01. Use L.T. to solve the following I.V.P. consisting of DE with IC

a. $y'' - 8y' + 15y = 9t \cdot e^{2t}$, $y(0) = 5$, $y'(0) = 10$

b. $y''' - 3y'' + 3y' - y = t^2 \cdot e^t$, $y(0) = 1$, $y'(0) = 0$, $y''(0) = 2$

c. $y'' + 6y' + 9y = 0$, $y(0) = 2$, $y'(0) = 9$

02. By using convolution theorem, find

a. $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$

b. $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$

c. $L^{-1}\left\{\frac{1}{(s^2+4)(s+1)^2}\right\}$

d. $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

e. $L^{-1}\left\{\frac{1}{(s+2)^2(s-2)}\right\}$

03. Find the inverse Laplace Transform of $f(t)$ of each of the following function $F(s)$:

a. $\frac{s+17}{(s-1)(s+3)}$ b. $\frac{2s+12}{s^2+6s+13}$ c. $\frac{s^3+6s^2+14s}{(s+2)^4}$

d. $\frac{s+1}{(s^2+1)(s^2+4)}$ e. $\frac{1}{s(s+1)^2}$

4. Evaluate : $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$

5. Evaluate : $L^{-1}\left\{\frac{s+2}{s^2(s+3)}\right\}$

6. Evaluate : $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

7. Evaluate : $L^{-1}\left\{\frac{s+1}{(s^2+2s+2)^2}\right\}$

8. Evaluate $L^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\}$

9. Evaluate $L^{-1}\left\{\frac{1}{(s^2+2s+5)^2}\right\}$

10. Evaluate $L^{-1}\left\{\frac{s}{(s+a)^2+5^2}\right\}$

11. Prove that, $L^{-1}\left\{\frac{a^3}{s^4-a^4}\right\} = \frac{1}{2}(\sinh at - \sin at)$

and hence evaluate, $L^{-1}\left\{\frac{64}{81s^4-256}\right\}$

12. Evaluate $L^{-1}\left\{\frac{3(s^2-2)^2}{2s^5}\right\}$

13. Prove that $L\left\{\frac{e^{-at}-e^{-bt}}{t}\right\} = \ln\left(\frac{s+b}{s+a}\right)$

and hence evaluate, $\int_0^\infty \frac{e^{-3t}-e^{-6t}}{t}\,dt$.

14. Evaluate $L\left\{\frac{\cos at - \cos bt}{t}\right\}$ and hence prove that $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt = \ln\frac{2}{3}$

15. Prove that $\int_0^\infty t \cdot e^{-3t} \sin t \, dt = \frac{3}{50}$

16. Prove that $\int_0^\infty t^2 e^{-4t} \cdot \sin 2t \, dt = \frac{11}{500}$

17. Evaluate, $L\{e^{2t}(3\sin 4t - 4\cos 4t)\}$

18. Evaluate, $L\{\cos 3t \cdot \cos 2t \cdot \cos t\}$

19. Evaluate, $L\{\cos^3 at\}$

20. Evaluate, $L\{\cosh at \cdot \cos bt\}$