Transformations

§1.6 Transformation

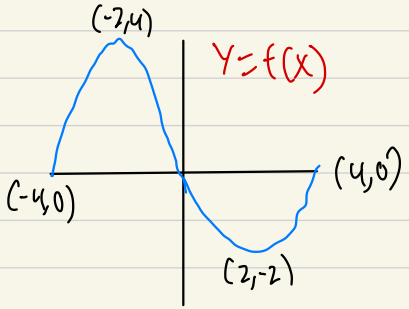
$y = f(x)$ ← original func

V. Scaling, H. Scaling, V. Shift

$y = -a\,f(-b(x-c)) + d$ $\quad a, b > 0$

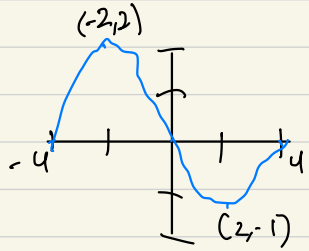
$-a$: reflect abt x-axis; $-b$: reflect abt y-axis; c: H. shift

EX

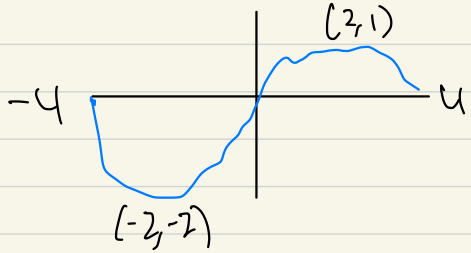


Sketch $y = -\frac{1}{2}f(x-1) + 3$

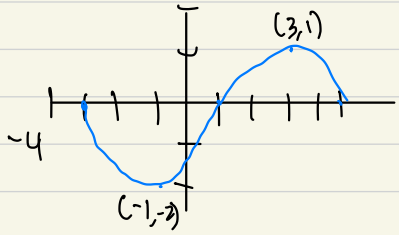
Soln: 1) V. Shrink by a factor of 2



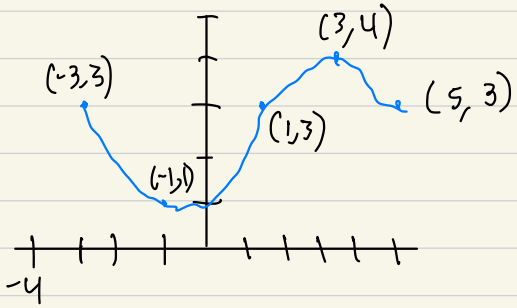
2) reflection about the x-axis



3) H. Shift 1 unit to right



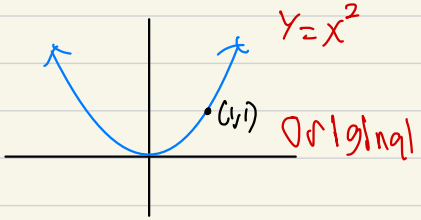
4) V. Shift 3 units up



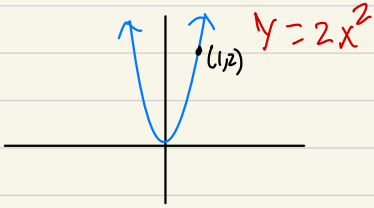
Ex

Sketch $y=2(x-1)^2+3$

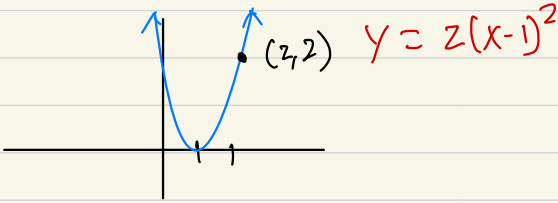
Soln



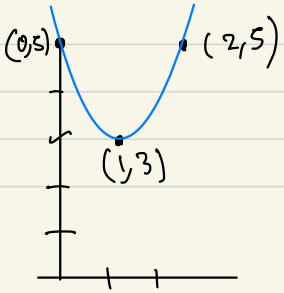
1) Vertical stretch by factor of 2



2) Horizontal Shift by 1 unit right



3) Vertical Shift 3 up



Combinations of functions

§1.7 Combinations of functions

let f & g be two functions.

we define

1) Sum/diff $(f \pm g)(x) = f(x) \pm g(x)$

$dom(f \pm g) = dom(f) \cap dom(g)$

2) Product $(fg)(x) = f(x)\, f(g)$

$dom(fg) = dom(f) \cap dom(g)$

3] Quotient $(f/g)(x) = f(x)/g(x)$

$dom(f/g) = dom(f) \cap dom(g)$

and $g(x) \neq 0$

ex

let $f(x) = 2x+3$ & $g(x) = x^2+2x-1$

Then $(f+g)(x) = f(x) + g(x)$

$= 2x+3 + x^2+2x-1$

$= x^2+4x+2$

$(fg)(x) = f(x)\, g(x)$

$= (2x+3)(x^2+2x-1)$

$= 2x^3 + 3x^2 + 4x^2 + 6x - 2x - 3$

$= 2x^3 + 7x^2 + 4x - 3$

$(f/g)(x) = f(x)/g(x)$

$= (2x+3)/(x^2+2-1)$

$dom\left(\frac{f}{g}\right)$ = quadratic eqn of f must be excluded

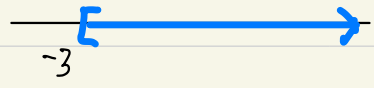
ex

let $f(x) = \sqrt{x+3}$ & $g(x) = \sqrt{3x-2}$

Then $(f+g)(x) = f(x) + g(x) = \sqrt{x+3} + \sqrt{3x-2}$

$\text{dom}(f+g) = ?$

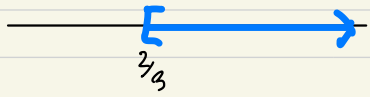
$\text{dom}(f) = [-3, \infty)$



want $x+3 \geq 0$

$x \geq -3$

$\text{dom}(g) = [\frac{2}{3}, \infty)$



want $3x-2 \geq 0$

$x \geq \frac{2}{3}$

$\text{dom}(f+g) = \text{dom}(f) \cap \text{dom}(g)$

$= [-3, \infty) \cap [\frac{2}{3}, \infty)$

$= [\frac{2}{3}, \infty)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+3}}{\sqrt{3x-2}} = \sqrt{\frac{x+3}{3x-2}}$

$\text{dom}(f) \cap \text{dom}(g) = [\frac{2}{3}, \infty)$

Solve $g(x) = 0$

$\sqrt{3x-2} = 0$

$3x-2 = 0$

$3x = 2$

$x = \frac{2}{3}$

$\text{dom}(f/g) = [\frac{2}{3}, \infty) - \{\frac{2}{3}\} = (\frac{2}{3}, \infty)$

Composite functions

Def let f and g be functions. The composition of f with g is defined by

$$(f \circ g)(x) = f(g(x))$$

EX

x	-2	-1	0	1	2
$f(x)$	-3	-1	1	3	5
$g(x)$	2	0	0	2	6

$$(f \circ g)(-2) = f(g(-2)) = f(2) = 5$$

$$(g \circ f)(-1) = g(f(-1)) = g(-1) = 0$$

$$(f \circ f)(-1) = f(f(-1)) = f(-1) = -1$$

$$(g \circ g)(1) = g(g(1)) = g(2) = 6$$

ex

$f(x) = 5x+6, \quad g(x) = 2x^2 - x - 1$

then $(f \circ g)(2) = f(g(2)) = f(2(2)^2 - 2 - 1)$

$= f(5) = 5(5) + 6$

$= 31$

$(f \circ g)(x) = f(g(x))$

$= f(2x^2 - x - 1)$

$= 5(2x^2 - x - 1) + 6$

$= 10x^2 - 5x - 5 + 6$

$= 10x^2 - 5x + 1$

$(g \circ f)(x) = g(f(x))$

$= g(5x+6)$

$= 2(5x+6)^2 - (5x+6) - 1$

$= 2(25x^2 + 60x + 36) - 5x - 6 - 1$

$= 50x^2 + 120x + 72 - 5x - 7$

$= 50x^2 + 115x + 65$

$= 5(x^2 + 23x + 13)$

In general, $f \circ g \neq g \circ f$.

non-communitive

Domain of $f \circ g$?

$(f \circ g)(x) = f(g(x))$

$\text{dom}(f \circ g) = \{ x \mid x \in \text{dom}(g) \ \& \ g(x) \in \text{dom}(f) \}$

ex

$f(x) = \frac{1}{x+1}$ and $g(x) = \frac{2}{x-1}$

find domain of $f \circ g$

$\text{dom}(g) = \{x \mid x \neq 1\} = (-\infty, 1) \cup (1, \infty)$

$f(g(x)) = \frac{1}{g(x)+1}$

Solve $g(x)+1 = 0$

$\frac{2}{x-1} + 1 = 0$

$1 = \frac{-2}{x-1}$

$x - 1 = -2$

$x = -1$

$\text{dom}(f \circ g) = \{x \mid x \neq 1, -1\}$

$= (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$