

Output tables for the test of Multiple comparisons.

June 12, 2025

1 Average rankings of Friedman test

Average ranks obtained by applying the Friedman procedure

Friedman statistic considering reduction performance (distributed according to chi-square with 15 degrees of freedom: 335.85049.

P-value computed by Friedman Test: 1.3431300516231204E-10.

Iman and Davenport statistic considering reduction performance (distributed according to F-distribution with 15 and 525 degrees of freedom: 57.579208.

P-value computed by Iman and Davenport Test: -2.220446049250313E-16.

Algorithm	Ranking
BestCyclicAssignment	12.75
BestNearest	7.125
CLARA	9.6389
CoefficientPropagation	11.6111
CyclicAssignment	14.1667
Farthest-First	3.4444
KMEANS	4.5139
NearestByCustomer	6.9444
NearestByDepot	10.9722
PAM	9.0556
Parallel	6.9444
RandomByElement	16
Simplified	6.0833
Sweep	6.1528
ThreeCriteriaClustering	3.3472
UPGMC	7.25

Table 1: Average Rankings of the algorithms

2 Post hoc comparisons

Results achieved on post hoc comparisons for $\alpha = 0.05$, $\alpha = 0.10$ and adjusted p-values.

2.1 P-values for $\alpha = 0.05$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417 .

Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.00082 .

Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000417 .

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	RandomByElement vs. ThreeCriteriaClustering	11.275305	0	0.000417
119	Farthest-First vs. RandomByElement	11.188667	0	0.00042
118	KMEANS vs. RandomByElement	10.23565	0	0.000424
117	CyclicAssignment vs. ThreeCriteriaClustering	9.641562	0	0.000427
116	CyclicAssignment vs. Farthest-First	9.554924	0	0.000431
115	RandomByElement vs. Simplified	8.837067	0	0.000435
114	RandomByElement vs. Sweep	8.775183	0	0.000439
113	CyclicAssignment vs. KMEANS	8.601907	0	0.000442
112	BestCyclicAssignment vs. ThreeCriteriaClustering	8.379124	0	0.000446
111	BestCyclicAssignment vs. Farthest-First	8.292486	0	0.00045
110	NearestByCustomer vs. RandomByElement	8.069702	0	0.000455
109	Parallel vs. RandomByElement	8.069702	0	0.000459
108	BestNearest vs. RandomByElement	7.908803	0	0.000463
107	RandomByElement vs. UPGMC	7.797412	0	0.000467
106	CoefficientPropagation vs. ThreeCriteriaClustering	7.364222	0	0.000472
105	BestCyclicAssignment vs. KMEANS	7.339469	0	0.000476
104	CoefficientPropagation vs. Farthest-First	7.277584	0	0.000481
103	CyclicAssignment vs. Simplified	7.203323	0	0.000485
102	CyclicAssignment vs. Sweep	7.141439	0	0.00049
101	NearestByDepot vs. ThreeCriteriaClustering	6.794888	0	0.000495
100	Farthest-First vs. NearestByDepot	6.70825	0	0.0005
99	CyclicAssignment vs. NearestByCustomer	6.435959	0	0.000505
98	CyclicAssignment vs. Parallel	6.435959	0	0.00051
97	CoefficientPropagation vs. KMEANS	6.324567	0	0.000515
96	BestNearest vs. CyclicAssignment	6.27506	0	0.000521
95	PAM vs. RandomByElement	6.188422	0	0.000526
94	CyclicAssignment vs. UPGMC	6.163668	0	0.000532
93	BestCyclicAssignment vs. Simplified	5.940885	0	0.000538
92	BestCyclicAssignment vs. Sweep	5.879001	0	0.000543
91	KMEANS vs. NearestByDepot	5.755233	0	0.000549
90	CLARA vs. RandomByElement	5.668595	0	0.000556
89	CLARA vs. ThreeCriteriaClustering	5.60671	0	0.000562
88	CLARA vs. Farthest-First	5.520073	0	0.000568
87	BestCyclicAssignment vs. NearestByCustomer	5.173521	0	0.000575
86	BestCyclicAssignment vs. Parallel	5.173521	0	0.000581
85	PAM vs. ThreeCriteriaClustering	5.086883	0	0.000588
84	BestCyclicAssignment vs. BestNearest	5.012622	0.000001	0.000595
83	Farthest-First vs. PAM	5.000245	0.000001	0.000602
82	CoefficientPropagation vs. Simplified	4.925984	0.000001	0.00061
81	BestCyclicAssignment vs. UPGMC	4.90123	0.000001	0.000617
80	CoefficientPropagation vs. Sweep	4.8641	0.000001	0.000625
79	CLARA vs. KMEANS	4.567056	0.000005	0.000633
78	CyclicAssignment vs. PAM	4.554679	0.000005	0.000641
77	NearestByDepot vs. RandomByElement	4.480418	0.000007	0.000649
76	NearestByDepot vs. Simplified	4.356649	0.000013	0.000658
75	NearestByDepot vs. Sweep	4.294765	0.000017	0.000667
74	CoefficientPropagation vs. NearestByCustomer	4.15862	0.000032	0.000676
73	CoefficientPropagation vs. Parallel	4.15862	0.000032	0.000685
72	KMEANS vs. PAM	4.047228	0.000052	0.000694
71	CLARA vs. CyclicAssignment	4.034851	0.000055	0.000704
70	BestNearest vs. CoefficientPropagation	3.997721	0.000064	0.000714
69	CoefficientPropagation vs. RandomByElement	3.911083	0.000092	0.000725
68	CoefficientPropagation vs. UPGMC	3.886329	0.000102	0.000735
67	NearestByCustomer vs. NearestByDepot	3.589285	0.000332	0.000746
66	NearestByDepot vs. Parallel	3.589285	0.000332	0.000758
65	ThreeCriteriaClustering vs. UPGMC	3.477893	0.000505	0.000769
64	BestNearest vs. NearestByDepot	3.428386	0.000607	0.000781
63	Farthest-First vs. UPGMC	3.391255	0.000696	0.000794
62	BestNearest vs. ThreeCriteriaClustering	3.366502	0.000761	0.000806
61	NearestByDepot vs. UPGMC	3.316994	0.00091	0.00082
60	BestCyclicAssignment vs. PAM	3.292241	0.000994	0.000833
59	BestNearest vs. Farthest-First	3.279864	0.001039	0.000847
58	NearestByCustomer vs. ThreeCriteriaClustering	3.205603	0.001348	0.000862
57	Parallel vs. ThreeCriteriaClustering	3.205603	0.001348	0.000877
56	CLARA vs. Simplified	3.168472	0.001532	0.000893
55	Farthest-First vs. NearestByCustomer	3.118965	0.001815	0.000909
54	Farthest-First vs. Parallel	3.118965	0.001815	0.000926
53	CLARA vs. Sweep	3.106588	0.001893	0.000943
52	BestCyclicAssignment vs. RandomByElement	2.896182	0.003777	0.000962
51	CyclicAssignment vs. NearestByDepot	2.846674	0.004418	0.00098
50	BestCyclicAssignment vs. CLARA	2.772413	0.005564	0.001
49	PAM vs. Simplified	2.648645	0.008082	0.00102
48	PAM vs. Sweep	2.58676	0.009688	0.001042
47	Sweep vs. ThreeCriteriaClustering	2.500123	0.012415	0.001064
46	Simplified vs. ThreeCriteriaClustering	2.438238	0.014759	0.001087
45	KMEANS vs. UPGMC	2.438238	0.014759	0.001111
44	Farthest-First vs. Sweep	2.413485	0.015801	0.001136
43	CLARA vs. NearestByCustomer	2.401108	0.016346	0.001163
42	CLARA vs. Parallel	2.401108	0.016346	0.00119
41	Farthest-First vs. Simplified	2.3516	0.018693	0.00122
40	BestNearest vs. KMEANS	2.326847	0.019973	0.00125
39	CoefficientPropagation vs. PAM	2.277339	0.022766	0.001282
38	CoefficientPropagation vs. CyclicAssignment	2.277339	0.022766	0.001316
37	BestNearest vs. CLARA	2.240209	0.025077	0.001351
36	KMEANS vs. NearestByCustomer	2.165948	0.030315	0.001389
35	KMEANS vs. Parallel	2.165948	0.030315	0.001429
34	CLARA vs. UPGMC	2.128817	0.033269	0.001471
33	NearestByCustomer vs. PAM	1.88128	0.059934	0.001515
32	PAM vs. Parallel	1.88128	0.059934	0.001563

2.2 P-values for $\alpha = 0.10$

Nemenyi's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833 .

Holm's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.001887 .

Shaffer's procedure rejects those hypotheses that have an unadjusted p-value ≤ 0.000833 .

i	algorithms	$z = (R_0 - R_i)/SE$	p	Holm
120	RandomByElement vs. ThreeCriteriaClustering	11.275305	0	0.000833
119	Farthest-First vs. RandomByElement	11.188667	0	0.00084
118	KMEANS vs. RandomByElement	10.23565	0	0.000847
117	CyclicAssignment vs. ThreeCriteriaClustering	9.641562	0	0.000855
116	CyclicAssignment vs. Farthest-First	9.554924	0	0.000862
115	RandomByElement vs. Simplified	8.837067	0	0.00087
114	RandomByElement vs. Sweep	8.775183	0	0.000877
113	CyclicAssignment vs. KMEANS	8.601907	0	0.000885
112	BestCyclicAssignment vs. ThreeCriteriaClustering	8.379124	0	0.000893
111	BestCyclicAssignment vs. Farthest-First	8.292486	0	0.000901
110	NearestByCustomer vs. RandomByElement	8.069702	0	0.000909
109	Parallel vs. RandomByElement	8.069702	0	0.000917
108	BestNearest vs. RandomByElement	7.908803	0	0.000926
107	RandomByElement vs. UPGMC	7.797412	0	0.000935
106	CoefficientPropagation vs. ThreeCriteriaClustering	7.364222	0	0.000943
105	BestCyclicAssignment vs. KMEANS	7.339469	0	0.000952
104	CoefficientPropagation vs. Farthest-First	7.277584	0	0.000962
103	CyclicAssignment vs. Simplified	7.203323	0	0.000971
102	CyclicAssignment vs. Sweep	7.141439	0	0.00098
101	NearestByDepot vs. ThreeCriteriaClustering	6.794888	0	0.00099
100	Farthest-First vs. NearestByDepot	6.70825	0	0.001
99	CyclicAssignment vs. NearestByCustomer	6.435959	0	0.00101
98	CyclicAssignment vs. Parallel	6.435959	0	0.00102
97	CoefficientPropagation vs. KMEANS	6.324567	0	0.001031
96	BestNearest vs. CyclicAssignment	6.27506	0	0.001042
95	PAM vs. RandomByElement	6.188422	0	0.001053
94	CyclicAssignment vs. UPGMC	6.163668	0	0.001064
93	BestCyclicAssignment vs. Simplified	5.940885	0	0.001075
92	BestCyclicAssignment vs. Sweep	5.879001	0	0.001087
91	KMEANS vs. NearestByDepot	5.755233	0	0.001099
90	CLARA vs. RandomByElement	5.668595	0	0.001111
89	CLARA vs. ThreeCriteriaClustering	5.60671	0	0.001124
88	CLARA vs. Farthest-First	5.520073	0	0.001136
87	BestCyclicAssignment vs. NearestByCustomer	5.173521	0	0.001149
86	BestCyclicAssignment vs. Parallel	5.173521	0	0.001163
85	PAM vs. ThreeCriteriaClustering	5.086883	0	0.001176
84	BestCyclicAssignment vs. BestNearest	5.012622	0.000001	0.00119
83	Farthest-First vs. PAM	5.000245	0.000001	0.001205
82	CoefficientPropagation vs. Simplified	4.925984	0.000001	0.00122
81	BestCyclicAssignment vs. UPGMC	4.90123	0.000001	0.001235
80	CoefficientPropagation vs. Sweep	4.8641	0.000001	0.00125
79	CLARA vs. KMEANS	4.567056	0.000005	0.001266
78	CyclicAssignment vs. PAM	4.554679	0.000005	0.001282
77	NearestByDepot vs. RandomByElement	4.480418	0.000007	0.001299
76	NearestByDepot vs. Simplified	4.356649	0.000013	0.001316
75	NearestByDepot vs. Sweep	4.294765	0.000017	0.001333
74	CoefficientPropagation vs. NearestByCustomer	4.15862	0.000032	0.001351
73	CoefficientPropagation vs. Parallel	4.15862	0.000032	0.00137
72	KMEANS vs. PAM	4.047228	0.000052	0.001389
71	CLARA vs. CyclicAssignment	4.034851	0.000055	0.001408
70	BestNearest vs. CoefficientPropagation	3.997721	0.000064	0.001429
69	CoefficientPropagation vs. RandomByElement	3.911083	0.000092	0.001449
68	CoefficientPropagation vs. UPGMC	3.886329	0.000102	0.001471
67	NearestByCustomer vs. NearestByDepot	3.589285	0.000332	0.001493
66	NearestByDepot vs. Parallel	3.589285	0.000332	0.001515
65	ThreeCriteriaClustering vs. UPGMC	3.477893	0.000505	0.001538
64	BestNearest vs. NearestByDepot	3.428386	0.000607	0.001563
63	Farthest-First vs. UPGMC	3.391255	0.000696	0.001587
62	BestNearest vs. ThreeCriteriaClustering	3.366502	0.000761	0.001613
61	NearestByDepot vs. UPGMC	3.316994	0.00091	0.001639
60	BestCyclicAssignment vs. PAM	3.292241	0.000994	0.001667
59	BestNearest vs. Farthest-First	3.279864	0.001039	0.001695
58	NearestByCustomer vs. ThreeCriteriaClustering	3.205603	0.001348	0.001724
57	Parallel vs. ThreeCriteriaClustering	3.205603	0.001348	0.001754
56	CLARA vs. Simplified	3.168472	0.001532	0.001786
55	Farthest-First vs. NearestByCustomer	3.118965	0.001815	0.001818
54	Farthest-First vs. Parallel	3.118965	0.001815	0.001852
53	CLARA vs. Sweep	3.106588	0.001893	0.001887
52	BestCyclicAssignment vs. RandomByElement	2.896182	0.003777	0.001923
51	CyclicAssignment vs. NearestByDepot	2.846674	0.004418	0.001961
50	BestCyclicAssignment vs. CLARA	2.772413	0.005564	0.002
49	PAM vs. Simplified	2.648645	0.008082	0.002041
48	PAM vs. Sweep	2.58676	0.009688	0.002083
47	Sweep vs. ThreeCriteriaClustering	2.500123	0.012415	0.002128
46	Simplified vs. ThreeCriteriaClustering	2.438238	0.014759	0.002174
45	KMEANS vs. UPGMC	2.438238	0.014759	0.002222
44	Farthest-First vs. Sweep	2.413485	0.015801	0.002273
43	CLARA vs. NearestByCustomer	2.401108	0.016346	0.002326
42	CLARA vs. Parallel	2.401108	0.016346	0.002381
41	Farthest-First vs. Simplified	2.3516	0.018693	0.002439
40	BestNearest vs. KMEANS	2.326847	0.019973	0.0025
39	CoefficientPropagation vs. PAM	2.277339	0.022766	0.002564
38	CoefficientPropagation vs. CyclicAssignment	2.277339	0.022766	0.002632
37	BestNearest vs. CLARA	2.240209	0.025077	0.002703
36	KMEANS vs. NearestByCustomer	2.165948	0.030315	0.002778
35	KMEANS vs. Parallel	2.165948	0.030315	0.002857
34	CLARA vs. UPGMC	2.128817	0.033269	0.002941
33	NearestByCustomer vs. PAM	1.88128	0.059934	0.00303
32	PAM vs. Parallel	1.88128	0.059934	0.003125

2.3 Adjusted p-values

i	hypothesis	unadjusted p	p_{Neme}	p_{Holm}	p_{Sha}
1	RandomByElement vs .ThreeCriteriaClustering	0	0	0	0
2	Farthest-First vs .RandomByElement	0	0	0	0
3	KMEANS vs .RandomByElement	0	0	0	0
4	CyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
5	CyclicAssignment vs .Farthest-First	0	0	0	0
6	RandomByElement vs .Simplified	0	0	0	0
7	RandomByElement vs .Sweep	0	0	0	0
8	CyclicAssignment vs .KMEANS	0	0	0	0
9	BestCyclicAssignment vs .ThreeCriteriaClustering	0	0	0	0
10	BestCyclicAssignment vs .Farthest-First	0	0	0	0
11	NearestByCustomer vs .RandomByElement	0	0	0	0
12	Parallel vs .RandomByElement	0	0	0	0
13	BestNearest vs .RandomByElement	0	0	0	0
14	RandomByElement vs .UPGMC	0	0	0	0
15	CoefficientPropagation vs .ThreeCriteriaClustering	0	0	0	0
16	BestCyclicAssignment vs .KMEANS	0	0	0	0
17	CoefficientPropagation vs .Farthest-First	0	0	0	0
18	CyclicAssignment vs .Simplified	0	0	0	0
19	CyclicAssignment vs .Sweep	0	0	0	0
20	NearestByDepot vs .ThreeCriteriaClustering	0	0	0	0
21	Farthest-First vs .NearestByDepot	0	0	0	0
22	CyclicAssignment vs .NearestByCustomer	0	0	0	0
23	CyclicAssignment vs .Parallel	0	0	0	0
24	CoefficientPropagation vs .KMEANS	0	0	0	0
25	BestNearest vs .CyclicAssignment	0	0	0	0
26	PAM vs .RandomByElement	0	0	0	0
27	CyclicAssignment vs .UPGMC	0	0	0	0
28	BestCyclicAssignment vs .Simplified	0	0	0	0
29	BestCyclicAssignment vs .Sweep	0	0	0	0
30	KMEANS vs .NearestByDepot	0	0.000001	0.000001	0.000001
31	CLARA vs .RandomByElement	0	0.000002	0.000001	0.000001
32	CLARA vs .ThreeCriteriaClustering	0	0.000002	0.000002	0.000001
33	CLARA vs .Farthest-First	0	0.000004	0.000003	0.000001
34	BestCyclicAssignment vs .NearestByCustomer	0	0.000028	0.00002	0.000001
35	BestCyclicAssignment vs .Parallel	0	0.000028	0.00002	0.000001
36	PAM vs .ThreeCriteriaClustering	0	0.000044	0.000031	0.000001
37	BestCyclicAssignment vs .BestNearest	0.000001	0.000064	0.000045	0.000001
38	Farthest-First vs .PAM	0.000001	0.000069	0.000048	0.000001
39	CoefficientPropagation vs .Simplified	0.000001	0.000101	0.000069	0.000001
40	BestCyclicAssignment vs .UPGMC	0.000001	0.000114	0.000077	0.000001
41	CoefficientPropagation vs .Sweep	0.000001	0.000138	0.000092	0.000001
42	CLARA vs .KMEANS	0.000005	0.000594	0.000391	0.000391
43	CyclicAssignment vs .PAM	0.000005	0.00063	0.000409	0.000409
44	NearestByDepot vs .RandomByElement	0.000007	0.000894	0.000574	0.000574
45	NearestByDepot vs .Simplified	0.000013	0.001585	0.001004	0.000913
46	NearestByDepot vs .Sweep	0.000017	0.002099	0.001312	0.001212
47	CoefficientPropagation vs .NearestByCustomer	0.000032	0.003842	0.002369	0.002369
48	CoefficientPropagation vs .Parallel	0.000032	0.003842	0.002369	0.002369
49	KMEANS vs .PAM	0.000052	0.006219	0.003732	0.003732
50	CLARA vs .CyclicAssignment	0.000055	0.006556	0.003879	0.003879
51	BestNearest vs .CoefficientPropagation	0.000064	0.007675	0.004477	0.004477
52	CoefficientPropagation vs .RandomByElement	0.000092	0.011026	0.00634	0.00634
53	CoefficientPropagation vs .UPGMC	0.000102	0.012213	0.00692	0.00692
54	NearestByCustomer vs .NearestByDepot	0.000332	0.03979	0.022216	0.022216
55	NearestByDepot vs .Parallel	0.000332	0.03979	0.022216	0.022216
56	ThreeCriteriaClustering vs .UPGMC	0.000505	0.060645	0.032849	0.032849
57	BestNearest vs .NearestByDepot	0.000607	0.072862	0.03886	0.03703
58	Farthest-First vs .UPGMC	0.000696	0.083488	0.043831	0.042424
59	BestNearest vs .ThreeCriteriaClustering	0.000761	0.091354	0.047199	0.046419
60	NearestByDepot vs .UPGMC	0.00091	0.10919	0.055505	0.055505
61	BestCyclicAssignment vs .PAM	0.000994	0.119271	0.059636	0.059636
62	BestNearest vs .Farthest-First	0.001039	0.124629	0.061276	0.061276
63	NearestByCustomer vs .ThreeCriteriaClustering	0.001348	0.161736	0.078172	0.078172
64	Parallel vs .ThreeCriteriaClustering	0.001348	0.161736	0.078172	0.078172
65	CLARA vs .Simplified	0.001532	0.183891	0.085816	0.085816
66	Farthest-First vs .NearestByCustomer	0.001815	0.217785	0.099818	0.099818
67	Farthest-First vs .Parallel	0.001815	0.217785	0.099818	0.099818
68	CLARA vs .Sweep	0.001893	0.227112	0.100308	0.099818
69	BestCyclicAssignment vs .RandomByElement	0.003777	0.453281	0.196422	0.196422
70	CyclicAssignment vs .NearestByDepot	0.004418	0.530143	0.225311	0.225311
71	BestCyclicAssignment vs .CLARA	0.005564	0.667708	0.278212	0.272612
72	PAM vs .Simplified	0.008082	0.969783	0.395995	0.395995
73	PAM vs .Sweep	0.009688	1.162595	0.465038	0.465038
74	Sweep vs .ThreeCriteriaClustering	0.012415	1.489804	0.583507	0.583507
75	Simplified vs .ThreeCriteriaClustering	0.014759	1.771085	0.678916	0.678916
76	KMEANS vs .UPGMC	0.014759	1.771085	0.678916	0.678916
77	Farthest-First vs .Sweep	0.015801	1.896095	0.695235	0.695235
78	CLARA vs .NearestByCustomer	0.016346	1.961462	0.702857	0.702857
79	CLARA vs .Parallel	0.016346	1.961462	0.702857	0.702857
80	Farthest-First vs .Simplified	0.018693	2.243141	0.766407	0.766407
81	BestNearest vs .KMEANS	0.019973	2.396811	0.798937	0.798937
82	CoefficientPropagation vs .PAM	0.022766	2.731916	0.887873	0.887873
83	CoefficientPropagation vs .CyclicAssignment	0.022766	2.731916	0.887873	0.887873
84	BestNearest vs .CLARA	0.025077	3.009284	0.927863	0.927863
85	KMEANS vs .NearestByCustomer	0.030315	3.637822	1.091346	1.091346
86	KMEANS vs .Parallel	0.030315	3.637822	1.091346	1.091346
87	CLARA vs .UPGMC	0.033269	3.992326	1.131159	1.131159
88	NearestByCustomer vs .PAM	0.059934	7.192055	1.977815	1.977815
89	PAM vs .Parallel	0.059934	7.192055	1.977815	1.977815