Output tables for the test of Multiple comparisons.

June 6, 2025

1 Average rankings of Friedman test

Average ranks obtained by applying the Friedman procedure

Friedman statistic considering reduction performance (distributed according to chi-square with 17 degrees of freedom: 339.627368.

P-value computed by Friedman Test: 1.560215290297151E-10.

Iman and Davenport statistic considering reduction performance (distributed according to F-distribution with 17 and 408 degrees of freedom: 95.476228.

P-value computed by Iman and Daveport Test: 7.771561172376096E-16.

Algorithm	Ranking
BestCyclicAssignment	13.04
BestNearest	5.58
CLARA	11.08
CoefficientPropagation	11.28
CyclicAssignment	14.76
Farthest-First	4.02
KMEANS	4.38
NearestByCustomer	6.4
NearestByDepot	10.2
PAM	17.68
Parallel	6.4
RandomByElement	15.56
RandomSequentialCyclic	15.44
SequentialCyclic	14.92
Simplified	5.24
Sweep	5.26
ThreeCriteriaClustering	3.68
UPGMC	6.08

Table 1: Average Rankings of the algorithms

2 Post hoc comparisons

Results achieved on post hoc comparisons for $\alpha = 0.05$, $\alpha = 0.10$ and adjusted p-values.

2.1 P-values for $\alpha = 0.05$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000327.
Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000641.
Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000327.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
153	PAM vs. ThreeCriteriaClustering	9.271726	0	0.000327
152	Farthest-First vs. PAM	9.046556	0	0.000329
151	KMEANS vs. PAM	8.80814	0	0.000331
150	PAM vs. Simplified	8.238591	0	0.000333
149	PAM vs. Sweep	8.225346	0	0.000336
148	BestNearest vs. PAM	8.013421	0	0.000338
147	RandomByElement vs. ThreeCriteriaClustering	7.867722	0	0.00034
146	RandomSequentialCyclic vs. ThreeCriteriaClustering	7.78825	0	0.000342
145	PAM vs. UPGMC	7.682288	0	0.000345
144	Farthest-First vs. RandomByElement	7.642552	0	0.000347
143	Farthest-First vs. RandomSequentialCyclic	7.56308	0	0.00035
142	NearestByCustomer vs. PAM	7.470362	0	0.000352
141	PAM vs. Parallel	7.470362	0	0.000355
140	SequentialCyclic vs. ThreeCriteriaClustering	7.443872	0	0.000357
139	KMEANS vs. RandomByElement	7.404136	0	0.00036
138	CyclicAssignment vs. ThreeCriteriaClustering	7.337909	0	0.000362
137	KMEANS vs. RandomSequentialCyclic	7.324664	0	0.000365
136	Farthest-First vs. SequentialCyclic	7.218701	0	0.000368
135	CyclicAssignment vs. Farthest-First	7.112739	0	0.00037
134	KMEANS vs. SequentialCyclic	6.980286	0	0.000373
133	CyclicAssignment vs. KMEANS	6.874323	0	0.000376
132	RandomByElement vs. Simplified	6.834587	0	0.000379
131	RandomByElement vs. Sweep	6.821342	0	0.000382
130	RandomSequentialCyclic vs. Simplified	6.755115	0	0.000385
129	RandomSequentialCyclic vs. Sweep	6.74187	0	0.000388
128	BestNearest vs. RandomByElement	6.609416	0	0.000391
127	BestNearest vs. RandomSequentialCyclic	6.529945	0	0.000394
126	SequentialCyclic vs. Simplified	6.410737	0	0.000397
125	SequentialCyclic vs. Sweep	6.397491	0	0.0004
124	CyclicAssignment vs. Simplified	6.304774	0	0.000403
123	CyclicAssignment vs. Sweep	6.291529	0	0.000407
122	RandomByElement vs. UPGMC	6.278283	0	0.00041
121	BestCyclicAssignment vs. ThreeCriteriaClustering	6.198811	0	0.000413
120	RandomSequentialCyclic vs. UPGMC	6.198811	0	0.000417
119	BestNearest vs. SequentialCyclic	6.185566	0	0.00042
118	BestNearest vs. CyclicAssignment	6.079604	0	0.000424
117	NearestByCustomer vs. RandomByElement	6.066358	0	0.000427
116	Parallel vs. RandomByElement	6.066358	0	0.000431
115	NearestByCustomer vs. RandomSequentialCyclic	5.986886	0	0.000435
114	Parallel vs. RandomSequentialCyclic	5.986886	0	0.000439
113	BestCyclicAssignment vs. Farthest-First	5.973641	0	0.000442
112	SequentialCyclic vs. UPGMC	5.854433	0	0.000446
111	CyclicAssignment vs. UPGMC	5.74847	0	0.00045
110	BestCyclicAssignment vs. KMEANS	5.735225	0	0.000455
109	NearestByCustomer vs. SequentialCyclic	5.642508	0	0.000459
108	Parallel vs. SequentialCyclic	5.642508	0	0.000463
107	CyclicAssignment vs. NearestByCustomer	5.536545	0	0.000467
106	CyclicAssignment vs. Parallel	5.536545	0	0.000472
105	BestCyclicAssignment vs. Simplified	5.165676	0	0.000476
104	BestCyclicAssignment vs. Sweep	5.152431	0	0.000481
103	CoefficientPropagation vs. ThreeCriteriaClustering	5.033223	0	0.000485
102	NearestByDepot vs. PAM	4.953751	0.000001	0.00049
101	BestCyclicAssignment vs. BestNearest	4.940506	0.000001	0.000495
100	CLARA vs. ThreeCriteriaClustering	4.90077	0.000001	0.0005
99	CoefficientPropagation vs. Farthest-First	4.808052	0.000002	0.000505
98	CLARA vs. Farthest-First	4.675599	0.000003	0.00051
97	BestCyclicAssignment vs. UPGMC	4.609373	0.000004	0.000515
96	CoefficientPropagation vs. KMEANS	4.569637	0.000005	0.000521
95	CLARA vs. KMEANS	4.437183	0.000009	0.000526
94	BestCyclicAssignment vs. NearestByCustomer	4.397447	0.000011	0.000532
93	BestCyclicAssignment vs. Parallel	4.397447	0.000011	0.000538
92	CLARA vs. PAM	4.370957	0.000012	0.000543
91	NearestByDepot vs. ThreeCriteriaClustering	4.317975	0.000016	0.000549
90	CoefficientPropagation vs. PAM	4.238504	0.000023	0.000556
89	Farthest-First vs. NearestByDepot	4.092805	0.000043	0.000562
88	CoefficientPropagation vs. Simplified	4.000088	0.000063	0.000568
87	CoefficientPropagation vs. Sweep	3.986842	0.000067	0.000575
86	CLARA vs. Simplified	3.867634	0.00011	0.000581
85	CLARA vs. Sweep	3.854389	0.000116	0.000588
84	KMEANS vs. NearestByDepot	3.854389	0.000116	0.000595
83	BestNearest vs. CoefficientPropagation	3.774917	0.00016	0.000602
82	BestNearest vs. CLARA	3.642464	0.00027	0.00061
81	NearestByDepot vs. RandomByElement	3.549747	0.000386	0.000617
80	NearestByDepot vs. RandomSequentialCyclic	3.470275	0.00052	0.000625
79	CoefficientPropagation vs. UPGMC	3.443784	0.000574	0.000633
78	CLARA vs. UPGMC	3.311331	0.000929	0.000641
77	NearestByDepot vs. Simplified	3.28484	0.00102	0.000649
76	NearestByDepot vs. Sweep	3.271595	0.001069	0.000658
75	CoefficientPropagation vs. NearestByCustomer	3.231859	0.00123	0.000667
74	CoefficientPropagation vs. Parallel	3.231859	0.00123	0.000676
73	NearestByDepot vs. SequentialCyclic	3.125896	0.001773	0.000685
72	CLARA vs. NearestByCustomer	3.099406	0.001939	0.000694
71	CLARA vs. Parallel	3.099406	0.001939	0.000704
70	BestCyclicAssignment vs. PAM	3.072915	0.00212	0.000714
69	BestNearest vs. NearestByDepot	3.05967	0.002216	0.000725
68	CyclicAssignment vs. NearestByDepot	3.019934	0.002528	0.000735
67	CLARA vs. RandomByElement	2.966952	0.003008	0.000746
66	CLARA vs. RandomSequentialCyclic	2.887481	0.003883	0.000758
65	CoefficientPropagation vs. RandomByElement	2.834499	0.00459	0.000769

2.2 P-values for $\alpha = 0.10$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000654.

Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.00137.

Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000654.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
153	PAM vs. ThreeCriteriaClustering	9.271726	0	0.000654
152	Farthest-First vs. PAM	9.046556	0	0.000658
151	KMEANS vs. PAM	8.80814	0	0.000662
150	PAM vs. Simplified	8.238591	0	0.000667
149	PAM vs. Sweep	8.225346	0	0.000671
148	BestNearest vs. PAM	8.013421	0	0.000676
147	RandomByElement vs. ThreeCriteriaClustering	7.867722	0	0.00068
146	RandomSequentialCyclic vs. ThreeCriteriaClustering	7.78825	0	0.000685
145	PAM vs. UPGMC	7.682288	0	0.00069
144	Farthest-First vs. RandomByElement	7.642552	0	0.000694
143	Farthest-First vs. RandomSequentialCyclic	7.56308	0	0.000699
142	NearestByCustomer vs. PAM	7.470362	0	0.000704
141	PAM vs. Parallel	7.470362	0	0.000709
140	SequentialCyclic vs. ThreeCriteriaClustering	7.443872	0	0.000714
139	KMEANS vs. RandomByElement	7.404136	0	0.000719
138	CyclicAssignment vs. ThreeCriteriaClustering	7.337909	0	0.000725
137	KMEANS vs. RandomSequentialCyclic	7.324664	0	0.00073
136	Farthest-First vs. SequentialCyclic	7.218701	0	0.000735
135	CyclicAssignment vs. Farthest-First	7.112739	0	0.000741
134	KMEANS vs. SequentialCyclic	6.980286	0	0.000746
133	CyclicAssignment vs. KMEANS	6.874323	0	0.000752
132	RandomByElement vs. Simplified	6.834587	0	0.000758
131	RandomByElement vs. Sweep	6.821342	0	0.000763
130	RandomSequentialCyclic vs. Simplified	6.755115	0	0.000769
129	RandomSequentialCyclic vs. Sweep	6.74187	0	0.000775
128	BestNearest vs. RandomByElement	6.609416	0	0.000781
127	BestNearest vs. RandomSequentialCyclic	6.529945	0	0.000787
126	SequentialCyclic vs. Simplified	6.410737	0	0.000794
125	SequentialCyclic vs. Sweep	6.397491	0	0.0008
124	CyclicAssignment vs. Simplified	6.304774	0	0.000806
123	CyclicAssignment vs. Sweep	6.291529	0	0.000813
122	RandomByElement vs. UPGMC	6.278283	0	0.00082
121	BestCyclicAssignment vs. ThreeCriteriaClustering	6.198811	0	0.000826
120	RandomSequentialCyclic vs. UPGMC	6.198811	0	0.000833
119	BestNearest vs. SequentialCyclic	6.185566	0	0.00084
118	BestNearest vs. CyclicAssignment	6.079604	0	0.000847
117	NearestByCustomer vs. RandomByElement	6.066358	0	0.000855
116	Parallel vs. RandomByElement	6.066358	0	0.000862
115	NearestByCustomer vs. RandomSequentialCyclic	5.986886	0	0.00087
114	Parallel vs. RandomSequentialCyclic	5.986886	0	0.000877
113	BestCyclicAssignment vs. Farthest-First	5.973641	0	0.000885
112	SequentialCyclic vs. UPGMC	5.854433	0	0.000893
111	CyclicAssignment vs. UPGMC	5.74847	0	0.000901
110	BestCyclicAssignment vs. KMEANS	5.735225	0	0.000909
109	NearestByCustomer vs. SequentialCyclic	5.642508	0	0.000917
108	Parallel vs. SequentialCyclic	5.642508	0	0.000926
107	CyclicAssignment vs. NearestByCustomer	5.536545	0	0.000935
106	CyclicAssignment vs. Parallel	5.536545	0	0.000943
105	BestCyclicAssignment vs. Simplified	5.165676	0	0.000952
104	BestCyclicAssignment vs. Sweep	5.152431	0	0.000962
103	CoefficientPropagation vs. ThreeCriteriaClustering	5.033223	0	0.000971
102	NearestByDepot vs. PAM	4.953751	0.000001	0.00098
101	BestCyclicAssignment vs. BestNearest	4.940506	0.000001	0.00099
100	CLARA vs. ThreeCriteriaClustering	4.90077	0.000001	0.001
99	CoefficientPropagation vs. Farthest-First	4.808052	0.000002	0.00101
98	CLARA vs. Farthest-First	4.675599	0.000003	0.00102
97	BestCyclicAssignment vs. UPGMC	4.609373	0.000004	0.001031
96	CoefficientPropagation vs. KMEANS	4.569637	0.000005	0.001042
95	CLARA vs. KMEANS	4.437183	0.000009	0.001053
94	BestCyclicAssignment vs. NearestByCustomer	4.397447	0.000011	0.001064
93	BestCyclicAssignment vs. Parallel	4.397447	0.000011	0.001075
92	CLARA vs. PAM	4.370957	0.000012	0.001087
91	NearestByDepot vs. ThreeCriteriaClustering	4.317975	0.000016	0.001099
90	CoefficientPropagation vs. PAM	4.238504	0.000023	0.001111
89	Farthest-First vs. NearestByDepot	4.092805	0.000043	0.001124
88	CoefficientPropagation vs. Simplified	4.000088	0.000063	0.001136
87	CoefficientPropagation vs. Sweep	3.986842	0.000067	0.001149
86	CLARA vs. Simplified	3.867634	0.00011	0.001163
85	CLARA vs. Sweep	3.854389	0.000116	0.001176
84	KMEANS vs. NearestByDepot	3.854389	0.000116	0.00119
83	BestNearest vs. CoefficientPropagation	3.774917	0.00016	0.001205
82	BestNearest vs. CLARA	3.642464	0.00027	0.00122
81	NearestByDepot vs. RandomByElement	3.549747	0.000386	0.001235
80	NearestByDepot vs. RandomSequentialCyclic	3.470275	0.00052	0.00125
79	CoefficientPropagation vs. UPGMC	3.443784	0.000574	0.001266
78	CLARA vs. UPGMC	3.311331	0.000929	0.001282
77	NearestByDepot vs. Simplified	3.28484	0.00102	0.001299
76	NearestByDepot vs. Sweep	3.271595	0.001069	0.001316
75	CoefficientPropagation vs. NearestByCustomer	3.231859	0.00123	0.001333
74	CoefficientPropagation vs. Parallel	3.231859	0.00123	0.001351
73	NearestByDepot vs. SequentialCyclic	3.125896	0.001773	0.00137
72	CLARA vs. NearestByCustomer	3.099406	0.001939	0.001389
71	CLARA vs. Parallel	3.099406	0.001939	0.001408
70	BestCyclicAssignment vs. PAM	3.072915	0.00212	0.001429
69	BestNearest vs. NearestByDepot	3.05967	0.002216	0.001449
68	CyclicAssignment vs. NearestByDepot	3.019934	0.002528	0.001471
67	CLARA vs. RandomByElement	2.966952	0.003008	0.001493
66	CLARA vs. RandomSequentialCyclic	2.887481	0.003883	0.001515
65	CoefficientPropagation vs. RandomByElement	2.834499	0.00459	0.001538

2.3 Adjusted p-values

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Sh}
1	PAM vs .ThreeCriteriaClustering	0	0	0	0
2	Farthest-First vs .PAM	0	0	0	0
3	KMEANS vs .PAM	0	0	0	0
4	PAM vs .Simplified	0	0	0	0
5	PAM vs .Sweep	0	0	0	0
6	BestNearest vs .PAM	0	0	0	0
7	RandomByElement vs .ThreeCriteriaClustering	0	0	0	0
8	RandomSequentialCyclic vs .ThreeCriteriaClustering	0	0	0	0
9	PAM vs .UPGMC	0	0	0	0
10	Farthest-First vs .RandomByElement	0	0	0	0
11	Farthest-First vs .RandomSequentialCyclic	0	0	0	0
12	NearestByCustomer vs .PAM	0	0	0	0
13	PAM vs .Parallel	0	0	0	0
14	SequentialCyclic vs .ThreeCriteriaClustering	0	0	0	0
15	KMEANS vs .RandomByElement	0	0	0	0
16	CyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
17	KMEANS vs .RandomSequentialCyclic	0	0	0	0
18	Farthest-First vs .SequentialCyclic	0	0	0	0
19	CyclicAssignment vs .Farthest-First	0	0	0	0
20	KMEANS vs .SequentialCyclic	0	0	0	0
21	CyclicAssignment vs .KMEANS	0	0	0	0
22	RandomByElement vs .Simplified	0	0	0	0
23	RandomByElement vs .Sweep	0	0	0	0
24	RandomSequentialCyclic vs .Simplified	0	0	0	0
25	RandomSequentialCyclic vs .Sweep	0	0	0	0
26	BestNearest vs .RandomByElement	0	0	0	0
27	BestNearest vs .RandomSequentialCyclic	0	0	0	0
28	SequentialCyclic vs .Simplified	0	0	0	0
29	SequentialCyclic vs .Sweep	0	0	0	0
30	CyclicAssignment vs .Simplified	0	0	0	0
31	CyclicAssignment vs .Sweep	0	0	0	0
32	RandomByElement vs .UPGMC	0	0	0	0
33	BestCyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
34	RandomSequentialCyclic vs .UPGMC	0	0	0	0
35	BestNearest vs .SequentialCyclic	0	0	0	0
36	BestNearest vs .CyclicAssignment	0	0	0	0
37	NearestByCustomer vs .RandomByElement	0	0	0	0
38	Parallel vs .RandomByElement	0	0	0	0
39	NearestByCustomer vs .RandomSequentialCyclic	0	0	0	0
40	Parallel vs .RandomSequentialCyclic	0	0	0	0
41	BestCyclicAssignment vs .Farthest-First	0	0	0	0
42	SequentialCyclic vs .UPGMC	0	0.000001	0.000001	0.000
43	CyclicAssignment vs .UPGMC	0	0.000001	0.000001	0.000
44	BestCyclicAssignment vs .KMEANS	0	0.000001	0.000001	0.000
45	NearestByCustomer vs .SequentialCyclic	0	0.000003	0.000002	0.000
46	Parallel vs .SequentialCyclic	0	0.000003	0.000002	0.000
47	CyclicAssignment vs .NearestByCustomer	0	0.000005	0.000003	0.000
48	CyclicAssignment vs .Parallel	0	0.000005	0.000003	0.000
49	BestCyclicAssignment vs .Simplified	0	0.000037	0.000025	0.000
50	BestCyclicAssignment vs .Sweep	0	0.000039	0.000027	0.000
51	CoefficientPropagation vs .ThreeCriteriaClustering	0	0.000074	0.00005	0.000
52	NearestByDepot vs .PAM	0.000001	0.000111	0.000074	0.000
53	BestCyclicAssignment vs .BestNearest	0.000001	0.000119	0.000079	0.000
54	CLARA vs .ThreeCriteriaClustering	0.000001	0.000146	0.000095	0.000
55	CoefficientPropagation vs .Farthest-First	0.000002	0.000233	0.000151	0.000
56	CLARA vs .Farthest-First	0.000003	0.000448	0.000287	0.000
57	BestCyclicAssignment vs .UPGMC	0.000004	0.000618	0.000392	0.000
58	CoefficientPropagation vs .KMEANS	0.000005	0.000748	0.000469	0.000
59	CLARA vs .KMEANS	0.000009	0.001394	0.000866	0.000
60	BestCyclicAssignment vs .NearestByCustomer	0.000011	0.001676	0.00103	0.001
61	BestCyclicAssignment vs .Parallel	0.000011	0.001676	0.00103	0.001
62	CLARA vs .PAM	0.000012	0.001893	0.001138	0.001
63	NearestByDepot vs .ThreeCriteriaClustering	0.000016	0.002409	0.001433	0.001
64	CoefficientPropagation vs .PAM	0.000023	0.003443	0.002025	0.001
65	Farthest-First vs .NearestByDepot	0.000043	0.006521	0.003793	0.003
66	CoefficientPropagation vs .Simplified	0.000063	0.009688	0.005572	0.005
67	CoefficientPropagation vs .Sweep	0.000067	0.010245	0.005825	0.005
68	CLARA vs .Simplified	0.00011	0.016814	0.009451	0.009
69	CLARA vs .Sweep	0.000116	0.017751	0.009862	0.009
70	KMEANS vs .NearestByDepot	0.000116	0.017751	0.009862	0.009
71	BestNearest vs .CoefficientPropagation	0.00016	0.024489	0.013285	0.013
72	BestNearest vs .CLARA	0.00027	0.041316	0.022143	0.022
73	NearestByDepot vs .RandomByElement	0.000386	0.058997	0.031234	0.031
74	NearestByDepot vs .RandomSequentialCyclic	0.00052	0.079549	0.041594	0.041
75	CoefficientPropagation vs .UPGMC	0.000574	0.087766	0.045317	0.045
76	CLARA vs .UPGMC	0.000929	0.142066	0.072426	0.072
77	NearestByDepot vs .Simplified	0.00102	0.156122	0.078571	0.077
78	NearestByDepot vs .Sweep	0.001069	0.163622	0.081276	0.081
79	CoefficientPropagation vs .NearestByCustomer	0.00123	0.188171	0.092241	0.089
80	CoefficientPropagation vs .Parallel	0.00123	0.188171	0.092241	0.089
81	NearestByDepot vs .SequentialCyclic	0.001773	0.271214	0.129403	0.129
82	CLARA vs .NearestByCustomer	0.001939	0.296681	0.139615	0.139
83	CLARA vs .Parallel	0.001939	0.296681	0.139615	0.139
84	BestCyclicAssignment vs .PAM	0.00212	0.324328	0.148385	0.148
85	BestNearest vs .NearestByDepot	0.002216	0.339019	0.152891	0.152
86	CyclicAssignment vs .NearestByDepot	0.002528	0.38683	0.171924	0.171
87	CLARA vs .RandomByElement	0.003008	0.460174	0.201514	0.201
88	CLARA vs .RandomSequentialCyclic	0.003883	0.594161	0.256305	0.256
89	CoefficientPropagation vs .RandomByElement	0.00459	0.702233	0.298334	0.298