Output tables for the test of Multiple comparisons.

June 12, 2025

1 Average rankings of Friedman test

Average ranks obtained by applying the Friedman procedure

Friedman statistic considering reduction performance (distributed according to chi-square with 15 degrees of freedom: 340.340686.

P-value computed by Friedman Test: 1.8857704287000843E-10.

Iman and Davenport statistic considering reduction performance (distributed according to F-distribution with 15 and 525 degrees of freedom: 59.661249.

P-value computed by Iman and Daveport Test: -2.220446049250313E-16.

Algorithm	Ranking
BestCyclicAssignment	12.75
BestNearest	7.125
CLARA	10
CoefficientPropagation	12.1111
CyclicAssignment	14.0556
Farthest-First	3.4722
KMEANS	4.5694
NearestByCustomer	6.9167
NearestByDepot	10.8889
PAM	8.9722
Parallel	6.9167
RandomByElement	15.9722
Simplified	6.1111
Sweep	6.1806
ThreeCriteriaClustering	3.4306
UPGMC	6.5278

Table 1: Average Rankings of the algorithms

2 Post hoc comparisons

Results achieved on post hoc comparisons for $\alpha = 0.05$, $\alpha = 0.10$ and adjusted p-values.

2.1 P-values for $\alpha = 0.05$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417.
Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.
Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	RandomByElement vs. ThreeCriteriaClustering	11.17629	0	0.000417
119	Farthest-First vs. RandomByElement	11.13916	0	0.00042
118	KMEANS vs. RandomByElement	10.161389	0	0.000424
117	CyclicAssignment vs. ThreeCriteriaClustering	9.468286	0	0.000427
116	CyclicAssignment vs. Farthest-First	9.431155	0	0.000431
115	RandomByElement vs. Simplified	8.787559	0	0.000435
114	RandomByElement vs. Sweep	8.725675	0	0.000439
113	CyclicAssignment vs. KMEANS	8.453385	0	0.000442
112	RandomByElement vs. UPGMC	8.416254	0	0.000446
111	BestCyclicAssignment vs. ThreeCriteriaClustering	8.304863	0	0.00045
110	BestCyclicAssignment vs. Farthest-First	8.267732	0	0.000455
109	NearestByCustomer vs. RandomByElement	8.069702	0	0.000459
108	Parallel vs. RandomByElement	8.069702	0	0.000463
107	BestNearest vs. RandomByElement	7.88405	0	0.000467
106	CoefficientPropagation vs. ThreeCriteriaClustering	7.735528	0	0.000472
105	CoefficientPropagation vs. Farthest-First	7.698397	0	0.000476
104	BestCyclicAssignment vs. KMEANS	7.289961	0	0.000481
103	CyclicAssignment vs. Simplified	7.079555	0	0.000485
102	CyclicAssignment vs. Sweep	7.017671	0	0.00049
101	CoefficientPropagation vs. KMEANS	6.720626	0	0.000495
100	CyclicAssignment vs. UPGMC	6.70825	0	0.0005
99	NearestByDepot vs. ThreeCriteriaClustering	6.646365	0	0.000505
98	Farthest-First vs. NearestByDepot	6.609235	0	0.00051
97	CyclicAssignment vs. NearestByCustomer	6.361698	0	0.000515
96	CyclicAssignment vs. Parallel	6.361698	0	0.000521
95	PAM vs. RandomByElement	6.23793	0	0.000526
94	BestNearest vs. CyclicAssignment	6.176045	0	0.000532
93	BestCyclicAssignment vs. Simplified	5.916132	0	0.000538
92	BestCyclicAssignment vs. Sweep	5.854247	0	0.000543
91	CLARA vs. ThreeCriteriaClustering	5.854247	0	0.000549
90	CLARA vs. Farthest-First	5.817117	0	0.000556
89	KMEANS vs. NearestByDepot	5.631464	0	0.000562
88	BestCyclicAssignment vs. UPGMC	5.544826	0	0.000568
87	CoefficientPropagation vs. Simplified	5.346797	0	0.000575
86	CLARA vs. RandomByElement	5.322043	0	0.000581
85	CoefficientPropagation vs. Sweep	5.284913	0	0.000588
84	BestCyclicAssignment vs. NearestByCustomer	5.198275	0	0.000595
83	BestCyclicAssignment vs. Parallel	5.198275	0	0.000602
82	BestCyclicAssignment vs. BestNearest	5.012622	0.000001	0.00061
81	CoefficientPropagation vs. UPGMC	4.975491	0.000001	0.000617
80	PAM vs. ThreeCriteriaClustering	4.938361	0.000001	0.000625
79	Farthest-First vs. PAM	4.90123	0.000001	0.000633
78	CLARA vs. KMEANS	4.839346	0.000001	0.000641
77	CoefficientPropagation vs. NearestByCustomer	4.62894	0.000004	0.000649
76	CoefficientPropagation vs. Parallel	4.62894	0.000004	0.000658
75	NearestByDepot vs. RandomByElement	4.529925	0.000006	0.000667
74	CyclicAssignment vs. PAM	4.529925	0.000006	0.000676
73	BestNearest vs. CoefficientPropagation	4.443287	0.000009	0.000685
72	NearestByDepot vs. Simplified	4.257634	0.000021	0.000694
71	NearestByDepot vs. Sweep	4.19575	0.000027	0.000704
70	KMEANS vs. PAM	3.92346	0.000087	0.000714
69	NearestByDepot vs. UPGMC	3.886329	0.000102	0.000725
68	CLARA vs. CyclicAssignment	3.614039	0.000301	0.000735
67	NearestByCustomer vs. NearestByDepot	3.539777	0.0004	0.000746
66	NearestByDepot vs. Parallel	3.539777	0.0004	0.000758
65	CLARA vs. Simplified	3.465516	0.000529	0.000769
64	CoefficientPropagation vs. RandomByElement	3.440763	0.00058	0.000781
63	CLARA vs. Sweep	3.403632	0.000665	0.000794
62	BestCyclicAssignment vs. PAM	3.366502	0.000761	0.000806
61	BestNearest vs. NearestByDepot	3.354125	0.000796	0.00082
60	BestNearest vs. ThreeCriteriaClustering	3.292241	0.000994	0.000833
59	BestNearest vs. Farthest-First	3.25511	0.001133	0.000847
58	NearestByCustomer vs. ThreeCriteriaClustering	3.106588	0.001893	0.000862
57	Parallel vs. ThreeCriteriaClustering	3.106588	0.001893	0.000877
56	CLARA vs. UPGMC	3.094211	0.001973	0.000893
55	Farthest-First vs. NearestByCustomer	3.069457	0.002144	0.000909
54	Farthest-First vs. Parallel	3.069457	0.002144	0.000926
53	BestCyclicAssignment vs. RandomByElement	2.871428	0.004086	0.000943
52	CyclicAssignment vs. NearestByDepot	2.82192	0.004774	0.000962
51	CoefficientPropagation vs. PAM	2.797167	0.005155	0.00098
50	ThreeCriteriaClustering vs. UPGMC	2.760036	0.005779	0.001
49	CLARA vs. NearestByCustomer	2.747659	0.006002	0.00102
48	CLARA vs. Parallel	2.747659	0.006002	0.001042
47	Farthest-First vs. UPGMC	2.722906	0.006471	0.001064
46	BestNearest vs. CLARA	2.562007	0.010407	0.001087
45	PAM vs. Simplified	2.54963	0.010784	0.001111
44	PAM vs. Sweep	2.487746	0.012856	0.001136
43	Sweep vs. ThreeCriteriaClustering	2.450615	0.014261	0.001163
42	BestCyclicAssignment vs. CLARA	2.450615	0.014261	0.00119
41	Farthest-First vs. Sweep	2.413485	0.015801	0.00122
40	Simplified vs. ThreeCriteriaClustering	2.388731	0.016907	0.00125
39	Farthest-First vs. Simplified	2.3516	0.018693	0.001282
38	BestNearest vs. KMEANS	2.277339	0.022766	0.001316
37	PAM vs. UPGMC	2.178325	0.029382	0.001351
36	KMEANS vs. NearestByCustomer	2.091687	0.036467	0.001389
35	KMEANS vs. Parallel	2.091687	0.036467	0.001429
34	CLARA vs. CoefficientPropagation	1.88128	0.059934	0.001471
33	NearestByCustomer vs. PAM	1.831773	0.066985	0.001515
32	PAM vs. Parallel	1.831773	0.066985	0.001563

2.2 P-values for $\alpha = 0.10$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.

Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.001724.

Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833.

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	RandomByElement vs. ThreeCriteriaClustering	11.17629	0	0.000833
119	Farthest-First vs. RandomByElement	11.13916	0	0.00084
118	KMEANS vs. RandomByElement	10.161389	0	0.000847
117	CyclicAssignment vs. ThreeCriteriaClustering	9.468286	0	0.000855
116	CyclicAssignment vs. Farthest-First	9.431155	0	0.000862
115	RandomByElement vs. Simplified	8.787559	0	0.00087
114	RandomByElement vs. Sweep	8.725675	0	0.000877
113	CyclicAssignment vs. KMEANS	8.453385	0	0.000885
112	RandomByElement vs. UPGMC	8.416254	0	0.000893
111	BestCyclicAssignment vs. ThreeCriteriaClustering	8.304863	0	0.000901
110	BestCyclicAssignment vs. Farthest-First	8.267732	0	0.000909
109	NearestByCustomer vs. RandomByElement	8.069702	0	0.000917
108	Parallel vs. RandomByElement	8.069702	0	0.000926
107	BestNearest vs. RandomByElement	7.88405	0	0.000935
106	CoefficientPropagation vs. ThreeCriteriaClustering	7.735528	0	0.000943
105	CoefficientPropagation vs. Farthest-First	7.698397	0	0.000952
104	BestCyclicAssignment vs. KMEANS	7.289961	0	0.000962
103	CyclicAssignment vs. Simplified	7.079555	0	0.000971
102	CyclicAssignment vs. Sweep	7.017671	0	0.00098
101	CoefficientPropagation vs. KMEANS	6.720626	0	0.00099
100	CyclicAssignment vs. UPGMC	6.70825	0	0.001
99	NearestByDepot vs. ThreeCriteriaClustering	6.646365	0	0.00101
98	Farthest-First vs. NearestByDepot	6.609235	0	0.00102
97	CyclicAssignment vs. NearestByCustomer	6.361698	0	0.001031
96	CyclicAssignment vs. Parallel	6.361698	0	0.001042
95	PAM vs. RandomByElement	6.23793	0	0.001053
94	BestNearest vs. CyclicAssignment	6.176045	0	0.001064
93	BestCyclicAssignment vs. Simplified	5.916132	0	0.001075
92	BestCyclicAssignment vs. Sweep	5.854247	0	0.001087
91	CLARA vs. ThreeCriteriaClustering	5.854247	0	0.001099
90	CLARA vs. Farthest-First	5.817117	0	0.001111
89	KMEANS vs. NearestByDepot	5.631464	0	0.001124
88	BestCyclicAssignment vs. UPGMC	5.544826	0	0.001136
87	CoefficientPropagation vs. Simplified	5.346797	0	0.001149
86	CLARA vs. RandomByElement	5.322043	0	0.001163
85	CoefficientPropagation vs. Sweep	5.284913	0	0.001176
84	BestCyclicAssignment vs. NearestByCustomer	5.198275	0	0.00119
83	BestCyclicAssignment vs. Parallel	5.198275	0	0.001205
82	BestCyclicAssignment vs. BestNearest	5.012622	0.000001	0.00122
81	CoefficientPropagation vs. UPGMC	4.975491	0.000001	0.001235
80	PAM vs. ThreeCriteriaClustering	4.938361	0.000001	0.00125
79	Farthest-First vs. PAM	4.90123	0.000001	0.001266
78	CLARA vs. KMEANS	4.839346	0.000001	0.001282
77	CoefficientPropagation vs. NearestByCustomer	4.62894	0.000004	0.001299
76	CoefficientPropagation vs. Parallel	4.62894	0.000004	0.001316
75	NearestByDepot vs. RandomByElement	4.529925	0.000006	0.001333
74	CyclicAssignment vs. PAM	4.529925	0.000006	0.001351
73	BestNearest vs. CoefficientPropagation	4.443287	0.000009	0.00137
72	NearestByDepot vs. Simplified	4.257634	0.000021	0.001389
71	NearestByDepot vs. Sweep	4.19575	0.000027	0.001408
70	KMEANS vs. PAM	3.92346	0.000087	0.001429
69	NearestByDepot vs. UPGMC	3.886329	0.000102	0.001449
68	CLARA vs. CyclicAssignment	3.614039	0.000301	0.001471
67	NearestByCustomer vs. NearestByDepot	3.539777	0.0004	0.001493
66	NearestByDepot vs. Parallel	3.539777	0.0004	0.001515
65	CLARA vs. Simplified	3.465516	0.000529	0.001538
64	CoefficientPropagation vs. RandomByElement	3.440763	0.00058	0.001563
63	CLARA vs. Sweep	3.403632	0.000665	0.001587
62	BestCyclicAssignment vs. PAM	3.366502	0.000761	0.001613
61	BestNearest vs. NearestByDepot	3.354125	0.000796	0.001639
60	BestNearest vs. ThreeCriteriaClustering	3.292241	0.000994	0.001667
59	BestNearest vs. Farthest-First	3.25511	0.001133	0.001695
58	NearestByCustomer vs. ThreeCriteriaClustering	3.106588	0.001893	0.001724
57	Parallel vs. ThreeCriteriaClustering	3.106588	0.001893	0.001754
56	CLARA vs. UPGMC	3.094211	0.001973	0.001786
55	Farthest-First vs. NearestByCustomer	3.069457	0.002144	0.001818
54	Farthest-First vs. Parallel	3.069457	0.002144	0.001852
53	BestCyclicAssignment vs. RandomByElement	2.871428	0.004086	0.001887
52	CyclicAssignment vs. NearestByDepot	2.82192	0.004774	0.001923
51	CoefficientPropagation vs. PAM	2.797167	0.005155	0.001961
50	ThreeCriteriaClustering vs. UPGMC	2.760036	0.005779	0.002
49	CLARA vs. NearestByCustomer	2.747659	0.006002	0.002041
48	CLARA vs. Parallel	2.747659	0.006002	0.002083
47	Farthest-First vs. UPGMC	2.722906	0.006471	0.002128
46	BestNearest vs. CLARA	2.562007	0.010407	0.002174
45	PAM vs. Simplified	2.54963	0.010784	0.002222
44	PAM vs. Sweep	2.487746	0.012856	0.002273
43	Sweep vs. ThreeCriteriaClustering	2.450615	0.014261	0.002326
42	BestCyclicAssignment vs. CLARA	2.450615	0.014261	0.002381
41	Farthest-First vs. Sweep	2.413485	0.015801	0.002439
40	Simplified vs. ThreeCriteriaClustering	2.388731	0.016907	0.0025
39	Farthest-First vs. Simplified	2.3516	0.018693	0.002564
38	BestNearest vs. KMEANS	2.277339	0.022766	0.002632
37	PAM vs. UPGMC	2.178325	0.029382	0.002703
36	KMEANS vs. NearestByCustomer	2.091687	0.036467	0.002778
35	KMEANS vs. Parallel	2.091687	0.036467	0.002857
34	CLARA vs. CoefficientPropagation	1.88128	0.059934	0.002941
33	NearestByCustomer vs. PAM	1.831773	0.066985	0.00303
32	PAM vs. Parallel	1.831773	0.066985	0.003125

2.3 Adjusted p-values

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Sha}
1	RandomByElement vs .ThreeCriteriaClustering	0	0	0	0
2	Farthest-First vs .RandomByElement	0	0	0	0
3	KMEANS vs .RandomByElement	0	0	0	0
4	CyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
5	CyclicAssignment vs .Farthest-First	0	0	0	0
6	RandomByElement vs .Simplified	0	0	0	0
7	RandomByElement vs .Sweep	0	0	0	0
8	CyclicAssignment vs .KMEANS	0	0	0	0
9	RandomByElement vs .UPGMC	0	0	0	0
10	BestCyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
11	BestCyclicAssignment vs .Farthest-First	0	0	0	0
12	NearestByCustomer vs .RandomByElement	0	0	0	0
13	Parallel vs .RandomByElement	0	0	0	0
14	BestNearest vs .RandomByElement	0	0	0	0
15	CoefficientPropagation vs .ThreeCriteriaClustering	0	0	0	0
16	CoefficientPropagation vs .Farthest-First	0	0	0	0
17	BestCyclicAssignment vs .KMEANS	0	0	0	0
18	CyclicAssignment vs .Simplified	0	0	0	0
19	CyclicAssignment vs .Sweep	0	0	0	0
20	CoefficientPropagation vs .KMEANS	0	0	0	0
21	CyclicAssignment vs .UPGMC	0	0	0	0
22	NearestByDepot vs .ThreeCriteriaClustering	0	0	0	0
23	Farthest-First vs .NearestByDepot	0	0	0	0
24	CyclicAssignment vs .NearestByCustomer	0	0	0	0
25	CyclicAssignment vs .Parallel	0	0	0	0
26	PAM vs .RandomByElement	0	0	0	0
27	BestNearest vs .CyclicAssignment	0	0	0	0
28	BestCyclicAssignment vs .Simplified	0	0	0	0
29	BestCyclicAssignment vs .Sweep	0	0.000001	0	0
30	CLARA vs .ThreeCriteriaClustering	0	0.000001	0	0
31	CLARA vs .Farthest-First	0	0.000001	0.000001	0
32	KMEANS vs .NearestByDepot	0	0.000002	0.000002	0.00000
33	BestCyclicAssignment vs .UPGMC	0	0.000004	0.000003	0.00000
34	CoefficientPropagation vs .Simplified	0	0.000011	0.000008	0.00000
35	CLARA vs .RandomByElement	0	0.000012	0.000009	0.00000
36	CoefficientPropagation vs .Sweep	0	0.000015	0.000011	0.0000
37	BestCyclicAssignment vs .NearestByCustomer	0	0.000024	0.000017	0.0000
38	BestCyclicAssignment vs .Parallel	0	0.000024	0.000017	0.0000
39	BestCyclicAssignment vs .BestNearest	0.000001	0.000064	0.000044	0.0000
40	CoefficientPropagation vs .UPGMC	0.000001	0.000078	0.000053	0.0000
41	PAM vs .ThreeCriteriaClustering	0.000001	0.000095	0.000063	0.0000
42	Farthest-First vs .PAM	0.000001	0.000114	0.000075	0.0000
43	CLARA vs .KMEANS	0.000001	0.000156	0.000102	0.0001
44	CoefficientPropagation vs .NearestByCustomer	0.000004	0.000441	0.000283	0.0002
45	CoefficientPropagation vs .Parallel	0.000004	0.000441	0.000283	0.0002
46	NearestByDepot vs .RandomByElement	0.000006	0.000708	0.000443	0.0004
47	CyclicAssignment vs .PAM	0.000006	0.000708	0.000443	0.0004
48	BestNearest vs .CoefficientPropagation	0.000009	0.001063	0.000647	0.0006
49	NearestByDepot vs .Simplified	0.000021	0.002479	0.001488	0.0014
50	NearestByDepot vs .Sweep	0.000027	0.003264	0.001931	0.0018
51	KMEANS vs .PAM	0.000087	0.010474	0.00611	0.0060
52	NearestByDepot vs .UPGMC	0.000102	0.012213	0.007022	0.0070
53	CLARA vs .CyclicAssignment	0.000301	0.036176	0.0205	0.020
54	NearestByCustomer vs .NearestByDepot	0.0004	0.048056	0.026831	0.0268
55	NearestByDepot vs .Parallel	0.0004	0.048056	0.026831	0.0268
56	CLARA vs .Simplified	0.000529	0.063506	0.034399	0.0343
57	CoefficientPropagation vs .RandomByElement	0.00058	0.069609	0.037125	0.0353
58	CLARA vs .Sweep	0.000665	0.079795	0.041893	0.0405
59	BestCyclicAssignment vs .PAM	0.000761	0.091354	0.047199	0.0464
60	BestNearest vs .NearestByDepot	0.000796	0.09554	0.048566	0.0485
61	BestNearest vs .ThreeCriteriaClustering	0.000994	0.119271	0.059636	0.0596
62	BestNearest vs .Farthest-First	0.001133	0.136018	0.066876	0.0668
63	NearestByCustomer vs .ThreeCriteriaClustering	0.001893	0.227112	0.109771	0.1097
64	Parallel vs .ThreeCriteriaClustering	0.001893	0.227112	0.109771	0.1097
65	CLARA vs .UPGMC	0.001973	0.236804	0.110509	0.1105
66	Farthest-First vs .NearestByCustomer	0.002144	0.257338	0.117946	0.1179
67	Farthest-First vs .Parallel	0.002144	0.257338	0.117946	0.1179
68	BestCyclicAssignment vs .RandomByElement	0.004086	0.490346	0.21657	0.2124
69	CyclicAssignment vs .NearestByDepot	0.004774	0.572844	0.248232	0.2482
70	CoefficientPropagation vs .PAM	0.005155	0.618635	0.26292	0.2629
71	ThreeCriteriaClustering vs .UPGMC	0.005779	0.693539	0.288975	0.2831
72	CLARA vs .NearestByCustomer	0.006002	0.720268	0.294109	0.2941
73	CLARA vs .Parallel	0.006002	0.720268	0.294109	0.2941
74	Farthest-First vs .UPGMC	0.006471	0.776526	0.304139	0.3041
75	BestNearest vs .CLARA	0.010407	1.248831	0.478719	0.4787
76	PAM vs .Simplified	0.010784	1.294048	0.485268	0.4852
77	PAM vs .Sweep	0.012856	1.542667	0.565645	0.5656
78	Sweep vs .ThreeCriteriaClustering	0.014261	1.711348	0.613233	0.6132
79	BestCyclicAssignment vs .CLARA	0.014261	1.711348	0.613233	0.6132
80	Farthest-First vs .Sweep	0.015801	1.896095	0.647833	0.6478
81	Simplified vs .ThreeCriteriaClustering	0.016907	2.028801	0.676267	0.6762
82	Farthest-First vs .Simplified	0.018693	2.243141	0.729021	0.7290
83	BestNearest vs .KMEANS	0.022766	2.731916	0.865107	0.8651
84	PAM vs .UPGMC	0.029382	3.525826	1.08713	1.0871
85	KMEANS vs .NearestByCustomer	0.036467	4.375986	1.312796	1.3127
86	KMEANS vs .Parallel	0.036467	4.375986	1.312796	1.3127
87	CLARA vs .CoefficientPropagation	0.059934	7.192055	2.037749	2.0377
88	NearestByCustomer vs .PAM	0.066985	8.038231	2.210513	2.2105
89	PAM vs .Parallel	0.066985	8.038231	2.210513	2.2105