Inverse Functions

§ 1.8

Inverse funcs

Def

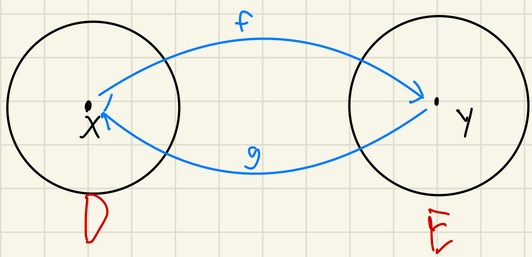
let $f: D \to E$ be a func. If there exists a func $g: E \to D$ such that

$$g(f(x)) = x, \quad x \in D$$

and $$f(g(y)) = y, \quad y \in E$$

then we can say that f has an inverse func and that g is the inverse of f.

notation: $g = f^{-1}$ (not $\frac{1}{f}$)



$$g(f(x)) = g(y) = x$$
$$f(g(y)) = f(x) = y$$

Ex

Show that $f(x) = 4x-7$ and $g(x) = \frac{x+7}{4}$ of each other

Soln:

$$g(f(x)) = \frac{f(x)+7}{4} = \frac{4x-7+7}{4} = \frac{4x}{4} = x$$

$$f(g(x)) = 4g(x) - 7 = 4\left(\frac{x+7}{4}\right) - 7 = x+7-7 = x$$

EX

let $S = \{0, 1, 2, 3, 4\}$

Define $f : S \to S$

x	0	1	2	3	4
$f(x)$	4	0	3	1	2

Find inverse of f

Soln:

$f^{-1} : S \to S$

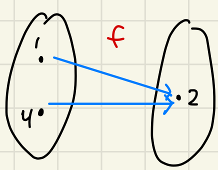
x	0	1	2	3	4
$f^{-1}(x)$	1	3	4	2	0

ex

x	0	1	2	3	4
$f(x)$	1	2	4	3	2

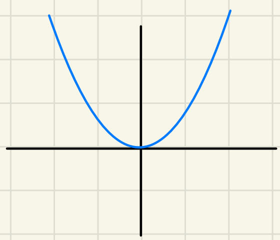
$f^{-1}(2) = 1$ or 4? ✗

f has no inverse



two-to-one func

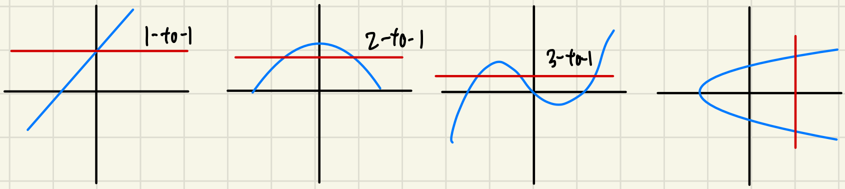
ex



2-to-1

ex

Which graphs represent func. that has an inverse?
(Horizontal line test)



yes — no — no — not a func (no)

☆ How to find f^{-1}?

1) replace $f(x)$ w/ y
2) interchange x & y
3) solve for y

ex

find the inverse of
$f(x) = 2x+3$

soln:

1) $y = 2x+3$
2) $x = 2y+3$
3) $2y = x-3$
$y = \frac{x-3}{2}$
4) $f^{-1}(x) = \frac{x-3}{2}$

ex

Find the Inverse of $f(x) = 2x^3 - 1$

Soln:

1) $y = 2x^3 - 1$
2) $x = 2y^3 - 1$
3) $2y^3 = x + 1$

$y^3 = \frac{x+1}{2}$

$y = \sqrt[3]{\frac{x+1}{2}}$

4) $f^{-1}(x) = \sqrt{\frac{x+1}{2}}$

ex

$f(x) = \frac{x+1}{x-2}$, $x \neq 2$

Inverse ?

1) $y = \frac{x+1}{x-2}$
2) $x = \frac{y+1}{y-2}$
3) $x(y-2) = y+1$

$xy - 2x = y + 1$

$xy - y = 2x + 1$

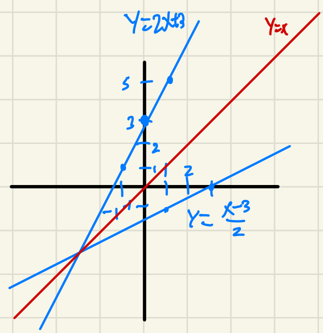
$y(x-1) = 2x+1$

$y = \frac{2x+1}{x-1}$

4) $f^{-1}(x) = \frac{2x+1}{x-1}$

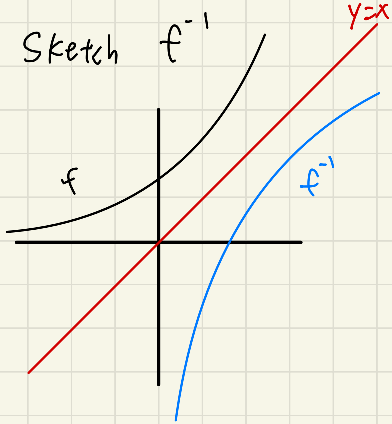
ex

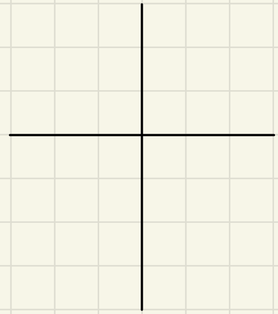
$f(x) = 2x + 3$, $f^{-1}(x) = \frac{x-3}{2}$



ex

Sketch f^{-1}

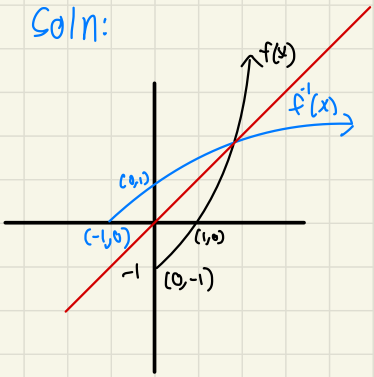




ex

Sketch $f(x)=x^2-1$, $x\ge 0$ and $f^{-1}(x)$

Soln:



1) $y = x^2-1,\ x\ge 0$
2) $x = y^2-1,\ y\ge 0$
3) $y^2 = x+1$

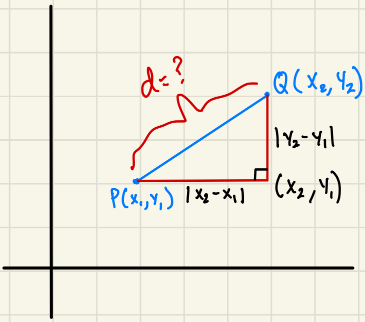
$y = \pm\sqrt{x+1}$

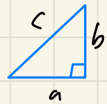
$y = \sqrt{x+1}$

4) $f^{-1}(x)=\sqrt{x+1}$

Distance & Midpoint

§1.9 Distance, Midpoint, circles





Pythagorean: $a^2+b^2=c^2$

$$d^2 = (|x_2-x_1|)^2 + (|y_2-y_1|)^2$$
$$d^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$
$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Distance formula

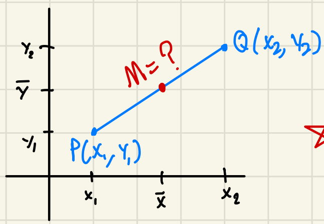
$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

ex

find dist between $(-1,4)$ and $(3,-2)$

$$d = \sqrt{(3+1)^2 + (-2-4)^2} = \sqrt{4^2+(-6)^2}$$
$$= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$$

Midpoint



$$\bar{x} = \frac{x_1+x_2}{2}$$
$$\bar{y} = \frac{y_1+y_2}{2}$$

☆ $M = (\bar{x}, \bar{y}) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

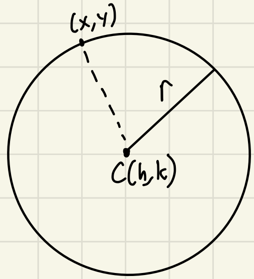
ex

find the midpoint of the line segment w/ endpoints $(-1,4)$ & $(3,-2)$

$$M = \left(\frac{-1+3}{2}, \frac{4-2}{2}\right) = (1,1)$$

Circles

Circles



Center: (h,k)

radius: r

equation: ?

using the distance formula

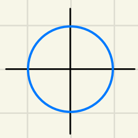
$$\sqrt{(x-h)^2+(y-k)^2}=r$$

$$(x-h)^2+(y-k)^2=r^2$$

Standard equation of a circle

ex

Find an eqn of the circle with center $(0,0)$ and radius 1



$$(x-0)^2+(y-0)^2=1^2$$

$$x^2+y^2=1$$

ex

Circle with center $(2,-3)$ and radius 2

$$(x-2)^2+(y+3)^2=2^2$$

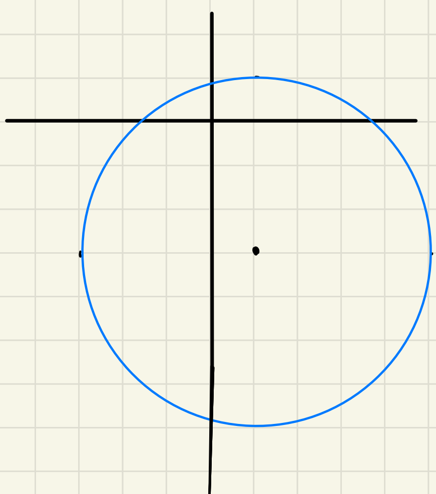
$$(x-2)^2+(y+3)^2=4$$

ex

Sketch a graph of $(x-1)^2+(y+3)^2=16$

$C=(1,-3)$

$r=4$



domain: $[-3,5]$

range: $[-7,1]$

ex

find the center and radius of

$$x^2+y^2+4x-6y-23=0$$

$$(x^2+4x)+(y^2+6y)=23$$

complete the square

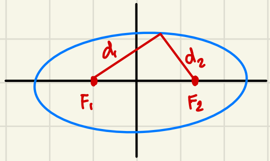
$$(x^2+4x+4)+(y^2+6y+9)=23+4+9=36$$

$$(x+2)^2+(y+3)^2=6^2$$

$C=(-2,3)$

$r=6$

§9.1 ellipse



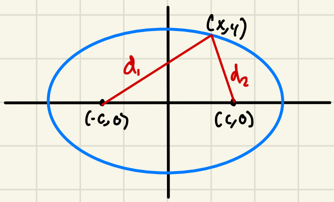
$d_1 + d_2 = \text{const.}$

Def

An Ellipse is a set of Points in the Plane the sum of whose distances from two fixed Points F_1 and F_2 (Foci) is Constant

Equation ?

center = (0,0)



$d_1 + d_2 = 2a$

$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$

algebra

$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$

let $b^2 = a^2 - c^2$

ellipse Standard form

$b^2x^2 + a^2y^2 = a^2b^2$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$