Isomorphism between Strong Fuzzy Relational Graphs based on k-formulae

L. Wendling, J. Desachy

Université Paul Sabatier - IRIT
118, route de Narbonne
31062 Toulouse Cedex, FRANCE (33)
email: wendling@irit.fr

Abstract. A new graph matching approach based on 1D information is presented. Each node of the matched graphs represents a fuzzy region (fuzzy segmentation step). Each couple of nodes is linked by a relational histogram which can be assumed to the attraction of two regions following a set of directions. This attraction is computed by a continuous function, depending on the distance of the matched objects. Each case of the histogram corresponds to a particular direction. Then, relational graph computed from strong scenes are matched.

1 Introduction

In this paper a new pattern recognition system, based on sample images, is presented. An object to be recognized is described by one scene or by a set of little images. Then, a fuzzy segmentation step is performed in order to split an image into a fuzzy partition which consists in a set of fuzzy regions. For each fuzzy region both topological and relational features can be computed (methods based on atomic regions and hierarchical trees have also been proposed in previous papers [6] [20]). In the proposed approach each couple of regions of the image is linked by a histogram of forces. So, an image is defined as a relational graph. The same process is applied to another images to match. Then, a new approach of matching based on strong relational graph is defined.

2 Pattern Recognition System

Our pattern recognition system (figure 1) consists in five parts.

1. Input data (grey levels or RGB color images).
2. Fuzzy segmentation: To split the image into a set of fuzzy regions.
3. Relations, features of regions.
4. A data base composed of typical relational graphs to match.
5. Decision part to give a distance measure between two matched scenes.

Each part performs a particular process to split the image into a relational graph and to take a decision: Scenes are similar or not.

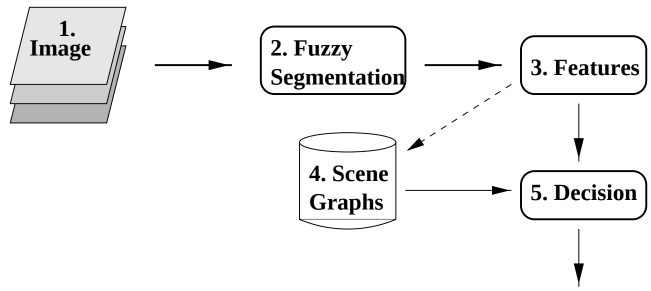

Fig. 1. System Description.

3 Fuzzy Region Definition

First part of the proposed system performs a fuzzy segmentation from the input image. The goal of fuzzy segmentation method is to manage with imprecise boundaries [18] [9] and to allow to a pixel to belong more ore less to a given region. Crisp segmentation [13] is a particular case of fuzzy segmentation and can be achieved with a max criteria applied to a fuzzy partition [3].

Most of the fuzzy segmentation methods are based on the definition of fuzzy partitions using fuzzy c-means algorithms [3]. Nevertheless these approaches give noisy and non-totally coherent results [8] [4].

A recent method proposed by Krishnapuram [10] seems to overcome these problems. His method is independent of the interclass distance and is based on a "good" membership profile [21]. The initialisation of the defined algorithm is fundamental to achieve partitions. Barny & al. [1] have shown that the use of c-means algorithm to define the input partition can fail by defining indentical clusters. This problem can be solve if possibilistic c-means algorithm [12] is used with a number of classes equal to 1.

First any point of the result cluster is set at 1. Then the most favourable cluster (defined from both validity criteria and partitions variations between two steps) is carried out. If such a class is found, points of cluster data which most verify this achieved cluster are removed from the image partition (clusters RGB in a color image, for example). Processing is runned again until the achievement of unconsistent clusters (too small for example) is performed. Currently clusters validity algorithms have generally high processing time with sometimes unconsistent results [8]. Moreover, it exists no mathematic models to define what is a "good" partition.

In the present system, this algorithm has been applied with a level cut criteria to decrease processing time (0.8s on a 100 MHz SUN SPARC 4). Then a partition composed of fuzzy clusters is achieved.

A fuzzy region is defined as being a set of connected pixels with a non-zero membership value. At this step, the system has defined the set of fuzzy regions (nodes of the graph). Then, these regions can linked with relational features to define a relational graph.

4 Fuzzy Relational Graph

First, each region is assumed to its centroid. Then, we can compute the 1D relation which links each couple of nodes.

In previous works, features, computed on pixels [16] or level-cuts [5] [7], are often used to distinguish objects. The main problem of these methods is to define the most significant features which depend on the application.

In this paper, a new relational matching based on histogram of angles and forces notion is proposed. Histogram of forces is a generalisation of Miyajima and Ralescu histogram of angles [15] with isotrop segments.

Let A and B two objects and θ a direction. The histogram value assumed to θ consists in a Riemann sum of combination of segments of A and B. An object is composed by a set of parallel segments (one pixel height) following a direction. The function apply between a segment of a region A and a segment of a region B, bear by the same straight line, takes into account the distance notion: The farer the objects, the lesser the value of the linking continuous function is. This process can be assumed to be the projection of the information onto an one dimension space in regard to both matched regions.

This approach allows to have a low processing time and to manage with non-disjoint and fuzzy objects. Such a function is entirely detailed in [14].

5 Matching between Strong Graphs

5.1 Graph Structure

It is well known that graph isomorphism problem is not deterministic (except for special kinds of graphs). The present approach manages with a particular class of graphs: Graphs with strong structure. The structure is achieved after computing histogram of forces between regions using their centroid.

In a graph G, an edge between two nodes s and t is single. A set of values is beared by each edge. Edges can be double, but as information, given by histogram of forces, is symmetric a preorder is set.

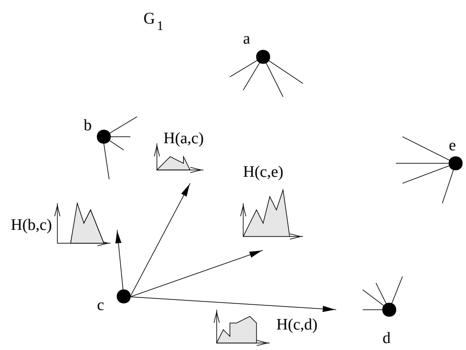

Fig. 2. Relational Graph.

5.2 Similarity Ratio

Let A be a histogram of forces linking vertex s to t in a graph G_1 and B be the histogram of forces linking vertex u to v in a graph G_2. A and B are superimposed in order to compute a distance between them from their common parts (figure 3).

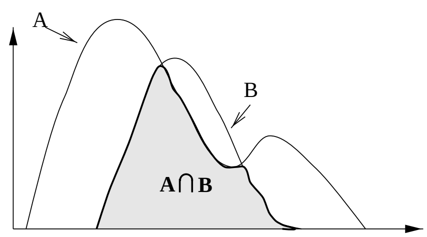

Fig. 3. Histogram Superimposition.

In the present application, a similarity ratio has been chosen to match A and B. It is obvious that other distance measures can be used. Nevertheless, the calculus of a similarity ratio is low processing time and takes into account all the histogram information. Let ν be the number of steps of each histogram, i.e. the number of digitized directions. The cardinal of intersection of histogram A and B is given by:

$$|A \cap B| = \sum_{i=1,\nu} min(A[i], B[i])$$

$|A|$ represents the cardinal of the histogram A. By definition, A and B can be null if nodes (fuzzy regions) s, t, u and v exist (that's due to the positive function applied). Then, the similarity ratio is computed as follows:

$$S(A, B) = \frac{|A \cap B|}{\max(|A|, |B|)}$$

5.3 k-formula Computation

Strong graphs, i.e. rigid structures, are taken into account.

Let θ be an orientation and let s and t be two nodes of a graph G_1. Let P_t and P_s be the respective projections of s and t following a directional straight line D_θ defined with a θ angle rotation from the frame image. If P_t is lower than P_s on D_θ (figure 4.b), an edge from t to s, denoted $t \rightarrow_{G_1} s$, is carried out. Vertice are sorted (quick sort method) following an orientation θ to construct the associated k-formula.

This step is carried out for all the nodes of the Graph G_1. The set of k-formulas which defined the graph G_1, following the direction θ, is so performed.

The same processing is carried out for all the nodes of graph G_2 and following a given direction. Then, a set of k-formulas [2] has been computed from each graph (figure 4.a). At last, the k-formulas of graph G_1 are matched with the K-formulas of graph G_2.

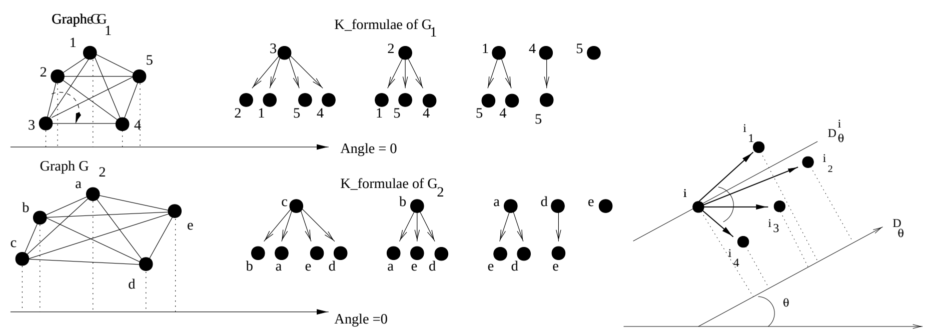

Fig. 4. k-formulae definition (projection).

The final recognition rate is given either by the mean similarity ratio following the optimal direction (which corresponds to a histogram shift) or by the minimum similarity ratio following the optimal direction.

The choice of the decision criteria (minimal or mean) can be brought by the number of nodes and depends to the kind of the current application. For example, to take a minimum criteria with graph belonging a large number of nodes can induce a drowning phenomena.

An application, of such an approach, to the case of real scenes matching is presented in section 6.

5.4 Matching Algorithm

The processing performed by our pattern recognition system can be summarized as follows:

```
Match(G1, G2)
Input: Two Graphs.
Output: Recognition Rate.
{
    Fuzzy Segmentation
    Regions Localization and Graph definition
    G1 and G2 Histogram of forces
    k-formulas definition of G1 (quicksort)
    Similarity = 0
    For any direction θ (histogram digitization) Do
    {
        k-formulas definition of G2 (quicksort)
        λ ⇐ Matching between k-formulas of G1 and G2
        Similarity ⇐ max(Similarity,λ)
    }
}
```

Fig. 5. Matching.

5.5 Complexity

The case of any k-formulae for a given graph is processed. Given an angle θ, comparisons between k-formulae of graphs G_1 and G_2 are carried out. This processing is performed for any θ and the recognition rate is set to the maximal similarity ratio.

The maximal complexity associated to the matching is in $\mathcal{O}(n^2)$ time (with $n = |G_1| = |G_2|$).

Nevertheless complexity of the method depends on the k-formulae definition. If a quicksort is used, a K-formulas with n_1 sons is build in $\mathcal{O}(n_1 \ln n_1)$ time. So, the building of all the k-formulas of a graph G, with $|G| = n+1$ needs $f = \sum_{i=1}^{n} i \ln i$ operations. Then, let us put down the following proposition:

Proposition 1. $\forall n \in \mathbf{N}^{\star}$, we have:

$$\frac{1}{2}\left(n^2 \ln n - \frac{n^2}{2} - \frac{1}{2}\right) \leq \sum_{i=1}^{n} i \ln i \leq \frac{1}{2}\left((n+1)^2 \ln(n+1) - \frac{(n+1)^2}{2} - \frac{1}{2}\right)$$

A simple demonstration by recurrence can be used to check these two inegalities. The function f is strictly increasing (and continuous). The boundaries functions have been defined by the integration of f following the rectangle method (area of f minored and majored by both minimal and maximal rectangle integration functions).

Finally, it is easy to deduce that the maximal complexity of the proposed approach is in $\mathcal{O}(n^2 \ln n)$ time.

6 Application with Color Scenes

An application of the previous described method is given now. It consists in matching two RGB scenes supposed strong.

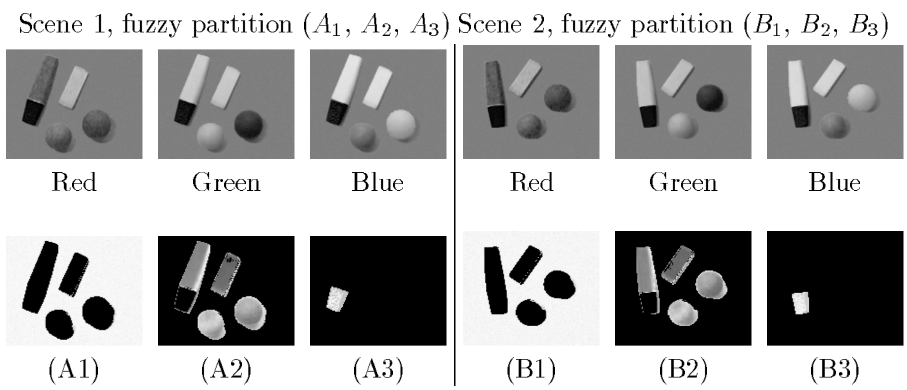

Fig. 6. Matched scenes.

Both acquisition height and orientation of images are different.

Two methods have been applied. First, an isomorphism search between graph G_1 (defined by the five regions included in clusters A_2 and A_3) and graph G_2 (clusters B_2 and B_3) is performed. For each couple of regions a histogram of forces is computed. The digitization step is equal to $1/256$. Using a lower step, value variation of the final rate is under 10^{-4} order.

Figures (i) and (ii) (next page) show similarity ratio variation from angle rotation in $[-\pi, \pi]$ interval.

A mean similarity rate of 89.84%, between the two scenes, is reached with a 28.13 degrees shift (case (i)).

Of course, if the number of regions is important then similarity ratio should be high even if a region is bad. So, it can be interesting to compute the minimal similarity ratio (following optimal angle) (case (ii)). In the present example, it reached 78.47%.

These curves (i) and (ii) show that an A^* heuristic can be useful to decrease processing time. In the present approach an A^* heuristic has been defined to find the most characteristic directed acyclic graph. This algorithm is based on distance minimal from an edge of graph G_1 and any of graph G_2 (up to a translation in the matched histograms of forces to find the best probable direction) and by building G_2 with an optimal cost.

The application of such a heuristic gives the same similarity ratio achieved result, but fastly. Moreover, this heuristic can be also applied to the case of strong subgraphs.

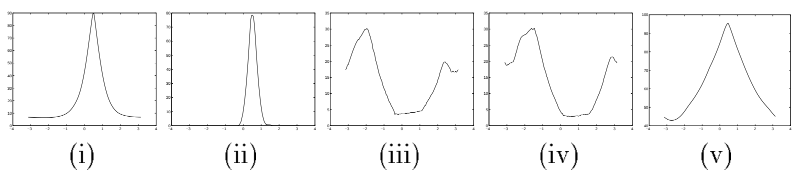

Fig. 7. (i), (ii), (v): Similarity ratio variation; (iii), (iv): Histogram of forces between clusters.

In a second approach, each cluster is assumed to be a single region. The histogram of forces linking the second and the third cluster on figures (iii) - A_2 with A_3 - and (iv) - B_2 with B_3 -. In this case, a maximal similarity ratio of 95.44% is reached for a 25.31 degrees shift.

The acquizition image height is fairly different and histogram normalized (to avoid the zoom factor) has given a weak improvement of 0.3%. Similarity ratio variations are given in figure (v).

This other approach is useful because it is not necessary to define each region (set of connected pixels with no zero membership value) contained in the clusters of a fuzzy partition. As a consequence, the number of matches is lower than in the previous method (ten time lower).

Such an approach can be assumed to be a distance measure between two models.

Hence this method is less discriminant than the previous one which takes into account any region of a partition. Then, relational information are lost. A better ratio is reached but it does not give a better idea of the reality. This result is relative to the partition quality. In the present approach the main information is located in the second cluster (B_2 and A_2).

If a cluster validity criteria is taken, which limits the number of regions per cluster and selects only the most dense part of the cluster, it is obvious to think that the number of clusters per partition should be increased and the number of regions per partition should be decreased. In this case, the second approach becomes more interesting because relational information is rather located in an inter-partition scheme.

It is possible to apply this kind of approach even with noised areas (the function used to defined histogram of forces take into account disjoint information). Moreover, in the previous approach, noise must be removed to keep "dense" areas, which induces lost of information.

The proposed approach has been applied with success on more complex scenes (up to twenty five nodes and seven clusters). When we take into account two small regions the present approach fail down. That is not the case when we work only with clusters.

Currently, our aim is to consider the search of maximal subgraph to improve the matching.

7 Conclusion

In this paper, a method of pattern recognition based on relational graphs and k-formulae definition has been proposed. This approach, which has a low processing time, has given interesting results using objects with strong structure.

Currently, the present algorithm is applied to more complex scenes (about a hundrend of nodes) and we try to extend the present approach to the case of subgraph matching in order to take into account the problem of small regions.

Nevertheless, the present approach is limited on a particular class of graphs. We try to generalize our approach to manage with non-totally ordonned graph. Our aim is to define a progressive algorithm managing with the case of strong structure to general structure.

References

1. M. Barni, V. Cappellini and A. Mecocci, *Comments on "A Possibilistic Approach to Clustering"*, IEEE Transactions on Fuzzy Systems, vol 4:3, 1996, pp 393-396
2. A.T. Berztiss, *A Backtrack Procedure for Isomorphism of Directed Graphs*, Journal of the Association for Computing Machinery, vol 20:3, 1973, pp 365-377
3. J.C. Bezdek, *Pattern Recognition with Fuzzy Objective Function Algorithms*, Plenum Press, New York, 1981
4. R.N. Dave, *Characterization and Detection of Noise in Clustering*, Pattern Recognition Letters, vol 12, 1991, pp 657-664
5. D. Dubois and M.C. Jaulent, *A General Approach to Parameter Evaluation in Fuzzy Digital Pictures*, Pattern Recognition Letters, vol 6, 1987, pp 251-261
6. P. Dhérété, L. Wendling and J. Desachy, *Fuzzy Segmentation and Astronomical Images Interpretation*, ICIAP'95-IAPR, Springer-Verlag, Lecture Notes in Computer Science 974, 1995, pp 21-27
7. R. Krishnapuram, J.M. Keller and Y. Ma, *Quantitative Analysis of Properties and Spatial Relations of Fuzzy Image Regions*, IEEE Transactions on Fuzzy Systems, vol 1:3, 1993, pp 222-233
8. R. Krishnapuram, *Generation of Membership Functions via Possibilistic Clustering*, 3rd IEEE Conference on Fuzzy System, vol 3, 1994, pp 902-908
9. J.M. Keller and C.L. Carpenter, *Image Segmentation in the Presence of Uncertainty*, International Journal of Intelligent System, vol 5:2, 1990, pp 193-208
10. R. Krishnapuram, *Fuzzy Clustering Methods in Computer Vision*, EUFIT'93, 1993, pp 720-730
11. R. Krishnapuram and J.M. Keller, *A Possibilistic Approach to Clustering*, IEEE Transactions on Fuzzy Systems, vol 1:2, 1993, pp 98-110
12. R. Krishnapuram and J.M. Keller, *The Possibilistic C-Means Algorithm: Insights and Recommendations*, IEEE Transactions on Fuzzy Systems, vol 4:3, 1996, pp 385-393
13. S. L. Horowitz and T. Pavlidis, *Picture Segmentation by a Directed Split and Merge Procedure*, 2^{nd} International Conference on Pattern Recognition, 1974, pp 424-433
14. P. Matsakis, L. Wendling and J. Desachy, *Représentation de la position relative d'objets 2D au moyen d'un histogramme de forces*, revue Traitement du Signal, accepté, 1997.
15. K. Miyajima and A. Ralescu, *Spatial Organization in 2D Segmented Images: Representation and Recognition of Primitive Spatial Relations*, Fuzzy Sets and Systems, vol. 65, 1994, pp 225-236
16. A. Rosenfeld, *Fuzzy Geometry: An Overview*, First IEEE Conference on Fuzzy Systems, 1992, pp 113-118
17. A. Rosenfeld, R. Hummel and S. Zucker, *Scene labelling by relaxation operations*,IEEE Transactions on Systems, Man and Cybernetics, SMC-6, 1976, pp 420-433
18. E.H. Ruspini, *A New Approach to Clustering*, Inform. and Control, vol 15, 1969, pp 22-32
19. L. Wendling, *Segmentation floue appliquée à la reconnaissance d'objets dans les images numériques*, Ph.D. thesis, Université Paul Sabatier, Toulouse, 1997
20. L. Wendling, A. Pariès and J. Desachy, *Pattern Recognition by Splitting Images into Trees of Fuzzy Regions*, International Journal of Intelligent Data Analysis, 1997
21. H. J. Zimmerman and P. Zysno, *Quantifying Vagueness in Decision Models*, European J. Operational Res., vol 22, 1985, pp 148-158