SMAI - HW-1

Gadela Kesav

20181010079

3. Linear Algebra

Q(1)

a) 15

b) >

c) <

(d) =

Q(2)

(a) $2\sqrt{6}$

(b) 60° or $\pi/3$

Q3)

(a) 2

(b) -4

Q1) $u \in \mathbb{R}^3$, in Q1 of xy-plane

$\Rightarrow u = \{x, y, 3\}$ where $3 = 0$ & $x > 0$ & $y > 0$

$\Rightarrow u = \{x, y, 0\}$, $\sqrt{x^2+y^2} = 3$

$v \in \mathbb{R}^3$, on +ve z-axis

$\Rightarrow v = \{a, b, c\}$ where $a = 0$ & $b = 0$ & $c > 0$

$\Rightarrow v = \{0, 0, c\}$, $\sqrt{c^2} = 5$

$$u \times v = \begin{vmatrix} i & j & k \\ x & y & 0 \\ 0 & 0 & c \end{vmatrix} = yc\hat{i} - xc\hat{j} + 0\hat{k}$$

$$= yc\hat{i} - xc\hat{j}$$

a) $\|u \times v\| = \sqrt{(yc)^2 + (xc)^2} = \sqrt{c^2} \times \sqrt{x^2+y^2} = 5 \times 3 = \underline{15}$ //

b) x-coordinate $= yc$ & $y > 0$, $c > 0 \Rightarrow \underline{yc > 0}$ // $\Rightarrow$ '>'

c) y-coordinate $= -xc$ & $x > 0$, $c > 0 \Rightarrow -xc < 0$ // $\Rightarrow$ '<'

d) z-coordinate $= 0$ // $\Rightarrow$ '='

Q2) Given $|u| = 2\sqrt{2}$, $|v| = 2\sqrt{2}$

$$\|u \mp v\| = \sqrt{|u|^2 + |v|^2 \mp 2u \cdot v} = 2\sqrt{2}$$

$$\Rightarrow |u|^2 + |v|^2 - 2u \cdot v = 8$$

$$\Rightarrow 8 + 8 - 2u \cdot v = 8 \Rightarrow 2u \cdot v = 8 \quad -①$$

(a)

$$\|u + v\| = \sqrt{|u|^2 + |v|^2 + 2u \cdot v} = \sqrt{8+8+8} = 2\sqrt{6}$$

(b) From ① $2u \cdot v = 8$

$\Rightarrow 2 \times |u| \times |v| \cos\theta = 8$

$\Rightarrow 2 \times 2\sqrt{2} \times 2\sqrt{2} \times \cos\theta = 8 \Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = (2n \pm \frac{1}{3})\pi$

$n \in \mathbb{Z}$

$\Rightarrow 60°$ in 3-D plane

{anti-clock/clock wise}

3

Q3)

a) $\begin{bmatrix} 1 & 3 & 2 \\ a & 6 & 2 \\ 0 & 9 & 5 \end{bmatrix}$

First make element in 1st colum 2nd row 0?

Gaussian elimination $\Rightarrow R_2 \rightarrow R_2 - R_1(a)$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 6-3a & 2-2a \\ 0 & 9 & 5 \end{bmatrix}$$

Now, $a_{22} \Rightarrow$ Need to be non-zero (or)

b) we will require row interchange

$\Rightarrow$ Row interchange if $6-3a = 0 \Rightarrow a = 2$ //

det of Matrix $\Rightarrow 5(6-3a) - 9(2-2a)$

$= 30 - 15a - 18 + 18a = 3a - 12$

singular $\Rightarrow$ det $= 0 \Rightarrow 3a + 12 = 0$

$\Rightarrow a = -4$ //