Partition Problem KPS

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The partition problem is formulated as follows: Let V be a finite set and *weight* be a function on V with positive integer values (this is an additive function). It is requested to find, if it exists, a partition of V, denoted V_1, V_2, such that $weight(V_1) = weight(V_2)$. A solution to this problem is provided in [1] by using a recognizer tissue P system with cell division and symport/antiport rules. We make the following notations: let $V = \{v_1, \ldots, v_n\}$, be a finite set, with $weight(v_i) = k_i$, where k_i is a positive integer, $1 \leq i \leq n$.

We provide a solution to the partition problem by using a classical approach in membrane systems, also illustrated in [1], which consists of the following stages: *working space generation*, *verification* and *solution generation*. In the first stage, an exponential space is generated in linear time; this consists of all the compartments that might contain a solution to the problem. Next, in the verification stage, it is checked, in every compartment, whether a solution has been produced. Finally, if at least one solution is found, then a special symbol, *yes*, is sent to the environment, labelled 0; otherwise, a *no* is sent to it.

We build the following skP system, which depends on n, for solving the partition problem (i.e., checking whether there is a partition, V_1, V_2, with $weight(V_1) = weight(V_2)$) and working in the maximally parallel manner:

$$\mathrm{sk}\Pi_P(n) = (A, L, IO, \mu, C_1, C_2, 0),$$

where

- A is the alphabet;
- $L = \{0, 1, 2\}$;
- IO consists of *yes*, *no*; at the end, after $n + 3$ steps, one of the two possible answers will be sent out;
- $C_1 = (1, w_{1,0}, R_1), C_2 = (2, w_{2,0}, R_2)$, where $w_{1,0} = S$, $w_{2,0} = A_1\mathrm{code}(n)$, with $\mathrm{code(n)} = v_1^{k_1} \ldots v_n^{k_n}$ being the code of the weights of the elements of V;
- μ is given by the graph with edge $(1, 2)$;
- R_1 and R_2 are given below:
  - R_1 contains:

$$r_{1,1} : S \to (yes, 0)\{\geq T\}, \quad r_{1,2} : S \to (no, 0)\{\geq F < T\};$$

    $r_{1,1}$ or $r_{1,2}$ sends into the environment the answer *yes* or *no*, respectively;
  - R_2 contains membrane division rules:

$$r_{2,i} : [A_i]_2 \to [B_iA_{i+1}]_2[A_{i+1}]_2, \quad 1 \leq i < n,$$

$$r_{2,n} : [A_n]_2 \to [B_n X]_2 [X]_2;$$

these rules generate all the subsets of V in n steps (i.e., 2^n subsets), each of them being a potential V_1 (with V_2 as its complement);
rewriting rules:

$$r_{2,i,j} : v_i v_j \to v\{= B_i \neq B_j = X \mid \neq B_i = B_j = X\}, \quad 1 \leq i < j \leq n,$$

$$r_{2,n+1} : X \to Y;$$

rewriting and communication rules:

$$r_{2,n+2} : Y \to (F, 1)\{\geq v_1 \mid \ldots \mid \geq v_n\},$$

$$r_{2,n+3} : Y \to (T, 1)\{< v_1 \ldots < v_n\}.$$

The skP system proceeds in the following steps:

- **Generation stage**: Starts with two compartments, C_1 and C_2. C_1, initially containing symbol S, will collect the output of the problem, either *yes* or *no*. C_2, containing a codification of set V with the weights of its elements, is divided by rules $r_{2,1}$ to $r_{2,n}$ to generate 2^n compartments C_2 corresponding to all subsets of V.
- **Verification stage**: In step $n+1$, each compartment C_2 pairs elements of $V_1 \subseteq V$ with elements of its complement as per their weights, using $r_{2,i,j}$, while X is transformed to Y by $r_{2,n+1}$. In step $n+2$, T or F is sent to C_1 based on whether all elements are paired or the weights differ, via $r_{2,n+3}$ or $r_{2,n+2}$.
- **Solution generation**: In step $n+3$, either *yes* or *no* is sent to the environment using $r_{1,1}$ or $r_{1,2}$.

We denote all possible partitions generated by the above model as $V_{1,l}, V_{2,l}$, $1 \leq l \leq 2^n$, with

$$V_{1,l} = \{a_{i,1}, \ldots, a_{i,p_l}\}, \quad V_{2,l} = \{a_{j,1}, \ldots, a_{j,q_l}\}, \quad p_l + q_l = n,$$

where $\mathit{weight}(a_h) = k_h$, $1 \leq h \leq n$. Let

$$m_l = \min\left\{ \sum_{1\leq h\leq p_l} k_{i_h}, \sum_{1\leq h\leq q_l} k_{j_h} \right\} \quad \text{and} \quad M = \max\{m_h \mid 1 \leq h \leq 2^n\}.$$

References

1. Díaz-Pernil, D., Gutiérrez-Naranjo, M., Pérez-Jiménez, M., Riscos-Núñez, A.: A linear time solution to the partition problem in a cellular tissue-like model. Journal of Computational and Theoretical Nanoscience **7**, 884–889 (2010). https://doi.org/10.1166/jctn.2010.1435