Notes

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January 14, 2021

1 Plummer model

Consider a Plummer model (Dejonghe, H.1987, MNRAS 224, 13) with potential with units r_s the Plummer scale radius (which sets the size of the cluster core), M the total mass of the cluster and $\bar{\tau}$ some unit time. Let ψ_s be defined by

$$\psi_\mathrm{s} = \frac{GM}{r_\mathrm{s}}, \tag{1}$$

for the central potential

$$\psi(r) = \frac{\psi_\mathrm{s}}{\sqrt{1+r^2}}. \tag{2}$$

Let use fix $G = 1\, r_\mathrm{s}^3 . M^{-1} . \bar{\tau}^{-2}$ in the new units so that $\psi_\mathrm{s} = 1\, r_\mathrm{s}^2 \cdot \bar{\tau}^{-2}$. This fixes the time unit $\bar{\tau}$, as we have the relation. Therefore, in those units the potential (per unit mass) is given by

$$\psi(r) = \frac{1}{\sqrt{1+r^2}}. \tag{3}$$

Define, given a radius r, the angular momentum $L(r, v_\mathrm{r}, v_\mathrm{t})$ and binding energy per unit mass $E(r, v_\mathrm{r}, v_\mathrm{t})$, functions of the radial velocity v_r and the tangential velocity $v_\mathrm{t} \geq 0$ (defined as $\boldsymbol{v} = \boldsymbol{v_\mathrm{r}} + \boldsymbol{v_\mathrm{t}} = v_\mathrm{r}\hat{\boldsymbol{r}} + \boldsymbol{v_\mathrm{t}}$), as

$$\begin{array}{rcl} E(r, v_\mathrm{r}, v_\mathrm{t}) & = & \psi(r) - \frac{1}{2}v_\mathrm{r}^2 - \frac{1}{2}v_\mathrm{t}^2, \\ L(r, v_\mathrm{r}, v_\mathrm{t}) & = & r \cdot v_\mathrm{t}, \end{array} \tag{4}$$

whose Jacobian is

$$\mathrm{Jac}_{(r,v_r,v_t)\to(r,E,L)} = \begin{pmatrix} \frac{\partial E}{\partial v_r} & \frac{\partial E}{\partial v_t} \\ \frac{\partial L}{v_{ar}} & \frac{\partial L}{\partial v_t} \end{pmatrix} = \begin{pmatrix} -v_r & -v_t \\ 0 & r \end{pmatrix} \Rightarrow |\mathrm{Jac}| = r|v_r|. \tag{5}$$