and denying the antecedent fits with the mental models prediction that participants would commit the affirming the consequent fallacy more frequently than the denying the antecedent fallacy. One possible explanation of these results is that people do not use *either* mental logic *or* mental models exclusively, but that they shift between strategies. The theory of affordances could explain this shift.

Affordances

Affordances are features of the environment which allow certain actions to be performed. These affordances interact with the abilities of an agent in that environment. The relation between the environment's affordances and the agent's abilities defines likely actions in that environment (Greeno, 1994).

The theory of affordances is typically applied to studying the *physical* environment in which people act (see, e.g., Charlotte & Uzzell, 2002; Ye, Cardwell, & Mark, 2009). However, Greeno (1994) pointed out that Gibson's theory of affordances can be applied more generally, and is open to the possibility of creating an affordance theory of behavior generally. In light of this, we examined whether the theory of affordances could be applied to the area of logical reasoning.

Particularly, we considered whether different types of conditional inferences afford different patterns of response in relation to the logical abilities of participants. One other aspect of affordance theory we would like to emphasize here is the possibility of attunement to constraints. In Greeno's (1994) description of the theory of affordances, people can become attuned to constraints in their environment. In attunement, people learn through experience to make distinctions between subtler differences in their environment which they were not aware of at first, and this affects their behavior in the environment. we would like to see if people could be attuned to constraints in logical reasoning, particularly, the constraint that conditionals are not equivalent to biconditionals.

Our main hypothesis was that people are able to make use of both mental logic and mental models as reasoning strategies. The results from the pilot experiment suggested that people use mental logic for MP and MT problems, and use mental models for AC and DA problems. It could be that because MP and MT are valid logical inferences, they afford mental logic reasoning; they can be proved by the application of rules to the premises. AC and DA do not afford the mental logic ability, however, because no inference rule will yield a fallacious conclusion. AC and DA do afford mental models, however, because the conclusions can be compared to the premises for consistency. This gives an account of logical reasoning that combines mental models and mental logic with the theory of affordances.

We tested this hypothesis in two parts. One part attempted to show that people are capable of reasoning with both mental models and mental logic, and the other part attempted to show that the theory of affordances can be applied to logical reasoning.

We experimentally manipulated which reasoning strategy participants use by giving them instructions that fit either mental models or mental logic. If there is a significant difference between the responses of the participants before and after the instruction in the direction predicted by the reasoning theory they were instructed in, it would suggest that people are capable of employing both reasoning strategies. If there is no significant difference, either people are not capable of shifting strategies, or my instructions were not sufficient to cause them to shift strategies.

The instructions in mental logic consisted of an explanation of the inference rules of Modus Ponens and Modus Tollens (See Appendix). The instructions in mental models condition consisted of the truth table for conditionals, which is analogous to the creation of models for conditionals. One further way to test to see which reasoning strategy participants are using was to present them with a valid proof of MT, an invalid proof of AC, and an invalid proof of DA and have them rate how convincing they found the proofs (see Appendix). Best (2005) used a similar proof-conviction task with correct and bogus proofs in the context of mental logic theory. We adapted the paradigm here. We predicted that participants given instructions about mental logic will be more convinced by the proof of MT and less convinced by the other proofs than participants instructed in mental models because the participants instructed in mental logic are using a rules-based reasoning strategy and will be more competent at evaluating rules-based proofs and bogus proofs of conditional inferences. Participants instructed in mental models will have experience with constructing truth tables, but not with rules-based strategies, and therefore will be neutral to the rules based proofs.

We attempted to show that the theory of affordances can explain logical reasoning by using counterexamples to attune participants to constraints in logical reasoning. When reasoning, participants do take counterexamples into account (Schroyens & Schaeken, 2008). These counterexamples will be an example of perceptual learning as it is understood by the theory of affordances. The main reason why people commit logical fallacies is that we tend to confuse conditionals with biconditionals (e.g., Braine & O’Brien, 1991). It may be possible to attune people to the distinction between conditional and biconditional statements by presenting them with an example which demonstrates that they cannot treat conditionals as biconditionals. The examples use items and categories people have experience with, rather than abstract words, to show the fallacious conclusions as fallacious. If the theory of affordances can be applied to logical reasoning, then after being presented with counterexamples, participants' performance on logic problems should improve compared to a group given no counterexamples.

Experiment 1

Method

Participants and Materials. After receiving IRB approval, we tested 32 undergraduates who received course credit for their participation. Materials included a stopwatch, packets of 32 logic problems (one problem per page) and instructions for each participant and answer sheets for each participant.

Design and Procedure. Problem type was an independent variable with four levels (MP, MT, AC, DA inferences), and participants’ responses were the dependent variable (multiple choice response with four possible answers). Problem type order was randomized then counterbalanced.

After participants received consent forms, the packets of logic problems, and answer sheets, the Experimenter read aloud the instructions. Participants selected answers for each of the 32 problems every 15 seconds. The experimenter used a stopwatch to call time for participants to turn the page to the next problem. Debriefing forms were handed out at the end of testing.

Results

Figure 3 shows the mean scores (out of 8) for each problem type. A one-way ANOVA found that there was a significant effect of question type $F(3, 36) = 29.58$, $p < .001$. Pairwise comparisons between the problem types showed that participants performed significantly better on MP problems than all other problem types, *ps* < .001. Performance on MT was significantly better than on AC, $p = .002$, and not significantly different from performance on DA. Performance on DA was significantly greater than performance on AC, $p < .001$.

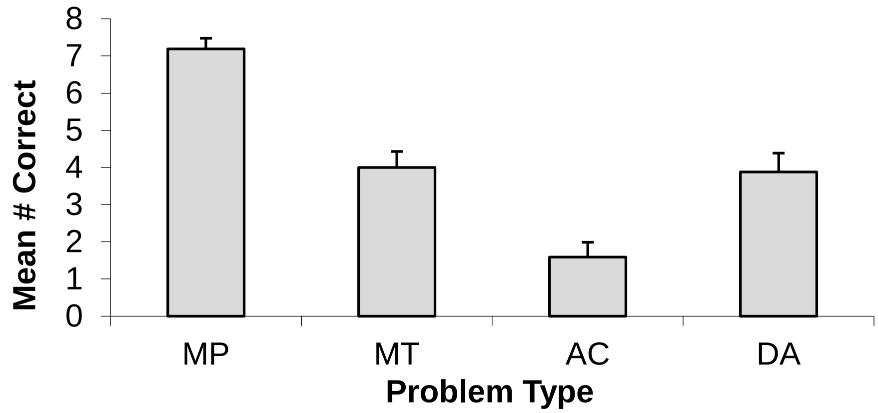

Figure 3. Mean accuracy by problem type.

Discussion

The results of this experiment replicate the results of our pilot study in that participants performed significantly better on MP problems than MT problems, and significantly better on DA problems than AC problems. These findings support aspects of the predictions of both mental logic and mental models.

Experiment 2

We manipulated training participants received; they either received or did not receive a counterexample (see Appendix).

Method

Participants and materials. Participants were 36 undergraduates who received either course credit or $10. We constructed 34 logic problems, made two packets of 16 logic problems (the other 2 problems were our counterexamples), and a packet of proofs, and provided answer sheets. We used a stopwatch to time the tasks.

Design and Procedure. The independent variables were instruction type (mental models or mental logic), counterexamples (either counterexamples or no counterexamples), proof type (MT, AC, and DA), and problem type (MP, MT, AC, DA). The dependent variable was responses to multiple choice problems and convincedness rating of the proofs. We also collected any comments people may have left about the proofs.

After participants signed the consent form, they were randomly assigned to the no-instructions group and the instruction group. Within the instructions group, we assigned participants to either a mental models instruction or a mental logic instruction. Within the no-instructions group, participants were randomly assigned to either a counterexample group or a no-counterexample group. All participants began by completing 16 logic problems. These included 4 problems of each conditional inference type, presented randomly with respect to type and counterbalanced with the second set of 16 problems. We used a time constraint of 15 seconds per problem, as in Experiment 1.

After completing the first16 problems, the mental models group received instructions from the experimenter about how to construct truth tables. Following the instructions, participants received another 16 logic problems. The no-instruction group completed an additional 16 logic problems after their first 16.

The mental logic instruction group received instructions about how to apply formal logical rules to premises. The experimenter explained the two valid conditional inference rules, modus ponens and modus tollens, in terms of the step-by-step logical proof of the inferences. Following the instructions, participants received another 16 logic problems.

Those in the counterexample group received two problems, one AC and one DA. The content of these problems were concrete items (as opposed to abstract ideas). Following these two problems, participants received another 16 problems. There was no break in format or time to indicate the difference between the counterexample problems and the others.

Participants in the no-counterexample group followed the same procedure as those in the no counterexample group, except that the two problems they received in the middle were identical to the surrounding problems: They had no counterexample.

After participants completed all of the logic problems, they received a proof-conviction task. Participants received (in randomized order) a correct proof of MT, a bogus proof of AC, and a bogus proof of DA (the proofs are bogus because there is no proof of a fallacy). Participants were asked to rate how convincing they find these proofs to be of the conclusion. To end the testing, participants received a debriefing form.

Results

Table 1 shows the means for the frequencies of correct responses on the problem types in the four conditions. As expected, participants performed very well on MP inferences, and, consistent with our pilot study, poorly on AC inferences. A 4 problem type (MP, MT, AC, DA), x 2 time (before and after manipulation) x 4 group (mental logic instruction, mental model instruction, counterexample, and no counterexample) mixed ANOVA found a main effect of problem type $F(3, 42) = 14.45$, $p < .001$. There was also a three way interaction between problem type, time, and group $F(9, 42) = 4.00$, $p < .001$. A t-test between the means of each problem type before and after manipulation showed a nearly significant difference between the before and after scores of the no counterexample condition on MT problems, $t(11) = 2.159$, $p = .054$, with performance on MT problems decreasing. In the mental logic instruction condition, there was a nearly significant decrease in performance on AC problems, $t(8) = 2.294$, $p = .051$, and a significant decrease in performance on MT problems, $t(8) = -.3.35$, $p < .01$.

Table 1

Means and Standard Deviations for Correct Responses (Out of 4).

Problem Type	Mental Logic				Mental Models			
	Before		After		Before		After	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
MP	3.89	.33	4.00	.000	4.00	.000	3.88	.35
MT	1.89	1.54	3.78	.67	1.25	1.17	1.50	1.51
AC	2.33	1.66	1.22	1.56	2.75	1.58	2.38	1.69
DA	2.00	1.80	1.67	1.80	3.36	.92	3.00	1.85
	Counterexample				No Counterexample			
	Before		After		Before		After	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
MP	3.77	.17	3.54	.21	3.58	.18	3.33	.21
MT	2.31	.40	2.69	.36	3.08	.41	2.25	.38
AC	2.39	.455	2.154	.417	2.17	.474	2.583	.434
DA	2.23	.410	2.39	.422	2.58	.427	2.58	.439

For the proof-convincedness ratings, we compared participants' rating (from 1 being not convinced to 5 being convinced) to a score of 3 (which was neutral). In the mental logic instruction condition, participants rated significantly higher than three for MT proofs $t(8) = 3.77$, $p < .05$, but showed no significant difference from a rating of 3 for the AC or DA proofs. In the mental model instruction condition, participants showed no significant difference from a rating of 3 for the MT proof, but ratings were significantly lower than three for the AC proof $t(7) = -5.61$, $p < .01$ and for the DA proof $t(7) = -4.33$, $p < .01$. In the counterexample condition, participants rated the MT proof significantly higher than 3, $t(12) = 2.79$, $p < .02$. In the no counterexample condition, participants' ratings were not significantly different from 3 in any of the proofs.

Discussion

In the control group (NC), there was a nearly significant drop in performance on MT problems, but no significant changes in the performance of any of the other problem types. We have reason to suspect that MT inferences are difficult by mental logic theory because they require more steps to complete than MP inferences. It has been suggested by mental model theorists (e.g., Evans et al., 1998) that the difficulty of MT problems could be explained by the fact that solving an MT problem requires reasoners to develop more models than in MP inferences. Perhaps the unexpected drop in performance in MT in the control group could be explained as a fatigue effect: as participants complete the 32-34 problems, they become fatigued and MT seems to be the most sensitive to this fatigue. MT, unlike AC and DA, is not a fallacy which people reason about incorrectly in a consistent way (and perhaps committing these fallacies is less draining on cognitive resources than correctly answering MT).

At first glance, it seems that there is no major effect of instructions on performance of the different problem types. However, when we consider the patterns of response between the before and after conditions in relation to the trend in the NC condition, it looks as though mental model instruction might have had some effect on mitigating fatigue in MT problems, while ML *improved* performance on MT problems. The counterexample condition also seemed to mitigate the effect of fatigue in MT problems; in the counterexample condition, participants improved in their performance on MT, although this was not significant. Thus, we have some support that counterexamples help to distinguish between conditionals and biconditionals, as found in the study of intentions by Juhos, Quelhas, and Byrne (2015).

In the proofs, it seems that the mental logic instructions helped participants correctly identify the proof of modus tollens as correct but did not help them or hurt them in identifying the proofs of affirming the consequent and denying the antecedent as incorrect. The mental model instructions helped participants identify affirming the consequent and denying the antecedent as incorrect, but did not help or hurt participants in recognizing the modus tollens proof as correct. The counterexample helped participants identify the MT proof as correct, but did not help with the AC or DA proofs. The means on the proof convincedness ratings for the no counterexample conditions, as expected, were not significantly different from a neutral response.

General Discussion

The results of Experiment 2 are interesting because instruction seemed to have an effect on MT problems (which appeared most sensitive to manipulation), but not in the way we expected. According to the mental logic prediction, people ought to perform well with MP, poorly with MT, and equally well on the fallacies. However, after the manipulation intended to encourage mental logic thinking, participants instead *improved* in MT problems, and performance decreased on AC problems. One possible explanation of this is that the instructions were ineffective; it could be that once participants had learned how MT problems are solved through the mental logic method of applying inferences rules to sets of premises, participants just memorized the conclusion that is supposed to follow from premises that look like they fit MT. Instead of looking at the premises, and working through the inference rules step-by-step to reach a conclusion, participants may have instead tried to recognize the premises as being similar to the ones that the MT inference works for, and then concluding with what MT would say. Some support for this interpretation is found in participants' decreased performance on AC: AC problems and MT problems share surface similarities because those premises both provide participants with information about the consequent of the conditional. If participants were just trying to apply the conclusion of MT wherever they met premises that looked like they fit MT, participants should have done well on MT (which they did) and poorly on AC (which they did) because AC looks similar to MT problems but it is a fallacy and so the correct answer for an MT problem will be an incorrect answer for an AC problem.

Participants’ performance on the proof-convincedness ratings also lends support to this interpretation. Our original prediction was that participants who received mental logic instruction should have rated all of the proofs correctly, because mental logic operates by inference rules, which are analogous to the steps in a formal proof. If this were correct, participants’ responses would have been significantly different from a rating of 3 (neutral, not convinced or unconvinced) for all three proofs, and participants would have rated the valid MT proof as a 5 (convinced) and the other two invalid proofs as a 1 (not convinced). Participants only showed a significant difference from a rating of 3 for the MT proof. This suggests that participants were able to recognize the MT proof from the mental logic instructions and identified it as correct, but because they did not apply inference rules in their analysis of the other proofs, participants were not able to identify them as incorrect.

The mental model instructions may have had some effect in mitigating fatigue on MT problems, but there were no significant changes in performance on any of the problem types before and after the instructions. Participants instructed in mental models showed a different pattern of response of the proof-convincedness ratings than any of the other conditions. Participants rated the two invalid proofs, AC and DA, significantly lower than a rating of 3, but the rating of the valid MT proof was not significantly different than 3. It is possible that the truth table instructions given in the mental model instruction helped participants identify the incorrect inferences as incorrect: in the truth table for the conditional, participants can see that when the consequent is true, the antecedent can be true or false, and when the antecedent is false, the consequent can be true or false. There is no evidence of this understanding in the participants' responses on the multiple choice questions, but perhaps without the time constraint in the multiple choice section, participants were able to use a truth table analysis to evaluate the proofs.

In the no counterexample condition in the proof-convincedness rating section there were no significant differences from a rating of 3. This makes sense because no participants in this condition received any manipulation. The counterexamples seemed to help people identify the MT proof as correct, but did not help with the AC and DA proofs. we had expected that if the counterexample helped with the proofs at all, it would have helped with identifying the AC and DA proofs as incorrect (because the concrete counterexamples themselves were AC and DA problems), so it is interesting that the counterexample improved performance on MT instead. we am not exactly sure why that happened, but it seemed that the counterexamples did have an effect on people's reasoning with the proofs.

Although the counterexample condition did not show the attunement effect we was looking for, and the instructions did not influence participants' responses in the second half of the problem set and on the proofs in the way we had anticipated, both manipulations did seem to have an effect on people's performance. Both instructions seemed to have an effect in mitigating fatigue on MT problems, and the mental logic in fact reversed this trend and improved people's performance. This suggests that people's performance in logical reasoning is sensitive to instruction, albeit not in the way we had predicted. Future research can investigate using other methods of instruction to attempt influencing people to use either mental models or mental logic. This research is encouraging because it supports the idea that the way people reason logically can be experimentally manipulated.

The fatigue effect we observed in participants' performance on MT problems was also interesting and unexpected. We had not observed the fatigue effect in experiment 1 and the pilot experiment because we had not compared a before-and-after condition. The fatigue effect is interesting because it suggests that people have the capacity to perform MT inferences correctly, but that doing so requires more cognitive resources than completing the other types of problems (correctly or incorrectly). Also, some of the instructions of the present experiment seemed to have an effect in mitigating or reversing the fatigue effect in MT reasoning. This effect would be an interesting topic for future research.

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Footnote

1. Natalia Maurer and Amar Sarkar collaborated with this pilot study.

Appendix

Example of a multiple choice item on the logic task:

If there is a car, then there is not a marble. There is not a car.
Therefore,
A. There is a car.
B. There is not a marble.
C. There is a marble.
D. There is not enough information to decide.

Proof-conviction task. Each participant will receive a packet of three conditional inferences followed by a proof of the conclusion. There is a scale from 1-5 of how convincing the participant felt the proof to be (1 = not convinced, 2 = skeptical, 3 = neutral, 4 = confident, 5 = convinced) and an area to write comments about why or why not the proof was convincing if the participant would like to comment.

Proof of MT:

1. If p, then q
2. Not q
3. Assume that p.
4. Therefore q (1, 3)
5. Therefore q and not q (2, 4).
6. Therefore not p. The assumption that p leads to a contradiction, so p is impossible.

Bogus proof of AC:

1. If p, then q.
2. q.
3. Assume that p.
4. Therefore q (1, 3).
5. Therefore q and q (2, 4).
6. Therefore q. The assumption that p is consistent with premise 2, so we conclude that p is the case.

Bogus proof of DA:

1. If p, then q.
2. not p.
3. Assume that q.
4. Therefore p (1, 3).
5. Therefore p and not p (2, 4).
6. Therefore not q. The assumption that q leads to a contradiction, so q is impossible.

Mental model Instructions: "Now that we have completed those 16 logic problems, we are going to learn about how to construct truth tables to assist you with the next 16 problems. A truth table shows what would be necessary for a statement to be true or to be false. For example, the statement “if p, then q” is true if both “p” is true and “q” is true. This is the standard truth condition for “if p, then q.” “If p then q” is false if “p” is true and “q” is false.

p	q	If p then q
T	T	T
T	F	F
F	T	T
F	F	T

When the standard truth condition (which is the first line of the truth table) and the second premise agree with each other it is likely that the conclusion of the first premise is true.”

Mental logic instructions: "Now that we have completed those 16 problems, we are going to learn about how to apply inference rules to premises in order to help you with the next 16. The first two sentences you saw in the problems are the premises, and the answer choices are the possible conclusions. Inference rules are rules that you can apply to premises of the right form in order to reach a conclusion. For example, if we have the sentences “if p, then q” and “p,” the inference rule called modus ponens lets us conclude q. If we have premises of the form “if p then q” and “not q,” the inference rule modus tollens allows us to conclude “not p.” Here is how MT works: assume for a minute that p. If p, then q (my modus ponens). Therefore q. But then it looks like we have both q and not q (from our original premises).But that’s impossible, so we must have been wrong to assume p in the first place, and therefore not p.”

Counter-example problems:

DA:

1. If Max is French, then Max is a human being.
2. Max is not French.

Therefore, Max is not a human being?

AC:

1. If Max has a cold, then Max has a runny nose.
2. Max has a runny nose.

Therefore Max has a cold?