

UNIT – 3 – Sampling Theory

1. Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from Normal distribution $N(\mu, 1)$ population. Show that $t = \frac{1}{n} \sum_{i=1}^n X_i^2$ is an unbiased estimator of $\mu^2 + 1$.
2. Let t_1 and t_2 be two unbiased estimators of θ . Show that estimator $t = at_1 + (1 - a)t_2$ is an unbiased estimator of θ .
3. Let T_1 and T_2 be two consistent estimators of μ_1 and μ_2 respectively. Prove that $aT_1 + bT_2$ is a consistent estimator of $a\mu_1 + b\mu_2$, where a and b are constant and independent of population.
4. If X_1, X_2 , and X_3 constitute a random sample of size 3 from normal population with mean μ and variance σ^2 . Find the most efficient estimator among the three statistics $t_1 = \frac{X_1 + X_2 + X_3}{3}$, $t_2 = \frac{X_1 + 2X_2 + X_3}{4}$ and $t_3 = X_1 + \frac{X_2 + X_3}{2}$.
5. Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from Normal distribution $N(\mu, \sigma^2)$ population. Prove that $t = \frac{\sum_{i=1}^n X_i}{n}$ is a good estimator of μ .
6. Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from a population with population density function $f(X, \theta) = \theta X^{\theta-1}; 0 < X < 1, \theta > 0$. Find the sufficient estimator for θ .
7. Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from $N(\mu, \sigma^2)$ population with p.d.f $f(X, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$. Find the maximum likelihood estimator of μ .
8. Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from $N(\mu, \sigma^2)$ population with probability density function $f(X, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$. Find the MLE of σ^2 .
9. Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from exponential distribution $f(X, \lambda) = \frac{1}{\lambda} e^{-\lambda X}, x > 0, \lambda < \infty$. Find the MLE of λ .

10. Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from uniform distribution with population density function $f(X, \theta) = \frac{1}{2\theta}, -\theta < X < \theta$. Obtain the estimator of θ by the method of moments.
11. The mean and variance of a random sample of 64 observations were computed as 160 and 100 respectively. Compute the 95% confidence limits for population mean.
12. A random sample of 700 units from a large consignment and in that 200 were damaged. Find 95% confidence limit for the proportion of damaged units in the consignment.
13. Out of 20000 customer saving account a sample of 600 accounts was taken to test the accuracy of hosting and balancing where in 45 mistakes were found assign limits with in no of defective case can be expected 95% level. Find out the confidence limit for 95% significant level.
14. A research worker wishes to estimate the mean of population by using sufficiently large sample. The probability is 0.95 that the sample mean will not differ from the true mean by more than 25% of the standard deviation. How large a sample should be taken?
15. A manufacturing concern to estimate the average amount purchase of its products in a month by the customers. If the standard deviation is Rs. 10. Find sample size, if the maximum error is not to exceed Rs. 3 with probability of 0.99.
16. A random sample of 100 articles selected from a batch of 2,000 articles shows that the average diameter of the articles 0.354 with standard deviation 0.048. Find 95% confidence interval for the average of this batch of 2,000 articles.
17. A random sample of 100 articles selected from a large batch of articles contain 5 defective articles. Set up 99 percent confidence limits for the proportion of defective items in the batch.
18. The mean and variance of a random sample of 64 observations were computed as 160 and 100 respectively. If the investigator wants to be 95% confidence that the error in the estimate of population mean should not exceed ± 1.4 , how many additional observations are required?

19. A random sample of 500 pineapples was taken from a large consignment and 65 of them were found to be bad. Show that the standard error of the proportion of bad ones in a sample of this size is 0.015.
20. A random sample of 100 articles selected from a large batch of articles contain 5 defective articles. If the batch contains 2669 items, set up 95% confidence interval for the population of defective items.
21. A research worker wishes to estimate the mean of population by using sufficiently large sample. The probability is 0.95 that the sample mean will not differ from the true mean by more than 25% of the standard deviation. How large a sample should be taken?

UNIT – 4 /5– Test of Hypothesis

1. A stenographer claims that she can take decision at the rate of 120 wpm. Can we reject her claim on the basis of 100 trials in which she demonstrates a mean of words with standard deviation of $\sigma=5\%$.
2. It is claimed that a random sample of 100 tyres with the mean life 15269kms is drawn from a population of tyres which has a mean life of 15200kms and standard deviation of 1248kms. Test the validity of the claim at 1% level of significance.
3. A weighing machine without any display was used by an average of 320 persons a day with a standard deviation of 50 persons. When an attractive display was used on the machine, the average for 100 days increased by 15 persons. Can we say that the display did not help much? Use a level of significance of 0.05.
4. A coin is tossed 900 times and had appeared 490 times. Does this result support the hypothesis that a coin is unbiased? Use 5% level of significance.
5. A sample of 400 parts manufactured by a factory, the number of defective parts was found to be 30. The company, however, claimed that at most 5% of their product is defective. Is the claim tenable?
6. In a random sample of 400 persons from a large population, 120 are females. Can it be said that males and females are in ratio 5:3 in the population? Use 1 % level of significance.
7. In big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers?
8. In order to make a survey of the buying habits, 2 makers A & B are chosen at 2 different part of city. 400 women shoppers are chosen are

random in market A. Their average daily expenditure on food is found to be Rs.250 with standard deviation Rs.40. The figure are Rs.220 and Rs.55 in the market B, where also 400 female shoppers are chosen at random. Test at 1% liberal of significance weather the daily food expenditure of the two population of shoppers are equal.

9. A radio shop sells, on an average 200 radios per day with standard deviation 50 radios. After an extensive advertising campaign, the management will compute the average sales for the next 25 days to see whether an improvement has occurred. Assume that the daily sales of radio is normally distributed. Test the hypothesis at 5 % level of significance if the sample average is 216.

10. Following information is related to 2 places A and B test. Whether there is any significance between their mean wages. Use $\alpha=5\%$.

	A	B
Mean Wages	47	49
Standard Deviation	28	40
No. of Workers	1000	1500

11. The mean yield of 2 sets of plots and their variability are as given below. Examine whether the difference in the variability in the yields in significance at 5% level of significance.

	Set of 40 plots	Set of 60 plots
Mean Yield per plot	1258 lb	1243 lb
S.D per plot	34	28

12. A company has head office at Kolkata and a branch at Mumbai. The personal director want to know if the workers at the two places would like the introduction of a new plan work and a survey has conducted for this purpose. Out of sample of 500 workers at Kolkata 62% favor the new plan. At Mumbai out of 400 workers 41% were against the new plan. Is there any significance difference b/w the two groups in their attitude towards the new plan at 5% level?
13. A machine puts out 16 imperfect articles in the sample of 500, after the machine is overhauled it puts out 3 imperfect articles in the batch of 100. Has the machine improved? use a 5% level of significance.
14. The manufacturer of television tubes knows from the past experience that the average life of a tube is 2,000 hours with a s.d of 200 hours. A sample of 100 tubes has an average life of 1,950 hours. Test at 5% LOS whether the sample came from a normal population of mean 2,000 hours.
15. What do you mean by (i) level of significance (ii) critical values

16. Explain the procedure for testing of hypothesis
17. In a city a sample of 1000 people were taken and out of them 540 are vegetarian and the rest are non-vegetarian. Can we say that both habits of eating are equally popular in the city?
18. The heights of six randomly chosen sailors are in inches : 63, 65, 68, 69, 71, 72. Those of 10 randomly 61, 62, 65, 66, 69, 69, 70, 71, 72, 73. Test whether the sailors are on the average taller than soldiers
19. In a random sample of 500 men 300 are found to be smokers. In another random sample of 1000 men 550 are found to be smokers. Do the data indicate that the two set of men are significantly different with respect to the prevalence of smoking among men
20. A simple sample of heights of 6,400 Englishmen has a mean of 67.85 inches and a s.d of 2.56 inches, while a simple sample of heights of 1,600 Indians has a mean of 68.55 inches and a s.d of 2.52 inches. Does the data indicate that Indians are on the average taller than Englishmen?
21. In one sample of 10 observations from a normal population, the sum of squares of deviations of the sample values from the sample mean is 102.4 and in another sample of 12 observations from another normal population the sum of squares of deviations of the sample values from the sample mean is 120. Examine whether the two normal populations have the same variances