

8/25/2022

§ P.6 Rational expressions

Def Rational expression

$$\frac{\text{Poly}}{\text{Poly}} = \frac{p(x)}{q(x)}, q(x) \neq 0$$

Ex Multiply

$$\begin{aligned} \frac{x-7}{x-1} \cdot \frac{x^2-1}{3x-21} &= \frac{(x-7)(x^2-1)}{(x-1)(3x-21)} \quad \leftarrow \text{diff of 2 squares} \\ &= \frac{(x-7)(x+1)(x-1)}{3(x-1)(x-7)} \quad \leftarrow 9x-21 \\ &= \frac{x+1}{3}, x \neq 1, x \neq 7 \end{aligned}$$

Ex divide

$$\begin{aligned} \frac{x^2-2x-8}{x^2-9} \div \frac{x-4}{x+3} &= \frac{x^2-2x-8}{x^2-9} \cdot \frac{x+3}{x-4} \\ &= \frac{(x^2-2x-8)(x+3)}{(x^2-9)(x-4)} = \frac{(x-4)(x+2)(x+3)}{(x-3)(x+3)(x-4)} \\ &= \frac{x+2}{x-3}, x \neq 3, x \neq -3, x \neq 4 \end{aligned}$$

Addition & subtraction

Ex

Find the least common denominator (LCD or LCM) of

$$\frac{1}{x^3-x^2}, \frac{2}{x^3-x}, \frac{3}{x^2+2x+1}$$

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Soln:

Step 1: factor the denominator of each fraction

$$\begin{aligned}x^3-x^2 &= x^2(x-1) \\x^3-x &= x(x+1)(x-1) \\x^2+2x+1 &= (x+1)^2\end{aligned}$$

Step 2: list the highest exponent of each different factor

$$x^2, (x-1), (x+1)^2$$

Step 3: The product of the factors in step 2 is the LCM (LCD)

Ans: $LCM = x^2(x-1)(x+1)^2$

Ex Add

$$\frac{x+3}{x^2+x-2} + \frac{2}{x^2-1}$$

Soln: 1. $x^2+x-2 = (x+2)(x-1)$

2. $x^2-1 = (x+1)(x-1)$

3. $(x+2)(x-1)(x+1)$

$$= \frac{x+3}{(x+2)(x-1)} \cdot \frac{x+1}{x+1} + \frac{2}{(x+1)(x-1)} \cdot \frac{x+2}{x+2}$$

Ex Add

cont.

$$= \frac{x+3}{(x+2)(x-1)} \cdot \frac{x+1}{x+1} + \frac{2}{(x+1)(x-1)} \cdot \frac{x+2}{x+2}$$

$$= \frac{(x+3)(x+1)}{(x+2)(x-1)(x+1)} + \frac{2(x+2)}{(x+1)(x-1)(x+2)}$$

$$= \frac{(x+3)(x+1) + 2(x+2)}{(x+2)(x-1)(x+1)}$$

$$= \frac{x^2 + 4x + 3 + 2x + 4}{(x+2)(x-1)(x+1)}$$

$$= \frac{x^2 + 6x + 7}{(x+2)(x-1)(x+1)} \quad \text{irreducible}$$

Try Subtract

$$\frac{x-8}{x^2-4} - \frac{2}{x^2-2x}$$

$$= \frac{x-8}{(x+2)(x-2)} - \frac{2}{x(x-2)} = \frac{x(x-8) - 2(x+2)}{x(x+2)(x-2)} = \frac{x^2-8x-2x-4}{x(x+2)(x-2)}$$

$$= \frac{x^2-10x-4}{x(x+2)(x-2)}$$

Ans: $\frac{x^2-10x-4}{x(x+2)(x-2)}$

Complex fraction

fraction made of fractions

Ex Simplify

$$\frac{1 + \frac{2}{x}}{1 - \frac{2}{x^2}}$$

LCM of x & x^2 is x^2

mult both num and denom by LCM

$$= \frac{1 + \frac{2}{x}}{1 - \frac{2}{x^2}} \cdot \frac{x^2}{x^2} = \frac{x^2 + 2x}{x^2 - 2}$$

§ P.7 Equations

$$\text{Expr} = \text{Expr}$$

Def Linear equation

$$\underbrace{ax+b=0}_{\text{deg 1 Polynomial}}, \quad a \neq 0$$

Ex solve

$$2x + 4 = 0$$

$$\underline{2x} = \underline{-4}$$

$$x = -2$$

Ex solve

$$\frac{x+2}{4} - \frac{x-1}{3} = 2 \quad \text{"linear"}$$

LCM of 4 and 3 is 12

$$12 \left(\frac{x+2}{4} - \frac{x-1}{3} \right) = 2 \cdot 12$$

$$3(x+2) - 4(x-1) = 24$$

$$3x + 6 - 4x + 4 = 24$$

$$-x + 10 = 24$$

$$-x = 14 \quad \div -1$$

$$x = 14$$

Ex solve the rational equation

$$\frac{3}{x+6} + \frac{1}{x-2} = \frac{4}{x^2+4x-12} = \frac{4}{(x+6)(x-2)}$$

$$x \neq 2, x \neq -6$$

LCM of $x+6, x-2, x^2+4x-12$ is $(x+6)(x-2)$

$$(x+6)(x-2) \left(\frac{3}{x+6} + \frac{1}{x-2} \right) = \frac{4}{(x+6)(x-2)} (x+6)(x-2)$$

$$(x+6)(x-2) \left(\frac{3}{x+6} + \frac{1}{x-2} \right) = \frac{4}{(x+6)(x-2)} (x+6)(x-2)$$

$$3x-6 + x+6 = 4$$

$$4x = 4$$

$$x = 1$$

Def
Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \quad \text{distance from 0}$$

e.g. $|5| = 5$ bc $5 \geq 0$
 $|-5| = 5$ bc $5 < 0$

fact

$$\text{If } |u| = c \text{ then } u = c \text{ or } u = -c$$

Ex

Solve the eqn involving abs value

$$5|1-4x| - 15 = 0$$

Step 1 Solve for $|u|$

$$5|1-4x| = 15$$

$$|1-4x| = 3$$

Step 2 Use $|u| = c \Rightarrow u = c \text{ or } \Rightarrow u = -c$

$$1-4x = 3 \quad \text{or} \quad 1-4x = -3$$

$$-4x = 2$$

$$x = -\frac{1}{2}$$

$$-4x = -4$$

$$x = 1$$

$$\text{Solution set: } \left\{ -\frac{1}{2}, 1 \right\}$$

Def
Quadratic equation

$$ax^2 + bx + c = 0, \quad a \neq 0$$

deg 2 Poly

$$A \cdot B = 0 \rightarrow A = 0 \text{ or } B = 0$$

Ex solve

$$x^2 - 3x + 2 = 0$$

Soln: $(x-2)(x-1) = 0 \rightarrow x-2=0 \text{ or } x-1=0$
 Soln set: $\{1, 2\}$ $x=2$ $x=1$

EX solve

$$4x^2 - 2x + 1 = 0$$

$$x^2 - \frac{1}{2}x + \frac{1}{16} = -\frac{1}{4} + \frac{1}{16}$$

$$(x - \frac{1}{4})^2 = -\frac{3}{16}$$

$$x - \frac{1}{4} = \pm \sqrt{-\frac{3}{16}}$$

$$x = \frac{1}{4} \pm \sqrt{-\frac{3}{16}} = \frac{1}{4} \pm \frac{\sqrt{3}}{4}i = \frac{1 \pm \sqrt{3}i}{4}$$

irreducible lets come back to it
complete the sq

$$i = \sqrt{-1}$$

EX solve

$$2(x-1)^2 - 4 = 0$$

$$2(x-1)^2 = 4$$

$$(x-1)^2 = 2$$

$$x-1 = \pm\sqrt{2}$$

$$x = 1 \pm \sqrt{2}$$

Soln set: $\{1+\sqrt{2}, 1-\sqrt{2}\}$

fact.

If $u^2 = d$ then $u = \pm\sqrt{d}$

Completing the square

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

$$\left(\frac{b}{2}\right)^2 = 3^2 = 9$$

EX solve

$$x^2 - 6x + 4 = 0$$

$$x^2 - 6x + 9 = -4 + 9$$

$$(x-3)^2 = 5$$

$$x-3 = \pm\sqrt{5}$$

$$x = 3 \pm \sqrt{5}$$

Soln set: $\{3+\sqrt{5}, 3-\sqrt{5}\}$

EX solve

$$ax^2 + bx + c = 0, a \neq 0$$

Soln:

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$
$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic formula

EX Solve
The radical
equation

$$\sqrt{2x+1} + 2 = x$$

Soln: Step 1. Solve for \sqrt{u}

$$\sqrt{2x+1} = x-2$$

Step 2. Square both sides

$$2x+1 = x^2 - 4x + 4$$

$$x^2 - 6x + 3 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 12}}{2} = \frac{6 \pm \sqrt{24}}{2} = 3 \pm \sqrt{6}$$

Step 3, Check

$$x = 3 + \sqrt{6} : \quad \begin{aligned} \sqrt{2x+1} + 2 &= x \\ \sqrt{2(3+\sqrt{6})+1} + 2 &= 3 + \sqrt{6} \end{aligned}$$