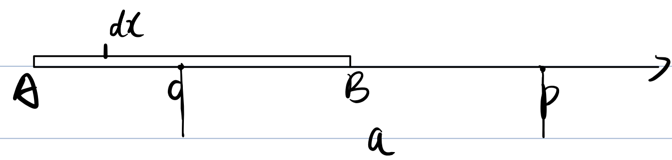
1. 有一均匀带电的细棒AB，长度为L，所带总电量为q

求细棒延长线上到棒中心的距离为a处的p点电势



解：在AB上取一处dx，距A点距离x

$$dq = \frac{q}{L}\cdot dx$$

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{q\cdot dx}{4\pi\varepsilon_0 L(\frac{L}{2}+a-x)}$$

$$V = \int_0^L dV$$

$$= \int_0^L \frac{q\cdot dx}{4\pi L\varepsilon_0(\frac{L}{2}+a-x)}$$

$$= \frac{q}{4\pi L\varepsilon_0}\cdot \ln(\frac{L}{2}+a-x)\Big|_L^0$$

$$= \frac{q}{4\pi L\varepsilon_0}\cdot \ln\frac{\frac{L}{2}+a}{a-\frac{L}{2}}$$

$$= \frac{q}{4\pi L\varepsilon_0}\cdot \ln\frac{2a+L}{2a-L}$$

2. 两个同心的均匀带电球面，半径分别为 $R_1=5.0cm$，$R_2=20.0cm$

已知内球面的电势为 $U_1=60V$，外球面的电势为 $U_2=-30V$

(1) 求内、外球面上所带电量

$$U_1=\int_r^{R_1}\vec{E}_1\cdot d\vec{r}+\int_{R_1}^{R_2}\vec{E}_2\cdot d\vec{r}+\int_{R_2}^{\infty}\vec{E}_3\cdot d\vec{r}$$
$$=0+\int_{R_1}^{R_2}\frac{q_1}{4\pi\varepsilon_0 r^2}dr+\int_{R_2}^{\infty}\frac{q_1+q_2}{4\pi\varepsilon_0 r^2}dr$$
$$=\frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{q_1+q_2}{4\pi\varepsilon_0 R_2}$$
$$=\frac{q_1}{4\pi\varepsilon_0 R_1}-\frac{q_1}{4\pi\varepsilon_0 R_2}+\frac{q_1+q_2}{4\pi\varepsilon_0 R_2}$$
$$=\frac{q_1}{4\pi\varepsilon_0 R_1}+\frac{q_2}{4\pi\varepsilon_0 R_2}$$
$$=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1}+\frac{q_2}{R_2}\right)$$

$$U_2=\int_{R_2}^{\infty}\vec{E}_3\cdot d\vec{r}$$
$$=\frac{q_1+q_2}{4\pi\varepsilon_0 R_2}$$

$$\begin{cases}U_1=\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q_1}{R_1}+\dfrac{q_2}{R_2}\right)=60V\\ U_2=\dfrac{q_1+q_2}{4\pi\varepsilon_0 R_2}=-30V\end{cases}$$

$$\Rightarrow\begin{cases}q_1\approx 6.7\times10^{-10}\,C\\ q_2\approx -1.3\times10^{-9}\,C\end{cases}$$

(2) 在两个球面之间何处的电势为 0

设球面之间半径为 r 处

$$U=\int_r^{R_2}\vec{E}_2 d\vec{r}+\int_{R_2}^{\infty}\vec{E}_3 d\vec{r}$$
$$=\int_r^{R_2}\frac{q_1}{4\pi\varepsilon_0 r^2}dr+\int_{R_2}^{\infty}\frac{q_1+q_2}{4\pi\varepsilon_0 r^2}dr$$
$$=\frac{q_1}{4\pi\varepsilon_0}\cdot\frac{1}{r}\Big|_{R_2}^{r}+\frac{q_1+q_2}{4\pi\varepsilon_0}\cdot\frac{1}{r}\Big|_{\infty}^{R_2}$$
$$=\frac{q_1}{4\pi\varepsilon_0 r}-\frac{q_1}{4\pi\varepsilon_0 R_2}+\frac{q_1+q_2}{4\pi\varepsilon_0 R_2}$$

$$= \frac{q_1}{4\pi\varepsilon_0 r} + \frac{q_2}{4\pi\varepsilon_0 R_2} = 0$$

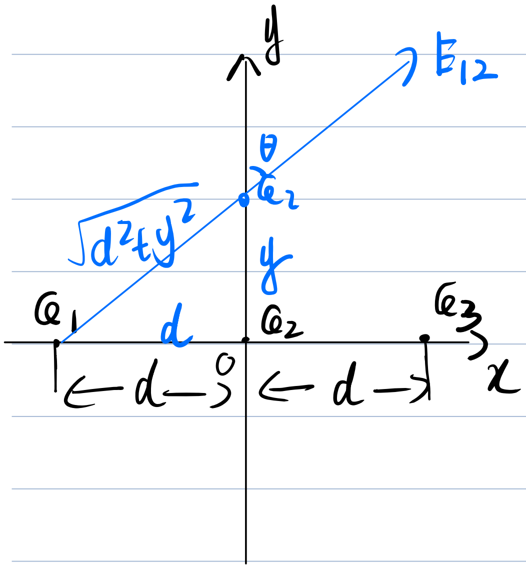
$$\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R^2}\right) = 0$$

$$\frac{6.7\times10^{-10}}{r} - \frac{1.3\times10^{-9}}{20\times10^{-2}} = 0$$

$$r = \frac{-1.34\times10^{-8}}{-1.3\times10^{-7}}$$

$$\approx 0.1\,m = 10\,cm$$

如图，有三个点电荷 Q_1，Q_2，Q_3 沿一条直线等间距分布，且 $Q_1=Q_3=Q$
已知其中任意点电荷所受动均为零，求在固定 Q_1 Q_3 的情况下，将 Q_2
从点O移到无限远处时，外力所做的功



由题意可知

∵ Q_1处所受动为零

且 $F_{21}=k\cdot\frac{Q_1Q_2}{d^2}$

$F_{31}=k\cdot\frac{Q_1Q_3}{4d^2}$

∴ $F_{21}+F_{31}=0$

$\Rightarrow k\frac{Q_1Q_2}{d^2}=-k\frac{Q_1Q_3}{4d^2}$

$Q_2=-\frac{1}{4}Q_3$

由对称性可知，y轴上点所受场强大小为 E_1 和 E_3 在y轴上之和

$$E_1=E_3=\frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{d^2+y^2}$$

$$E_y=E_1\cdot\cos\theta$$

$$=\frac{Q}{4\pi\varepsilon_0(d^2+y^2)}\cdot\frac{y}{\sqrt{d^2+y^2}}$$

$$=\frac{Q\cdot y}{4\pi\varepsilon_0(d^2+y^2)^{\frac{3}{2}}}$$

$$E=2E_y$$

$$=\frac{Q\cdot y}{2\pi\varepsilon_0(d^2+y^2)^{\frac{3}{2}}}$$

$$A=-\int_0^{\infty}Q_2\cdot\vec{E}\cdot d\vec{l}$$

$$=-\int_0^{\infty}\left(-\frac{1}{4}Q\right)\cdot\frac{Q\cdot y}{2\pi\varepsilon_0(d^2+y^2)^{\frac{3}{2}}}dy$$

$$= \frac{Q^2}{8\pi\varepsilon_0} \int_0^\infty \frac{y}{(d^2+y^2)^{\frac{3}{2}}}\, dy$$

$$= \frac{Q^2}{8\pi\varepsilon_0} \cdot \frac{1}{2} \int_0^\infty \frac{1}{(d^2+y^2)^{\frac{3}{2}}}\, d(y^2+d^2)$$

$$= \frac{-Q^2}{8\pi\varepsilon_0} \cdot \frac{1}{\sqrt{d^2+y^2}} \Big|_0^\infty$$

$$= \frac{Q^2}{8\pi\varepsilon_0 d}$$