STELLAR EVOLUTION

all about MESA

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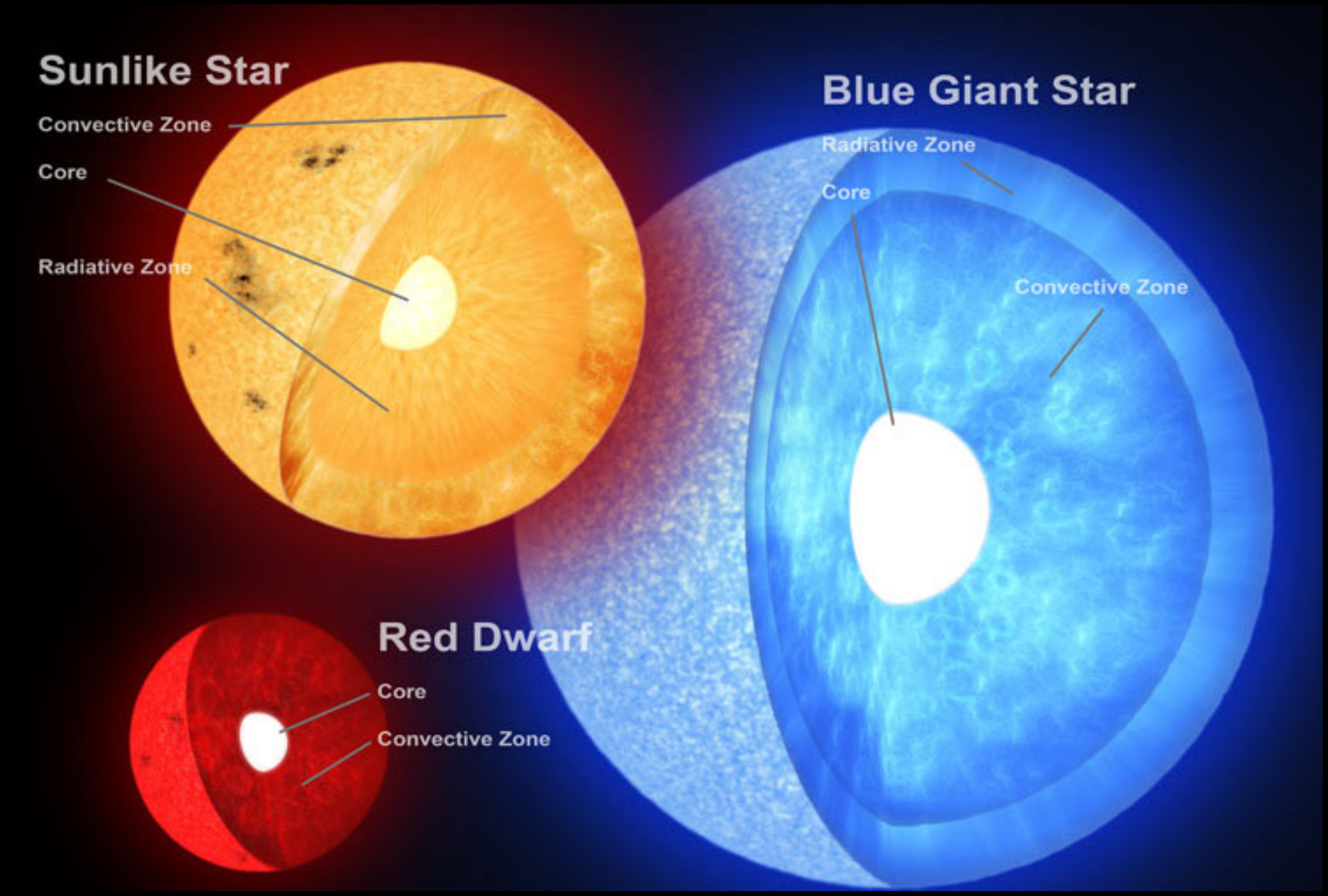
What is a star?



Stars are giant balls of gas that avoid collapsing onto themselves under the effect of gravity thanks to nuclear burning. The life of a star is a continuous fight against gravity.

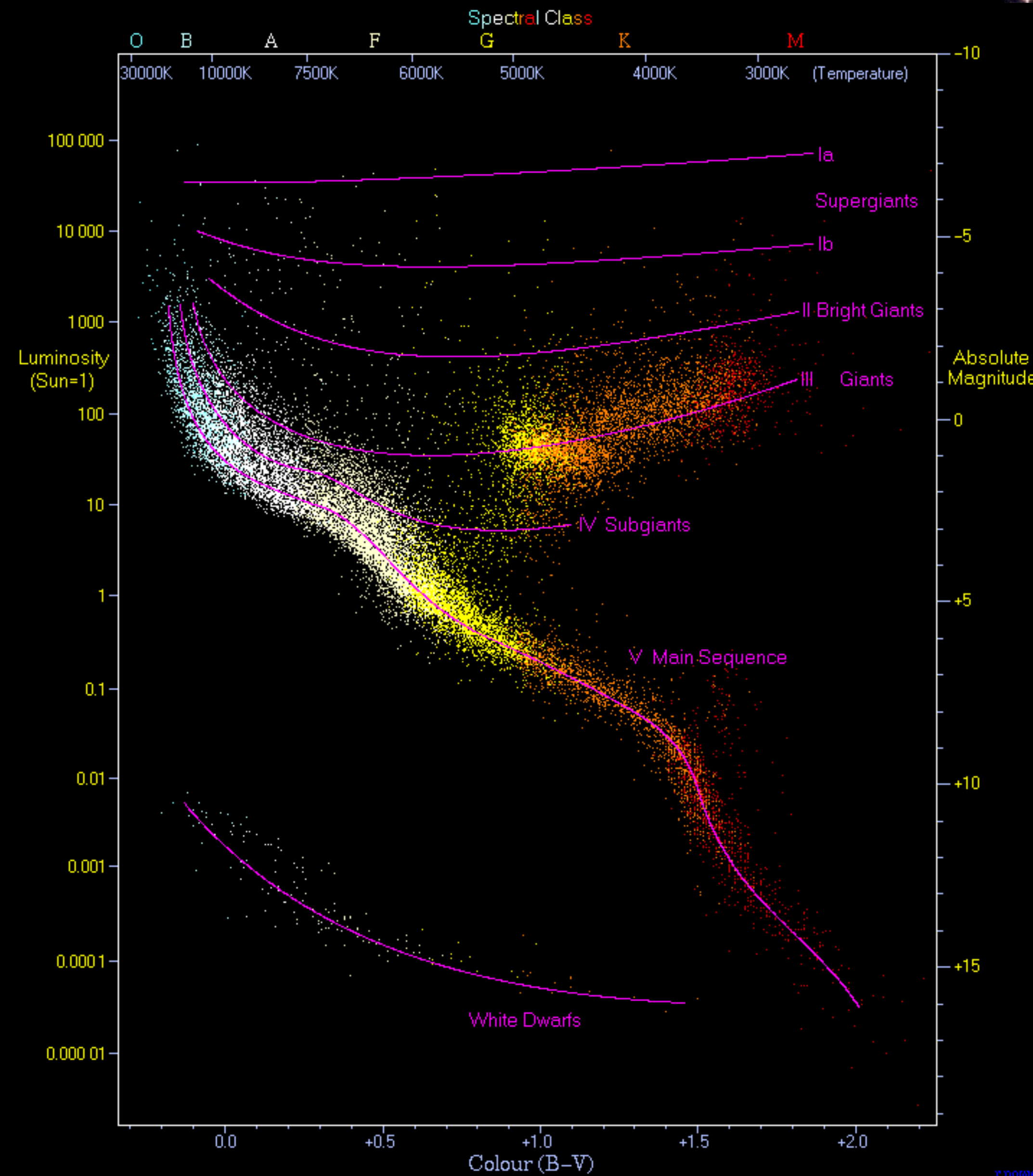
What is a star?

The main parameters that distinguish one star from another are:

- mass
- age
- composition

The most important parameter is mass. The initial mass of a star will determine:

- how long the star is going to last,
- where it's going to spend most of its time on the main sequence,
- which elements it's going to burn in its core
- its final destination



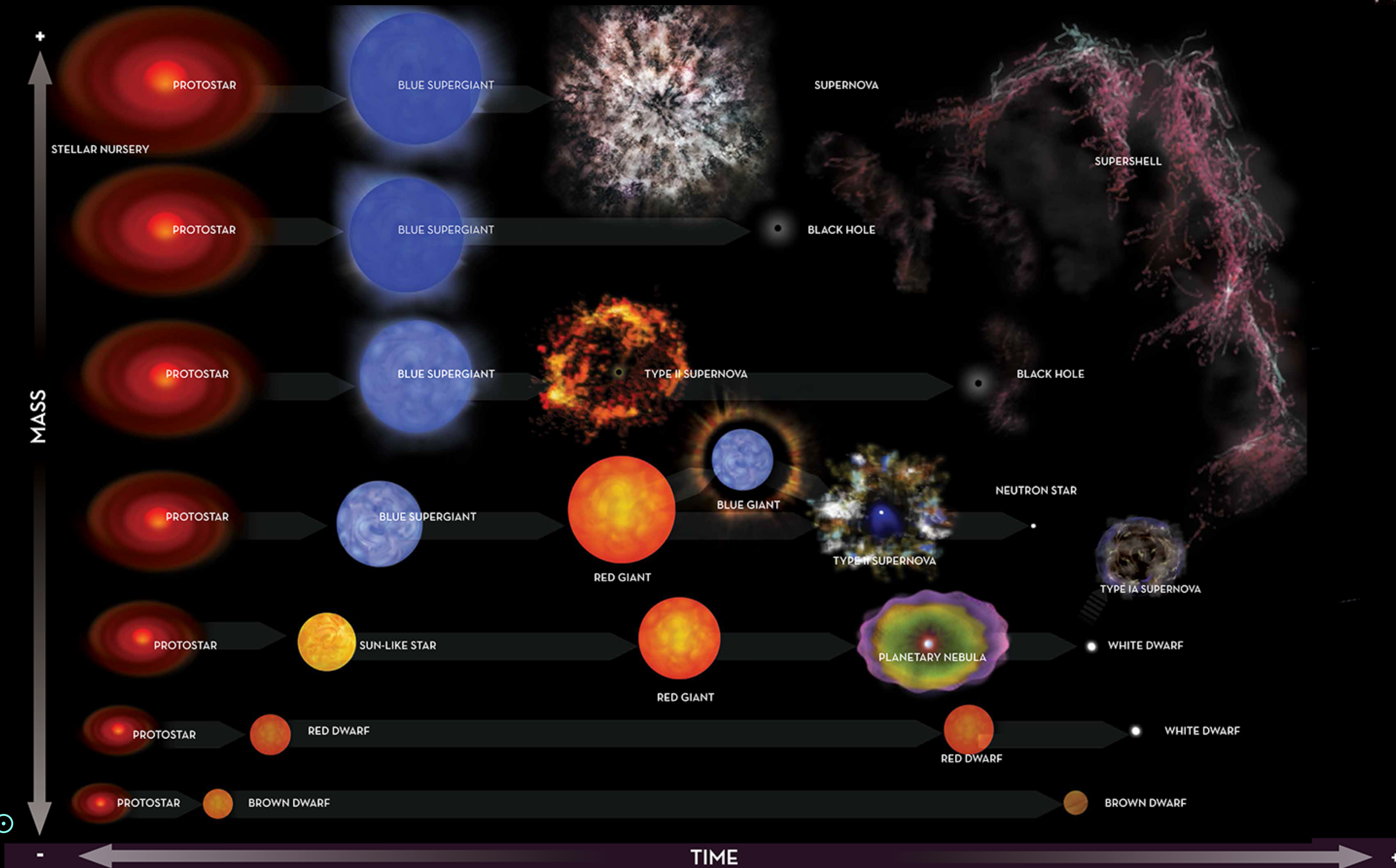
What is a star?

$\gtrsim 25\, M_\odot$

$\lesssim 25\, M_\odot$

$\lesssim 8\, M_\odot$

$\lesssim 0.08\, M_\odot$



What is MESA?

MESA is an extremely flexible code that aims to simulate most of the different phases of a star's life. And planets.

- It's a state of the art, 1D stellar evolution code.
- It's completely open source. Anyone can download the source code.
- It's modular, and easily customizable.
- It uses modern techniques for mass loss/gain, convection, diffusion etc.
- It employs up-to-date, wide-ranging, flexible, and independently useable microphysics modules.
- It is capable of calculating the evolution of stars in a wide range of environments, compact objects, giant planets, asteroseismology …

Equations of stellar structure
Hydrostatic equilibrium

MESA divides the star in shells along the radial direction and solves the equations of stellar structure at each step. Let's see the basic.

The outward force due to the pressure gradient within the star is exactly balanced by the inward force due to gravity.

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$$

where M_r is the cumulative mass inside the shell at and G is the gravitational constant. The cumulative mass increases with radius according to the mass continuity equation:

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

Equations of stellar structure
Energy equilibrium

But to solve these equations we need some more information about the equation of state and about the temperature.

In every shell, the difference between the energy flux coming in the shell and going out needs to be equal to the energy produced in the shell itself.

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \left(\epsilon - \epsilon_\nu\right)$$

where ϵ_ν is the luminosity produced in the form of neutrinos (which usually escape the star without interacting with ordinary matter) per unit mass. Outside the core of the star, where nuclear reactions occur, no energy is generated, so the luminosity is constant.

Equations of stellar structure
Energy equilibrium

The relation between the energy transport and the temperature gradient is decided by the amount of energy that needs to be transported and by the way it is transported

$$L_r(r) = -4\pi r^2 K_{th}(r) \frac{dT}{dr}$$

where K_{th} is thermal conductivity. K_{th} depends on the type of heat transfer. The heat transfer in a star can be of two types:

1. **Microscopic transfer:** the heat is carried by particles
    - *conduction*: electrons and ions
    - *radiative* transfer: photons
2. **Macroscopic transfer**: the heat is carried by *convection*.

Equations of stellar structure
Heat transfer

The thermal conductivity in case of radiative transfer is given by:

$$K_{th} = -\frac{16\sigma T^3(r)}{3\kappa\rho}$$

Convection is harder to treat as it is a turbulent phenomenon, and it arises when the temperature gradient is higher than the *adiabatic* gradient:

$$\left|\frac{dT}{dr}\right| > \left|\frac{dT}{dr}\right|_{ad} = \left(\frac{1-\gamma}{\gamma}\right)\frac{T}{P}\frac{GM_r\rho(r)}{r^2}$$

Equations of stellar structure
Missing ingredients

Let's summarize the equations:

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$$

$$\frac{dM_r}{dr} = 4\pi r^2\rho$$

Energy generation

Neutrino rates

$$\frac{dL_r}{dr} = 4\pi r^2\rho\left(\epsilon - \epsilon_\nu\right)$$

$$L_r(r) = -\frac{64\pi r^2\sigma T^3(r)}{3\kappa\rho}\frac{dT}{dr}$$

Opacities

Is it that easy?

The answer is, of course, not.

- The equation that we saw are for perfect equilibrium and they don't have time derivatives, but we know that stars change, so those equations do not always hold.
- MESA is a 1D code, but stars are not always spherical.
- The timescales that the code has to deal with are very different:
    1. nuclear timescale:

$$\tau_N = \frac{0.007 X M_\odot c^2}{L_\odot} \approx 6 \times 10^{10}\mathrm{yr}$$

    2. thermal timescale:

$$\tau_{KH} = \frac{3GM_\odot^2}{5R_\odot L_\odot} \approx 1.8 \times 10^{7}\mathrm{yr}$$

    3. dynamical timescale:

$$\tau_D = \sqrt{\frac{1}{G\rho}} = \sqrt{\frac{4\pi R_\odot^3}{3GM_\odot}} \approx 54 \text{ minutes}$$

Don’t stars always change slowly?

Stars usually change on the nuclear timescale, but

- Sometimes the nuclear timescale is short, even hours.
- Protostars have no nuclear reactions yet, so they change on a thermal timescale.
- White dwarfs no longer have nuclear reactions, so they change on a thermal timescale.
- Supernovae evolve on a dynamical timescale.
- For example, have you ever wondered why there are so few bright stars in the sky that are the same colour of the Sun?

Don't stars always change slowly?

- Most of the bright stars at night are giants: bluer or redder than the Sun but not the colour of the Sun.
- For stars more than 50% more massive than the Sun, when nuclear burning shuts off in the core, it shuts off entirely, and the star evolves on a thermal timescale and passes from blue-white to yellow in a few thousand years.