矩阵建模法习题及题解

一。矩阵建模法基础

1. 已知系统由下列方程组成：其中 u 为输入，y 为输出，q 为迟延算子，试用矩阵建模法求输出与输入之间的关系 W=y/u。

解：列出矩阵方程

$$\left.\begin{array}{l} x_1 = u - 0.1x_4, \\ x_2 = qx_1, \\ x_3 = x_2 + 2x_1 + 0.3x_4, \\ x_4 = qx_3 \\ y = 0.5x_3 + x_4 \end{array}\right\} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ y \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -0.1 & 0 \\ q & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0.3 & 0 \\ 0 & 0 & q & 0 & 0 \\ 0 & 0 & 0.5 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u \Rightarrow \mathbf{X = QX + PU}$$

程序为：

```
clear, syms q
Q(2,1)=q; Q(1,4)=-0.1;Q(3,1)=2;Q(3,2)=1;Q(3,4)=0.3;
Q(4,3)=q; Q(5,3)=0.5; Q(5,4)=1;Q(5,5)=0;P=[1;0;0;0;0];
W=inv(eye(5)-Q)*P; pretty(W(5))
```

运行结果：

$$\frac{y}{u} = \frac{25\,q^2 + 10\,q + 10}{10 - q^2 + q} = \frac{1 + q + 2.5\,q^2}{1 + 0.1q - 0.1q^2}$$

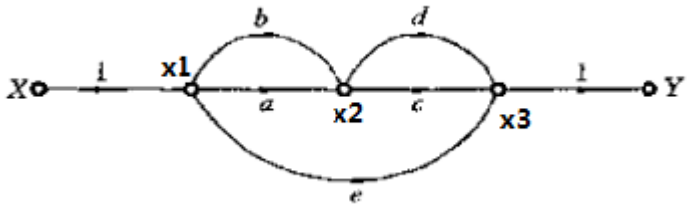
2. 求右图的 W=Y/X

解：方程为：

$$\left.\begin{array}{l} x1 = X + bx2 \\ x2 = ax1 + dY \\ Y = cx2 + ex1 \end{array}\right\} \Rightarrow \begin{bmatrix} x1 \\ x2 \\ Y \end{bmatrix} = \begin{bmatrix} 0 & b & 0 \\ a & 0 & d \\ e & c & 0 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ Y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} X$$

程序为：

```
syms a b c d e
Q=[0,b,0;a,0,d;e,c,0], P=[1;0;0]
W=(eye(3)-Q)\P; W(3)
```



运行结果：W(3)为

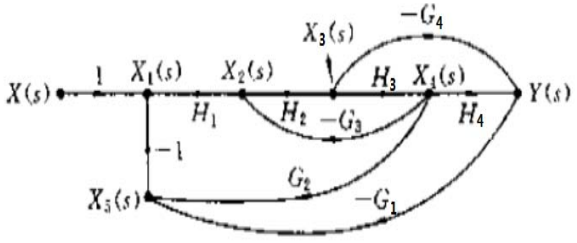
```
-(c*a+e)/(a*b-1+d*e*b+d*c)
```

与书上用梅森公式求出的答案一致：

$$H(s) = \frac{Y(s)}{X(s)} = \frac{ac + e}{1 - (ab + cd + bde)} = \frac{ac + e}{1 - ab - cd - bde}$$

二、信号流图复杂题（取自郑君里“信号与系统”第三版 11 章，可对照梅森公式结果）

3. 求右列流图的转移函数 $H(s) = \frac{Y(s)}{X(s)}$



解：方程组为：

```
x1=x-x5
x2=H1x1-G3x4
x3=H2x2-G4Y
x4=H3x3
x5=G2x4-G1Y
y=H4x4
```

矩阵模型为：

$$\left.\begin{array}{l} x_1=x-x_5 \\ x_2=H_1x_1-G_3x_4 \\ x_3=H_2x_2-G_4Y \\ x_4=H_3x_3 \\ x_5=G_2x_4-G_1Y \\ y=H_4x_4 \end{array}\right\} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ y \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 \\ H_1 & 0 & 0 & -G_3 & 0 & 0 \\ 0 & H_2 & 0 & 0 & 0 & -G_4 \\ 0 & 0 & H_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_2 & 0 & -G_1 \\ 0 & 0 & 0 & H_4 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x \Rightarrow \mathbf{X = QX + PU}$$

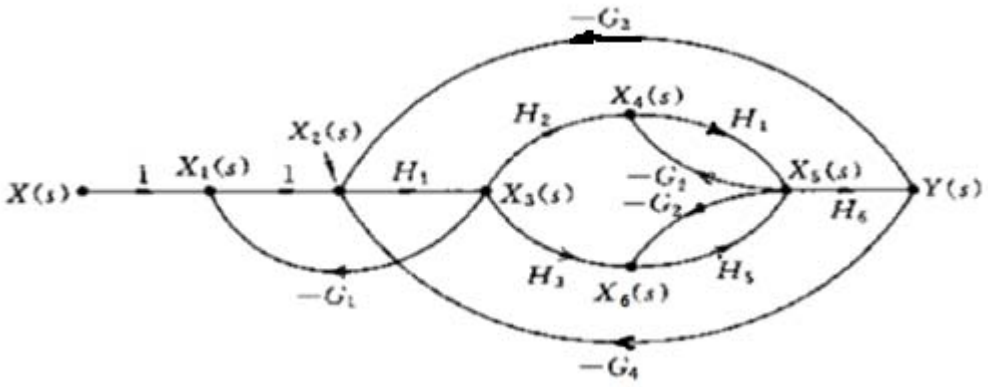
程序为：

```
clear, syms G1 G2 G3 G4 H1 H2 H3 H4
Q(2,1)=H1, Q(1,5)=-1,Q(2,4)=-G3,Q(3,2)=H2,Q(3,6)=-G4,
Q(4,3)=H3,Q(5,4)=G2,Q(5,6)=-G1,Q(6,4)=H4,Q(6,6)=0,
P(1,1)=1,P(6,1)=0,
W=(eye(6)-Q)\P, pretty(W(6))
```

$$W(6)=\frac{H4\ H3\ H2\ H1}{G2\ H3\ H2\ H1 - H2\ H1\ G1\ H4\ H3 + G3\ H3\ H2 + 1 + G4\ H4\ H3}$$

书上的结果：$$H(s)=\frac{Y(s)}{X(s)}=\frac{H_1H_2H_3H_4}{1+H_2H_3G_3+H_3H_4G_4+H_1H_2H_3G_2-H_1H_2H_3H_4G_1}$$

4. 求下列流图的转移函数 $H(s)=\dfrac{Y(s)}{X(s)}$



解:方程组:

解:列出方程组并化为矩阵模型:

$$\left.\begin{array}{l} x1=x-G1x3; \\ x2=x1-G4y-G3y \\ x3=H1x2 \\ x4=H2x3-G2x5 \\ x5=H4x4+H5x6 \\ x6=H3x3-G2x5 \\ y=H6x5 \end{array}\right\} \Rightarrow \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \\ x6 \\ y \end{bmatrix} = \begin{bmatrix} 0 & 0 & -G1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -G4-G3 \\ 0 & H1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & H2 & 0 & -G2 & 0 & 0 \\ 0 & 0 & 0 & H4 & 0 & H5 & 0 \\ 0 & 0 & H3 & 0 & -G2 & 0 & 0 \\ 0 & 0 & 0 & 0 & H6 & 0 & 0 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \\ x6 \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x \Rightarrow \mathbf{X = QX + PU}$$

程序为：

```
clear, syms H1 H2 H3 H4 H5 H6 G1 G2 G3 G4
Q(1,3)=-G1;Q(2,1)=1;Q(2,7)=-G4-G3;
Q(3,2)=H1;Q(4,3)=H2;Q(4,5)=-G2;
Q(5,4)=H4;Q(5,6)=H5;Q(6,3)=H3;Q(6,5)=-G2;
```

Q(7,5)=H6;Q(7,7)=0,

P=[1;0;0;0;0;0;0],

W=inv(eye(7)-Q)*P; pretty(simple(W(7)))

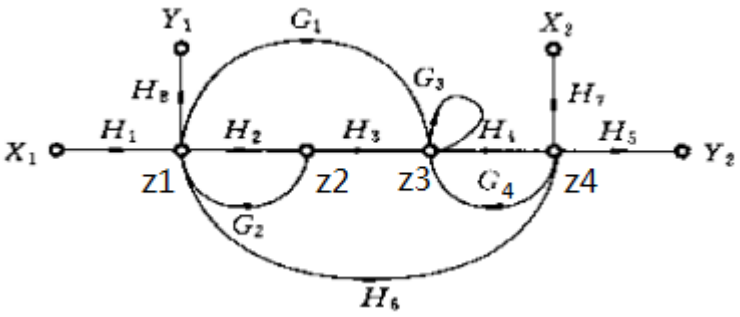
运行结果：

$$\frac{\text{H6 H1 (H3 H5 + H2 H4)}}{\text{((G1 + G1 H4 G2 + G1 H5 G2 + H6 G4 H5 H3 + H6 G4 H2 H4+ H6 G3 H5 H3 + H6 G3 H2 H4) H1 + 1 + H4 G2 + H5 G2)}}$$

用梅森公式结果是：

$$W(s)=\frac{H_1H_6(H_2H_4+H_3H_5)}{1+H_1G_1+G_2(H_4+H_5)+H_1H_6(G_3+G_4)(H_2H_4+H_3H_5)+H_1G_1G_2(H_4+H_5)}$$

5. 求右边所示流图的系统的转移函数 H(s)=Y(s)/X(s).



解：方程组为：

z1=H1x1+G2x2+G1z3

z2=H2z1,z3=H3z2+G3z3+G4z4

z4=H4z3+H7x2+H6z1

Y1=H8z1,

Y2=H5z4

$$\left.\begin{array}{l}z1=H1x1+G2z2+G1z3\\ z2=H2z1\\ z3=H3z2+G3z3+G4z4\\ z4=H4z3+H7x2+H6z1\\ Y1=H8z1\\ Y2=H5z4\end{array}\right\}\Rightarrow\begin{bmatrix}z1\\z2\\z3\\z4\\Y1\\Y2\end{bmatrix}=\begin{bmatrix}0&G2&G1&0&0&0\\H2&0&0&0&0&0\\0&H3&G3&G4&0&0\\H6&0&H4&0&0&0\\H8&0&0&0&0&0\\0&0&0&H5&0&0\end{bmatrix}\begin{bmatrix}z1\\z2\\z3\\z4\\Y1\\Y2\end{bmatrix}+\begin{bmatrix}H1&0\\0&0\\0&0\\0&H7\\0&0\\0&0\end{bmatrix}\begin{bmatrix}x1\\x2\end{bmatrix}\Rightarrow \mathbf{Z=QZ+PX}$$

clear,

syms H1 H2 H3 H4 H5 H6 H7 H8 G1 G2 G3 G4

Q(1,2)=G2; Q(1,3)=G1;Q(2,1)=H2;

Q(3,2)=H3; Q(3,3)=G3; Q(3,4)=G4;

Q(4,1)=H6; Q(4,3)=H4;Q(5,1)=H8;

Q(6,4)=H5; Q(6,6)=0;

P(1,1)=H1; P(4,2)=H7;P(6,2)=0;

W=Z/X=inv(eye(6)-Q)*P

$$Y/X=\begin{bmatrix}Y1/X1 & Y1/X2\\ Y2/X1 & Y2/X2\end{bmatrix}=\frac{\begin{bmatrix}\text{(G4 H4 - 1 + G3) H8 H1} & \text{- G4 H8 G1 H7}\\ \text{-H5 (H3 H4 H2 - H6 G3 + H6) H1} & \text{H5 (H3 H2 G1 - 1 + G3 + H2 G2 - H2 G2 G3) H7}\end{bmatrix}}{\text{H3 H2 G1 + G1 G4 H6 + H2 G2 - H2 G2 G3 + G4 H4 - 1 - G2 G4 H4 H2 + G3}}$$

书上答案为：

$$H_{21}=\frac{H_1H_2H_3H_4H_5+H_1H_6H_5(1-G_3)}{1-(H_2G_2+H_4G_4+H_2H_3G_1+G_3+G_4G_1H_6)+(H_2G_2H_4G_4+H_2G_2G_3)}$$

用梅森公式结果

$$\Delta_5=1-(H_2G_2+H_4G_4+H_2H_3G_1+G_3+G_4G_1H_6)+(H_2G_2H_4G_4+H_2G_2G_3)$$

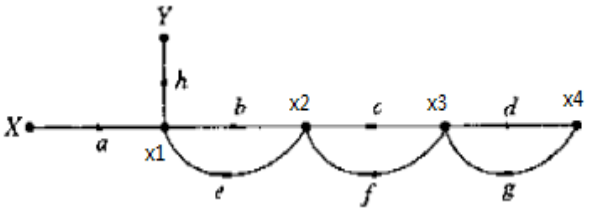
$$H_{11}(s)=\frac{Y_1(s)}{X_1(s)}=\frac{H_1H_6(1-G_3-H_4G_4)}{\Delta_5}$$

$$H_{21}(s)=\frac{Y_2(s)}{X_1(s)}=\frac{H_1H_2H_3H_4H_5+H_1H_6H_5(1-G_3)}{\Delta_b}$$

$$H_{12}(s)=\frac{Y_1(s)}{X_2(s)}=\frac{H_7G_4G_1H_8}{\Delta_b}$$

$$H_{22}(s)=\frac{Y_2(s)}{X_2(s)}=\frac{H_7H_5(1-H_2G_2-G_3-H_2H_3G_1+H_2G_2G_3)}{\Delta_b}$$

6. 求右列流图的转移函数 Y/X：



解：方程为：

x1=ax+ex2;x2=bx1+fx3;

x3=cx2+gx4;

x4=dx3;Y=hx1;

矩阵模型为：

$$\left.\begin{array}{l}x1=aX+ex2;\\ x2=bx1+fx3;\\ x3=cx2+gx4;\\ x4=dx3;\\ Y=hx1;\end{array}\right\}\Rightarrow\begin{bmatrix}x1\\x2\\x3\\x4\\Y\end{bmatrix}=\begin{bmatrix}0&e&0&0&0\\b&0&f&0&0\\0&c&0&g&0\\0&0&d&0&0\\h&0&0&0&0\end{bmatrix}\begin{bmatrix}x1\\x2\\x3\\x4\\Y\end{bmatrix}+\begin{bmatrix}a\\0\\0\\0\\0\end{bmatrix}X\Rightarrow \mathbf{Z=QZ+PX}$$

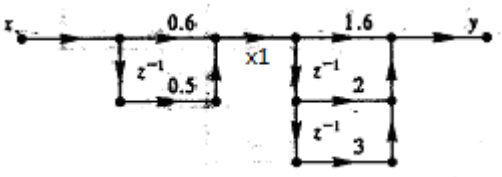
程序为：

```
clear, syms a b c d e f g h
Q(1,2)=e;Q(2,1)=b;Q(2,3)=f;Q(3,2)=c;Q(3,4)=g;
Q(4,3)=d;Q(5,1)=h;Q(5,5)=0;
P=[a;0;0;0;0];
W=inv(eye(5)-Q)*P;pretty(W(5))
```

运行结果为：$W(5)=\dfrac{a\,h\,(1-d\,g-c\,f)}{1+d\,g\,b\,e-b\,e-c\,f-d\,g}$

三、数字信号处理中滤波器系统函数

7. 求右图所示滤波器的转移函数 W=y/x.



解：令 $q=z^{-1}$,方程成为：

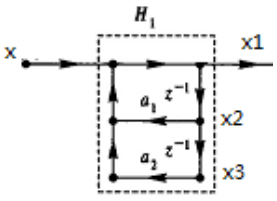
x1=0.6x+0.5qx=(0.6+0.5q)x;

y=1.6x1+2qx1+3q*q*x1=$(1.6+2q+3q^2)$x1

得到：

W=y/x=(Y/x1)(x1/x)=$(1.6+2q+3q^2)$*(0.6+0.5q)

8. 求右图中的 H1=x1/x。



解：x1=x+a1x2+a2x3

x2=qx1,

x3=qx2

$$\left.\begin{array}{l}x1=x+a1x2+a2x3\\ x2=qx1,\\ x3=qx2\end{array}\right\}\Rightarrow\begin{bmatrix}x1\\x2\\x3\end{bmatrix}=\begin{bmatrix}0&a_1&a_2\\q&0&0\\0&q&0\end{bmatrix}\begin{bmatrix}x1\\x2\\x3\end{bmatrix}+\begin{bmatrix}1\\0\\0\end{bmatrix}x\Rightarrow \mathbf{X=QX+PU}$$

```
clear, syms a1 a2 q
Q=[0,a1,a2;q,0,0;0,q,0];P=[1;0;0];
```

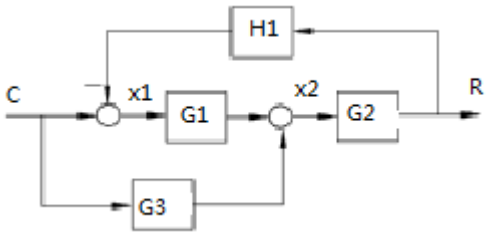
H1=inv(eye(3)-Q)*P, pretty(W(3))

H2=b0+b1q+b2q^2

程序运行结果 $H1=\dfrac{q^2}{1 - a1q - a2\,q^2}$

四、自动控制系统结构图变换

9. 右图所示自动控制框图，试用矩阵建模法求系统传递函数 R/C.



解：列写方程：

x1=C-H1R;

x2=G1x1+G3C

R=G2x2

化为矩阵模型：

$$\left.\begin{array}{l}\text{x1=C-H1R;}\\ \text{x2=G1x1+G3C}\\ \text{R=G2x2}\end{array}\right\}\Rightarrow\begin{bmatrix}x1\\x2\\R\end{bmatrix}=\begin{bmatrix}0&0&H1\\G1&0&0\\0&G2&0\end{bmatrix}\begin{bmatrix}x1\\x2\\R\end{bmatrix}+\begin{bmatrix}1\\G3\\0\end{bmatrix}C\Rightarrow \mathbf{X=QX+PC}\Rightarrow\begin{bmatrix}x1/C\\x2/C\\R/C\end{bmatrix}=\mathbf{inv(I-Q)*P}$$

clear, syms G1 G2 G3 H1

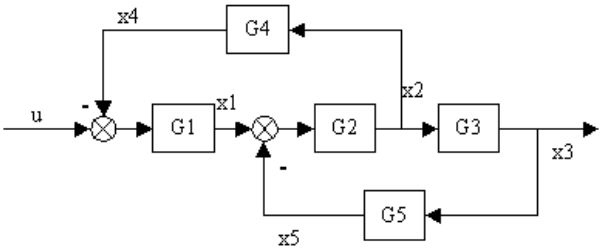
Q=[0,0,H1;G1,0,0;0,G2,0],P=[1;G3;0]

W=inv(eye(3)-Q)*P, pretty(simplify(W(3)))

运行结果：

$$R/C=W(3)=\frac{\text{G2 (G1 + G3)}}{\text{1 - G1 H1 G2}}$$

10. 控制系统结构图如右，试求以 u 为输入，x1,x2,x3,x4 为输出之传递函数矩阵。



解：方程组为：

x1=G1*(u-x4)

x2=G2(x1-x5)

x3=G3x2

x4=G4x2

x5=G5x3

$$\left.\begin{array}{l}x_1=G_1(u-x_4)\\ x_2=G_2(x_1-x_5)\\ x_3=G_3x_2\\ x_4=G_4x_2\\ x_5=G_5x_3\end{array}\right\}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}=\begin{bmatrix}0&0&0&-G_1&0\\G_2&0&0&0&-G_2\\0&G_3&0&0&0\\0&G_4&0&0&0\\0&0&G_5&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}+\begin{bmatrix}G_1\\0\\0\\0\\0\end{bmatrix}u\Rightarrow \mathbf{X=QX+Pu}$$

程序为：

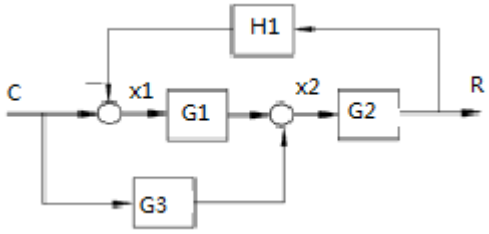
syms G1 G2 G3 G4 G5

Q=[0,0,0,-G1,0;G2,0,0,0,-G2;0,G3,0,0,0;0,G4,0,0,0;0,0,G5,0,0],P=[G1;0;0;0;0],

W=(eye(5)-Q)\P,pretty(W)

结果：$$W=\frac{1}{1-G_2\,G_1\,G_4+G_2\,G_5\,G_3}\begin{bmatrix}G_1\,(1+G_2\,G_5\,G_3)\\ G_2\,G_1\\ G_3\,G_2\,G_1\\ G_4\,G_2\,G_1\\ G_5\,G_3\,G_2\,G_1\end{bmatrix}$$

11. 右图所示自动控制框图，试用矩阵建模法求系统传递函数 R/C.



解：方程组为：

x1=C-H1R

x2=G1x1+G3C

R=G2x2

矩阵模型为：

$$\left.\begin{matrix}\text{x1=C-H1R}\\ \text{x2=G1x1+G3C}\\ \text{R=G2x2}\end{matrix}\right\}\Rightarrow\begin{bmatrix}x1\\ x2\\ R\end{bmatrix}=\begin{bmatrix}0 & 0 & -H1\\ G1 & 0 & 0\\ 0 & G2 & 0\end{bmatrix}\begin{bmatrix}x1\\ x2\\ R\end{bmatrix}+\begin{bmatrix}1\\ G3\\ 0\end{bmatrix}C\Rightarrow \mathbf{X=QX+PC}\Rightarrow\begin{bmatrix}x1/C\\ x2/C\\ R/C\end{bmatrix}=\mathbf{inv(I-Q)*P}$$

```
clear, syms G1 G2 G3 H1
Q=[0,0,-H1;G1,0,0;0,G3,0],P=[1;G2;0]
W=inv(eye(3)-Q)*P, pretty(W(3))
```

$$\frac{R}{C}=\frac{G1G3+G2\,G3}{1+G1\,H1\,G3}=\frac{(G1+G2)G3}{1+G1\,H1\,G3}$$