Output tables for the test of Multiple comparisons.

June 12, 2025

1 Average rankings of Friedman test

Average ranks obtained by applying the Friedman procedure

Friedman statistic considering reduction performance (distributed according to chi-square with 15 degrees of freedom: 307.053922.

P-value computed by Friedman Test: 1.3455703218312465E-10.

Iman and Davenport statistic considering reduction performance (distributed according to F-distribution with 15 and 525 degrees of freedom: 46.134656.

P-value computed by Iman and Daveport Test: -2.220446049250313E-16.

Algorithm	Ranking
BestCyclicAssignment	12.5278
BestNearest	6.625
CLARA	11.0278
CoefficientPropagation	9.8333
CyclicAssignment	14.3611
Farthest-First	5.1944
KMEANS	5.5694
NearestByCustomer	6.3333
NearestByDepot	11.2222
PAM	9.1111
Parallel	6.3333
RandomByElement	15.9722
Simplified	5.3889
Sweep	5.5694
ThreeCriteriaClustering	6.4306
UPGMC	4.5

Table 1: Average Rankings of the algorithms

2 Post hoc comparisons

Results achieved on post hoc comparisons for $\alpha = 0.05$, $\alpha = 0.10$ and adjusted p-values.

2.1 P-values for $\alpha = 0.05$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417.
Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000806.
Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	RandomByElement vs. UPGMC	10.223273	0	0.000417
119	Farthest-First vs. RandomByElement	9.604431	0	0.00042
118	RandomByElement vs. Simplified	9.431155	0	0.000424
117	KMEANS vs. RandomByElement	9.270256	0	0.000427
116	RandomByElement vs. Sweep	9.270256	0	0.000431
115	CyclicAssignment vs. UPGMC	8.787559	0	0.000435
114	NearestByCustomer vs. RandomByElement	8.58953	0	0.000439
113	Parallel vs. RandomByElement	8.58953	0	0.000442
112	RandomByElement vs. ThreeCriteriaClustering	8.502892	0	0.000446
111	BestNearest vs. RandomByElement	8.329616	0	0.00045
110	CyclicAssignment vs. Farthest-First	8.168717	0	0.000455
109	CyclicAssignment vs. Simplified	7.995441	0	0.000459
108	CyclicAssignment vs. KMEANS	7.834542	0	0.000463
107	CyclicAssignment vs. Sweep	7.834542	0	0.000467
106	BestCyclicAssignment vs. UPGMC	7.153816	0	0.000472
105	CyclicAssignment vs. NearestByCustomer	7.153816	0	0.000476
104	CyclicAssignment vs. Parallel	7.153816	0	0.000481
103	CyclicAssignment vs. ThreeCriteriaClustering	7.067178	0	0.000485
102	BestNearest vs. CyclicAssignment	6.893902	0	0.00049
101	BestCyclicAssignment vs. Farthest-First	6.534974	0	0.000495
100	BestCyclicAssignment vs. Simplified	6.361698	0	0.0005
99	BestCyclicAssignment vs. Sweep	6.200799	0	0.000505
98	BestCyclicAssignment vs. KMEANS	6.200799	0	0.00051
97	PAM vs. RandomByElement	6.114161	0	0.000515
96	NearestByDepot vs. UPGMC	5.990393	0	0.000521
95	CLARA vs. UPGMC	5.817117	0	0.000526
94	BestCyclicAssignment vs. NearestByCustomer	5.520073	0	0.000532
93	BestCyclicAssignment vs. Parallel	5.520073	0	0.000538
92	CoefficientPropagation vs. RandomByElement	5.470565	0	0.000543
91	BestCyclicAssignment vs. ThreeCriteriaClustering	5.433435	0	0.000549
90	Farthest-First vs. NearestByDepot	5.37155	0	0.000556
89	BestCyclicAssignment vs. BestNearest	5.260159	0	0.000562
88	NearestByDepot vs. Simplified	5.198275	0	0.000568
87	CLARA vs. Farthest-First	5.198275	0	0.000575
86	NearestByDepot vs. Sweep	5.037376	0	0.000581
85	KMEANS vs. NearestByDepot	5.037376	0	0.000588
84	CLARA vs. Simplified	5.024999	0.000001	0.000595
83	CLARA vs. Sweep	4.8641	0.000001	0.000602
82	CLARA vs. KMEANS	4.8641	0.000001	0.00061
81	CoefficientPropagation vs. UPGMC	4.752708	0.000002	0.000617
80	CyclicAssignment vs. PAM	4.678447	0.000003	0.000625
79	CLARA vs. RandomByElement	4.406157	0.000011	0.000633
78	NearestByCustomer vs. NearestByDepot	4.356649	0.000013	0.000641
77	NearestByDepot vs. Parallel	4.356649	0.000013	0.000649
76	NearestByDepot vs. ThreeCriteriaClustering	4.270011	0.00002	0.000658
75	NearestByDepot vs. RandomByElement	4.232881	0.000023	0.000667
74	CLARA vs. NearestByCustomer	4.183373	0.000029	0.000676
73	CLARA vs. Parallel	4.183373	0.000029	0.000685
72	CoefficientPropagation vs. Farthest-First	4.133866	0.000036	0.000694
71	PAM vs. UPGMC	4.109112	0.00004	0.000704
70	BestNearest vs. NearestByDepot	4.096735	0.000042	0.000714
69	CLARA vs. ThreeCriteriaClustering	4.096735	0.000042	0.000725
68	CoefficientPropagation vs. CyclicAssignment	4.034851	0.000055	0.000735
67	CoefficientPropagation vs. Simplified	3.96059	0.000075	0.000746
66	BestNearest vs. CLARA	3.92346	0.000087	0.000758
65	CoefficientPropagation vs. Sweep	3.799691	0.000145	0.000769
64	CoefficientPropagation vs. KMEANS	3.799691	0.000145	0.000781
63	Farthest-First vs. PAM	3.49027	0.000483	0.000794
62	PAM vs. Simplified	3.316994	0.00091	0.000806
61	PAM vs. Sweep	3.156095	0.001599	0.00082
60	KMEANS vs. PAM	3.156095	0.001599	0.000833
59	CoefficientPropagation vs. NearestByCustomer	3.118965	0.001815	0.000847
58	CoefficientPropagation vs. Parallel	3.118965	0.001815	0.000862
57	BestCyclicAssignment vs. RandomByElement	3.069457	0.002144	0.000877
56	BestCyclicAssignment vs. PAM	3.044704	0.002329	0.000893
55	CoefficientPropagation vs. ThreeCriteriaClustering	3.032327	0.002427	0.000909
54	CLARA vs. CyclicAssignment	2.970443	0.002974	0.000926
53	BestNearest vs. CoefficientPropagation	2.859051	0.004249	0.000943
52	CyclicAssignment vs. NearestByDepot	2.797167	0.005155	0.000962
51	NearestByCustomer vs. PAM	2.475369	0.01331	0.00098
50	PAM vs. Parallel	2.475369	0.01331	0.001
49	BestCyclicAssignment vs. CoefficientPropagation	2.401108	0.016346	0.00102
48	PAM vs. ThreeCriteriaClustering	2.388731	0.016907	0.001042
47	BestNearest vs. PAM	2.215455	0.026729	0.001064
46	BestNearest vs. UPGMC	1.893657	0.058271	0.001087
45	NearestByDepot vs. PAM	1.88128	0.059934	0.001111
44	ThreeCriteriaClustering vs. UPGMC	1.720381	0.085363	0.001136
43	CLARA vs. PAM	1.708005	0.087636	0.001163
42	NearestByCustomer vs. UPGMC	1.633743	0.102313	0.00119
41	Parallel vs. UPGMC	1.633743	0.102313	0.00122
40	BestCyclicAssignment vs. CyclicAssignment	1.633743	0.102313	0.00125
39	CyclicAssignment vs. RandomByElement	1.435714	0.151084	0.001282
38	BestCyclicAssignment vs. CLARA	1.336699	0.181321	0.001316
37	BestNearest vs. Farthest-First	1.274815	0.202375	0.001351
36	CoefficientPropagation vs. NearestByDepot	1.237684	0.215833	0.001389
35	BestCyclicAssignment vs. NearestByDepot	1.163423	0.244658	0.001429
34	BestNearest vs. Simplified	1.101539	0.270662	0.001471
33	Farthest-First vs. ThreeCriteriaClustering	1.101539	0.270662	0.001515
32	CLARA vs. CoefficientPropagation	1.064409	0.287144	0.001563

2.2 P-values for $\alpha = 0.10$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.

Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.001695.

Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	RandomByElement vs. UPGMC	10.223273	0	0.000833
119	Farthest-First vs. RandomByElement	9.604431	0	0.00084
118	RandomByElement vs. Simplified	9.431155	0	0.000847
117	KMEANS vs. RandomByElement	9.270256	0	0.000855
116	RandomByElement vs. Sweep	9.270256	0	0.000862
115	CyclicAssignment vs. UPGMC	8.787559	0	0.00087
114	NearestByCustomer vs. RandomByElement	8.58953	0	0.000877
113	Parallel vs. RandomByElement	8.58953	0	0.000885
112	RandomByElement vs. ThreeCriteriaClustering	8.502892	0	0.000893
111	BestNearest vs. RandomByElement	8.329616	0	0.000901
110	CyclicAssignment vs. Farthest-First	8.168717	0	0.000909
109	CyclicAssignment vs. Simplified	7.995441	0	0.000917
108	CyclicAssignment vs. KMEANS	7.834542	0	0.000926
107	CyclicAssignment vs. Sweep	7.834542	0	0.000935
106	BestCyclicAssignment vs. UPGMC	7.153816	0	0.000943
105	CyclicAssignment vs. NearestByCustomer	7.153816	0	0.000952
104	CyclicAssignment vs. Parallel	7.153816	0	0.000962
103	CyclicAssignment vs. ThreeCriteriaClustering	7.067178	0	0.000971
102	BestNearest vs. CyclicAssignment	6.893902	0	0.00098
101	BestCyclicAssignment vs. Farthest-First	6.534974	0	0.00099
100	BestCyclicAssignment vs. Simplified	6.361698	0	0.001
99	BestCyclicAssignment vs. Sweep	6.200799	0	0.00101
98	BestCyclicAssignment vs. KMEANS	6.200799	0	0.00102
97	PAM vs. RandomByElement	6.114161	0	0.001031
96	NearestByDepot vs. UPGMC	5.990393	0	0.001042
95	CLARA vs. UPGMC	5.817117	0	0.001053
94	BestCyclicAssignment vs. NearestByCustomer	5.520073	0	0.001064
93	BestCyclicAssignment vs. Parallel	5.520073	0	0.001075
92	CoefficientPropagation vs. RandomByElement	5.470565	0	0.001087
91	BestCyclicAssignment vs. ThreeCriteriaClustering	5.433435	0	0.001099
90	Farthest-First vs. NearestByDepot	5.37155	0	0.001111
89	BestCyclicAssignment vs. BestNearest	5.260159	0	0.001124
88	NearestByDepot vs. Simplified	5.198275	0	0.001136
87	CLARA vs. Farthest-First	5.198275	0	0.001149
86	NearestByDepot vs. Sweep	5.037376	0	0.001163
85	KMEANS vs. NearestByDepot	5.037376	0	0.001176
84	CLARA vs. Simplified	5.024999	0.000001	0.00119
83	CLARA vs. Sweep	4.8641	0.000001	0.001205
82	CLARA vs. KMEANS	4.8641	0.000001	0.00122
81	CoefficientPropagation vs. UPGMC	4.752708	0.000002	0.001235
80	CyclicAssignment vs. PAM	4.678447	0.000003	0.00125
79	CLARA vs. RandomByElement	4.406157	0.000011	0.001266
78	NearestByCustomer vs. NearestByDepot	4.356649	0.000013	0.001282
77	NearestByDepot vs. Parallel	4.356649	0.000013	0.001299
76	NearestByDepot vs. ThreeCriteriaClustering	4.270011	0.00002	0.001316
75	NearestByDepot vs. RandomByElement	4.232881	0.000023	0.001333
74	CLARA vs. NearestByCustomer	4.183373	0.000029	0.001351
73	CLARA vs. Parallel	4.183373	0.000029	0.00137
72	CoefficientPropagation vs. Farthest-First	4.133866	0.000036	0.001389
71	PAM vs. UPGMC	4.109112	0.00004	0.001408
70	BestNearest vs. NearestByDepot	4.096735	0.000042	0.001429
69	CLARA vs. ThreeCriteriaClustering	4.096735	0.000042	0.001449
68	CoefficientPropagation vs. CyclicAssignment	4.034851	0.000055	0.001471
67	CoefficientPropagation vs. Simplified	3.96059	0.000075	0.001493
66	BestNearest vs. CLARA	3.92346	0.000087	0.001515
65	CoefficientPropagation vs. Sweep	3.799691	0.000145	0.001538
64	CoefficientPropagation vs. KMEANS	3.799691	0.000145	0.001563
63	Farthest-First vs. PAM	3.49027	0.000483	0.001587
62	PAM vs. Simplified	3.316994	0.00091	0.001613
61	PAM vs. Sweep	3.156095	0.001599	0.001639
60	KMEANS vs. PAM	3.156095	0.001599	0.001667
59	CoefficientPropagation vs. NearestByCustomer	3.118965	0.001815	0.001695
58	CoefficientPropagation vs. Parallel	3.118965	0.001815	0.001724
57	BestCyclicAssignment vs. RandomByElement	3.069457	0.002144	0.001754
56	BestCyclicAssignment vs. PAM	3.044704	0.002329	0.001786
55	CoefficientPropagation vs. ThreeCriteriaClustering	3.032327	0.002427	0.001818
54	CLARA vs. CyclicAssignment	2.970443	0.002974	0.001852
53	BestNearest vs. CoefficientPropagation	2.859051	0.004249	0.001887
52	CyclicAssignment vs. NearestByDepot	2.797167	0.005155	0.001923
51	NearestByCustomer vs. PAM	2.475369	0.01331	0.001961
50	PAM vs. Parallel	2.475369	0.01331	0.002
49	BestCyclicAssignment vs. CoefficientPropagation	2.401108	0.016346	0.002041
48	PAM vs. ThreeCriteriaClustering	2.388731	0.016907	0.002083
47	BestNearest vs. PAM	2.215455	0.026729	0.002128
46	BestNearest vs. UPGMC	1.893657	0.058271	0.002174
45	NearestByDepot vs. PAM	1.88128	0.059934	0.002222
44	ThreeCriteriaClustering vs. UPGMC	1.720381	0.085363	0.002273
43	CLARA vs. PAM	1.708005	0.087636	0.002326
42	NearestByCustomer vs. UPGMC	1.633743	0.102313	0.002381
41	Parallel vs. UPGMC	1.633743	0.102313	0.002439
40	BestCyclicAssignment vs. CyclicAssignment	1.633743	0.102313	0.0025
39	CyclicAssignment vs. RandomByElement	1.435714	0.151084	0.002564
38	BestCyclicAssignment vs. CLARA	1.336699	0.181321	0.002632
37	BestNearest vs. Farthest-First	1.274815	0.202375	0.002703
36	CoefficientPropagation vs. NearestByDepot	1.237684	0.215833	0.002778
35	BestCyclicAssignment vs. NearestByDepot	1.163423	0.244658	0.002857
34	BestNearest vs. Simplified	1.101539	0.270662	0.002941
33	Farthest-First vs. ThreeCriteriaClustering	1.101539	0.270662	0.00303
32	CLARA vs. CoefficientPropagation	1.064409	0.287144	0.003125

2.3 Adjusted p-values

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Sha}
1	RandomByElement vs .UPGMC	0	0	0	0
2	Farthest-First vs .RandomByElement	0	0	0	0
3	RandomByElement vs .Simplified	0	0	0	0
4	KMEANS vs .RandomByElement	0	0	0	0
5	RandomByElement vs .Sweep	0	0	0	0
6	CyclicAssignment vs .UPGMC	0	0	0	0
7	NearestByCustomer vs .RandomByElement	0	0	0	0
8	Parallel vs .RandomByElement	0	0	0	0
9	RandomByElement vs .ThreeCriteriaClustering	0	0	0	0
10	BestNearest vs .RandomByElement	0	0	0	0
11	CyclicAssignment vs .Farthest-First	0	0	0	0
12	CyclicAssignment vs .Simplified	0	0	0	0
13	CyclicAssignment vs .KMEANS	0	0	0	0
14	CyclicAssignment vs .Sweep	0	0	0	0
15	BestCyclicAssignment vs .UPGMC	0	0	0	0
16	CyclicAssignment vs .NearestByCustomer	0	0	0	0
17	CyclicAssignment vs .Parallel	0	0	0	0
18	CyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
19	BestNearest vs .CyclicAssignment	0	0	0	0
20	BestCyclicAssignment vs .Farthest-First	0	0	0	0
21	BestCyclicAssignment vs .Simplified	0	0	0	0
22	BestCyclicAssignment vs .Sweep	0	0	0	0
23	BestCyclicAssignment vs .KMEANS	0	0	0	0
24	PAM vs .RandomByElement	0	0	0	0
25	NearestByDepot vs .UPGMC	0	0	0	0
26	CLARA vs .UPGMC	0	0.000001	0.000001	0.00000
27	BestCyclicAssignment vs .NearestByCustomer	0	0.000004	0.000003	0.00000
28	BestCyclicAssignment vs .Parallel	0	0.000004	0.000003	0.00000
29	CoefficientPropagation vs .RandomByElement	0	0.000005	0.000004	0.00000
30	BestCyclicAssignment vs .ThreeCriteriaClustering	0	0.000007	0.000005	0.00000
31	Farthest-First vs .NearestByDepot	0	0.000009	0.000007	0.00000
32	BestCyclicAssignment vs .BestNearest	0	0.000017	0.000013	0.0000
33	NearestByDepot vs .Simplified	0	0.000024	0.000018	0.0000
34	CLARA vs .Farthest-First	0	0.000024	0.000018	0.0000
35	NearestByDepot vs .Sweep	0	0.000057	0.000041	0.0000
36	KMEANS vs .NearestByDepot	0	0.000057	0.000041	0.0000
37	CLARA vs .Simplified	0.000001	0.00006	0.000042	0.0000
38	CLARA vs .Sweep	0.000001	0.000138	0.000095	0.0000
39	CLARA vs .KMEANS	0.000001	0.000138	0.000095	0.0000
40	CoefficientPropagation vs .UPGMC	0.000002	0.000241	0.000163	0.0001
41	CyclicAssignment vs .PAM	0.000003	0.000347	0.000231	0.0002
42	CLARA vs .RandomByElement	0.000011	0.001263	0.000831	0.0008
43	NearestByCustomer vs .NearestByDepot	0.000013	0.001585	0.00103	0.0010
44	NearestByDepot vs .Parallel	0.000013	0.001585	0.00103	0.0010
45	NearestByDepot vs .ThreeCriteriaClustering	0.00002	0.002346	0.001486	0.0014
46	NearestByDepot vs .RandomByElement	0.000023	0.002769	0.00173	0.0016
47	CLARA vs .NearestByCustomer	0.000029	0.003447	0.002125	0.0020
48	CLARA vs .Parallel	0.000029	0.003447	0.002125	0.0020
49	CoefficientPropagation vs .Farthest-First	0.000036	0.004281	0.002568	0.0025
50	PAM vs .UPGMC	0.00004	0.004766	0.00282	0.0027
51	BestNearest vs .NearestByDepot	0.000042	0.005028	0.002933	0.0028
52	CLARA vs .ThreeCriteriaClustering	0.000042	0.005028	0.002933	0.0028
53	CoefficientPropagation vs .CyclicAssignment	0.000055	0.006556	0.003715	0.0037
54	CoefficientPropagation vs .Simplified	0.000075	0.008972	0.005009	0.0050
55	BestNearest vs .CLARA	0.000087	0.010474	0.005761	0.0057
56	CoefficientPropagation vs .Sweep	0.000145	0.017385	0.009417	0.0094
57	CoefficientPropagation vs .KMEANS	0.000145	0.017385	0.009417	0.0094
58	Farthest-First vs .PAM	0.000483	0.057904	0.0304	0.0294
59	PAM vs .Simplified	0.00091	0.10919	0.056415	0.0555
60	PAM vs .Sweep	0.001599	0.191876	0.097537	0.0975
61	KMEANS vs .PAM	0.001599	0.191876	0.097537	0.0975
62	CoefficientPropagation vs .NearestByCustomer	0.001815	0.217785	0.107078	0.1070
63	CoefficientPropagation vs .Parallel	0.001815	0.217785	0.107078	0.1070
64	BestCyclicAssignment vs .RandomByElement	0.002144	0.257338	0.122235	0.1222
65	BestCyclicAssignment vs .PAM	0.002329	0.279492	0.130429	0.1304
66	CoefficientPropagation vs .ThreeCriteriaClustering	0.002427	0.291212	0.133472	0.1334
67	CLARA vs .CyclicAssignment	0.002974	0.356845	0.16058	0.1546
68	BestNearest vs .CoefficientPropagation	0.004249	0.509893	0.225203	0.2209
69	CyclicAssignment vs .NearestByDepot	0.005155	0.618635	0.268075	0.2680
70	NearestByCustomer vs .PAM	0.01331	1.597184	0.678803	0.6788
71	PAM vs .Parallel	0.01331	1.597184	0.678803	0.6788
72	BestCyclicAssignment vs .CoefficientPropagation	0.016346	1.961462	0.800931	0.8009
73	PAM vs .ThreeCriteriaClustering	0.016907	2.028801	0.811521	0.8115
74	BestNearest vs .PAM	0.026729	3.207461	1.256256	1.2562
75	BestNearest vs .UPGMC	0.058271	6.992463	2.680444	2.6804
76	NearestByDepot vs .PAM	0.059934	7.192055	2.697021	2.6970
77	ThreeCriteriaClustering vs .UPGMC	0.085363	10.243577	3.755978	3.7559
78	CLARA vs .PAM	0.087636	10.516261	3.768327	3.7683
79	NearestByCustomer vs .UPGMC	0.102313	12.277528	4.297135	4.2971
80	Parallel vs .UPGMC	0.102313	12.277528	4.297135	4.2971
81	BestCyclicAssignment vs .CyclicAssignment	0.102313	12.277528	4.297135	4.2971
82	CyclicAssignment vs .RandomByElement	0.151084	18.130051	5.892267	5.8922
83	BestCyclicAssignment vs .CLARA	0.181321	21.758502	6.890192	6.8901
84	BestNearest vs .Farthest-First	0.202375	24.284969	7.487866	7.4878
85	CoefficientPropagation vs .NearestByDepot	0.215833	25.899971	7.769991	7.7699
86	BestCyclicAssignment vs .NearestByDepot	0.244658	29.358933	8.563022	8.5630
87	BestNearest vs .Simplified	0.270662	32.479449	9.202511	9.2025
88	Farthest-First vs .ThreeCriteriaClustering	0.270662	32.479449	9.202511	9.2025
89	CLARA vs .CoefficientPropagation	0.287144	34.457236	9.202511	9.2025