UNIT - 4/5

1) A stenographer claims that she can take decision at the rate of 120 wpm. Can we reject her claim on the basis of 100 trails in which she demonstrate a mean of words with standard deviation of $\alpha = 5\%$.

A) $\mu_0 = 120$ $\quad\quad H_0 : \mu = 120$

$n = 100$ $\quad\quad H_1 : \mu \neq 120$

$\alpha = 5\%$

$\sigma = 0.05 \times 120 = 6$

$$z = \frac{\bar{x}^2 - \mu_0}{\sqrt{\frac{\sigma^2}{n}}} \Rightarrow$$

2) It is claimed that the random sample of 100 types with the mean life 15269 kms is drawn from a population of typtyres which has a mean life of 15200 kms and standard deviation of 1248 kms. Test the validity of the claim at 1% level of significance.

A) $n = 100$ $\quad\quad H_0 : \mu = 15200$

$\bar{x} = 15269$ $\quad\quad H_1 : \mu \neq 15200$

$\mu_0 = 15200$

$\sigma = 1248$

$\alpha = 0.01$ $\quad\quad z = \frac{\bar{x} - \mu_0}{S.E.}$; $S.E = \sqrt{\frac{\sigma^2}{n}} \Rightarrow \frac{1248}{10} \Rightarrow 124.8$

$$\Rightarrow z = \frac{15269 - 15200}{124.8} \Rightarrow 0.552 < 2.576$$

$\therefore$ So the null hypothesis is accepted

3) A weighing machine without any display is used by an average of 320 persons a day with a standard deviation of 50 persons. When an attractive display was used on the machine, the average for 100 days increased by 15 persons. Can we say that the display did not help much? $\alpha = 0.05$.

A) $\mu_0 = 320$ person/day

$\sigma = 50$ person

$n = 100$

$\bar{x} = 320 + 15 = 335$

$\alpha = 0.05$

$H_0 : \mu = 320$

$H_1 : \mu \ngtr 320$

$$z = \frac{\bar{x} - \mu_0}{\sqrt{\frac{\sigma^2}{n}}} \quad ; \quad \sqrt{\frac{\sigma^2}{n}} = \frac{50}{10} \Rightarrow 5$$

$$\Rightarrow z = \frac{335 - 320}{5} \Rightarrow \frac{15}{5} \Rightarrow 3 > \underline{1.645}$$

$\therefore$ The null hypothesis is rejected and alternate hypothesis is accepted.

4) A coin is tossed 900 times and head appeared 490 times. Does the result support the hypothesis that the coin is unbaised? Use 5% level of significance.

A) $n = 900$

$x = 490$

$p = 0.5$

$\alpha = 0.05$

$H_0 : P = 0.5$ (unbaised)

$H_1 : P \neq 0.5$

$$z = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} \quad ; \quad \hat{p} = \frac{x}{n} = \frac{490}{900} \Rightarrow 0.54$$

$$\Rightarrow \frac{0.544 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{900}}} \Rightarrow 2.66 > \underline{1.96}$$

$\therefore$ The null hypothesis is rejected and alternate hypothesis is accepted.

5) A sample of 400 parts manufactured by a factory, the number of defective parts was found to be 30. The Company, however claimed that at most 5% of their product is defective. ~~At~~ Is the claim tenable?

A) $n = 400$

$x = 30$

$\hat{p} = \frac{x}{n} = \frac{30}{400} = 0.075$

Company claim: $P_0 < 0.05$

$\alpha = 0.05$

$H_0: P = 0.05$

$H_1: P > 0.05$

$$Z = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} \quad ; \quad SE = \sqrt{\frac{P_0(1-P_0)}{n}} \Rightarrow \sqrt{\frac{0.05 \times 0.95}{400}} \Rightarrow 0.0109$$

$$\Rightarrow Z = \frac{0.075 - 0.05}{0.0109} \Rightarrow 2.29 > 1.645$$

∴ Null hypothesis rejected and alternative hypothesis is accepted.

6) In a random sample of 400 person from a large population, 120 are females. Can it ~~said~~ that ~~male~~ males and females are in ratio 5:3 in the population? use 1% of level of significance

A) $n = 400$

$x = 120$

$\alpha = 0.01$

$H_0: P = 0.375$

$H_1: P \neq 0.375$

$P_0 = \frac{3}{3+5} = \frac{3}{8} = 0.375$

$\hat{p} = \frac{x}{n} = \frac{120}{400} = 0.3$

$$Z = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} \quad ; \quad SE = \sqrt{\frac{P_0(1-P_0)}{n}} \Rightarrow \sqrt{\frac{0.375 \times 0.625}{400}} \Rightarrow 0.024$$

$$\Rightarrow Z = \frac{0.3 - 0.375}{0.024} \Rightarrow -3.10 \Rightarrow |Z| > 2.576$$

∴ Hence Null hypothesis is rejected and alternate hypothesis is accepted.

7) In big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that majority of men in the city smokes

A) $x = 325$

$n = 600$

$\hat{p} = \frac{325}{600} = 0.541$

$0.541 > 0.5$

∴ Hence the conclusion is true

H_0 is accepted.

9) A radio shop sells, on an average 200 radio per day with standard deviation 50. After an extensive advertising campaign, the management will compute the average sales for the next 25 days to see whether an improvement has occurred. Assume that the daily sales of significance if the sample average is 216

A) $\mu_0 = 200$

$\sigma = 50$

$n = 25$

$\bar{x} = 216$

$\alpha = 0.05$

$H_0 : \mu = 200$

$H_1 : \mu > 200$

$$Z = \frac{\bar{x} - \mu_0}{\sqrt{\frac{\sigma^2}{n}}} \quad ; \quad SE = \frac{\sigma}{\sqrt{n}} = \frac{50}{\sqrt{25}} = 10$$

$$\Rightarrow Z = \frac{216 - 200}{10} \Rightarrow 1.6 < \underline{1.645}$$

∴ Hence the null hypothesis is accepted.

10) Following information is related to 2 places A & B test whether there is any significance between their mean wages. use $\alpha = 5\%$.

	mean wage	standard deviation	no. of workers
A	47	28	1000
B	49	40	1500

A) $\bar{x}_A = 47$, $S_A = 28$, $n_A = 1000$

$\bar{x}_B = 49$, $S_B = 40$, $n_B = 1500$

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A \neq \mu_B$

$$Z = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}} \quad ; \quad SE = \sqrt{\frac{28^2}{1000} + \frac{40^2}{1500}} \Rightarrow 1.360$$

$$\Rightarrow Z = \frac{47 - 49}{1.360} \Rightarrow -1.47 \Rightarrow |Z| < \underline{1.96}$$

∴ Hence the null hypothesis is accepted

12) A company has head office at kolkata and a branch at mumbai. The personal director want to know if the workers at the two places would like the introduction of the new plan work and a survey has conducted for this purpose. Out of sample of 500 workers at kolkata 62% favor the new plan. At mumbai out of 400 workers 41% were against the new plan. Is there any significance difference b/w the two groups in their attitude towards the new plan at 5% level

A) $n_1 = 500$, $\hat{P}_1 = 0.62$ $\quad H_0 : P_1 = P_2$

$n_2 = 400$, $\hat{P}_2 = 0.59$ $\quad H_1 : P_1 \neq P_2$

$\alpha = 0.05$

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}(1-\hat{P})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \; ; \quad \hat{P} = \frac{x_1 + x_2}{n_1 + n_2} \Rightarrow \frac{546}{900} \simeq 0.6067$$

$$SE = \sqrt{0.6067 \times 0.3933\left(\frac{1}{500} + \frac{1}{400}\right)} \Rightarrow 0.0328$$

$$\Rightarrow Z = \frac{0.62 - 0.59}{0.0328} \Rightarrow 0.915 < 1.96$$

∴ Hence null hypothesis is accepted.

13) A machine puts out 16 imperfect article in the sample of 500, after the machine is overhauled ~~2 we~~ it puts out 3 imperfect articles in the batch of 100. Has the machine improved? use a 5% level significance.

A) $n_1 = 500$, $x_1 = 16 \Rightarrow \hat{P}_1 = 16/500 \Rightarrow 0.032$

$n_2 = 100$, $x_2 = 3 \Rightarrow \hat{P}_2 = 3/100 \Rightarrow 0.030$

$\alpha = 0.05$

$H_0 : P_1 = P_2$, $H_1 : P_1 > P_2$

$$\hat{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{16+3}{600} \Rightarrow 0.0316 \; ; \; SE = \sqrt{\hat{P}(1-\hat{P})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$= 0.01918$$

$$\Rightarrow Z = \frac{\hat{P}_2 - \hat{P}_1}{SE} \Rightarrow \frac{0.030 - 0.032}{0.01918} \Rightarrow -0.1043$$

$\Rightarrow |Z| < 1.645$

∴ Hence the null hypothesis is accepted.

15) what do you mean by (i) level of significance (ii) critical value.

A) The level of significance, usually denoted by α (alpha) is the maximum probability of making a Type I error - that is rejecting a true null hypothesis.

⇒ The critical value of ~~the~~ is the cutoff point (or boundary) on the statistical distribution that separates the ~~reg~~ rejection from the acceptance region for the null hypothesis.

16) Explain the procedure of testing the hypothesis.

A) Testing of hypothesis involve

* state the null & alternate hypothesis
  - H_0 ⇒ statement of no difference.
  - H_1 ⇒ It is what we want to test.
* choose the level of significance.
* select the appropriate test statistic.
* Compute the test ~~statistic~~ by

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

* find the critical value and define the rejection region
* Make the decision
* Draw a Conclusion.

17) In a city a sample of 1000 people were taken and out of them 540 are vegetarian and the rest are non veg. Can we say that both habit of eating are equally popular in the city.

A) $n = 1000$, $x = 540$ ⇒ $\hat{p} = 540/1000$ ⇒ 0.54

$H_0 : P = 0.5$

$H_1 : P \neq 0.5$

$$\Rightarrow Z = \frac{\hat{P} - P_0}{SE}, \quad SE = \sqrt{\frac{P_0(1-P_0)}{n}} \Rightarrow \sqrt{\frac{0.5 \times 0.5}{1000}} \Rightarrow 0.0158$$

$$\Rightarrow Z = \frac{\hat{P} - P_0}{SE} \Rightarrow \frac{0.54 - 0.5}{0.0158} \simeq \underline{2.53 > 1.96}$$

∴ Hence the null hypothesis is rejected & alternate hypothesis is accepted.

19) In a random sample of 500 men 300 are found to be smoker. In another random sample of 1000 men 550 are found to be smoker. Do the data indicate that the two set of men are significantly difference with respect to the privalence of smoking among men.

A) $n_1 = 500$, $x_1 = 300$, $\hat{P}_1 = 0.60$ $\quad$ $H_0: P_1 = P_1$

$n_2 = 1000$, $x_2 = 550$, $\hat{P}_2 = 0.55$ $\quad$ $H_1: P_1 \neq P_2$

$$\hat{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{850}{1500} = 0.5666$$

$$SE = \sqrt{\hat{P}(1-\hat{P})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \Rightarrow 0.0271$$

$$\Rightarrow Z = \frac{\hat{P}_1 - \hat{P}_2}{SE} \Rightarrow \frac{0.60 - 0.55}{0.02714} \Rightarrow 1.84 < 1.96$$

∴ Hence The null hypothesis is accepted.

20) A simple sample of height of 6400 Englishman has mean of 67.85 inches and a s.d of 2.56 inch, while a simple sample of height of 1600 indians has a mean of 68.55 inches and the s.d is 2.52 inches. Does the data indicates that Indians are on the average taller then Englishmen?

A) $n_1 = 6400$, $x_1 = 67.85$, $S_1 = 2.56$ $\quad$ $H_0: \mu_1 = \mu_2$

$n_2 = 1600$, $x_2 = 68.55$, $S_2 = 2.52$ $\quad$ $H_1: \mu_1 > \mu_2$

$$SE = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \Rightarrow 0.070661$$

$$Z = \frac{x_2 - x_1}{SE} \Rightarrow \frac{68.55 - 67.85}{0.070661} \Rightarrow 9.906 \gg 1.645$$

∴ Hence null hypothesis is rejected.

21) In one sample of 10 observations from a normal population, the sum of square of deviations of the sample values from the sample mean is 102.4 and in another sample of 12 observations from another normal population the sum of square of deviations of the sample value from the sample mean is 120. Examin whether the two normal populations have the same variance.

A) $n_1 = 10, \quad \Sigma(x-\bar{x})^2 = 102.4$

$n_2 = 12, \quad \Sigma(y-\bar{y})^2 = 120$

$$s_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1 - 1} \Rightarrow 11.377$$

$$s_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2 - 1} = \frac{120}{11} \Rightarrow 10.9090...$$

$$F = \frac{s_1^2}{s_2^2} = \frac{11.377}{10.9090} \Rightarrow 1.04296$$

$df_1 = 9, \ df_2 = 11.$

$H_0 : \sigma_1^2 = \sigma_2^2$ if $F > F_{1-\alpha/2}(9, 11)$

The observed $F \simeq 1.043$ is well inside the acceptance interval $(0.33, 3.3)$.

∴ Hence null Hypothesis is accepted.