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Super-Eddington accretion discs around Kerr black holes

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ABSTRACT

We calculate the structure of the accretion disc around a rapidly rotating black hole with a super-Eddington accretion rate. The luminosity and height of the disc are reduced by the advection effect. In the case of large viscosity parameter, $\alpha > 0.03$, the accretion flow deviates strongly from thermodynamic equilibrium and overheats in the central region. With increasing accretion rate, the flow temperature steeply increases, reaches maximum, and then falls off. The maximum is achieved in the advection-dominated regime of accretion. The maximum temperature in the disc around a massive black hole of $M = 10^8\ \mathrm{M}_\odot$ with $\alpha = 0.3$ is of order 3×10^8 K. The discs with large accretion rates can emit X-rays in quasars as well as in galactic black hole candidates.

Key words: accretion, accretion discs – black hole physics – hydrodynamics – radiation mechanisms: thermal – relativity.

1 INTRODUCTION

Black hole accretion discs are believed to be the mechanism of energy release in active galactic nuclei and in some X-ray binaries. Their luminosity is constrained by the Eddington limit, $L_\mathrm{E} = 4\pi G M m_\mathrm{p} c/\sigma_\mathrm{T}$ (M is the black hole mass), and the accretion rate, $\dot{M}$, is usually expressed in the Eddington units, $\dot{m} = \dot{M}c^2/L_\mathrm{E}$. The luminosity becomes equal to the Eddington limit at $\dot{m}_\mathrm{cr} = \eta^{-1}$, where η is the radiative efficiency of the disc. Accretion rates of order $\dot{m}_\mathrm{cr}$ are likely to feed the huge energy release in quasars, and $\dot{m} \sim \dot{m}_\mathrm{cr}$ is implied in many models of the soft X-ray excess observed in quasar spectra (e.g. Czerny & Elvis 1987; Dörrer et al. 1996; Szuszkiewicz, Malkan & Abramowicz 1996).

According to the standard model (Shakura 1972; Shakura & Sunyaev 1973), the effective temperature of the disc surface scales as $T^\mathrm{s}_\mathrm{eff} \propto (\dot{m}/M)^{1/4}$. A special feature of discs with $\dot{m} \sim \dot{m}_\mathrm{cr}$ is that their temperature can be much greater than $T^\mathrm{s}_\mathrm{eff}$ provided the viscosity parameter α is large enough (Shakura & Sunyaev 1973). In the inner region of such a disc the inflow time-scale is shorter than the time-scale for relaxation to thermodynamic equilibrium, and the accretion flow is overheated. The overheating might be strong enough for the transition from the standard radiation-dominated disc to the hot two-temperature ion-pressure-supported regime of accretion proposed by Shapiro, Lightman & Eardley (1976). This transition has been discussed in recent works (Liang & Wandel 1991; Artemova et al. 1996; Björnsson et al. 1996).

The standard model can be a good approximation only if $\dot{m} < \dot{m}_\mathrm{cr}$. When the accretion rate approaches $\dot{m}_\mathrm{cr}$ the main assumption of the model, that the viscously released energy is radiated away locally, becomes inadequate. Inside some radius r_t, the radiation produced is trapped by the flow and advected eventually to the black hole instead of being radiated away. This radius can be estimated by taking the internal energy density, ε, and the density, ρ, of the disc from Shakura & Sunyaev (1973). Then one finds that the radial flux of internal energy $= \dot{M}\varepsilon/\rho$ exceeds the total flux of radiation emitted between r and $2r$ at

$$r_\mathrm{t} \approx \dot{m} r_\mathrm{g} \sqrt{1 - \left(\frac{3 r_\mathrm{g}}{r_\mathrm{t}}\right)^{1/2}}, \tag{1}$$

where $r_\mathrm{g} = 2GM/c^2$ is the gravitational radius. For $\dot{m} = \dot{m}_\mathrm{cr} = 12$ equation (1) yields $r_\mathrm{t} \approx 7.1 r_\mathrm{g}$. A substantial portion of the released energy must be swallowed by the black hole even when $\dot{m} < \dot{m}_\mathrm{cr}$. On the other hand, the relative height (H/r) of the Shakura–Sunyaev disc equals $\dot{m}/27$ at the maximum. Advection therefore becomes essential before the accretion flow becomes quasi-spherical, and this effect can be investigated in an extended version of the standard model, retaining the vertically integrated approximation. The corresponding set of equations was proposed

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by Paczyński & Bisnovatyi-Kogan (1981). Numerical solution of these equations gives an adequate description for the inner transonic edge of the accretion flow and shows that the disc remains relatively thin at moderately super-Eddington accretion rates (Abramowicz et al. 1988; Chen & Taam 1993). This made natural the extension of the standard model to the super-Eddington advection-dominated regime, the so-called 'slim' accretion disc.

All the models of the super-Eddington slim disc employed the pseudo-Newtonian approximation to the black hole gravitational field (Paczyński & Wiita 1980) which is good only in the case of a Schwarzschild black hole. Recently, the equations of a relativistic slim disc have been derived by Lasota (1994, see also Abramowicz et al. 1996) and applied to another type of advection-dominated accreton flow, optically thin and hot.

In the present paper we solve the relativistic equations of the super-Eddington disc. In the case of a rapidly rotating black hole the relativistic effects become especially important. The sonic radius is typically inside the ergosphere, close to the black hole horizon. The released power increases steeply when the hole spin approaches its extreme value, $a_*=Jc/M^2G \rightarrow 1$ (J is the angular momentum of the black hole). In this paper we calculate the disc structure for the case $a_*=0.998$ and compare it with the case of a non-rotating black hole, $a_*=0$.

We pay particular attention to discs with a large viscosity parameter α. In the previously investigated pseudo-Newtonian models, the super-Eddington disc was assumed to be in thermodynamic equilibrium, which is a good approximation in the case of small α. We find that a significant deviation from equilibrium occurs when $\alpha \gtrsim 0.03$. Then the flow overheats in the central region. We find that the temperature of the flow rises steeply at $\dot{m} > \dot{m}_{\rm cr}$ and reaches a maximum in the advection-dominated regime of accretion. This maximum temperature is especially high in the case of a rapidly rotating black hole.

In the next section, we write down the equations of the relativistic advective disc. These equations describe the radiation-dominated flow and neglect the thermal pressure of the accreting plasma. Their solution yields the radiation density, plasma density and height of the disc within the equipartition radius of the standard model, $r_{\rm eq}$, at which the radiation pressure equals the plasma pressure. The transition from the standard model to advective accretion occurs at $r_{\rm t} \ll r_{\rm eq}$. We describe the numerical method, present the solution, and discuss the structure of the disc in Section 3. Then, in Section 4, we calculate the temperature of the accreting plasma. Finally, we check that inside $r_{\rm eq}$ our radiation-dominated models never approach the ion-pressure-supported regime of accretion. The results are summarized in Section 5.

2 RELATIVISTIC DISC EQUATIONS

In the description of the gas flow around a Kerr black hole we use the Boyer–Lindquist coordinates $x^i=(t, r, \theta, \varphi)$. The metric tensor g_{ij} of Kerr space–time is given e.g. by Misner, Thorne & Wheeler (1973). The disc is assumed to lie at the equatorial plane of the Kerr geometry. The four-velocity of the accreting gas has components $u^i=(u^t, u^r, u^\theta, u^\varphi)$ in the Boyer–Lindquist coordinates, and $u^\theta=0$. The angular velocity of the gas rotation equals $\Omega=u^\varphi/u^t$, and the Lorentz factor measured in the frame of local observers with zero angular momentum equals $\gamma=u^t(-g^{tt})^{-1/2}$.

To obtain the simplest model of the relativistic disc with advection we employ the vertically integrated (slim) approximation. The slim disc equations are written for the following thermodynamic quantities: surface rest-mass density Σ, surface energy densty U (that includes both the rest mass energy Σc^2 and internal energy Π) and vertically integrated pressure P. The dimensionless specific enthalpy is defined as $\mu=(U+P)/\Sigma c^2$. The equation of state in the radiation-dominated disc is $\Pi \approx 3P$. F^+ denotes the surface rate of viscous heating, and F^- denotes the radiation flux radiated from both faces of the disc. All the thermodynamic quantities and both fluxes F^-, F^+ are measured in the comoving frame.

The main equations express the conservation laws for baryon number, energy, angular momentum and radial momentum. We include in the equations the inertial mass associated with internal energy accumulated in the flow (Beloborodov, Abramowicz & Novikov 1997).

(i) *Baryon conservation*

$$2\pi r c u^r \Sigma = -\dot{M} \tag{2}$$

(ii) *Conservation of angular momentum*

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[\mu\left(\frac{\dot{M}u_\varphi}{2\pi}+2\nu\Sigma r\sigma^r_\varphi\right)\right]=\frac{F^-}{c^2}\, r u_\varphi, \tag{3}$$

where ν is the kinematic viscosity, and σ^r_φ is the shear,

$$\sigma^r_\varphi=\frac{1}{2}\, g^{rr} g_{\varphi\varphi}\sqrt{-g^{tt}}\,\gamma^3\,\frac{\mathrm{d}\Omega}{\mathrm{d}r}.$$

(iii) *First law of thermodynamics*

$$F^+-F^-=cu^r\left(\frac{\mathrm{d}\Pi}{\mathrm{d}r}-\xi\,\frac{\Pi+P}{\Sigma}\,\frac{\mathrm{d}\Sigma}{\mathrm{d}r}\right), \tag{4}$$

where $\xi \approx 1$ is a numerical factor accounting for non-homogeneity of the disc in the vertical direction. Hereafter we set $\xi=1$. The surface heating rate is given by

$$F^+=2\nu\Sigma\mu\sigma^2c^2, \quad \sigma^2=\frac{1}{2}\, g^{rr} g_{\varphi\varphi}(-g^{tt})\gamma^4\left(\frac{\mathrm{d}\Omega}{\mathrm{d}r}\right)^2.$$

(iv) *Radial Euler equation*

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}r}(u_r u^r)=-\frac{1}{2}\frac{\partial g_{\varphi\varphi}}{\partial r}\, g^{tt}\gamma^2(\Omega-\Omega_{\rm K}^+)(\Omega-\Omega_{\rm K}^-)$$

$$-\frac{1}{c^2\Sigma\mu}\frac{\mathrm{d}P}{\mathrm{d}r}-\frac{F^+u_r}{c^3\Sigma\mu}, \tag{5}$$

where $\Omega_{\rm K}^\pm$ are the Keplerian angular velocities,

$$\Omega_{\rm K}^\pm=\pm\frac{c}{r(2r/r_g)^{1/2}\pm(a_* r_g/2}.$$

The set of disc structure equations becomes closed when the viscosity, ν, and radiative cooling, F^-, are specified. The standard α-prescription for viscosity is $\nu=\alpha c_s H$, where α is a constant and $c_s=c(P/U)^{1/2}$ is the isothermal sound speed. The half-thickness of the disc, H, should be estimated from the vertical balance condition.

(v) *Vertical balance*

Near the black hole the relativistic effects become important and the tidal force contracting the disc in the vertical direction depends on Ω. At $\Omega=\Omega_K^+$ the vertical tidal acceleration in the comoving tetrade equals (e.g. Riffert & Herold 1995)

$$a_{(\theta)}=\frac{zr_g(r^2-a_* r_g\sqrt{2r_g r}+0.75a_*^2 r_g^2)}{r^3(2r^2-3r_g r+a_* r_g\sqrt{2r_g r})}=\frac{zr_g}{2r^3}J(a_*, r), \tag{6}$$

where $z=\sqrt{g_{\theta\theta}}(\theta-\pi/2)$ is the height in the disc and J is the relativistic correction factor becoming unity at $r\gg r_g$. Then the typical half-thickness of the disc can be estimated as

$$H^2=\frac{P}{U}\frac{2r^3}{r_g J}. \tag{7}$$

More accurate expression takes into account the deviation $\Delta\Omega=\Omega-\Omega_K^+$ of gas rotation from Keplerian (Abramowicz, Lanza & Percival 1997). We will consider only the disc region outside the sonic radius where the correction to estimation (7) connected with $\Delta\Omega$ does not exceed several per cent, and hereafter we use this estimation.

(vi) *The radiative losses*

The time-scale for photon diffusion from the interior of the disc to its surface equals $t_D=H\tau_0/c$ where $\tau_0=\sigma_T\Sigma/2m_p$. This is the typical leaking time for radiation trapped inside the disc, and the radiative losses can be written as

$$F^-=\chi\frac{m_p c\Pi}{\sigma_T\Sigma H}, \tag{8}$$

where $\chi\sim 1$ is a numerical factor. In the standard model with vertical structure governed by radiation diffusion this factor equals $2/\sqrt{3}$. In the advection-dominated region, t_D exceeds the inflow time-scale t_a and the detailed treatment of stationary diffusion is not relevant: $t_a<t_D$ means that the gas accretes faster than a stationary vertical distribution of trapped radiation could establish. However, the estimation (8) gives a right limit $F^-\ll F^+$ at $r\ll r_t$. Indeed, when $t_a\ll t_D$ we have $\Pi\approx F^+t_a$, hence $F^-/F^+\approx t_a/t_d\ll 1$. The radiative losses are 'turned off' inside r_t, and the exact value of F^- is not important for the hydrodynamical behaviour of the flow. A detailed calculation of F^- would require 2D simulation of the radiation diffusion in the disc with a specified vertical distribution for the viscous energy release.

For definiteness we hereafter choose $\chi=2/\sqrt{3}$ in equation (8).

(vii) *Global energy conservation*

The luminosity of the disc is related to the accretion rate by (Beloborodov et al. 1997)

$$L^-=-\frac{2\pi}{c}\int_{r_s}^{\infty}u_t F^- r\,dr\approx\dot{M}c^2\left(1+\mu_s\frac{u_t^s}{c}\right), \tag{9}$$

where index 's' refers to the inner transonic edge of the disc. The advection effect means that the luminosity is less than the total power released in the disc,

$$L^+=-\frac{2\pi}{c}\int_{r_s}^{\infty}u_t F^+ r\,dr.$$

The power 'swallowed' by the black hole equals $L_{adv}=L^+-L^-$.

3 NUMERICAL SOLUTION

To solve numerically the disc structure equations, we choose three independent variables Ω, c_s and $\zeta=\mu(2\nu\Sigma r\sigma_\varphi^r+\dot{M}u_\varphi/2\pi)$. From equations (3) and (4) we have

$$\frac{d\zeta}{dr}=\frac{F^-}{c^2}ru_\varphi, \tag{10}$$

$$\frac{P}{\Sigma^2}\frac{d\Sigma}{dr}-3\frac{d}{dr}\left(\frac{P}{\Sigma}\right)=\frac{2\pi rc}{\dot{M}}(F^+-F^-). \tag{11}$$

Expressing u_φ, $F^\pm$, P and Σ in terms of Ω, c_s and ζ we get two differential equations for the three unknowns. The third differential equation is the radial equation (5). We solve the equations (5), (10) and (11) with external boundary conditions, c_s^{out}, Ω_{out}, ζ^{out} at radius r_{out} such that $r_t\ll r_{out}\ll r_{eq}$. The values of c_s^{out} and $\Omega_{out}=\Omega_K^+$ are taken from the relativistic version of the standard model (Novikov & Thorne 1973; Page & Thorne 1974) with corrected vertical balance (Riffert & Herold 1995). The value of ζ^{out} is the eigenvalue of the problem, see below.

We use the relaxation technique. As a first approximation we assume $\Omega(r)=\Omega_K^+(r)$ and solve the equations (10) and (11) for c_s and ζ. Substituting this solution into the radial equation (5), we calculate $\Omega'(x)$, needed to fulfill this equation. Then we perform the relaxation step, changing $\Omega(r)$ to $\Omega(r)+\varepsilon\delta\Omega(r)$ where $\delta\Omega=\Omega'-\Omega$ and $\varepsilon<1$ is the relaxation parameter. With new $\Omega(r)$ we solve again the equations (10) and (11), and so on until $\delta\Omega$ converges to zero at all r. This method is numerically unstable unless some smoothing procedure is used to suppress the numerical oscillations in $\delta\Omega(r)$. We have found the needed procedure as a combination of the two-point and three-point smoothing, and determined $\Delta\Omega=\Omega-\Omega_K^+$ with accuracy $\sim$0.1 per cent.

This technique allows us to obtain the solution of equations (5), (10) and (11) for any given ζ^{out}. Then we adjust ζ^{out} to fulfill the regularity condition at the sonic radius, r_s, and find the transonic solution. The regularity condition can be written as $N=D=0$ where N and D are the numerator and denominator in the radial equation with explicitly expressed derivative of velocity, $du^r/dr=N/D$. In this way, we find r_s with $\sim$1 per cent accuracy and calculate the disc structure at $r>r_s$ (inside r_s the gas is almost in free-fall).

Example solutions outside r_s are shown in Fig. 1 for the cases $a_*=0$ and $a_*=0.998$ (the represented magnitudes are independent of the black hole mass). In these models $\alpha=0.1$. To a good accuracy, the solutions with different α' can be obtained from the solutions with $\alpha=0.1$ using the simple scaling law $\Sigma'=\Sigma\alpha/\alpha'$, $P'=P\alpha/\alpha'$, with the same $H(r)$, $\Delta\Omega(r)$ and $c_s(r)$. Essential deviation from this law appears only when $\alpha\to 1$. Then the sonic radius moves outwards, being well beyond the radius of the Keplerian marginally stable orbit, $r_{\rm ms}$. At small α, $r_{\rm s} < r_{\rm ms}$.

The relativistic version of the standard model is shown by dashed lines in Fig. 1. The accretion flow deviates from this model at the trapping radius $r_{\rm t}$. Inside $r_{\rm t}$ the condition $\dot{M}c^2\mu > 2\pi r^2 F^-$ takes place, which means that the radial flux of internal energy exceeds the local radiative losses, i.e. the flow is advection-dominated. The trapping radius is practically independent of α; the dependence on $\dot{m}$ is given later (in Fig. 3). From Fig. 1 one can see that (i) the advection effect reduces the height H below the standard value, so that the relative disc height, H/r, is kept modest even at $\dot{m} \gtrsim 10\dot{m}_{\rm cr}$, (ii) the Thomson optical depth of the disc, τ_0, has a minimum $\sim 100(\alpha/0.1)^{-1}$ at $\dot{m} \sim \dot{m}_{\rm cr}$, and (iii) the maximum deviation from Keplerian rotation is ~ 20 per cent.

The critical accretion rate $\dot{m}_{\rm cr}$ equals 17.5 in the case of $a_* = 0$ and 3.11 in the case of $\alpha_* = 0.998$. At $\dot{m} > \dot{m}_{\rm cr}$ a substantial fraction of the total energy released in the disc is advected into the black hole; see Fig. 2.

Note that in the standard disc with $\dot{m} < \dot{m}_{\rm cr}$ the density $n = \tau_0/\sigma_{\rm T} H \propto \dot{m}^{-2}$. In the advection-dominated regime

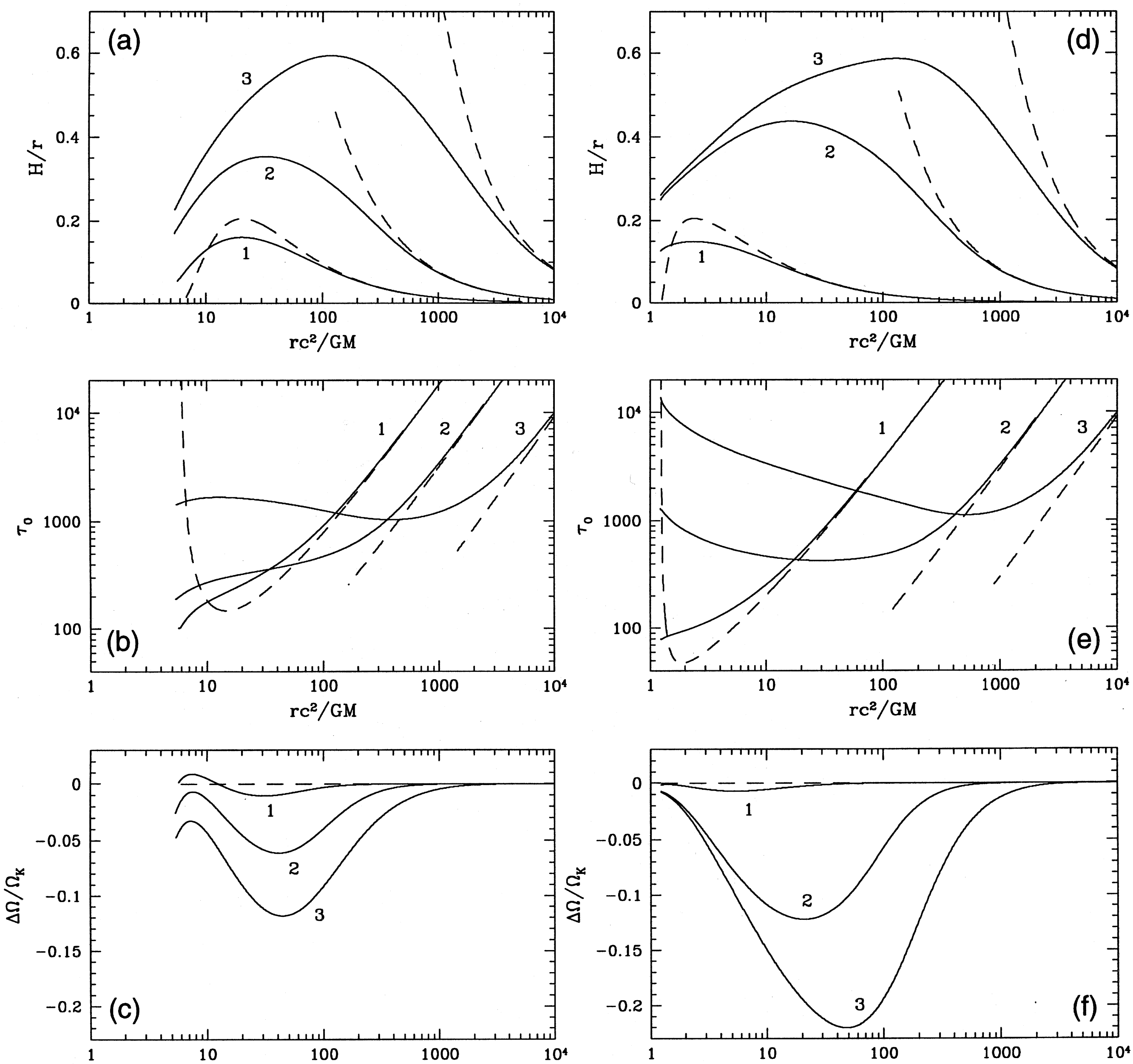

Figure 1. The structure of the disc around a black hole with $a_* = 0$ (panels a–c) and $a_* = 0.998$ (panels d–f) in the case of viscosity parameter $\alpha = 0.1$. The upper panels show the relative height of the disc, the panels in the middle show the Thomson optical depth of the flow, and the bottom panels show the deviation from Keplerian rotation. All these quantities are independent of the black hole mass. Curves with different numbers correspond to different accretion rates: $1 - \dot{m} = \dot{m}_{\rm cr}$ ($\dot{m}_{\rm cr} = 17.5$ in the case of $a_* = 0$ and $\dot{m}_{\rm cr} = 3.11$ in the case of $a_* = 0.998$), $2 - \dot{m} = 100$, $3 - \dot{m} = 1000$. The dashed lines display the relativistic standard model.

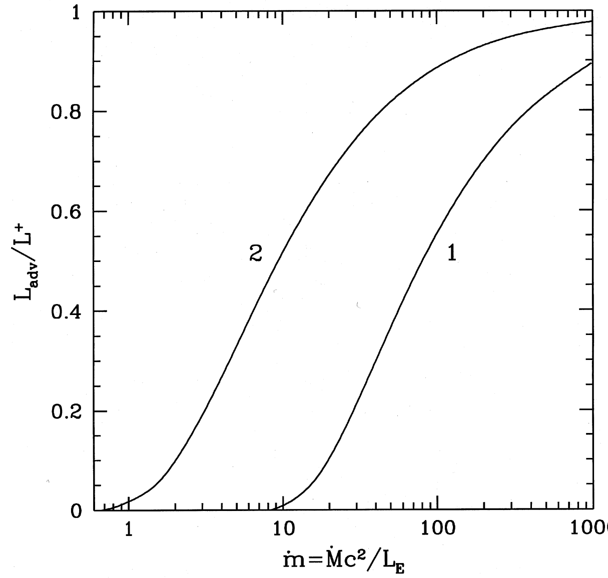

Figure 2. The fraction of the total power released in the disc that is eventually advected into the black hole [see item (vii) of Section 2]. Curves 1 and 2 correspond to $a_*=0$ and $a_*=0.998$ respectively.

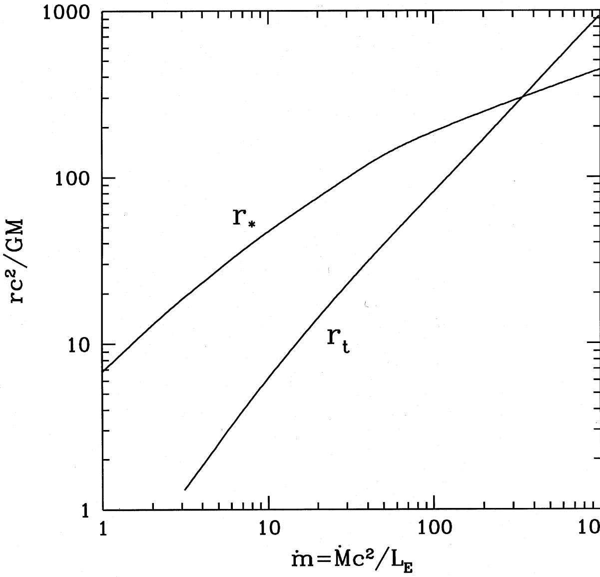

Figure 3. The radius of the overheated region r_* versus $\dot{m}$ in the case of $\alpha=0.3$, $M=10^8\,\mathrm{M_\odot}$, $a_*=0.998$. For comparison the trapping radius r_t is also plotted.

($\dot{m}>\dot{m}_{cr}$, $r\ll r_t$), the density scales as $\dot{m}$. It means that at $\dot{m}\sim\dot{m}_{cr}$ there must be a minimum of the density. At this minimum strong overheating of the flow is possible, as discussed in the next section.

4 OVERHEATING IN THE CENTRAL REGION

The accretion disc can be considered as a two-fluid flow 'plasma + radiation'. In the case of small viscosity, the disc is dense and the plasma and radiation are close to thermodynamic equilibrium at common temperature $T\approx T_{eff}=(w/a_r)^{1/4}$ where $w=3P/2H$ is the radiation density calculated in Section 3, and $a_r=7.56\times10^{-15}$ is the radiation constant. [Note that T_{eff} inside the disc differs from the surface effective temperature $T^s_{eff}=(F^-/2\sigma)^{1/4}$ where $\sigma=a_rc/4$ is the Stefan–Boltzmann constant.] However, when $\alpha>0.03$ and $\dot{m}$ approaches $\dot{m}_{cr}$ a strong deviation from equilibrium occurs in the inner region of the disc. In this case the accreting plasma has low density and therefore low emission capability. As a result the plasma does not manage to reprocess the released energy into Planckian radiation, and the balance between heating and heat removal through radiative cooling maintains temperature $T\gg T_{eff}$. Then the radiation is concentrated in the Wien peak of temperature T and its density w is much less than the corresponding Planckian value $w_{pl}=a_rT^4$, i.e. the flow is far from the blackbody state.

4.1 Radius of the overheated region

In the outer 'blackbody' region of the disc the free–free emission capability $\dot{w}_{ff}$ of the plasma with $T=T_{eff}$ exceeds the heating rate $\dot{w}^+=F^+/2H$. The boundary r_* between the blackbody and the overheated regions can be estimated from the condition

$$\dot{w}^+=\dot{w}_{ff}(T_{eff})\quad\text{at}\quad r=r_*,\tag{12}$$

$$\dot{w}_{ff}=1.6\times10^{-27}n^2\sqrt{T}.$$

In the case of exact thermodynamic equilibrium free–free emission would be balanced by free–free absorption, and $\dot{w}_{ff}$ can be written as $\dot{w}_{ff}=cn\bar{\sigma}_{ff}w_{pl}$. $\bar{\sigma}_{ff}$ introduced in this way has the meaning of the effective cross-section for absorption of the Planckian radiation; it is ≈4 times the Rosseland-averaged σ_{ff}.

Radius r_* is usually estimated from the condition that the effective optical depth of the flow $\tau_*=(\tau_0\tau_{ff})^{1/2}=1$ where $\tau_{ff}=Hn\bar{\sigma}_{ff}$ (Rosseland σ_{ff} is also often taken instead of $\bar{\sigma}_{ff}$). For the standard disc this condition is equivalent to equation (12). In that case, the radiation density inside the disc equals $w\approx\dot{w}^+t_D$ where $t_D=\tau_0H/c$ is the diffusion time-scale. Then equality (12) with $w\approx w_{pl}$ gives $\tau_*\approx1$, and the decoupling of w below w_{pl} happens if $\tau_*<1$. The condition $\tau_*<1$ means that the deviation from thermodynamic equilibrium in the standard disc occurs if the bulk of the radiation diffuses out without absorption and re-emission.

This condition, however, is not relevant in the advection-dominated regime. In this case, the radiation is advected rather than escaping, and the time-scale for free–free absorption should be compared with the inflow time-scale rather than the diffusion time-scale. Then the condition $\tau_*<1$ should be replaced by $\tau_*<(t_D/t_a)^{1/2}\sim r_t/r$, which follows from the equation (12).

We have calculated r_* from equation (12) for the case $\alpha=0.3$, $M=10^8\,\mathrm{M_\odot}$, $a_*=0.998$. In Fig. 3, r_* is shown versus $\dot{m}$ and compared with the trapping radius r_t. One can see that the entire advection-dominated region is overheated at $\dot{m}<300$.

4.2 Cooling rate inside r_*

The overheated plasma inside r_* is cooled by Comptonized free–free emission with rate

$$\dot{w}_{\rm cool}=A\dot{w}_{\rm ff}, \qquad r\ll r_*, \tag{13}$$

where $A(n, T)$ is the amplification factor resulting from Compton upscattering of photons emitted at low energies $h\nu<kT$. This factor is given by (e.g., Rybicki & Lightman 1979)

$$A=1+\tfrac{3}{4}\ln^2 x_{\rm coh}, \tag{14}$$

where $x_{\rm coh}=h\nu_{\rm coh}/kT$ is the dimensionless photon energy below which the absorption time-scale is shorter than the time-scale for shifting in frequency resulting from Comptonization. $x_{\rm coh}$ is determined by the equation $\sigma_{\rm ff}(x_{\rm coh})=8\theta\sigma_{\rm T}$, where $\theta=kT/m_{\rm e}c^2$ and $\sigma_{\rm ff}(x)=1.8\times10^{-33}\ln(2.25/x)(1-{\rm e}^{-x})x^{-3}\theta^{-7/2}n\sigma_{\rm T}$. Expression (14) for the Compton enhancement factor assumes that photons with $x>x_{\rm coh}$ upscatter to a Wien peak before they can escape the disc or advect to the black hole. The time-scale for upscattering to the Wien peak is $t_{\rm C}=\ln(x_{\rm coh}^{-1})/8\theta n\sigma_{\rm T}c$. Hence, $t_{\rm C}/_{\rm D}\sim\ln(x_{\rm coh}^{-1})/y$, where $y=4\theta\tau_0^2$ is the Kompaneets parameter. In the considered situation $y\gg1$ and photons with $x>x_{\rm coh}$ do Comptonize to the Wien peak before they can escape. Inside the trapping radius $t_{\rm C}$ should be compared with $t_{\rm a}$. We check in the calculated models that $t_{\rm C}$ is shorter than $t_{\rm a}$ as well.

4.3 Heating = cooling balance

Everywhere in the disc the plasma temperature is determined by the heating = cooling balance $\dot{w}^+=\dot{w}_{\rm cool}$. Inside r_* $\dot{w}_{\rm cool}$ is given by equation (13). Outside r_* the cooling rate is equal to the difference between free–free emission and absorption, being proportional to a small deviation of $w_{\rm pl}$ from w: $\dot{w}_{\rm cool}=\dot{w}_{\rm ff}(1-w/w_{\rm pl})$. The transition at r_* can be smoothly described using the following interpolation for the heating = cooling balance:

$$\dot{w}^+=\dot{w}_{\rm ff}\left[\frac{w}{w_{\rm pl}}+A\left(1-\frac{w}{w_{\rm pl}}\right)\right]\left(1-\frac{w}{w_{\rm pl}}\right). \tag{15}$$

In the limit $w\approx w_{\rm pl}$ this equation yields $\dot{w}^+=\dot{w}_{\rm ff}(1-w/w_{\rm pl})$ while in the limit $w\ll w_{\rm pl}$ it transforms into $\dot{w}^+=A\dot{w}_{\rm ff}$.

4.4 Results

Using the balance (15), we have calculated the temperature in the disc around a massive black hole, $M=10^8\,{\rm M}_\odot$, with spins $a_*=0$ and $a_*=0.998$. The results are shown in Figs 4 and 5 for the case $\alpha=0.3$. The strong overheating $T\gg T_{\rm eff}$ occurs inside the radius r_*. The maximum temperature is reached in the innermost region, $\sim3\times10^8$ K for a rapidly rotating black hole and $\sim10^7$ K for a Schwarzschild black hole. In Figs 6 and 7 we show the dependence of the maximum temperature on the accretion rate $\dot{m}$ in the cases of $\alpha=0.03$ and $\alpha=0.3$.

A question of interest is whether the protons can be thermally decoupled from the electrons at such temperatures. The time-scale for Coulomb energy exchange between protons of temperature $T_{\rm p}$ and electrons of temperature $T_{\rm e}$ is given by (Landau & Lifshitz 1981)

$$t_{\rm ep}=\sqrt{\frac{\pi}{2}}\frac{m_{\rm p}}{m_{\rm e}}\left(\frac{kT_{\rm e}}{mc^2}\right)^{3/2}\frac{1}{\ln\Lambda\, n\sigma_{\rm T}c}\approx12.5\,\frac{T_{\rm e}^{3/2}}{n},$$

where $\ln\Lambda\approx20$ is the Coulomb logarithm. Even in the hottest models with $\alpha=0.3$ and $a_*=0.998$, $t_{\rm ep}\ll t_{\rm a}$ and the accreting plasma can be well described in the one-temperature approximation $T_{\rm e}\approx T_{\rm p}\approx T$.

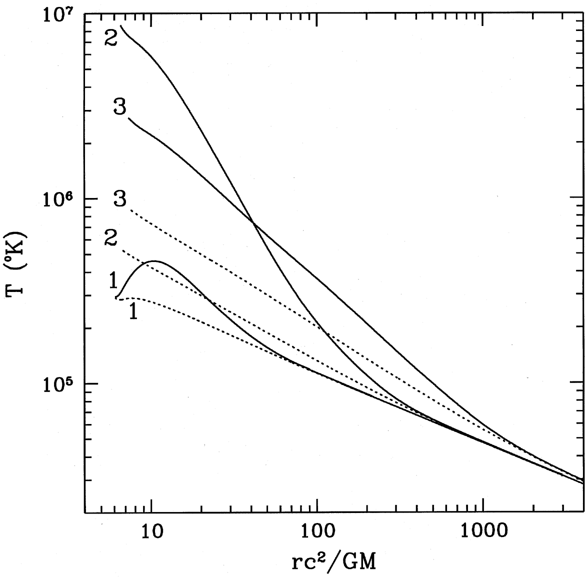

Figure 4. The temperature of the disc with $\alpha=0.3$ in the case of $a_*=0$: $1-\dot{m}=10$, $2-\dot{m}=100$, $3-\dot{m}=1000$. The dotted lines show the effective temperature $T_{\rm eff}$ of the radiation field in the disc.

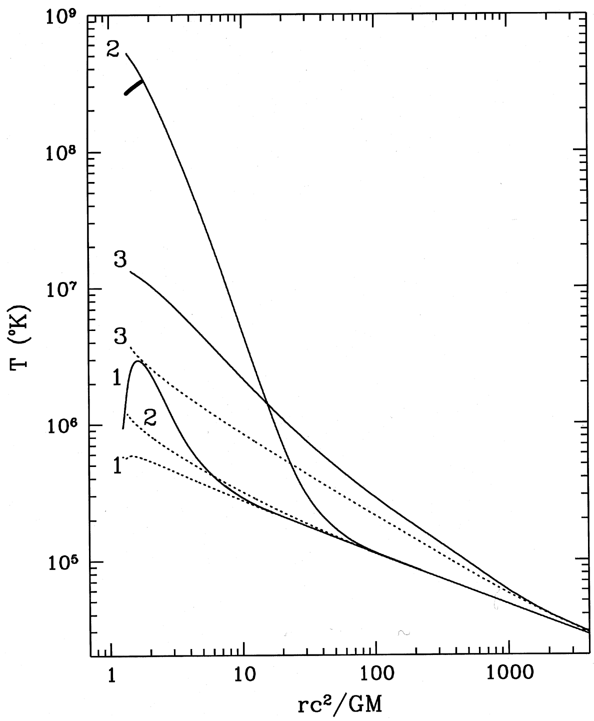

Figure 5. The temperature of the disc with $\alpha=0.3$ in the case of $a_*=0.998$: $1-\dot{m}=1$, $2-\dot{m}=10$, $3-\dot{m}=1000$. The dotted lines display the effective temperature $T_{\rm eff}$ of the radiation in the disc. The heavy line shows the temperature for which $\nu_B=\nu_{\rm coh}$ at $B=B_{\rm eq}$ (see the text).

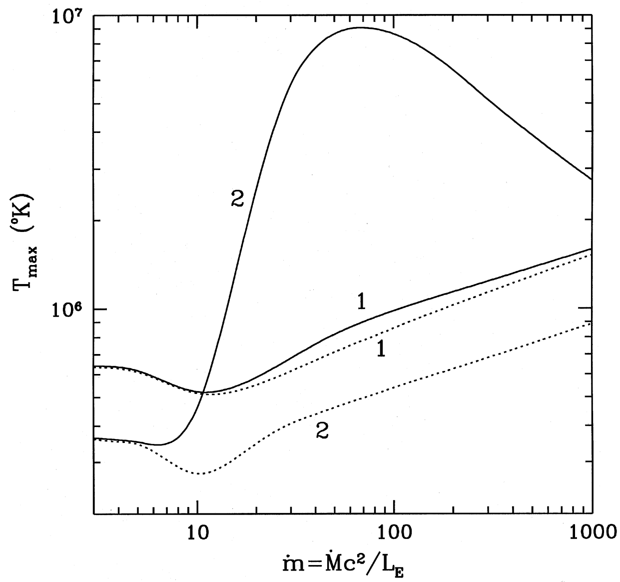

Figure 6. The maximum temperature in the disc as a function of the accretion rate for the case of $a_*=0$: $1-\alpha=0.03$, $2-\alpha=0.3$. The dotted curves show the corresponding T_{eff}.

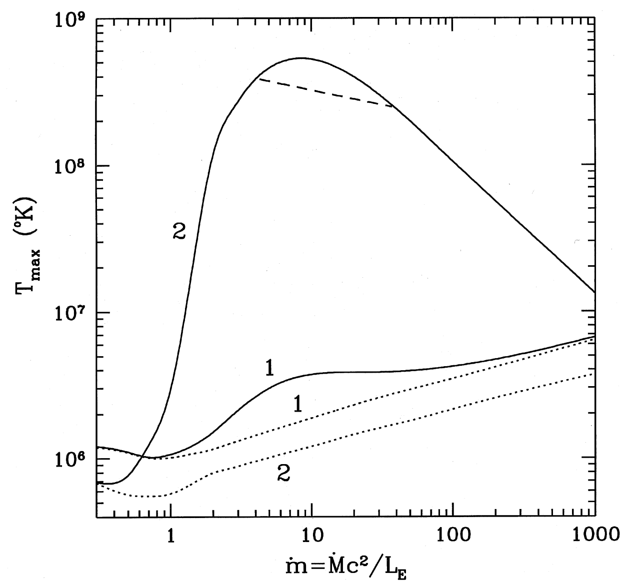

Figure 7. The maximum temperature in the disc as a function of the accretion rate for the case of $a_*=0.998$: $1-\alpha=0.03$, $2-\alpha=0.3$. Above the dashed line ν_B exceeds ν_{coh} in the inner hottest region of the disc.

We then compared the thermal plasma pressure, $p_{gas}=2nkT$, with the radiation pressure, $p_{rad}=w/3$. Even in the hottest region of the disc with $\alpha=0.3$, the plasma pressure does not exceed $\sim 10^{-3}p_{rad}$, so the calculated radiation-dominated models are self-consistent. Note that the accretion flows with $\alpha\to 1$ are unlikely (it would imply turbulence of scale H with sound speed). Anyway, such flows could not be much hotter because at higher temperatures additional cooling mechanisms become important: Comptonization of the cyclotron radiation and collective waves in the plasma, as noted by Shakura & Sunyaev (1973). Cyclotron radiation and collective waves are additional sources of soft photons that can be upscattered to a Wien peak and cool the plasma efficiently. Roughly, the cyclotron source becomes important when the gyrofrequency $\nu_B=eB/2\pi m_e c$ exceeds ν_{coh} (more detailed treatment including higher cyclotron harmonics was given in Gnedin & Sunyaev 1973). In a similar way, Comptonization of collective waves can be important when $\nu_{pl}\gtrsim\nu_{coh}$, where $\nu_{pl}=(ne^2/\pi m_e)^{1/2}$ is the plasma frequency. If the magnetic field is comparable to the equipartition value $B_{eq}=(8\pi w)^{1/2}$, the gyrofrequency exceeds ν_{coh} in the disc sooner than the plasma frequency does. In Fig. 5 the short heavy line shows the temperature at which the gyrofrequency equals ν_{coh}. The corresponding maximal temperature in the disc is shown by the dashed line in Fig. 7.

5 CONCLUSIONS

We have investigated the relativistic accretion disc around Kerr black hole with a super-Eddington accretion rate. The disc has a modest relative height, $H/r<0.4$ up to $\dot{m}\sim 10\dot{m}_{cr}$, and its luminosity is near the Eddington limit. The bulk of the energy released in the inner region of the disc is advected into the black hole. The angular velocity of rotation deviates from the Keplerian one by up to 20 per cent. We paid particular attention to the case of large viscosity parameter $\alpha>0.03$. In this case the accretion flow deviates from thermodynamic equilibrium and overheats in the central region. The hottest flow has accretion rate $\dot{m}\sim 3\dot{m}_{cr}$. We have calculated the maximum temperature in the disc around a massive black hole, $M=10^8$ $M_\odot$, with $\alpha=0.3$. In the case of a Schwarzschild black hole it equals $\sim 10^7$ K. In the case of a rapidly rotating black hole, the maximum temperature is of order 3×10^8 K. It far exceeds the effective temperature corresponding to thermodynamic equilibrium. However, it is not large enough for thermal decoupling of the protons and transition to the ion-pressure-dominated regime of accretion.

The employed vertically integrated model gives the approximate characteristics of the bulk of the gas flowing in the disc, and does not describe the physical conditions in the upper layers (in the 'skin' of the disc) where the spectrum of the emerging radiation is formed. To evaluate the spectrum of the super-Eddington disc, more detailed treatment of the vertical energy transfer is needed, which is likely to demand 2D simulation of the advective region. The disc spectrum may also be strongly affected by corona activity above the surface. It is clear, however, that the surface temperature of the flow with large α and large accretion rate must be much hotter than the effective surface temperature $T^s\mathrm{eff}=(F^-/2\sigma)^{1/4}$, and the activity of the corona may only make the spectrum harder. X-ray emission in the innermost region of the super-Eddington disc may help to reconcile observed quasar spectra with the accretion disc model.

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