Output tables for the test of Multiple comparisons.

June 12, 2025

1 Average rankings of Friedman test

Average ranks obtained by applying the Friedman procedure

Friedman statistic considering reduction performance (distributed according to chi-square with 15 degrees of freedom: 319.223039.

P-value computed by Friedman Test: 1.7722832320899987761E-10.

Iman and Davenport statistic considering reduction performance (distributed according to F-distribution with 15 and 525 degrees of freedom: 50.606759.

P-value computed by Iman and Daveport Test: 1.1224956004586499E-91.

Algorithm	Ranking
BestCyclicAssignment	12.2778
BestNearest	6.9306
CLARA	10.8333
CoefficientPropagation	11.25
CyclicAssignment	13.8333
Farthest-First	3.1944
KMEANS	4.4028
NearestByCustomer	6.8056
NearestByDepot	10.8889
PAM	8.4167
Parallel	6.8056
RandomByElement	15.9722
Simplified	6.1389
Sweep	6.3194
ThreeCriteriaClustering	4.0417
UPGMC	7.8889

Table 1: Average Rankings of the algorithms

2 Post hoc comparisons

Results achieved on post hoc comparisons for $\alpha = 0.05$, $\alpha = 0.10$ and adjusted p-values.

2.1 P-values for $\alpha = 0.05$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417.
Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000862.
Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	Farthest-First vs. RandomByElement	11.386697	0	0.000417
119	RandomByElement vs. ThreeCriteriaClustering	10.631709	0	0.00042
118	KMEANS vs. RandomByElement	10.309911	0	0.000424
117	CyclicAssignment vs. Farthest-First	9.480663	0	0.000427
116	RandomByElement vs. Simplified	8.762806	0	0.000431
115	CyclicAssignment vs. ThreeCriteriaClustering	8.725675	0	0.000435
114	RandomByElement vs. Sweep	8.601907	0	0.000439
113	CyclicAssignment vs. KMEANS	8.403877	0	0.000442
112	NearestByCustomer vs. RandomByElement	8.168717	0	0.000446
111	Parallel vs. RandomByElement	8.168717	0	0.00045
110	BestCyclicAssignment vs. Farthest-First	8.094456	0	0.000455
109	BestNearest vs. RandomByElement	8.057326	0	0.000459
108	BestCyclicAssignment vs. ThreeCriteriaClustering	7.339469	0	0.000463
107	RandomByElement vs. UPGMC	7.203323	0	0.000467
106	CoefficientPropagation vs. Farthest-First	7.17857	0	0.000472
105	BestCyclicAssignment vs. KMEANS	7.017671	0	0.000476
104	Farthest-First vs. NearestByCustomer	6.856772	0	0.000481
103	CyclicAssignment vs. Simplified	6.856772	0	0.000485
102	CLARA vs. Farthest-First	6.807264	0	0.00049
101	PAM vs. RandomByElement	6.733003	0	0.000495
100	CyclicAssignment vs. Sweep	6.695873	0	0.0005
99	CoefficientPropagation vs. ThreeCriteriaClustering	6.423582	0	0.000505
98	CyclicAssignment vs. NearestByCustomer	6.262683	0	0.00051
97	CyclicAssignment vs. Parallel	6.262683	0	0.000515
96	BestNearest vs. CyclicAssignment	6.151292	0	0.000521
95	CoefficientPropagation vs. KMEANS	6.101784	0	0.000526
94	NearestByDepot vs. ThreeCriteriaClustering	6.101784	0	0.000532
93	CLARA vs. ThreeCriteriaClustering	6.052277	0	0.000538
92	KMEANS vs. NearestByDepot	5.779986	0	0.000543
91	CLARA vs. KMEANS	5.730479	0	0.000549
90	BestCyclicAssignment vs. Simplified	5.470565	0	0.000556
89	BestCyclicAssignment vs. Sweep	5.309666	0	0.000562
88	CyclicAssignment vs. UPGMC	5.297289	0	0.000568
87	BestCyclicAssignment vs. NearestByCustomer	4.876477	0.000001	0.000575
86	BestCyclicAssignment vs. Parallel	4.876477	0.000001	0.000581
85	CyclicAssignment vs. PAM	4.826969	0.000001	0.000588
84	BestCyclicAssignment vs. BestNearest	4.765085	0.000002	0.000595
83	Farthest-First vs. PAM	4.653693	0.000003	0.000602
82	CLARA vs. RandomByElement	4.579432	0.000005	0.00061
81	CoefficientPropagation vs. Simplified	4.554679	0.000005	0.000617
80	NearestByDepot vs. RandomByElement	4.529925	0.000006	0.000625
79	CoefficientPropagation vs. Sweep	4.39378	0.000011	0.000633
78	NearestByDepot vs. Simplified	4.232881	0.000023	0.000641
77	CoefficientPropagation vs. RandomByElement	4.208127	0.000026	0.000649
76	Farthest-First vs. UPGMC	4.183373	0.000029	0.000658
75	CLARA vs. Simplified	4.183373	0.000029	0.000667
74	NearestByDepot vs. Sweep	4.071982	0.000047	0.000676
73	CLARA vs. Sweep	4.022474	0.000058	0.000685
72	CoefficientPropagation vs. NearestByCustomer	3.96059	0.000075	0.000694
71	CoefficientPropagation vs. Parallel	3.96059	0.000075	0.000704
70	BestCyclicAssignment vs. UPGMC	3.911083	0.000092	0.000714
69	PAM vs. ThreeCriteriaClustering	3.898706	0.000097	0.000725
68	BestNearest vs. CoefficientPropagation	3.849199	0.000119	0.000735
67	NearestByCustomer vs. NearestByDepot	3.638792	0.000274	0.000746
66	NearestByDepot vs. Parallel	3.638792	0.000274	0.000758
65	CLARA vs. NearestByCustomer	3.589285	0.000332	0.000769
64	CLARA vs. Parallel	3.589285	0.000332	0.000781
63	KMEANS vs. PAM	3.576908	0.000348	0.000794
62	BestNearest vs. NearestByDepot	3.527401	0.00042	0.000806
61	BestNearest vs. CLARA	3.477893	0.000505	0.00082
60	BestCyclicAssignment vs. PAM	3.440763	0.00058	0.000833
59	ThreeCriteriaClustering vs. UPGMC	3.428386	0.000607	0.000847
58	BestNearest vs. Farthest-First	3.329371	0.00087	0.000862
57	BestCyclicAssignment vs. RandomByElement	3.292241	0.000994	0.000877
56	Farthest-First vs. NearestByCustomer	3.21798	0.001291	0.000893
55	Farthest-First vs. Parallel	3.21798	0.001291	0.000909
54	KMEANS vs. UPGMC	3.106588	0.001893	0.000926
53	CoefficientPropagation vs. UPGMC	2.995196	0.002743	0.000943
52	Farthest-First vs. Sweep	2.78479	0.005356	0.000962
51	NearestByDepot vs. UPGMC	2.673398	0.007509	0.00098
50	CLARA vs. CyclicAssignment	2.673398	0.007509	0.001
49	Farthest-First vs. Simplified	2.623891	0.008693	0.00102
48	CLARA vs. UPGMC	2.623891	0.008693	0.001042
47	CyclicAssignment vs. NearestByDepot	2.623891	0.008693	0.001064
46	BestNearest vs. ThreeCriteriaClustering	2.574384	0.010042	0.001087
45	CoefficientPropagation vs. PAM	2.524876	0.011574	0.001111
44	NearestByCustomer vs. ThreeCriteriaClustering	2.462992	0.013778	0.001136
43	Parallel vs. ThreeCriteriaClustering	2.462992	0.013778	0.001163
42	CoefficientPropagation vs. CyclicAssignment	2.302093	0.02133	0.00119
41	BestNearest vs. KMEANS	2.252586	0.024285	0.00122
40	NearestByDepot vs. PAM	2.203078	0.027589	0.00125
39	CLARA vs. PAM	2.153571	0.031274	0.001282
38	KMEANS vs. NearestByCustomer	2.141194	0.032258	0.001316
37	KMEANS vs. Parallel	2.141194	0.032258	0.001351
36	Sweep vs. ThreeCriteriaClustering	2.029802	0.042377	0.001389
35	PAM vs. Simplified	2.029802	0.042377	0.001429
34	CyclicAssignment vs. RandomByElement	1.906034	0.056646	0.001471
33	Simplified vs. ThreeCriteriaClustering	1.868903	0.061636	0.001515
32	PAM vs. Sweep	1.868903	0.061636	0.001563

2.2 P-values for $\alpha = 0.10$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.

Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.001852.

Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	Farthest-First vs. RandomByElement	11.386697	0	0.000833
119	RandomByElement vs. ThreeCriteriaClustering	10.631709	0	0.00084
118	KMEANS vs. RandomByElement	10.309911	0	0.000847
117	CyclicAssignment vs. Farthest-First	9.480663	0	0.000855
116	RandomByElement vs. Simplified	8.762806	0	0.000862
115	CyclicAssignment vs. ThreeCriteriaClustering	8.725675	0	0.00087
114	RandomByElement vs. Sweep	8.601907	0	0.000877
113	CyclicAssignment vs. KMEANS	8.403877	0	0.000885
112	NearestByCustomer vs. RandomByElement	8.168717	0	0.000893
111	Parallel vs. RandomByElement	8.168717	0	0.000901
110	BestCyclicAssignment vs. Farthest-First	8.094456	0	0.000909
109	BestNearest vs. RandomByElement	8.057326	0	0.000917
108	BestCyclicAssignment vs. ThreeCriteriaClustering	7.339469	0	0.000926
107	RandomByElement vs. UPGMC	7.203323	0	0.000935
106	CoefficientPropagation vs. Farthest-First	7.17857	0	0.000943
105	BestCyclicAssignment vs. KMEANS	7.017671	0	0.000952
104	Farthest-First vs. NearestByCustomer	6.856772	0	0.000962
103	CyclicAssignment vs. Simplified	6.856772	0	0.000971
102	CLARA vs. Farthest-First	6.807264	0	0.00098
101	PAM vs. RandomByElement	6.733003	0	0.00099
100	CyclicAssignment vs. Sweep	6.695873	0	0.001
99	CoefficientPropagation vs. ThreeCriteriaClustering	6.423582	0	0.00101
98	CyclicAssignment vs. NearestByCustomer	6.262683	0	0.00102
97	CyclicAssignment vs. Parallel	6.262683	0	0.001031
96	BestNearest vs. CyclicAssignment	6.151292	0	0.001042
95	CoefficientPropagation vs. KMEANS	6.101784	0	0.001053
94	NearestByDepot vs. ThreeCriteriaClustering	6.101784	0	0.001064
93	CLARA vs. ThreeCriteriaClustering	6.052277	0	0.001075
92	KMEANS vs. NearestByDepot	5.779986	0	0.001087
91	CLARA vs. KMEANS	5.730479	0	0.001099
90	BestCyclicAssignment vs. Simplified	5.470565	0	0.001111
89	BestCyclicAssignment vs. Sweep	5.309666	0	0.001124
88	CyclicAssignment vs. UPGMC	5.297289	0	0.001136
87	BestCyclicAssignment vs. NearestByCustomer	4.876477	0.000001	0.001149
86	BestCyclicAssignment vs. Parallel	4.876477	0.000001	0.001163
85	CyclicAssignment vs. PAM	4.826969	0.000001	0.001176
84	BestCyclicAssignment vs. BestNearest	4.765085	0.000002	0.00119
83	Farthest-First vs. PAM	4.653693	0.000003	0.001205
82	CLARA vs. RandomByElement	4.579432	0.000005	0.00122
81	CoefficientPropagation vs. Simplified	4.554679	0.000005	0.001235
80	NearestByDepot vs. RandomByElement	4.529925	0.000006	0.00125
79	CoefficientPropagation vs. Sweep	4.39378	0.000011	0.001266
78	NearestByDepot vs. Simplified	4.232881	0.000023	0.001282
77	CoefficientPropagation vs. RandomByElement	4.208127	0.000026	0.001299
76	Farthest-First vs. UPGMC	4.183373	0.000029	0.001316
75	CLARA vs. Simplified	4.183373	0.000029	0.001333
74	NearestByDepot vs. Sweep	4.071982	0.000047	0.001351
73	CLARA vs. Sweep	4.022474	0.000058	0.00137
72	CoefficientPropagation vs. NearestByCustomer	3.96059	0.000075	0.001389
71	CoefficientPropagation vs. Parallel	3.96059	0.000075	0.001408
70	BestCyclicAssignment vs. UPGMC	3.911083	0.000092	0.001429
69	PAM vs. ThreeCriteriaClustering	3.898706	0.000097	0.001449
68	BestNearest vs. CoefficientPropagation	3.849199	0.000119	0.001471
67	NearestByCustomer vs. NearestByDepot	3.638792	0.000274	0.001493
66	NearestByDepot vs. Parallel	3.638792	0.000274	0.001515
65	CLARA vs. NearestByCustomer	3.589285	0.000332	0.001538
64	CLARA vs. Parallel	3.589285	0.000332	0.001563
63	KMEANS vs. PAM	3.576908	0.000348	0.001587
62	BestNearest vs. NearestByDepot	3.527401	0.00042	0.001613
61	BestNearest vs. CLARA	3.477893	0.000505	0.001639
60	BestCyclicAssignment vs. PAM	3.440763	0.00058	0.001667
59	ThreeCriteriaClustering vs. UPGMC	3.428386	0.000607	0.001695
58	BestNearest vs. Farthest-First	3.329371	0.00087	0.001724
57	BestCyclicAssignment vs. RandomByElement	3.292241	0.000994	0.001754
56	Farthest-First vs. NearestByCustomer	3.21798	0.001291	0.001786
55	Farthest-First vs. Parallel	3.21798	0.001291	0.001818
54	KMEANS vs. UPGMC	3.106588	0.001893	0.001852
53	CoefficientPropagation vs. UPGMC	2.995196	0.002743	0.001887
52	Farthest-First vs. Sweep	2.78479	0.005356	0.001923
51	NearestByDepot vs. UPGMC	2.673398	0.007509	0.001961
50	CLARA vs. CyclicAssignment	2.673398	0.007509	0.002
49	Farthest-First vs. Simplified	2.623891	0.008693	0.002041
48	CLARA vs. UPGMC	2.623891	0.008693	0.002083
47	CyclicAssignment vs. NearestByDepot	2.623891	0.008693	0.002128
46	BestNearest vs. ThreeCriteriaClustering	2.574384	0.010042	0.002174
45	CoefficientPropagation vs. PAM	2.524876	0.011574	0.002222
44	NearestByCustomer vs. ThreeCriteriaClustering	2.462992	0.013778	0.002273
43	Parallel vs. ThreeCriteriaClustering	2.462992	0.013778	0.002326
42	CoefficientPropagation vs. CyclicAssignment	2.302093	0.02133	0.002381
41	BestNearest vs. KMEANS	2.252586	0.024285	0.002439
40	NearestByDepot vs. PAM	2.203078	0.027589	0.0025
39	CLARA vs. PAM	2.153571	0.031274	0.002564
38	KMEANS vs. NearestByCustomer	2.141194	0.032258	0.002632
37	KMEANS vs. Parallel	2.141194	0.032258	0.002703
36	Sweep vs. ThreeCriteriaClustering	2.029802	0.042377	0.002778
35	PAM vs. Simplified	2.029802	0.042377	0.002857
34	CyclicAssignment vs. RandomByElement	1.906034	0.056646	0.002941
33	Simplified vs. ThreeCriteriaClustering	1.868903	0.061636	0.00303
32	PAM vs. Sweep	1.868903	0.061636	0.003125

2.3 Adjusted p-values

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Sha}
1	Farthest-First vs .RandomByElement	0	0	0	0
2	RandomByElement vs .ThreeCriteriaClustering	0	0	0	0
3	KMEANS vs .RandomByElement	0	0	0	0
4	CyclicAssignment vs .Farthest-First	0	0	0	0
5	RandomByElement vs .Simplified	0	0	0	0
6	CyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
7	RandomByElement vs .Sweep	0	0	0	0
8	CyclicAssignment vs .KMEANS	0	0	0	0
9	NearestByCustomer vs .RandomByElement	0	0	0	0
10	Parallel vs .RandomByElement	0	0	0	0
11	BestCyclicAssignment vs .Farthest-First	0	0	0	0
12	BestNearest vs .RandomByElement	0	0	0	0
13	BestCyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
14	RandomByElement vs .UPGMC	0	0	0	0
15	CoefficientPropagation vs .Farthest-First	0	0	0	0
16	BestCyclicAssignment vs .KMEANS	0	0	0	0
17	Farthest-First vs .NearestByDepot	0	0	0	0
18	CyclicAssignment vs .Simplified	0	0	0	0
19	CLARA vs .Farthest-First	0	0	0	0
20	PAM vs .RandomByElement	0	0	0	0
21	CyclicAssignment vs .Sweep	0	0	0	0
22	CoefficientPropagation vs .ThreeCriteriaClustering	0	0	0	0
23	CyclicAssignment vs .NearestByCustomer	0	0	0	0
24	CyclicAssignment vs .Parallel	0	0	0	0
25	BestNearest vs .CyclicAssignment	0	0	0	0
26	CoefficientPropagation vs .KMEANS	0	0	0	0
27	NearestByDepot vs .ThreeCriteriaClustering	0	0	0	0
28	CLARA vs .ThreeCriteriaClustering	0	0	0	0
29	KMEANS vs .NearestByDepot	0	0.000001	0.000001	0.00000
30	CLARA vs .KMEANS	0	0.000001	0.000001	0.00000
31	BestCyclicAssignment vs .Simplified	0	0.000005	0.000004	0.00000
32	BestCyclicAssignment vs .Sweep	0	0.000013	0.00001	0.00000
33	CyclicAssignment vs .UPGMC	0	0.000014	0.00001	0.0000
34	BestCyclicAssignment vs .NearestByCustomer	0.000001	0.00013	0.000094	0.0000
35	BestCyclicAssignment vs .Parallel	0.000001	0.00013	0.000094	0.0000
36	CyclicAssignment vs .PAM	0.000001	0.000166	0.000118	0.0001
37	BestCyclicAssignment vs .BestNearest	0.000002	0.000227	0.000159	0.0001
38	Farthest-First vs .PAM	0.000003	0.000391	0.000271	0.0002
39	CLARA vs .RandomByElement	0.000005	0.000559	0.000382	0.0003
40	CoefficientPropagation vs .Simplified	0.000005	0.00063	0.000425	0.0004
41	NearestByDepot vs .RandomByElement	0.000006	0.000708	0.000472	0.0004
42	CoefficientPropagation vs .Sweep	0.000011	0.001337	0.00088	0.0008
43	NearestByDepot vs .Simplified	0.000023	0.002769	0.0018	0.0018
44	CoefficientPropagation vs .RandomByElement	0.000026	0.00309	0.001983	0.0018
45	Farthest-First vs .UPGMC	0.000029	0.003447	0.002183	0.0020
46	CLARA vs .Simplified	0.000029	0.003447	0.002183	0.0020
47	NearestByDepot vs .Sweep	0.000047	0.005594	0.003449	0.0033
48	CLARA vs .Sweep	0.000058	0.006911	0.004204	0.0041
49	CoefficientPropagation vs .NearestByCustomer	0.000075	0.008972	0.005383	0.0053
50	CoefficientPropagation vs .Parallel	0.000075	0.008972	0.005383	0.0053
51	BestCyclicAssignment vs .UPGMC	0.000092	0.011026	0.006432	0.0063
52	PAM vs .ThreeCriteriaClustering	0.000097	0.011605	0.006673	0.0066
53	BestNearest vs .CoefficientPropagation	0.000119	0.014221	0.008058	0.0080
54	NearestByCustomer vs .NearestByDepot	0.000274	0.03287	0.018353	0.0183
55	NearestByDepot vs .Parallel	0.000274	0.03287	0.018353	0.0183
56	CLARA vs .NearestByCustomer	0.000332	0.03979	0.021553	0.0215
57	CLARA vs .Parallel	0.000332	0.03979	0.021553	0.0215
58	KMEANS vs .PAM	0.000348	0.041722	0.021904	0.0215
59	BestNearest vs .NearestByDepot	0.00042	0.050359	0.026019	0.0255
60	BestNearest vs .CLARA	0.000505	0.060645	0.030828	0.0308
61	BestCyclicAssignment vs .PAM	0.00058	0.069609	0.034805	0.0348
62	ThreeCriteriaClustering vs .UPGMC	0.000607	0.072862	0.035824	0.0358
63	BestNearest vs .Farthest-First	0.00087	0.104451	0.050485	0.0504
64	BestCyclicAssignment vs .RandomByElement	0.000994	0.119271	0.056654	0.0566
65	Farthest-First vs .NearestByCustomer	0.001291	0.154916	0.072294	0.0722
66	Farthest-First vs .Parallel	0.001291	0.154916	0.072294	0.0722
67	KMEANS vs .UPGMC	0.001893	0.227112	0.1022	0.0984
68	CoefficientPropagation vs .UPGMC	0.002743	0.329122	0.145362	0.1426
69	Farthest-First vs .Sweep	0.005356	0.642749	0.278525	0.2785
70	NearestByDepot vs .UPGMC	0.007509	0.901044	0.382944	0.3829
71	CLARA vs .CyclicAssignment	0.007509	0.901044	0.382944	0.3829
72	Farthest-First vs .Simplified	0.008693	1.043179	0.425965	0.4259
73	CLARA vs .UPGMC	0.008693	1.043179	0.425965	0.4259
74	CyclicAssignment vs .NearestByDepot	0.008693	1.043179	0.425965	0.4259
75	BestNearest vs .ThreeCriteriaClustering	0.010042	1.205026	0.461927	0.4619
76	CoefficientPropagation vs .PAM	0.011574	1.388869	0.520826	0.5208
77	NearestByCustomer vs .ThreeCriteriaClustering	0.013778	1.653396	0.606245	0.6062
78	Parallel vs .ThreeCriteriaClustering	0.013778	1.653396	0.606245	0.6062
79	CoefficientPropagation vs .CyclicAssignment	0.02133	2.559591	0.895857	0.8958
80	BestNearest vs .KMEANS	0.024285	2.914234	0.995697	0.9956
81	NearestByDepot vs .PAM	0.027589	3.310708	1.103569	1.1035
82	CLARA vs .PAM	0.031274	3.752861	1.21968	1.2196
83	KMEANS vs .NearestByCustomer	0.032258	3.871007	1.225819	1.2258
84	KMEANS vs .Parallel	0.032258	3.871007	1.225819	1.2258
85	Sweep vs .ThreeCriteriaClustering	0.042377	5.085195	1.525558	1.5255
86	PAM vs .Simplified	0.042377	5.085195	1.525558	1.5255
87	CyclicAssignment vs .RandomByElement	0.056646	6.797495	1.925957	1.9259
88	Simplified vs .ThreeCriteriaClustering	0.061636	7.396349	2.033996	2.03399
89	PAM vs .Sweep	0.061636	7.396349	2.033996	2.03399