Mathematical Property of Arithmetic Mean.

Property No-1 :- The Algebraic sum of the deviation of the given ~~from their Arithm~~ set of observation from their Arithmetic mean is zero.

$$\sum f_i(x_i - \bar{x}) = 0$$

$$\sum f_i(x_i - \bar{x}) = \sum f_i x_i - f_i \bar{x}$$

$$= \sum f_i x_i - \bar{x}\sum f_i$$

$$= \sum f_i x_i - \frac{\sum f_i x_i}{\sum f_i} \times \sum f_i$$

$$= \sum f_i x_i - \sum f_i x_i$$

$$= 0$$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Property No-2 :- If m, n are the size and x, y are the respective mean of two groups then the mean of z of the combined group size is $\bar{x} = \frac{m\bar{x} + n\bar{y}}{m+n}$.

where, $\bar{x}$ = mean of x

$\bar{y}$ = mean of y

m = no. of data in x

n = no. of data in y

$m+n$ = no. of data in z

Proof : sum of m observation and its mean

$$\bar{x} = \frac{x_1 + x_2 + \cdots\cdots + x_m}{m}$$

$$x_1 + x_2 + \cdots\cdots x_m = m\bar{x}$$

similarly of n observation and its mean.

$$\bar{y} = \frac{y_1 + y_2 + \cdots\cdots + y_n}{n}$$

$$y_1 + y_2 + \cdots\cdots + y_n = n\bar{y}$$

Sum of x observation and y observation is

$$\bar{z} = \frac{x_1 + x_2 + x_3 \cdots\cdots + x_m + y_1 + y_2 + y_3 \cdots\cdots + y_n}{m+n}.$$

$\bar{z} = \frac{m\bar{x} + n\bar{y}}{m+n}$. Generalised formula $= \frac{m_1x_1 + m_2x_2 + \cdots + m_nx_n}{m_1 + m_2 + m_3 \cdots + m_n}$.

Property No-3 :

The sum of the square of the deviations of the given set of observation is minimum when taken from the arithmetic mean

Let $S = \sum f_i (x_i - A)^2$

$S_1 = \sum f_i (x_i - \bar{x})^2$

then, $S_1 < S$

Proof, Here $S = \sum f_i (x_i - A)^2$

$$\frac{dS}{dA} = 2\sum f_i (x_i - A)(-1)$$

$$2\sum f_i (x_i - A) = 0$$

$$\cancel{x_i - A}$$

$$\sum f_i x_i - \sum f_i A = 0$$

$A \sum f_i = \sum f_i x_i$ critical point $\bar{x}$

$$A = \frac{\sum f_i x_i}{\sum f_i} = \bar{x}$$

$$\frac{d^2S}{dA^2} = -2\Sigma f_i x_i - 2\Sigma f_i A$$

$$-2\frac{d}{dA} = -2[\Sigma f_i x_i - \Sigma f_i A]$$

$$-2 \times -\Sigma f_i = 2\Sigma f_i > 0.$$

Hence, ~~the~~ at the function, $S = \Sigma f_i (x_i - A)^2$ the critical point is $\bar{x}$ which is minimum at $\boxed{2\Sigma f_i < 0}$

Mathematically,

$$S_{min} = 2\Sigma f_i > 0$$