LAPLACE TRANSFORMATION

1. $\left(\sqrt{t} \pm \frac{1}{\sqrt{t}}\right)^3$

2. $\sin^2 kt$

3. $4e^{5t} + 6t^3 - 3\sin 4t + 2\cos 2t$

4. $\cos^3 at$

5. $\cos 3t \cdot \cos 2t \cdot \cos t$

6. $\sin \sqrt{t}$

7. If $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right)$, find $L\left\{\frac{\sin at}{t}\right\}$

8. If $L\{f(t)\} = \frac{20 - 4s}{s^2 - 4s + 20}$, find $L\{f(3t)\}$

9. If $L\{\sin t\} = \frac{1}{s^2+1}$, find $L\{\sin 3t\}$

10. $(t+2)^2 e^t$

11. $e^{-4t}\cosh 2t$

12. $\sinh at \cdot \sin at$

13. $e^{2t}(3\sin 4t - 4\cos 4t)$

14. $e^{-t}\sin^2 t$

15. $t \sin at$

16. $t \cos at$

17. $\dfrac{t^{n-1}}{1-e^{-t}}$

18. If $L\{f(t)\} = F(s)$ then proove that

a. $L\{\cosh at \cdot f(t)\} = \frac{1}{2}[F(s-a) + F(s+a)]$

b. $L\{\sinh at \cdot f(t)\} = \frac{1}{2}[F(s-a) - F(s+a)]$

19. $e^{-2t} \cdot \sin^3 t$

20. $\sin^4 t \cdot e^{2t}$

21. $e^{4t} \sin 2t \cdot \cos t$

22. t^n

23. $\sin^2 t$

24. $\cos at$

25. $t^2 \sin at$

26. $t \cosh at$

27. $\int_0^t e^{-u} \cdot \cos u \, du$

28. $\int_0^t e^u \cdot \frac{\sin u}{u} \, du$

29. $\int_0^t \frac{e^{-4u} \cdot \sin 3u}{u} \, du$

30. $\cosh t \int_0^t e^u \cosh u \, du$

31. $e^{-4t} \int_0^t \frac{\sin 3u}{u} \, du$

32. $t . \cos at$

33. $t^2 \sin t$

34. $t . \sinh 2t$

35. $t(3 \sin 2t - 2 \cos 2t)$

36. $t^3 . \cos t$

37. $t . e^{at} . \sin at$

38. $\int_0^\infty t . e^{-3t} . \sin t \, dt = \frac{3}{50}$ (Prove)

39. $\int_0^\infty t^3 . e^{-t} . \sin t \, dt = 0$ (Prove)

40. $\int_0^\infty e^{-3t} . t . \cos t \, dt = \frac{2}{25}$ (Prove)

41. $\int_0^\infty e^{-2t} . \sin^3 t \, dt = \frac{6}{65}$ (Prove)

42. $\frac{t - e^{2t}}{t}$

43. $\frac{\sinh t}{t}$

44. $\frac{1 - \cos at}{t}$

45. $\frac{\sin 3t \cdot \cos t}{t}$

46. Show that $\int_0^\infty \frac{e^{-3t} - e^{-6t}}{t} = \ln 2$

47. Show that $\int_0^\infty \frac{\sin^2 t}{t^2}\, dt = \frac{\pi}{2}$

48. Find the L.T of $\left\{ \int_0^t e^t \cdot \frac{\sin t}{t}\, dt \right\}$

49. Show that $\int_0^\infty e^{-t} \cdot \frac{\sin^2 t}{t}\, dt = \frac{1}{2} \ln 5$

50. $\int_0^\infty e^{-2t} \frac{2\sin t - 3\sinh t}{t}\, dt = 2\cot^{-1} 2 + \frac{3}{2} \log\left(\frac{1}{3}\right)$

INVERSE LAPLACE TRANSFORMATION

51. $\frac{3}{s+4}$

52. $\frac{8s}{s^2+16}$

53. $\frac{1}{2s-5}$

54. $\frac{6}{s^2+4}$

55. $\frac{3s-12}{s^2+8}$

56. $(2s-5)/(s^2-9)$

57. $s^{-\frac{7}{2}}$

58. $\frac{s+1}{s^{4/3}}$

59. $\left(\frac{\sqrt{s}-1}{s}\right)^2$

60. $\frac{3s-8}{4s^2+25}$

61. $\frac{5s+10}{9s^2-16}$

62. $\frac{3(s^2-1)^2}{2s^5} + \frac{4s-18}{9-s^2} + \frac{(s+1)(2-s^{1/2})}{s^{5/2}}$

63. $\frac{1}{s}\sin\left(\frac{1}{s}\right)$

64. $\frac{1}{s}e^{-1/s}$

65. If $L^{-1}\left\{e^{-1/s}/\sqrt{s}\right\} = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$, show that

$$L^{-1}\left\{e^{-a/s}/\sqrt{s}\right\} = \frac{\cos 2\sqrt{at}}{\sqrt{\pi t}} \cdot \text{when } a>0.$$

66. If $L^{-1}\{F(s)\} = f(t)$ Prove that

$$L^{-1}\{F(as+b)\} = \frac{1}{a}e^{-(b/a)t} \cdot f\left(\frac{t}{a}\right) \text{ where } a>0.$$

67. If $L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \frac{t\sin t}{2}$, show that

$$L^{-1}\left\{\frac{8s}{(4s^2+1)^2}\right\} = \frac{1}{2}\sin\frac{t}{2}.$$

68. If $L^{-1}\left\{\frac{s^2-1}{(s^2+1)^2}\right\} = t\cos t$ Show that

$$L^{-1}\left\{\frac{9s^2-1}{(9s^2+1)^2}\right\} = \frac{1}{9}\cos\frac{t}{3}.$$

69. $\frac{5}{(s+2)^5}$

70. $\frac{4s+12}{s^2+8s+16}$

71. $\frac{s}{(s+1)^5}$

72. $\frac{s}{(s+1)^{5/2}}$

73. $\frac{1}{\sqrt[3]{8s-27}}$

74. $\frac{3s-14}{s^2-4s+8}$

75. $\frac{5s-2}{s^2+4s+8}$

76. $\frac{3s+2}{4s^2+12s+9}$

77. $\frac{8s+20}{s^2-12s+32}$

78. $\frac{1}{(s^2-12s+5)^2}$

79. $\frac{1}{(s-a)^n}$, $n = 1, 2, 3$

80. $\frac{s}{(s^2+a^2)^2}$

81. $\frac{1}{s^2+4s+5}$

82. $\log\left(\frac{s+a}{s+b}\right)$

83. $\tan^{-1}\frac{2}{s^2}$

84. $\log\frac{s(s+1)}{(s^2+4)}$

85. $\log\left(1-\frac{a^2}{s^2}\right)$

86. $\log\left(\frac{s+1}{s-1}\right)$

87. $\cot^{-1}\left(\frac{s+3}{2}\right)$

88. Evaluate: $L^{-1}\left\{\int_s^\infty\left(\frac{1}{u}-\frac{1}{u+1}\right)du\right\}$

89. Evaluate: $L^{-1}\left\{\int_s^\infty \ln\left(\frac{u+2}{u+1}\right)du\right\}$

90. Evaluate: $L^{-1}\left\{\int_s^\infty \tan^{-1}\left(\frac{2}{u^2}\right)du\right\}$

91. $\frac{s^2}{s^2+1}$

92. $\frac{s}{(s^2+1)^2}$

93. $\frac{1}{s^3(s^2+1)}$

94. $\frac{s}{(s-2)^5(s+1)}$

95. $\frac{s^2}{(s^2-4s+5)^2}$

96. $\frac{1}{s}\left(\frac{s-a}{s+a}\right)$

97. $\frac{1}{s^2}\left(\frac{s+1}{s^2+1}\right)$

98. $\frac{1}{s^4-2s^3}$

99. $\frac{1}{s(s^2+a^2)}$

100. $\frac{1}{s(s+a)^3}$

101. $\frac{1}{s^2(s^2+a^2)}$

102. $\frac{s-2}{s^2+5s+6}$

103. $\frac{2s^2-4}{(s+1)(s-2)(s-3)}$

104. $\frac{s^2-7s+24}{s^3-7s^2+14s-8}$

105. $\frac{s+17}{(s-1)(s+3)}$

106. $\frac{5s}{s^2+4s+4}$

107. $\frac{5}{(s-2)^4}$

108. $\frac{7}{(2s+1)^3}$

109. $\frac{s+2}{s^2+4s+7}$

110. $\frac{2s+12}{s^2+6s+13}$

111. $\frac{s^2+9s-9}{s^3-9s}$

112. $\frac{s}{(s^2-2s+2)(s^2+2s+2)}$

113. $\frac{3s^3-3s^2-40s+36}{(s^2-4)^2}$

114. $\frac{2s^3-s^2-1}{(s+1)^2(s^2+1)^2}$

115. $\frac{5s^2-7s+17}{(s-1)(s^2+4)}$

116. $\frac{2s^2+15s+7}{(s+1)^2(s-2)}$

117. $\frac{s+1}{(s^2+1)(s^2+4)}$

118. $\frac{10}{s(s^2-2s+5)}$

119. $\frac{1}{s(s+1)^2}$

120. $\frac{s^2+8s+27}{(s+1)(s^2+4s+13)}$

121. $\frac{s}{s^4+s^2+1}$

122. $\frac{s}{s^4+4a^4}$

123. $\frac{5s+3}{(s-1)(s^2+2s+5)}$

124. $\frac{s^2}{s^4+4a^4}$

125. $\frac{1}{(s+a)(s+b)}$

126. $\frac{1}{s(s^2+9)}$

127. $\frac{1}{s^2(s+3)}$

128. $\frac{s^2}{(s^2+a^2)^2}$

129. $\frac{s}{(s^2+a^2)^3}$

130. $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$

131. $\frac{1}{(s+1)(s^2+1)}$

132. $\frac{1}{(s+1)(s+9)^2}$

133. $\frac{1}{s^2(s-a)}$

134. $\frac{s}{(s^2+a^2)^2}$

135. $\frac{a}{s^2(s^2+a^2)}$

136. $\frac{1}{(s^2+4)(s+1)^2}$

137. $\frac{1}{s^3(s^2+1)}$

138. $\frac{1}{s^2(s^2-a^2)}$

139. $\frac{1}{(s+2)^2(s-2)}$

140. $y'-y=0$, general solution

141. $y' - y = e^{3t}$, $y(0) = 2$

142. $y'' + y' = 0$, general solution

143. $y'' + y = 2e^{t}$, $y(0) = 0$, $y'(0) = 2$

144. $y'' - 6y' + 9y = 0$, $y(0) = 0$, $y'(0) = 9$

145. $y'' + 4y = 9t$, $y(0) = 0$, $y'(0) = 7$

146. $y'' + 7y' + 10y = 4e^{-3t}$, $y(0) = 0$, $y'(0) = -1$

147. $y'' - 8y' + 15y = 9te^{2t}$, $y(0) = 5$, $y'(0) = 10$

148. $y'' + y = t \cos 2t$, $y(0) = 0$, $y'(0) = 0$

149. $y'' + n^2 y = a \sin(nt + \theta)$, $y(0) = y'(0) = 0$

150. $y'' + y = \sin t \cdot \sin 2t$, $y(0) = 1$, $y'(0) = 0$

151. $y'' + y = e^{-2t} \cdot \sin t$, $y(0) = 0$, $y'(0) = 0$

152. $y''' + 4y'' + 5y' + 2y = 10 \cos t$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 0$

153. $y''' - y = e^{t}$, $y(0) = y'(0) = y''(0) = 0$

154. $y^{iv} - 16y = 30 \sin t$, $y''(0) = 0$, $y'''(0) = -18$,
$y''(\pi) = 0$, $y'''(\pi) = -18$.

155. $\frac{dx}{dt} = 2x - 3y$; $\frac{dy}{dt} = y - 2x$, $x(0) = 8$, $y(0) = 3$

156. $\frac{dx}{dt} + y \sin t$; $\frac{dy}{dt} + x = \cancel{\cot}\ \cos t$, $x(0) = 2$, $y(0) = 0$

157. $\frac{dx}{dt} - 6x + 3y = 8e^t$; $\frac{dy}{dt} - 2x - y = 4e^t$

$x(0) = -1$, $y(0) = 0$

158. $\frac{2dx}{dt} + \frac{4dy}{dt} + x - y = 3e^t$;

$\frac{dx}{dt} + \frac{dy}{dt} + 2x + 2y = e^t$, $x(0) = 1$, $y(0) = 0$

159. $\frac{dx}{dt} - \frac{dy}{dt} - 2x + 2y = 1 - 2t$

$\frac{d^2x}{dt^2} + 2x + x = 2$, $x(0) = y(0) = x'(0) = 0$

160. $\frac{dx}{dt} + 2\frac{d^2y}{dt^2} = e^{-t}$

$\frac{dx}{dt} + 2x - y = 1$, $x(0) = y(0) = y'(0) = 0$

161. $\frac{3dx}{dt} + \frac{dy}{dt} + 2x = 1$

$\frac{dx}{dt} + 4\frac{dy}{dt} + 3y = 0$, $x(0) = 3$, $y(0) = 0$

162. $\frac{dx}{dt} = y + e^t$; $\frac{dy}{dt} = \sin t - x$, $x(0) = 1$, $y(0) = 0$

163. $\frac{d^2x}{dt^2} = 2x + 3y + e^{2t}$; $\frac{d^2y}{dt^2} = -x - 2y$

$x(0) = y(0) = 1$, $x'(0) = y'(0) = 0$

164. $\frac{d^2x}{dt^2} + \frac{dy}{dt} + 3x = 15e^{-t}$

$\frac{d^2y}{dt^2} - \frac{4dx}{dt} + 3y = 15 \sin 2t$

$x(0) = 35$, $x'(0) = -48$, $y(0) = 27$, $y'(0) = -55$