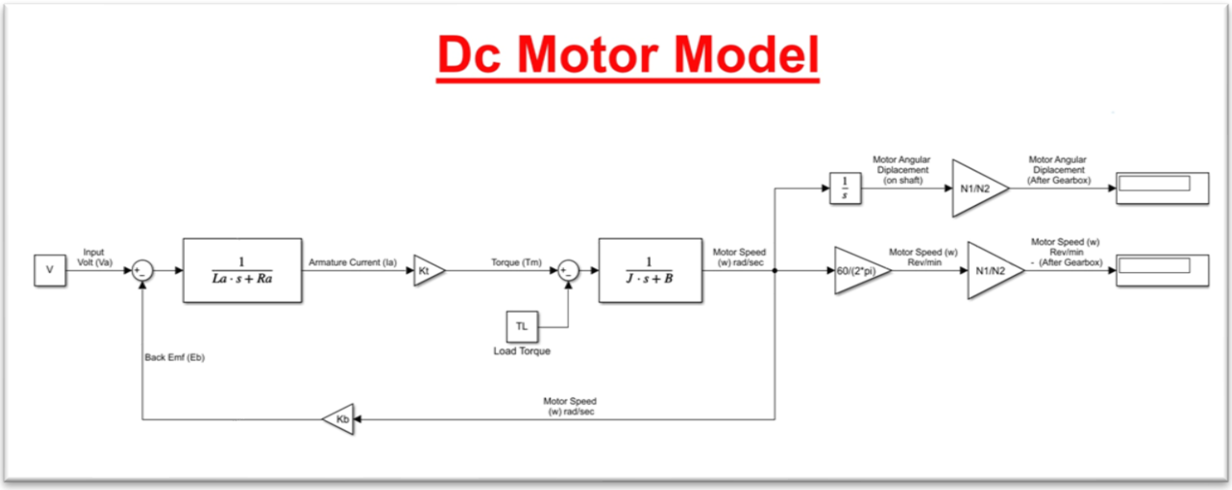
Mathematical Model of Dc Motor &Representation it

By Using Simulink



By

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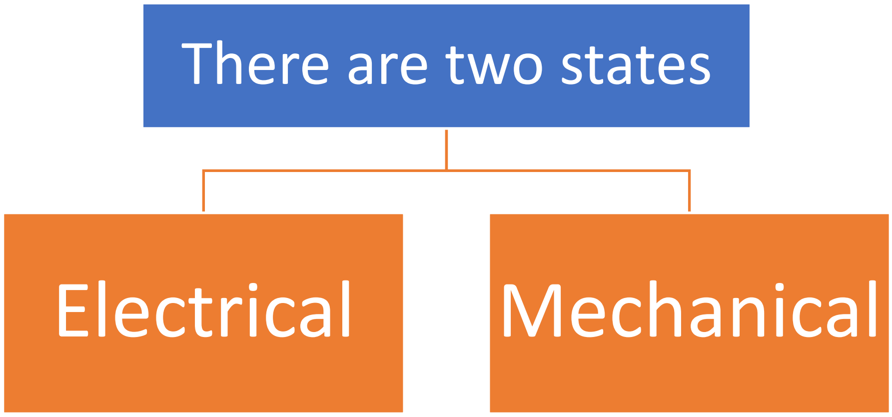

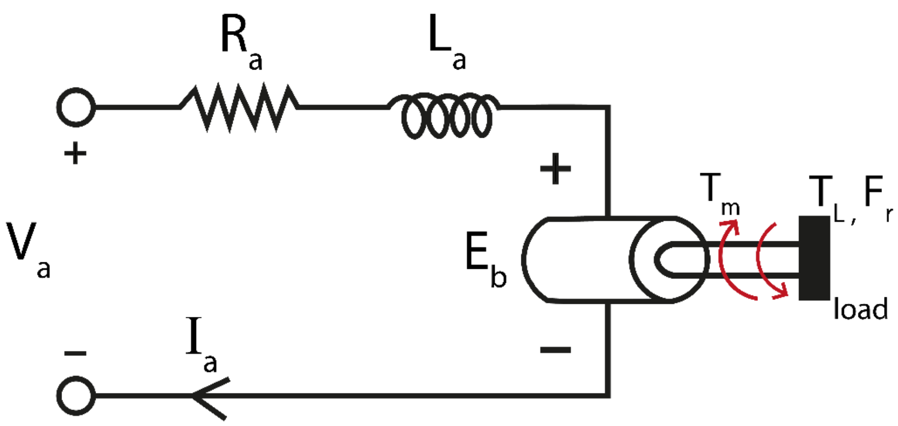

Figure 1 - The equivalent circuit for a DC motor

The usage parameters:

- V_a : $Input\ Voltage$
- R_a : $Armature\ Resistance$
- L_a : $Coil\ Resistance$
- I_a : $Armature\ Current$
- E_b : $Back\ EMF$
- T_m : $Torque\ generated\ by\ motor$
- T_L : $Load\ Torque$
- F_r : $Friction\ Force$
- B : $Friction$ coefficient
- J : $Motor\ Body\ Inertia$
- ω_n: $Motor\ Angular\ Speed$ $(\dot{\boldsymbol{\theta}})$
- θ_m: $Motor\ Angular\ Displacement$ $(\boldsymbol{\theta})$

Electrical:

$$\boxed{V_a = I_a R_a + L_a \frac{di}{dt} + E_b \qquad (1)}$$

Mechanical

$$\sum f = m.a$$

$$\sum T = J\ddot{\theta}$$

$$T_m - T_L - T_{friction} = J\frac{\omega_n}{dt}$$

$$\boxed{T_m = T_L + B.\omega_n + J\frac{\omega_n}{dt} \qquad (2)}$$

By taking laplace for two equations (1), (2)

$$V_a(s) = I_a(s)\,[\,R_a(s) + L_a(s).S\,] + E_b(s) \qquad (3)$$

$$I_a(s) = \frac{V_a(s) - E_b(s)}{R_a(s) + L_a(s).S} \qquad (4)$$

$$T_m = T_L + \omega_n\,[\,B + J.S\,] \qquad (5)$$

$$\omega_n = \frac{T_m - T_L}{[\,B + J.S\,]} \qquad (6)$$

$$\boldsymbol{T_m = K_t \, . \, I_a} \qquad (7)$$

$$\boldsymbol{I_a = \frac{T_m}{K_t}} \qquad (8)$$

$$\boldsymbol{E_b = K_b \, . \, \omega_n} \qquad (9)$$

From equations (3) , (8) , (9)

$$V_a(s) = \frac{\boldsymbol{T_m}}{\boldsymbol{K_t}}\,[\,R_a(s) + \ L_a(s).S\,] \ + \ \boldsymbol{K_b}\ .\ \boldsymbol{\omega_n} \qquad (10)$$

From equation (5), let that the load torque is = 0, ($T_L = 0$)

$$T_m = \omega_n\,[\,B + J.S\,] \qquad (11)$$

From equation (11) into equation (10):

$$V_a(s) = \frac{\omega_n\,[\,B + J.S\,]}{\boldsymbol{K_t}}\,[\,R_a(s) + \ L_a(s).S\,] + \ \boldsymbol{K_b}\ .\ \boldsymbol{\omega_n} \qquad (\mathbf{12})$$

$$V_a(s).\boldsymbol{K_t} = \omega_n\,[\,[\,B.R_a(s) + S(B.L_a(s) + R_a(s).J) + \ J.L_a(s).S^2\,] + \ \boldsymbol{K_b}.\boldsymbol{K_t}\ ] \quad (\mathbf{13})$$

$$\frac{\omega_n(s)}{V_a(s)} = \frac{\dfrac{\boldsymbol{K_t}}{J.L_a(s)}}{\dfrac{B.R_a(s)}{J.L_a(s)} + \dfrac{B.L_a(s) + R_a(s).J}{J.L_a(s)}.S + \ S^2 + \dfrac{\boldsymbol{K_b}.\boldsymbol{K_t}}{J.L_a(s)}} \qquad (14)$$

By simplifying equation (14) , we get the transfer function of DC motor between angular speed as output and the input voltage as an input.

$$\frac{\omega_n(s)}{V_a(s)} = \frac{\dfrac{\boldsymbol{K_t}}{J.L_a(s)}}{S^2 + \dfrac{B.L_a(s) + R_a(s).J}{J.L_a(s)}.S \ + \left(\dfrac{B.R_a(s) + \ \boldsymbol{K_b}.\boldsymbol{K_t}}{J.L_a(s)}\right)} \qquad (15)$$

The transfer function of speed control system $\left(\frac{\omega_n(s)}{V_a(s)} = \frac{\dot{\theta}(s)}{V_a(s)}\right)$:

If there a gearbox, we should to multiply by the gearbox ratio to get the speed of motor on the outer shaft:

$$\frac{\omega_n(s)}{V_a(s)} = \frac{\frac{\boldsymbol{K_t}}{J.L_a(s)}}{S^2 + \frac{B.L_a(s) + R_a(s).J}{J.L_a(s)}.S + \left(\frac{B.R_a(s) + \boldsymbol{K_b}.\boldsymbol{K_t}}{J.L_a(s)}\right)} \times \frac{N_1}{N_2} \quad (16)$$

The transfer function of position control system $\left(\frac{\boldsymbol{\theta}(s)}{V_a(s)}\right)$:

We can multiply the transfer function of speed control by integrator to get the transfer function of position control:

$$\frac{\boldsymbol{\theta}(s)}{V_a(s)} = \frac{\frac{\boldsymbol{K_t}}{J.L_a(s)} \times \frac{N_1}{N_2}}{S^2 + \frac{B.L_a(s) + R_a(s).J}{J.L_a(s)}.S + \frac{B.R_a(s)}{J.L_a(s)} + \frac{\boldsymbol{K_b}.\boldsymbol{K_t}}{J.L_a(s)}} \times \frac{1}{S} \quad (17)$$

$$\frac{\boldsymbol{\theta}(s)}{V_a(s)} = \frac{\frac{\boldsymbol{K_t}}{J.L_a(s)} \times \frac{N_1}{N_2}}{S^3 + \frac{B.L_a(s) + R_a(s).J}{J.L_a(s)}.S^2 + \left(\frac{B.R_a(s) + \boldsymbol{K_b}.\boldsymbol{K_t}}{J.L_a(s)}\right).S} \quad (18)$$

From equations (4),(6),(8),(9), can represents the DC motor model in Simulink as fig:

$$I_a(s) = \frac{V_a(s) \; - \; E_b(s)}{R_a(s) + \; L_a(s).S} \qquad (4)$$

$$\omega_n = \frac{T_m - T_L}{[\,B + J.S\,]} \qquad (6)$$

$$\boldsymbol{I_a = \frac{T_m}{K_t}} \qquad \mathbf{(8)}$$

$$\boldsymbol{E_b = K_b \,.\, \omega_n} \qquad \mathbf{(9)}$$

by applying motors parameters as in [1]

V = 12
Ra = 7.2
La =0.0917
Kt = 0.1236
Kb =0.1236
B = 0.0004
J = 0.0007046
TL = 0
N1 = 1
N2 = 1

Dc Motor Model

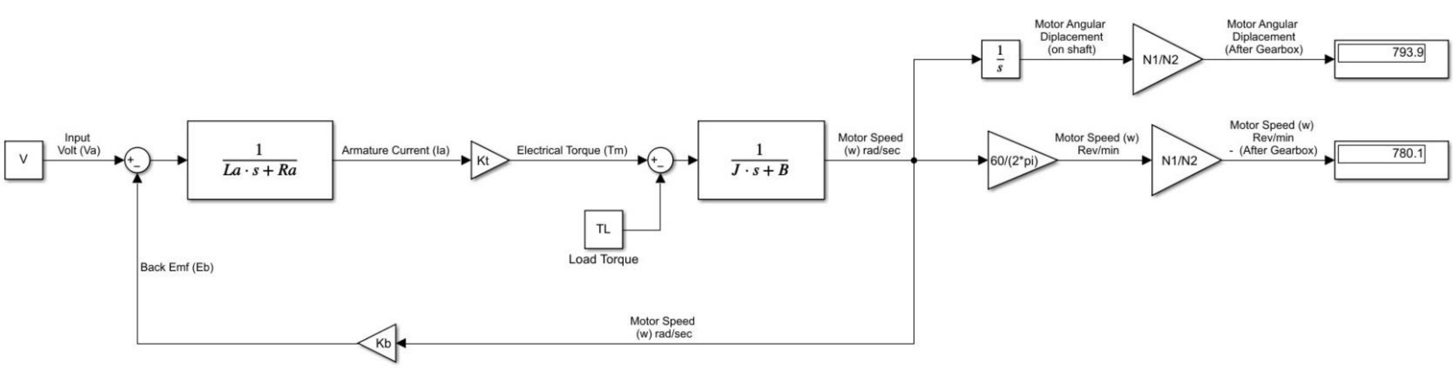

Figure 2 - Simulink model for DC motor

Figure 3 - Graph illustrate the relation between the speed and torque

References:

1- Mehta, S., & Chiasson, J. (1997). *Nonlinear control of a series DC motor: theory and experiment. Proceedings of the 1997 American Control Conference (Cat. No.97CH36041)*. doi:10.1109/acc.1997.611799

Dc Motor Model