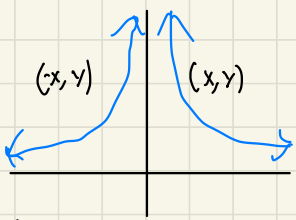
Even & odd functions

Due dates

HW1 → 9/11
HW2 → Sunday
Test1 →
2 hours for Test.
HW3 → next Sunday 9/18
open book/open note

§1.3 Even/odd functions

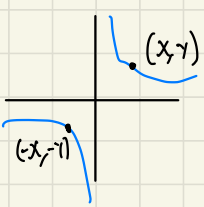
Last time



Symm. w.r.t. the y-axis

$f(x) = y$
$f(-x) = y$ > Same
$f(x) = f(-x)$ def of Even

Def even func: A func is even if $f(-x) = f(x)$ for all $x \in \text{dom}(f)$



$f(x) = y$
$f(-x) = -y$
$-f(-x) = y$
$f(x) = -f(x)$ def of odd

Def odd func: f is an odd func if $f(x) = -f(x)$ for all $x \in \text{dom}(f)$

Ex $f(x) = x^3$ is odd
b.c. $f(-x) = (-x)^3 = -x^3 = -f(x)$

Ex

$f(x) = x^3 - 2x$

Soln: $f(-x) = (-x)^3 - 2(-x)$

$= -x^3 + 2x$

$= -(x^3 - 2x)$

$= -f(x)$

therefore f is odd

Ex

$f(x) = x^4 + 2x^2$

$f(-x) = (-x)^4 + 2(-x)^2$

$= x^4 + 2x^2$

$= f(x)$

therefore f is even

Ex

$f(x) = x^3 + x^2$

$f(-x) = (-x)^3 + (-x)^2$

$= -x^3 + x^2$

$\neq -f(x)$

$\neq f(x)$

therefore f is neither odd nor even

Difference Quotient

$$\frac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

Ex

$f(x) = 2x^2 - x + 1$

D.Q. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2(x+h)^2 - (x+h) + 1 - (2x^2 - x + 1)}{h}$

$= \dfrac{2(x^2 + 2xh + h^2) - x - h + 1 - 2x^2 + x - 1}{h}$

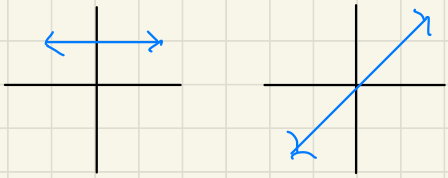
$= \dfrac{2x^2 + 4xh + 2h^2 - h - 2x^2}{h} = \dfrac{h(4x + 2h - 1)}{h}$

$= 4x + 2h - 1$

Linear Functions

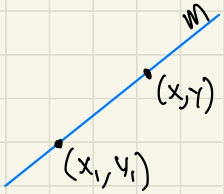
§1.4 linear functions

e.g. $f(x)=c$, $f(x)=ax+b$



Def Slope

The Slope of the line thru $P(x_1,y_1)$ & $Q(x_2,y_2)$ is $m=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$



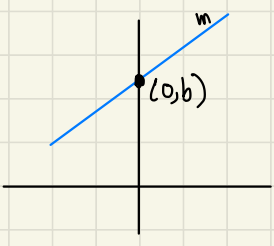
Pick x & y on line

$m=\frac{y-y_1}{x-x_1}$

$m(x-x_1)=y-y_1$

Point-Slope Form

$y-y_1=m(x-x_1)$



$y-b=m(x-0)$

$y=mx+b$

Slope-intercept form

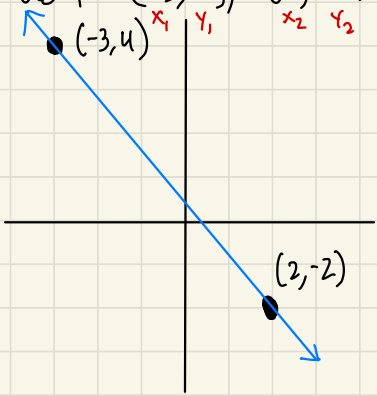
$y=mx+b$

$m>0$ $m<0$ $m=0$ $m=$ Undefined

Ex

Find an equation of the line passing through $(-3,4)$, $(2,-2)$

x_1 y_1 x_2 y_2



Slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-4}{2-(-3)} = \frac{-6}{5}$

Point-Slope Form $y - y_1 = m(x - x_1)$

$y - 4 = -\frac{6}{5}(x+3)$

General form $ax + by + c = 0$

ex

find the slope and the y-intercept of

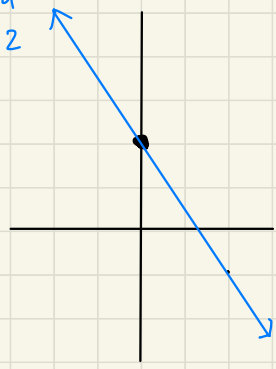
$3x + 2y - 4 = 0$

$2y = -3x + 4$

$y = -\frac{3}{2}x + 2$

Slope $m = -\frac{3}{2}$

Y-intercept $b = 2$



Ex

Sketch $3x+2y-4=0$ use x-and y-intercept

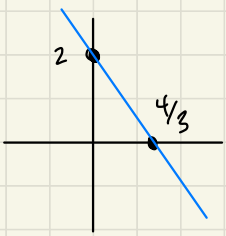
y-int = 2 from prev example

$3x+y(0)-4=0$

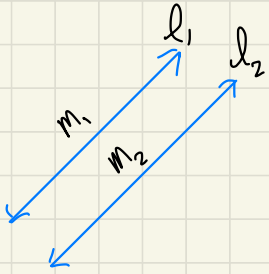
$3x=4$

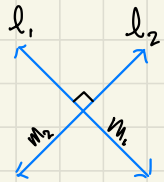
$x=4/3$

x-int = $4/3$



§1.5 more on slope





$l_1 \parallel l_2$

$m_1 = m_2$

$l_1 \perp l_2$

$m_2 = -\frac{1}{m_1}$

$m_1 = -\frac{1}{m_2}$

$m_1 \cdot m_2 = -1$

Ex

Find an equation of the line passing thru (1,2) and

a) Parallel to $x+2y=1$

b) Perp to $x+2y=1$

$2y=-x+1$

$y=-\frac{1}{2}x+\frac{1}{2}$

$m_1=-\frac{1}{2}$

$m_2=m_1$

a) $y-2=-\frac{1}{2}(x-1)$

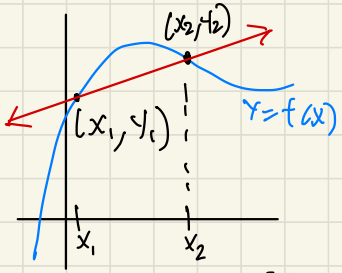
b) $y-2=2(x-1)$

$m_2=\frac{-1}{m_1}=\frac{1}{(-\frac{1}{2})}=2$

Def Average Rate of Change

AROC

Average rate of change of a function is the slope of a secant line between 2 points.



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

AROC

EX

a ball rolling down a ramp.

$s(t) = 5t^2$ ft, t in sec

Find the average velocity $t_1 = 2$, $t_2 = 3$

$$A.V. = \frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{s(3) - s(2)}{3 - 2} =$$

$s(3) = 5 \cdot 3^2 = 45$

$s(2) = 5 \cdot 2^2 = 20$

$$= \frac{45 - 20}{1} = 25 \frac{ft}{sec}$$

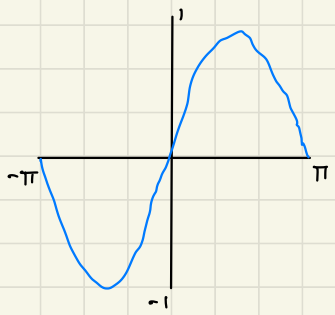
Transformation of functions

§1.6 Transformation of functions

$y = f(x)$

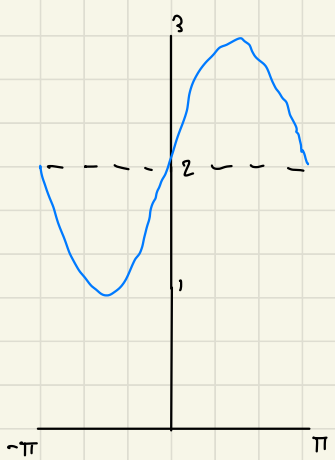
- Vertical Shift: $y = f(x) + d$
- horizontal Shift: $y = f(x-c)$ if $c>0$ shift right, $c<0$ shift left
- vertical Scaling: $y = af(x)$ if $a>1$ stretching, $0<a<1$ shrinking $\} a>0$
- horizontal Scaling: $y = f(bx)$ if $b>1$ Shrinking $\} b>0$
- Reflection about the x-axis: $y = -f(x)$
- Reflection about the y-axis: $y = f(-x)$

Ex $y = \sin(x), \quad -\pi \le x \le \pi$



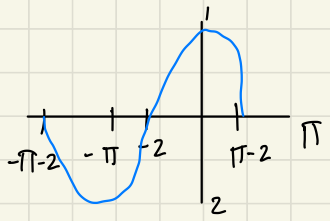
Sketch a) $y = \sin(x) + 2$

Shift 2 units UP



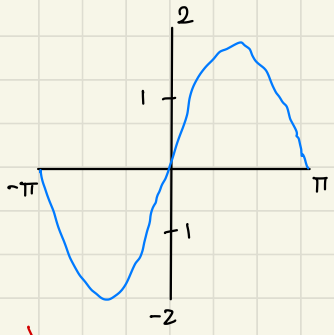
b) $f(x) = \sin(x+2)$

Shift 2 units left



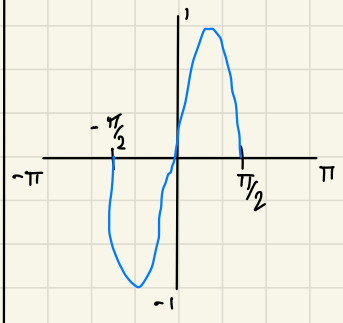
c) $f(x) = 2\sin(x)$

Vertically scale by a factor of 2



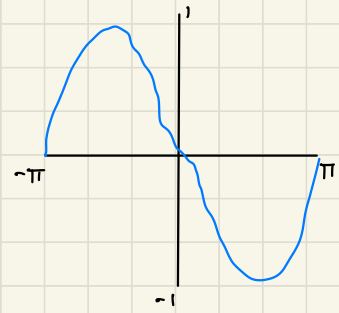
d) $f(x) = \sin(2x)$

horizontal shrink by factor of $1/2$



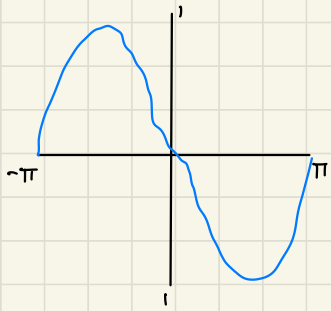
e) $f(x) = -\sin(x)$

reflection over x-axis



f) $y = \sin(-x)$

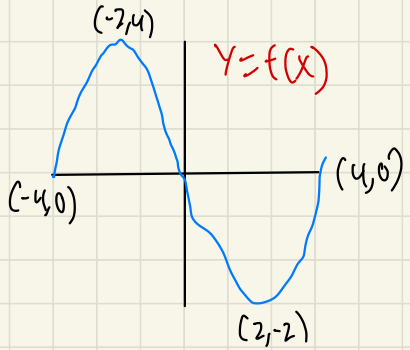
reflection over y-axis



$\sin(-x) = -\sin(x)$?

Yes! sin is an odd function

EX



Sketch $y = -\frac{1}{2}f(x-1)+3$