Output tables for the test of Multiple comparisons.

June 12, 2025

1 Average rankings of Friedman test

Average ranks obtained by applying the Friedman procedure

Friedman statistic considering reduction performance (distributed according to chi-square with 15 degrees of freedom: 278.220588.

P-value computed by Friedman Test: 1.2288892126122164E-10.

Iman and Davenport statistic considering reduction performance (distributed according to F-distribution with 15 and 525 degrees of freedom: 37.198191.

P-value computed by Iman and Daveport Test: 1.2162314073751133E-72.

Algorithm	Ranking
BestCyclicAssignment	8.7778
BestNearest	7.5972
CLARA	10.125
CoefficientPropagation	7.3889
CyclicAssignment	13.8056
Farthest-First	5.5833
KMEANS	5.5139
NearestByCustomer	8.5972
NearestByDepot	12.1389
PAM	10.1389
Parallel	8.5972
RandomByElement	15.6528
Simplified	7.7083
Sweep	7.4167
ThreeCriteriaClustering	5.1389
UPGMC	1.8194

Table 1: Average Rankings of the algorithms

2 Post hoc comparisons

Results achieved on post hoc comparisons for $\alpha = 0.05$, $\alpha = 0.10$ and adjusted p-values.

2.1 P-values for $\alpha = 0.05$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417.
Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000685.
Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	RandomByElement vs. UPGMC	12.327337	0	0.000417
119	CyclicAssignment vs. UPGMC	10.681217	0	0.00042
118	RandomByElement vs. ThreeCriteriaClustering	9.369271	0	0.000424
117	NearestByDepot vs. UPGMC	9.195995	0	0.000427
116	KMEANS vs. RandomByElement	9.035096	0	0.000431
115	Farthest-First vs. RandomByElement	8.973212	0	0.000435
114	CyclicAssignment vs. ThreeCriteriaClustering	7.723151	0	0.000439
113	PAM vs. UPGMC	7.41373	0	0.000442
112	CLARA vs. UPGMC	7.401353	0	0.000446
111	CyclicAssignment vs. KMEANS	7.388976	0	0.00045
110	CoefficientPropagation vs. RandomByElement	7.364222	0	0.000455
109	RandomByElement vs. Sweep	7.339469	0	0.000459
108	CyclicAssignment vs. Farthest-First	7.327092	0	0.000463
107	BestNearest vs. RandomByElement	7.17857	0	0.000467
106	RandomByElement vs. Simplified	7.079555	0	0.000472
105	NearestByCustomer vs. RandomByElement	6.287437	0	0.000476
104	Parallel vs. RandomByElement	6.287437	0	0.000481
103	NearestByDepot vs. ThreeCriteriaClustering	6.23793	0	0.000485
102	BestCyclicAssignment vs. UPGMC	6.200799	0	0.00049
101	BestCyclicAssignment vs. RandomByElement	6.126538	0	0.000495
100	NearestByCustomer vs. UPGMC	6.0399	0	0.0005
99	Parallel vs. UPGMC	6.0399	0	0.000505
98	KMEANS vs. NearestByDepot	5.903755	0	0.00051
97	Farthest-First vs. NearestByDepot	5.841871	0	0.000515
96	CoefficientPropagation vs. CyclicAssignment	5.718102	0	0.000521
95	CyclicAssignment vs. Sweep	5.693348	0	0.000526
94	BestNearest vs. CyclicAssignment	5.532449	0	0.000532
93	CyclicAssignment vs. Simplified	5.433435	0	0.000538
92	Simplified vs. UPGMC	5.247782	0	0.000543
91	BestNearest vs. UPGMC	5.148767	0	0.000549
90	Sweep vs. UPGMC	4.987868	0.000001	0.000556
89	CoefficientPropagation vs. UPGMC	4.963115	0.000001	0.000562
88	CLARA vs. RandomByElement	4.925984	0.000001	0.000568
87	PAM vs. RandomByElement	4.913607	0.000001	0.000575
86	CyclicAssignment vs. NearestByCustomer	4.641317	0.000003	0.000581
85	CyclicAssignment vs. Parallel	4.641317	0.000003	0.000588
84	BestCyclicAssignment vs. CyclicAssignment	4.480418	0.000007	0.000595
83	PAM vs. ThreeCriteriaClustering	4.455664	0.000008	0.000602
82	CLARA vs. ThreeCriteriaClustering	4.443287	0.000009	0.00061
81	CoefficientPropagation vs. NearestByDepot	4.232881	0.000023	0.000617
80	NearestByDepot vs. Sweep	4.208127	0.000026	0.000625
79	KMEANS vs. PAM	4.121489	0.000038	0.000633
78	CLARA vs. KMEANS	4.109112	0.00004	0.000641
77	Farthest-First vs. PAM	4.059605	0.000049	0.000649
76	CLARA vs. Farthest-First	4.047228	0.000052	0.000658
75	BestNearest vs. NearestByDepot	4.047228	0.000052	0.000667
74	NearestByDepot vs. Simplified	3.948213	0.000079	0.000676
73	Farthest-First vs. UPGMC	3.354125	0.000796	0.000685
72	KMEANS vs. UPGMC	3.292241	0.000994	0.000694
71	CLARA vs. CyclicAssignment	3.279864	0.001039	0.000704
70	CyclicAssignment vs. PAM	3.267487	0.001085	0.000714
69	BestCyclicAssignment vs. ThreeCriteriaClustering	3.242733	0.001184	0.000725
68	NearestByCustomer vs. NearestByDepot	3.156095	0.001599	0.000735
67	NearestByDepot vs. Parallel	3.156095	0.001599	0.000746
66	NearestByDepot vs. RandomByElement	3.131342	0.00174	0.000758
65	NearestByCustomer vs. ThreeCriteriaClustering	3.081834	0.002057	0.000769
64	Parallel vs. ThreeCriteriaClustering	3.081834	0.002057	0.000781
63	BestCyclicAssignment vs. NearestByDepot	2.995196	0.002743	0.000794
62	ThreeCriteriaClustering vs. UPGMC	2.958066	0.003096	0.000806
61	BestCyclicAssignment vs. KMEANS	2.908558	0.003631	0.00082
60	BestCyclicAssignment vs. Farthest-First	2.846674	0.004418	0.000833
59	KMEANS vs. NearestByCustomer	2.747659	0.006002	0.000847
58	KMEANS vs. Parallel	2.747659	0.006002	0.000862
57	Farthest-First vs. NearestByCustomer	2.685775	0.007236	0.000877
56	Farthest-First vs. Parallel	2.685775	0.007236	0.000893
55	CoefficientPropagation vs. PAM	2.450615	0.014261	0.000909
54	CLARA vs. CoefficientPropagation	2.438238	0.014759	0.000926
53	PAM vs. Sweep	2.425861	0.015272	0.000943
52	CLARA vs. Sweep	2.413485	0.015801	0.000962
51	Simplified vs. ThreeCriteriaClustering	2.289716	0.022038	0.00098
50	BestNearest vs. PAM	2.264963	0.023515	0.001
49	BestNearest vs. CLARA	2.252586	0.024285	0.00102
48	BestNearest vs. ThreeCriteriaClustering	2.190701	0.028473	0.001042
47	PAM vs. Simplified	2.165948	0.030315	0.001064
46	CLARA vs. Simplified	2.153571	0.031274	0.001087
45	Sweep vs. ThreeCriteriaClustering	2.029802	0.042377	0.001111
44	CoefficientPropagation vs. ThreeCriteriaClustering	2.005049	0.044958	0.001136
43	KMEANS vs. Simplified	1.955541	0.050519	0.001163
42	Farthest-First vs. Simplified	1.893657	0.058271	0.00119
41	BestNearest vs. KMEANS	1.856527	0.063379	0.00122
40	BestNearest vs. Farthest-First	1.794642	0.072711	0.00125
39	CLARA vs. NearestByDepot	1.794642	0.072711	0.001282
38	NearestByDepot vs. PAM	1.782266	0.074706	0.001316
37	KMEANS vs. Sweep	1.695628	0.089956	0.001351
36	CoefficientPropagation vs. KMEANS	1.670874	0.094747	0.001389
35	CyclicAssignment vs. RandomByElement	1.64612	0.099739	0.001429
34	Farthest-First vs. Sweep	1.633743	0.102313	0.001471
33	CoefficientPropagation vs. Farthest-First	1.60899	0.107619	0.001515
32	CyclicAssignment vs. NearestByDepot	1.485221	0.137485	0.001563

2.2 P-values for $\alpha = 0.10$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.

Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.001471.

Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	RandomByElement vs. UPGMC	12.327337	0	0.000833
119	CyclicAssignment vs. UPGMC	10.681217	0	0.00084
118	RandomByElement vs. ThreeCriteriaClustering	9.369271	0	0.000847
117	NearestByDepot vs. UPGMC	9.195995	0	0.000855
116	KMEANS vs. RandomByElement	9.035096	0	0.000862
115	Farthest-First vs. RandomByElement	8.973212	0	0.00087
114	CyclicAssignment vs. ThreeCriteriaClustering	7.723151	0	0.000877
113	PAM vs. UPGMC	7.41373	0	0.000885
112	CLARA vs. UPGMC	7.401353	0	0.000893
111	CyclicAssignment vs. KMEANS	7.388976	0	0.000901
110	CoefficientPropagation vs. RandomByElement	7.364222	0	0.000909
109	RandomByElement vs. Sweep	7.339469	0	0.000917
108	CyclicAssignment vs. Farthest-First	7.327092	0	0.000926
107	BestNearest vs. RandomByElement	7.17857	0	0.000935
106	RandomByElement vs. Simplified	7.079555	0	0.000943
105	NearestByCustomer vs. RandomByElement	6.287437	0	0.000952
104	Parallel vs. RandomByElement	6.287437	0	0.000962
103	NearestByDepot vs. ThreeCriteriaClustering	6.23793	0	0.000971
102	BestCyclicAssignment vs. UPGMC	6.200799	0	0.00098
101	BestCyclicAssignment vs. RandomByElement	6.126538	0	0.00099
100	NearestByCustomer vs. UPGMC	6.0399	0	0.001
99	Parallel vs. UPGMC	6.0399	0	0.00101
98	KMEANS vs. NearestByDepot	5.903755	0	0.00102
97	Farthest-First vs. NearestByDepot	5.841871	0	0.001031
96	CoefficientPropagation vs. CyclicAssignment	5.718102	0	0.001042
95	CyclicAssignment vs. Sweep	5.693348	0	0.001053
94	BestNearest vs. CyclicAssignment	5.532449	0	0.001064
93	CyclicAssignment vs. Simplified	5.433435	0	0.001075
92	Simplified vs. UPGMC	5.247782	0	0.001087
91	BestNearest vs. UPGMC	5.148767	0	0.001099
90	Sweep vs. UPGMC	4.987868	0.000001	0.001111
89	CoefficientPropagation vs. UPGMC	4.963115	0.000001	0.001124
88	CLARA vs. RandomByElement	4.925984	0.000001	0.001136
87	PAM vs. RandomByElement	4.913607	0.000001	0.001149
86	CyclicAssignment vs. NearestByCustomer	4.641317	0.000003	0.001163
85	CyclicAssignment vs. Parallel	4.641317	0.000003	0.001176
84	BestCyclicAssignment vs. CyclicAssignment	4.480418	0.000007	0.00119
83	PAM vs. ThreeCriteriaClustering	4.455664	0.000008	0.001205
82	CLARA vs. ThreeCriteriaClustering	4.443287	0.000009	0.00122
81	CoefficientPropagation vs. NearestByDepot	4.232881	0.000023	0.001235
80	NearestByDepot vs. Sweep	4.208127	0.000026	0.00125
79	KMEANS vs. PAM	4.121489	0.000038	0.001266
78	CLARA vs. KMEANS	4.109112	0.00004	0.001282
77	Farthest-First vs. PAM	4.059605	0.000049	0.001299
76	CLARA vs. Farthest-First	4.047228	0.000052	0.001316
75	BestNearest vs. NearestByDepot	4.047228	0.000052	0.001333
74	NearestByDepot vs. Simplified	3.948213	0.000079	0.001351
73	Farthest-First vs. UPGMC	3.354125	0.000796	0.00137
72	KMEANS vs. UPGMC	3.292241	0.000994	0.001389
71	CLARA vs. CyclicAssignment	3.279864	0.001039	0.001408
70	CyclicAssignment vs. PAM	3.267487	0.001085	0.001429
69	BestCyclicAssignment vs. ThreeCriteriaClustering	3.242733	0.001184	0.001449
68	NearestByCustomer vs. NearestByDepot	3.156095	0.001599	0.001471
67	NearestByDepot vs. Parallel	3.156095	0.001599	0.001493
66	NearestByDepot vs. RandomByElement	3.131342	0.00174	0.001515
65	NearestByCustomer vs. ThreeCriteriaClustering	3.081834	0.002057	0.001538
64	Parallel vs. ThreeCriteriaClustering	3.081834	0.002057	0.001563
63	BestCyclicAssignment vs. NearestByDepot	2.995196	0.002743	0.001587
62	ThreeCriteriaClustering vs. UPGMC	2.958066	0.003096	0.001613
61	BestCyclicAssignment vs. KMEANS	2.908558	0.003631	0.001639
60	BestCyclicAssignment vs. Farthest-First	2.846674	0.004418	0.001667
59	KMEANS vs. NearestByCustomer	2.747659	0.006002	0.001695
58	KMEANS vs. Parallel	2.747659	0.006002	0.001724
57	Farthest-First vs. NearestByCustomer	2.685775	0.007236	0.001754
56	Farthest-First vs. Parallel	2.685775	0.007236	0.001786
55	CoefficientPropagation vs. PAM	2.450615	0.014261	0.001818
54	CLARA vs. CoefficientPropagation	2.438238	0.014759	0.001852
53	PAM vs. Sweep	2.425861	0.015272	0.001887
52	CLARA vs. Sweep	2.413485	0.015801	0.001923
51	Simplified vs. ThreeCriteriaClustering	2.289716	0.022038	0.001961
50	BestNearest vs. PAM	2.264963	0.023515	0.002
49	BestNearest vs. CLARA	2.252586	0.024285	0.002041
48	BestNearest vs. ThreeCriteriaClustering	2.190701	0.028473	0.002083
47	PAM vs. Simplified	2.165948	0.030315	0.002128
46	CLARA vs. Simplified	2.153571	0.031274	0.002174
45	Sweep vs. ThreeCriteriaClustering	2.029802	0.042377	0.002222
44	CoefficientPropagation vs. ThreeCriteriaClustering	2.005049	0.044958	0.002273
43	KMEANS vs. Simplified	1.955541	0.050519	0.002326
42	Farthest-First vs. Simplified	1.893657	0.058271	0.002381
41	BestNearest vs. KMEANS	1.856527	0.063379	0.002439
40	BestNearest vs. Farthest-First	1.794642	0.072711	0.0025
39	CLARA vs. NearestByDepot	1.794642	0.072711	0.002564
38	NearestByDepot vs. PAM	1.782266	0.074706	0.002632
37	KMEANS vs. Sweep	1.695628	0.089956	0.002703
36	CoefficientPropagation vs. KMEANS	1.670874	0.094747	0.002778
35	CyclicAssignment vs. RandomByElement	1.64612	0.099739	0.002857
34	Farthest-First vs. Sweep	1.633743	0.102313	0.002941
33	CoefficientPropagation vs. Farthest-First	1.60899	0.107619	0.00303
32	CyclicAssignment vs. NearestByDepot	1.485221	0.137485	0.003125

2.3 Adjusted p-values

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Sha}
1	RandomByElement vs .UPGMC	0	0	0	0
2	CyclicAssignment vs .UPGMC	0	0	0	0
3	RandomByElement vs .ThreeCriteriaClustering	0	0	0	0
4	NearestByDepot vs .UPGMC	0	0	0	0
5	KMEANS vs .RandomByElement	0	0	0	0
6	Farthest-First vs .RandomByElement	0	0	0	0
7	CyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
8	PAM vs .UPGMC	0	0	0	0
9	CLARA vs .UPGMC	0	0	0	0
10	CyclicAssignment vs .KMEANS	0	0	0	0
11	CoefficientPropagation vs .RandomByElement	0	0	0	0
12	RandomByElement vs .Sweep	0	0	0	0
13	CyclicAssignment vs .Farthest-First	0	0	0	0
14	BestNearest vs .RandomByElement	0	0	0	0
15	RandomByElement vs .Simplified	0	0	0	0
16	NearestByCustomer vs .RandomByElement	0	0	0	0
17	Parallel vs .RandomByElement	0	0	0	0
18	NearestByDepot vs .ThreeCriteriaClustering	0	0	0	0
19	BestCyclicAssignment vs .UPGMC	0	0	0	0
20	BestCyclicAssignment vs .RandomByElement	0	0	0	0
21	NearestByCustomer vs .UPGMC	0	0	0	0
22	Parallel vs .UPGMC	0	0	0	0
23	KMEANS vs .NearestByDepot	0	0	0	0
24	Farthest-First vs .NearestByDepot	0	0.000001	0.000001	0
25	CoefficientPropagation vs .CyclicAssignment	0	0.000001	0.000001	0.00000
26	CyclicAssignment vs .Sweep	0	0.000001	0.000001	0.00000
27	BestNearest vs .CyclicAssignment	0	0.000004	0.000003	0.00000
28	CyclicAssignment vs .Simplified	0	0.000007	0.000005	0.00000
29	Simplified vs .UPGMC	0	0.000018	0.000014	0.0000
30	BestNearest vs .UPGMC	0	0.000031	0.000024	0.0000
31	Sweep vs .UPGMC	0.000001	0.000073	0.000055	0.0000
32	CoefficientPropagation vs .UPGMC	0.000001	0.000083	0.000062	0.0000
33	CLARA vs .RandomByElement	0.000001	0.000101	0.000074	0.0000
34	PAM vs .RandomByElement	0.000001	0.000107	0.000078	0.0000
35	CyclicAssignment vs .NearestByCustomer	0.000003	0.000415	0.000298	0.0002
36	CyclicAssignment vs .Parallel	0.000003	0.000415	0.000298	0.0002
37	BestCyclicAssignment vs .CyclicAssignment	0.000007	0.000894	0.000626	0.0006
38	PAM vs .ThreeCriteriaClustering	0.000008	0.001004	0.000694	0.0006
39	CLARA vs .ThreeCriteriaClustering	0.000009	0.001063	0.000726	0.0007
40	CoefficientPropagation vs .NearestByDepot	0.000023	0.002769	0.001869	0.0018
41	NearestByDepot vs .Sweep	0.000026	0.00309	0.00206	0.0020
42	KMEANS vs .PAM	0.000038	0.004517	0.002974	0.0029
43	CLARA vs .KMEANS	0.00004	0.004766	0.003098	0.0030
44	Farthest-First vs .PAM	0.000049	0.005899	0.003785	0.0035
45	CLARA vs .Farthest-First	0.000052	0.006219	0.003939	0.0037
46	BestNearest vs .NearestByDepot	0.000052	0.006219	0.003939	0.0037
47	NearestByDepot vs .Simplified	0.000079	0.009448	0.005827	0.0056
48	Farthest-First vs .UPGMC	0.000796	0.09554	0.05812	0.0573
49	KMEANS vs .UPGMC	0.000994	0.119271	0.071563	0.0715
50	CLARA vs .CyclicAssignment	0.001039	0.124629	0.073739	0.0716
51	CyclicAssignment vs .PAM	0.001085	0.130208	0.075955	0.0748
52	BestCyclicAssignment vs .ThreeCriteriaClustering	0.001184	0.142067	0.081688	0.0816
53	NearestByCustomer vs .NearestByDepot	0.001599	0.191876	0.10873	0.1087
54	NearestByDepot vs .Parallel	0.001599	0.191876	0.10873	0.1087
55	NearestByDepot vs .RandomByElement	0.00174	0.208812	0.114846	0.1148
56	NearestByCustomer vs .ThreeCriteriaClustering	0.002057	0.246875	0.133724	0.1337
57	Parallel vs .ThreeCriteriaClustering	0.002057	0.246875	0.133724	0.1337
58	BestCyclicAssignment vs .NearestByDepot	0.002743	0.329122	0.172789	0.1673
59	ThreeCriteriaClustering vs .UPGMC	0.003096	0.371491	0.191937	0.1888
60	BestCyclicAssignment vs .KMEANS	0.003631	0.435719	0.221491	0.2214
61	BestCyclicAssignment vs .Farthest-First	0.004418	0.530143	0.265071	0.2650
62	KMEANS vs .NearestByCustomer	0.006002	0.720268	0.354132	0.3541
63	KMEANS vs .Parallel	0.006002	0.720268	0.354132	0.3541
64	Farthest-First vs .NearestByCustomer	0.007236	0.868341	0.412462	0.4124
65	Farthest-First vs .Parallel	0.007236	0.868341	0.412462	0.4124
66	CoefficientPropagation vs .PAM	0.014261	1.711348	0.784368	0.7843
67	CLARA vs .CoefficientPropagation	0.014759	1.771085	0.796988	0.7843
68	PAM vs .Sweep	0.015272	1.832652	0.809421	0.7941
69	CLARA vs .Sweep	0.015801	1.896095	0.821641	0.8216
70	Simplified vs .ThreeCriteriaClustering	0.022038	2.644533	1.123926	1.1239
71	BestNearest vs .PAM	0.023515	2.821798	1.175749	1.1522
72	BestNearest vs .CLARA	0.024285	2.914234	1.189979	1.1899
73	BestNearest vs .ThreeCriteriaClustering	0.028473	3.416809	1.366723	1.3667
74	PAM vs .Simplified	0.030315	3.637822	1.424813	1.4248
75	CLARA vs .Simplified	0.031274	3.752861	1.438597	1.4385
76	Sweep vs .ThreeCriteriaClustering	0.042377	5.085195	1.906948	1.9069
77	CoefficientPropagation vs .ThreeCriteriaClustering	0.044958	5.39494	1.978145	1.9781
78	KMEANS vs .Simplified	0.050519	6.062304	2.172326	2.1723
79	Farthest-First vs .Simplified	0.058271	6.992463	2.447362	2.4473
80	BestNearest vs .KMEANS	0.063379	7.605423	2.59852	2.5985
81	BestNearest vs .Farthest-First	0.072711	8.725283	2.908428	2.9084
82	CLARA vs .NearestByDepot	0.072711	8.725283	2.908428	2.9084
83	NearestByDepot vs .PAM	0.074706	8.964712	2.908428	2.9084
84	KMEANS vs .Sweep	0.089956	10.79477	3.328387	3.3283
85	CoefficientPropagation vs .KMEANS	0.094747	11.369589	3.410877	3.4108
86	CyclicAssignment vs .RandomByElement	0.099739	11.968679	3.490865	3.4908
87	Farthest-First vs .Sweep	0.102313	12.277528	3.490865	3.4908
88	CoefficientPropagation vs .Farthest-First	0.107619	12.91423	3.551413	3.5514
89	CyclicAssignment vs .NearestByDepot	0.137485	16.498225	4.399527	4.3995