Functional Dependency

⇒ Functional Dependency is a relationship that exist between two set of attribute.

⇒ It typically exist between primary key and key attribute or non-key attribute within a table.

$$X \xrightarrow{\text{determines}} Y$$

Determinant   determines   Dependent.

- X determines Y
- Y is determined by X.

⇒ It is denoted by $X \longrightarrow Y$.

⇒ **Example:**

- Assume that we have a student table that contains attributes RollNo, Name, Class, Mark, Address.
- Here RollNo attributes can uniquely identify the Name attribute of student table because if we know the RollNo, we can tell that name associated with it.
- Functional Dependency can be written as RollNo ⟶ Name.
- Here we can say Name is functionally dependent on RollNo.

#

x	y
1	A
1	A
2	B
3	C
4	D

$x \longrightarrow y$

FD: $x \longrightarrow y$

where $t_1.x = t_2.x$

then $t_2.y = t_2.y$

Student :-

Roll No	Name	Marks	Dept	Course
1.	a	78	CS	C_1
2.	b	60	EE	C_1
3	a	78	CS	C_2
4	b	60	EE	C_3
5	c	80	IT	C_3
6	d	80	EC	C_2

i) $\underset{x}{\underline{\text{RollNo}}} \longrightarrow \underset{y}{\underline{\text{Name}}}$

where $t_1 . x = t_2 . x$

then $t_2 . y = t_2 . y$.

As the where condition is ~~not~~ satisfied, ~~therefore~~ But then condition exist.

Thus, RollNo can determine marks.

Hence it is functionally dependent.

ii) Name $\longrightarrow$ RollNo.

∴ It is not funtionally dependent.

iii) RollNo $\longrightarrow$ Marks.

Yes it is functionally dependent.

iv) Dept $\longrightarrow$ Course.

No it is not functionally dependent.

iv) Marks $\longrightarrow$ Dept.

No ~~Yes~~ it is not funtionally dependent.

v) ~~RollNo~~ $\longrightarrow$

v) $\underbrace{\text{RollNo, Name}}_{x} \longrightarrow \underbrace{\text{Mark}}_{y}$

As there is no equal value in RollNo, Name . So, it can determines the marks .

Thus it is functionally dependent .

vi) Name $\longrightarrow$ Mark .

Yes it is functionally dependent .

vii) Name, Marks $\longrightarrow$ Dept .

Yes it is functionally dependent .

viii) Name, Marks $\longrightarrow$ Dept, Course .

No, it is not functionally dependent .

TYPES OF FUNCTIONAL DEPENDENCY :-

1) TRIVIAL FUNCTIONAL DEPENDENCY
2) NON-TRIVIAL FUNCTIONAL DEPENDENCY

1) Trivial Functional Dependency -

$A \longrightarrow B$, $B \subseteq A$.

$\Rightarrow$ A determines B has trivial functional dependency if B is the subset of A .

$\Rightarrow$ $A \rightarrow A$, if the attribute can determines itself .

2) Non-Trivial Functional Dependency -

$\Rightarrow$ $A \rightarrow B$, A determines B has non-trivial functional dependency, if B is not a subset of A .

$\Rightarrow$ When $A \cap B = \phi$, then $A \rightarrow B$ is called complete Non-trivial

Example :-

1) {Emp_id, Emp_Name} $\longrightarrow$ Emp_id

⇒ It is trivial function.

2) SID $\longrightarrow$ SNAME

⇒ It is not trivial function.

INFERENCE RULE :- (ARMSTRONG'S AXIOMS RULE)

⇒ The Armstrong's axioms are the basic inference rule.

⇒ Armstrong's axioms are used to conclude functional dependencies on a relational database.

⇒ The inference rule is a type of assertion. It can apply to a set of FD (functional dependency) to derive other FD.

⇒ Using the inference rule, we can derive additional functional dependency from the initial set.

⇒ ~~In the reflexi~~

Rule 1 (Reflexivity Rule) :-

⇒ In the reflexive rule, if Y is a subset of X, then X determines Y.

⇒ Notation: If $Y \subseteq X$ then $X \longrightarrow Y$ and $X \rightarrow X$ also.

Ex:

$X = \{A, B, C, D\}$.

$Y = \{C, A, B\}$

Rule 2 (Transitive Rule):-

⇒ In the transitive rule, if X determines Y and Y determines Z, then X must determine Z.

⇒ If $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$.

⇒ Example:
If RollNo → RegdNo and RegdNo → Name, then RollNo → Name also holds.

Rule 3 (Augmentation Rule):-

⇒ The augmentation is also called as a partial dependency. In augmentation, if X determines Y, then XZ determines YZ for any Z.

⇒ If $X \rightarrow Y$ then $XZ \rightarrow YZ$

⇒ Example:
for $R(ABCD)$, if $A \rightarrow B$, then $AC \rightarrow BC$

Rule 4 (Union Rule):-

⇒ Union rule says, if X determines Y and X determines Z, then X must determine Y and Z.

⇒ If $X \rightarrow Y$ and $X \rightarrow Z$ then $X \rightarrow YZ$.

Rule 5 (Decomposition Rule):-

⇒ Decomposition rule is also known as project rule. It is the reverse of union rule.

⇒ This rule say, if X determines Y and Z then X determines Y and X determines Z.

⇒ If $X \rightarrow YZ$ then $X \rightarrow Y$ and $X \rightarrow Z$.

Rule 6 (Pseudo transitivity Rule):-

⇒ In Pseudo transitive Rule, if X determines Y and WY determines Z, then WX determines Z.

⇒ If $X \rightarrow Y$ and $WY \rightarrow Z$ then $WX \rightarrow Z$.

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Q R(A,B,C,D,E)

FD: {A → B, B → C, C → D, D → E}

~~A A → B, C, D, E.~~

A → A, B → B, C → C, D → D, E → E.

A → C, B → D, C → E, D → D.

A → B, B → C, C → D

A → D, B → E

A → E,

i) A → A, B, C, D, E

ii) B → C, D, E, B

iii) C → D, E, C

iv) D → DE

v) E → E

vi) $(AB)^+$ → ABCDE

vii)

Q R(A,B,C,D,E)

FD: {A → B, B → C, C → D, D → E}

A → A, B → B, C → C, D → D, E → E

A → C, B → C, C → D, D → E

A → B, B → D, C → E

A → D, B → E,

A → E,

i) A → ABCDE

ii) B → BCDE

iii) C → CDE

iv) D → DE

v) E → E

$X^+ \longrightarrow$ Set of attributes.

$\Rightarrow$ It will contain set of attribute determine by X.

Now,

CK $A^+ \rightarrow \{A, B, C, D, E\}$ → SK , $B^+ = \{B, C, D, E\}$

$(AD)^+ \rightarrow \{A, D, B, C, E\}$ → SK , $(CD)^+ = \{C, D, E\}$

$(AB)^+ \rightarrow \{A, B, C, D, E\} \rightarrow$ SK , $BCDE^+ = \{B, C, D, E\}$

Super Key :-

$\Rightarrow$ Super Key is a set of attribute whose closure contain all the attribute of given relation.

Q2 $R(A, B, C, D, E)$

FD : $\{A \rightarrow B, D \rightarrow E\}$.

~~A → A, B~~

$ABCDE^+ = \{A, B, C, D, E\} \longrightarrow$ SK

$ABDE^+ = \{A, B, D, E\}$ ×

$ACDE^+ = \{A, C, D, E, B\} \rightarrow$ SK

$ACD^+ = \{A, C, D, E, B\} \rightarrow$ SK $\longrightarrow$ Candidate Key.

$A^+ = \{A, B\}$, $\{AC\}^+ = \{A, B, C\}$

$C^+ = \{C\}$, $\{AD\}^+ = \{A, B, D, E\}$

$D^+ = \{D, E\}$, $\{CD\}^+ = \{C, D, E\}$

* $ABCDE^+$

$A^+ = \{A, B\}$	$AB^+ = \{A, B\}$	$BD^+ = \{B, D, E\}$	$ACE^+ = \{A, C, E$
$B^+ = \{B, E$	$AC^+ = \{A, B, C\}$	$BE^+ = \{B, E\}$	
$C^+ = \{C\}$.	$AD^+ = \{A, B, D, E\}$	$CD^+ = \{C, D, E\}$	$ADE^+ = \{A, D, E$
$D^+ = \{D, E\}$	$AE^+ = \{A, B, E\}$	$CE^+ = \{C, E\}$,	
$E^+ = \{E\}$	$BC^+ = \{B, C\}$	$DE^+ = \{D, E\}$.	
$ABCD^+ = \{A, B, C, D, E\}$	$ABC^+ = \{A, B, C\}$	$ABE^+ = \{A, B, E\}$	
	$ABD^+ = \{A, B, D, E\}$	$ACD^+ = \{A, C, D, E$	

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Q. R(A, B, C, D) –

FD: {A → B, B → C, C → A}.

$A\cancel{B}CD^+ = \{A, B, C, D\} \longrightarrow SK.$

$ACD^+ = \{A, B, C, D\} \longrightarrow SK.$ / $A\cancel{C}D^+ = \{A, B, C, D\}$

$AD^+ = \{A, B, C, D\}$.

$CD^+ = \{C, D\}$

$ABD^+ = \{A, B, C, D\}$ = {A

$AD^+ = \{A, B, C, D\}$.

* $ABCD^+$

* $A^+ = \{A, B, C\}$	$AB^+ = \{A, B, C\}$	✓ $BD^+ = \{B, C, A, D\}$
$B^+ = \{B, C, A\}$	$AC^+ = \{A, B, C\}$	✓ $CD^+ = \{C, A, B, D\}$
$C^+ = \{C, A, B\}$	✓ $AD^+ = \{A, B, C, D\}$	
* $D^+ = \{D\}$	$BC^+ = \{B, C, A\}$	

$ABC^+ = \{A, B, C\}$

✓ $ACD^+ = \{A, B, C, D\}$

✓ $BCD^+ = \{B, C, D, A\}$

✓ $ABD^+ = \{A, B, C, D\}$

✓ $ABCD^+ = \{A, B, C, D\}$.

B

Prime Attribute = {A, D} C, B

Candidate Key => AD → CK

↓

CD (Candidate Key).

↓

BD (Candiate Key)

↓

AD → CK

Q. $R(A,B,C,D)$

FD: $\{AB \to CD,\ D \to B,\ C \to A\}$.

$\begin{cases} AB \to D \\ AB \to C \end{cases}$

$\cancel{A}BCD^+ = \{A,B,C,D\}$

$\cancel{B}CD^+ = \{D,B,C,A\}$

$CD^+ = \{C,A,D,B\}$

Prime Attribute = $\langle C, D, A,$

Candidate Key = CD (Candidate Key)

$\downarrow$

AD

$\downarrow$

AB

Q. $R(A, B, C, D)$

$\begin{cases} AB \to C \\ AB \to D \end{cases}$

FD: $\{AB \to CD,\ D \to B,\ C \to A\}$

$ABCD^+ \{A, B, C, D\}$

$AB\cancel{D}^+ = \{A, B, C, D\}$.

$AB = \{A, B, C, D\}$ $\rightarrow$ SK, $\rightarrow$ CK

$A^+ = \{A\}$

$B^+ = \{B\}$

AB

$\downarrow$

CB

$\downarrow$

AB

$\downarrow$

CD

$\downarrow$

Q $R(A, B, C, D, E, F)$

i) FD: $\{AB \to C, C \to DE, E \to F, D \to A, C \to B\}$

ii) FD: $\{AB \to C, C \to DE, E \to F, \cancel{A \to A}, \cancel{C \to B}\}$.

$AB\cancel{C}DEF^+ = \{A, B, C, D, E, F\}$

$ABDE\cancel{F}^+ = \{A, B, C, D, E, F\}$

$AB\cancel{D}E^+ = \{A, B, C, D, E, F\}$

$AB\cancel{E}^+ = \{B, D, E, F, C, A\}$

$AB^+ = \{A, B, C, D, E, F\}$.

$A^+ = \{A\}$

$B^+ = \{B\}$

$AB \to DE$
$AB \to F$
$C \to F$
$C \to D$
$C \to E$.

Prime Attribute $(A, B, \cancel{D}, \cancel{C})$

AB → DB → AB → C → AB → AC

AB → DB → DC → C^+

$C^+ = \{C, D, A, B, E, F\}$

$DB^+ = \{D, B, A, C, E, F\}$

$D^+ = \{D, A\}$

$B^+ = \{B$

Candidate Key.
↳ AB, DB, C

$DE^+ = \{A, D, E, F\}$

$AC^+ = \{A, C, B, D, E, F\}$

$A^+ = \{A\}$

$C^+ = \{C, B, D, E, F, A\}$

Q $R(A,B,C,D,E)$

FD: $\{A \rightarrow B,\ B \rightarrow E,\ C \rightarrow D\}$

$A^+ = \{A,B,E\}$

$B^+ = \{B,E\}$

$C^+ = \{C,D\}$.

$D^+ = \{D\}$.

$E^+ = \{E\}$.

$A\cancel{B}CDE^+ = \{A,B,C,D,E\}$.

$AC\cancel{D}E^+ = \{A,B,C,D,E\}$.

$AC\cancel{E}^+ = \{A,B,C,D,E\}$.

$AC^+ = \{A,B,E,C,D\}$. Prime Attribute $\langle A,C \rangle$

$A^+ = \{A,B,E\}$ $AC \rightarrow$ (Candidate Key)

$C^+ = \{C,D\}$.

Q $R(A,B,C,D,E,F)$

FD: $\{A \rightarrow BC,\ B \rightarrow D,\ C \rightarrow DE,\ BC \rightarrow F\}$

$\left\{ \begin{array}{l} A \rightarrow B,\ A \rightarrow C \\ C \rightarrow D,\ C \rightarrow E \\ A \rightarrow F \end{array} \right.$

$A\cancel{B}\cancel{C}DEF^+ = \{A,B,C,D,E,F\}$.

$ABC\cancel{D}EF^+ = \{A,B,C,D,E,F\}$

$ABCE\cancel{F}^+ = \{A,B,C,D,E,F\}$.

$A\cancel{B}\cancel{C}E^+ = \{A,B,C,D,E,F\}$.

$AE^+ = \{A,B,C,D,E,F\}$. Prime Attribute $\langle A,E \rangle$.

$A^+ = \{A,B,C,D,E,F\}$. $A \rightarrow$ Candidate Key

$E^+ = \{E\}$

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Q $R(P, Q, R, S, T, U, V, W, X, Y)$

FD: $\{PQ \rightarrow R,\ PS \rightarrow VW,\ QS \rightarrow TU,\ P \rightarrow X,\ W \rightarrow Y\}$

$\{PS \rightarrow V,\ PS \rightarrow W,\ QS \rightarrow T,\ QS \rightarrow U,\ PS \rightarrow Y$

$PQ\cancel{R}STUVWXY^+ = \{P, Q, R, S, T, U, V, W, X, Y\}$

$PQS\cancel{T}\cancel{U}VWXY^+ = \{P, Q, R, S, T, U, V, W, X, Y\}$.

$PQSVW\cancel{X}Y^+ = \{P, Q, R, S, V, W, T, U, X, Y\}$.

$PQSVW\cancel{Y}^+ = \{P, Q, R, S, V, W, Y, T, U, X\}$

$PQS\cancel{V}\cancel{W}^+ = \{P, Q, R, S, V, W, T, U, X, Y\}$

$PQS\cancel{V}^+ = \{P, Q, R, V, W, T, U, X, Y\}$.

$P^+ = \{P, X\}$

$Q^+ = \{Q\}$

$S^+ = \{S\}$

$PQ^+ = \{P, Q, R, X\}$

$PS^+ = \{P, S, X, V, W, Y\}$

Prime Attribute

$\langle P, Q, S \rangle$

$PQS \rightarrow$ Candidate Key