UNIT – 3 – Sampling Thtory

1. Let $X_1, X_2, X_3, \dots . X_n$ be a random sample from Normal distribution $N(\mu, 1)$ population. Show that $t = \sum_{i=0}^{n} X_i^2$ is an unbiased estimator of $\mu^2 + 1$.

2. Let t_1 and t_2 be two unbiased estimators of θ. Show that estimator $t = at_1 + (1-a)t_2$ is an unbiased estimator of θ.

3. Let T_1 and T_2 be two consistent estimators of μ_1 and μ_2 respectively. Prove that aT_1+bT_2 is a consistent estimator of $a\mu_1 + b\mu_2$, where a and b are constant and independent of population.

4. If $X_1, X_2,$ and X_3 constitute a random sample of size 3 from normal population with mean μ and variance σ^2. Find the most efficient estimator among the three statistics $t_1 = \frac{X_1 + X_2 + X_3}{3}$, $t_2 = \frac{X_1 + 2X_2 + X_3}{4}$ and $t_3 = X_1 + \frac{X_2 + X_3}{2}$.

5. Let $X_1, X_2, X_3, \dots . X_n$ be a random sample from Normal distribution $N(\mu, \sigma^2)$ population. Prove that $t = \frac{\sum_{i=0}^{n} X_i}{n}$ is a good estimator of μ.

6. Let $X_1, X_2, X_3, \dots . X_n$ be a random sample from a population with population density function $f(X, \Theta) = \Theta X^{\Theta - 1}; 0 < X < 1, \Theta > 0$. Find the sufficient estimator for Θ.

7. Let $X_1, X_2, X_3, \dots . X_n$ be a random sample from $N(\mu, \sigma^2)$ population with p.d.f $f(X, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$. Find the maximum likelihood estimator of μ.

8. Let $X_1, X_2, X_3, \dots . X_n$ be a random sample from $N(\mu, \sigma^2)$ population with probability density function $f(X, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$. Find the MLE of σ^2

9. Let $X_1, X_2, X_3, \dots . X_n$ be a random sample from exponential distribution $f(X, \lambda) = \frac{1}{\lambda} e^{-\lambda X}, x > 0, \lambda < \infty$. Find the MLE of λ.

10. Let $X_1, X_2, X_3, \dots . X_n$ be a random sample from uniform distribution with population density function $f(X, \theta) = \frac{1}{2\theta}, -\theta < X < \theta$. Obtain the estimator of θ by the method of moments.

11. The mean and variance of a random sample of 64 observation were computed as 160 and 100 respectively. Compute the 95% confidence limits for population mean.

12. A random sample of 700 units from a large consignment and in that 200 were damaged. Find 95% confidence limit for the proportion of damage units in the consignment.

13. Out of 20000 customer saving account a sample of 600 account was taken to test the accuracy of hosting and balancing where in 45 mistakes were found assign limits with in no of defective case can be expected 95% level. Find out the confidential limit for 95% significant level.

14. A research worker wishes to estimate the mean of population by using sufficiently large sample. The probability is 0.95 that the sample mean will not differ from the true mean by more than 25% of the standard deviation. How large a sample should be taken?

15. A manufacturing concern to estimate the average amount purchase of its products in a month by the customers. If the standard deviation is Rs. 10. Find sample size, if the maximum error is not to exceed Rs. 3 with probability of 0.99.

16. A random sample of 100 articles selected from a batch of 2,000 articles shows that the average diameter of the articles 0.354 with standard deviation 0.048. Find 95% confidence interval for the average of this batch of 2,000 articles.

17. A random sample of 100 articles selected from a large batch of articles contain 5 defective articles. Set up 99 percent confidence limits for the proportion of defective items in the batch.