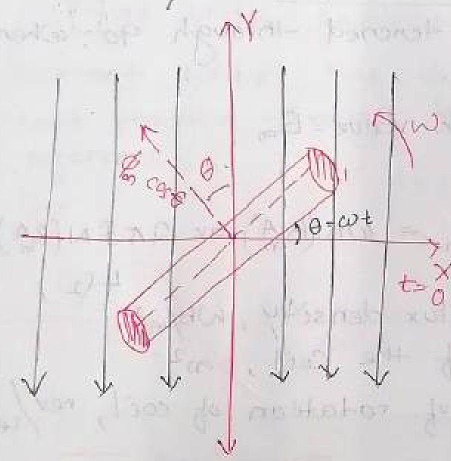
UNIT-II
SINGLE - PHASE AC Circuits

A.C. fundamentals →

Alternating voltage may be generated by rotating a coil in a magnetic field or by rotating a magnetic field within a stationary coil.



Consider a rectangular coil having N turns & rotating in a uniform magnetic field, with an angular velocity of ω rad/sec

[Ang. velocity :-

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Let time (t) → x-axis.

ϕ_m → Max. flux w.r.t x-axis.

In time-t sec, this coil rotates through an angle $\theta = \omega t$. In this deflected position, the component of the flux which is $\perp^r$ to the plane of coil is

$$\phi = \phi_m \cos \omega t$$

Hence flux linkages of coil at any time are

$$N\phi = N\phi_m \cos \omega t$$

According to Faraday's laws of Electro-magnetic induction, the emf induced in the coil is given by the rate of change of flux-linkages of coil. Hence, the value of induced emf at this instant i.e $\theta = \omega t$

or instantaneous value of the emf is

$$e = -\frac{d}{dt}(N\phi) = -N\frac{d}{dt}(\phi_m \cos\omega t)$$

$$= -N\phi_m\omega(-\sin\omega t)$$

$$e = \omega N\phi_m \sin\omega t \text{ volt}$$

$$= \omega N\phi_m \sin\theta \quad \text{--- (1)}$$

when coil has turned through 90°; when $\theta = 90°$ then $\sin\theta = 1$.

so e has max. value $= E_m$.

eq^n (1) become

$$E_m = \omega N\phi_m = \omega N(B_m A) \text{ or } 2\pi f N(B_m A) \quad \text{--- (2)}$$

∵ $B_m \rightarrow$ Max. flux density, Wb/m^2
$A \rightarrow$ area of the coil, m^2
$f \rightarrow$ freq of rotation of coil, rev/sec

By putting eq^n (2) in (1)

$$\boxed{e = E_m \sin\theta = E_m \sin\omega t} \quad \text{--- (3)}$$

$$\parallel^{ly} \quad \boxed{i = I_m \sin\omega t} \quad \text{--- (4)}$$

eq^n of induced alternating current.

Some Common terms →

a) Cycle :- One complete set of +ve & -ve values of alternating quantity is known as cycle.

→ One complete cycle = 360° or 2π radians.

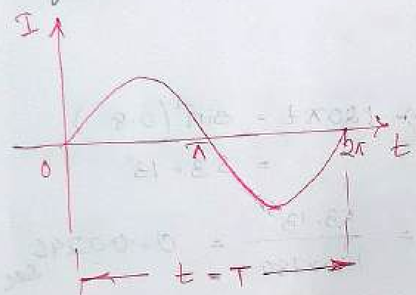
b) Time-period ⇒ The time-taken by an alternating quantity to complete one cycle is time-period (T).

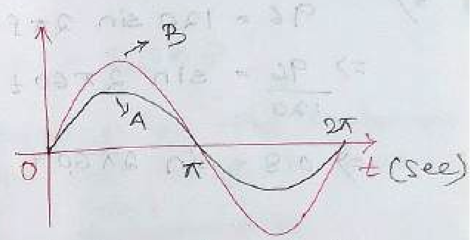
c) Frequency ⇒ The number of cycles/sec is called the freq. of unit = Hertz (Hz). 19

$$\boxed{f = \frac{1}{T}} \text{ or } \boxed{T = \frac{1}{f}}$$

d) Amplitude ⇒ The max. value, +ve or −ve of an alternating qty is known as its amplitude.

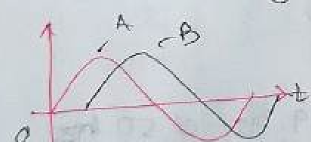
e) Phase ⇒ The fraction of T of that alternating current which has elapsed since the current last passed through the zero position of reference.





[A & B are in same phase]

f) Phase difference ⇒ "The angular phase diff. betn the max. possible value of the 2 alternating quantities having same freq".



[diff. phase or Out of phase]

1) An alternating current of freq 60 Hz has a max. value of 120 A. Write down the eqn for its instantaneous value. Reckoning time from the instant the current is zero & is becoming +ve. Find a) instantaneous value after 1/360 sec (b) time taken to reach 96 A for the 1st time.

sol^n $\quad f = 60\ hz$

$\omega = 2\pi f = 2\pi \times 60 = 377.1$

Instantaneous current eq^n is

$$I = I_m \sin \omega t$$

$$= 120 \sin 120\pi t$$

a) when $t = \frac{1}{360}$ s then

$$I = 120 \sin\left(120\pi \times \frac{1}{360}\right) = 120 \sin\left(\frac{\pi}{3}\right)$$

$$= \underline{103.9\ A} \quad (\text{in rad})$$

b) $\quad 96 = 120 \sin 2\pi f t$

$$\Rightarrow \frac{96}{120} = \sin 2\pi 60 t$$

$$\Rightarrow 0.8 = \sin 2\pi 60 t \Rightarrow 120\pi t = \sin^{-1}(0.8)$$

$$= 53.13^\circ$$

$$t = \frac{53.13}{120\pi} = \frac{53.13}{120 \times 180} = 0.00246 \text{ sec}$$

2) An ac is given by $i = 141.4 \sin 314t$, find out max. value, freq & time period.

$$i = 141.4 \sin 314 t$$

$$\Rightarrow i = I_m \sin \omega t$$

$I_m = 141.4\ A$, $\quad \omega = 314$

$$\Rightarrow 2\pi f = 314$$

$$\Rightarrow f = \frac{314}{2\pi} = 49.95 \simeq 50\ \text{hz}$$

$$T = \frac{1}{f} = \underline{0.02 \text{ sec}}$$

Root - Mean - Square (RMS) Value :-

The rms value of an alternating current is given by "steady (dc) current which when flowing through a given circuit for a given time produces the same heat as produced by AC when flowing through the same ckt for the same time."

↳ It's also known as the effective or virtual value of alternating current.

Mathematical expression :-

The standard form of a sinusoidal alternating current is

$$i = I_m \sin \omega t = I_m \sin \theta$$

The mean of the squares of the instaneous values of current over one complete cycle is

$$I_{rms}^2 = \int_0^{2\pi} \frac{i^2 d\theta}{2\pi} = \int_0^{2\pi} \frac{I_m^2 \sin^2\theta \, d\theta}{2\pi}$$

square root or $I_{rms} = \sqrt{\int_0^{2\pi} \frac{I_m^2 \sin^2\theta}{2\pi} d\theta}$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\int_0^{2\pi} 1 . d\theta - \int_0^{2\pi} \cos 2\theta \, d\theta\right]}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\theta\Big|_0^{2\pi} - \frac{\sin 2\theta}{2}\Big|_0^{2\pi}\right]}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[(2\pi - 0) - \frac{1}{2}(\sin 4\pi - \sin 0)\right]}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[2\pi\right]} = \sqrt{\frac{I_m^2}{2}} \Rightarrow \boxed{I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m}$$

RMS value of a complex wave ⇒

The rms value of a complex wave is equal to the square root of the sum of the squares of the rms value of its individual elements.

For eg.

$$i = 12 \sin \omega t + 6 \sin\left(3\omega t - \frac{\pi}{6}\right) + 4 \sin\left(5\omega t + \frac{\pi}{3}\right)$$

$$I_{rms} = \sqrt{\left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2}$$

$$= 9.74\ A$$

Average value ⇒ I_{avg}

"That steady current which transfers across any circuit the same charge as is transferred by that AC during same time"

$$I_{avg} = \frac{i_1 + i_2 \cdots + i_n}{n}$$

or

$$I_{avg} = \int_0^{\pi} \frac{i\, d\theta}{\pi}$$

$$= \int_0^{\pi} \frac{(I_m \sin\theta)}{\pi}\, d\theta$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{\pi}\left[-\cos\pi + \cos 0\right]$$

$$= \frac{I_m}{\pi}\left(+1 + (1)\right) = \frac{2I_m}{\pi}$$

$$\boxed{I_{avg} = \frac{2I_m}{\pi}} \text{ or } \frac{I_m}{\left(\frac{\pi}{2}\right)} = 0.637\, I_m$$

circuits

1) An alternating current varying sinusoidally with a freq of 50 Hz has an rms value of 20A. Write down the eq^n for the instantaneous value & find this value (a) 0.0025 sec (b) 0.0125 sec after passing through a positive max. value. At what time, measured from a +ve max. value, will the instaneous current be 14.14 A ?

$I_{rms} = 20$ A ; $f = 50$ Hz

$\Rightarrow I_m = \sqrt{2} I_{rms} = 28.2$ A ; $\omega = 2\pi f = 100\pi$ rad/s

$i = I_m \sin \omega t$

$= 28.2 \sin 100\pi t$ A

a) when $t = 0.0025$ s

$i = 28.2 \, \cancel{\sin}\, \cos \left[100\pi \times 0.0025\right]$

$= 19.94 \simeq 20$ A

ii) when $t = 0.0125$ s

$i = 28.2 \cos (100\pi \times 0.0125) = -19.94 \simeq -20$ A

iii) when $i = 14.14$ A

$14.14 = 28.2 \cos (100\pi t) \Rightarrow 0.5 = \cos (100\pi t)$

$\Rightarrow 100\pi t = \cancel{30°}\ 60°$

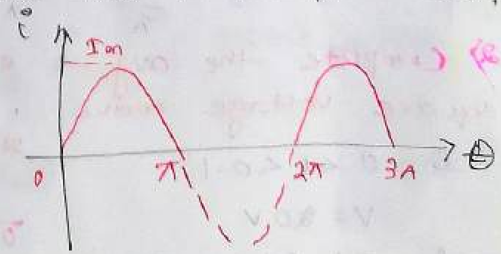
$t = \dfrac{60}{100\pi}$

$= \dfrac{1}{300}$ s.

RMS value of half-wave rectified AC current

↳ One half-cycle has been suppressed.

$I_{rms} = \sqrt{\int_0^{\pi} \dfrac{i^2 d\theta}{2\pi}}$



$= \sqrt{\int_0^{\pi} \dfrac{I_m^2 \sin^2\theta}{2\pi} d\theta}$

$= \sqrt{\dfrac{I_m^2}{2\pi} \int_0^{\pi} \left(\dfrac{1-\cos 2\theta}{2}\right) d\theta}$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\int_0^{\pi} 1.d\theta - \int_0^{\pi} \cos 2\theta \, d\theta\right]}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[(\pi - 0) - \frac{\sin 2\theta}{2}\Big|_0^{\pi}\right]}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\pi - \left\{\frac{\sin 2\pi - \sin 0}{2}\right\}\right]}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\pi\right]}$$

$$I_{rms} = \sqrt{\frac{I_m^2}{4}} \Rightarrow \boxed{I_{rms} = \frac{I_m}{2} = 0.5 I_m}$$

Similarly avg value $\boxed{I_{av} = \frac{I_m}{\pi}}$

1) An alternating voltage $e = 200 \sin 314t$ is applied to a device which offers an ohmic resistance of 20Ω to the flow of current in one dir^n, while preventing the flow of current in opposite dir^n. Calculate RMS value, avg value over one cycle.

Given :- $V_m = 200\,v$ $\quad e = 200 \sin 314t$

$R = 20\Omega$ $\quad V = V_m \sin \omega t$

$\omega = 314$

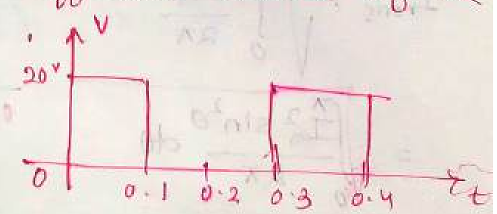
$$I_m = \frac{V_m}{R} = \frac{200}{20} = 10\ A$$

for one cycle,

$$I_{rms} = \frac{I_m}{2} = \frac{10}{2} = 5\ A$$

$$I_{avg} = \frac{I_m}{\pi} = \frac{10}{\pi} = 3.18\ A$$

2) Compute the avg. & effective values of the square voltage wave



for $0 < t < 0.1$

$V = 20\ V$

for $0.1 < t < 0.3$

$V = 0$

$T = 0.3\ s$

$$V_{avg} = \frac{1}{T}\int_0^T v\,dt = \frac{1}{0.3}\int_0^{0.1} 20\,dt$$

$$= \frac{1}{0.3}(20 \times 0.1) = 6.67\ V$$

$$v^2 = \frac{1}{T}\int_0^T v^2 dt = \frac{1}{0.3}\int_0^{0.1} 400\,dt = \frac{400}{0.3} \times 0.1 = 133.3$$

$V_{rms} = 11.5\ V$

Complex algebra ⇒

There are diff. forms of representation
→ i) symbolic notation
ii) Trigonometrical form
iii) Exponential " iv) Polar form.

$Z = a + jb \Rightarrow |Z| = \sqrt{a^2+b^2}$ $\underset{angle}{\angle z} = \phi = \tan^{-1}\left(\frac{b}{a}\right)$

∴ j = complex operator.
a = real b = img part.
= in-phase = Quadrature

→ Symbol j is used to indicate the counter clockwise rotation of a vector through 90°. It's assigned a value of $\sqrt{(1)}$

$j = \sqrt{-1} \Rightarrow \boxed{j^2 = -1}$

$\boxed{\frac{1}{j} = -j}$

i) Trigonometric form = $E = |E|(\cos\theta + j\sin\theta)$

ii) Exponential " = $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

$$E = |E|\, e^{\pm j\theta}$$

iii) Polar form = $E\angle\theta$

1) Write the equivalent exponential & polar forms of $(3+j4)$. $E = a_1 + jb$

Magnitude $= \sqrt{a^2+b^2} = \sqrt{9+16} = 5$

phase $= \theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$

Exponential form $= 5e^{j53.1^\circ}$

Polar $= 5\angle 53.1^\circ$

Addition $\Rightarrow E = E_1 + E_2 = (a_1+jb_1) + (a_2+jb_2)$

$= (a_1+a_2) + j(b_1+b_2)$

magnitude $= \sqrt{(a_1+a_2)^2 + (b_1+b_2)^2}$

Phase $\theta = \tan^{-1}\left(\frac{b_1+b_2}{a_1+a_2}\right)$

Subtraction $\Rightarrow E = E_1 - E_2 = (a_1+jb_1) - (a_2-jb_2)$

$= (a_1-a_2) + j(b_1-b_2)$

$|E| = \sqrt{(a_1-a_2)^2+(b_1-b_2)^2}$ // $\theta = \tan^{-1}\left(\frac{b_1-b_2}{a_1-a_2}\right)$

Multiplication $\Rightarrow$ (Rect. form)

$E = E_1 \times E_2 = (a_1+jb_1) \times (a_2+jb_2)$

$= (a_1a_2 - b_1b_2) + j(a_1b_2 + b_1a_2)$

Division $\rightarrow$ (Rect. form)

$$E = \frac{E_1}{E_2} = \frac{a_1+jb_1}{a_2+jb_2} = \frac{(a_1+jb_1)(a_2-jb_2)}{(a_2+jb_2)(a_2-jb_2)}$$

$$= \left(\frac{a_1a_2+a_1b_2}{a_2^2+b_2^2}\right) + j\left(\frac{b_1a_2+a_1b_2}{a_2^2+b_2^2}\right)$$

1) Add the following vectors given in rectangular form &

$A = 16 + j12 \quad B = -6 + j10.4$

$Z = A + B = (16+j12) + (-6+j10.4)$

$= (16-6) + j(12+10.4)$

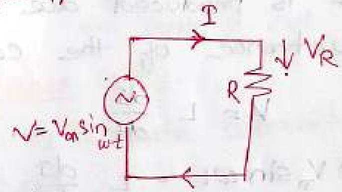
$= 10 + j\,22.4$

mag $|Z| = \sqrt{10^2 + 22.4^2} = 25.5$

$\theta = \tan^{-1}\left(\frac{22.4}{10}\right) = 65.95'$

AC through Pure R, L & C ⇒

AC through Pure resistance :-



Let $V = V_m \sin \omega t$ —(1)

I = rms value of the resultant Current

Voltage drop across R = $V_R = IR$

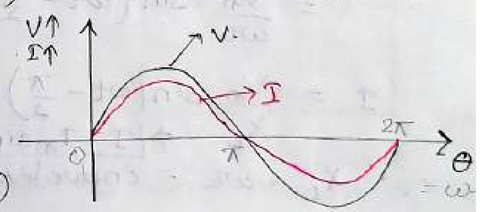
$= I_m$

$V = IR$

$\Rightarrow V_m \sin \omega t = IR$

$\Rightarrow I = \frac{V_m \sin \omega t}{R} \Rightarrow I = I_m \sin \omega t$ — (2)

from eq^n (1) & (2) ; Both V & I are in Same phase.



⟨Same Phase⟩

* Instantaneous Power $P = VI$

$= (V_m \sin \omega t)(I_m \sin \omega t)$

$= V_m I_m \sin^2 \omega t$

$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$

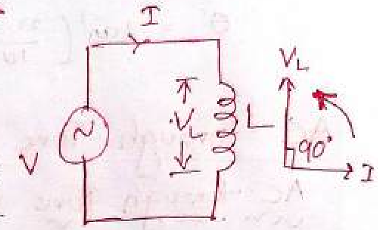
$= \underbrace{\frac{V_m I_m}{2}}_{\text{Const. Part (Real)}} - \underbrace{\frac{V_m I_m}{2}\cos 2\omega t}_{\text{Pulsating Component}}$ ——— (3)

for a Complete cycle, the avg value of $\frac{V_m I_m}{2}\cos 2\omega t$ is Zero. Hence,

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$\boxed{P = V_{rms} \times I_{rms}} \quad \text{--- (4)}$$

AC through Pure Inductance →



Whenever an alternating Voltage is applied to a Purely inductive coil, a back emf is produced due to self-inductance of the coil.

[The ckt which contains only L is called a Pure inductive ckt. Here, the current lags behind the voltage by 90°.]

$$V = L\frac{dI}{dt}$$

$$\Rightarrow V_m \sin\omega t = L\frac{dI}{dt}$$

$$\Rightarrow \frac{dI}{dt} = \frac{V_m}{L}\sin\omega t$$

$$\Rightarrow \int dI = \int \frac{V_m}{L}\sin\omega t\, dt$$

$$\Rightarrow I = \frac{V_m}{\omega L}(-\cos\omega t)$$

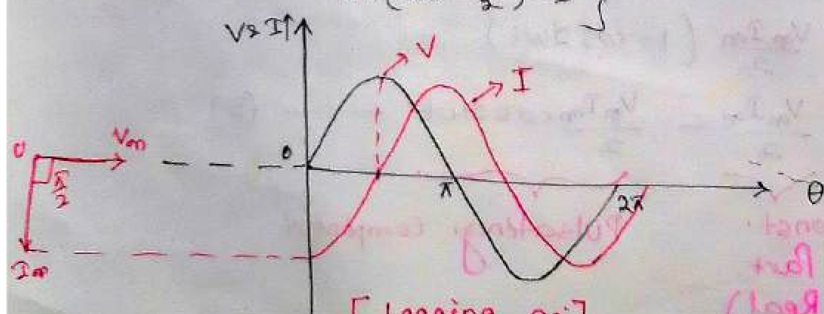
$$= \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$I = \frac{V_m}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{--- 5(a)}$$

$$\Rightarrow \boxed{I = I_m \sin\left(\omega t - \frac{\pi}{2}\right)} \quad \text{--- 5(b)}$$

$\therefore X_L = \omega L$ = inductive reactance.

$I_m = \frac{V_m}{X_L}$ {Value of I will be max. when $\sin(\omega t - \frac{\pi}{2}) = 1$}



[Lagging 90°]

Power in Pure inductive ckt :-

$P = VI$

$= (V_m \sin\omega t)(I_m \sin(\omega t - \frac{\pi}{2}))$

$= V_m I_m \sin\omega t \cdot \sin(\omega t - \frac{\pi}{2})$

$= \frac{V_m I_m}{2}[2\sin\omega t \cdot \sin(\omega t - \frac{\pi}{2})]$

$= \frac{V_m I_m}{2} [\cos(\omega t - \omega t + \frac{\pi}{2}) - \cos(\omega t + \omega t - \frac{\pi}{2})]$

$= \frac{V_m I_m}{2} [\cos\frac{\pi}{2} - \cos(2\omega t - \frac{\pi}{2})]$

$\boxed{\cancel{P = 0}}$

$= \frac{V_m I_m}{2}[-\sin 2\omega t] \Rightarrow \boxed{P = 0}$ — (6)

in pure L ckt.

2nd method

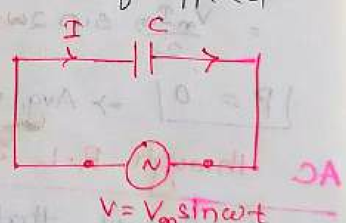
$= -\frac{V_m I_m}{2} \sin\omega t . \cos\omega t$

$= -\frac{V_m I_m}{2} . \sin 2\omega t$

$P = 0$

AC through Pure Capacitance Ckt :-

* the ckt containing only a pure capacitor is known as pure capacitor ckt.
* The current leads the voltage by an angle of 90 degree.
* The capacitor works as a storage device.
* If it's connected to the direct supply, it gets charged equal to the value of applied voltage.
* Let alternating volt. applied



$V = V_m \sin\omega t$ — (1)

Charge of the Capacitor at any instant of time

$\Rightarrow q = CV$ — (2)(a)

$= C(V_m \sin\omega t)$ — 2(b)

current flowing through the ckt

$i = \frac{dq}{dt}$

$= \frac{d}{dt} C V_m \sin\omega t$ {using eqn 2(b)}

$= C V_m \omega \cos\omega t = \frac{V_m}{(1/\omega C)} \sin(\omega t + \frac{\pi}{2})$

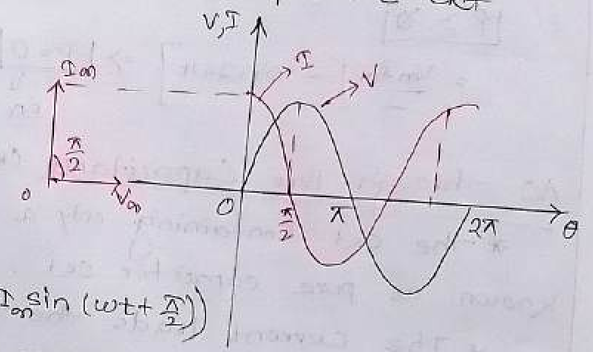
$$I = \frac{V_m}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right) \quad -(3)$$

$\because \boxed{X_C = \frac{1}{\omega C}}$ = Capacitive reactance

Value of I will be max when $\sin\left(\omega t + \frac{\pi}{2}\right) = 1$

So $\boxed{I_m = \frac{V_m}{X_C}}$

eq^n (3) became $\boxed{I = I_m \sin\left(\omega t + \frac{\pi}{2}\right)}$ $-(4)$ for pure Capacitive ckt



$P = VI$

$= (V_m \sin \omega t)\left(I_m \sin\left(\omega t + \frac{\pi}{2}\right)\right)$

$= V_m I_m \sin \omega t \cos \omega t$

$= \frac{V_m I_m}{2}\left[2 \sin \omega t \cdot \cos \omega t\right]$

$= \frac{V_m I_m}{2} \sin 2\omega t$

$\boxed{P = 0}$ → Avg. power in capacitive ckt = Zero

AC through R-L Series Ckt

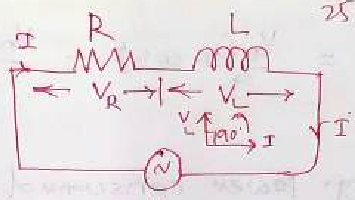
* A Circuit that contains a pure resistance (R) connected in series with a coil having Pure inductance (L) is known as RL Series Ckt.

* Let an AC supply vol 'V' is applied, the Current I flows in the Circuit &

$V_R = IR$

= Voltage drop across R

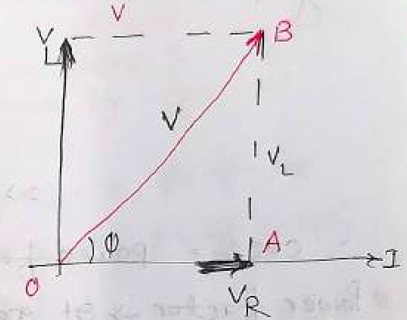
(in phase with I)



$V_L = I \cdot X_L$

= Voltage drop across coil

(ahead of I by 90°)



ΔOAB

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$

$$\Rightarrow \frac{V}{I} = \sqrt{R^2 + X_L^2} \Rightarrow \boxed{Z = \sqrt{R^2 + X_L^2}}$$

$\Downarrow$

Impedance, unit = Ω

$$\boxed{(\text{Impedance})^2 = (\text{resistance})^2 + (\text{reactance})^2}$$

Phase angle :- In RL series circuit the I lags voltage by 90° angle known as phase angle.

$$\tan\theta = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \text{ or}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{X_L}{R}\right)}$$

Power :-

$$V = V_m \sin\omega t \quad \text{--- (1)}$$

$$i = I_m \sin(\omega t - \phi) \quad \because I_m = \frac{V_m}{Z}$$

$$P = Vi$$

$$= (V_m \sin\omega t)(I_m \sin(\omega t - \phi))$$

$$= \frac{V_m I_m}{2}\left[2 \cdot \sin\omega t \cdot \sin(\omega t - \phi)\right]$$

$$= \frac{V_m I_m}{2}\left[\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)\right]$$

$\Rightarrow P = \frac{V_m I_m}{2} \cos\phi - \frac{V_m I_m}{2} \cos(2\omega t - \phi)$

Avg power consumed $P = \frac{V_m I_m}{2} \cos\phi$

$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$

$= V_{rms} I_{rms} \cos\phi$

$\Rightarrow \boxed{P = VI \cos\phi}$

$\because \cos\phi$ = power factor of the circuit.

Power factor → It may be defined as

i) cosine of the angle of lead or lag

ii) the ratio $\frac{R}{Z} = \frac{\text{resistance}}{\text{impedance}}$

iii) $= \frac{\text{true power}}{\text{Apparent Power}} = \frac{\text{watt}}{\text{Volt. amp}} = \frac{W}{VA}$

Active power (P or W)

Power which is actually dissipated in the circuit resistance

$P = I^2 R = VI\cos\phi$ watts.

Reactive Power (Q)

Power developed in the inductive reactance of the circuit.

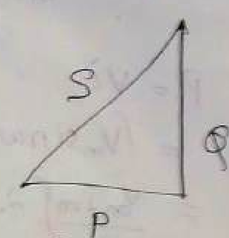
$Q = I^2 X_L = I^2 \cdot Z \sin\phi = I \cdot (IZ) \sin\phi = VI \sin\phi$

Apparent Power (S)

Product of rms values of applied voltage & circuit current.

$S = VI = (IZ) \cdot I = I^2 Z$

$S = \sqrt{P^2 + Q^2}$



(Power Δ)

① In a Series circuit containing pure resistance & a pure inductance, the current & voltage are expressed as ; $\frac{26}{2}$ 1

$$i(t) = 5\sin\left(314t + \frac{2\pi}{3}\right) \quad \& \quad V(t) = 15\sin\left(314t + \frac{5\pi}{6}\right)$$

a) What's the impedance of the CKt ?
b) What's the value of the resistance ?
c) What's the inductance in henry ?
d) What's the avg power drawn by CKt ?
e) What's the power factor ?

$i(t) = 5\sin\left(314t + \frac{2\pi}{3}\right) \Rightarrow I_m = 5 \; ; \; \phi = \frac{2\pi}{3} = 120^\circ$

$v(t) = 15\sin\left(314t + \frac{5\pi}{6}\right) \Rightarrow V_m = 15 \; ; \; \phi = \frac{5\pi}{6} = 150^\circ$

a) Impedance $\boxed{Z = \frac{V_m}{I_m} = 3\Omega}$

Phase diff. bet $V(t)$ & $i(t)$ = 30°. It means Current lags behind Voltage by 30° i.e RL CKt.

b) $\frac{R}{Z} = \cos\phi \Rightarrow R = Z\cos\phi$

$= 3\cos 30^\circ$

$= 2.598 \simeq \underline{2.6\ \Omega}$

c) $X_L = \omega L$ or $Z\sin\phi$

$= 3\sin 30^\circ = 1.5$

$\Rightarrow \cancel{2\pi f L = 1.5}$ $\quad \omega L = 1.5$

$\Rightarrow \cancel{2\pi x}$ $\quad \Rightarrow 314L = 1.5 \Rightarrow L = \frac{1.5}{314} = 0.00477$ H

or $\underline{4.77}$ mH

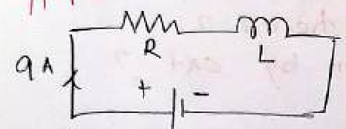
d) $P = I^2R = I_{rms}^2 R = \left(\frac{5}{\sqrt{2}}\right)^2 \times 2.6$

$= \frac{25}{2} \times 2.6 = \cancel{05}\ 32.5$ W

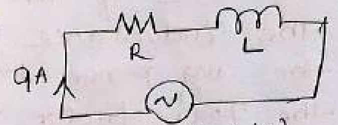
e) Power factor $= \cos\phi = \cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$ (lag)

2) The potential diff. measured across a coil 4.5 V, when it carries a direct current of 9 A.

⇒ The same coil when carries an alternating current of 9 A at 25 Hz, the potential diff is 24 V. Find the current, the power & the p.f when it's supplied by 50 V, 50 Hz supply.



4.5v (dc)



24V, 25Hz (AC)

i) for dc,

freq = 0

$$R = \frac{V}{I} = \frac{4.5}{9} = 0.5\ \Omega$$

ii) when ac, $I_{rms} = 9$ A,

$$Z = \frac{V}{I} = \frac{24}{9} = 2.66\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} \Rightarrow 2.66 = \sqrt{0.5^2 + X_L^2}$$ $\left[X_L = \sqrt{Z^2 - R^2} \right.$

$$\Rightarrow X_L = \sqrt{2.66^2 - 0.5^2} = 2.62\ \Omega$$

$$\Rightarrow 2\pi f L = 2.62$$

$$\Rightarrow L = \frac{2.62}{2\pi \times 25} = 0.0167\ H$$

At 50 V, 50 Hz,

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.0167 = 5.24\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(0.5)^2 + (5.24)^2} = 5.26\ \Omega$$

$$I = \frac{V}{Z} = \frac{50}{5.26} = 9.5\ A$$

$$P = I^2 R = (9.5)^2 0.5 = 45 W$$

$$P.f = \cos\phi = \frac{R}{Z} = \frac{0.5}{5.26} = 0.095$$

1

27

(3) In a particular R-L Series circuit a voltage of 10V at 50 Hz produces a current of 700 mA while the same voltage at 75 Hz produces 500 mA. What are the values of R & L?

Sol^n

$V = 10V$, $I = 700\,mA$

$f = 50 Hz$,

i) $Z = \sqrt{R^2 + X_L^2}$ $\because X_L = 2\pi f L = 2\pi \times 50 L = 100\pi L$

$I = \frac{V}{Z} = \frac{10}{\sqrt{R^2 + (100\pi L)^2}}$

$\Rightarrow 700 \times 10^{-3} = \frac{10}{\sqrt{R^2 + (100\pi L)^2}} \Rightarrow \left(\sqrt{R^2 + (100\pi L)^2}\right) = \frac{10}{700 \times 10^{-3}}$

$\Rightarrow R^2 + (100\pi L)^2 = \left(\frac{10}{700 \times 10^{-3}}\right)^2$

$\Rightarrow R^2 + 98696 L^2 = 204.08$ — (1) 204.08

$\Rightarrow$

ii) $f = 75$ Hz & $I = 500\,mA$

$I = \frac{V}{Z} = \frac{10}{\sqrt{R^2 + (2\pi 75 L)^2}}$

$\Rightarrow 500\,mA = \frac{10}{\sqrt{R^2 + (150\pi L)^2}}$

$\Rightarrow \left(\frac{10}{500 \times 10^{-3}}\right)^2 = R^2 + (150\pi L)^2 \Rightarrow 400 = R^2 + 222066 L^2$ — (2)

$\Rightarrow$ Subtracting eq^n (1) from (2),

$$\begin{aligned} R^2 + 222066 L^2 &= 400 \\ - \; R^2 + 98696 L^2 &= 204.08 \\ \hline 123370 L^2 &= 195.92 \end{aligned}$$

$\Rightarrow L^2 = 0.001588$

$L = 0.0398$ H

or 39.8 mH $\simeq 40$ mH

putting this value in eq^n (2)

$$R^2 + 222066(0.0398)^2 = 400$$

$$\Rightarrow R^2 = 400 - 351.76$$

$$= 48.24$$

$$\boxed{R = 6.95\,\Omega}$$

4) A series cut consists of a resistance of 6Ω & an inductive reactance of 8Ω. A potential diff. of 141.4 V (rms) is applied to it. At a certain instant the applied voltage is 100V & is increasing. Find at this current, i) the current ii) the voltage drop across the resistance iii) voltage drop across inductive reactance.

$R = 6\Omega$, $X_L = 8\Omega$ $\quad V_{rms} = 141.4$ V

$V = 100$ V

$|Z| = \sqrt{6^2+8^2} = 10$

$\phi = \tan^{-1}\left(\frac{8}{6}\right) = 53.13$

$$Z = R + jX_L = 6 + j8 = 10\angle 53.13^\circ$$

$$V_m = \sqrt{2}\,V_{rms} = 141.4\sqrt{2} = 199.96 \simeq 200\text{v}$$

$$V = V_m \sin\omega t = 200 \sin\omega t \quad -(1)$$

$$I_m = \frac{V_m}{Z} = \frac{200}{10} = 20$$

Currents lags behind voltage by 53.13°

So $i = 20\sin(\omega t - 53.13^\circ) \quad -(2)$

i) when the voltage is 100 v

$$V = 200\sin\omega t$$

$$\Rightarrow 100 = 200 \sin\omega t$$

$$\Rightarrow \omega t = \sin^{-1}\left(\frac{100}{200}\right) = 30^\circ$$

so current at this point from eq^n (2)

$$i = 20\sin(30^\circ - 53.13^\circ)$$

$$= -7.856\text{ A}$$

ii) Voltage drop across R

$= iR = -7.856 \times 6$

$= -47.13 \ V$

iii) Voltage drop across inductive reactance.

$V_L = I_m X_L$

$= 20 \times 8 = 160 \ V$

$V_L = iX_L = 20 \sin(\omega t - 53.13) \times 8 \angle 90$

$= 20 \angle -53.13 \times 8 \angle 90^\circ$

$= 160 \angle 90^\circ - 53.13^\circ$

$= 160 \angle 36.87^\circ \approx 160 \angle 36.9^\circ$

Hence at $\omega t = 30^\circ$, the voltage drop is

$V_L = 160 \sin(30 + 36.9)$

$= 147.17 \ V$

5) A 200V, 50 Hz. inductive circuit takes a current of 10 A, lagging 30 degree. find (i) resistance ii) reactance iii) inductance of coil.

$V = 200 V, \ f = 50 HZ, \ I = 10 A, \ \phi = 30^\circ (lag)$

$Z = \frac{V}{I} = \frac{200}{10} = 20 \ \Omega$

i) $R = Z \cos\phi = 20 \cos 30^\circ = 17.32 \ \Omega$

ii) $X_L = Z \sin\phi = 20 \sin 30^\circ = 10 \ \Omega$

iii) $X_L = 2\pi f L \Rightarrow L = \frac{X_L}{2\pi f} = \frac{10}{2 \times 3.14 \times 50} = 0.0318 \ H$

6) A Capacitor of capacitance 79.5 uF is connected in series with a non-inductive resistance of 30 Ω across a 100V, 50Hz. find (i) impedance ii) current iii) Phase angle iv) Equation for instant. Value of I

$C = 79.5 \times 10^{-6} f, \ R = 30, \ V = 100, \ f = 50$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 79.5 \times 10^{-6}} = 40 \ \Omega$

i) $Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = 50 \ \Omega$

ii) $I = \frac{V}{Z} = \frac{100}{50} = 2\,A$

iii) $\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{40}{30}\right) = 53^\circ$ lead

iv) $I = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = \sqrt{2}I = \sqrt{2} \times 2 = 2.83\,A$

$\omega = 2\pi f = 2 \times 3.14 \times 50 = 314$ rad/sec

$i = I_m \sin(\omega t + \phi)$

$= 2.83 \sin(314t + 53^\circ)$

(7) In a given RL circuit, $R = 3.5\,\Omega$ & $L = 0.1\,H$. Find (i) the Current through the circuit and (ii) Power factor if a 50 Hz voltage $V = 220\angle 30^\circ$ is applied across the Circuit.

$R = 3.5\,\Omega$, $L = 0.1\,H$, $f = 50\,Hz$,

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\,\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(3.5)^2 + (31.42)^2} = 31.6$$

$$Z = 31.6 \angle \tan^{-1}\left(\frac{31.42}{3.5}\right) = 31.6\angle 83.65^\circ$$

$$I = \frac{V}{Z} = \frac{220\angle 30^\circ}{31.6\angle 83.65} = 6.96\angle -53.65$$

Phase angle betn v & i $= 53.65 + 30^\circ = 83.65^\circ$ (lag)

$$p.f = \cos 83.65 = 0.11 \text{ (lag)}$$

(8) A 2-element Series circuit is connected across an ac Source $e = 200\sqrt{2}\sin(\omega t + 20)\,v$. The current in the circuit is found as $10\sqrt{2}\cos(314t - 25)$. find the parameters of the Circuit.

$$e = 200\sqrt{2}\sin(\omega t + 20) = 200\sqrt{2}\angle 20^\circ$$

$$i = 10\sqrt{2}\cos(314t - 25)$$

$$= 10\sqrt{2}\sin(314t - 25 + 90)$$

$$= 10\sqrt{2}\sin(314t + 65^\circ)$$

$$= 10\sqrt{2}\angle 65^\circ$$

$$Z = \frac{e}{i} \text{ or } \frac{V}{i} = \frac{200\sqrt{2}\angle 20}{10\sqrt{2}\angle 65} = 20\angle -45^\circ = 14.1 - j14.1$$

$R = 14.1\,\Omega$ $\qquad X_C = 14.1\,\Omega$

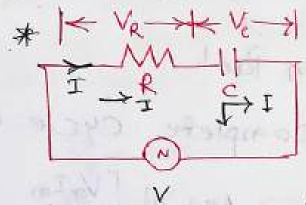
$$\rightarrow \frac{1}{2\pi f C} = 14.1$$

$$C = \frac{1}{2\pi \times 50 \times 14.1} = 226\,\mu F$$

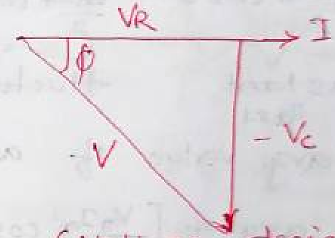
AC through RC → ⟨Series RC circuits⟩

* A circuit that contains pure R connected in series with a pure C is known as RC series ckt.

*



⟨RC series ckt⟩



(Voltage triangle)
or
(Phasor diagram)

Drop across $R = V_R = IR$
(in phase with I)

Drop across Capacitor $V_C = IX_C$ [lagging I by $\frac{\pi}{2}$] Pg-541

$\therefore X_C = \frac{1}{2\pi f C}$

As X_C is taken -ve, V_C is shown along -ve direction of Y-axis.

Now $V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2}$

$= I\sqrt{R^2 + X_C^2}$

$\Rightarrow \frac{V}{I} = \sqrt{R^2 + X_C^2} \Rightarrow \boxed{Z = \sqrt{R^2 + X_C^2}}$ Impedance of Series RC circuit

. From Voltage Δ, it is found that I lead V by an angle ϕ, this angle is called phase angle.

$\tan\phi = \frac{-V_C}{V_R} = \frac{-IX_C}{IR} = \frac{-X_C}{R}$

or $\phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$

If $V = V_m \sin\omega t$
$i = I_m \sin(\omega t + \phi)$

Then $P = Vi$
$= (V_m \sin\omega t)(I_m \sin(\omega t + \phi))$

$$= \frac{V_m I_m}{2}\left[2\sin \underset{B}{\omega t}\cdot \sin(\underset{A}{\omega t}+\phi)\right]$$

$$= \frac{V_m I_m}{2}\left[\cos(\omega t + \phi - \omega t) - \cos(\omega t+\omega t+\phi)\right]$$

$$= \underbrace{\frac{V_m I_m}{2}\cos\phi}_{\text{constant Part}} - \underbrace{\frac{V_m I_m}{2}\cos(2\omega t+\phi)}_{\text{fluctuation Part}}$$

$2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

$\sin A \cdot \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$

The avg. value of a complete cycle is

$$P = \text{avg of}\left[\frac{V_m I_m}{2}\cos\phi\right] - \text{Avg of}\left[\frac{V_m I_m}{2}\cos(2\omega t+\phi)\right]$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi - 0$$

$$P = V_{rms} I_{rms} \cos\phi$$

$\boxed{P = VI\cos\phi}$ where $\cos\phi$ = Power factor of the ckt.

1) In a circuit, the applied voltage is 100V and is found to lag the current of 10 A by 30°. i) Is the p.f lagging or leading? ii) What's the value of p.f? iii) Is the circuit inductive or capacitive? iv) What is the value of active & reactive power in the circuit?

Given $V = 100V$, $I = 10\,A$, $\phi = 30°$

i) As given voltage is lagging behind I, so current leads voltage.

p.f is leading.

ii) $p.f = \cos\phi = \cos 30° = 0.866$ (lead)

iii) As I leads V, ckt is capacitive.

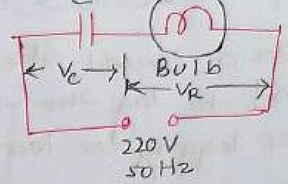
iv) Active power $= VI\cos\phi$

$= 100 \times 10 \times 0.866 = 866\ W$

Reactive power $= VI\sin\phi$

$= 100 \times 10 \times \sin 30° = 500\ VAR$

② A tungsten filament bulb rated at 500 W, 100 V is to be Connected to series with a capacitance across 200 V, 50 Hz supply. Calculate a) the value of capacitor such that the voltage & power consumed by the bulb are according to the rating of the bulb, b) the power factor of current drawn from the supply. c) draw the phasor diagram.



$V_R = 100\,V$

$P = VI \Rightarrow 500 = 100I$

$I = \frac{500}{100} = 5A$

a) $V_c = \sqrt{220^2 - 100^2} \Rightarrow IX_c = 195.9 \simeq 196$

$\Rightarrow 5X_c = 196$

$\Rightarrow X_c = 39.2\,\Omega$

$\Rightarrow \frac{1}{2\pi f C} = 39.2 \Rightarrow C = \frac{1}{2\pi \times 50 \times 39.2}$

$= 8.12 \times 10^{-5}\,F$

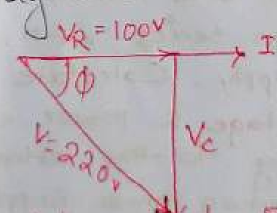
$\simeq 81.2\,\mu f$

b) $P.f = \cos\phi = \frac{V_R}{V}$

$= \frac{100}{220} = 0.454$

$P.f = 0.454$

c) Phasor diagram →



③ A pure resistance of 50 Ω is in series with a pure capacitance of 100 μF. The series comb' is connected across 100 V, 50 Hz supply. find a) the impedance (b) I c) P.f d) phase angle e) voltage across R, f) Vol. across C,

$R = 50\,\Omega,\ C = 100\,\mu f \qquad V = 100\,V,\ f = 50\,Hz$

a) $Z = \sqrt{R^2 + X_c^2} \qquad \therefore X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$

$= \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = \frac{10^6}{100\pi \times 100} = 32\ \Omega$

$Z = \sqrt{50^2 + 32^2} = 59.4\ \Omega$

b) $I = \frac{V}{Z} = \frac{100}{59.4} = 1.683\ A$

c) $P.f = \frac{R}{Z} = \frac{50}{59.4} = 0.842$

d) $\phi = \cos^{-1}(P.f) = \cos^{-1}(0.842)$

(4) In a circuit, the applied voltage is 100V and found to lag the current of 10 A by 30°. i) Is the P.f lagging or leading? ii) what is the value of P.f. iii) Is the circuit inductive or capacitive. iv) what's the value of active & reactive power in the circuit.

$\phi = 30°$ (lag) $I = 10\ A$

i) Current is leading the voltage by 30°. Hence PF is leading.

ii) $pf = \cos\phi = \cos 30° = 0.866$

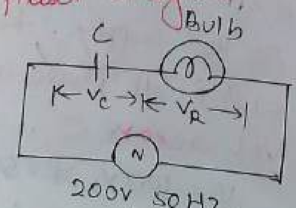
iii) ~~As~~ I leads V, circuit is Capacitive.

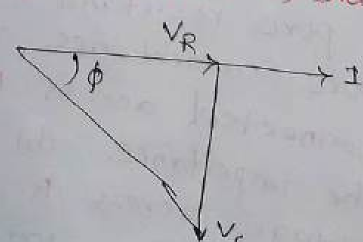
iv) Active Power $P = VI\cos\phi$

$= 100 \times 10 \times 0.866 = 866\ W$

v) Reactive Power $Q = VI\sin\phi = 100 \times 10 \times \sin 30 = 500$ VAR

(5) A tungsten filament bulb rated at 500 W, 100 V is to be connected in series with a capacitor across 200 V, 50 Hz supply. Calculate a) value of C such that the voltage & power consumed by the bulb are according to the rating of bulb, b) the PF of the I drawn from supply. c) Draw the phasor diagram.





$V = 200\,V,\ f = 50\,Hz,\ P = 500\,W,\ V_R = 100$

$$V_c = \sqrt{V^2 - V_R^2} = \sqrt{200^2 - 100^2} = 196$$

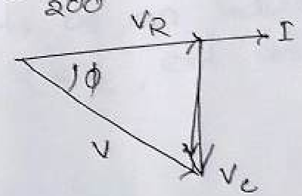
$$\Rightarrow I X_c = 196 \Rightarrow X_c = \frac{196}{5} = 39.2$$

$$\Rightarrow \frac{1}{2\pi f C} = 39.2 \Rightarrow \frac{1}{2\pi \times 50 \times 39.2} = C$$

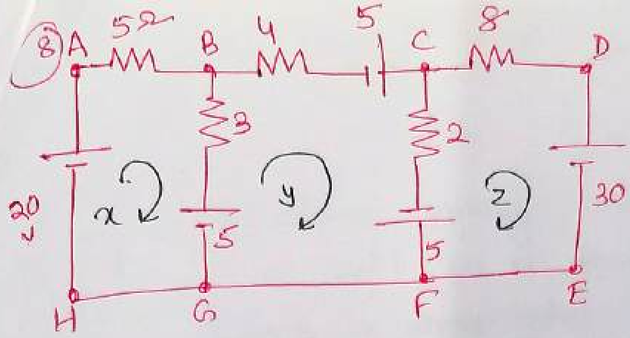
$$\Rightarrow \boxed{C = 81\,\mu F}$$

b) $p.f = \cos\phi = \frac{V_R}{V} = \frac{100}{200} = 0.455$

c) Phasor diagram =



8)



find the current supplied by each battery in the shown ckt.

Using KVL in ABGHA

$$20 - 5x - 3(x-y) - 5 = 0$$

$$\Rightarrow 15 - 5x - 3x + 3y = 0 \Rightarrow 8x - 3y = 15 \quad \text{— (1)}$$

in BCFGB,

$$5 - 3(y-x) - 4y + 5 - 2(y-z) + 5 = 0$$

$$\Rightarrow 15 - 3y + 3x - 4y - 2y + 2z = 0$$

$$\Rightarrow 15 + 3x - 9y + 2z = 0 \Rightarrow 3x - 9y + 2z = -15 \quad \text{— (2)}$$

in CDEFC,

$$-5 - 2(z-y) - 8z - 30 = 0$$

$$\Rightarrow -35 - 2z + 2y - 8z = 0$$

$$\Rightarrow -10z + 2y = 35 \Rightarrow 2y - 10z = 35 \quad \text{— (3)}$$

Solving eq^n (1), (2), (3)

$$8x - 3y = 15$$

$$3x - 9y + 2z = -15$$