

ENGR-241 Transfer Function Lab

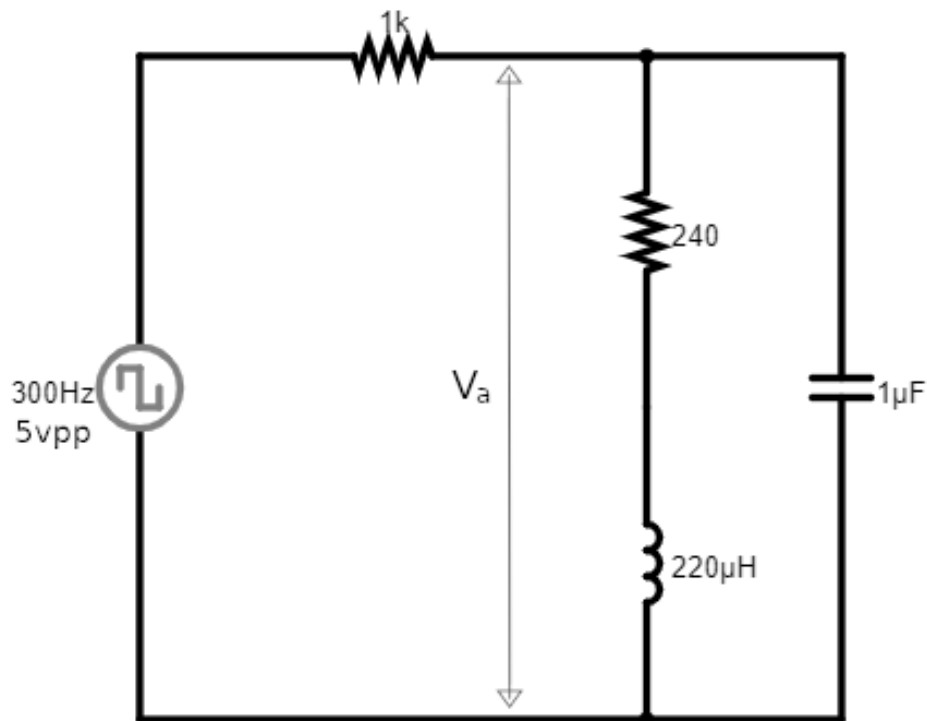
Jeremy Munson, Lauren Speirs & Andrew Henrikson

March 19, 2021

Overview

For this lab we calculated the transfer function then built the required circuit from the lab guidelines. We then calculated the voltage across the capacitor and compared our results from our circuits output to the oscilloscope. We then built the circuit using Orcad and analyzed the circuit using the Laplace function in Pspice and compared its output to our previous results.

Circuit Diagrams



Our suggested values and measured values for our components used are shown in the table below. The percent error is also listed.

Components	Suggested	Measured	% Diff
Resistor 1	1k Ω	0.998k Ω	-.2%
Resistor 2	240 Ω	241 Ω	0.4%
Inductor	220 μH	199 μH	-10.6%
Capacitor	1 μF	0.95 μF	-5.3%

Calculations

As a note, throughout these calculations the numbers are exact, not approximations.

Finding $H(s)$

The circuit may be solved for $V(s)$ by using a simple voltage divider in the s domain.

Find the equivalent impedance $Z_1 = (240\Omega + s \cdot L) \parallel \frac{1}{s \cdot c} = (s \cdot c + (240\Omega + s \cdot L)^{-1})^{-1}$

Which makes $Z_{total} = Z_1 + 1k\Omega$ and the voltage divider equation yields $V_a = V_s \cdot \frac{Z_1}{Z_{total}}$

Solving for $H(s) = \frac{V_a}{V_s} = \frac{Z_1}{Z_{total}}$ gives us

$$H(s) = \frac{(220 \times 10^{-6}s + 240)}{0.22s^2 + 240,000,000 + 1240}$$

Finding $V_a(s)$

We may simply multiply the transfer function by V_s^1 to find V_a

$$V(a) \cdot \frac{s}{5} = \frac{(220 \times 10^{-6}s + 240)}{0.22s^2 + 240,000,000 + 1240}$$

$$V(a) = \frac{5}{s} \cdot \frac{(220 \times 10^{-6}s + 240)}{0.22s^2 + 240,000,000 + 1240}$$

¹The input function, $5u(t)$ transforms to $5/s$

Finding $v_a(t)$

In order to find the time-domain expression for v_a we must do an inverse laplace transform of $V_a(s)$. The function $V_a(s)$ is not transformable in it's current form, so we will expand it with partial fractions.

Using partial fraction decomposition we find:

$$K_1 = 0.967743$$

$$K_2 = -0.972384$$

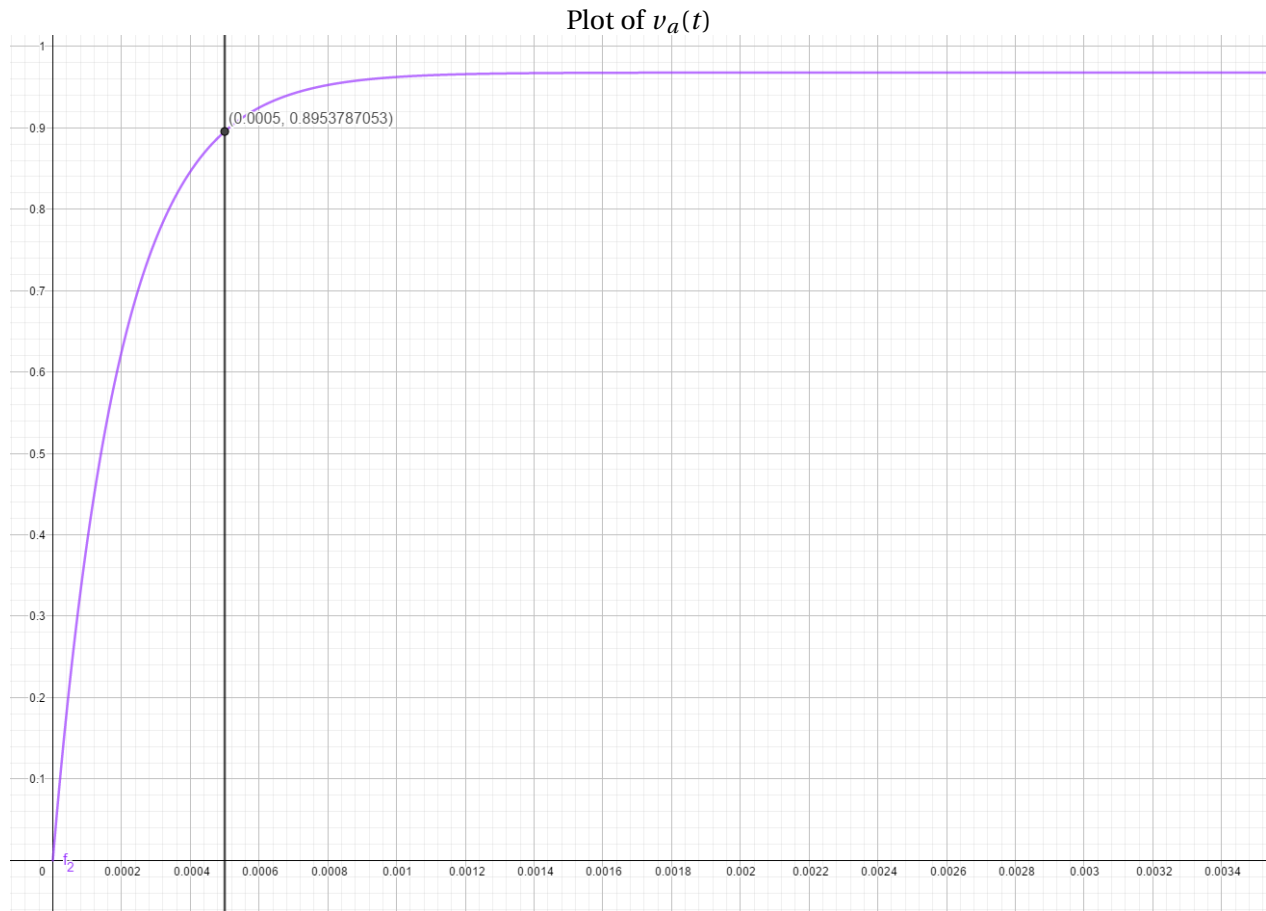
$$K_3 = 4.646087 \cdot 10^{-3}$$

This gives us an expanded expression in the s-domain of:

$$F(s) = \frac{0.967743}{s} - \frac{0.972384}{s+5186.57} + \frac{4.646087 \cdot 10^{-3}}{s+1.8672 \cdot 10^6}$$

Performing this inverse Laplace transform of the function gives a final expression in the time domain of:

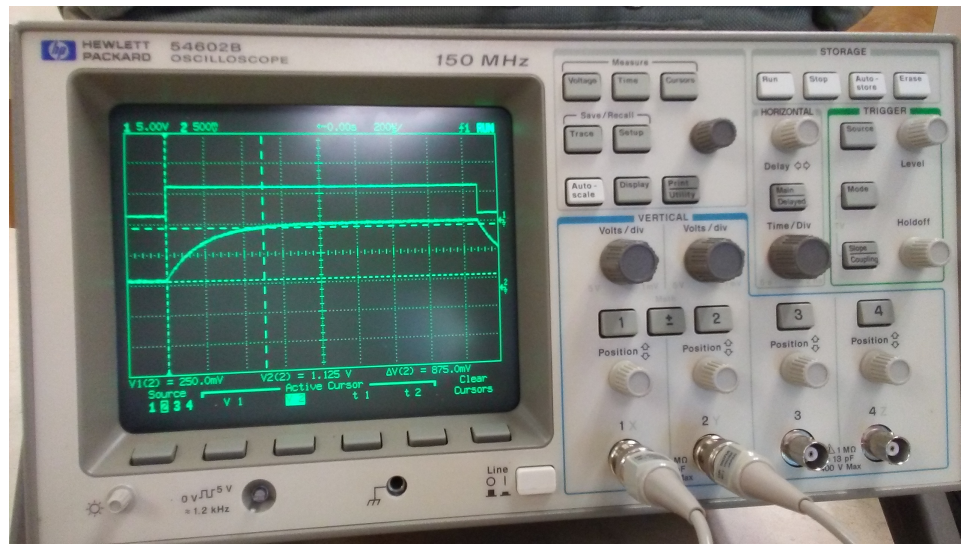
$$f(t) = (0.967743 - 0.972384 \cdot e^{-5186.57t} + 0.00464087 \cdot e^{-1.872 \cdot 10^6 t})u(t)$$



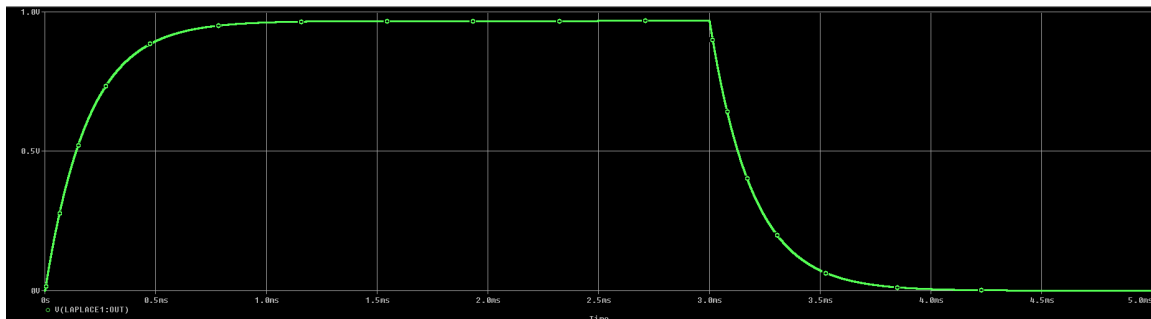
Procedure

The circuit was simple to construct using the breadboard. Aside from constructing the circuit as normal, we used the LCR meter to select a capacitor and an inductor that were as close to the exact values specified as possible. After constructing the circuit, measurements were taken in the usual manner using the oscilloscope.

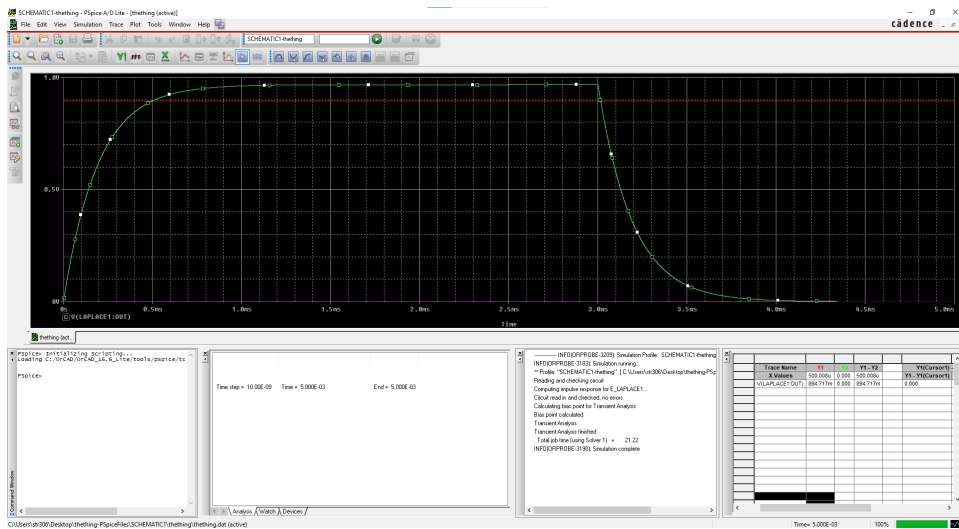
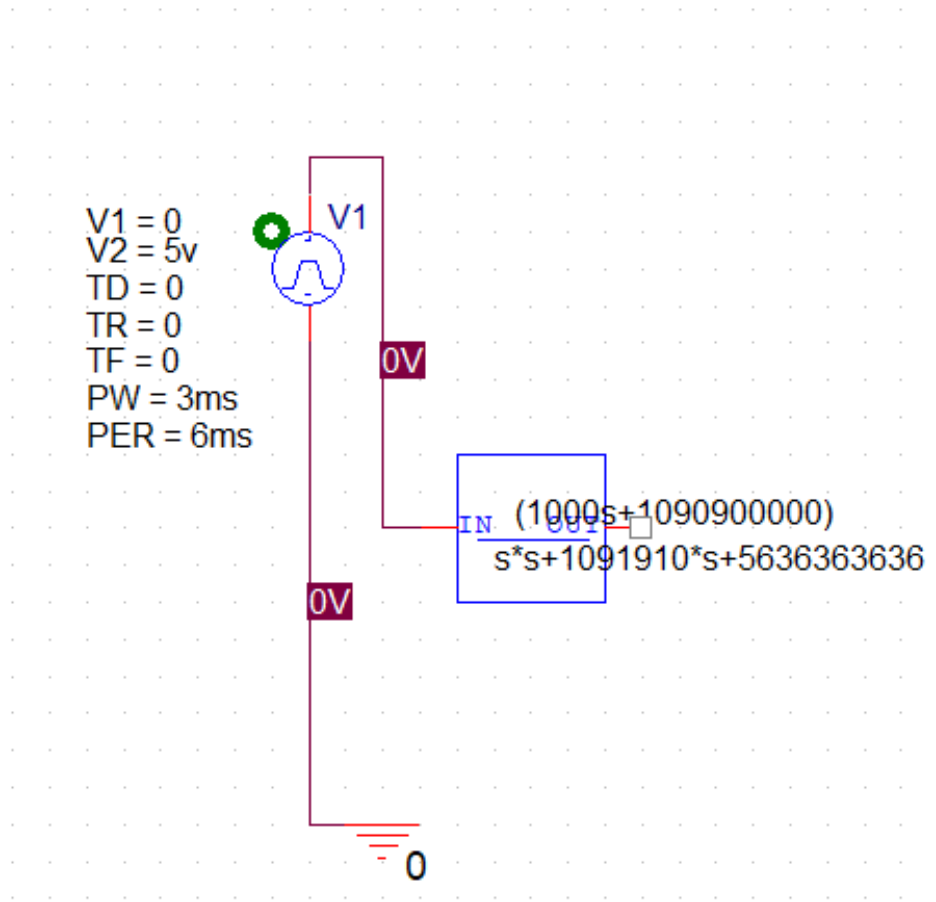
The circuits output is shown below.



We then performed simulations of the same circuit in Orcad. The graph below is the output from the simulation.



We subsequently used the Laplace feature in Orcad to verify our calculations.



Error Analysis

Error in this lab was extremely low. The largest error coming from the circuit we built due to not its components not being ideal. Also setting the frequency exactly at the required value is very difficult due to the touchy nature of the signal generator's controls. We were able to minimize the error from the components by measuring their real values on the LCR meter. All three functions approach approximately the same value at about 0.96V. The

real circuit had slightly lower values than the simulated circuits.

Conclusion

The lab was performed without issue. We were able to successfully calculate, simulate and build the circuit. Our results were excellent with minimal error.