Output tables for the test of Multiple comparisons.

June 12, 2025

1 Average rankings of Friedman test

Average ranks obtained by applying the Friedman procedure

Friedman statistic considering reduction performance (distributed according to chi-square with 15 degrees of freedom: 314.33701.

P-value computed by Friedman Test: 1.730716681080935E-10.

Iman and Davenport statistic considering reduction performance (distributed according to F-distribution with 15 and 525 degrees of freedom: 48.753211.

P-value computed by Iman and Daveport Test: 2.220446049250313E-16.

Algorithm	Ranking
BestCyclicAssignment	12.1111
BestNearest	6.9028
CLARA	11.3611
CoefficientPropagation	10.1667
CyclicAssignment	14.25
Farthest-First	4.1389
KMEANS	4.5694
NearestByCustomer	6.75
NearestByDepot	11.1944
PAM	9.2222
Parallel	6.75
RandomByElement	15.9722
Simplified	6.0278
Sweep	5.9306
ThreeCriteriaClustering	4.5417
UPGMC	6.1111

Table 1: Average Rankings of the algorithms

2 Post hoc comparisons

Results achieved on post hoc comparisons for $\alpha = 0.05$, $\alpha = 0.10$ and adjusted p-values.

2.1 P-values for $\alpha = 0.05$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417.
Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000847.
Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	Farthest-First vs. RandomByElement	10.545071	0	0.000417
119	RandomByElement vs. ThreeCriteriaClustering	10.186143	0	0.00042
118	KMEANS vs. RandomByElement	10.161389	0	0.000424
117	CyclicAssignment vs. Farthest-First	9.010343	0	0.000427
116	RandomByElement vs. Sweep	8.948458	0	0.000431
115	RandomByElement vs. Simplified	8.861821	0	0.000435
114	RandomByElement vs. UPGMC	8.787559	0	0.000439
113	CyclicAssignment vs. ThreeCriteriaClustering	8.651414	0	0.000442
112	CyclicAssignment vs. KMEANS	8.62666	0	0.000446
111	NearestByCustomer vs. RandomByElement	8.218225	0	0.00045
110	Parallel vs. RandomByElement	8.218225	0	0.000455
109	BestNearest vs. RandomByElement	8.082079	0	0.000459
108	CyclicAssignment vs. Sweep	7.41373	0	0.000463
107	CyclicAssignment vs. Simplified	7.327092	0	0.000467
106	CyclicAssignment vs. UPGMC	7.252831	0	0.000472
105	BestCyclicAssignment vs. Farthest-First	7.104309	0	0.000476
104	BestCyclicAssignment vs. ThreeCriteriaClustering	6.74538	0	0.000481
103	BestCyclicAssignment vs. KMEANS	6.720626	0	0.000485
102	CyclicAssignment vs. NearestByCustomer	6.683496	0	0.00049
101	CyclicAssignment vs. Parallel	6.683496	0	0.000495
100	BestNearest vs. CyclicAssignment	6.547351	0	0.0005
99	CLARA vs. Farthest-First	6.435959	0	0.000505
98	Farthest-First vs. NearestByDepot	6.287437	0	0.00051
97	CLARA vs. ThreeCriteriaClustering	6.077031	0	0.000515
96	CLARA vs. KMEANS	6.052277	0	0.000521
95	PAM vs. RandomByElement	6.015146	0	0.000526
94	NearestByDepot vs. ThreeCriteriaClustering	5.928508	0	0.000532
93	KMEANS vs. NearestByDepot	5.903755	0	0.000538
92	BestCyclicAssignment vs. Sweep	5.507696	0	0.000543
91	BestCyclicAssignment vs. Simplified	5.421058	0	0.000549
90	CoefficientPropagation vs. Farthest-First	5.37155	0	0.000556
89	BestCyclicAssignment vs. UPGMC	5.346797	0	0.000562
88	CoefficientPropagation vs. RandomByElement	5.173521	0	0.000568
87	CoefficientPropagation vs. ThreeCriteriaClustering	5.012622	0.000001	0.000575
86	CoefficientPropagation vs. KMEANS	4.987868	0.000001	0.000581
85	CLARA vs. Sweep	4.839346	0.000001	0.000588
84	BestCyclicAssignment vs. NearestByCustomer	4.777462	0.000002	0.000595
83	BestCyclicAssignment vs. Parallel	4.777462	0.000002	0.000602
82	CLARA vs. Simplified	4.752708	0.000002	0.00061
81	NearestByDepot vs. Sweep	4.690824	0.000003	0.000617
80	CLARA vs. UPGMC	4.678447	0.000003	0.000625
79	BestCyclicAssignment vs. BestNearest	4.641317	0.000003	0.000633
78	NearestByDepot vs. Simplified	4.604186	0.000004	0.000641
77	NearestByDepot vs. UPGMC	4.529925	0.000006	0.000649
76	Farthest-First vs. PAM	4.529925	0.000006	0.000658
75	CyclicAssignment vs. PAM	4.480418	0.000007	0.000667
74	NearestByDepot vs. RandomByElement	4.257634	0.000021	0.000676
73	PAM vs. ThreeCriteriaClustering	4.170997	0.00003	0.000685
72	KMEANS vs. PAM	4.146243	0.000034	0.000694
71	CLARA vs. RandomByElement	4.109112	0.00004	0.000704
70	CLARA vs. NearestByCustomer	4.109112	0.00004	0.000714
69	CLARA vs. Parallel	4.109112	0.00004	0.000725
68	BestNearest vs. CLARA	3.972967	0.000071	0.000735
67	NearestByCustomer vs. NearestByDepot	3.96059	0.000075	0.000746
66	NearestByDepot vs. Parallel	3.96059	0.000075	0.000758
65	BestNearest vs. NearestByDepot	3.824445	0.000131	0.000769
64	CoefficientPropagation vs. Sweep	3.774938	0.00016	0.000781
63	CoefficientPropagation vs. Simplified	3.6883	0.000226	0.000794
62	CoefficientPropagation vs. CyclicAssignment	3.638792	0.000274	0.000806
61	CoefficientPropagation vs. UPGMC	3.614039	0.000301	0.00082
60	BestCyclicAssignment vs. RandomByElement	3.440763	0.00058	0.000833
59	CoefficientPropagation vs. NearestByCustomer	3.044704	0.002329	0.000847
58	CoefficientPropagation vs. Parallel	3.044704	0.002329	0.000862
57	PAM vs. Sweep	2.933312	0.003354	0.000877
56	BestNearest vs. CoefficientPropagation	2.908558	0.003631	0.000893
55	PAM vs. Simplified	2.846674	0.004418	0.000909
54	PAM vs. UPGMC	2.772413	0.005564	0.000926
53	CyclicAssignment vs. NearestByDepot	2.722906	0.006471	0.000943
52	BestCyclicAssignment vs. PAM	2.574384	0.010042	0.000962
51	CLARA vs. CyclicAssignment	2.574384	0.010042	0.00098
50	BestNearest vs. Farthest-First	2.462992	0.013778	0.001
49	Farthest-First vs. NearestByCustomer	2.326847	0.019973	0.00102
48	Farthest-First vs. Parallel	2.326847	0.019973	0.001042
47	NearestByCustomer vs. PAM	2.203078	0.027589	0.001064
46	PAM vs. Parallel	2.203078	0.027589	0.001087
45	BestNearest vs. ThreeCriteriaClustering	2.104064	0.035373	0.001111
44	BestNearest vs. KMEANS	2.07931	0.037589	0.001136
43	BestNearest vs. PAM	2.066933	0.03874	0.001163
42	NearestByCustomer vs. ThreeCriteriaClustering	1.967918	0.049077	0.00119
41	Parallel vs. ThreeCriteriaClustering	1.967918	0.049077	0.00122
40	KMEANS vs. NearestByCustomer	1.943165	0.051996	0.00125
39	KMEANS vs. Parallel	1.943165	0.051996	0.001282
38	CLARA vs. PAM	1.906034	0.056646	0.001316
37	BestCyclicAssignment vs. CyclicAssignment	1.906034	0.056646	0.001351
36	NearestByDepot vs. PAM	1.757512	0.078831	0.001389
35	Farthest-First vs. UPGMC	1.757512	0.078831	0.001429
34	BestCyclicAssignment vs. CoefficientPropagation	1.732758	0.083139	0.001471
33	Farthest-First vs. Simplified	1.683251	0.092327	0.001515
32	Farthest-First vs. Sweep	1.596613	0.110352	0.001563

2.2 P-values for $\alpha = 0.10$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.

Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.001695.

Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	Farthest-First vs. RandomByElement	10.545071	0	0.000833
119	RandomByElement vs. ThreeCriteriaClustering	10.186143	0	0.00084
118	KMEANS vs. RandomByElement	10.161389	0	0.000847
117	CyclicAssignment vs. Farthest-First	9.010343	0	0.000855
116	RandomByElement vs. Sweep	8.948458	0	0.000862
115	RandomByElement vs. Simplified	8.861821	0	0.00087
114	RandomByElement vs. UPGMC	8.787559	0	0.000877
113	CyclicAssignment vs. ThreeCriteriaClustering	8.651414	0	0.000885
112	CyclicAssignment vs. KMEANS	8.62666	0	0.000893
111	NearestByCustomer vs. RandomByElement	8.218225	0	0.000901
110	Parallel vs. RandomByElement	8.218225	0	0.000909
109	BestNearest vs. RandomByElement	8.082079	0	0.000917
108	CyclicAssignment vs. Sweep	7.41373	0	0.000926
107	CyclicAssignment vs. Simplified	7.327092	0	0.000935
106	CyclicAssignment vs. UPGMC	7.252831	0	0.000943
105	BestCyclicAssignment vs. Farthest-First	7.104309	0	0.000952
104	BestCyclicAssignment vs. ThreeCriteriaClustering	6.74538	0	0.000962
103	BestCyclicAssignment vs. KMEANS	6.720626	0	0.000971
102	CyclicAssignment vs. NearestByCustomer	6.683496	0	0.00098
101	CyclicAssignment vs. Parallel	6.683496	0	0.00099
100	BestNearest vs. CyclicAssignment	6.547351	0	0.001
99	CLARA vs. Farthest-First	6.435959	0	0.00101
98	Farthest-First vs. NearestByDepot	6.287437	0	0.00102
97	CLARA vs. ThreeCriteriaClustering	6.077031	0	0.001031
96	CLARA vs. KMEANS	6.052277	0	0.001042
95	PAM vs. RandomByElement	6.015146	0	0.001053
94	NearestByDepot vs. ThreeCriteriaClustering	5.928508	0	0.001064
93	KMEANS vs. NearestByDepot	5.903755	0	0.001075
92	BestCyclicAssignment vs. Sweep	5.507696	0	0.001087
91	BestCyclicAssignment vs. Simplified	5.421058	0	0.001099
90	CoefficientPropagation vs. Farthest-First	5.37155	0	0.001111
89	BestCyclicAssignment vs. UPGMC	5.346797	0	0.001124
88	CoefficientPropagation vs. RandomByElement	5.173521	0	0.001136
87	CoefficientPropagation vs. ThreeCriteriaClustering	5.012622	0.000001	0.001149
86	CoefficientPropagation vs. KMEANS	4.987868	0.000001	0.001163
85	CLARA vs. Sweep	4.839346	0.000001	0.001176
84	BestCyclicAssignment vs. NearestByCustomer	4.777462	0.000002	0.00119
83	BestCyclicAssignment vs. Parallel	4.777462	0.000002	0.001205
82	CLARA vs. Simplified	4.752708	0.000002	0.00122
81	NearestByDepot vs. Sweep	4.690824	0.000003	0.001235
80	CLARA vs. UPGMC	4.678447	0.000003	0.00125
79	BestCyclicAssignment vs. BestNearest	4.641317	0.000003	0.001266
78	NearestByDepot vs. Simplified	4.604186	0.000004	0.001282
77	NearestByDepot vs. UPGMC	4.529925	0.000006	0.001299
76	Farthest-First vs. PAM	4.529925	0.000006	0.001316
75	CyclicAssignment vs. PAM	4.480418	0.000007	0.001333
74	NearestByDepot vs. RandomByElement	4.257634	0.000021	0.001351
73	PAM vs. ThreeCriteriaClustering	4.170997	0.00003	0.00137
72	KMEANS vs. PAM	4.146243	0.000034	0.001389
71	CLARA vs. RandomByElement	4.109112	0.00004	0.001408
70	CLARA vs. NearestByCustomer	4.109112	0.00004	0.001429
69	CLARA vs. Parallel	4.109112	0.00004	0.001449
68	BestNearest vs. CLARA	3.972967	0.000071	0.001471
67	NearestByCustomer vs. NearestByDepot	3.96059	0.000075	0.001493
66	NearestByDepot vs. Parallel	3.96059	0.000075	0.001515
65	BestNearest vs. NearestByDepot	3.824445	0.000131	0.001538
64	CoefficientPropagation vs. Sweep	3.774938	0.00016	0.001563
63	CoefficientPropagation vs. Simplified	3.6883	0.000226	0.001587
62	CoefficientPropagation vs. CyclicAssignment	3.638792	0.000274	0.001613
61	CoefficientPropagation vs. UPGMC	3.614039	0.000301	0.001639
60	BestCyclicAssignment vs. RandomByElement	3.440763	0.00058	0.001667
59	CoefficientPropagation vs. NearestByCustomer	3.044704	0.002329	0.001695
58	CoefficientPropagation vs. Parallel	3.044704	0.002329	0.001724
57	PAM vs. Sweep	2.933312	0.003354	0.001754
56	BestNearest vs. CoefficientPropagation	2.908558	0.003631	0.001786
55	PAM vs. Simplified	2.846674	0.004418	0.001818
54	PAM vs. UPGMC	2.772413	0.005564	0.001852
53	CyclicAssignment vs. NearestByDepot	2.722906	0.006471	0.001887
52	BestCyclicAssignment vs. PAM	2.574384	0.010042	0.001923
51	CLARA vs. CyclicAssignment	2.574384	0.010042	0.001961
50	BestNearest vs. Farthest-First	2.462992	0.013778	0.002
49	Farthest-First vs. NearestByCustomer	2.326847	0.019973	0.002041
48	Farthest-First vs. Parallel	2.326847	0.019973	0.002083
47	NearestByCustomer vs. PAM	2.203078	0.027589	0.002128
46	PAM vs. Parallel	2.203078	0.027589	0.002174
45	BestNearest vs. ThreeCriteriaClustering	2.104064	0.035373	0.002222
44	BestNearest vs. KMEANS	2.07931	0.037589	0.002273
43	BestNearest vs. PAM	2.066933	0.03874	0.002326
42	NearestByCustomer vs. ThreeCriteriaClustering	1.967918	0.049077	0.002381
41	Parallel vs. ThreeCriteriaClustering	1.967918	0.049077	0.002439
40	KMEANS vs. NearestByCustomer	1.943165	0.051996	0.0025
39	KMEANS vs. Parallel	1.943165	0.051996	0.002564
38	CLARA vs. PAM	1.906034	0.056646	0.002632
37	BestCyclicAssignment vs. CyclicAssignment	1.906034	0.056646	0.002703
36	NearestByDepot vs. PAM	1.757512	0.078831	0.002778
35	Farthest-First vs. UPGMC	1.757512	0.078831	0.002857
34	BestCyclicAssignment vs. CoefficientPropagation	1.732758	0.083139	0.002941
33	Farthest-First vs. Simplified	1.683251	0.092327	0.00303
32	Farthest-First vs. Sweep	1.596613	0.110352	0.003125

2.3 Adjusted p-values

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Shaj}
1	Farthest-First vs .RandomByElement	0	0	0	0
2	RandomByElement vs .ThreeCriteriaClustering	0	0	0	0
3	KMEANS vs .RandomByElement	0	0	0	0
4	CyclicAssignment vs .Farthest-First	0	0	0	0
5	RandomByElement vs .Sweep	0	0	0	0
6	RandomByElement vs .Simplified	0	0	0	0
7	RandomByElement vs .UPGMC	0	0	0	0
8	CyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
9	CyclicAssignment vs .KMEANS	0	0	0	0
10	NearestByCustomer vs .RandomByElement	0	0	0	0
11	Parallel vs .RandomByElement	0	0	0	0
12	BestNearest vs .RandomByElement	0	0	0	0
13	CyclicAssignment vs .Sweep	0	0	0	0
14	CyclicAssignment vs .Simplified	0	0	0	0
15	CyclicAssignment vs .UPGMC	0	0	0	0
16	BestCyclicAssignment vs .Farthest-First	0	0	0	0
17	BestCyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
18	BestCyclicAssignment vs .KMEANS	0	0	0	0
19	CyclicAssignment vs .NearestByCustomer	0	0	0	0
20	CyclicAssignment vs .Parallel	0	0	0	0
21	BestNearest vs .CyclicAssignment	0	0	0	0
22	CLARA vs .Farthest-First	0	0	0	0
23	Farthest-First vs .NearestByDepot	0	0	0	0
24	CLARA vs .ThreeCriteriaClustering	0	0	0	0
25	CLARA vs .KMEANS	0	0	0	0
26	PAM vs .RandomByElement	0	0	0	0
27	NearestByDepot vs .ThreeCriteriaClustering	0	0	0	0
28	KMEANS vs .NearestByDepot	0	0	0	0
29	BestCyclicAssignment vs .Sweep	0	0.000004	0.000003	0.00000
30	BestCyclicAssignment vs .Simplified	0	0.000007	0.000005	0.00000
31	CoefficientPropagation vs .Farthest-First	0	0.000009	0.000007	0.00000
32	BestCyclicAssignment vs .UPGMC	0	0.000011	0.000008	0.00000
33	CoefficientPropagation vs .RandomByElement	0	0.000028	0.00002	0.0000
34	CoefficientPropagation vs .ThreeCriteriaClustering	0.000001	0.000064	0.000047	0.0000
35	CoefficientPropagation vs .KMEANS	0.000001	0.000073	0.000053	0.0000
36	CLARA vs .Sweep	0.000001	0.000156	0.000111	0.0001
37	BestCyclicAssignment vs .NearestByCustomer	0.000002	0.000213	0.000149	0.0001
38	BestCyclicAssignment vs .Parallel	0.000002	0.000213	0.000149	0.0001
39	CLARA vs .Simplified	0.000002	0.000241	0.000165	0.0001
40	NearestByDepot vs .Sweep	0.000003	0.000327	0.00022	0.0002
41	CLARA vs .UPGMC	0.000003	0.000347	0.000231	0.0002
42	BestCyclicAssignment vs .BestNearest	0.000003	0.000415	0.000273	0.0002
43	NearestByDepot vs .Simplified	0.000004	0.000497	0.000323	0.0003
44	NearestByDepot vs .UPGMC	0.000006	0.000708	0.000454	0.0004
45	Farthest-First vs .PAM	0.000006	0.000708	0.000454	0.0004
46	CyclicAssignment vs .PAM	0.000007	0.000894	0.000559	0.0005
47	NearestByDepot vs .RandomByElement	0.000021	0.002479	0.001529	0.0014
48	PAM vs .ThreeCriteriaClustering	0.00003	0.003639	0.002214	0.0021
49	KMEANS vs .PAM	0.000034	0.004056	0.002433	0.0024
50	CLARA vs .RandomByElement	0.00004	0.004766	0.00282	0.0027
51	CLARA vs .NearestByCustomer	0.00004	0.004766	0.00282	0.0027
52	CLARA vs .Parallel	0.00004	0.004766	0.00282	0.0027
53	BestNearest vs .CLARA	0.000071	0.008518	0.004827	0.0048
54	NearestByCustomer vs .NearestByDepot	0.000075	0.008972	0.005009	0.0050
55	NearestByDepot vs .Parallel	0.000075	0.008972	0.005009	0.0050
56	BestNearest vs .NearestByDepot	0.000131	0.015728	0.008519	0.0085
57	CoefficientPropagation vs .Sweep	0.00016	0.019206	0.010243	0.0097
58	CoefficientPropagation vs .Simplified	0.000226	0.027091	0.014223	0.0137
59	CoefficientPropagation vs .CyclicAssignment	0.000274	0.03287	0.016983	0.0167
60	CoefficientPropagation vs .UPGMC	0.000301	0.036176	0.018389	0.0183
61	BestCyclicAssignment vs .RandomByElement	0.00058	0.069609	0.034805	0.0348
62	CoefficientPropagation vs .NearestByCustomer	0.002329	0.279492	0.137417	0.1374
63	CoefficientPropagation vs .Parallel	0.002329	0.279492	0.137417	0.1374
64	PAM vs .Sweep	0.003354	0.40244	0.191159	0.1911
65	BestNearest vs .CoefficientPropagation	0.003631	0.435719	0.203336	0.2033
66	PAM vs .Simplified	0.004418	0.530143	0.242982	0.2429
67	PAM vs .UPGMC	0.005564	0.667708	0.300469	0.2893
68	CyclicAssignment vs .NearestByDepot	0.006471	0.776526	0.342966	0.3364
69	BestCyclicAssignment vs .PAM	0.010042	1.205026	0.522178	0.5221
70	CLARA vs .CyclicAssignment	0.010042	1.205026	0.522178	0.5221
71	BestNearest vs .Farthest-First	0.013778	1.653396	0.688915	0.6751
72	Farthest-First vs .NearestByCustomer	0.019973	2.396811	0.978698	0.9786
73	Farthest-First vs .Parallel	0.019973	2.396811	0.978698	0.9786
74	NearestByCustomer vs .PAM	0.027589	3.310708	1.296694	1.2966
75	PAM vs .Parallel	0.027589	3.310708	1.296694	1.2966
76	BestNearest vs .ThreeCriteriaClustering	0.035373	4.244749	1.591781	1.5917
77	BestNearest vs .KMEANS	0.037589	4.510666	1.653911	1.6539
78	BestNearest vs .PAM	0.03874	4.648856	1.66584	1.6658
79	NearestByCustomer vs .ThreeCriteriaClustering	0.049077	5.889293	2.061253	2.0612
80	Parallel vs .ThreeCriteriaClustering	0.049077	5.889293	2.061253	2.0612
81	KMEANS vs .NearestByCustomer	0.051996	6.239554	2.079851	2.0798
82	KMEANS vs .Parallel	0.051996	6.239554	2.079851	2.0798
83	CLARA vs .PAM	0.056646	6.797495	2.15254	2.1525
84	BestCyclicAssignment vs .CyclicAssignment	0.056646	6.797495	2.15254	2.1525
85	NearestByDepot vs .PAM	0.078831	9.459672	2.837901	2.8379
86	Farthest-First vs .UPGMC	0.078831	9.459672	2.837901	2.8379
87	BestCyclicAssignment vs .CoefficientPropagation	0.083139	9.976639	2.837901	2.8379
88	Farthest-First vs .Simplified	0.092327	11.079186	3.046776	3.0467
89	Farthest-First vs .Sweep	0.110352	13.242242	3.531265	3.5312