Output tables for the test of Multiple comparisons.

June 12, 2025

1 Average rankings of Friedman test

Average ranks obtained by applying the Friedman procedure

Friedman statistic considering reduction performance (distributed according to chi-square with 15 degrees of freedom: 322.185662.

P-value computed by Friedman Test: 1.658975179452682E-10.

Iman and Davenport statistic considering reduction performance (distributed according to F-distribution with 15 and 525 degrees of freedom: 51.771147.

P-value computed by Iman and Daveport Test: 3.4350375403090855E-93.

Algorithm	Ranking
BestCyclicAssignment	13.0278
BestNearest	6.7639
CLARA	10
CoefficientPropagation	11.5972
CyclicAssignment	14.1667
Farthest-First	3.25
KMEANS	4.2639
NearestByCustomer	7.3333
NearestByDepot	11
PAM	8.4722
Parallel	7.3333
RandomByElement	15.1944
Simplified	6.5
Sweep	6.5139
ThreeCriteriaClustering	3.3472
UPGMC	7.2361

Table 1: Average Rankings of the algorithms

2 Post hoc comparisons

Results achieved on post hoc comparisons for $\alpha = 0.05$, $\alpha = 0.10$ and adjusted p-values.

2.1 P-values for $\alpha = 0.05$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417.
Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000847.
Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	Farthest-First vs. RandomByElement	10.644086	0	0.000417
119	RandomByElement vs. ThreeCriteriaClustering	10.557448	0	0.00042
118	KMEANS vs. RandomByElement	9.740576	0	0.000424
117	CyclicAssignment vs. Farthest-First	9.7282	0	0.000427
116	CyclicAssignment vs. ThreeCriteriaClustering	9.641562	0	0.000431
115	CyclicAssignment vs. KMEANS	8.82469	0	0.000435
114	BestCyclicAssignment vs. Farthest-First	8.713298	0	0.000439
113	BestCyclicAssignment vs. ThreeCriteriaClustering	8.62666	0	0.000442
112	BestCyclicAssignment vs. KMEANS	7.809789	0	0.000446
111	RandomByElement vs. Simplified	7.747905	0	0.00045
110	RandomByElement vs. Sweep	7.735528	0	0.000455
109	BestNearest vs. RandomByElement	7.512744	0	0.000459
108	CoefficientPropagation vs. Farthest-First	7.438483	0	0.000463
107	CoefficientPropagation vs. ThreeCriteriaClustering	7.351846	0	0.000467
106	RandomByElement vs. UPGMC	7.091932	0	0.000472
105	NearestByCustomer vs. RandomByElement	7.005294	0	0.000476
104	Parallel vs. RandomByElement	7.005294	0	0.000481
103	Farthest-First vs. NearestByDepot	6.906279	0	0.000485
102	CyclicAssignment vs. Simplified	6.832018	0	0.00049
101	CyclicAssignment vs. Sweep	6.819641	0	0.000495
100	NearestByDepot vs. ThreeCriteriaClustering	6.819641	0	0.0005
99	BestNearest vs. CyclicAssignment	6.596858	0	0.000505
98	CoefficientPropagation vs. KMEANS	6.534974	0	0.00051
97	CyclicAssignment vs. UPGMC	6.176045	0	0.000515
96	CyclicAssignment vs. NearestByCustomer	6.089407	0	0.000521
95	CyclicAssignment vs. Parallel	6.089407	0	0.000526
94	CLARA vs. Farthest-First	6.015146	0	0.000532
93	KMEANS vs. NearestByDepot	6.002769	0	0.000538
92	PAM vs. RandomByElement	5.990393	0	0.000543
91	CLARA vs. ThreeCriteriaClustering	5.928508	0	0.000549
90	BestCyclicAssignment vs. Simplified	5.817117	0	0.000556
89	BestCyclicAssignment vs. Sweep	5.80474	0	0.000562
88	BestCyclicAssignment vs. BestNearest	5.581957	0	0.000568
87	BestCyclicAssignment vs. UPGMC	5.161144	0	0.000575
86	CLARA vs. KMEANS	5.111637	0	0.000581
85	CyclicAssignment vs. PAM	5.074506	0	0.000588
84	BestCyclicAssignment vs. NearestByCustomer	5.074506	0	0.000595
83	BestCyclicAssignment vs. Parallel	5.074506	0	0.000602
82	Farthest-First vs. PAM	4.653693	0.000003	0.00061
81	CLARA vs. RandomByElement	4.62894	0.000004	0.000617
80	PAM vs. ThreeCriteriaClustering	4.567056	0.000005	0.000625
79	CoefficientPropagation vs. Simplified	4.542302	0.000006	0.000633
78	CoefficientPropagation vs. Sweep	4.529925	0.000006	0.000641
77	BestNearest vs. CoefficientPropagation	4.307142	0.000017	0.000649
76	BestCyclicAssignment vs. PAM	4.059605	0.000049	0.000658
75	NearestByDepot vs. Simplified	4.010098	0.000061	0.000667
74	NearestByDepot vs. Sweep	3.997721	0.000064	0.000676
73	CoefficientPropagation vs. UPGMC	3.886329	0.000102	0.000685
72	CoefficientPropagation vs. NearestByCustomer	3.799691	0.000145	0.000694
71	CoefficientPropagation vs. Parallel	3.799691	0.000145	0.000704
70	BestNearest vs. NearestByDepot	3.774938	0.00016	0.000714
69	KMEANS vs. PAM	3.750184	0.000177	0.000725
68	NearestByDepot vs. RandomByElement	3.737807	0.000186	0.000735
67	CLARA vs. CyclicAssignment	3.713053	0.000205	0.000746
66	Farthest-First vs. NearestByCustomer	3.638792	0.000274	0.000758
65	Farthest-First vs. Parallel	3.638792	0.000274	0.000769
64	NearestByCustomer vs. ThreeCriteriaClustering	3.552154	0.000382	0.000781
63	Parallel vs. ThreeCriteriaClustering	3.552154	0.000382	0.000794
62	Farthest-First vs. UPGMC	3.552154	0.000382	0.000806
61	ThreeCriteriaClustering vs. UPGMC	3.465516	0.000529	0.00082
60	NearestByDepot vs. UPGMC	3.354125	0.000796	0.000833
59	NearestByCustomer vs. NearestByDepot	3.267487	0.001085	0.000847
58	NearestByDepot vs. Parallel	3.267487	0.001085	0.000862
57	CoefficientPropagation vs. RandomByElement	3.205603	0.001348	0.000877
56	BestNearest vs. Farthest-First	3.131342	0.00174	0.000893
55	CLARA vs. Simplified	3.118965	0.001815	0.000909
54	CLARA vs. Sweep	3.106588	0.001893	0.000926
53	BestNearest vs. ThreeCriteriaClustering	3.044704	0.002329	0.000943
52	Farthest-First vs. Sweep	2.908558	0.003631	0.000962
51	Farthest-First vs. Simplified	2.896182	0.003777	0.00098
50	BestNearest vs. CLARA	2.883805	0.003929	0.001
49	Sweep vs. ThreeCriteriaClustering	2.82192	0.004774	0.00102
48	CyclicAssignment vs. NearestByDepot	2.82192	0.004774	0.001042
47	Simplified vs. ThreeCriteriaClustering	2.809544	0.004961	0.001064
46	CoefficientPropagation vs. PAM	2.78479	0.005356	0.001087
45	KMEANS vs. NearestByCustomer	2.735283	0.006233	0.001111
44	KMEANS vs. Parallel	2.735283	0.006233	0.001136
43	BestCyclicAssignment vs. CLARA	2.698152	0.006973	0.001163
42	KMEANS vs. UPGMC	2.648645	0.008082	0.00119
41	CLARA vs. UPGMC	2.462992	0.013778	0.00122
40	CLARA vs. NearestByCustomer	2.376354	0.017485	0.00125
39	CLARA vs. Parallel	2.376354	0.017485	0.001282
38	CoefficientPropagation vs. CyclicAssignment	2.289716	0.022038	0.001316
37	NearestByDepot vs. PAM	2.252586	0.024285	0.001351
36	BestNearest vs. KMEANS	2.227832	0.025892	0.001389
35	KMEANS vs. Sweep	2.005049	0.044958	0.001429
34	KMEANS vs. Simplified	1.992672	0.046297	0.001471
33	BestCyclicAssignment vs. RandomByElement	1.930788	0.053509	0.001515
32	BestCyclicAssignment vs. NearestByDepot	1.807019	0.070759	0.001563

2.2 P-values for $\alpha = 0.10$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.

Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.001852.

Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	Farthest-First vs. RandomByElement	10.644086	0	0.000833
119	RandomByElement vs. ThreeCriteriaClustering	10.557448	0	0.00084
118	KMEANS vs. RandomByElement	9.740576	0	0.000847
117	CyclicAssignment vs. Farthest-First	9.7282	0	0.000855
116	CyclicAssignment vs. ThreeCriteriaClustering	9.641562	0	0.000862
115	CyclicAssignment vs. KMEANS	8.82469	0	0.00087
114	BestCyclicAssignment vs. Farthest-First	8.713298	0	0.000877
113	BestCyclicAssignment vs. ThreeCriteriaClustering	8.62666	0	0.000885
112	BestCyclicAssignment vs. KMEANS	7.809789	0	0.000893
111	RandomByElement vs. Simplified	7.747905	0	0.000901
110	RandomByElement vs. Sweep	7.735528	0	0.000909
109	BestNearest vs. RandomByElement	7.512744	0	0.000917
108	CoefficientPropagation vs. Farthest-First	7.438483	0	0.000926
107	CoefficientPropagation vs. ThreeCriteriaClustering	7.351846	0	0.000935
106	RandomByElement vs. UPGMC	7.091932	0	0.000943
105	NearestByCustomer vs. RandomByElement	7.005294	0	0.000952
104	Parallel vs. RandomByElement	7.005294	0	0.000962
103	Farthest-First vs. NearestByDepot	6.906279	0	0.000971
102	CyclicAssignment vs. Simplified	6.832018	0	0.00098
101	CyclicAssignment vs. Sweep	6.819641	0	0.00099
100	NearestByDepot vs. ThreeCriteriaClustering	6.819641	0	0.001
99	BestNearest vs. CyclicAssignment	6.596858	0	0.00101
98	CoefficientPropagation vs. KMEANS	6.534974	0	0.00102
97	CyclicAssignment vs. UPGMC	6.176045	0	0.001031
96	CyclicAssignment vs. NearestByCustomer	6.089407	0	0.001042
95	CyclicAssignment vs. Parallel	6.089407	0	0.001053
94	CLARA vs. Farthest-First	6.015146	0	0.001064
93	KMEANS vs. NearestByDepot	6.002769	0	0.001075
92	PAM vs. RandomByElement	5.990393	0	0.001087
91	CLARA vs. ThreeCriteriaClustering	5.928508	0	0.001099
90	BestCyclicAssignment vs. Simplified	5.817117	0	0.001111
89	BestCyclicAssignment vs. Sweep	5.80474	0	0.001124
88	BestCyclicAssignment vs. BestNearest	5.581957	0	0.001136
87	BestCyclicAssignment vs. UPGMC	5.161144	0	0.001149
86	CLARA vs. KMEANS	5.111637	0	0.001163
85	CyclicAssignment vs. PAM	5.074506	0	0.001176
84	BestCyclicAssignment vs. NearestByCustomer	5.074506	0	0.00119
83	BestCyclicAssignment vs. Parallel	5.074506	0	0.001205
82	Farthest-First vs. PAM	4.653693	0.000003	0.00122
81	CLARA vs. RandomByElement	4.62894	0.000004	0.001235
80	PAM vs. ThreeCriteriaClustering	4.567056	0.000005	0.00125
79	CoefficientPropagation vs. Simplified	4.542302	0.000006	0.001266
78	CoefficientPropagation vs. Sweep	4.529925	0.000006	0.001282
77	BestNearest vs. CoefficientPropagation	4.307142	0.000017	0.001299
76	BestCyclicAssignment vs. PAM	4.059605	0.000049	0.001316
75	NearestByDepot vs. Simplified	4.010098	0.000061	0.001333
74	NearestByDepot vs. Sweep	3.997721	0.000064	0.001351
73	CoefficientPropagation vs. UPGMC	3.886329	0.000102	0.00137
72	CoefficientPropagation vs. NearestByCustomer	3.799691	0.000145	0.001389
71	CoefficientPropagation vs. Parallel	3.799691	0.000145	0.001408
70	BestNearest vs. NearestByDepot	3.774938	0.00016	0.001429
69	KMEANS vs. PAM	3.750184	0.000177	0.001449
68	NearestByDepot vs. RandomByElement	3.737807	0.000186	0.001471
67	CLARA vs. CyclicAssignment	3.713053	0.000205	0.001493
66	Farthest-First vs. NearestByCustomer	3.638792	0.000274	0.001515
65	Farthest-First vs. Parallel	3.638792	0.000274	0.001538
64	NearestByCustomer vs. ThreeCriteriaClustering	3.552154	0.000382	0.001563
63	Parallel vs. ThreeCriteriaClustering	3.552154	0.000382	0.001587
62	Farthest-First vs. UPGMC	3.552154	0.000382	0.001613
61	ThreeCriteriaClustering vs. UPGMC	3.465516	0.000529	0.001639
60	NearestByDepot vs. UPGMC	3.354125	0.000796	0.001667
59	NearestByCustomer vs. NearestByDepot	3.267487	0.001085	0.001695
58	NearestByDepot vs. Parallel	3.267487	0.001085	0.001724
57	CoefficientPropagation vs. RandomByElement	3.205603	0.001348	0.001754
56	BestNearest vs. Farthest-First	3.131342	0.00174	0.001786
55	CLARA vs. Simplified	3.118965	0.001815	0.001818
54	CLARA vs. Sweep	3.106588	0.001893	0.001852
53	BestNearest vs. ThreeCriteriaClustering	3.044704	0.002329	0.001887
52	Farthest-First vs. Sweep	2.908558	0.003631	0.001923
51	Farthest-First vs. Simplified	2.896182	0.003777	0.001961
50	BestNearest vs. CLARA	2.883805	0.003929	0.002
49	Sweep vs. ThreeCriteriaClustering	2.82192	0.004774	0.002041
48	CyclicAssignment vs. NearestByDepot	2.82192	0.004774	0.002083
47	Simplified vs. ThreeCriteriaClustering	2.809544	0.004961	0.002128
46	CoefficientPropagation vs. PAM	2.78479	0.005356	0.002174
45	KMEANS vs. NearestByCustomer	2.735283	0.006233	0.002222
44	KMEANS vs. Parallel	2.735283	0.006233	0.002273
43	BestCyclicAssignment vs. CLARA	2.698152	0.006973	0.002326
42	KMEANS vs. UPGMC	2.648645	0.008082	0.002381
41	CLARA vs. UPGMC	2.462992	0.013778	0.002439
40	CLARA vs. NearestByCustomer	2.376354	0.017485	0.0025
39	CLARA vs. Parallel	2.376354	0.017485	0.002564
38	CoefficientPropagation vs. CyclicAssignment	2.289716	0.022038	0.002632
37	NearestByDepot vs. PAM	2.252586	0.024285	0.002703
36	BestNearest vs. KMEANS	2.227832	0.025892	0.002778
35	KMEANS vs. Sweep	2.005049	0.044958	0.002857
34	KMEANS vs. Simplified	1.992672	0.046297	0.002941
33	BestCyclicAssignment vs. RandomByElement	1.930788	0.053509	0.00303
32	BestCyclicAssignment vs. NearestByDepot	1.807019	0.070759	0.003125

2.3 Adjusted p-values

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Sha}
1	Farthest-First vs .RandomByElement	0	0	0	0
2	RandomByElement vs .ThreeCriteriaClustering	0	0	0	0
3	KMEANS vs .RandomByElement	0	0	0	0
4	CyclicAssignment vs .Farthest-First	0	0	0	0
5	CyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
6	CyclicAssignment vs .KMEANS	0	0	0	0
7	BestCyclicAssignment vs .Farthest-First	0	0	0	0
8	BestCyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
9	BestCyclicAssignment vs .KMEANS	0	0	0	0
10	RandomByElement vs .Simplified	0	0	0	0
11	RandomByElement vs .Sweep	0	0	0	0
12	BestNearest vs .RandomByElement	0	0	0	0
13	CoefficientPropagation vs .Farthest-First	0	0	0	0
14	CoefficientPropagation vs .ThreeCriteriaClustering	0	0	0	0
15	RandomByElement vs .UPGMC	0	0	0	0
16	NearestByCustomer vs .RandomByElement	0	0	0	0
17	Parallel vs .RandomByElement	0	0	0	0
18	Farthest-First vs .NearestByDepot	0	0	0	0
19	CyclicAssignment vs .Simplified	0	0	0	0
20	CyclicAssignment vs .Sweep	0	0	0	0
21	NearestByDepot vs .ThreeCriteriaClustering	0	0	0	0
22	BestNearest vs .CyclicAssignment	0	0	0	0
23	CoefficientPropagation vs .KMEANS	0	0	0	0
24	CyclicAssignment vs .UPGMC	0	0	0	0
25	CyclicAssignment vs .NearestByCustomer	0	0	0	0
26	CyclicAssignment vs .Parallel	0	0	0	0
27	CLARA vs .Farthest-First	0	0	0	0
28	KMEANS vs .NearestByDepot	0	0	0	0
29	PAM vs .RandomByElement	0	0	0	0
30	CLARA vs .ThreeCriteriaClustering	0	0	0	0
31	BestCyclicAssignment vs .Simplified	0	0.000001	0.000001	0
32	BestCyclicAssignment vs .Sweep	0	0.000001	0.000001	0.00000
33	BestCyclicAssignment vs .BestNearest	0	0.000003	0.000002	0.00000
34	BestCyclicAssignment vs .UPGMC	0	0.000029	0.000021	0.0000
35	CLARA vs .KMEANS	0	0.000038	0.000027	0.00003
36	CyclicAssignment vs .PAM	0	0.000047	0.000033	0.00003
37	BestCyclicAssignment vs .NearestByCustomer	0	0.000047	0.000033	0.00003
38	BestCyclicAssignment vs .Parallel	0	0.000047	0.000033	0.00003
39	Farthest-First vs .PAM	0.000003	0.000391	0.000267	0.00026
40	CLARA vs .RandomByElement	0.000004	0.000441	0.000298	0.00029
41	PAM vs .ThreeCriteriaClustering	0.000005	0.000594	0.000396	0.00039
42	CoefficientPropagation vs .Simplified	0.000006	0.000668	0.00044	0.0004
43	CoefficientPropagation vs .Sweep	0.000006	0.000708	0.00046	0.0004
44	BestNearest vs .CoefficientPropagation	0.000017	0.001985	0.001273	0.00119
45	BestCyclicAssignment vs .PAM	0.000049	0.005899	0.003736	0.00353
46	NearestByDepot vs .Simplified	0.000061	0.007283	0.004552	0.0043
47	NearestByDepot vs .Sweep	0.000064	0.007675	0.004733	0.00460
48	CoefficientPropagation vs .UPGMC	0.000102	0.012213	0.007429	0.00733
49	CoefficientPropagation vs .NearestByCustomer	0.000145	0.017385	0.010431	0.01043
50	CoefficientPropagation vs .Parallel	0.000145	0.017385	0.010431	0.01043
51	BestNearest vs .NearestByDepot	0.00016	0.019206	0.011203	0.01104
52	KMEANS vs .PAM	0.000177	0.021205	0.012193	0.01219
53	NearestByDepot vs .RandomByElement	0.000186	0.022276	0.012623	0.01263
54	CLARA vs .CyclicAssignment	0.000205	0.024573	0.01372	0.0137
55	Farthest-First vs .NearestByCustomer	0.000274	0.03287	0.018079	0.01807
56	Farthest-First vs .Parallel	0.000274	0.03287	0.018079	0.01807
57	NearestByCustomer vs .ThreeCriteriaClustering	0.000382	0.045851	0.024454	0.02330
58	Parallel vs .ThreeCriteriaClustering	0.000382	0.045851	0.024454	0.02330
59	Farthest-First vs .UPGMC	0.000382	0.045851	0.024454	0.02330
60	ThreeCriteriaClustering vs .UPGMC	0.000529	0.063506	0.032282	0.03228
61	NearestByDepot vs .UPGMC	0.000796	0.09554	0.04777	0.0477
62	NearestByCustomer vs .NearestByDepot	0.001085	0.130208	0.064019	0.0640
63	NearestByDepot vs .Parallel	0.001085	0.130208	0.064019	0.0640
64	CoefficientPropagation vs .RandomByElement	0.001348	0.161736	0.076825	0.07683
65	BestNearest vs .Farthest-First	0.00174	0.208812	0.097445	0.09744
66	CLARA vs .Simplified	0.001815	0.217785	0.099818	0.0998
67	CLARA vs .Sweep	0.001893	0.227112	0.1022	0.0998
68	BestNearest vs .ThreeCriteriaClustering	0.002329	0.279492	0.123442	0.1211
69	Farthest-First vs .Sweep	0.003631	0.435719	0.188812	0.1888
70	Farthest-First vs .Simplified	0.003777	0.453281	0.192644	0.19264
71	BestNearest vs .CLARA	0.003929	0.471483	0.196451	0.19264
72	Sweep vs .ThreeCriteriaClustering	0.004774	0.572844	0.233911	0.2339
73	CyclicAssignment vs .NearestByDepot	0.004774	0.572844	0.233911	0.2339
74	Simplified vs .ThreeCriteriaClustering	0.004961	0.595341	0.233911	0.2339
75	CoefficientPropagation vs .PAM	0.005356	0.642749	0.246387	0.24638
76	KMEANS vs .NearestByCustomer	0.006233	0.747921	0.28047	0.2804
77	KMEANS vs .Parallel	0.006233	0.747921	0.28047	0.2804
78	BestCyclicAssignment vs .CLARA	0.006973	0.836707	0.29982	0.2998
79	KMEANS vs .UPGMC	0.008082	0.969783	0.339424	0.33943
80	CLARA vs .UPGMC	0.013778	1.653396	0.56491	0.5649
81	CLARA vs .NearestByCustomer	0.017485	2.098161	0.699387	0.69938
82	CLARA vs .Parallel	0.017485	2.098161	0.699387	0.69938
83	CoefficientPropagation vs .CyclicAssignment	0.022038	2.644533	0.837435	0.83743
84	NearestByDepot vs .PAM	0.024285	2.914234	0.898556	0.89853
85	BestNearest vs .KMEANS	0.025892	3.107007	0.932102	0.93210
86	KMEANS vs .Sweep	0.044958	5.39494	1.573524	1.57353
87	KMEANS vs .Simplified	0.046297	5.555686	1.574111	1.5741
88	BestCyclicAssignment vs .RandomByElement	0.053509	6.421117	1.765807	1.76580
89	BestCyclicAssignment vs .NearestByDepot	0.070759	8.491113	2.264297	2.26429