Output tables for the test of Multiple comparisons.

June 6, 2025

1 Average rankings of Friedman test

Average ranks obtained by applying the Friedman procedure

Friedman statistic considering reduction performance (distributed according to chi-square with 17 degrees of freedom: 369.690526.

P-value computed by Friedman Test: 1.3182654967636154E-10.

Iman and Davenport statistic considering reduction performance (distributed according to F-distribution with 17 and 408 degrees of freedom: 160.41687.

P-value computed by Iman and Daveport Test: -4.440892098500626E-16.

Algorithm	Ranking
BestCyclicAssignment	6.92
BestNearest	6.4
CLARA	17.52
CoefficientPropagation	11.88
CyclicAssignment	4.74
Farthest-First	15.72
KMEANS	14.88
NearestByCustomer	5.52
NearestByDepot	10.54
PAM	17.44
Parallel	5.7
RandomByElement	1.7
RandomSequentialCyclic	4.4
SequentialCyclic	4.32
Simplified	7.52
Sweep	8.76
ThreeCriteriaClustering	13.64
UPGMC	13.4

Table 1: Average Rankings of the algorithms

2 Post hoc comparisons

Results achieved on post hoc comparisons for $\alpha = 0.05$, $\alpha = 0.10$ and adjusted p-values.

2.1 P-values for $\alpha = 0.05$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000327.
Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000658.
Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000327.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
153	CLARA vs. RandomByElement	10.477051	0	0.000327
152	PAM vs. RandomByElement	10.42407	0	0.000329
151	Farthest-First vs. RandomByElement	9.284972	0	0.000331
150	CLARA vs. SequentialCyclic	8.741914	0	0.000333
149	KMEANS vs. RandomByElement	8.728668	0	0.000336
148	CLARA vs. RandomSequentialCyclic	8.688932	0	0.000338
147	PAM vs. SequentialCyclic	8.688932	0	0.00034
146	PAM vs. RandomSequentialCyclic	8.635951	0	0.000342
145	CLARA vs. CyclicAssignment	8.463762	0	0.000345
144	CyclicAssignment vs. PAM	8.41078	0	0.000347
143	CLARA vs. NearestByCustomer	7.947194	0	0.00035
142	RandomByElement vs. ThreeCriteriaClustering	7.907458	0	0.000352
141	NearestByCustomer vs. PAM	7.894213	0	0.000355
140	CLARA vs. Parallel	7.827986	0	0.000357
139	PAM vs. Parallel	7.775005	0	0.00036
138	RandomByElement vs. UPGMC	7.748514	0	0.000362
137	Farthest-First vs. SequentialCyclic	7.549834	0	0.000365
136	Farthest-First vs. RandomSequentialCyclic	7.496853	0	0.000368
135	BestNearest vs. CLARA	7.3644	0	0.00037
134	BestNearest vs. PAM	7.311419	0	0.000373
133	CyclicAssignment vs. Farthest-First	7.271683	0	0.000376
132	BestCyclicAssignment vs. CLARA	7.020021	0	0.000379
131	KMEANS vs. SequentialCyclic	6.993531	0	0.000382
130	BestCyclicAssignment vs. PAM	6.96704	0	0.000385
129	KMEANS vs. RandomSequentialCyclic	6.94055	0	0.000388
128	Farthest-First vs. NearestByCustomer	6.755115	0	0.000391
127	CoefficientPropagation vs. RandomByElement	6.74187	0	0.000394
126	CyclicAssignment vs. KMEANS	6.715379	0	0.000397
125	Farthest-First vs. Parallel	6.635907	0	0.0004
124	CLARA vs. Simplified	6.622662	0	0.000403
123	PAM vs. Simplified	6.56968	0	0.000407
122	KMEANS vs. NearestByCustomer	6.198811	0	0.00041
121	SequentialCyclic vs. ThreeCriteriaClustering	6.172321	0	0.000413
120	BestNearest vs. Farthest-First	6.172321	0	0.000417
119	RandomSequentialCyclic vs. ThreeCriteriaClustering	6.119339	0	0.00042
118	KMEANS vs. Parallel	6.079604	0	0.000424
117	SequentialCyclic vs. UPGMC	6.013377	0	0.000427
116	RandomSequentialCyclic vs. UPGMC	5.960396	0	0.000431
115	CyclicAssignment vs. ThreeCriteriaClustering	5.894169	0	0.000435
114	NearestByDepot vs. RandomByElement	5.854433	0	0.000439
113	BestCyclicAssignment vs. Farthest-First	5.827942	0	0.000442
112	CLARA vs. Sweep	5.801452	0	0.000446
111	PAM vs. Sweep	5.74847	0	0.00045
110	CyclicAssignment vs. UPGMC	5.735225	0	0.000455
109	BestNearest vs. KMEANS	5.616017	0	0.000459
108	Farthest-First vs. Simplified	5.430583	0	0.000463
107	NearestByCustomer vs. ThreeCriteriaClustering	5.377601	0	0.000467
106	BestCyclicAssignment vs. KMEANS	5.271639	0	0.000472
105	Parallel vs. ThreeCriteriaClustering	5.258393	0	0.000476
104	NearestByCustomer vs. UPGMC	5.218657	0	0.000481
103	Parallel vs. UPGMC	5.09945	0	0.000485
102	CoefficientPropagation vs. SequentialCyclic	5.006732	0.000001	0.00049
101	CoefficientPropagation vs. RandomSequentialCyclic	4.953751	0.000001	0.000495
100	KMEANS vs. Simplified	4.874279	0.000001	0.0005
99	BestNearest vs. ThreeCriteriaClustering	4.794807	0.000002	0.000505
98	CoefficientPropagation vs. CyclicAssignment	4.728581	0.000002	0.00051
97	RandomByElement vs. Sweep	4.675599	0.000003	0.000515
96	BestNearest vs. UPGMC	4.635863	0.000004	0.000521
95	CLARA vs. NearestByDepot	4.622618	0.000004	0.000526
94	Farthest-First vs. Sweep	4.609373	0.000004	0.000532
93	NearestByDepot vs. PAM	4.569637	0.000005	0.000538
92	BestCyclicAssignment vs. ThreeCriteriaClustering	4.450429	0.000009	0.000543
91	BestCyclicAssignment vs. UPGMC	4.291485	0.000018	0.000549
90	CoefficientPropagation vs. NearestByCustomer	4.212013	0.000025	0.000556
89	NearestByDepot vs. SequentialCyclic	4.119296	0.000038	0.000562
88	CoefficientPropagation vs. Parallel	4.092805	0.000043	0.000568
87	NearestByDepot vs. RandomSequentialCyclic	4.066314	0.000048	0.000575
86	Simplified vs. ThreeCriteriaClustering	4.053069	0.000051	0.000581
85	KMEANS vs. Sweep	4.053069	0.000051	0.000588
84	Simplified vs. UPGMC	3.894125	0.000099	0.000595
83	RandomByElement vs. Simplified	3.854389	0.000116	0.000602
82	CyclicAssignment vs. NearestByDepot	3.841144	0.000122	0.00061
81	CLARA vs. CoefficientPropagation	3.735181	0.000188	0.000617
80	CoefficientPropagation vs. PAM	3.6822	0.000231	0.000625
79	BestNearest vs. CoefficientPropagation	3.629219	0.000284	0.000633
78	BestCyclicAssignment vs. RandomByElement	3.457029	0.000546	0.000641
77	Farthest-First vs. NearestByDepot	3.430539	0.000602	0.000649
76	NearestByCustomer vs. NearestByDepot	3.324576	0.000886	0.000658
75	BestCyclicAssignment vs. CoefficientPropagation	3.28484	0.00102	0.000667
74	Sweep vs. ThreeCriteriaClustering	3.231859	0.00123	0.000676
73	NearestByDepot vs. Parallel	3.205368	0.001349	0.000685
72	BestNearest vs. RandomByElement	3.112651	0.001854	0.000694
71	Sweep vs. UPGMC	3.072915	0.00212	0.000704
70	SequentialCyclic vs. Sweep	2.940462	0.003277	0.000714
69	CoefficientPropagation vs. Simplified	2.887481	0.003883	0.000725
68	RandomSequentialCyclic vs. Sweep	2.887481	0.003883	0.000735
67	KMEANS vs. NearestByDepot	2.874235	0.00405	0.000746
66	BestNearest vs. NearestByDepot	2.741782	0.006111	0.000758
65	CLARA vs. UPGMC	2.728537	0.006362	0.000769

2.2 P-values for $\alpha = 0.10$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000654.

Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.001389.

Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000654.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
153	CLARA vs. RandomByElement	10.477051	0	0.000654
152	PAM vs. RandomByElement	10.42407	0	0.000658
151	Farthest-First vs. RandomByElement	9.284972	0	0.000662
150	CLARA vs. SequentialCyclic	8.741914	0	0.000667
149	KMEANS vs. RandomByElement	8.728668	0	0.000671
148	CLARA vs. RandomSequentialCyclic	8.688932	0	0.000676
147	PAM vs. SequentialCyclic	8.688932	0	0.00068
146	PAM vs. RandomSequentialCyclic	8.635951	0	0.000685
145	CLARA vs. CyclicAssignment	8.463762	0	0.00069
144	CyclicAssignment vs. PAM	8.41078	0	0.000694
143	CLARA vs. NearestByCustomer	7.947194	0	0.000699
142	RandomByElement vs. ThreeCriteriaClustering	7.907458	0	0.000704
141	NearestByCustomer vs. PAM	7.894213	0	0.000709
140	CLARA vs. Parallel	7.827986	0	0.000714
139	PAM vs. Parallel	7.775005	0	0.000719
138	RandomByElement vs. UPGMC	7.748514	0	0.000725
137	Farthest-First vs. SequentialCyclic	7.549834	0	0.00073
136	Farthest-First vs. RandomSequentialCyclic	7.496853	0	0.000735
135	BestNearest vs. CLARA	7.3644	0	0.000741
134	BestNearest vs. PAM	7.311419	0	0.000746
133	CyclicAssignment vs. Farthest-First	7.271683	0	0.000752
132	BestCyclicAssignment vs. CLARA	7.020021	0	0.000758
131	KMEANS vs. SequentialCyclic	6.993531	0	0.000763
130	BestCyclicAssignment vs. PAM	6.96704	0	0.000769
129	KMEANS vs. RandomSequentialCyclic	6.94055	0	0.000775
128	Farthest-First vs. NearestByCustomer	6.755115	0	0.000781
127	CoefficientPropagation vs. RandomByElement	6.74187	0	0.000787
126	CyclicAssignment vs. KMEANS	6.715379	0	0.000794
125	Farthest-First vs. Parallel	6.635907	0	0.0008
124	CLARA vs. Simplified	6.622662	0	0.000806
123	PAM vs. Simplified	6.56968	0	0.000813
122	KMEANS vs. NearestByCustomer	6.198811	0	0.00082
121	SequentialCyclic vs. ThreeCriteriaClustering	6.172321	0	0.000826
120	BestNearest vs. Farthest-First	6.172321	0	0.000833
119	RandomSequentialCyclic vs. ThreeCriteriaClustering	6.119339	0	0.00084
118	KMEANS vs. Parallel	6.079604	0	0.000847
117	SequentialCyclic vs. UPGMC	6.013377	0	0.000855
116	RandomSequentialCyclic vs. UPGMC	5.960396	0	0.000862
115	CyclicAssignment vs. ThreeCriteriaClustering	5.894169	0	0.00087
114	NearestByDepot vs. RandomByElement	5.854433	0	0.000877
113	BestCyclicAssignment vs. Farthest-First	5.827942	0	0.000885
112	CLARA vs. Sweep	5.801452	0	0.000893
111	PAM vs. Sweep	5.74847	0	0.000901
110	CyclicAssignment vs. UPGMC	5.735225	0	0.000909
109	BestNearest vs. KMEANS	5.616017	0	0.000917
108	Farthest-First vs. Simplified	5.430583	0	0.000926
107	NearestByCustomer vs. ThreeCriteriaClustering	5.377601	0	0.000935
106	BestCyclicAssignment vs. KMEANS	5.271639	0	0.000943
105	Parallel vs. ThreeCriteriaClustering	5.258393	0	0.000952
104	NearestByCustomer vs. UPGMC	5.218657	0	0.000962
103	Parallel vs. UPGMC	5.09945	0	0.000971
102	CoefficientPropagation vs. SequentialCyclic	5.006732	0.000001	0.00098
101	CoefficientPropagation vs. RandomSequentialCyclic	4.953751	0.000001	0.00099
100	KMEANS vs. Simplified	4.874279	0.000001	0.001
99	BestNearest vs. ThreeCriteriaClustering	4.794807	0.000002	0.00101
98	CoefficientPropagation vs. CyclicAssignment	4.728581	0.000002	0.00102
97	RandomByElement vs. Sweep	4.675599	0.000003	0.001031
96	BestNearest vs. UPGMC	4.635863	0.000004	0.001042
95	CLARA vs. NearestByDepot	4.622618	0.000004	0.001053
94	Farthest-First vs. Sweep	4.609373	0.000004	0.001064
93	NearestByDepot vs. PAM	4.569637	0.000005	0.001075
92	BestCyclicAssignment vs. ThreeCriteriaClustering	4.450429	0.000009	0.001087
91	BestCyclicAssignment vs. UPGMC	4.291485	0.000018	0.001099
90	CoefficientPropagation vs. NearestByCustomer	4.212013	0.000025	0.001111
89	NearestByDepot vs. SequentialCyclic	4.119296	0.000038	0.001124
88	CoefficientPropagation vs. Parallel	4.092805	0.000043	0.001136
87	NearestByDepot vs. RandomSequentialCyclic	4.066314	0.000048	0.001149
86	Simplified vs. ThreeCriteriaClustering	4.053069	0.000051	0.001163
85	KMEANS vs. Sweep	4.053069	0.000051	0.001176
84	Simplified vs. UPGMC	3.894125	0.000099	0.00119
83	RandomByElement vs. Simplified	3.854389	0.000116	0.001205
82	CyclicAssignment vs. NearestByDepot	3.841144	0.000122	0.00122
81	CLARA vs. CoefficientPropagation	3.735181	0.000188	0.001235
80	CoefficientPropagation vs. PAM	3.6822	0.000231	0.00125
79	BestNearest vs. CoefficientPropagation	3.629219	0.000284	0.001266
78	BestCyclicAssignment vs. RandomByElement	3.457029	0.000546	0.001282
77	Farthest-First vs. NearestByDepot	3.430539	0.000602	0.001299
76	NearestByCustomer vs. NearestByDepot	3.324576	0.000886	0.001316
75	BestCyclicAssignment vs. CoefficientPropagation	3.28484	0.00102	0.001333
74	Sweep vs. ThreeCriteriaClustering	3.231859	0.00123	0.001351
73	NearestByDepot vs. Parallel	3.205368	0.001349	0.00137
72	BestNearest vs. RandomByElement	3.112651	0.001854	0.001389
71	Sweep vs. UPGMC	3.072915	0.00212	0.001408
70	SequentialCyclic vs. Sweep	2.940462	0.003277	0.001429
69	CoefficientPropagation vs. Simplified	2.887481	0.003883	0.001449
68	RandomSequentialCyclic vs. Sweep	2.887481	0.003883	0.001471
67	KMEANS vs. NearestByDepot	2.874235	0.00405	0.001493
66	BestNearest vs. NearestByDepot	2.741782	0.006111	0.001515
65	CLARA vs. UPGMC	2.728537	0.006362	0.001538

2.3 Adjusted p-values

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Sho}
1	CLARA vs .RandomByElement	0	0	0	0
2	PAM vs .RandomByElement	0	0	0	0
3	Farthest-First vs .RandomByElement	0	0	0	0
4	CLARA vs .SequentialCyclic	0	0	0	0
5	KMEANS vs .RandomByElement	0	0	0	0
6	CLARA vs .RandomSequentialCyclic	0	0	0	0
7	PAM vs .SequentialCyclic	0	0	0	0
8	PAM vs .RandomSequentialCyclic	0	0	0	0
9	CLARA vs .CyclicAssignment	0	0	0	0
10	CyclicAssignment vs .PAM	0	0	0	0
11	CLARA vs .NearestByCustomer	0	0	0	0
12	RandomByElement vs .ThreeCriteriaClustering	0	0	0	0
13	NearestByCustomer vs .PAM	0	0	0	0
14	CLARA vs .Parallel	0	0	0	0
15	PAM vs .Parallel	0	0	0	0
16	RandomByElement vs .UPGMC	0	0	0	0
17	Farthest-First vs .SequentialCyclic	0	0	0	0
18	Farthest-First vs .RandomSequentialCyclic	0	0	0	0
19	BestNearest vs .CLARA	0	0	0	0
20	BestNearest vs .PAM	0	0	0	0
21	CyclicAssignment vs .Farthest-First	0	0	0	0
22	BestCyclicAssignment vs .CLARA	0	0	0	0
23	KMEANS vs .SequentialCyclic	0	0	0	0
24	BestCyclicAssignment vs .PAM	0	0	0	0
25	KMEANS vs .RandomSequentialCyclic	0	0	0	0
26	Farthest-First vs .NearestByCustomer	0	0	0	0
27	CoefficientPropagation vs .RandomByElement	0	0	0	0
28	CyclicAssignment vs .KMEANS	0	0	0	0
29	Farthest-First vs .Parallel	0	0	0	0
30	CLARA vs .Simplified	0	0	0	0
31	PAM vs .Simplified	0	0	0	0
32	KMEANS vs .NearestByCustomer	0	0	0	0
33	SequentialCyclic vs .ThreeCriteriaClustering	0	0	0	0
34	BestNearest vs .Farthest-First	0	0	0	0
35	RandomSequentialCyclic vs .ThreeCriteriaClustering	0	0	0	0
36	KMEANS vs .Parallel	0	0	0	0
37	SequentialCyclic vs .UPGMC	0	0	0	0
38	RandomSequentialCyclic vs .UPGMC	0	0	0	0
39	CyclicAssignment vs .ThreeCriteriaClustering	0	0.000001	0	0
40	NearestByDepot vs .RandomByElement	0	0.000001	0.000001	0.0000
41	BestCyclicAssignment vs .Farthest-First	0	0.000001	0.000001	0.0000
42	CLARA vs .Sweep	0	0.000001	0.000001	0.0000
43	PAM vs .Sweep	0	0.000001	0.000001	0.0000
44	CyclicAssignment vs .UPGMC	0	0.000001	0.000001	0.0000
45	BestNearest vs .KMEANS	0	0.000003	0.000002	0.0000
46	Farthest-First vs .Simplified	0	0.000009	0.000006	0.0000
47	NearestByCustomer vs .ThreeCriteriaClustering	0	0.000012	0.000008	0.0000
48	BestCyclicAssignment vs .KMEANS	0	0.000021	0.000014	0.0000
49	Parallel vs .ThreeCriteriaClustering	0	0.000022	0.000015	0.0000
50	NearestByCustomer vs .UPGMC	0	0.000028	0.000019	0.0000
51	Parallel vs .UPGMC	0	0.000052	0.000035	0.0000
52	CoefficientPropagation vs .SequentialCyclic	0.000001	0.000085	0.000056	0.0000
53	CoefficientPropagation vs .RandomSequentialCyclic	0.000001	0.000111	0.000074	0.0000
54	KMEANS vs .Simplified	0.000001	0.000167	0.000109	0.0001
55	BestNearest vs .ThreeCriteriaClustering	0.000002	0.000249	0.000161	0.0001
56	CoefficientPropagation vs .CyclicAssignment	0.000002	0.000346	0.000222	0.0002
57	RandomByElement vs .Sweep	0.000003	0.000448	0.000284	0.0002
58	BestNearest vs .UPGMC	0.000004	0.000544	0.000341	0.0003
59	CLARA vs .NearestByDepot	0.000004	0.00058	0.00036	0.0003
60	Farthest-First vs .Sweep	0.000004	0.000618	0.00038	0.000
61	NearestByDepot vs .PAM	0.000005	0.000748	0.000454	0.0004
62	BestCyclicAssignment vs .ThreeCriteriaClustering	0.000009	0.001311	0.000788	0.0007
63	BestCyclicAssignment vs .UPGMC	0.000018	0.002715	0.001615	0.0010
64	CoefficientPropagation vs .NearestByCustomer	0.000025	0.003873	0.002278	0.0022
65	NearestByDepot vs .SequentialCyclic	0.000038	0.005814	0.003382	0.0033
66	CoefficientPropagation vs .Parallel	0.000043	0.006521	0.00375	0.003
67	NearestByDepot vs .RandomSequentialCyclic	0.000048	0.007308	0.004155	0.0040
68	Simplified vs .ThreeCriteriaClustering	0.000051	0.007734	0.004347	0.0042
69	KMEANS vs .Sweep	0.000051	0.007734	0.004347	0.0042
70	Simplified vs .UPGMC	0.000099	0.015079	0.008279	0.0082
71	RandomByElement vs .Simplified	0.000116	0.017751	0.00963	0.0095
72	CyclicAssignment vs .NearestByDepot	0.000122	0.018737	0.010042	0.0100
73	CLARA vs .CoefficientPropagation	0.000188	0.0287	0.015194	0.0151
74	CoefficientPropagation vs .PAM	0.000231	0.035378	0.018498	0.0184
75	BestNearest vs .CoefficientPropagation	0.000284	0.043495	0.022458	0.0224
76	BestCyclicAssignment vs .RandomByElement	0.000546	0.083563	0.042601	0.0420
77	Farthest-First vs .NearestByDepot	0.000602	0.092165	0.046384	0.0457
78	NearestByCustomer vs .NearestByDepot	0.000886	0.135486	0.0673	0.067
79	BestCyclicAssignment vs .CoefficientPropagation	0.00102	0.156122	0.07653	0.0744
80	Sweep vs .ThreeCriteriaClustering	0.00123	0.188171	0.091011	0.0897
81	NearestByDepot vs .Parallel	0.001349	0.206381	0.09847	0.098
82	BestNearest vs .RandomByElement	0.001854	0.283685	0.133499	0.1334
83	Sweep vs .UPGMC	0.00212	0.324328	0.150505	0.1483
84	SequentialCyclic vs .Sweep	0.003277	0.501417	0.229406	0.2294
85	CoefficientPropagation vs .Simplified	0.003883	0.594161	0.267955	0.2679
86	RandomSequentialCyclic vs .Sweep	0.003883	0.594161	0.267955	0.2679
87	KMEANS vs .NearestByDepot	0.00405	0.619661	0.271355	0.2713
88	BestNearest vs .NearestByDepot	0.006111	0.934935	0.403305	0.4033
89	CLARA vs .UPGMC	0.006362	0.973325	0.413504	0.4135