

8/30/2022

Model with equations

Test and assignments

make sure to sign up for math lab

1 assignment a week

§ 7.8

word problems

Model with Equations

Simple interest formula

$$A = P + Prt = P(1 + rt)$$

$$I = Prt$$

I = interest = $A - P$

A = total,

P = Principal

r = interest rate

t = # of years

EX

A Part of \$7000 dollars was borrowed at 6% simple interest annually and the remainder at 8%. If the total interest after 3 years is \$1380 How much was borrowed at each rate?

Soln: 1) Introduce a variable x to represent 1 of the unknown quantities.

let x = amount borrowed at 6%.

2) Express any other unknown in terms of x .

$\$7000 - x$ is the amount borrowed at 8%.

3) write an equation in x that models the problem.

total interest = 1380

$$x r_1 t + (7000 - x) r_2 t = 1380$$

$$x(0.06)(3) + (7000 - x)(0.08)(3) = 1380$$

4) Solve for x and answer the problem's question.

$$0.18x + 0.24(7000 - x) = 1380$$

$$0.18x + 1680 - 0.24x = 1380$$

$$-0.06x = -300$$

$$x = 5000$$

$$7000 - 5000 = 2000$$

Ans:

\$5000 was borrowed at 6% and
\$2000 was borrowed at 8%.

§ p.9
Inequalities

Inequalities

Def Set

A Set is a collection of objects. Each object is called an element (or member) of the set.

e.g. $A = \{a, b, c\}$ is a set that contains three letters a , b , and c

Notation

$a \in A$ "a is a member of A"
 $d \notin A$ "d is not a member of A"

Ex

the solution set of $(x-2)(x-3)=0$ is $\{2, 3\}$

Ex

Let $E =$ the set of all even integers

$$E = \{\dots, -4, -2, 0, 2, 4, \dots\}$$

$$E = \{0, \pm 2, \pm 4, \pm 6, \dots\}$$

Set Builder notation

$$\{x \mid P(x)\}$$

↑ Property that defines the set

EX cont.

$$E = \{x \mid x \text{ is an even number}\}$$

EX

$$\begin{aligned} E &= \{x \mid x^2 - 5x + 6 = 0\} \\ &= \{x \mid (x-2)(x-3) = 0\} \\ &= \{2, 3\} \end{aligned}$$

Def interval notation

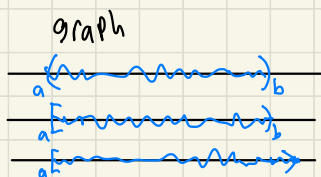
$$(a, b) = \{x \mid a < x < b\}$$

$$[a, b) = \{x \mid a \leq x < b\}$$

$$[a, \infty) = \{x \mid x \geq a\}$$

$$[a, b] = \{x \mid a \leq x \leq b\}$$

$$(-\infty, \infty) = \{x \mid -\infty < x < \infty\} = \mathbb{R} = \text{the set of all real numbers.}$$



Def Union

Let A and B be sets.

The Union of A and B is the set $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
All elements from both A and B

Def intersection

The Intersection of A and B is the set
 $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

EX

$$\text{Let } A = \{1, 2, 3, 4, 5\}$$

$$\text{Let } B = \{4, 5, 6, 7\}$$

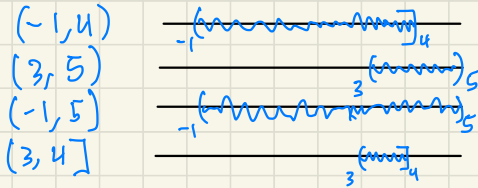
then

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$$

$$A \cap B = \{4, 5\}$$

EX

$$\begin{aligned}(-1, 4) \cup (3, 5) &= (-1, 5) \\ (-1, 4) \cap (3, 5) &= (3, 4)\end{aligned}$$




Solving linear
Inequalities

EX

$$\text{Solve } 3 - 2x \leq 11$$


$$\begin{aligned}-3 & \quad -3 \\ -2x & \leq 8 \\ \frac{-2x}{-2} & \leq \frac{8}{-2} \\ x & \geq -4\end{aligned}$$

graph 
Interval $[-4, \infty)$
Set $\{x \mid x \geq -4\}$

EX solve

$$12\left(\frac{x+3}{4}\right) \geq \left(\frac{x-2}{3} + \frac{1}{4}\right) 12 \quad \text{LCM} = 12$$

$$\begin{aligned}3x + 9 & \geq 4x - 8 + 3 \\ -3x + 9 & \quad -3x + 6 - 3 \\ -3 & \geq x \quad \text{OR} \quad x \leq 14\end{aligned}$$

graph 
Interval $(-\infty, 14]$
Set $\{x \mid x \leq 14\}$

Solving Inequalities with absolute value

Facts

1. If $|u| < c$ then $-c < u < c$
2. If $|u| > c$ then $u > c$ or $u < -c$

EX Solve

$$-2|3x+5|+7 \geq -13$$

Soln:

Step 1) solve for $|u|$

$$-2|3x+5| \geq -20$$

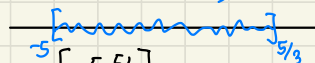
$$|3x+5| \leq 10$$

Step 2) use fact 1 or 2

$$\text{Fact 1 } -10 \leq 3x+5 \leq 10$$

$$-15 \leq \frac{3x}{3} \leq \frac{5}{3}$$

$$-5 \leq x \leq \frac{5}{3}$$

Graph 

Interval

$$[-5, \frac{5}{3}]$$

Set

$$\{x \mid -5 \leq x \leq \frac{5}{3}\}$$

EX Solve

$$7 < |5-2x|$$

Soln:

Step 1) already done

Step 2) fact 2

$$|5-2x| > 7$$

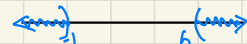
$$5-2x > 7 \quad \text{OR} \quad 5-2x < -7$$

$$-2x > 2$$

$$-2x < -12$$

$$x < -1$$

$$x > 6$$

Graph 

Interval $(-\infty, -1) \cup (6, \infty)$

Set

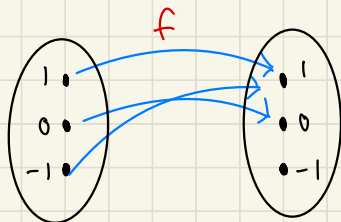
$$\{x \mid x < -1 \quad \text{OR} \quad x > 6\}$$

Ch. 1.2 Functions and their graphs

Def function

A **function** is a rule that assigns each element $x \in D$ to exactly **one** element $y \in E$

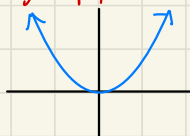
e.g.



Table

x	-1	0	1
y	1	0	1

graph



Formula

$$f(x) = x^2$$

Relation

Set of ordered pairs
 $R = \{(-1, 1), (0, 0), (1, 1)\}$

Domain
Range

$\{ \text{all in puts} \}$ x values
 $\{ \text{all out puts} \}$ y values

Ex

Domain of $R = \{-1, 0, 1\}$
 Range of $R = \{0, 1\}$

EX

Consider the relation $R = \{(6, 1), (6, 2), (8, 3), (9, 4)\}$

$$\text{dom}(R) = \{6, 8, 9\}$$

$$\text{ran}(R) = \{1, 2, 3, 4\}$$

Is R a function?

no, It has 6 going to two different values.

Equations

ex

is $y = x^2$ a function in x ? **Yes**

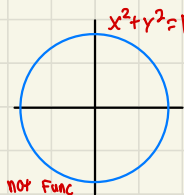
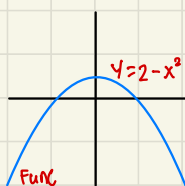
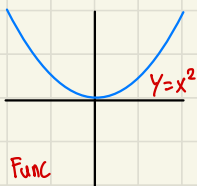
is $x^2 + y = 2$ a function in x ? **Yes**
 $y = 2 - x^2$

is $x^2 + y^2 = 1$ a function in x ? **NO**

$$y^2 = 1 - x^2$$

$$y = \pm \sqrt{1 - x^2}$$

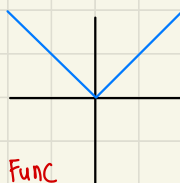
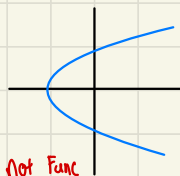
↑
two solutions



Vertical line test

If any vertical line intersects a graph more than 1 point, then the graph does not define y as a function of x .

EX



Evaluate a function

given $f(x) = 2x$

a) find $f(-3)$, $f(0)$, $f(2)$

$$f(-3) = 2(-3) = -6$$

$$f(0) = 2(0) = 0$$

$$f(2) = 2(2) = 4$$

b) Suppose $f(x) = 9$ find x

$$2(x) = 9$$

$$x = 9/2$$