Math501

Tutorial 2

Arguments in PL

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- <u>Conditional Statement:</u>

Conditional statementt (Material implication): $p \to q$ $\equiv \neg p \lor q$

$\equiv$

converse of $p \to q$: $q \to p$ $\equiv \neg q \lor p$

contrapositive of $p \to q$: $\neg q \to \neg p$ $\equiv q \lor \neg p$

inverse of $p \to q$: $\neg p \to \neg q$ $\equiv p \lor \neg q$

$\equiv$

- **p:** The weather is nice
- **q:** I will go to sokhna
- **Conditional statement:** If the weather is nice, then I will go to sokhna : $p \to q \equiv \neg p \lor q$
- **Converse statement:** I will go to sokhna only if the weather is nice: $\boxed{q \to p} \equiv \neg q \lor p$
- **Contrapositive statement:** If I didn't go to sokhna, then the weather isn't nice: $\boxed{\neg q \to \neg p} \equiv q \lor \neg p$
- **Inverse statement:** If the weather isn't nice, then I will not go to sokhna: $\boxed{\neg p \to \neg q} \equiv p \lor \neg q$

• Arguments:

- An argument is a finite set of statements.
- The last statement is the conclusion.
- All the preceding statements are the premises (hypothesis).
- A mathematical proof is an argument.

• Form of an Argument:

- Let the **premises** be $P_1, P_2, \ldots, P_n$.
- Let the **conclusion** be q.
- The argument form is $\underbrace{(P_1 \wedge P_2 \wedge \cdots \wedge P_n)}_{\text{Premises}} \rightarrow \underbrace{q}_{\text{Conclusion}}$ or

$$\begin{array}{l} \left.\begin{array}{l} P_1 \\ P_2 \\ \vdots \\ P_n \end{array}\right\} \text{Premises (hypothesis)} \\ \hline \therefore q \} \text{ Conclusion} \end{array}$$

- **Validity of an Argument:**

always true

- The argument form is **valid** if $(P_1 \wedge P_2 \wedge \cdots \wedge P_n) \rightarrow q$ is a tautology
- Otherwise, the argument form is invalid.
- An invalid argument form is called a fallacy.

Remember:

The implication is always true, except when the antecedent is true & the Consequent is false

$(t \rightarrow f)$

Note:

- Statements can be either true or false.
- Arguments can be either valid or invalid.

- **Rules of inference:**

Rules of inference: Other rules are missing (e.g., commutativity, associativity, double negation, De Morgan's, etc.)

Name (abbreviation) of the rule:		The rule:
Implication	(IMP)	$P \to Q \equiv \neg P \vee Q$.
Contrapositive	(CONT)	$P \to Q \equiv \neg Q \to \neg P$.
Modus Ponens	(MP)	from P and $P \to Q$, deduce Q.
Modus Tollens	(MT)	from $P \to Q$ and $\neg Q$, deduce $\neg P$.
Simplification	(SIMP)	from $P \wedge Q$, deduce P (or deduce Q).
Conjunction	(CONJ)	from P and Q, deduce $P \wedge Q$.
Hypothetical Syllogism	(HS)	from $P \to Q$ and $Q \to R$, deduce $P \to R$.
Disjunctive Syllogism	(DS)	from $P \vee Q$ and $\neg P$, deduce Q.
Addition	(ADD)	from P, deduce $P \vee Q$.
Negation Introduction	(NI)	from $P \to Q$ and $P \to \neg Q$, deduce $\neg P$.
Negation Elimination	(NE)	from P and $\neg P$, deduce Q.

- **Rules of inference:**

Modus Ponens (**MP**)	$\begin{array}{l} p \\ p \to q \\ \hline \therefore\ q \end{array}$	Modus Tollens (**MT**)	$\begin{array}{l} \neg q \\ p \to q \\ \hline \therefore\ \neg p \end{array}$
Hypothetical Syllogism (**HS**)	$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore\ p \to r \end{array}$	Disjunctive Syllogism (**DS**)	$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore\ q \end{array}$
Addition	$\begin{array}{l} p \\ \hline \therefore\ p \vee q \end{array}$	Simplification	$\begin{array}{l} p \wedge q \\ \hline \therefore\ p \end{array}$
Conjunction	$\begin{array}{l} p \\ q \\ \hline \therefore\ p \wedge q \end{array}$	Resolution	$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore\ q \vee r \end{array}$

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 7 Logical Equivalences Involving Conditional Statements.

$$p \rightarrow q \equiv \neg p \vee q$$
$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$
$$p \vee q \equiv \neg p \rightarrow q$$
$$p \wedge q \equiv \neg(p \rightarrow \neg q)$$
$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$
$$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$$
$$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$$
$$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$$
$$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$
$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$
$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$
$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

Reference: Textbook
Discrete Mathematics and Its Applications - SEVENTH EDITION - **Kenneth H. Rosen**

• Rules of Logical Equivalence:

- Commutativity:
  1. $p \vee q \equiv q \vee p.$
  2. $p \wedge q \equiv q \wedge p.$
- Associativity:
  1. $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  2. $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- De Morgan's laws:
  1. $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  2. $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- Distributive Properties:
  1. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
  2. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- Implication: $p \rightarrow q \equiv \neg p \vee q$
- Double Negation: $\neg\neg p \equiv p$
- Equivalence: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

- **Formal proof:**

➢ A formal proof is used to proof that a given argument is valid or in other words to prove that the conclusion follows from the hypothesis.

➢ A formal proof is a sequence of statements in which each statement is either a hypothesis or the result of applying a predefined set of derivation rules (inference rules or logically equivalent rules).

➢ Inference rules are used to drive new statements from previous statements in the proof.

➢ Inference rules are valid (always true).

Remember:

The argument form is $\underbrace{(P_1 \wedge P_2 \wedge \cdots \wedge P_n)}_{\text{hypothesis}} \rightarrow \underbrace{q}_{\text{conclusion}}$ or

$$\left.\begin{array}{l} P_1 \\ P_2 \\ \vdots \\ P_n \end{array}\right\} \text{hypothesis}$$
$$\overline{\therefore q} \; \} \text{ conclusion}$$

- **Summary to prove the validity of an argument:**

Rule of thumb:

used only to prove that an argument is valid.

① Truth table
* Construct a truth table with valuation $= 2^n$, n propositions & check that the argument is a tautology.

② Boolean Algebra
* use the rules of logical equivalences to prove that the argument will always be true.

③ Interpretations
* If possible to make the argument $(t \to f)$ then the argument is invalid.
* If impossible to make the argument $(t \to f)$ then the argument is valid.

Steps:
① Start by setting conseq. to false
② Try to make the antecedent true
③ If possible to be $t \to f$ $\therefore$ invalid

④ Rules of inference
* use
① hypothesis
② Rules of inference
③ Rules of logical equivalences
In order to reach conclusion.

Prove the validity of the Disjunctive Syllogism (DS)

$$[(P \vee Q) \wedge \neg P] \rightarrow Q$$

a. Use truth table
b. Use Boolean Algebra (Rules of Logical Equivalence)
c. Use Interpretation
d. Use Formal proof

a) using truth table:

P	Q	P∨Q	¬P	(P∨Q)∧¬P	[(P∨Q)∧¬P]→Q
t	t	t	f	f	t
t	f	t	f	f	t
f	t	t	t	t	t
f	f	f	t	f	t

∴ Valid

Note:
To prove the validity of an argument, you can use any approach unless mentioned a specific approach that you must follow.

b) using boolean algebra:

$$[(P \vee Q) \wedge \neg P] \rightarrow Q$$

$$\equiv \neg[(P \vee Q) \wedge \neg P] \vee Q$$

$$\equiv \neg[(\cancel{P \wedge \neg P}) \vee (Q \wedge \neg P)] \vee Q$$

$\bot$

$$\equiv \neg(Q \wedge \neg P) \vee Q$$

$$\equiv \neg Q \vee P \vee Q$$

$$\equiv P \vee T$$

$$\equiv T \quad \therefore \text{valid}$$

c) using interpretation:

* The main idea is if you are able to state that the argument is impossible to be $t \rightarrow f$ then the argument is valid.

Why!?

because the implication is always true except when true $\rightarrow$ false.

Thus if it is impossible to be $t \rightarrow f$ then the argument is valid.

Thus we are aiming to set the consequence to false & check if possible to set the antecedent to true.

* $I(Q) = f$
* check if possible to set the antecedent to true!?

* If $I(P)=t$ then $I(P \vee Q)=t$
$I(\neg P)=f$
thus $I((P \vee Q) \wedge \neg P)=f$
thus antecedent failed to be true if $I(P)=t$.

* If $I(P)=f$ then $I(P \vee Q)=f$
$I(\neg P)=t$
then $I((P \vee Q) \wedge \neg P)=f$
thus antecedent failed to be true if $I(P)=f$.

Thus impossible to obtain

argument is valid.

d) Using Formal Proof:

1] $P \vee Q$ (hypothesis)
2] $\neg P$ (hypothesis)
3] Q (1,2 Resolution)

$$\text{Prove that } \underbrace{(\underbrace{((A \to B) \vee \neg D \vee C)} \wedge \underbrace{A} \wedge \underbrace{\neg B} \wedge \underbrace{E})}_{\text{hypothesis}} \to \underbrace{(D \to C)}_{\text{conclusion}} \text{ is a valid argument.}$$

a. Construct formal proof
b. Use Interpretation

Rules of inference: Other rules are missing (e.g., commutativity, associativity, double negation, De Morgan's, etc.)

Name (abbreviation) of the rule:		The rule:
Implication	(IMP)	$P \to Q \equiv \neg P \vee Q$.
Contrapositive	(CONT)	$P \to Q \equiv \neg Q \to \neg P$.
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Addition	(ADD)	from P, deduce $P \vee Q$.
Negation Introduction	(NI)	from $P \to Q$ and $P \to \neg Q$, deduce $\neg P$.
Negation Elimination	(NE)	from P and $\neg P$, deduce Q.

a) Construct formal proof:

1. $(A \to B) \vee (\neg D \vee C)$ (hypothesis)
2. $A \wedge \neg B$ (hypothesis)
3. $\neg(\neg A \vee B)$ (2, De Morgan's)
4. $\neg(A \to B)$ (3, Implication)
5. $\neg D \vee C$ (1, 4, DS)
6. $D \to C$ (5, Implication)

Note: In this approach (Construct formal proof) we use the hypothesis, rules of inference & rules of logical equivalences in order to reach the conclusion.

b) using interpretation:

$$\overbrace{(((A\to B)\vee\neg D\vee C)\wedge A\wedge\neg B\wedge E)}^{F}\to\overbrace{(D\to C)}^{G}$$

To prove the validity of the argument we will discuss the cases such that $\mathfrak{I}(D\to C)=$ false

We aim to find out that Impossible to make the antecedent true such that implication $(F\to G)$ to be valid.

$\mathfrak{I}(D\to C)=f$ thus $\mathfrak{I}(D)=t$ & $\mathfrak{I}(C)=f$

* for the antecedent to be true:

$\mathfrak{I}(A)=t$ & $\mathfrak{I}(\neg B)=t \Rightarrow \mathfrak{I}(B)=f$ & $\mathfrak{I}(E)=t$

$\therefore \mathfrak{I}(A\to B)=f$ $\therefore \mathfrak{I}(\neg D\vee C)=f$

$\therefore \mathfrak{I}((A\to B)\vee\neg D\vee C)=f$

Thus the compound statement $(A\to B)\vee\neg D\vee C$ will be false thus antecedent will never be true. Thus $F\to G$ is valid.

Let A, B, C, D, and E correspond to the propositions:

A: THE QUESTION IS EASY

B: I AM HAPPY

C: I AM BATMAN

D: MONEY SOLVES ALL PROBLEMS

E: LOGIC CHANGES THE WAY I THINK

Prove that we can conclude: if MONEY SOLVES ALL PROBLEMS, then I AM BATMAN. from the following hypotheses: $D \to C$

- LOGIC CHANGES THE WAY I THINK E
- THE QUESTION IS EASY but I AM SAD $A \wedge \neg B$
- I AM BATMAN or it is not the case that MONEY SOLVES ALL PROBLEMS, or if THE QUESTION IS EASY, then I AM HAPPY. $C \vee \neg D \vee (A \to B)$

$$\therefore \Big[\underbrace{\big([(A \to B) \vee \neg D \vee C] \wedge A \wedge \neg B \wedge E\big)}_{\text{hypothesis}} \longrightarrow \underbrace{(D \to C)}_{\text{conclusion}}\Big] \text{ Argument}$$

$\Rightarrow$ Already proved using 2 different approaches (Slides 14&15)

Practice Assignment 2:

Exercise 1–1 "Foundational!"

What rule of inference[1] is used in each of these arguments?

a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major. Addition $(M) \to M \vee C$

$(M \wedge C) \to M$ Simplification

b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.

c) If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed. $[(r \to c) \wedge r] \to c$ Modus ponens

d) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today. $[(S \to C) \wedge \neg C] \to \neg S$ Modus tollens

e) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

$[(W \to S) \wedge (S \to b)] \to (W \to b)$ Hypothetical Syllogism

(f) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials. $[p \wedge q] \rightarrow q$ Simplification

(g) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous. $[(p \vee q) \wedge \neg p] \rightarrow q$ Disjunctive Syllogism

(h) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard. $[p \wedge (p \rightarrow q)] \rightarrow q$ Modus Ponens

(i) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum. $p \rightarrow (p \vee q)$ Addition

(j) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material. $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ Hypothetical Syllogism

Exercise 1–3 “Basics-2”

Determine whether or not an argument of the following form is valid:

(a)
$$\begin{array}{l} p \to q \\ \neg p \\ \hline \therefore \neg q \end{array}$$

$$[(p \to q) \wedge \neg p] \to \neg q$$

using interpretation :

$g(\neg q) = f$ then $g(q) = t$

* if $g(p) = t$ then $g(p \to q) = t$, $g(\neg p) = f$ ∴ $g((p \to q) \wedge \neg p) = f$
* if $g(p) = f$ then $g(p \to q) = t$, $g(\neg p) = t$ ∴ $g((p \to q) \wedge \neg p) = t$

Thus from $g(q) = t$ & $g(p) = f$ the antecedent is true & the consequent is false thus the implication is false. Thus the argument is invalid.

(b)

$p \to r$
$q \to r$
$\neg(p \vee q)$
$\therefore \neg r$

$$\underbrace{[(p \to r) \wedge (q \to r) \wedge \neg(p \vee q)]}_{\Gamma} \to \underbrace{\neg r}_{G}$$

* using interpretation:

$I(\neg r) = f$ then $I(r) = t$

* if $I(p) = f$ & $I(q) = f$ then $I(\neg(p \vee q)) = t$, $I(q \to r) = t$, $I(p \to r) = t$

Thus if $I(r) = t$, $I(p) = f$ & $I(q) = f$ then the antecedent will be true and the consequent is false. Thus the argument is invalid.

Exercise 1–4

Without using truth tables, prove that $\underbrace{(p \vee \neg q) \vee \neg r}_{F} \to \underbrace{((q \wedge r) \to p)}_{G}$.

* using formal proof:

1] $(p \vee \neg q) \vee \neg r$ (hypothesis)
2] $p \vee (\neg q \vee \neg r)$ (1, associativity)
3] $p \vee \neg (q \wedge r)$ (2, DeMorgan's)
4] $\neg (q \wedge r) \vee p$ (3, commutativity)
5] $(q \wedge r) \to p$ (4, Implication)

* using Interpretation:

we want to try to set $I(F)=$ true & $I(G)=$ false.

$I(q)=t$
$I(r)=t$
$\therefore I(q \wedge r)=t$
$I(p)=f$
$\therefore I((q \wedge r) \to p)=f$
$\therefore I(p \vee \neg q)=f$
$\therefore I((p \vee \neg q) \vee \neg r)=f$

$\therefore$ It is impossible to be $t \to f$
then the argument is valid.

Exercise 1–5 "The Crowning Question!"

Determine whether or not the following argument is valid. Prove your answer.

She is a Math Major or a Computer Science Major.
If she does not know discrete math, she is not a Math Major.
If she knows discrete math, she is smart.
She is not a Computer Science Major.
Therefore, she is smart.

* propositions :

- M : Math major
- C : computer Science major
- D : Discrete math
- S : Smart

* Translation:

$$[(M \vee C) \wedge (\neg D \rightarrow \neg M) \wedge (D \rightarrow S) \wedge \neg C] \rightarrow S$$

* using rules of inference :

1. $M \vee C$ (hyp.)
2. $\neg D \rightarrow \neg M$ (hyp.)
3. $D \rightarrow S$ (hyp.)
4. $\neg C$ (hyp.)
5. $\neg C \rightarrow M$ (1, implication)
6. M (4,5 MP)
7. $M \rightarrow D$ (2, Contrapositive)
8. D (6,7 MP)
9. S (3,8 MP)

Rules of inference: Other rules are missing (e.g., commutativity, associativity, double negation, De Morgan's, etc.)

Name (abbreviation) of the rule:		The rule:
Implication	(IMP)	$P \rightarrow Q \equiv \neg P \vee Q$.
Contrapositive	(CONT)	$P \rightarrow Q \equiv \neg Q \rightarrow \neg P$.
Modus Ponens	(MP)	from P and $P \rightarrow Q$, deduce Q.
Modus Tollens	(MT)	from $P \rightarrow Q$ and $\neg Q$, deduce $\neg P$.
Simplification	(SIMP)	from $P \wedge Q$, deduce P (or deduce Q).
Conjunction	(CONJ)	from P and Q, deduce $P \wedge Q$.
Hypothetical Syllogism	(HS)	from $P \rightarrow Q$ and $Q \rightarrow R$, deduce $P \rightarrow R$.
Disjunctive Syllogism	(DS)	from $P \vee Q$ and $\neg P$, deduce Q.
Addition	(ADD)	from P, deduce $P \vee Q$.
Negation Introduction	(NI)	from $P \rightarrow Q$ and $P \rightarrow \neg Q$, deduce $\neg P$.
Negation Elimination	(NE)	from P and $\neg P$, deduce Q.

Exercise 1–6 “Basics-3”

For each of these sets of premises, what relevant conclusion or conclusions can be drawn?
Explain the rules of inference used to obtain each conclusion from the premises. Try to use English statements and also try using symbols. Work on the level of natural language understanding by humans and symbolically as much as you can.

h s w

(a) “If I play hockey, then I am sore the next day.” “I use the whirlpool if I am sore.” “I did not use the whirlpool.”

$(h \to s) \land (s \to w) \land \neg w$

using rules of inference:

1. $\neg w$ (hypothesis)
2. $s \to w$ (hypothesis)
3. $\neg s$ (1,2 Modus tollens)
4. $h \to s$ (hypothesis)
5. $h \to w$ (2,4 hypothetical syllogism)
6. $\neg h$ (3,4 Modus tollens)

or $\neg h$ (1,5 Modus tollens)

∴ Conclusions:
I didn't play hockey
& I am not sore next day.

Exercise 1–7 (VIP)

Use "resolution" to infer that the hypotheses: "*I left my notes in the library* or *I finished the rough draft of the paper*" and "*I did not leave my notes in the library or I revised the bibliography*" imply that: "*I finished the rough draft of the paper or I revised the bibliography*".

L F R

* Propositions:

L : left my notes in the library

F : finished the rough draft of the paper

R : revised the bibliography

* Translation :

$$[(L \vee F) \wedge (\neg L \vee R)] \rightarrow F \vee R$$

* Resolution:

$$\begin{array}{l} L \vee F \\ \neg L \vee R \\ \hline \therefore F \vee R \end{array}$$

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Exercise 1–10

Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?

(a) If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.

(b) If n is a real number with $n > 3$, then $n^2 > 9$. Suppose that $n^2 \le 9$. Then $n \le 3$.

(c) If n is a real number with $n > 2$,then $n^2 > 4$. Suppose that $n \le 2$. Then $n^2 \le 4$.

p, q

a] $\underbrace{[(p \to q) \wedge q]}_{F} \to \underbrace{p}_{G}$

$g(p) = f$

$g(q) = t$

$\therefore g(p \to q) = t$

$\therefore g((p \to q) \wedge q) = t$

Therefore, antecedent is true & consequence is false Thus the argument is invalid.

* If you are able to set F to be true & G to be false thus the argument is invalid.

$[(p \to q) \wedge p] \to q$ MP

$g(q) = f$

$g(p) = t$

$\therefore g(p \to q) = f$

$\therefore g((p \to q) \wedge p) = f$

Therefore, antecedent is impossible to be true, thus the argument is valid.

Thank you.

Any Questions!