The Astrophysical Journal, 516:420–424, 1999 May 1

SELF-SIMILAR SOLUTION OF OPTICALLY THICK ADVECTION-DOMINATED FLOWS

Jian-Min Wang
Astronomy Department, Beijing Normal University; and Chinese Academy of Sciences, Peking University Joint Astrophysical Center (CAS-PKU.BAC), Beijing 100871, People's Republic of China; agnwjm@bnu.edu.cn

AND

You-Yuan Zhou
Center for Astrophysics, University of Science and Technology of China, Hefei 230026, People's Republic of China; zyy@cfa.ustc.edu.cn
Received 1998 June 1; accepted 1998 December 8

ABSTRACT

The photons released by the viscosity process in a disk with accretion rate exceeding the Eddington critical value will be trapped and restored as entropy in the accreting gas rather than being radiated from the disk surface, since the advection cooling dominates the diffusion cooling in such a flow. In this paper a self-similar solution of optically thick advection-dominated flow has been obtained. This kind of flow shows some interesting characteristics that differ from both the optically thin advection-dominated disk and the standard disk. Such a flow is thermally stable. The Bernoulli number is not positive in a wide range of parameters unless the viscosity α is extremely small. The shape always stays slim and is independent of the accretion rate ($H/r \approx 1/\sqrt{5}$). The gas rotates with sub-Keplerian angular velocity $\Omega \approx \Omega_K/\sqrt{5}$ (for smaller viscosity). The luminosity of the disk weakly depends on the accretion rate, and the maximum luminosity is about 4.0×10^{37} (ergs s^{-1}) $(M/M_\odot)$ (less than the Eddington luminosity), although the accretion rate is super-Eddington. This provides a method for estimating the mass of a black hole. The validity of this solution has been tested and found to be suitable in most of the accretion disk (within the photon trapping radius). Its potential applications to the *saturated* accreting systems are pointed out.

Subject headings: accretion, accretion disks — black hole physics — hydrodynamics

1. INTRODUCTION

Accretion disks are expected to occur in a large number of various celestial objects, for example, X-ray binaries, protostars, and active galactic nuclei (AGNs) (see review by Frank, King, & Raine 1992). In the standard model of an accretion disk (Shakura & Sunyaev 1973; Novikov & Thorne 1973), the gravitational energy will be locally efficiently radiated from the disk surface, and the gas keeps its Keplerian rotation because the interactions between neighboring radial annuli are neglected. However, advection has recently come to be thought of as an important process and results in a structure different from that of the standard model. The advection process physically means that the generated energy via viscous dissipation is restored as entropy of the accreting gas rather than being radiated. The advection effect is very important for the cases of both low and high accretion rate, since the radiation efficiently decreases under these circumstances.

The transonic flow also deals involves the advection process because of the importance of radial motion of accreting gas. The original work on such issues was done by Paczyński & Bisnovatyi-Kogan (1981) and Muchotrzeb & Paczyński (1982), and extensively by Matsumoto et al. (1984), Abramowicz et al. (1988), Szuszkiewicz (1988), and Chen & Taam (1993). These authors focused their attention on the optically thick disk numerically. The angular velocity of gas is much lower than the Keplerian, and gas becomes transonic at a certain location depending on the parameters, such as accretion rate and viscosity coefficient. Instead of the standard model of accretion disks, the slim disk model has been applied to fit big blue bumps (BBBs) of AGNs (Szuszkiewicz, Abramowicz, & Malkan 1996; Wang & Zhou 1996; Wang & Szuszkiewicz 1999). It is expected to interpret the statistical properties of BBBs via accretion disk theory (Zhou et al. 1997).

Recently much more attention has been paid to the optically thin advection-dominated disk (Chen et al. 1995; Abramowicz et al. 1995; Narayan, Kato, & Honma 1997; Chen, Abramowicz, & Lasota 1997; Blandford & Begelman 1998; and others) stemming from the self-similar solution of Narayan & Yi (1994, 1995). The most prominent characteristics of such a flow are significantly sub-Keplerian, thermally stable, and capable of simultaneously producing outflow. Such a flow has been applied to several kinds of objects, for example, Sgr A* (Narayan, Yi, & Mahadevan 1995) and NGC 4258 (Lasota et al. 1996), which are the *starving* accretion systems with low accretion rates.

The existing two-dimensional self-similar solutions for the geometrically thick disk (Gilham 1981; Begelman & Meier 1982; Anderson 1987) neglect the inertial term, $v_r\, dv_r/dr$, describing the dynamical importance of the accretion velocity v_r in the radial motion equation, and others. In this paper we focus our attention on the disk with high accretion rate in which photon trapping and advection dominate over surface diffusion cooling. We show the presence of self-similar behavior of the optically thick advection-dominated flow by using the height-averaged equations of the disk, and point out the potential applications to the *saturated* accretion systems.

2. THE SELF-SIMILAR SOLUTION

The basic model employed here is similar to the slim disk model, but we consider an extreme case in which the blackbody radiation from the disk surface is so inefficient that advection cooling dominates over surface cooling because the high accretion rate leads to photon trapping in the disk

(Begelman & Meier 1982). It is assumed that the steady disk in a Newtonian potential well is of azimuthal symmetry. The Keplerian angular velocity is $\Omega_{\rm K} = (GM/r^3)^{1/2}$, where G is the gravitational constant and r is the distance from the central object with mass M. Following Shakura & Sunyaev (1973), we take the kinematic coefficient of shear viscosity to be $\nu = \alpha c_s H$, where c_s is the sound speed, α the viscosity parameter, and H the half-height of the disk. The basic equations describing the slim disk are given by Muchotrzeb & Paczyński (1982), but here the coefficients B_i with unit order neglected, read

$$\dot{M} = 4\pi r H \rho v_r \,, \tag{1}$$

$$\frac{P}{\rho} = H^2 \Omega_{\rm K}^2 \,, \tag{2}$$

$$\dot{M}(l - l_{\rm in}) = 4\pi r^2 H \alpha P \,, \tag{3}$$

$$\frac{1}{\rho}\frac{dP}{dr} - (\Omega^2 - \Omega_k^2) r + v_r \frac{dv_r}{dr} + \frac{P}{\rho}\frac{d\ln \Omega_k}{dr} = 0 \,, \tag{4}$$

where $\dot{M}$ is the accretion rate, P the total pressure, ρ the mass density, v_r the radial velocity of the flow, Ω the angular velocity, l the angular momentum, and $l_{\rm in}$ the eigenvalue of angular momentum at the inner boundary. The last term of the left-hand side of equation (4) is the correction for the decrease of the radial component of gravitational force away from the equator (Matsumoto et al. 1984). The law of energy conservation should be added to the above equations. The advection cooling may dominate the diffusion cooling, and reads (Abramowicz et al. 1988)[1]

$$\begin{aligned} Q_{\rm adv} &= -T\frac{\dot{M}}{4\pi r}\frac{dS}{dr} \\ &= \frac{\dot{M}}{4\pi r}\left(\frac{P}{\rho}\right)\left[(4 - 0.75\beta)\frac{d\ln\rho}{dr} - (12 - 10.5\beta)\frac{d\ln T}{dr}\right] \\ &= \frac{\dot{M}}{4\pi r^2}\left(\frac{P}{\rho}\right)\xi \,, \end{aligned} \tag{5}$$

where $\xi = (4 - 0.75\beta)\gamma_\rho - (12 - 10.5\beta)\gamma_T$, β is the ratio of gas to total pressure, $\gamma_\rho = d\ln\rho/d\ln r$, and $\gamma_T = d\ln T/d\ln r$. In this paper we take $\beta = 0$, since the radiation pressure dominates over the gas; thus $\xi = 4\gamma_\rho - 12\gamma_T$. In this paper we consider the extreme case in which the generated energy is balanced by the advection cooling,

$$Q_{\rm vis} = Q_{\rm adv} = \frac{\dot{M}}{4\pi r^2}\left(\frac{P}{\rho}\right)\xi \,, \tag{6}$$

where $Q_{\rm vis}$ is the energy generation per area via the viscosity dissipation, and follows from

$$Q_{\rm vis} = \frac{\dot{M}(l - l_{\rm in})}{4\pi r}\left(-\frac{d\Omega}{dr}\right) = \frac{nf\dot{M}\Omega^2}{4\pi} \,; \tag{7}$$

here $n = -d\ln\Omega/d\ln r$ and $f = 1 - l_{\rm in}/l$ ($f = 1$ in Narayan & Yi 1994). Strictly speaking, the radial motion and energy conservation equations may be solved by supplementing the right boundary condition. Denoting $\gamma_p = d\ln P/d\ln r$, and $\gamma_v = d\ln v_r/d\ln r$, the radial motion equation will reduce to an algebraic equation as

$$\left(\gamma_p - \frac{3}{2}\right)\frac{P}{\rho} - (\Omega^2 - \Omega_{\rm K}^2) r^2 + \gamma_v v_r^2 = 0 \,. \tag{8}$$

Now the two differential equations simultaneously reduce to algebraic ones, since the flow becomes self-similar in structure. After some algebraic manipulations we get a set of self-similar solutions as follows:

$$P = \left(\frac{\xi f}{n}\right)^{1/2} \frac{\dot{M}\Omega_{\rm K}}{4\pi\alpha r} \propto r^{-5/2} \,, \tag{9}$$

$$\rho = \frac{\gamma_0^2}{4\pi\alpha}\left(\frac{\xi^3}{n^3 f}\right)^{1/2} \frac{\dot{M}}{\Omega_{\rm K} r^3} \propto r^{-3/2} \,, \tag{10}$$

$$v_r = \frac{n\alpha}{\xi\gamma_0} r\Omega_{\rm K} \propto r^{-1/2} \,, \tag{11}$$

$$\frac{H}{r} = \frac{1}{\gamma_0}\left(\frac{nf}{\xi}\right)^{1/2} = \text{constant} \,, \tag{12}$$

$$\Omega = \frac{\Omega_{\rm K}}{\gamma_0} \propto r^{-3/2} \,, \tag{13}$$

$$c_s = \left(\frac{nf}{\xi}\right)^{1/2} \frac{r\Omega_{\rm K}}{\gamma_0} \propto r^{-1/2} \,, \tag{14}$$

where γ_0 is

$$\gamma_0 = \left[1 - \frac{nf}{\xi}\left(\gamma_p - \frac{3}{2}\right) - \frac{n^2\alpha^2\gamma_v}{\xi^2}\right]^{1/2} \,. \tag{15}$$

The self-similar solutions are self-consistent if we neglect the boundary condition at the inner edge of the disk, i.e., the parameter $f = 1 - l_{\rm in}/l = 1$ as in Narayan & Yi (1994). From these solutions we have $\gamma_p = -5/2$, $\gamma_v = -1/2$, $n = 3/2$, $\gamma_\rho = -3/2$, $\gamma_T = -5/8$ (see below) and $\xi = 3/2$. Thus we get $\gamma_0 = (5 + \alpha^2/2)^{1/2} \approx \sqrt{5}$ (for $\alpha \ll 1$), which very weakly depends on the viscosity parameter α. The above self-similar solutions have the same power index of radius with optically thin advection-dominated flow, but its properties are different from those of the latter.

3. CONCLUSIONS AND DISCUSSION

In the advection-dominated flow the entropy of accreting gas becomes very high, and diffusion cooling from the disk surface becomes less important. Such a flow reduces to self-similar structure, although the structures at larger disk radius may be quite different. This is important from the observational viewpoint because this implies that all the spectra are the same in the high-energy band although the spectra of objects are different in the lower band. One of the general properties of advection-dominated flow is sub-Keplerian. The weak dependence of the parameter γ_0 on α reflects that the pressure gradient governs the flow over viscosity at high accretion rate.

The pressure is directly proportional to the accretion rate, and inversely proportional to the viscosity α. High viscosity, i.e., large α, leads the accreting gas to be close to free-fall velocity, and thus reduces the radial pressure. From

[1] The original formula in Abramowicz et al. (1988) reads $\xi = (4 - \beta)\gamma_\rho - (12 - 10.5\beta)\gamma_T$, which includes an error and should be corrected to the present form, as pointed out by the referee.

equation (9) we have

$$P=\left(\frac{4\sqrt{2}c^2}{\kappa_{\rm es}r_{g\odot}}\right)\left(\frac{\xi f}{n}\right)\left(\frac{\dot m}{m}\right)\alpha^{-1}x^{-5/2}$$
$$=3.2\times 10^{16}\left(\frac{\dot m}{m}\right)\alpha^{-1}x^{-5/2}\ {\rm dyn\ cm^{-2}}\,, \qquad (16)$$

where $\kappa_{\rm es}=0.34$, $r_{g\odot}=2GM_\odot/c^2=2.95\times 10^5$, $\dot m$ is the accretion rate normalized by the critical rate $\dot M_{\rm Edd}=64\pi GM/c\kappa_{\rm es}$ (Muchotrzeb & Paczyński 1982), m denotes $M/M_\odot$, and $x=r/r_g$ (where $r_g=2GM/c^2$). Since the radiation pressure dominates the gas pressure ($P=aT^4/3$, where $a=7.56\times 10^{-15}$ is the blackbody constant), the midplane temperature in the radial direction can be simply written as

$$T_{\rm c}=1.37\times 10^8\left(\frac{\dot m}{m}\right)^{1/4}\left(\frac{5}{x}\right)^{5/8}\left(\frac{0.001}{\alpha}\right)^{1/4}\ {\rm K}\,. \qquad (17)$$

This temperature is much lower than the virial temperature $T_v=(m_p c^2/2k)x^{-1}\approx 5.0\times 10^{12}x^{-1}$ K, whereas that of the optically thin advection-dominated case is close to the virial temperature.

The density distribution is $\rho\propto r^{-3/2}$, whereas in the radiation pressure–dominated region of a standard disk it is $\rho\propto r^{1.5}$. The density increases with accretion rate, and decreases linearly with α . This is due to the increase of v_r with α as seen from equation (11). The radial velocity v_r depends linearly on α. It is also important to note that the radial velocity, local height H, and angular velocity are independent of accretion rate $\dot M$ but depend only on the viscosity parameter α. The disk shape keeps slim with a universal constant $H/r\approx 1/\sqrt{5}$ (see eq. [12]), and decreases with increasing α. In this extreme case the shape of the disk is independent of the accretion rate. The angular velocity Ω ($\approx\Omega_{\rm K}/\sqrt{5}$ for small viscosity) is always less than the local Keplerian angular velocity. This is important in the stabilization of the disk.

The instability of such a flow can be discussed using the relationship between the surface density Σ and the accretion rate $\dot M$. From equations (10) and (12) we have

$$\Sigma=\left(\frac{\gamma_0\xi}{2\pi n\alpha}\right)\frac{\dot M}{r^2\Omega_{\rm K}}\,. \qquad (18)$$

In the standard model, we have $\Sigma\propto\dot M^{-1}$ (Shakura & Sunyaev 1973), which implies thermal instability in the radiation pressure–dominated region. Equation (18) clearly states $\Sigma\propto\dot M$, suggesting that such a flow is thermally stable although the radiation pressure dominates. The classical instability of the standard model is removed if advection cooling dominates. This property is similar to that of a slim disk as discussed by Abramowicz et al. (1988). From equations (7) and (13), the lower angular velocity Ω leads $Q_{\rm vis}$ to be smaller than that in the standard accretion disk by a factor of $1/\gamma_0^2$. Advection cooling plays two important roles: balancing and lowering the generated energy. These are the reasons why advection cooling is able to stabilize the pressure-dominated region of the disk.

That the Bernoulli number is positive for a wide range of parameters is one of the most prominent properties of optically thin advection-dominated flow as argued by Narayan & Yi (1994), suggesting that the accretion process simultaneously produces a jet. The Bernoulli number is defined as the sum of the kinetic energy, the potential energy, and the enthalpy of accreting gas. The enthalpy is obtained from $h=P/\rho+\epsilon=c_s^2+RT/(\gamma-1)$, where R ($=8.31\times 10^7$) is the gas constant and γ is the ratio of specific heats. The dimensionless Bernoulli number is given by

$$b=\frac{1}{v_{\rm K}^2}\left(\frac{1}{2}v_r^2+\frac{1}{2}\Omega^2r^2-\Omega_{\rm K}^2r^2+c_s^2+\frac{RT}{\gamma-1}\right)$$
$$=3.77\times 10^{-5}\left(\frac{\dot m}{m}\right)^{1/4}\frac{x^{3/8}}{(\gamma-1)\alpha^{1/4}}-\frac{7}{10+\alpha^2}\,. \qquad (19)$$

The Bernoulli number must be negative for a wide ranges of the parameters γ, $\dot m$, m, and α except in the case of extremely small α and the outer region $x>10^3$. This property is contrary to the case of optically thin flow. Thus the optically thick advection-dominated flow may not simultaneously produce outflow for a wide range of parameters. Equation (19) will return that of Narayan & Yi (1994) when we use the equation of state of an ideal gas as $c_s^2=RT$. However, it should be mentioned that the viscosity α may be a function of accretion rate as employed in Abramowicz et al. (1988), $\alpha\propto\exp(-\dot m)$, which implies that α drastically decreases with the increases of $\dot m$. Under the present condition (exceeding the critical value of the accretion rate) the viscosity may be much smaller than usual, since the accretion rate is too high to stimulate the turbulent viscosity. We keep this as a subject for future investigation.

The self-similar solutions of the optically thick advection-dominated flow have several potential applications to astrophysical objects which are powered by such a flow, especially to the *saturated* accretion systems, as the optically thin flow is suitable to a *starving* system. First of all, such a condition is possible in young stellar objects and symbiotic stars in outburst (Popham et al. 1993).

Second, the calculations of nucleosynthesis in an accretion disk (Jin, Arnett, & Chakrabarti 1989; Arai & Hashimoto 1992, 1995) should employ the detailed structure of a slim disk. It is easy to study this important process in disks using this simple self-similar solution. The newly formed accretion disk after supernova explosion is thought to have a high accretion rate $\dot M=10^6\dot M_{\rm Edd}$ (Mineshige et al. 1997). Under such conditions, nuclear reaction will take place and thus change the chemical composition of the supernova remnant. Meanwhile the chemical composition changes will result in changes in the opacity of the disk, thus affecting the structure of the disk. It is believed that the self-similar solution will be useful for discussion of the phenomena relative to the feedback to the central supernova remnant via accretion.

Third, the present self-similar solution may be applied to some of the X-ray Galactic and extragalactic sources. When mass accretion is higher than a certain value, the radiation-dominated regime takes place and the temperature of the accretion flow is much less than the virial one (see eq. [17]). Thus the whole system is quite cold and we see the soft state of the source, whereas it has been suggested that the hard radiation is produced as a result of the high kinetic energy of the bulk motion inflow to the black hole (Grove et al. 1998; Shrader & Titarchuk 1998; Vilhu & Nevalainen 1998). Recent observations reveal a new type of active galactic nuclei, i.e., narrow-line Seyfert 1 galaxies, whose properties are similar to those of Seyfert 1 galaxies but with narrow lines and the analog of the soft state of the Galactic sources (Boller, Brandt, & Fink 1996; Comastri et al. 1998; Shrader & Titarchuk 1998). This kind of object is thought to be powered by a less massive black hole with high accretion rate exceeding the Eddington rate. It is preferable to fit the big blue bump of a narrow-line Seyfert 1 galaxy using this self-similar solution, since it is in analytical form and its structure is thermally stable. The presence of a self-similar solution indicates that the flow with high accretion rate has the same behavior within a certain radius as that given by equation (22). Thus the boundary condition is less important for such a flow. Although the radiation is inefficient for such a disk, the properties of the luminosity from the disk are interesting. The present one-dimensional self-similar solution cannot provide the vertical structure without any assumption. Here we follow Laor & Netzer (1989), who assume that the energy generation by viscosity is proportional to the density; then the effective temperature of the disk, $T_{\rm eff}$, can be obtained from the diffusion approximation as $T_{\rm eff} = T_c/(1 + 0.75\tau_0)^{1/4} \approx T_c/\tau_0^{1/4}$ ($\tau_0 \gg 1$; see eq. [25] in Laor & Netzer 1989), where $\tau_0 = \kappa_{\rm es}\Sigma$ is the optical depth of the disk, namely,

$$T_{\rm eff} = \left(\frac{3c^4}{4a\kappa_{\rm es}GM_\odot}\right)^{1/4}\left(\frac{nf}{8\xi}\right)^{1/8}\gamma_0^{-1/4}m^{-1/4}x^{-1/2} \approx 2.1\times10^7 m^{-1/4}x^{-1/2}\ {\rm K} \tag{20}$$

for small α. It is interesting to note that the effective temperature is independent of the accretion rate $\dot{M}$ and very weakly dependent on the viscosity (through ξ), whereas $T_{\rm eff} \propto \dot{M}^{1/4}$ in the standard accretion disk. The luminosity can be found by integrating over the disk surface:

$$L = \frac{6\pi\sigma GM_\odot}{a\kappa_{\rm es}}\left(\frac{nf}{4\xi}\right)^{1/2}\left(\frac{m}{\gamma_0}\right)\ln\left(\frac{x_{\rm out}}{3}\right) \approx 4.0\times10^{37} m\ {\rm ergs\ s^{-1}}\,, \tag{21}$$

where σ is the Stefan-Boltzmann constant, and the outer radius of the disk, $x_{\rm out}$, is taken to be 7.2×10^2; here we are already taking the inner radius to be 3 Schwarzschild radii. The calculation of radiated luminosity is based on the blackbody radiation. It can be found that this luminosity weakly depends on $x_{\rm out}$, which may be determined by the photon trapping radius (see below). Thus the maximum luminosity is $L_{\rm max} \approx 4.0\times10^{37}m$ (ergs s^{-1}) less than the Eddington luminosity, whereas the flow is super-Eddington. The accretion luminosity of the disk *cannot* exceed the Eddington luminosity because of the photon trapping in high accretion rate. The radiation from the disk surface is of particular interest because the radiated luminosity is independent of the accretion rate and very weakly dependent on the viscosity α. This gives a method for determining the mass of the central object from the observed luminosity, namely, $M_{\rm BH} \approx 2.5\times10^6\,M_\odot\,(L/10^{44}\ {\rm ergs\ s^{-1}})$. Here we would like to point out that this method is valid in a rough estimation of black hole mass, owing to the approximation in the estimation of effective temperature (eq. [20]). It is important to note in equation (20) that the effective temperature is free of the accretion rate and is weakly dependent on the viscosity α and the mass of the black hole. The effective temperature distribution suggests that the observed spectra of objects are almost the same, i.e., there is a universal spectral form (also, the Comptonization may be very important, since the temperature of electrons at the optical depth $\tau \approx 1$ in such a flow is much higher than that in the standard disk). Detailed calculations of emergent spectra from such a disk are expected to explain the properties of BBBs (Zhou & Yu 1992; Walter & Fink 1993; Walter et al. 1994; Zhou et al. 1997). The present solution supplemented by energy transfer in the vertical direction will provide the emergent spectrum of the disk, but it is beyond the scope of this paper. We will use this simple solution to discuss these problems in the future.

In our derivations we completely neglect the diffusion cooling from the disk surface. Let us test this approximation. The timescale of photon diffusion in the z-direction is $\tau_{\rm dif} \approx \kappa_{\rm es}H\Sigma/c$, while the viscous timescale for the advection-dominated flow is $\tau_{\rm vis} \approx \pi r^2\Sigma/\dot{M}$. The photon trapping appears when $\tau_{\rm dif} > \tau_{\rm vis}$, namely,

$$\left(\frac{R_{\rm tr}}{r_g}\right) = \frac{\kappa_{\rm es}\dot{M}}{\pi c}\left(\frac{H}{r}\right) = 7.2\times10^2\left(\frac{\dot{m}}{50}\right). \tag{22}$$

It can be seen that the large trapping radius shows the general validity of our model for the accretion disk. It thus well represents the behavior of flow of high accretion rates. The presence of the self-similar solution suggests that all the flow with high accretion rate will have the same behavior within the trapping radius despite the quite different structure outside the trapping radius. The neglect of self-gravitation of the disk in this paper is valid because the disk density is much lower than the mean density of the black hole within the trapping radius of the disk.

The self-similar solutions are invalid close to the inner region, since we take the parameter f to be unity, and they break down if the mass accretion rate is 0.1. Nevertheless, our self-similar solutions are a good approximation for the optically thick flow dominated by advection, like that for the optically thin flow advocated by Narayan & Yi (1994), where the inner boundary condition is neglected.

The authors are very grateful to the anonymous referee for constructive suggestions and help in clarifying several points in the original version, especially the corrected formula for ξ in equation (5). Ewa Szuszkiewicz is acknowledged for her useful comments. J. M. W. expresses his thanks to F. Yuan and J. H. Wu for their discussions. This research is supported by the Pandeng Plan of the Department of Science and Technology of China, and the Natural Science Foundation of China under grant 19803002.

REFERENCES

Abramowicz, M. A., Chen, X., Kato, S., Lasota, J.-P., & Regev, O. 1995, ApJ, 438, L37
Abramowicz, M. A., Czerny, B., Lasota, J.-P., & Szuszkiewicz, E. 1988, ApJ, 332, 646
Anderson, M. 1987, MNRAS, 227, 623
Arai, K., & Hashimoto, M. 1992, A&A, 254, 191
———. 1995, A&A, 302, 191
Begelman, M. C., & Meier, D. L. 1982, ApJ, 253, 873
Blandford, R. D., & Begelman, M. C. 1988, MNRAS, in press
Boller, T., Brandt, W. N., & Fink, H. 1996, A&A, 305, 53
Chen, X., Abramowicz, M. A., & Lasota, J.-P. 1997, ApJ, 476, 61
Chen, X., Abramowicz, M. A., Lasota, J.-P., Narayan, R., & Yi, I. 1995, ApJ, 443, L61
Chen, X., & Taam, R. 1993, ApJ, 412, 254
Comastri, A., et al. 1998, A&A, in press
Frank, J., King, A. R., & Raine, D. J. 1992, Accretion Power in Astrophysics (Cambridge: Cambridge Univ. Press)
Gilham, S. 1981, MNRAS, 195, 755

Grove, J. E., Johnson, W. N., Kroeger, R. A., McNaron, K., & Skibo, J. G. 1998, ApJ, 500, 899
Jin, L., Arnett, W. D., & Chakrabarti, S. K. 1989, ApJ, 336, 572
Laor, A., & Netzer, H. 1989, MNRAS, 238, 897
Lasota, J.-P., et al. 1996, ApJ, 462, 142
Matsumoto, R., Kato, S., Fukue, J., & Okazaki, A. T. 1984, PASJ, 36, 71
Mineshige, S., et al. 1997, ApJ, 489, 227
Muchotrzeb, B., & Paczyński, B. 1982, Acta Astron., 32, 1
Narayan, R., Kato, S., & Honma, F. 1997, ApJ, 476, 49
Narayan, R., & Yi, I. 1994, ApJ, 428, L13
———. 1995, ApJ, 444, 231
Narayan, R., Yi, I., & Mahadevan, R. 1995, Nature, 374, 623
Novikov, I. D., & Thorne, K. S. 1973, in Black Holes, ed. C. D. Dewitt & B. Dewitt (New York: Gordon & Breach), 345
Paczyński, B., & Bisnovatyi-Kogan, G. 1981, Acta Astron., 31, 283
Popham, R., Narayan, R., Hartmann, L., & Kenyon, J. 1993, ApJ, 415, L127
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Shrader, C., & Titarchuk, L. 1998, ApJ, 499, L31
Szuszkiewicz, E. 1988, Ph.D. thesis, SISSA
Szuszkiewicz, E., Abramowicz, M., & Malkan, M. A. 1996, ApJ, 458, 474
Vilhu, O., & Nevalainen, J. 1998, ApJ, 508, L85
Walter, R., & Fink, H. H. 1993, A&A, 274, 105
Walter, R., et al. 1994, A&A, 285, 119
Wang, J.-M., & Szuszkiewicz, E. 1999, in preparation
Wang, J.-M., & Zhou, Y.-Y. 1996, ApJ, 469, 564
Zhou, Y.-Y., & Yu, K. N. 1992, ApJ, 400, 430
Zhou, Y.-Y., Yu, K. N., Young, E. C. M., Wang, J.-M., & Ma, E. 1997, ApJ, 475, L9