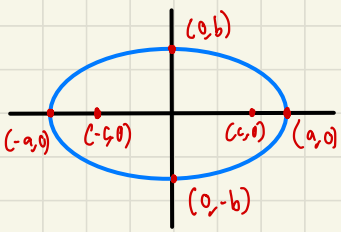
9/15/22

Conic Sections

LRC ~1 extra credit (10 visits)

9.1 The ellipse

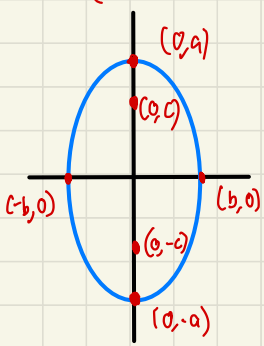


$a \geq b > 0$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$c^2 = a^2 + b^2$

vertices $(\pm a, 0)$
foci $(\pm c, 0)$



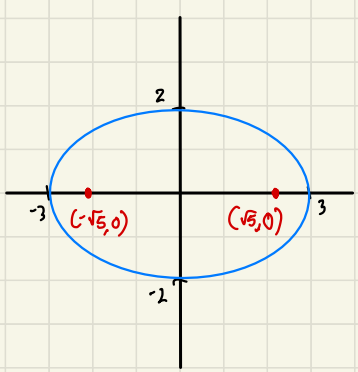
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

vertices $(0, \pm a)$
foci $(0, \pm c)$

EX Sketch $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Soln: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a^2 = 9 \rightarrow a = 3$
$b^2 = 4 \rightarrow b = 2$
$c^2 = a^2 - b^2 = 9 - 4 = 5$
$c = \sqrt{5}$
Center $(0,0)$
vertices $(\pm a, 0) = (\pm 3, 0)$
foci $(\pm c, 0) = (\pm \sqrt{5}, 0)$



Ex

find the standard form an ellipse with foci at $(\pm 1, 0)$ and vertices at $(\pm 2, 0)$

soln: horizontal ellipses

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a = 2$

$c = 1$

$c^2 = a^2 - b^2$

$b^2 = a^2 - c^2$

$b^2 = 4 - 1$

$b = \sqrt{3}$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

https://www.desmos.com/calculator/aprl48vsul

Ellipse centered at (h, k)

Horizontal

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Vertical

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$a \geq b > 0$

EX

Sketch $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$

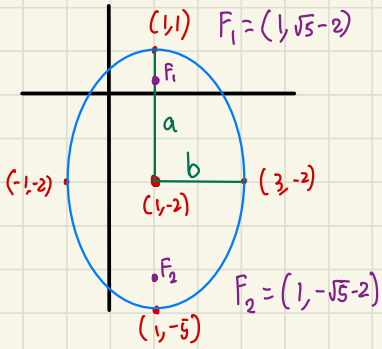
Soln:

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Center is $(h,k) = (1,-2)$

$a^2 = 9 \rightarrow a = 3$

$b^2 = 4 \rightarrow b = 2$



Vertices $(0, \pm a)$

1 right, 2 down

$(0+1, \pm 3 - 2)$

$(1,1)$ & $(1,-5)$

Foci $(0, \pm c)$

$c^2 = a^2 - b^2$

$c^2 = 9 - 4 = 5$

$c = \sqrt{5}$

$(0, \pm \sqrt{5})$

right 1, down 2

$(0+1, \pm \sqrt{5} - 2)$

$(1, \sqrt{5}-2)$ & $(1, -\sqrt{5}-2)$

Ex Sketch the ellipse

$4x^2 + 36y^2 + 40x - 288y + 532 = 0$

$(4x^2 + 40x) + (36y^2 - 288y) = -532$

$4(x^2 + 10x) + 36(y^2 - 8y) = -532$

$4(x^2 + 10x + 25) + 36(y^2 - 8y + 16) = -532 + 4 \cdot 25 + 36 \cdot 16$

$4(x+5)^2 + 36(y-4)^2 = 144$

$\frac{4(x+5)^2}{144} + \frac{36(y-4)^2}{144} = 1$

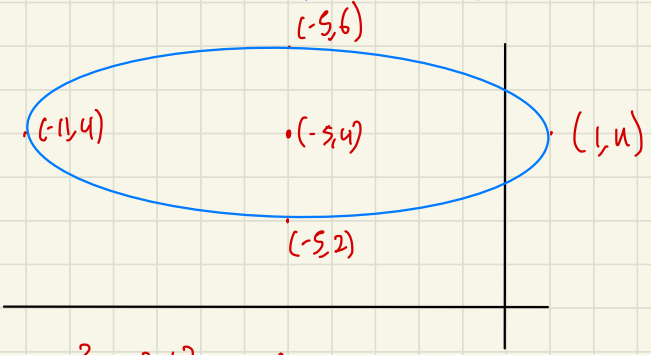
$\frac{(x+5)^2}{36} + \frac{(y-4)^2}{4} = 1$

(Horizontal $36 > 4$)

$a^2 = 36 \rightarrow a = 6$

$b^2 = 4 \rightarrow b = 2$

$(h,k) = (-5,4)$



$c^2 = a^2 - b^2 = 36 - 4 = 32$

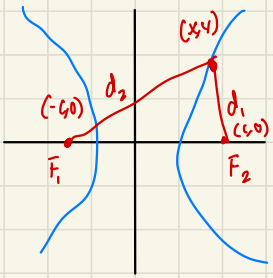
$c = \sqrt{32}$

vertecies: $(-11,4)$ & $(1,4)$

foci: $(\pm\sqrt{32} - 5, 4)$

Hyperbola

§ 9.2 hyperbola



$|d_2 - d_1| = 2a$ Constant

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

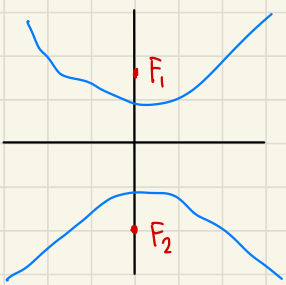
$c^2 = a^2 + b^2$

https://www.desmos.com/calculator/qbxnskqy84

foci $(\pm c, 0)$

vertices $(\pm a, 0)$

asymptote $y = \pm b/a\, x$



$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

foci $(0, \pm c)$

vertices $(0, \pm a)$

asymptote $y = \pm \frac{a}{b} x$

EX find the vertice and foci of the hyperbola $\frac{y^2}{9} - \frac{x^2}{16} = 1$

Soln:

vertices $(0, \pm a)$

$a^2 = 9 \Rightarrow a = 3$

$(0, \pm 3)$

$(0,3)$ & $(0,3)$

foci $(0, \pm c)$

$c^2 = a^2 + b^2$

$c^2 = 9 + 16 = 25$

$c = 5$

$(0, \pm 5) \rightarrow (0,5)$ & $(0,-5)$

Ex Find an eqn of the hyperbola with foci $(\pm 5, 0)$ and vertices $(\pm 4, 0)$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$a = 4, \quad c = 5$

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$b^2 = 25 - 16 = 9$

$b = 3$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Ex Sketch $9y^2 - 4x^2 = 36$

$$\frac{9y^2}{36} - \frac{4x^2}{36} = 1$$

$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$

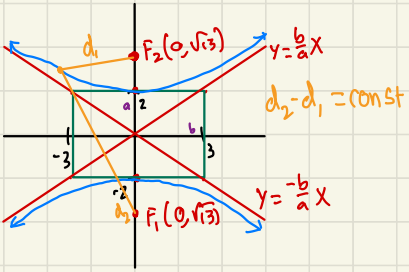
$a = 2$

$b = 3$

foci $(0, \pm c)$

$c^2 = a^2 + b^2 = 4 + 9 = 13$

$c = \sqrt{13}$



Ex Sketch

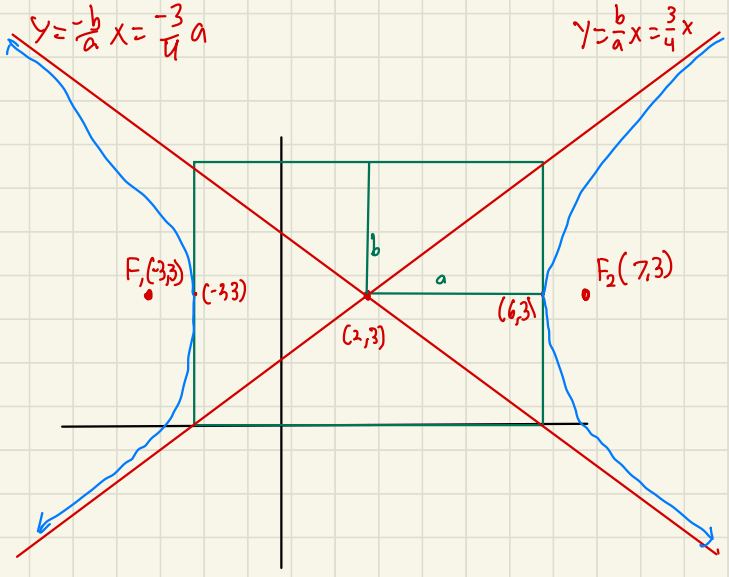
$$\frac{(x+2)^2}{16} - \frac{(y-3)^2}{9} = 1$$

Soln:

Center $(h,k) = (2,3)$

$a^2 = 16$ $\quad b^2 = 9$

$a = 4$ $\quad b = 3$



foci

$c^2 = a^2 + b^2 = 16 + 9 = 25$

$c = 5$

$(\pm 5, 0)$

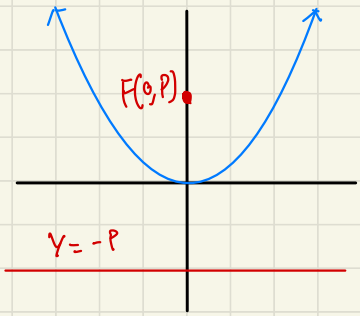
$+2, +3$

$(\pm 5 + 2, +3)$

$(7,3)$ & $(-3,-3)$

Parabola

§9.3 Parabola

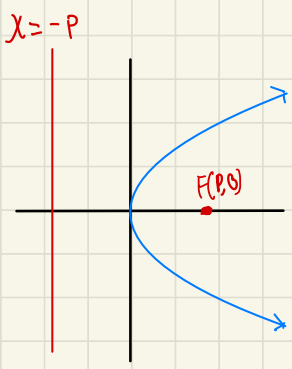


$x^2 = 4py$

focus $F=(0,p)$

Directrix: $y=-p$

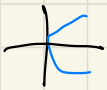
$p>0$  $p<0$ 



$y^2 = 4px$

focus $F=(p,0)$

Directrix $y=-p$

$p>0$  $p<0$ 

ex

find focus and directrix

$y^2 = 12x$

$4p = 12$

$p = 3$

focus $F=(3,0)$

directrix $x=-3$

ex

find an equation of a Parabola w/ $F=(0,-2)$ & directrix $y=2$

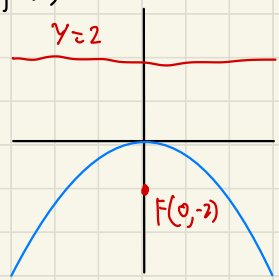
Soln: focus $(0,p)=(0,-2)$

$p=-2$

$x^2=4py$

$=4(-2)y$

$=-8y$



ex

Find the vertex, focus, and the directrix of the Parabola $y^2+2y+12x-23=0$

$y^2=4px$ vertex $(0,0)$

$(y-k)^2=4p(x-h)$ vertex (h,k)

$y^2+2y=-12x+23$

$y^2+2y+1=-12x+24$

$(y+1)^2=-12(x-2)$

$(h,k)=(2,-1)$

$-12(x-2)=4p(x-h)$

$-3=p$