Math501

Tutorial 1

Propositional logic

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- Course Assessment:

10 % Assignments (In class Assignments - Best 4 out of 5)
20 % Quizzes (Best 2 out of 3)
30 % Midterm Exam
40 % Final Exam

- What is Discrete Mathematics?

is mathematics that deals with discrete objects.

Discrete objects are those which are separated from each other. i.e. integers, rational no.'s (a/b), houses, people, etc.

"Discrete" → countable data → not continuous data

in our course discrete objects : Natural no.'s, propositions, functions, relations,…

- Building blocks of propositional logic:
  - **Propositions:** are statements associated with a truth value (either true or false).
  - **Logical connectives / Logical operators :** connect propositions(statements) to form compound statements. (used to describe the connection between two statements).
- Logical connectives:

$P \wedge q$

1. $a \wedge b$ **AND** (conjunction/ and / but / moreover/ also/ in addition/ nevertheless)
2. $a \vee b$ **OR** (Disjunction / inclusive or) $P \vee q$
3. $a \oplus b$ **XOR** (exclusive or) (meaning: either a or b, but not both) $P \oplus q \equiv (P \to \neg q) \wedge (\neg P \to q)$
4. $\neg a$ (Negation / It is not the case that a) $\neg P$
5. $a \to b$ (Implication / Conditional); a :antecedent , b :consequent $P \to q \equiv \neg P \vee q$
6. $a \leftrightarrow b$ (Equivalence / Biconditional) $P \leftrightarrow q \equiv (P \to q) \wedge (q \to P)$
7. $\top$ (always true) Tautology (always true)
8. $\bot$ (always false) Contradiction (always false)

- P: The weather is nice
- q: I will go to Sokhna

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- Rules:

a	T	$a \vee T$	$a \wedge T$	$\perp$	$a \vee \perp$	$a \wedge \perp$
t	t	t	t	f	t	f
f	t	t	f	f	f	f

1. $a \vee \top \equiv \top$
2. $a \wedge \top \equiv a$
3. $a \vee \perp \equiv a$
4. $a \wedge \perp \equiv \perp$
5. $a \wedge \neg a \equiv \perp$
6. $a \vee \neg a \equiv \top$
7. $\neg(a \wedge b) \equiv \neg a \vee \neg b$
8. $\neg(a \vee b) \equiv \neg a \wedge \neg b$
9. $a \rightarrow b \equiv \neg a \vee b$
10. $a \leftrightarrow b \equiv (a \rightarrow b) \wedge (b \rightarrow a)$

De Morgan's law (rules 7 and 8)

- <u>Rules:</u>

1. $a \vee \top \equiv \top$
2. $a \wedge \top \equiv a$
3. $a \vee \bot \equiv a$
4. $a \wedge \bot \equiv \bot$
5. $a \wedge \neg a \equiv \bot$
6. $a \vee \neg a \equiv \top$
7. $\neg(a \wedge b) \equiv \neg a \vee \neg b$
8. $\neg(a \vee b) \equiv \neg a \wedge \neg b$
9. $a \to b \equiv \neg a \vee b$
10. $a \leftrightarrow b \equiv (a \to b) \wedge (b \to a)$

De Morgan's law

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \rightarrow q \equiv \neg p \vee q$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$

$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Reference: Textbook
Discrete Mathematics and Its Applications - SEVENTH EDITION - **Kenneth H. Rosen**

- Truth table:

a	$\top$	$\bot$	$a \vee \top$	$a \wedge \top$	$a \vee \bot$	$a \wedge \bot$
t	t	f	t	t	t	f
f	t	f	t	f	f	f

- Terminology:
  - **Valuation:** is defined for propositions it can be either true or false.
  - **Interpretation:** meaning of a formula, it can be true or false.
    - $I(\top)=t$ (always true)
    - $I(\bot)=f$ (always false)
    - $I(\neg p) = \begin{cases} t \\ f \end{cases}$

- <u>Main aim:</u>

1. Translate English language → Logic
2. Problem → model it → deduce
3. We don't care about truth
4. Language composed of
   - syntax : what kind of sentences I can build by the language.
   - semantic: the meaning of the sentence.

- **Tautologies, Contradictions, and Contingencies:**

❖ **Tautology** is a statement that is true under any truth assignment to its variables (iff all interpretations true)
Example: $p \vee \neg p$, $(p \wedge p) \leftrightarrow p$, $p \rightarrow (q \rightarrow p)$

❖ **Contradiction** is a statement that is false under any truth assignment to its variables (iff all interpretations false)
Example: $p \wedge \neg p$, $p \leftrightarrow \neg(p \vee p)$

❖ **Satisfiable** is a statement that is not a contradiction (iff at least one interpretation is true)
Example: p

❖ **Falsifiable** is a statement that is not a tautology (iff at least one interpretation is false)
Example: $p \rightarrow q$

❖ **Contingency** is a statement that is both falsifiable and satisfiable (iff at least one interpretation is true & at least one interpretation is false)
Example: $p \vee (q \wedge \neg r)$

- Truth table:

$2^2 \langle^t_f = 4$ No. of valuations $= 2^n \rightarrow$ no. of propositions
$\langle^t_f$ or

p	q	pvq	p∧q	˥p	p→q	p↔q	˥pvq	q→p	(p→q)∧(q→p)	p⊕q
t	t	t	t	f	t	t	t	t	t	f
t	f	t	f	f	f	f	f	t	f	t
f	t	t	f	t	t	f	t	f	f	t
f	f	f	f	t	t	t	t	t	t	f

P: The weather is nice
q: I will go to Sokhna

if the weather is nice, then I will go to Sokhna.
$P \rightarrow q$

- Truth table:

p	q	pvq	p∧q	˥p	p→q	p↔q	˥pvq	q→p	(p→q)∧(q→p)	p⊕q
t	t	t	t	f	t	t	t	t	t	f
t	f	t	f	f	f	f	f	t	f	t
f	t	t	f	t	t	f	t	f	f	t
f	f	f	f	t	t	t	t	t	t	f

Example:

Determine which of the following are equivalent to each other:

(a) $(P \land Q) \lor (\neg P \land \neg Q)$

(b) $\neg P \lor Q$

(c) $(P \lor \neg Q) \land (Q \lor \neg P)$

(d) $\neg(P \lor Q)$

(e) $(Q \land P) \lor \neg P$

Solution:

p	q	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$	$\neg P \vee Q$	$(P \vee \neg Q) \wedge (Q \vee \neg P)$	$\neg(P \vee Q)$	$(Q \wedge P) \vee \neg P$
f	f	t	t	t	t	t
f	t	f	t	f	f	t
t	f	f	f	f	f	f
t	t	t	t	t	f	t

$(P \wedge Q) \vee (\neg P \wedge \neg Q) \equiv (P \vee \neg Q) \wedge (Q \vee \neg P)$

$(\neg P \vee Q) \equiv ((Q \wedge P) \vee \neg P)$

- **Precedence of Logical Operators:**

1. $\neg a$ (Negation / It is not the case that a)
2. $a \wedge b$ **AND** (conjunction / and / but / moreover/ also / in addition)
3. $a \vee b$ **OR** (Disjunction / inclusive or)
4. $a \rightarrow b$ (Implication / Conditional); a :antecedent , b :consequent
5. $a \leftrightarrow b$ (Equivalence / Biconditional)

- Boolean Algebra:

Example 1:

Show that $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

note:

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$LHS \equiv \neg(p \vee (\neg p \wedge q))$

$\equiv \neg(\underbrace{(p \vee \neg p)}_{T} \wedge (p \vee q))$

$\equiv \neg(p \vee q)$

$\equiv \neg p \wedge \neg q \equiv RHS$

- Boolean Algebra:

Example 2:

Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

$$\equiv \neg(p \wedge q) \vee (p \vee q)$$

$$\equiv (\neg p \vee \neg q) \vee (p \vee q)$$

$$\equiv (\underbrace{\neg p \vee p}_{T}) \vee (\underbrace{\neg q \vee q}_{T})$$

$$\equiv T \quad \therefore \text{tautology} \#$$

Note:

$$p \rightarrow q \equiv \neg p \vee q$$

- <u>Implication:</u>

p q

➢ If the weather is nice, then I will go to sokhna

Antecedent <u>or</u> Premises <u>or</u> Hypothesis

Consequent <u>or</u> conclusion

- **p:** The weather is nice
- **q:** I will go to sokhna
- If the weather is nice, then I will go to sokhna: $p \rightarrow q$
- **If p then q**
- **p implies q**
- **p therefore q**
- **q if p** $p \rightarrow q$
- **p is a sufficient condition for q**
- **p only if q** $p \rightarrow q$
- **q provided that p**
- **q follows from p**
- **q is a necessary condition for p**
- **q when p**
- **q unless ˥p**
- **q whenever p**

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- <u>Examples:</u>

L O

① . If there is life, there is oxygen.

$$L \to O$$

- Life is a sufficient condition for oxygen.

$$L \to O$$

- oxygen is a necessary condition for life

$$\neg O \to \neg L \equiv L \to O$$

$$\equiv O \vee \neg L \qquad \equiv \neg L \vee O$$

2

- $\underbrace{\text{If it's sunny}}_{p}$, then $\underbrace{\text{I wear a hat}}_{q}$.

$$p \to q \equiv \neg p \vee q$$

- If $\underbrace{\text{I'm not wearing a hat}}$, then $\underbrace{\text{it's not sunny}}$

$$\neg q \to \neg p \equiv q \vee \neg p$$

$\equiv$

- $\underbrace{\text{I wear a hat}}_{q}$ $\boxed{\text{only if}}$ $\underbrace{\text{it's sunny}}_{p}$.

$$q \to p \equiv \neg q \vee p$$

- If $\underbrace{\text{it isn't sunny}}_{\neg p}$, then there's no $\underbrace{\text{way that I'm wearing a hat}}_{\neg q}$.

$$\neg p \to \neg q \equiv p \vee \neg q$$

$\equiv$

- $\underbrace{x \text{ is greater than } 2}_{p}$ only if $\underbrace{x \text{ is greater than } 1}_{q}$.

$$p \to q$$

- Biconditional: $p \Leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$q \to p$ $p \to q$

  - p is necessary and sufficient for q

  $p \to q$
  - if p then q, and conversely $q \to p$
  - p iff q ; (iff : if and only if)

- q if p $p \to q$
- q only if p $q \to p$
- p if & only if q

$(q \to p) \wedge (p \to q)$

$\hookrightarrow$ p if q $\hookrightarrow$ p only if q

- <u>Conditional Statement:</u>

Conditional statement: $p \to q$ $\equiv \neg p \vee q$

$\equiv$

converse of $p \to q$: $q \to p$ $\equiv \neg q \vee p$

contrapositive of $p \to q$: $\neg q \to \neg p$ $\equiv q \vee \neg p$

inverse of $p \to q$: $\neg p \to \neg q$ $\equiv p \vee \neg q$

$\equiv$

- **p:** The weather is nice
- **q:** I will go to sokhna
- **Conditional statement:** If the weather is nice, then I will go to sokhna : $p \to q \equiv \neg p \vee q$
- **Converse statement:** I will go to sokhna only if the weather is nice: $\boxed{q \to p} \equiv \neg q \vee p$
- **Contrapositive statement:** If I didn't go to sokhna, then the weather isn't nice: $\boxed{\neg q \to \neg p} \equiv q \vee \neg p$
- **Inverse statement:** If the weather isn't nice, then I will not go to sokhna: $\boxed{\neg p \to \neg q} \equiv p \vee \neg q$

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Example:

Let p and q be the propositions

p : It is below freezing.
q : It is snowing.

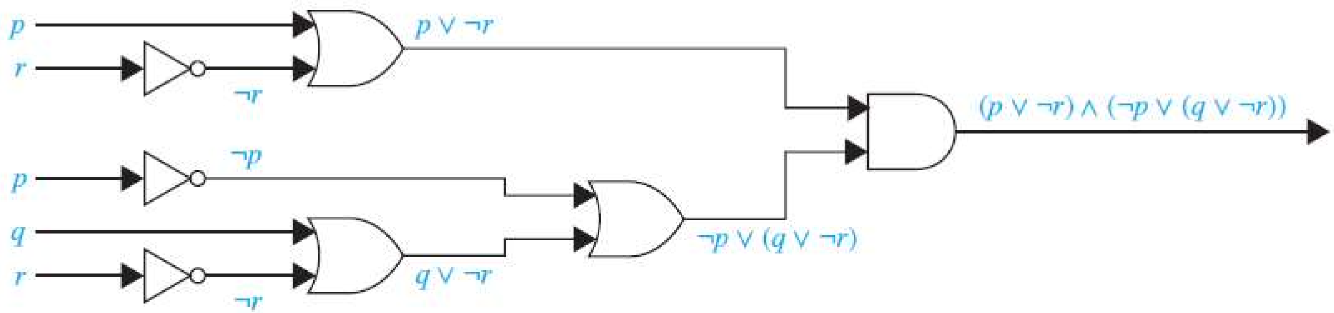
Write these propositions using p and q and logical connectives (including negations).

a) It is below freezing and snowing. $p \wedge q$
b) It is below freezing but not snowing. $p \wedge \neg q$
c) It is not below freezing and it is not snowing. $\neg p \wedge \neg q$
d) It is either snowing or below freezing (or both). $p \vee q$
e) If it is below freezing, it is also snowing. $p \rightarrow q$
f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing. $(p \vee q) \wedge (p \rightarrow \neg q)$
g) That it is below freezing is necessary and sufficient for it to be snowing. $q \leftrightarrow p$

- <u>Applications of Propositional Logic:</u>

Example:

Build a digital circuit that produces the output $(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))$ when given input bits p, q, and r.



Solution: To construct the desired circuit, we build separate circuits for $p \vee \neg r$ and for $\neg p \vee (q \vee \neg r)$ and combine them using an AND gate. To construct a circuit for $p \vee \neg r$, we use an inverter to produce $\neg r$ from the input r. Then, we use an OR gate to combine p and $\neg r$. To build a circuit for $\neg p \vee (q \vee \neg r)$, we first use an inverter to obtain $\neg r$. Then we use an OR gate with inputs q and $\neg r$ to obtain $q \vee \neg r$. Finally, we use another inverter and an OR gate to get $\neg p \vee (q \vee \neg r)$ from the inputs p and $q \vee \neg r$.

To complete the construction, we employ a final AND gate, with inputs $p \vee \neg r$ and $\neg p \vee (q \vee \neg r)$.

Practice Assignment 1:

Statement that could be either true or false

Exercise 1–1

T/F

Which of these sentences are propositions? What are the truth values of those that are propositions?

(a) Boston is the capital of Massachusetts. Yes, T

(b) Miami is the capital of Florida. Yes, F

(c) $2 + 3 = 5$. Yes, T

(d) $5 + 7 = 10$. Yes, F

(e) $x + 2 = 11$. NO

(f) Answer this question. NO

Exercise 1–4

What is the negation of each of these propositions?

p

(a) Steve has more than 100 GB free disk space on his laptop. $\neg p$: Steve doesn't have more than 100 GB free disk space on his laptop.

(b) Zach blocks e-mails (p) and texts from Jennifer (q). $\neg(p \land q) \equiv \neg p \lor \neg q$: Zach doesn't block emails from Jennifer or he doesn't block texts from Jennifer.

(c) $7 \times 11 \times 13 = 999$. $7 \times 11 \times 13 \neq 999$

p

(d) Diane rode her bicycle 100 miles on Sunday. $\neg p$: Diane didn't ride her bike 100 miles on Sunday.

Exercise 1–5

p, q, and r be the propositions

- p: You get an A on the final exam.
- q: You do every exercise in this book.
- r: You get an A in this class.

(a) You get an A in this class, but you do not do every exercise in this book. $r \wedge \neg q$

(b) You get an A on the final, you do every exercise in this book, and you get an A in this class. $p \wedge q \wedge r$

(c) To get an A in this class, $\boxed{\text{it is necessary}}$ for you to get an A on the final. $r \rightarrow p$

(d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class. $p \wedge \neg q \wedge r$

(e) Getting an A on the final and doing every exercise in this book $\boxed{\text{is sufficient}}$ for getting an A in this class. $(p \wedge q) \rightarrow r$

(f) You will get an A in this class $\boxed{\text{if and only if}}$ you either do every exercise in this book or you get an A on the final.

$r \leftrightarrow (q \vee p)$

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Exercise 1–6

Write each of these statements in the form “if p, then q”.

(a) I will remember to send you the address $\boxed{\text{only if}}$ you send me an e-mail message. $p \to q$: If I remember to send you the address, then you will have to send me an email message.

(b) To be a citizen of this country, it is sufficient that you were born in the United States. $p \to q$: If you were born in the U.S, then you are a citizen of this country.

(c) If you keep your textbook, it will be a useful reference in your future courses. $p \to q$

(d) The Red Wings will win the Stanley Cup if their goalie plays well. $p \to q$

(e) That you get the job implies that you had the best credentials. $p \to q$

(f) The beach erodes whenever there is a storm. $p \to q$

(g) It is necessary to have a valid password to log on to the server. $p \to q$

(h) You will reach the summit unless you begin your climb too late. $p \to q$

(i) You will get a free ice cream cone, provided that you are among the first 100 customers tomorrow.

(c) If you keep your textbook, then it will be a useful reference in your future courses.

(d) If their goalie plays well, then the Red wings will win the Stanley cup.

(e) If you get the job, then you had the best credentials

(f) If there is a storm, then the beach erodes.

(g) If you log on to the server, then you have a valid password

(h) If you don't begin your climb too late, then you will reach the summit.

(i) If you are among the first 100 customers tomorrow, then you will get a free ice cream cone.

Exercise 1–8

Explain, without using a truth table, why $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.

&

Clause 1 : $p \vee q \vee r$

Clause 2 : $\neg p \vee \neg q \vee \neg r$

Case 1 : $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ true

if any of the 3 propositions (p,q,r) is true then clause 1 will always be true & if any of the 3 propositions (p,q,r) is false then its negation is true then clause 2 will always be true. Therefore,

$T \wedge T \equiv T$

Case 2 : $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ false

* if p,q,r are true then clause 1 will always be true, however clause 2 will always be false. Thus $T \wedge F \equiv F$
* if p,q,r are false then clause 1 will always be false thus $F \wedge T \equiv F$

Exercise 1–9

What is the value of x after each of these statements is encountered in a computer program, if x = 1 before the statement is reached?

(a) if $x + 2 = 3$ then $x := x + 1$ True; $x = 2$

(b) if $(x + 1 = 3)$ OR $(2x + 2 = 3)$ then $x := x + 1$ False; $x = 1$

(c) if $(2x + 3 = 5)$ AND $(3x + 4 = 7)$ thenx $:= x + 1$ True; $x = 2$

(d) if $(\underbrace{x + 1 = 2}_{p})$ XOR $(\underbrace{x + 2 = 3}_{q})$ then $x := x + 1$ False; $x = 1$ XOR "either p or q but not both"

(e) if $x < 2$ then $x := x + 1$ True; $x = 2$

Exercise 1–18

always true

Show that each of these conditional statements is a tautology by using truth tables.

$\because p \to q \equiv \neg p \vee q$

(a) $[\neg p \wedge (p \vee q)] \to q$

(b) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

(c) $[p \wedge (p \to q)] \to q$

(d) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

Approach 1: (using Truth table)

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
t	t	f	t	f	t
t	f	f	t	f	t
f	t	t	t	t	t
f	f	t	f	f	t

$\therefore$ Tautology

Approach 2: (Boolean Algebra)

$\neg[\neg p \wedge (p \vee q)] \vee q$

$\neg[(\underbrace{\cancel{\neg p \wedge p}}_{\bot}) \vee (\neg p \wedge q)] \vee q$

$\neg[\neg p \wedge q] \vee q$

$\to p \vee \underbrace{\neg q \vee q}_{T}$

$p \vee T \equiv T$

$\therefore$ Tautology

Exercise 1–23

Determine whether each of these compound propositions is satisfiable.

(a) $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$

(b) $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg s) \wedge (\neg p \vee \neg r \vee \neg s) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg r \vee \neg s)$

(c) $(p \vee q \vee r) \wedge (p \vee \neg q \vee \neg s) \wedge (q \vee \neg r \vee s) \wedge (\neg p \vee r \vee s) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg r)(\neg p \vee \neg q \vee s) \wedge (\neg p \vee \neg r \vee \neg s)$

(a) The main aim is to find out the truth assignments for p, q, r, s such that the compound proposition is true. Thus each clause must be true. As we can see p occurs in 4 clauses out of the 5 clauses. Thus, setting $\tau(p) = t$

we remain with clause 5 : $\neg p \vee \neg q \vee \neg s$ in order to be true either set $\tau(q) = f$ or $\tau(s) = f$.

$\therefore$ $\tau(p) = t$, $\tau(q) = f$ & $\tau(r)$ & $\tau(s)$ could be whatsoever the statement will always be true!

Exercise 1–25

As a reward for saving his daughter from pirates, the King has given you the opportunity to win a treasure hidden inside one of three trunks. The two trunks that do not hold the treasure are empty. To win, you must select the correct trunk. Trunks 1 and 2 are each inscribed with the message "This trunk is empty," and Trunk 3 is inscribed with the message "The treasure is in Trunk 2." The Queen, who never lies, tells you that only one of these inscriptions is true, while the other two are wrong. Which trunk should you select to win?

* fact 1: Treasure hidden in one of three trunks & the other two are empty.
* fact 2: Only one inscribed message is true & other two are false.

Inscribed message P_i on T_i:

P_1 on T_1: T_1 is empty
P_2 on T_2: T_2 is empty
P_3 on T_3: Treasure in T_2

P_1	P_2	P_3	Consistent?
T	F	F	Inconsistent
F	T	F	Consistent
F	F	T	Inconsistent

→ T_2 isn't empty & Treasure not in T_2
→ T_1 isn't empty & T_2 isn't empty & we have the fact① that only one trunk has the treasure.

Note 1:
* fact's are always true.
* Inscribed message might be true or false. You have to study all possible scenario's & check the consistency of the statements.

Note 2: we narrowed down the truth table using fact②

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Exercise 1–27

Suppose that in Exercise 25 there are treasures in two of the three trunks. The inscriptions on Trunks 1, 2, and 3 are "This trunk is empty," "There is a treasure in Trunk 1," and "There is a treasure in Trunk 2." For each of these statements, determine whether the Queen who never lies could state this, and if so, which two trunks the treasures are in.

Total no. of valuations $= 2^3 = 8$

(a) "All the inscriptions are false." NO, Inscribed messages P_1 & P_2 are contradicting each other.

(b) "Exactly one of the inscriptions is true." yes, if $P_1: F$, $P_2: T$, $P_3: F$ Treasure in T_1 & T_3

(c) "Exactly two of the inscriptions are true." yes, if $P_1: F$, $P_2: T$, $P_3: T$ Treasure in T_1 & T_2 or $P_1: T$, $P_2: F$, $P_3: T$ Treasure in T_2 & T_3

(d) "All three inscriptions are true." NO, Inscribed message P_1 & P_2 are contradicting

* fact 1: Treasure in two of three trunks.

Inscribed message P_i on T_i:

- P_1 on T_1: T_1 is empty
- P_2 on T_2: Treasure in T_1

] Contradiction

- P_3 on T_3: Treasure in T_2

	P_1	P_2	P_3	Consistent ?	Treasure in Trunk
(a) P_1 & P_2 Contradiction	F	F	F	Inconsistent	
	F	F	T	Inconsistent	
(b)	F	T	F	Consistent	T_1 & T_3
(c)	F	T	T	Consistent	T_1 & T_2
Contradicts fact 1	T	F	F	Inconsistent	
(c)	T	F	T	Consistent	T_2 & T_3
	T	T	F	Inconsistent	
(d) P_1 & P_2 Contradiction	T	T	T	Inconsistent	

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Exercises 28-32 relate to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusions?

Exercise 1–28

A says "At least one of us is a knave" and B says nothing.

Claim A: At least one of us is lying (one or both lying)

Note:

* knave: always lying
* Knight: always telling truth
* If A is lying then claim A must be false.
* If A is telling truth then claim A must be true.

Assume A is knight & B is knight

A	B	Claim A	Consistent?
F	F	T	Inconsistent
F	T	T	Inconsistent
T	F	T	Consistent
T	T	F	Inconsistent

If A is lying then claim A cannot be true!

If A is telling truth then claim A cannot be false!

Thus according to the consistent row A is knight & B is knave

Exercise 1–34

Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known?

(a) Either Kevin or Heather, or both, are chatting. $K \vee H \equiv \neg K \to H$

(b) Either Randy or Vijay, but not both, are chatting. $(R \vee V) \wedge \neg(R \wedge V) \equiv (R \vee V) \wedge (\neg R \vee \neg V) \equiv (\neg R \to V) \wedge (R \to \neg V)$

(c) If Abby is chatting, so is Randy. $A \to R$

(d) Vijay and Kevin are either both chatting or neither is. $(V \wedge K) \vee (\neg V \wedge \neg K) \equiv (\neg V \vee K) \wedge (\neg K \vee V) \equiv (V \to K) \wedge (K \to V)$

(e) If Heather is chatting, then so are Abby and Kevin. $H \to (A \wedge K) \equiv \neg H \vee (A \wedge K) \equiv (\neg H \vee A) \wedge (\neg H \vee K) \equiv (H \to A) \wedge (H \to K)$

Explain your reasoning.

Step 1: all statements are converted to conditional statement.

Step 2: In order to attain all statements & since all statements are anded together then each st. must be true.

- If $T(H) = t$ then $T(A)$ & $T(K)$ must be true so that $\boxed{T(H \to A) \text{ \& } T(H \to K)}$ are true. (e) accordingly, $T(R)$ must be true to satisfy $\boxed{T(A \to R) = t}$. (c) Thus V must be false; (b) $\boxed{T(R \to \neg V) = t}$
  $\to \boxed{T(\neg K \to H) = t}$ & (a)

however, $\boxed{I(K \to V) = \text{false}}$ (d)

- If $I(H) = f$ then $I(K)$ must be true; $\overbrace{I(\neg K \to H) = \text{true}}^{a}$; accordingly $I(V)$ must be true such that $\underbrace{I(V \to K) = \text{true} \;\&\; I(K \to V) = \text{true}}$. Then $I(R)$ must be false such that $\underbrace{I(R \to \neg V) = \text{true} \text{ (d) } \& \; I(\neg R \to V) = \text{true}}_{(b)}$.

Consequently, $I(A)$ must be false such that $\underbrace{I(A \to R) = \text{true}}_{(c)}$, & already (e) $I(H \to A)$ & $I(H \to K)$ are both true. Thus all statements are true. Therefore

$$
\begin{aligned}
I(H) &= f \\
I(K) &= T \\
I(V) &= T \\
I(R) &= f \\
I(A) &= f
\end{aligned}
$$

Note:

$I(\ldots)$

↳ Stands for interpretation!

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Best of luck !!

Thank you.

Any Questions!