ASSIGNMENT - 4

01. If $\bar{A} = t^2\hat{i} - t\hat{j} + (2t+1)\hat{k}$
and $\bar{B} = (2t-3)\hat{i} + \hat{j} - t\hat{k}$
find (a) $\frac{d}{dt}(\bar{A}\cdot\bar{B})$ (b) $\frac{d}{dt}(\bar{A}\times\bar{B})$ (c) $\frac{d}{dt}|\bar{A}+\bar{B}|$
(d) $\frac{d}{dt}\left(\bar{A}\times\frac{d\bar{B}}{dt}\right)$ at $t=1$

02. Given the curve $x = t^2+2$, $y = 4t-5$, $z = 2t^2 - 6t$
find the unit tangent vector at the point $t=2$.

03. Find the angle between the directions of the velocity and acceleration vectors at time t of a particle with position vector $\bar{r} = (t^2+1)\hat{i} - 2t\hat{j} + (t^2-1)\hat{k}$

04. Prove that $\frac{d}{du}(\bar{A}\times\bar{B}) = \bar{C}\times(\bar{A}\times\bar{B})$ if

$$\frac{d\bar{A}}{du} = \bar{C}\times\bar{A} \text{ and } \frac{d\bar{B}}{du} = \bar{C}\times\bar{B}$$

05. Find the directional derivative of $f(x,y,z) = 4e^{2x-y+z}$ at the point $(1,1,-1)$ in the direction toward the point $(-3,5,6)$.

06. Find the directional derivative of $f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$ at , the point $(3,0,4)$ in the direction toward the point $(1,1,1)$.

07. Find a unit normal to the surface $x^2y + 2xz = 4$ at the point $(2,-2,3)$

08. Determine a unit vector normal to the surface $xy^3z^2 = 4$ at the point $(-1,-1,2)$

09. Find the maximal directional derivative of x^3y^2z at $(1,-2,3)$

10. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z = x^2+y^2-3$ at the point $(2,-1,2)$.

11. If $\nabla f = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ find $f(x,y,z)$ if $f(1,-2,2)=4$.

12. Define directional derivative.

13. Define Divergence.

14. Define gradient.

15. Define curl.

16. Show that $\nabla\cdot(f\bar{A}) = 5f$ where $f = x^2+y^2+z^2$ and $\bar{A} = x\hat{i}+y\hat{j}+z\hat{k}$

18. If $f = x^2yz$ and $g = xy-3z^2$, Calculate $\nabla(\nabla f\cdot\nabla g)$!

19. Show that, $\nabla\cdot(\nabla f) = \frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}$

20. Show that $\operatorname{curl}(fv) = (\operatorname{grad} f)\times v + f\operatorname{curl} v$

21. Show that $\operatorname{div}(fv) = f\cdot\operatorname{div}(v) + v\cdot\nabla f$