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Modelling the spin equilibrium of neutron stars in LMXBs without gravitational radiation

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ABSTRACT

In this paper we discuss the spin-equilibrium of accreting neutron stars in LMXBs. We demonstrate that, when combined with a naive spin-up torque, the observed data leads to inferred magnetic fields which are at variance with those of galactic millisecond radiopulsars. This indicates the need for either additional spin-down torques (eg. gravitational radiation) or an improved accretion model. We show that a simple consistent accretion model can be arrived at by accounting for radiation pressure in rapidly accreting systems (above a few percent of the Eddington accretion rate). In our model the inner disk region is thick and significantly sub-Keplerian, and the estimated equilibrium periods are such that the LMXB neutron stars have properties that accord well with the galactic millisecond radiopulsar sample. The implications for future gravitational-wave observations are also discussed briefly.

1 INTRODUCTION

In the last few years the evidence in favour of the notion that neutron stars are spun up to millisecond periods in accreting systems has strengthened significantly. The discovery of the millisecond X-ray pulsar SAX J1808.4-3658 in an accreting low-mass X-ray binary (LMXB) provided the long anticipated missing link between the general LMXB population and the millisecond radio pulsars. Since then four similar system has been observed, further strengthening the connection (Wijnands 2003). Furthermore, the link between the twin-peak separation of the kHz quasiperiodic oscillations (QPOs) seen in a number of systems and the spin of the neutron star has become somewhat clearer (although the underlying mechanism is still under debate) following the observation of QPOs in SAX J1808.4-3658. It now appears as if the QPO separation is either equal to or half the spin period (Miller 2003).

When the first indications of rapidly spinning neutron stars in LMXBs were discussed more than five years ago, the results suggested that the systems were clustered in a surprisingly narrow range of spin frequencies 250-370 Hz. As such spin rates are far below the predicted break-up limit of about 1 kHz, the data pointed towards the presence of a mechanism that could counteract the accretion spin-up torque. The obvious candidate — the interaction between the accretion disk and the magnetosphere of the neutron star — was discussed by White & Zhang (1997). Their results seemed to indicate the need for an unanticipated link between the accretion rate and the magnetic field strength. Since there is no reason to expect such fine-tuning in these systems, Bildsten (1998) argued that an additional spin-down mechanism may be in operation. He proposed that this torque could be provided by gravitational-wave emission, and that the required asymmetries would be induced in the neutron star crust by accretion. This idea echoed earlier suggestions by Papaloizou & Pringle (1978) and Wagoner (1984) that neutron stars may reach a spin-equilibrium with gravitational waves balancing the accretion torque.

The possibility that accreting neutron stars may radiate gravitational waves is of great interest given the generation of groundbased interferometers (LIGO, GEO600, TAMA300 and VIRGO) that is now reaching design sensitivity. It has been recognized that there are three distinct mechanisms that may be able to generate gravitational waves at the required rate. First of all, a more detailed study by Ushomirsky et al. (2000) suggests that the accretion induced crustal asymmetry proposed by Bildsten remains viable. The second possibility is that the stars spin fast enough that the gravitational-wave driven instability of the r-mode oscillations in the neutron star core is activated (see Andersson (2003) for references). Finally, Cutler (2002) has suggested that an internal toroidal magnetic field could lead to unstable free precession resulting in the star "tipping over" and becoming an orthogonal rotator, an efficient gravitational-wave source.

The present investigation is motivated by the following facts:

- The observational data has improved considerably since the original discussions in 1997-98. We now know that the LMXBs are not clustered in as narrow a range of spins as was originally thought, the current range being 250-620 Hz. It is relevant to ask to what extent the more recent data supports the need for an additional spin-down torque, eg. gravitational radiation, in these systems.
- A question that does not seem to have attracted much interest concerns whether a more refined model of the interaction between the accretion disk and the magnetosphere of the neutron star would be able to provide a satisfactory description of the LMXBs. After all, many important physical mechanisms were not accounted for in the analysis of White & Zhang (1997) and it may be wise not to refine the various gravitational-wave scenarios before their relevance is investigated.
- If we suppose that the LMXBs radiate gravitational waves at a significant level, then we need to address many difficult issues associated with the detection of such signals. A key issue concerns the spin-evolution of the system. Can we assume that the spin-period remains stable on a time-scale of a few months? After all, the signal needs to be integrated for at least two weeks in order to be detectable in the noisy data-stream. If the system tends to wander, as the data for slower spinning systems suggests (Bildsten et al. 1997), then we need to be able to model the accretion torque reliably.

In this paper we aim to address the second of these points. We discuss the argument that an additional spin-down torque is needed in the LMXBs, and provide a more detailed accretion model that is able to describe these systems without particular fine-tuning of the magnetic field. From this exercise we conclude that it may not be appropriate to assume that the neutron stars in LMXBs radiate gravitational waves at a rate that exactly balances the accretion spin-up torque expected for a non-magnetic star. We do not think this should be taken as meaning that these systems are irrelevant for gravitational-wave physics. The proposed mechanisms for generating gravitational radiation should certainly still work. Yet, our discussion makes it clear that modelling these systems is significantly more difficult than has been assumed so far (at least in the gravitational-wave community). Of course, by constructing a more detailed accretion model, we are beginning to address this issue.

2 LMXBS AND THE "STANDARD" ACCRETION MODEL

In the simplest models of accreting non-magnetic stars it is assumed that matter falling onto the surface of the star provides a torque proportional to the angular momentum associated with a Keplerian orbit at the stars equator;

$$N \approx \dot{M}\sqrt{GMR} \tag{1}$$

where M is the mass, R the radius and $\dot{M}$ the accretion rate. Despite it being well-known that this torque only provides an order-of-magnitude estimate, it has been used in most studies of gravitational waves from LMXBs so far. The line of reasoning has been that, if the neutron star is at spin equilibrium, then the radiated gravitational waves provide an equal and opposite torque. The strength of the gravitational waves can be inferred from the X-ray luminosity, since (assuming that the gravitational potential released by the infalling matter is radiated as X-rays)

$$L_X \approx \frac{GM\dot{M}}{R} \tag{2}$$

provides a link between the observations and the mass accretion rate.

Accretion onto a magnetised star is different since the pressure of the infalling gas is counteracted by the magnetic pressure. By balancing these two pressures (for spherical infall) one obtains the so-called magnetosphere radius

$$R_M \approx 7.8 \left(\frac{B_0}{10^8 \text{ G}}\right)^{4/7} \left(\frac{R}{10 \text{ km}}\right)^{12/7} \left(\frac{M}{1.4M_\odot}\right)^{-1/7} \left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{-2/7} \text{ km} \tag{3}$$

inside which with the flow of matter is likely to be dominated by the magnetic field. For a strongly magnetised star the magnetic field is expected to channel the accreting matter onto the polar caps (Frank et al. 2002), while the situation may be more complex for a weakly magnetised object.

An approximation of the maximum accretion rate we should expect follows from balancing the pressure due to spherically infalling gas to that of the emerging radiation. This leads to the Eddington limit;

$$\dot{M}_{\rm Edd} \approx 1.5 \times 10^{-8} \left(\frac{R}{10 \text{ km}}\right) \frac{M_\odot}{\text{yr}} \tag{4}$$

with associated X-ray luminosity

$$L_X \approx 1.8 \times 10^{38} \left(\frac{M}{1.4M_\odot}\right) \left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right) \text{ erg/s} \tag{5}$$

From these estimates we see that, for accretion at a fraction ϵ of the Eddington rate, eg. $\dot{M} = \epsilon \dot{M}_{\rm Edd}$, the magnetic field must be accounted for (in the sense that $R_M > R$) as long as it is stronger than

$$B_0 \geq 1.6 \times 10^8 \epsilon^{1/2} \mathrm{G} \tag{6}$$

Since observations indicate that rapidly rotating neutron stars have magnetic fields of the order of 10^8 G, and many transient LMXBs accrete with $\epsilon \sim 0.01$, we infer that the magnetic field is likely to play a role in these systems.

The interaction between a geometrically thin disk and the neutron star magnetosphere is a key ingredient in the standard model for accretion. The basic picture is that of a rotating magnetised neutron star surrounded by a magnetically threaded accretion disk, see Figure 2 for a schematic illustration. In the magnetosphere, accreting matter follows the magnetic field lines and gives up angular momentum on reaching the surface, exerting a spin-up torque. The material torque at the inner edge of the disk is usually approximated by

$$N = \dot{M}\sqrt{GMR_M} \tag{7}$$

It is important to note that this torque can be significantly stronger than the rough estimate for non-magnetic stars.

Meanwhile, outside the co-rotation radius,

$$R_c \approx 17 \left(\frac{P}{1 \text{ ms}}\right)^{2/3} \left(\frac{M}{1.4M_\odot}\right)^{1/3} \text{ km} \tag{8}$$

the field lines rotate faster than the local Keplerian speed, resulting in a negative torque. If $R_M > R_c$ the accretion flow will be centrifugally inhibited and matter may be ejected from the system. It is easy to see that this will happen if the spin period becomes very short, or the rate of flux of material onto the magnetosphere drops. This is known as the propeller regime. As accreting matter is flung away from the star in this phase, the star experiences a spin-down torque. To account for this effect we alter the material torque according to

$$N = \dot{M}R_M^2[\Omega_\mathrm{K}(R_M) - \Omega] = \dot{M}\sqrt{GMR_M}\left[1 - \left(\frac{R_M}{R_c}\right)^{3/2}\right] \tag{9}$$

where Ω is the spin frequency of the star and and Ω_K is the angular velocity of a particle in a Keplerian orbit;

$$\Omega_\mathrm{K}(r) = \left(\frac{GM}{r^3}\right)^{1/2} \tag{10}$$

Even though this expression only accounts for the propeller regime in a phenomenological way, it agrees with the expectation that accretion will not spin the star up beyond the point $R_M = R_c$. This leads to the equilibrium period

$$P_\mathrm{eq} \approx 0.30 \left(\frac{B_0}{10^8 \text{ G}}\right)^{6/7} \left(\frac{R}{10 \text{ km}}\right)^{18/7} \left(\frac{M}{1.4M_\odot}\right)^{-5/7} \left(\frac{\dot{M}}{\dot{M}_\mathrm{Edd}}\right)^{-3/7} \text{ ms} \tag{11}$$

Conversely, given an observed spin period we can (assuming that the system is at equilibrium) deduce the neutron star's magnetic field.

Let us now compare this estimate with the observational data, summarised in Table 1. To do this we need both stellar spin rates and an estimate of accretion rates. Let us first consider spin rates. For the X-ray pulsars, we will use the measured pulsar frequency ν_psr. For those sources that are not pulsars but have burst oscillations, we will assume that the measured burst oscillation frequency ν_burst is the stellar spin frequency. For the third class of sources, which exhibit neither pulsations nor burst oscillations, we will use the separation of the kHz QPOs, $\Delta\nu_\mathrm{QPO}$, as an estimate of the stellar spin. As can be seen from those pulsars and burst oscillation sources that also have kHz QPOs, this estimate is not exact, as the spin frequency can be as much as double the kHz QPO separation. In addition the kHz QPO separation is variable. For the purposes of this paper, however, we will estimate stellar spin as the midpoint of the observed range of $\Delta\nu_\mathrm{QPO}$ for all sources that do not show burst oscillations or pulsations. This places an upper limit on the inferred magnetic field. The inferred field would be lower if the true spin rate were greater than $\Delta\nu_\mathrm{QPO}$.

The accretion rate can be estimated from the X-ray luminosity, which can be highly variable. It is clear that the estimated equilibrium period is shortest when the accretion rate is highest (alternatively for a given spin rate the inferred magnetic field is maximal). In this paper we will assume that the observed spin rate is the equilibrium period associated with the maximum accretion rate for a given source, even for sources that are transient or highly variable[1]. This make sense if one assumes that the main contribution to the spin-up torque is associated with the phase when the star accretes at the fastest rate. Hence, the accretion rates given in Table 1 are estimates of maximum accretion rates. In addition we will assume $R = 10$ km and $M = 1.4M_\odot$ for all systems.

Figure 1 compares the model's predictions for LMXB magnetic fields with the inferred magnetic fields for the millisecond radio pulsars. The agreement is good for LMXBs accreting at the level of $10^{-2}\dot{M}_\mathrm{Edd}$ and below. The model does not, however, perform well for systems accreting with $\dot{M} \approx \dot{M}_\mathrm{Edd}$. The figure shows that the estimated magnetic fields appear to be too large for the systems accreting near the Eddington limit, mainly objects for which the spin rate was inferred from the kHz

[1] See however Lamb & Yu (2004) for a discussion of whether neutron stars do reach equilibrium.

Source	Source type	$\nu_{\rm psr}$ (Hz)	$\nu_{\rm burst}$ (Hz)	$\Delta\nu_{\rm QPO}$ (Hz)	$\dot{M}/\dot{M}_{Edd}$ (%)
SAX J1808.4-3658	P(T)	401 [1]	401 [2]	$\sim$ 200 [3]	4 [4]
XTE J1751-305	P(T)	435 [5]			11 [4]
XTE J0929-314	P(T)	185 [6]			3 [4]
XTE J1807-294	P(T)	191 [7]		$\sim$ 190 [8]	2 [4]
XTE J1814-338	P(T)	314 [9]	314 [10]		4 [4]
IGR J00291+5934	P(T)	599 [11]			5 [12]
4U 1608-522	A(T)		619 [13]	225–325 [14]	60 [15]
SAX J1750.8-2980	A(T)		601 [16]	$\approx$ 317 [17]	10 [15]
4U 1636-536	A		582 [18]	242–323 [19]	16 [15]
MXB 1658-298	U(T)		567 [20]		10 [15]
Aql X-1 (1908+005)	A(T)		549 [21]		50 [15]
KS 1731-260	A(T)		524 [22]	250–270 [23]	40 [15]
SAX J1748.9-2021	U(T)		410 [24]		25 [15]
4U 1728-34	A		363 [25]	274–350 [26]	7 [15]
4U 1702-429	A		330 [27]	328–338 [27]	6 [28]
4U 1916-053	A		270 [29]	290,348 [30]	7 [30,31]
GX 340+0 (1642-455)	Z			280–410 [32]	$\sim$ 100 [28]
Cyg X-2 (2142+380)	Z			346 [33]	$\sim$ 100 [28]
4U 1735-44	A			296–341 [34]	15 [28]
4U 0614+09	A			240–360 [35]	1 [28]
GX 5-1 (1758-250)	Z			232–344 [36]	$\sim$ 100[28]
4U 1820-30	A			230–350 [37]	30 [28]
Sco X-1 (1617-155)	Z			240–310 [38]	$\sim$ 100 [28]
GX 17+2 (1813-140)	Z			239–308 [39]	$\sim$ 100 [28]
XTE J2123-058	A(T)			255–275 [40,41]	16 [40,41]
GX 349+2 (1702-363)	Z			266 [42]	$\sim$ 100 [28]

Table 1. Data for rapidly rotating neutron stars (with spins above 100 Hz), with references given in square brackets. Source type classifications are P (pulsar), A (Atoll), Z (Z source) or U (Unknown) (Hasinger & van der Klis 1989; van der Klis 2004). (T) indicates that the source is transient. The frequencies given are pulsar spin frequency ($\nu_{\rm psr}$), burst oscillation frequency ($\nu_{\rm burst}$) and separation between the two kHz Quasi-Periodic Oscillations ($\Delta\nu_{\rm QPO}$). The accretion rates shown are estimates of maximum accretion rate, as discussed in the main text. References: [1] Wijnands & van der Klis (1998), [2] Chakrabarty et al. (2003), [3] Wijnands et al. (2003), [4] Galloway et al. (2004), [5] Markwardt et al. (2002), [6] Remillard et al. (2002), [7] Markwardt et al. (2003), [8] C.B.Markwardt, private communication, [9] Markwardt & Swank (2003), [10] Strohmayer et al. (2003), [11] Markwardt, Swank & Strohmayer (2004), [12] Galloway et al. (2005) [13] Hartman et al. (2003),[14] Mendez et al. (1998), [15] D.K.Galloway, private communication, [16] Kaaret et al. (2002), [17] Natalucci et al. (1999), [18] Giles et al. (2002), [19] Jonker et al. (2002), [20] Wijnands et al. (2001), [21] Zhang et al. (1998),[22] Smith et al. (1997), [23] Wijnands & van der Klis (1997), [24] Kaaret et al. (2003), [25] Strohmayer et al. (1996), [26] Migliari et al. (2003), [27] Markwardt et al. (1999), [28] Ford et al. (2000), [29] Galloway et al. (2001), [30] Boirin et al. (2000), [31] Smale et al. (1988), [32] Jonker et al. (2000), [33] Wijnands et al. (1998), [34] Ford et al. (1998), [35] van Straaten et al. (2003), [36] Jonker et al. (2002),[37] Zhang et al. (1998), [38] Mendez & van der Klis (2000), [39] Homan et al. (2002), [40] Homan et al. (1999), [41] Tomsick et al. (1999), [42] Zhang et al. (1998)

QPO separation. This is, essentially, the conclusion drawn by Bildsten (1998). There seems to be a need for an additional spin-down torque in the systems that accrete at near-Eddington rates. Note that, if we had assumed that the true spin-rate was twice $\Delta\nu_{\rm QPO}$, as indicated in some of the systems exhibiting burst oscillations, then the inferred magnetic field would be roughly half those indicated, which would still be problematic.

3 A MAGNETICALLY THREADED DISK

The interaction between an accretion disk and a spinning compact object involves much poorly known physics. The key issues were discussed in a number of seminal papers in the late 1970s (Ghosh et al. (1977); Ghosh & Lamb (1978, 1979a,b), see also Frank et al. (2002) for an excellent introduction). Although much effort has been invested in this area of research since then — after all, accretion is a cornerstone of astrophysics — these early papers remain the "standard" description of the problem.

In this Section we will focus on the contribution to the accretion torque from a magnetically threaded, thin disk. Our description is based on the work by Wang (1987, 1995) and Yi et al. (1997) (see also Yi & Wheeler (1998) and Yi & Grindlay (1998)).

We begin by pointing out that our previous description of the accretion problem was somewhat inconsistent since our various estimates, eg., of the size of the magnetosphere, were based on spherical infall of matter. The model can be improved, albeit at the cost of introducing several largely unknown parameters. First of all, we need a description of the viscosity in

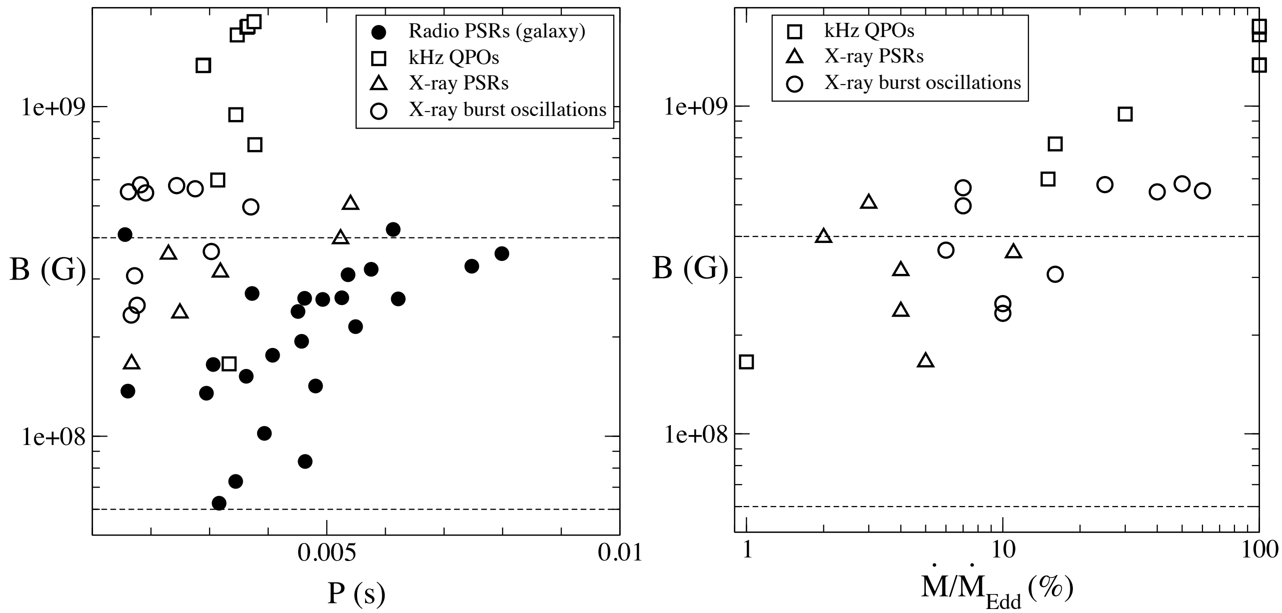

Figure 1. Comparing the neutron stars in LMXBs to the millisecond radio pulsar population. We include all millisecond radiopulsars (with periods below 10 ms) in the galaxy. Millisecond pulsar in globular clusters are excluded since a significant sample of them are seen to spin up, an effect likely due to motion relative to the core of the globular cluster (Phinney 1993). which makes the magnetic fields inferred for them dubious. In the left panel we compare the inferred magnetic field for the galactic millisecond pulsars, to those inferred for accreting neutron stars using the simplest estimate for the spin-equilibrium [$B(P_{\rm eq})$ is inferred from Eq. (11)]. The radio pulsars are shown as filled circles, systems showing burst oscillations are represented by open circles, data from systems where the spin period is estimated from the kHz QPO separation are open squares and the accreting X-ray pulsars are shown as open triangles. We also indicate the (rough) range of magnetic fields for the galactic radio pulsars $6 \times 10^7 - 4 \times 10^8$ G. The right panel relates the inferred magnetic fields for the accreting systems to the accretion rate (in % of the Eddington rate). This figure indicates that the fields are most seriously overestimated for the fastest accreting systems. [Radio pulsar data taken from the radio pulsar catalogue http://www.atnf.csiro.au/research/pulsar/psrcat/. Accreting neutron star data determined from Table 1.]

the disk. Viscosity is the main agent that dissipates energy and angular momentum, and thus enables matter to flow towards the central object. Since the microphysical viscosity (likely due to the magnetorotational instability in some form) is difficult to characterise, it is common to use the so-called α-viscosity introduced by Shakura & Sunyaev (1973), i.e. let the kinematic viscosity be parametrised as

$$\nu = \alpha c_s H = \alpha \frac{c_s^2}{\Omega_{\rm K}} \tag{12}$$

Here c_s is the sound speed in the disk and $H \sim c_s/\Omega_{\rm K}$ is the vertical scale height. In this description, ν is a function of r since both c_s and $\Omega_{\rm K}$ vary with position, but α is taken to be constant. In effect, this leads to a model where the viscosity ensures that the disk remains Keplerian as matter and angular momentum is transferred through the disk.

In the case of a magnetically threaded disk, we need to provide a description of the interaction between the disk flow and the magnetic field. Figure 2 provides a schematic illustration of the problem. To provide a detailed model of this complicated physics problem is, however, not a simple task. Nevertheless, one may hope that a somewhat simplistic description will be able to capture the main features of the complete problem.

From the φ-component of the Euler equations for the disk flow we can estimate the radius at which magnetic stresses balance the material stresses. We thus find

$$\dot{M} \frac{d}{dr}\left[\Omega_{\rm K}(r) r^2\right] = -r^2 B_\varphi B_z \tag{13}$$

where the mass transfer rate $\dot{M}$ will be assumed constant throughout the disk. This relation illustrates the difficulty involved in constructing a consistent model. If we consider a thin accretion disk, then the z-component of the magnetic field can be taken to be that associated with a rotating dipole (with dipole moment μ)

$$B_z = -\frac{\mu}{r^3} = -B_0 \left(\frac{R}{r}\right)^3 \tag{14}$$

where B_0 is the surface field of the star. Even though the field may be much more complicated close to the stellar surface the dipole contribution will dominate far away. The problem is associated with B_φ. This component, which vanishes in the

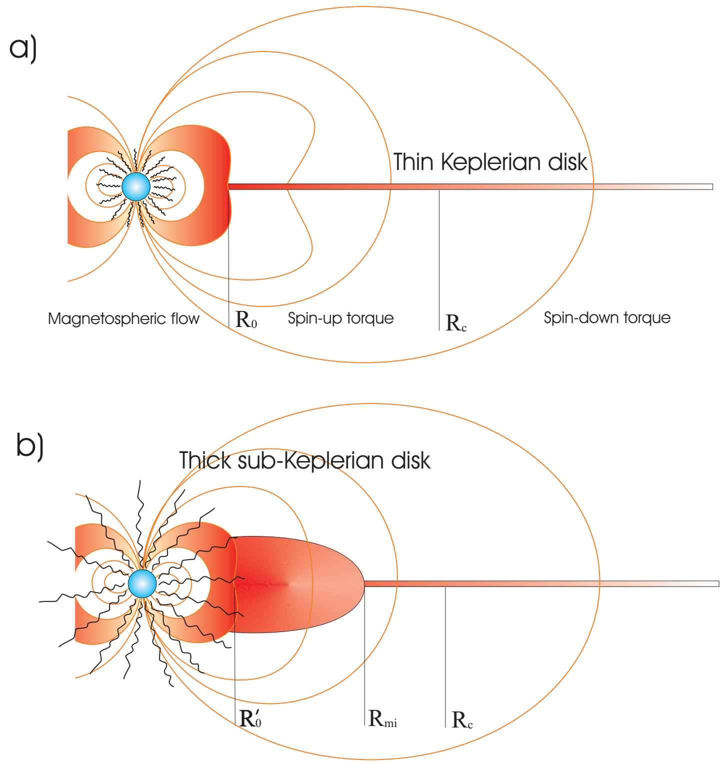

Figure 2. A schematic illustration of the accretion problem for a magnetically threaded disk. a) The standard thin disk picture, see Frank et al. (2002). b) The proposed model for rapidly accreting systems. Radiation pressure leads to a thick, sub-Keplerian disk in the inner region (between R_{mi} and R_0').

absence of a disk, represents the degree to which the magnetic field is dragged along with the matter flow. It is this interaction which leads to the torque on the star that we are aiming to model.

From the MHD induction equation we find that (Mestel 2004)

$$\partial_t B_\varphi \approx \frac{B_\varphi}{\tau_\varphi} \approx \nabla \times (\vec{v} \times \vec{B}) = \gamma(\Omega - \Omega_{\rm K}) B_z \tag{15}$$

where the star (and the magnetic field) is rotating at the constant rate Ω. In this equation, it is assumed that the disk flow changes from quasi-rigid to Keplerian over a lengthscale R/γ, with $\gamma \geq 1$. Wang (1995) has considered several different mechanisms for the timescale τ_φ (and by implication the toroidal component of the magnetic field). He concludes that the various models lead to quite similar predictions for the accretion torque. This is fortunate, since it means that the model is not very sensitive to the unknown physics. Here we will assume that the main mechanism that prevents the field from being dragged along with the flowing matter is turbulent diffusion. This leads to (Wang 1995),

$$\tau_\varphi \approx \frac{H}{\alpha c_s} \approx \frac{1}{\alpha \Omega_{\rm K}} \tag{16}$$

and consequently

$$B_\varphi \approx \frac{\gamma}{\alpha} \frac{\Omega - \Omega_{\rm K}}{\Omega_{\rm K}} B_z \tag{17}$$

We can now return to Eq. (13) and determine the "inner" edge of the accretion disk R_0, at which the matter flow departs significantly from a Keplerian profile;

$$\left(\frac{R_0}{R_c}\right)^{7/2} = \frac{2N_c}{\dot{M}\sqrt{GMR_c}} \left[1 - \left(\frac{R_0}{R_c}\right)^{3/2}\right] \tag{18}$$

where R_c is the co-rotation radius and we have defined

$$N_c = \frac{\gamma}{\alpha} \frac{\mu^2}{R_c^3} = \frac{\gamma}{\alpha} B_z^2 R_c^3 = \frac{\gamma}{\alpha} B_0^2 \frac{R^6}{R_c^3} \tag{19}$$

Where B_z is the magnetic field at R_c and B_0 is the field at the surface of the star as before.

We can also account for the torque due to the magnetically threaded disk outside R_0. The corresponding torque follows (essentially) from integrating Eq. (13) and we get

$$N_{\rm disk} = -\int_{R_0}^{\infty} B_\varphi B_z r^2 dr = -\frac{N_c}{3}\left[2\left(\frac{R_c}{R_0}\right)^{3/2} - \left(\frac{R_c}{R_0}\right)^3\right] \tag{20}$$

As discussed previously, the region $R_0 < r < R_c$ contributes a (positive) spin-up torque, while the region $R_c < r < \infty$ provides a (negative) spin-down torque, cf. Figure 2.

Finally, assuming that the matter gives up all its angular momentum (relative to the frame of the star) upon reaching R_0, i.e. that the matter flows along the field lines like “beads on a wire” in the region where the magnetic field dominates the flow[2] , we find that the total accretion torque is

$$N = \dot{M}\sqrt{GMR_0}\left[1 - \left(\frac{R_c}{R_0}\right)^{3/2}\right] + N_{\rm disk} = \frac{1}{3}\dot{M}\sqrt{GMR_0}\left[\frac{7/2 - 7(R_0/R_c)^{3/2} + 3(R_0/R_c)^3}{1-(R_0/R_c)^{3/2}}\right] \tag{21}$$

This result shows that the system reaches spin-equilibrium ($N = 0$) when

$$\left(\frac{R_0}{R_c}\right)^{3/2} = \frac{7-\sqrt{7}}{6} \longrightarrow R_0 \approx 0.8 R_c \tag{22}$$

This should be compared to the result of Wang (1995). The difference arises from the fact that Wang uses Eq. (7) rather than Eq. (9) for the material torque at the inner edge of the disk (now at R_0 instead of R_M).

Having added the spin-down torque exerted on the star by the outer parts of the disk we find that the system reaches equilibrium slightly before R_0 reaches R_c. Nevertheless, the predicted spin-period at equilibrium

$$P_{\rm eq} \approx 0.44\left(\frac{\alpha}{\gamma}\right)^{-3/7}\left(\frac{B_0}{10^8\ {\rm G}}\right)^{6/7}\left(\frac{R}{10\ {\rm km}}\right)^{18/7}\left(\frac{M}{1.4M_\odot}\right)^{-5/7}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{-3/7}\ {\rm ms} \tag{23}$$

does not differ much from the more naive prediction provided by Eq. (11). Of course, the actual spin period at equilibrium now depends explicitly on the ratio α/γ. Unfortunately, both these parameters are largely unknown. In addition, there are many uncertainties (at the level of factors of order unity) in the model.

In order to proceed we note that the viscosity parameter α is usually assumed to lie in the range $0.01 - 0.3$ (Frank et al. 2002), while γ has been assumed to be of order unity (Wang 1987). If we consider values in this range, what does the model imply for the magnetic fields of the accreting LMXB neutron stars? From Eq. (23) we find that a canonical neutron star will have equilibrium of 3 ms if

$$B_0 \approx 9.4\times 10^8\left(\frac{\alpha}{\gamma}\right)^{1/2}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{1/2}\ {\rm G} \tag{24}$$

We see that for $\alpha/\gamma \approx 0.1$ a star accreting at the Eddington rate is predicted to have a magnetic field within the range deduced for the millisecond radiopulsars. On the other hand, a star accreting at 1% of this rate will require a larger value of order $\alpha/\gamma \approx 1$ in order to lie in the range indicated in Figure 1. This means that the inclusion of the torques from a magnetically threaded thin disk is, in principle, sufficient to remove the direct need for an addition spin-down mechanism like gravitational radiation in these systems. Of course, this is achieved at the cost of introducing the poorly constrained parameters α and γ. If we want to adjust these parameters in such a way that the inferred magnetic fields agree with those for the radio pulsars in Figure 1 we essentially need to introduce a suitable $B_\varphi = B_\varphi(\dot{M})$. Despite this possibility, we do not think that the thin-disk model is entirely satisfactory. As we will argue in the next section, additional physics should be included in order to describe the fastest accreting systems. In essence, this means that we will only rely on the thin disk model for systems accreting below a few percent of the Eddington rate. From the above estimates we see that these systems are adequately described if we take $\alpha/\gamma \approx 1$. Hence, this will be our canonical value from now on.

4 THICK DISKS NEAR EDDINGTON ACCRETION

The thin disk model we have discussed so far is able to explain many features of accreting neutron star systems. Yet we will see that it cannot be relied upon for rapidly spinning stars accreting near the Eddington limit. Given this, it is meaningful to ask what the crucial missing piece of physics is. At this point, the most naive assumption in our discussion concerns the accretion torque arising from the inner edge of the disk, at R_0. While it seems reasonable to assume that the matter moves along the magnetic field lines in the inner region for low rates of accretion, it is not so clear that this model will work for faster accretors. Several mechanisms may alter the picture. Obvious possibilities are: radiation pressure from the emerging X-rays, the near balance between centrifugal and gravitational forces for rapidly spinning stars, heating of the disk in the inner region etcetera.

[2] In reality the problem is expected to be significantly more complicated, with the answer depending on the detailed physics in an extended transition region.

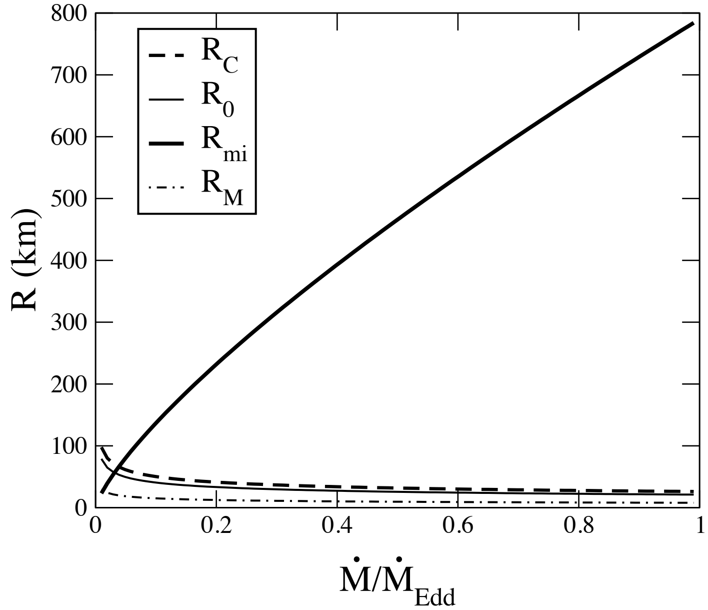

Figure 3. The main lengthscales in the accretion problem. The relevant parameters are taken to be $\alpha/\gamma = 1$, $B_0 = 10^8$ G and the star is assumed spin at a rate corresponding to equilibrium for thin disk accretion. The standard magnetosphere radius (for spherical accretion) R_M is shown as a thin dash-dotted line, and the corresponding radius for a thin magnetically threaded disk R_0 is a thin solid line. The co-rotation radius R_c, at which a Keplerian disk co-rotates with the star, is a thick dashed line. Finally, the distance at which radiation pressure balances gas pressure R_{mi} is shown as a thick solid line. The figure shows clearly that $R_{mi} >> R_c > R_0 > R_M$ above a few percent of the Eddington accretion rate. This suggests that radiation pressure must be accounted for, likely leading to a thickening of the disk and a sub-Keplerian flow in the inner region.

As a first stab at including these effects we will consider the radiation pressure. One can show that radiation pressure balances the gas pressure at a radius (Frank et al. 2002; Padmanabhan 2001)

$$R_{mi} = 880\alpha^{2/21} \left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{16/21} \left(\frac{M}{M_\odot}\right)^{1/3} f^{64/21} \text{ km} \tag{25}$$

where

$$f = \left[1 - \left(\frac{R}{r}\right)^{1/2}\right]^{1/4} \tag{26}$$

Strictly speaking, this result holds only for non-magnetic disks, but one can argue that it remains a good approximation also in the magnetic case (Campbell & Heptinstall 1998). Moreover, it is easy to show that the factor involving f will be near unity apart from in the absolute vicinity of the stellar surface. Hence, we can use

$$R_{mi} \approx 880\alpha^{2/21} \left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{16/21} \left(\frac{M}{1.4M_\odot}\right)^{1/3} \text{ km} \tag{27}$$

as a good approximation. Let us contrast this to the standard radius of the magnetosphere, R_M. We find that $R_{mi} = R_M$ when

$$\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right) \approx 2 \times 10^{-2} \alpha^{-1/11} \left(\frac{B_0}{10^8 \rm G}\right)^{6/11} \tag{28}$$

The key lengthscales in the problem are illustrated in Fig. 3. This figure shows that, for a neutron star with a weak magnetic field (a typical millisecond pulsar) radiation pressure will be important for accretion rates above a few percent of the Eddington rate.

We thus have to ask how the radiation pressure affects the model outlined in the previous section. Phenomenologically, the disk is likely to expand leading to the flow becoming sub-Keplerian. In fact, that the thin disk model is unstable in a region where the radiation pressure dominates the gas pressure was demonstrated a long time ago by Lightman & Eardley (1974) (see also Shapiro et al. (1976)). In order to account for this quantitatively, let us consider the following model. The thin disk description is relevant outside R_{mi}, and hence describes systems accreting below the critical rate. For faster accretors, there will exist an inner region inside R_{mi} where the disk is no longer thin. To model this region we follow Yi et al. (1997), and assume that the flow is such that $\Omega = A\Omega_{\rm K}$ with $A \leq 1$ [3]. Before moving on we should note that the study of Yi et al. (1997) pertains to advection dominated accretion below a critical accretion rate, while our model concerns radiation pressure dominated disks above a critical accretion rate. This may seem a cause for concern, especially since advection dominated flows are almost exclusively used in discussions of slowly accreting systems. However, as pointed out by Narayan & Yi (1994), the corresponding solution to the equations describing the accretion problem is likely to be relevant also for rapid accretion. Furthermore, the model is sufficiently simple to serve our present purposes. A more detailed analysis that supports the basic principles behind our model has been carried out by Campbell & Heptinstall (1998).

Repeating the arguments from the thin disk analysis, we find a new co-rotation radius $A^{2/3}R_c$ and the inner edge of the thick disk region R_0' is now determined from

$$\left(\frac{R_0'}{A^{2/3}R_c}\right)^{7/2} = \frac{2N_c'}{A^{4/3}\dot{M}\sqrt{GMR_c}}\left[1-\frac{1}{A}\left(\frac{R_0'}{R_c}\right)^{3/2}\right] \tag{29}$$

where we have defined

$$N_c' = \frac{\gamma}{\alpha}\frac{\mu^2}{A^2R_c^3} = \frac{\gamma}{\alpha}B_z^2A^2R_c^3 \tag{30}$$

The torque from the inner disk region follows from

$$N_{\rm thick} = -\int_{R_0'}^{R_{mi}} B_\varphi B_z r^2 dr = \frac{\mu^2\gamma}{\alpha}\int_{R_0'}^{R_{mi}}\frac{1}{r^4}\left[1-\frac{1}{A}\left(\frac{r}{R_c}\right)^{3/2}\right]dr \tag{31}$$

while the outer (thin) disk contributes a torque

$$N_{\rm thin} = -\int_{R_{mi}}^{\infty} B_\varphi B_z r^2 dr = \frac{\mu^2\gamma}{\alpha}\int_{R_{mi}}^{\infty}\frac{1}{r^4}\left[1-\left(\frac{r}{R_c}\right)^{3/2}\right]dr \tag{32}$$

Working out the algebra, we find that the total torque can be written

$$N = \dot{M}\sqrt{GMR_0'}\frac{A}{1-\bar{\omega}}\left\{\frac{7}{6}-\frac{7\bar{\omega}}{3}+\bar{\omega}^2+\frac{A(1-A)}{3}\left(\frac{R_c}{R_{mi}}\right)^{3/2}\bar{\omega}^2\right\} \tag{33}$$

where

$$\bar{\omega} = \frac{1}{A}\left(\frac{R_0'}{R_c}\right)^{3/2} \tag{34}$$

In this slightly more complicated model, the system will reach spin-equilibrium when

$$\frac{7}{6}-\frac{7\bar{\omega}}{3}+\bar{\omega}^2+\frac{A(1-A)}{3}\left(\frac{R_c}{R_{mi}}\right)^{3/2}\bar{\omega}^2 = 0 \tag{35}$$

Since we must have $R_0' < R_{mi}$ we are always interested in the smallest of the two roots to this quadratic. The problem simplifies considerably if we note that

$$\frac{R_c}{R_{mi}} \approx 2\times10^{-2}\alpha^{-2/21}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{-16/21}\left(\frac{P}{1\text{ ms}}\right)^{2/3} \tag{36}$$

for a canonical neutron star, cf. Fig. 3. This means that, for a sizeable fraction of the Eddington accretion rate and millisecond spin periods, we have equilibrium when

$$\bar{\omega} \approx \frac{7-\sqrt{7}}{6} \longrightarrow R_0' \approx 0.8A^{2/3}R_c \tag{37}$$

From this we can infer the spin period at equilibrium;

$$P_{\rm eq} \approx 0.44A^{-10/7}\left(\frac{\alpha}{\gamma}\right)^{-3/7}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{-3/7}\left(\frac{B_0}{10^8\text{ G}}\right)^{6/7}\left(\frac{M}{1.4M_\odot}\right)^{-5/7}\left(\frac{R}{10\text{ km}}\right)^{18/7}\text{ ms} \tag{38}$$

This result differs from the thin-disk model only by the factor of A. However, it is easy to see that this is a key factor which may lead to considerable differences in the predicted equilibrium spin periods.

To complete the thick disk model, we need to estimate the coefficient A which describes the nature of the sub-Keplerian flow. To do this, we consider the radial Euler equation which (for a thin disk) can be approximated by (Frank et al. 2002)

[3] This model should provide an acceptable representation of the inner disk flow, but there are important caveats: All radiation dominated configurations tend to be subject to thermal and convective instabilities and hence may not be stationary, see eg. Szuszkiewicz & Miller (2001) for discussion.

$$v_r \frac{\partial v_r}{\partial r} - \frac{v_\varphi^2}{r} \approx -\frac{1}{\rho}\frac{\partial}{\partial r}\left(p + \frac{B^2}{8\pi}\right) - \frac{GM}{r^2} + \frac{B_\varphi^2}{4\pi\rho r} \tag{39}$$

This equation will remain approximately relevant in the case of a thick disk provided that it is interpreted as a height average (Narayan & Yi 1995a,b). Apart from very near the Eddington accretion rate the dominant velocity component is v_φ. The situation near $\dot{M}_{\rm Edd}$ is complicated by the fact that the matter in the disk becomes highly virialised. In our thick disk model, we expect the radiation pressure to dominate in the region $R_0' < r < R_{mi}$. (It is worth noting the difference between the radial and azimuthal Euler equations here. In the latter the axisymmetric radiation pressure will not play a role and the magnetic and viscous stresses dominate.)

We express the radiation pressure gradient in terms of the co-moving radiation flux L_{co} (Miller 1990; Mitra 1992)

$$\frac{dp_{\rm rad}}{dr} = -\frac{\kappa\rho}{c}\frac{L_{co}}{4\pi r^2} \tag{40}$$

where κ is the opacity of the matter. Since the Eddington luminosity follows from

$$L_{\rm Edd} = \frac{4\pi GMc}{\kappa} \tag{41}$$

we have

$$\frac{dp_{\rm rad}}{dr} = -\rho\frac{GM}{r^2}\frac{L_{co}}{L_{\rm Edd}} \tag{42}$$

Using this relation in Eq. (39) we see that the velocity profile becomes sub-Keplerian with

$$v_\varphi \approx A\sqrt{\frac{GM}{r}} \qquad \text{where} \qquad A = \sqrt{1 - \frac{L_{co}}{L_{\rm Edd}}} \tag{43}$$

As a rough approximation we can assume that the co-moving flux is equal to the stationary flux observed at infinity $L_X = GM\dot{M}/r$ where r is the distance to the source. Then

$$\frac{L_{co}}{L_{\rm Edd}} \approx \frac{\dot{M}}{\dot{M}_{\rm Edd}} \tag{44}$$

and we see that

$$v_\varphi \approx Ar\Omega_K \quad \text{with} \quad A = \sqrt{1 - \frac{\dot{M}}{\dot{M}_{\rm Edd}}} \tag{45}$$

(effects due to eg. beaming have obviously not been included in this estimate).

The results we obtain by combining this approximation with the predicted equilibrium period for the thick disk model are illustrated in Figure 4. This figure shows that the thick disk model leads to significantly longer equilibrium spins for rapidly accreting systems. Conversely, we can use Eq. (38) to deduce a system accreting at 90% of the Eddington rate and which is observed to spin with a 3 ms period, should have a magnetic field of $B \approx 1.4 \times 10^8$ G. A field of this strength would put this system well within the range of fields inferred for the millisecond radio pulsars, cf. Figure 4. The figure shows that our thick disk model leads to predicted magnetic fields for the LMXBs which accord well with those of the galactic millisecond radio pulsars. (In order to infer the magnetic fields shown in Figure 4 we have assumed that the fastest accreting systems have $\dot{M} = 0.95\dot{M}_{\rm Edd}$. This is somewhat ad hoc, but it should be noted that the model breaks down, in the sense that $A \to 0$ which leads to $P_{\rm eq}$ diverging, as $\dot{M} \to \dot{M}_{\rm Edd}$. There is also significant uncertainty in the accretion rates given in Table 1.)

The fact that radiation pressure will affect accretion disk structure, and hence the spin period of neutron stars in LMXBs, has previously been discussed by several authors (White & Stella (1988); Ghosh & Lamb (1991, 1992), see also Miller et al. (1998); Psaltis & Chakrabarty (1999)). These models, like ours, give lower inferred magnetic fields for high accretion rate sources when radiation pressure is taken into account. One issue associated with the previous models is that if one assumes spin equilibrium, the models predict a strong correlation between magnetic field and accretion rate (see for example Eq. (25) of Miller et al. (1998)). No direct measurements of LMXB magnetic fields have yet been made, so this correlation cannot be tested, but the physical basis for such a strong relation is at best unclear. In fact, this has been one of the arguments against magnetic spin equilibrium models (Bildsten 1998; Ushomirsky et al. 2000). The model outlined in this paper also predicts a correlation between magnetic field and accretion rate. The "dependence" of B on $\dot{M}$ is however weaker, due to the dependence on accretion rate of the factor A. This illustrates that small modifications to the accretion model may be able to remove some of the perceived difficulties associated with magnetic equilibrium models.

We conclude this section with a brief discussion of the observational consequences of this model with regard to the detection of X-ray pulsars. Naively one expects the X-ray pulsars to have higher inferred magnetic fields than the other, non-pulsing, LMXBs (Cumming et al. 2001). As is clear from Figure 4, the thick disk model does not lead to the pulsars clustering at higher magnetic fields than the other sources. One possibility, suggested by Titarchuk et al. (2002), is that we are prevented

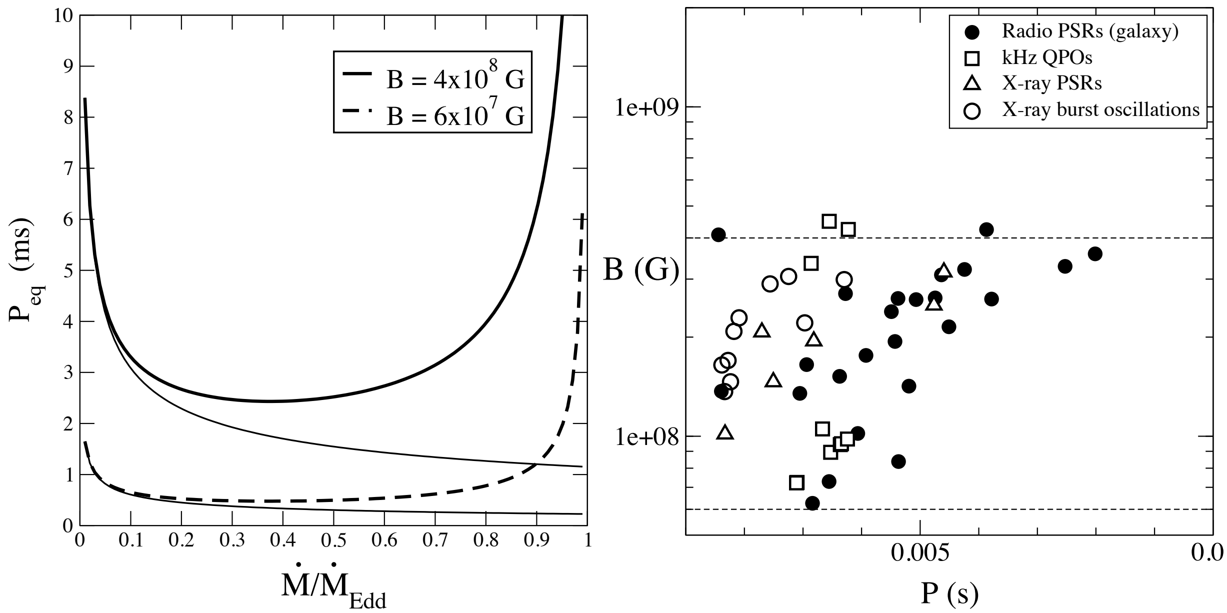

Figure 4. The predicted spin periods at equilibrium for the thick disk model, for $\alpha = \gamma = 1$. In the left panel we show $P_{\rm eq}$ as function of the accretion rate for magnetic fields which bracket the range for the millisecond radio pulsars: $B = 6 \times 10^7$ G (thick dashed curve) and $B = 4 \times 10^8$ G (thick solid curve). For comparison we also show the prediction of the naive model where spinup ceases at $R_c = R_M$ (thin solid curve). The right panel compares the inferred magnetic fields for LMXBs to those of the radio pulsars and should be compared to the right panel in Figure 1.

from seeing pulsations in many systems due to atmospheric scattering. A preliminary study by Krauss & Chakrabarty (2004) suggests that the scattering hypothesis may not be borne out by the data, but this is an area of ongoing research.

5 CONCLUSIONS

We have discussed the accretion spin-equilibrium for neutron stars in LMXBs. The outcome of this study is a more detailed model of the accretion torques and an appreciation that it is possible to construct a reasonably simple and consistent model for these systems without invoking additional spin-down torques due to, for example, gravitational radiation. This result is not particularly surprising. After all, the accretion problem is extremely complex (Frank et al. 2002), and the torques considered in the studies that argued for the need for an additional spin-down mechanism (see White & Zhang (1997) and Bildsten (1998)) were somewhat simplistic.

Of course, our results should not be taken as proof that the LMXBs do not radiate gravitational waves. The various proposed mechanisms for generating asymmetries in rapidly spinning, accreting neutron stars remain (essentially) as viable as before. The key difference is that we have eliminated the rationale for locking the gravitational radiation luminosity to the non-magnetic torque $\dot{M}\sqrt{GMR}$, which has been used as an order of magnitude estimate in most studies to date. In our picture, one would not be able to infer how the spin down due to gravitational radiation combines with the accretion torque from the observed spin periods. This alleviates some "problems" with the gravitational-wave models. In the case of accretion induced asymmetries in the crust (Bildsten 1998), one can show that the quadrupole deformation required to balance accretion is

$$\epsilon \approx 10^{-7} \left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{1/2} \left(\frac{P}{3 \text{ ms}}\right)^{5/2} \tag{46}$$

This should be compared to the maximum deformation that the crust can sustain, which according to Ushomirsky et al. (2000) can be approximated as

$$\epsilon_{\rm max} < 5 \times 10^{-7} \left(\frac{u_{\rm break}}{10^{-2}}\right) \tag{47}$$

where the breaking strain $u_{\rm break}$ is usually (based on results for terrestrial materials) assumed to be in the range $10^{-4} - 10^{-2}$. These estimates show that the breaking strain must be near the upper limit of the expected range in order for these asymmetries to balance near Eddington accretion in a star spinning at a period of a few milliseconds. By weakening the accretion torque, while at the same time not altering the mechanism generating the asymmetry (eg. the accretion rate), this issue is made less critical.

Our results also impacts the suggestion that the gravitational waves are emitted by unstable r-mode oscillations in the stellar fluid. In this case, the r-modes are expected to become unstable below a critical rotation period $P_{\rm crit}$. The point at which the instability becomes relevant depends on many complicated issues concerning viscosity, superfluidity etcetera (see Andersson (2003) for a discussion) but it is plausible that $P_{\rm crit} \approx 2-3$ ms. In the context of the present model, we obviously need $P_{\rm crit} > P_{\rm eq}$ in order for the r-mode instability to be relevant. Considering the results illustrated in the left panel of Figure 4, we expect that the instability may come into operation in weak magnetic field systems which are neither very slow nor very fast accretors.

The most important next step in modelling the LMXBs concerns the variability in the spin with varying accretion rate. The spin of accreting X-ray pulsars is known to vary considerably (Bildsten et al. 1997) on a timescale which is roughly similar to the variations in the accretion rate. But the data also suggests that there may not be a direct link between increased X-ray flux and an increase in the spin-up torque. It is important to understand this variability in general. This is also a very important issue for attempts to search for gravitational waves from the LMXBs. Any variability on timescales shorter than the observation time that remains unaccounted for will likely lead to a significant loss in signal-to-noise ratio. Our aim is to turn our attention to this challenging problem in the near future.

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