QP Code: RJ22BTECH019 Reg. No | | | | | | | | | | AR 22

GIET UNIVERSITY, GUNUPUR – 765022

B. Tech (Second Semester) Examinations, July - 2023

22BBSBS12001 – Engineering Mathematics - II

(Common to all branches except Biotechnology)

Time: 3 hrs Maximum: 70 Marks

Answer all questions

(The figures in the right hand margin indicate marks)

PART – A **(2 x 5 = 10 Marks)**

Q.1. Answer *ALL* questions	CO #	Blooms Level
a. Form a partial differential equation by Elimination of arbitrary function $z = f(x^2 - y^2)$	CO1	K1
b. Laplace transformation of $e^{3t} * \sin 2t$	CO2	K2
c. Find the gradient of the function $f = [x^2, y^2, z^2]$	CO3	K2
d. Check the exactness of the differential $F = 2xy^2dx + 2x^2ydy + dz$	CO4	K1
e. Find the Laplace transformation of Unit Step function.	CO2	K1

PART – B **(15 x 4 = 60 Marks)**

Answer *ALL* questions	Marks	CO #	Blooms Level
2. a. Solve $qz - p^2y - q^2y = 0$ by using Charpits Method.	10	CO1	K3
b. Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$	5	CO1	K2
(OR)			
c. Solve $q - px - p^2 = 0$ by using Charpits Method.	10	CO1	K3
d. Solve the problem by Lagrange's Method $(y + z)p + (z + x)q = x + y$	5	CO1	K2
3.a. Using convolution theorem, find Laplace inverse transformation of the following $\dfrac{S^2}{(S^2 + a^2)(S^2 + b^2)}$	8	CO2	K3
b. Solve the following integral equation $y(t) = te^t - 2e^t\int_0^t e^{-\tau}y(\tau)d\tau$	7	CO2	K2
(OR)			
c. Solve the differential equation using Laplace Transformation, $y'' + 2y' - 3y = \sin t,\ y(0) = 0,\ y'(0) = 0$	8	CO2	K3
d. Find the Laplace inverse transformation of the following $\dfrac{3S + 4}{(S^2 + 4S + 5)^2}$	7	CO2	K2
4.a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - 3 = z$ at the point (2.-1,2)	8	CO3	K2
b. Prove that $div(f\vec{V}) = f div\vec{V} + \vec{V}.\nabla f$	7	CO3	K2
(OR)			
c. Find the tangent and unit tangent vector of the curve $r(t) = \cosh t\,\hat{i} + 2\sinh t\,\hat{j}$ at the point (1/3,4/3,0)	8	CO3	K2
d. Find $\nabla^2 f$, if $f = z - \sqrt{x^2 + y^2}$	7	CO3	K2

Q.No.	Question	Marks	CO	K
5.a.	Calculate $\int_C F(r).ds$ where $f=\sqrt{2+x^2+3y^2}$ $C:r=[t,t,t^2]\ 0\le t\le 3$	7	CO4	K2
b.	By using Greens theorem evaluate $\int_C F(r).dr$ where $F=\frac{e^y}{x}\hat{i}+e^y\ln x+2x\ \hat{j}$ where $R: 1+x^4\le y\le 2$	8	CO4	K3
	(OR)			
c.	Determine whether the line integral $\int_{(1,0,1)}^{(0,\frac{\pi}{2},1)} 2xyz^2dx+(x^2z^2+z\cos yz)dy+(2x^3yz+y\cos yz)dz$ is independent of path integration, if so evaluate it.	7	CO4	K3
d.	Using Gauss divergence theorem, evaluate the integral $\iint_S F.ndA$ of $F=[x^3,y^3,z^3]$ and S is the sphere $x^2+y^2+z^2=9$	8	CO4	K3

--- End of Paper ---

GIET UNIVERSITY, GUNUPUR

B.Tech (Second Semester) Examinations

Subject Name: Engg. Mathematics-II

Subject Code: 22BBSBS12001

Question Paper Code: RJ22BTECH019

PART-A

Q.1. Answer ALL questions.

a. We have to given that $Z = f(x^2 - y^2)$

Now, do the partial derivative w.r.t x & y.

We get $\frac{\partial z}{\partial x} = f'(x^2 - y^2) 2x \quad \frac{\partial z}{\partial y} = f'(x^2 - y^2)(-2y)$

We know that $\frac{\partial z}{\partial x} = P \quad \frac{\partial z}{\partial y} = q$

$P = 2x f'(x^2 - y^2)$ —— ① $q = -2y f'(x^2 - y^2)$ —— ②

Now deviding equation ① & ② we get that

$$\frac{P}{q} = \frac{2x f'(x^2 - y^2)}{-2y f'(x^2 - y^2)} \Rightarrow \frac{P}{q} = \frac{x}{-y}$$

$\Rightarrow -Py = qx$

$\Rightarrow \boxed{-Py - qx = 0}$ or $\boxed{Py + qx = 0}$

Which is the required solution.

b. We have to given that $F(t) = e^{3t} * \sin 2t$

We know that the convolution theorem.

By the property of convolution theorm, we know that

$$\int f(t) * g(t) = f(s) * g(s)$$

c. We have to given that $f = [x^2, y^2, z^2]$

i.e $f = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$

We know that Gradient of $f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$

$\nabla f = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$

d. We have to given that

$$F = 2xy^2dx + 2x^2ydy + dz.$$

Let $F = f_1 dx + f_2 dy + f_3 dz$

$f_1 = 2xy^2 \quad f_2 = 2x^2y \quad f_3 = 1$

For the Exactness $\frac{\partial f_3}{\partial y} = \frac{\partial f_2}{\partial z} \quad \frac{\partial f_1}{\partial z} = \frac{\partial f_3}{\partial x} \quad \frac{\partial f_2}{\partial x} = \frac{\partial f_1}{\partial y}$

$\frac{\partial f_3}{\partial y} = \frac{\partial (1)}{\partial y} = 0, \quad \frac{\partial f_2}{\partial z} = \frac{\partial}{\partial z}(2x^2y) = 0, \quad \frac{\partial f_1}{\partial z} = \frac{\partial}{\partial z}(2xy^2) = 0$

$\frac{\partial f_3}{\partial x} = \frac{\partial}{\partial x}(1) = 0 \quad \frac{\partial f_2}{\partial x} = \frac{\partial (2x^2y)}{\partial x} = 4xy \quad \frac{\partial f_1}{\partial y} = \frac{\partial}{\partial y}(2xy^2) = 4xy$

$\therefore \frac{\partial f_3}{\partial x} = \frac{\partial f_1}{\partial z}, \quad \frac{\partial f_1}{\partial z} = \frac{\partial f_3}{\partial x}, \quad \frac{\partial f_2}{\partial x} = \frac{\partial f_1}{\partial y}$

Hence it is Exact.

e. We know that Unitstep function

$$u(t-a) = u_a(t) = \begin{cases} 1, & t \geq a \\ 0, & t < a \end{cases}$$

$$L[u(t-a)] = \int_0^\infty e^{-st} u(t-a)\,dt$$

$$= \int_0^a e^{-st} \cdot 0\,dt + \int_a^\infty e^{-st} \cdot 1\,dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_a^\infty = 0 - \frac{e^{-as}}{-s} = \frac{e^{-as}}{s}$$

$$\therefore \boxed{L[u(t-a)] = \frac{e^{-as}}{s}}$$

PART-B

Answer ALL questions.

2. a. We have to given that

$$qz - p^2y - q^2y = 0$$

We have to solve this by Charpits Method.

$$qz - y(p^2+q^2) = 0$$

$$\Rightarrow qz = (p^2+q^2)y \quad \text{This is the Given PDE.}$$

Let $f(x,y,z,p,q) = (p^2+q^2)y - qz = 0$ ——— (1)

By Charpit's Method

$$\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q} = \frac{dx}{-f_p} = \frac{dy}{-f_q}$$

From eq^n (1) we get that,

$\frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = p^2+q^2 \quad \frac{\partial f}{\partial z} = -q \quad \frac{\partial f}{\partial p} = 2py \quad \frac{\partial f}{\partial q} = 2yq - z$

We have to given that,

$$x^2(y-z)p + y^2(z-x)q = z^2(x-y).$$

We know that $Pp+Qq=R$

Here $P = x^2(y-z)$ $Q = y^2(z-x)$ $R = z^2(x-y)$

The Auxiliary Equation is $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

i.e $$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)}$$

Now deviding $\frac{1}{x^2}, \frac{1}{y^2}, \frac{1}{z^2}$ on neumator & denominator respectively

$$\frac{dx/x^2}{\frac{x^2(y-z)}{x^2}} = \frac{dy/y^2}{\frac{y^2(z-x)}{y^2}} = \frac{dz/z^2}{\frac{z^2(x-y)}{z^2}}$$

$$\frac{dx/x^2}{y-z} = \frac{dy/y^2}{z-x} = \frac{dz/z^2}{x-y}$$

$l = \frac{1}{x^2}$ $m = \frac{1}{y^2}$ $n = \frac{1}{z^2}$

$\int l\,dx + m\,dy + n\,dz = 0 \Rightarrow \int \frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz = 0$

$\Rightarrow +\frac{1}{x} - \frac{1}{y} - \frac{1}{z} = C$

$\Rightarrow \boxed{\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = C_1}$

Now, Deviding $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$, on neumator & denominator respectively.

$$\frac{dx/x}{\frac{x^2(y-z)}{x}} = \frac{dy/y}{\frac{y^2(x-z)}{y}} = \frac{dz/z}{\frac{z^2(x-y)}{z}}$$

$$\Rightarrow \frac{dx/x}{x(y-z)} = \frac{dy/y}{yx-yz} = \frac{dz/z}{zx-zy}$$

$l = \frac{1}{x}$ $m = \frac{1}{y}$ $n = \frac{1}{z}$

$\int l\,dx + m\,dy + n\,dz = 0 \Rightarrow \int \frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = C$

$\Rightarrow \log x + \log y + \log z = C$

$\Rightarrow \log(xyz) = \log c_2$

$\Rightarrow \boxed{xyz = C_2}$

$\therefore$ The required solution is $f\left(\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right), xyz\right) = 0$ ANS

The Charpits AE are

$$\frac{dp}{-pq} = \frac{dq}{p^2+q^2-q^2} = \frac{dz}{-2p^2y-2q^2y+qz} = \frac{dx}{-2py} = \frac{dy}{-2qy}$$

Now taking first two constraints,

$$\frac{dp}{-pq} = \frac{dq}{p^2} \Rightarrow \frac{dp}{q} = -\frac{dq}{p}$$

$$pdp = -qdq \quad \text{on Integrating}$$

$$\int pdp = -\int qdq$$

$$\frac{p^2}{2} = -\frac{q^2}{2} + C_1 \quad \frac{p^2}{2} + \frac{q^2}{2} = C_1 \quad p^2+q^2 = 2C_1 = C_2$$

$$\boxed{p^2+q^2 = C_2} \quad \text{———} ②$$

Now put eqⁿ ② in eqⁿ ① we get that,

$$C_2y - qz = 0 \Rightarrow C_2y = qz$$

$$\boxed{q = \frac{C_2y}{z}}$$

Now eqⁿ ② becomes

$$p^2 + \frac{C_2^2y^2}{z^2} = C_2 \Rightarrow p^2 = C_2 - \frac{C_2^2y^2}{z^2}$$

$$\Rightarrow p = \sqrt{\frac{C_2z^2 - C_2^2y^2}{z^2}} = \frac{\sqrt{C_2}}{z}\cdot\sqrt{z^2-C_2y^2}$$

$\because$ z depends on x & y

$$dz = pdx + qdy$$

$$dz = \frac{\sqrt{C_2}}{z}\cdot\sqrt{z^2-C_2y^2}\,dx + \frac{C_2y}{z}\,dy$$

$$\Rightarrow zdz = \sqrt{C_2}\cdot\sqrt{z^2-C_2y^2}\,dx + C_2y\,dy$$

$$\Rightarrow zdz - C_2ydy = \sqrt{C_2}\cdot\sqrt{z^2-C_2y^2}\,dx \qquad \text{Let } \sqrt{C_2} = c$$

$$\Rightarrow zdz - c^2ydy = c\sqrt{z^2-C_2y^2}\,dx \qquad \Rightarrow C_2 = c^2$$

$$\Rightarrow \frac{zdz - c^2ydy}{\sqrt{z^2-c^2y^2}} = c\,dx$$

$$\Rightarrow \frac{d(z^2/2) - d(c^2y^2/2)}{\sqrt{z^2-c^2y^2}} = c\,dx \Rightarrow \frac{1}{2}\frac{d(z^2-c^2y^2)}{\sqrt{z^2-c^2y^2}} = c\,dx$$

On Integrating bothsides

$$\frac{1}{2}\int\frac{d(z^2-c^2y^2)}{\sqrt{z^2-c^2y^2}} = \int c\,dx \qquad \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

$$d\sqrt{x} = \frac{1}{2}\frac{dx}{\sqrt{x}}$$

$$\sqrt{z^2-c^2y^2} = cx + c_3$$

$$\Rightarrow z^2 - c^2y^2 = (cx+c_3)^2 \quad \text{Squaring bothsides.}$$

$$\Rightarrow z^2 = c^2y^2 + (cx+c_3)^2$$

Which is the required solution.

(OK)

We have to given that

$$q - px - p^2 = 0$$

The given $f(x,y,z,p,q) = q - px - p^2 = 0$ —— ①

$f_x = -p \quad f_y = 0 \quad f_z = 0 \quad f_p = -x - 2p \quad f_q = 1$

The Auxiliary equation of Charpits method is

$$\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q} = \frac{dx}{-f_p} = \frac{dy}{-f_q}$$

So, $\frac{dp}{-p+p(0)} = \frac{dq}{0+q(0)} = \frac{dz}{-p(-x-2p) - q(1)} = \frac{dx}{-(-x-2p)} = \frac{dy}{-1}$

$\Rightarrow \frac{dp}{-p} = \frac{dq}{0} = \frac{dz}{px + 2p^2 - q} = \frac{dx}{x+2p} = \frac{dy}{-1}$

Now, $\frac{dp}{-p} = \frac{dy}{-1} \Rightarrow \frac{dp}{p} = \frac{dy}{1}$

$\int \frac{dp}{p} = \int dy \quad \log p = y + c$

$dq = 0$

$\int dq = \int 0$

$\boxed{q = a}$

Now put 'q' in eqn ① we get that

$$a - px - p^2 = 0$$
$$a - p(x - p) = 0$$
$$a = p(x - p)$$
$$\Rightarrow -p^2 + px = a$$
$$\Rightarrow -p^2 - px - a = 0$$

$p^2 + px + a = 0$

$$\Rightarrow p = \frac{-x \pm \sqrt{x^2 - 4.1.a}}{2.1} = \frac{-x \pm \sqrt{x^2 - 4a}}{2}$$

We know that $dz = pdx + qdy$

$$\int dz = \int \frac{-x \pm \sqrt{x^2 - 4a}}{2} dx + \int a\,dy$$

$$\Rightarrow z = \frac{1}{2}\int -x + \sqrt{x^2 - (2\sqrt{a})^2}\, dx + a\int dy$$

$$\Rightarrow z = \frac{1}{2}\left[-\frac{x^2}{2} + \frac{x}{2}\sqrt{x^2 - (2\sqrt{a})^2} + \frac{(2\sqrt{a})^2}{2}\log|2 + \sqrt{x^2 - 4a}| + ay + c\right.$$

$$\Rightarrow \boxed{z = -\frac{x^2}{4} + \frac{x}{4}\sqrt{x^2 - 4a} + a\log|2 + \sqrt{x^2 - 4a}| + ay}$$

Which is the required solution.

5.a. Calculate $\int_C F(r).ds$ wher...

d. We have to given that,
$(y+z)p+(z+x)q = x+y$
By using the Langrange's method $Pp+Qq=R$

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

Here $P = y+z$, $Q = z+x$, $R = x+y$

The Auxilliary Equation is $\frac{dx}{y+z} = \frac{dy}{x+z} = \frac{dz}{x+y}$

Now $\frac{dx-dy}{y-x} = \frac{dy-dz}{z-y} = \frac{dx+dy+dz}{2(x+y+z)}$

$\Rightarrow \frac{dx-dy}{y-x} = \frac{dy-dz}{z-y}$ & $\frac{dx-dy}{y-x} = \frac{dx+dy+dz}{2(x+y+z)}$

$\Rightarrow \frac{dx-dy}{x-y} = \frac{dy-dz}{y-z}$ & $-\left(\frac{dx-dy}{x-y}\right) = \frac{dx+dy+dz}{2(x+y+z)}$

$-2\left(\frac{dx-dy}{x-y}\right) = \frac{dx+dy+dz}{(x+y+z)}$

$\int \frac{dx-dy}{x-y} = \int \frac{dy-dz}{y-z}$ $\quad\Rightarrow -2\int \frac{dx-dy}{x-y} = \int \frac{d(x+y+z)}{x+y+z}$

$\log(x-y) = \log(y-z) + \log c$ $\quad\Rightarrow -2\log(x-y) = \log(x+y+z) + \log c$

$\Rightarrow \log \frac{x-y}{y-z} = \log c_1$ $\quad\Rightarrow \log(x-y)^{-2} = \log(x+y+z) + \log c$

$\Rightarrow \boxed{\frac{x-y}{y-z} = c_1}$ $\quad\Rightarrow \boxed{c_2 = (x+y+z)(x-y)^2}$

$\therefore$ The required solution is $f\left(\frac{x-y}{y-z}, (x+y+z)(x-y)^2\right) = ($

3. a. Here $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$

$$L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right] = L^{-1}\left[\frac{s}{s^2+a^2} * \frac{s}{s^2+b^2}\right]$$

Let $L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$ $\quad L^{-1}\left[\frac{s}{s^2+b^2}\right] = \cos bt$

$$L^{-1}\left[\frac{s}{s^2+a^2} * \frac{s}{s^2+b^2}\right] = \int_0^t \cos au \cdot \cos b(t-u)\,du$$

$\cos A \cdot \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$$L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right] = \frac{1}{2}\int_0^t [\cos(au+bt-bu) + \cos(au-bt+bu)]\,du$$

[illegible] + [illegible]$_2$ = [illegible]

[illegible] + [illegible]$_2$ = [illegible]

$$= \frac{1}{2}\int_0^t \left[\left(\cos((a-b)u+bt\right) + \left(\cos((a+b)u-bt\right)\right]du$$

$$= \frac{1}{2}\left[\frac{\sin(a-b)u+bt}{a-b}\right]_0^t + \frac{1}{2}\left[\frac{\sin(a+b)u-bt}{a+b}\right]_0^t$$

$$= \frac{1}{2}\left[\frac{\sin(a-b)t+bt}{a-b} - \frac{\sin(a-b).0+bt}{a-b}\right] + \frac{1}{2}\left[\frac{\sin(a+b)t-bt}{a+b} - \frac{\sin(a+b).0-bt}{a+b}\right]$$

$$= \frac{1}{2}\left[\frac{\sin at}{a-b} - \frac{\sin bt}{a-b}\right] + \frac{1}{2}\left[\frac{\sin at}{a+b} - \frac{\sin(-bt)}{a+b}\right]$$

$$= \frac{1}{2}\left[\frac{\sin at}{a-b} - \frac{\sin bt}{a-b} + \frac{\sin at}{a+b} + \frac{\sin bt}{a+b}\right]$$

$$= \frac{1}{2}\left[\left(\frac{\sin at}{a-b} + \frac{\sin at}{a+b}\right) - \left(\frac{\sin bt}{a-b} - \frac{\sin bt}{a+b}\right)\right]$$

$$= \frac{1}{2}\left[\frac{2a\sin at}{a^2-b^2} - \frac{2b\sin bt}{a^2-b^2}\right]$$

$$= \frac{1}{2}\left[\frac{2[a\sin at - b\sin bt]}{a^2-b^2}\right] = \frac{a\sin at - b\sin bt}{a^2-b^2}$$

$$\therefore L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right] = \frac{a\sin at - b\sin bt}{a^2-b^2}$$

We have to given that

$$y(t) = te^t - 2e^t\int_0^t e^{-\tau}y(\tau)\,d\tau$$

Now, Integrating part in here also there.
Taking Laplace transform both sides.

$$L[y(t)] = L[te^t] - L\left[2e^t\int_0^t e^{-\tau}y(\tau)\,d\tau\right]$$

$$Y(s) = \frac{1}{(s-1)^2} - 2\left[L\{e^t\}.L\{y(t)\}\right]$$

$$Y(s) = \frac{1}{(s-1)^2} - 2\,\frac{1}{s-1}\cdot Y(s)$$

$$\Rightarrow Y(s) + \frac{2}{s-1}Y(s) = \frac{1}{(s-1)^2} \Rightarrow Y(s)\left[1+\frac{2}{s-1}\right] = \frac{1}{(s-1)^2}$$

$$\Rightarrow \left[\frac{s-1+2}{s-1}\right]Y(s) = \frac{1}{(s-1)^2} \qquad Y(s)\left[\frac{s+1}{s-1}\right] = \frac{1}{(s-1)^2}$$

$$Y(s) = \frac{1}{(s-1)^2}\times\frac{s-1}{s+1} \Rightarrow Y(s) = \frac{1}{(s+1)(s-1)} = \frac{1}{s^2-1}$$

$$Y(s) = \frac{1}{s^2-1} = \frac{1}{s^2-1^2} \qquad L^{-1}[Y(s)] = L^{-1}\left[\frac{1}{s^2-1^2}\right] \Rightarrow \boxed{y(t) = \sinh t}$$

Hence the required solution is $y(t) = \sinh t$

5.a. Calculate $\int_C F(r).ds$ where $f=\sqrt{2+x^2+3y^2}$ C

b. By using Green...

OR

c. We have to given that

$y''+2y'-3y=\sin t \quad y(0)=0 \quad y'(0)=0$

Taking L-T of both sides of the equation, we get

$L[y'']+2L[y']-3L[y]=L[\sin t]$

$\Rightarrow s^2\bar{y}(s)-sy(0)-y'(0)+2[s\bar{y}(s)-y(0)]-3\bar{y}(s)=\frac{1}{s^2+1}$

Now using initial condition

$(s^2+2s-3)\bar{y}(s)=\frac{1}{s^2+1}$

$\Rightarrow [(s-1)(s+3)]\bar{y}(s)=\frac{1}{s^2+1} \qquad \bar{y}(s)=\frac{1}{(s^2+1)(s+3)(s-1)}$

$\frac{1}{(s^2+1)(s+3)(s-1)}=\frac{As+B}{s^2+1}+\frac{C}{s+3}+\frac{D}{s-1}$

$1=(As+B)(s+3)(s-1)+C(s-1)(s^2+1)+D(s+3)(s^2+1)$

$\Rightarrow 1=As(s^2+2s-3)+B(s^2+2s-3)+C(s^3+s-s^2-1)+D(s^2+s+3s^2+3)$

$\Rightarrow 1=A(s^3+2s^2-3s)+B(s^2+2s-3)+C(s^3-s^2+s-1)+D(s^3+3s^2+s+3)$

Put $s=-3$, $1=C(-3-1)(9+1)$

$1=-40C \qquad \boxed{C=\frac{-1}{40}}$

$s=1 \quad 1=D(1+3)(1+1) \quad 1=8D \quad \boxed{D=1/8}$

Similarly $A=\frac{-1}{10} \quad B=\frac{-1}{5}$

Now $\bar{y}(s)=\frac{-1}{10}\left(\frac{s}{s^2+1}\right)-\frac{1}{5}\left(\frac{1}{s^2+1}\right)-\frac{1}{40}\left(\frac{1}{s+3}\right)+\frac{1}{8}\left(\frac{1}{s-1}\right)$

$L^{-1}[\bar{y}(s)]=\frac{-1}{10}L^{-1}\left[\frac{s}{s^2+1}\right]-\frac{1}{5}L^{-1}\left[\frac{1}{s^2+1}\right]-\frac{1}{40}L^{-1}\left[\frac{1}{s+3}\right]+\frac{1}{8}L^{-1}\left[\frac{1}{s-1}\right]$

$y(t)=-\frac{1}{10}\cos t-\frac{1}{5}\sin t-\frac{1}{40}e^{-3t}+\frac{1}{8}e^{t}$

d. We have to given that $f(s)=\frac{3s+4}{(s^2+4s+5)^2}$

$f(s)=$

4. a) We have to given that

$$x^2+y^2+z^2=9 \qquad x^2+y^2-3=z$$

at a point $(2,-1,2)$.

Let $f = x^2+y^2+z^2-9 \qquad g = x^2+y^2-3-z$

$\nabla f = 2x\hat{i}+2y\hat{j}+2z\hat{k} \qquad \nabla g = 2x\hat{i}+2y\hat{j}-\hat{k}$

∇f at point $(2,-1,2)$ $\nabla f\big|_{(2,-1,2)} = 4\hat{i}-2\hat{j}+4\hat{k}$

∇g at point $(2,-1,2)$ $\nabla g\big|_{(2,-1,2)} = 4\hat{i}-2\hat{j}-\hat{k}$

$\nabla f.\nabla g = (4\hat{i}-2\hat{j}+4\hat{k}).(4\hat{i}-2\hat{j}-\hat{k})$

$= 16+4-4 = 20-4 = 16$

$|\nabla f| = \sqrt{(4)^2+(-2)^2+(4)^2} = \sqrt{36} = 6$

$|\nabla g| = \sqrt{(4)^2+(-2)^2+(-1)^2} = \sqrt{21}$

We know that, $\cos\alpha = \dfrac{\nabla f.\nabla g}{|\nabla f|.|\nabla g|}$

$\alpha = \cos^{-1}\left(\dfrac{\nabla f.\nabla g}{|\nabla f.\nabla g}\right) = \cos^{-1}\left(\dfrac{16}{6\sqrt{21}}\right)$

$\Rightarrow \alpha = \cos^{-1}\left(\dfrac{8}{3\sqrt{21}}\right)$

b. We have to given that

$$\text{div}(f\vec{V}) = f\,\text{div}\,\vec{V} + \vec{V}\nabla f$$

Let $\vec{V} = V_1\hat{i}+V_2\hat{j}+V_3\hat{k}$

$\text{div}(f\vec{V}) = \vec{\nabla}(f\vec{V})$

$= \left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)[f(V_1\hat{i}+V_2\hat{j}+V_3\hat{k})]$

$= \left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)(fV_1\hat{i}+fV_2\hat{j}+fV_3\hat{k})$

$$= \frac{\partial}{\partial x}(f v_1) + \frac{\partial}{\partial y}(f v_2) + \frac{\partial}{\partial z}(f v_3)$$

$$= f\frac{\partial v_1}{\partial x} + v_1\frac{\partial f}{\partial x} + f\frac{\partial v_2}{\partial y} + v_2\frac{\partial f}{\partial y} + v_3\frac{\partial f}{\partial z} + f\frac{\partial v_3}{\partial z}$$

$$= f\left(\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}\right) + \left(v_1\frac{\partial f}{\partial x} + v_2\frac{\partial f}{\partial y} + v_3\frac{\partial f}{\partial z}\right)$$

$$= f\left[\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).(v_1\hat{i} + v_2\hat{j} + v_3\hat{k})\right]$$
$$+ \left[\left(\hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}\right).(v_1\hat{i} + v_2\hat{j} + v_3\hat{k})\right]$$

$$= f(\vec{\nabla}.\vec{v}) + (\text{grad} f)\vec{v}$$
$$= f \,\text{div}\vec{v} + \vec{v}\,\nabla f \quad \square$$

[OR]

c. We have to given that
the curve is $r(t) = \cosh t\,\hat{i} + 2\sinh t\,\hat{j}$
at the point $P(\frac{1}{3}, \frac{4}{3}, 0)$.
Here P corresponds to $t = 1.098612$

$r'(t) = \sinh t\,\hat{i} + 2\cosh t\,\hat{j}$

$$u(t) = \frac{r'}{|r'|} = \frac{\sinh t\,\hat{i} + 2\cosh t\,\hat{j}}{\sqrt{\sinh^2 t + \cosh^2 t}}$$

$$u(t)\Big|_{(\frac{1}{3},\frac{4}{3},0)} = \frac{(\frac{4}{3}, \frac{5}{3}, 0)}{\sqrt{(\frac{4}{3})^2 + (\frac{5}{3})^2 + (0)^2}} = \frac{3}{\sqrt{41}}\left(\frac{4}{3}, \frac{5}{3}, 0\right) = \left(\frac{4}{\sqrt{41}}, \frac{5}{\sqrt{4}\ldots}\right.$$

d. We have to given that, $f = z - \sqrt{x^2+y^2}$
We have to find $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

$$\frac{\partial f}{\partial x} = \frac{-x}{\sqrt{x^2+y^2}} \qquad \frac{\partial f}{\partial y} = \frac{-y}{\sqrt{x^2+y^2}} \qquad \frac{\partial f}{\partial z} = 1$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{-y^2}{(x^2+y^2)^{3/2}} \qquad \frac{\partial^2 f}{\partial y^2} = \frac{-x^2}{(x^2+y^2)^{3/2}} \qquad \frac{\partial^2 f}{\partial z^2} = 0$$

Hence Laplacian of f be

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = -\frac{(x^2+y^2)}{(x^2+y^2)^{3/2}} = \frac{-1}{(x^2+y^2)^{1/2}}$$

ANS

a. We have to given that

$$f = \sqrt{2+x^2+3y^2} \qquad C: r = [t, t, t^2]$$

Here $\qquad 0 \le t \le 3.$

$$r(t) = t\hat{i} + t\hat{j} + t^2\hat{k}$$

$$\frac{dr}{dt} = \hat{i} + \hat{j} + 2t\hat{k}$$

$$\left(\frac{ds}{dt}\right)^2 = \frac{dr}{dt}\cdot\frac{dr}{dt} = (\hat{i}+\hat{j}+2t\hat{k})\cdot(\hat{i}+\hat{j}+2t\hat{k})$$

$$= 1+1+4t^2 = 2+4t^2$$

$$\therefore \left(\frac{ds}{dt}\right)^2 = 2+4t^2 \qquad \therefore \frac{ds}{dt} = \sqrt{2+4t^2}$$

$$ds = \sqrt{2+4t^2}\,dt$$

$$\int_C f(r)\,ds = \int_0^3 \sqrt{2+3t^2+t^2}\cdot\sqrt{2+4t^2}\,dt = \int_0^3 \sqrt{2+4t^2}\cdot\sqrt{2+4t^2}\,dt$$

$$= \int_0^3 (2+4t^2)\,dt = 2[t]_0^3 + 4\left[\frac{t^3}{3}\right]_0^3$$

$$= 2[3-0] + \frac{4}{3}[3^3-0^3] = 6+\frac{4}{3}[27] = 6+\frac{108}{3}$$

$$= 6+36 = 42$$

We have to given that,

$$F = \frac{e^y}{x}\hat{i} + e^y \ln x \cancel{+} 2x\hat{j}$$

$$R: 1+x^4 \le y \le 2.$$

Here, $F = F_1\hat{i} + F_2\hat{j} \qquad F_1 = \frac{e^y}{x} \qquad F_2 = e^y \ln x + 2x$

$$\frac{\partial F_2}{\partial x} = \frac{e^y}{x} + 2 \qquad \frac{\partial F_1}{\partial y} = \frac{e^y}{x}$$

$R: 1+x^4 \le y \le 2$

$1+x^4 = 2 \qquad y = 2$

$x^4 - 1 = 0$

$(x^2)^2 - (1^2)^2 = 0$

$(x^2+1)(x^2-1) = 0$

$x^2 = 1,\; x^2 = -1$ Not possible

$\boxed{x = \pm 1}$

$$\int_{-1}^{1}\int_{1+x^4}^{2} 2\,dy\,dx$$

$$= 2\int_{-1}^{1} [y]_{1+x^4}^{2}\,dx$$

$$= 2\int_{-1}^{1} (2-1-x^4)\,dx$$

$$= 2\int_{-1}^{1} (1-x^4)\,dx = 4\int_0^1 (1-x^4)\,dx = 4\left[x-\frac{x^5}{5}\right]_0^1 = 4\left[1-\frac{1}{5}\right]$$

$$= 4\left[\frac{4}{5}\right] = \frac{16}{5}$$

(OR)

c. We have to given that,

$$\int_{(1,0,1)}^{(0,\frac{\pi}{2},1)} 2xyz^2\,dx + (x^2z^2 + z\cos yz)\,dy + (2x^3yz + y\cos yz)\,dz$$

Let $F_1 = 2xyz^2$ $\quad F_2 = x^2z^2 + z\cos yz$ $\quad F_3 = 2x^3yz + y\cos yz$

$\frac{\partial F_1}{\partial z} = 4xyz$ $\quad \frac{\partial F_1}{\partial y} = 2xz^2$ $\quad \frac{\partial F_2}{\partial z} = 2x^2z + \cos yz - zy\sin yz$

$\frac{\partial F_2}{\partial x} = 2xz^2$ $\quad \frac{\partial F_3}{\partial y} = 2x^3z + \cos yz - yz\sin yz$

$\frac{\partial F_3}{\partial x} = 6x^2yz$

$$\therefore \frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}$$

d. We have to given that,

$F = [x^3, y^3, z^3]$ $\quad$ Sphere is $x^2+y^2+z^2=9$

Let $x = 3\cos v.\cos u$

$y = 3\cos v\ \sin u$ $\quad 0 \le u \le 2\pi$

$z = 3\sin v$ $\quad -\frac{\pi}{2} \le v \le \frac{\pi}{2}$ $\quad -\frac{\pi}{2} \le v \le \frac{\pi}{2}$

$N = r_u \times r_v$

$r = 3\cos v.\cos u\hat{i} + 3\cos v.\sin u\hat{j} + 3\sin v\hat{k}$

$r_u = -3\cos v.\sin u\hat{i} + 3\cos v.\cos u\hat{j} + 0\hat{k}$

$r_v = -3\cos u.\sin v\hat{i} - 3\sin v.\sin u\hat{j} + 3\cos v\hat{k}$

$N = 9[\cos^2 v.\cos u\hat{i} + \cos^2 v\sin u\hat{j} + \cos v\sin v\hat{k}]$

$n\,dA = n\,du\,dv$

$F(s).N = 9\times 27[\cos^5 v\cos^4 u + \cos^5 v\sin^4 u + \cos v\sin^4 v]$

$$\therefore \iint_S F(s)\cdot N\,du\,dv = 9\times 27\int_{-\pi/2}^{\pi/2}\int_0^{2\pi} (\cos^5 v\cos^4 u + \cos^5 v\sin^4 u + \cos v.\sin^4 v)\,du\,dv$$

$$= \frac{2916\pi}{5}$$

Short Answer Type

1. What is the significance of Gauss divergence theorm?

Ans: The significance of Gauss divergence theorm is it convert a surface integration into a triple integration and vice versa.

→ It can be interpreted as a conservation law, which states that the volume integral over all the sources and sinks is equal to the net flow through the volume's boundary.

2. Using Green's theorm show that the area A of the plane region bounded by a curve C is given by

$A = \oint_C x\,dy - y\,dx.$

Proof: For a parametric curve $x = f(t)$ and $y = g(t)$ then the area is $A = \frac{1}{2}\oint_C x\,dy - y\,dx.$

We know that the Green's theorm is

$$\iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dx\,dy = \int_C f\,dx + g\,dy$$

Putting $f = -y$ and $g = x$ we get

$$\iint_R (1+1)\,dx\,dy = \int_C -y\,dx + x\,dy$$

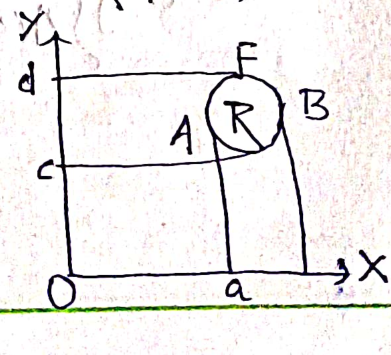
$$\Rightarrow 2\iint_R dx\,dy = \int_C x\,dy - y\,dx \Rightarrow 2A = \int_C x\,dy - y\,dx$$

$$\Rightarrow \boxed{A = \frac{1}{2}\oint_C x\,dy - y\,dx} \qquad \left[\because A = \iint_R dx\,dy\right]$$

3. State Green's theorm in a plane.

Let R be a closed bounded region in XY-plane whose boundary 'C' consists of finitely many smooth curves. Let $f(x,y)$ and $g(x,y)$ be functions that are continous and have continous partial derivatives. $\frac{\partial g}{\partial x}$ & $\frac{\partial f}{\partial y}$ every where in some domain R then

$$\int_C f\,dx + g\,dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dx\,dy$$



4. State Gauss divergence theorem.

Let V be a Closed and bounded region in space whose boundary is a smooth orientable surface S. Let $F(x,y,z)$ be a vector function which is continous and has continous first partial derivatives in some domain containing V then

$$\iiint_V div(F)dV = \iint_S F.\hat{n}\, dA$$

Where $\hat{n}$ is the outer unit normal vector of S.

5. State Stoke's Theorem.

Ans: Let S be a piecewise smooth oriented surface in space and let the boundary of S be a piecewise and smooth simple closed curve C. Let $F(x,y,z)$ be a continous vector function that has continous first partial derivatives in space containing 'S'. Then in a certain domain

$$\iint_S Curl F \hat{n}\, dA = \oint_C F.r'(s)ds$$

Where $\hat{n}$ is the unit normal of S.

6. What is the surface Area of the surface S whose equation is $f(x,y,z)=0$

Ans: The given equation is $f(x,y,z)=0$

Let the surface take $z = f(x,y)$.

Then its parametric form is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Put $x = u$, $y = v$, $z = f(u,v)$

So $\vec{r} = u\hat{i} + v\hat{j} + f(u,v)\hat{k}$ $\Rightarrow r_u = \hat{i} + f_u\hat{k}$

$r_v = \hat{j} + f_v\hat{k}$

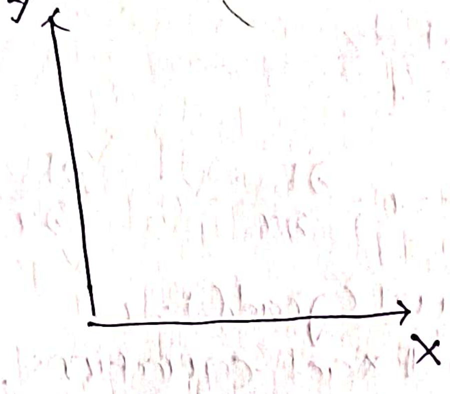
$$\therefore r_u \times r_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & f_u \\ 0 & 1 & f_v \end{vmatrix} = -f_u\hat{i} - f_v\hat{j} + \hat{k}$$

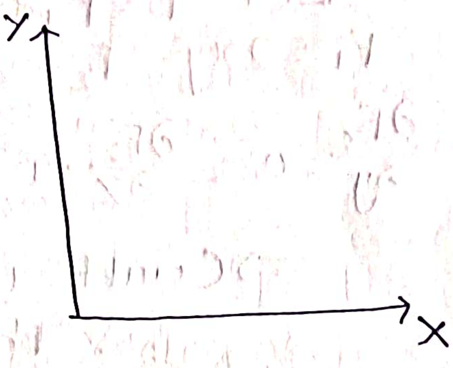
$\Rightarrow |r_u \times r_v| = \sqrt{1+(f_u)^2+(f_v)^2}$ $\Rightarrow |r_u \times r_v| = \sqrt{1+(f_x)^2+(f_y)^2}$

Hence $$\iint_S A(S)dA = \iint_S \sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}\, dxdy$$

7. Change the order of the integration $\int_0^2 \int_0^y f(x,y)\,dx\,dy$.

Ans:





$0 \le x \le 2$

$$\int_0^2 \int_x^2 f(x,y)\,dy\,dx$$

8. Find the unit normal vector to the right circular Cylinder.

Ans: $x^2+y^2=r^2 \qquad f(x,y,z) = x^2+y^2-r^2=0$

$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = (2x, 2y, 0) \qquad |\nabla f| = \sqrt{(2x)^2+(2y)^2} = 2\sqrt{x^2+y^2}$

$\hat{n} = \frac{\nabla f}{|\nabla f|} = \left(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}, 0\right)$

9. Find the unit normal vector to the sphere centre at origin and radius a.

Ans: $x^2+y^2+z^2=a^2 \qquad f = x^2+y^2+z^2-a^2$

$\nabla f = [2x, 2y, 2z] \qquad |\nabla f| = \sqrt{(2x)^2+(2y)^2+(2z)^2} = 2\sqrt{x^2+y^2+z^2}$
$= 2a$

$\hat{n} = \frac{\nabla f}{|\nabla f|} = \frac{(2x,2y,2z)}{2a} = \left(\frac{x}{a}, \frac{y}{a}, \frac{z}{a}\right)$

10. Check the Exactness of the differential
$F = 2xy^2dx + 2x^2ydy + dz$.

Ans: Let $F = F_1dx + F_2dy + F_3dz$

$F_1 = 2xy^2 \quad F_2 = 2x^2y \quad F_3 = 1$

$\frac{\partial F_3}{\partial y} = 0 = \frac{\partial F_2}{\partial z} = 0 \qquad \frac{\partial F_1}{\partial z} = 0 = \frac{\partial F_3}{\partial x} = 0 \qquad \frac{\partial F_2}{\partial x} = 4xy = \frac{\partial F_1}{\partial y}$

$\Rightarrow$ Curl F = 0 i.e curl(grad f) = 0.

$\therefore$ This is an Exact equations.

11. Find the area of the region in the first Quadrant within the cardiode $r = a(1-\cos\theta)$.

Ans: For the first Quadrant θ Varies from 0 to $\pi/2$.

$$A = \frac{1}{2}\oint_c r^2 d\theta = \frac{1}{2}\int_0^{\pi/2} a^2(1-\cos\theta)^2 d\theta$$

$$= \frac{a^2}{2}\int_0^{\pi/2}(1+\cos^2\theta - 2\cos\theta)d\theta$$

$$= \frac{a^2}{2}\int_0^{\pi/2}[1-2(\cos\theta)+\cos^2\theta]d\theta$$

$$= \frac{a^2}{2}\left[\int_0^{\pi/2}(1-2\cos\theta)d\theta + \int_0^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)d\theta\right]$$

$$= \frac{a^2}{2}[\theta - 2\sin\theta]_0^{\pi/2} + \frac{a^2}{2}\left[\frac{\theta}{2}+\frac{1}{4}\sin 2\theta\right]_0^{\pi/2}$$

$$= a^2\left(\frac{3\pi}{8}\right) - a^2 = a^2\left(\frac{3\pi}{8}-1\right).$$

12. Find the surface Area of the sphere $x^2+y^2+z^2=1$

$x^2+y^2+z^2 = 1^2$

$x^2+y^2+z^2 = r^2$

Subtract $\quad r^2 - 1 = 0 \Rightarrow r^2 = 1^2 \quad \boxed{r=1}$

Surface Area $= \iint ds = \int_0^{2\pi}\int_0^{\pi} r^2\sin\theta\, d\theta\, d\phi = \int_0^{2\pi}\int_0^{\pi} 1^2 \sin\theta\, d\theta\, d\phi$

$= \int_0^{2\pi}\int_0^{\pi}\sin\theta\, d\theta\, d\phi = \int_0^{2\pi}[-\cos\theta]_0^{\pi} d\phi$

$= \int_0^{2\pi}[-\cos\pi - \cos 0]d\phi = \int_0^{2\pi}[-(-1)-(-1)]d\phi$

$= \int_0^{2\pi} 2\, d\phi = 2\int_0^{2\pi} d\phi = 2[\phi]_0^{2\pi} = 2\times 2\pi = 4\pi$