waves and oscillations

1. write the unit and dimension of force constant.

Ans:- $|K| = \frac{F_R}{y}$

$= \frac{M^1L^1T^{-2}}{L^1} = M^1L^0T^{-2}$

SI = N/M and CGS = Dyn/cm

2. Find the positions where restoring force becomes maximum and minimum zero.

Ans:- The force is extreme at extreme point and minimum at mean point.

3. A SHM is given by $\frac{\partial^2 y}{\partial t^2} + 49\pi^2 y = 0$. Find frequency of oscillator.

Ans:- $\frac{\partial^2 y}{\partial t^2} + 49\pi^2 y = 0$

$\omega_0^2 = 49\pi^2 = (7\pi)^2$

$\omega = 2\pi f$

$\Rightarrow 7\pi = 2\pi f$

$\Rightarrow f = \frac{7\pi}{2\pi} = \frac{7}{2}$ Hz

4. A SHM is given by $\frac{\partial^2 x}{\partial t^2} + 25\pi^2 x = 0$. Find frequency of oscillator.

Ans:- $\frac{\partial^2 x}{\partial t^2} + 25\pi^2 x = 0$

$\omega_0^2 = 25\pi^2 = (5\pi)^2$

$\omega = 2\pi f$

$\Rightarrow 5\pi = 2\pi f$

$\Rightarrow f = \frac{5\pi}{2\pi} = \frac{5}{2}$ Hz

5. A SHM is given by $4\frac{\partial^2 x}{\partial t^2} + 64\pi^2 x = 0$. Find frequency of oscillator.

Ans:- $4\frac{\partial^2 x}{\partial t^2} + 64\pi^2 x = 0$

$\Rightarrow \frac{4}{4}\frac{\partial^2 x}{\partial t^2} + \frac{64}{4}\pi^2 x = 0$

$\Rightarrow \frac{\partial^2 x}{\partial t^2} + 16\pi^2 x = 0$

$\omega_0^2 = 16\pi^2 = (4\pi)^2$

$\omega = 2\pi f$

$\Rightarrow 4\pi = 2\pi f$

$\Rightarrow f = \frac{4\pi}{2\pi} = 2$ Hz

6. An oscillator oscillates with angular frequency 6 radian per second and amplitude 7cm. Find the velocities at positions 1cm, 2cm, 3cm from mean positions and also find the maximum and minimum velocities.

Ans:- $V = \omega_0\sqrt{A^2 - y^2}$

At mean point, $V = 6\sqrt{7^2 - 0} = 6 \times 7 = 42$ cm/s (maximum)

At extreme point, $V = 0$ (minimum)

At 1cm, $V = 6\sqrt{7^2 - 1^2} = 24\sqrt{3}$ cm/s $= 41.56$ cm/s

At 2cm, $V = 6\sqrt{7^2 - 2^2} = 18\sqrt{3}$ cm/s $= 40.25$ cm/s

At 3cm, $V = 6\sqrt{7^2 - 3^2} = 37.94$ cm/s

⑦ mathematically prove that the graph between velocity and displacement is an ellipse.

Ans:- $y = A\sin(\omega_0 t + \phi)$ $\quad$ $V = A\omega_0\cos(\omega_0 t + \phi)$

$\Rightarrow \frac{y}{A} = \sin(\omega_0 t + \phi)$ --- ① $\quad$ $\Rightarrow \frac{V}{A\omega_0} = \cos(\omega_0 t + \phi)$ --- ②

squaring and adding eqⁿ ① and ⑪, we get

$$\left(\frac{y}{A}\right)^2 + \left(\frac{V}{A\omega_0}\right)^2 = \sin^2(\omega_0 t + \phi) + \cos^2(\omega_0 t + \phi)$$

$$\Rightarrow \frac{y^2}{A^2} + \frac{V^2}{(A\omega_0)^2} = 1 \Rightarrow \text{ellipse}$$

(Hence proved)

⑧ Find the graph between velocity and acceleration.

Ans:- $V = A\omega_0\cos(\omega_0 t + \phi)$ $\quad$ $a = -A\omega_0^2\sin(\omega_0 t + \phi)$

$\Rightarrow \frac{V}{A\omega_0} = \cos(\omega_0 t + \phi)$ --- ① $\quad$ $\Rightarrow \frac{-a}{A\omega_0^2} = \sin(\omega_0 t + \phi)$ --- ②

squaring and adding eqⁿ ① and ②, we get

$$\frac{V^2}{(A\omega_0)^2} + \frac{a^2}{(A\omega_0^2)^2} = 1 \Rightarrow \text{ellipse}$$

(Hence proved)

⑨ Find total energy of the oscillator given by $y = 10\sin 2\pi t + \frac{\pi}{4}$ and $m = 2gm$.

Ans:- $y = 10\sin\boxed{2\pi t + \frac{\pi}{4}}$

$A = 10 \quad m = 2gm$

$w = 2\pi$

$TE = \frac{1}{2} \times m \times \omega_0^2 \times A^2$

$= \frac{1}{2} \times 2 \times (2\pi)^2 \times (10)^2$

$= 4\pi^2 \times 100$

$= 400\pi^2$

$= 3947.84$ erg

⑩ Prove that the total energy of an oscillator is constant.

Ans:- Total energy = Kinetic energy + Potential energy

$= \frac{1}{2} m\omega_0^2(A^2 - y^2) + \frac{1}{2} m\omega_0^2 y^2$

$= \frac{1}{2} m\omega_0^2 A^2$

As m, ω_0 and A are constant.

∴ Total energy is also constant.

⑪ The amplitude of an oscillator is 12cm. Find the position where kinetic energy is equal to potential energy.

Ans:- $KE = PE$

$\Rightarrow \cancel{\frac{1}{2}}\cancel{m}\cancel{\omega_0^2}(A^2 - y^2) = \cancel{\frac{1}{2}}\cancel{m}\cancel{\omega_0^2} y^2$

$\Rightarrow A^2 - y^2 = y^2$

$\Rightarrow A^2 = 2y^2$

$\Rightarrow y = \frac{A}{\sqrt{2}} = \frac{12}{\sqrt{2}} = 8.485$ cm

(12) set a differential equation of D.H.M?

Ans:- Number of forces acting on the body or oscillator in a viscuss medium to execute damping harmonic motion are:-

(i) restoring force, $F_R = -ky$

(ii) Frictional / Resistive force, $F_f = -bv$

$\therefore$ Net force acting on the is

$$F_{Net} = F_R + F_f$$

$$\Rightarrow ma = -ky - bv$$

$$\Rightarrow m\frac{d^2y}{dt^2} = -ky - b\frac{dy}{dt} \quad \left\{\because a = \frac{dv}{dt} = \frac{d}{dt}\frac{dy}{dt} = \frac{d^2y}{dt^2}\right\}$$

$$\Rightarrow \frac{d^2y}{dt^2} = -\left(\frac{k}{m}\right)y - \left(\frac{b}{m}\right)\frac{dy}{dt}$$

$$\Rightarrow \frac{d^2y}{dt^2} + \left(\frac{k}{m}\right)y + \left(\frac{b}{m}\right)\frac{dy}{dt} = 0$$

Let $\frac{k}{m} = \omega_0^2$ and $\frac{b}{m} = 2\beta$

$$\Rightarrow \frac{d^2y}{dt^2} + \omega_0^2 y + 2\beta\frac{dy}{dt} = 0$$

(13) write the differential equation of D.H.M?

Ans:- $\frac{d^2y}{dt^2} + \omega_0^2 y + 2\beta\frac{dy}{dt} = 0$

(14) write the physical significance of damping coefficient.

Ans:- $\beta = \frac{b}{2m}$

The damping coefficient indicates/signifies the nature of damping on the oscillator. Larger the mass, small is the damping and vice-versa.

(15) write the unit and dimension of damping coefficient.

Ans:- $F_f = -bv$

$$b = \frac{F}{v} = \frac{M^1L^1T^{-2}}{L^1T^{-1}} = M^1L^0T^{-1}$$

$$\beta = \frac{b}{2m} = \frac{M^1L^0T^{-1}}{M^1} = T^{-1} = sec^{-1} = Hz$$

unit is sec^{-1} and dimension is T^{-1}.

(16) Find the positions where the damping/Resistive force becomes maximum and minimum zero.

Ans:- $F_f = -bv$

Friction force is maximum at mean position ~~becau~~ because v is maximum.

Friction force is minimum zero at extreme position because v is minimum.

Friction force is minimum zero at extreme position because v is minimum.

(17) A damping oscillator of mass 5g and angular frequency of 7 radian/sec oscillates in critical condition with velocity 10 cm/sec. Find the magnitude of resistive / friction force.

Ans :- in critical Damping motion $\beta^2 \simeq \omega_0^2$

$\beta^2 \simeq 7^2$

$\beta \simeq 7$

$|F_f| = bv$

$= 2m\beta v$

$= 2 \times 5 \times 7 \times 10$

$= 700$ dyne

(18) Derive the solution of under damping motion.

Ans :- $y = A_1 e^{(-\beta + \sqrt{\beta^2 - \omega_0^2})t} + A_2 e^{(-\beta - \sqrt{\beta^2 - \omega_0^2})t}$

Condition $= \beta^2 < \omega_0^2$

$= \beta^2 - \omega_0^2 = -ve$

Now $\sqrt{\beta^2 - \omega_0^2} = \sqrt{-(\omega_0^2 - \beta^2)} = \sqrt{-\omega^2} = \pm i\omega$

$y = A_1 e^{(\beta + i\omega)t} + A_2 e^{(\beta - i\omega)t}$

$y = e^{-\beta t} \left[A_1 e^{i\omega t} + A_2 e^{-i\omega t}\right]$

$y = A e^{-\beta t} \sin(\omega t + \phi)$

(19) Write mathematical expression or formula for the following characteristics of DHM.

Ans :- (1) $a = A e^{-\beta t}$

(2) $\tau_m = \frac{1}{\beta}$

(3) $\lambda_d = \beta T$

(4) $E = E_0 e^{-2\beta T}$

(5) $\tau_R = \frac{1}{2\beta}$

(6) $Q = \omega \tau_R = \frac{\omega}{2\beta}$

(20) A damped oscillator is given by $Y = 16 e^{-0.5t} \sin(7\pi t + \frac{\pi}{4})$. Find $\tau_m, \lambda_d, \tau_R, Q$.

Ans :- Given $A = 16$, $\beta = +0.5$, $\omega = 7\pi$, $\phi = \frac{\pi}{4}$

$\tau_m = \frac{1}{\beta} = \frac{1}{0.5} = 2$ sec

$\lambda_d = \beta T = 0.5 \frac{2\pi}{7\pi} = 0.14$ $\left\{\because T = \frac{2\pi}{\omega}\right\}$

$\tau_R = \frac{1}{2\beta} = 1$ sec

$Q = \frac{\omega}{2\beta} = \frac{7\pi}{2 \times 0.5} = 21.99$

(21) The Q factor of a physical device of frequency 256 Hz is 5120. Find Relaxation time.

Ans:- $\tau_R = \frac{Q}{\omega} = \frac{5120}{2\pi \times 256} = 3.18$ sec

(22) The amplitude of a damped oscillator reduces to $\frac{1}{10}$ of its original after making 25 number of oscillation. If the time period is 2 sec. Find amplitude, τ_m, λ_d, τ_R, Q.

Ans:- $a_n = Ae^{-n\beta T}$

$\Rightarrow a_{25} = Ae^{-25.\beta.2}$

$\Rightarrow \frac{1}{10}\cancel{A} = \cancel{A}e^{-50\beta}$

$\Rightarrow 10 = e^{50\beta}$

$\Rightarrow \ln(10) = \ln(e^{50\beta})$

$\Rightarrow 2.3 = 50\beta$

$\Rightarrow \beta = \frac{2.3}{50} = 0.046$

$\tau_m = \frac{1}{\beta} = 21.73$ sec

$\lambda_d = \beta T = 0.046 \times 2 = 0.092$

$\tau_R = \frac{1}{2\beta} = 10.86$ sec

$Q = \frac{2\pi}{T} \times \tau_R = 34.1$

(23) $n = 50$, $t = 1.5$ sec, amplitude reduces to $\frac{1}{30}$ of original. Find τ_m, τ_R, λ_d, Q.

Ans:- $a_n = Ae^{-n\beta t}$

$\Rightarrow a_{50} = Ae^{-50 \times \beta \times 1.5}$

$\Rightarrow \frac{1}{30}\cancel{A} = \cancel{A}e^{-75\beta}$

$\Rightarrow 30 = e^{75\beta}$

$\Rightarrow \ln(30) = \ln(e^{75\beta})$

$\Rightarrow 3.40 = 75\beta$

$\Rightarrow \beta = \frac{3.40}{75} = 0.045$

$\tau_m = \frac{1}{\beta} = \frac{1}{0.034} = 22.22$ sec

$\tau_R = \frac{1}{2\beta} = \frac{1}{2 \times 0.034} = 11.11$ sec

$\lambda_d = \beta T = 0.045 \times 1.5 = 0.0675$

$Q = \frac{2\pi}{T} \times \tau_R = 46.55$

(24) The energy of the damped oscillator reduces to $\frac{1}{100}$ of original after making 100 number of oscillation. If the time period is 1 sec. Find τ_m, τ_R, λ_d, Q.

Ans:- $E_n = E_0 e^{-2n\beta t}$

$\Rightarrow \frac{1}{100}\cancel{E_0} = \cancel{E_0} e^{-2 \times 100 \times \beta \times 1}$

$\Rightarrow 100 = e^{200\beta}$

$\Rightarrow \ln(100) = \ln(e^{200\beta})$

$\Rightarrow 4.60 = 200\beta$

$\Rightarrow \beta = \frac{4.60}{200} = 0.023$

$\tau_m = \frac{1}{\beta} = 43.47$

$\tau_R = \frac{1}{2\beta} = 21.74$

$\lambda_d = \beta T = 0.023 \times 1 = 0.023$

$Q = \frac{2\pi}{T} \times \tau_R = 136.59$

(25) Discuss the three types of damped motion with examples.

Ans:- 1. Over Damped motion:- When the amplitude of oscillation of an oscillator decreases slowly and the body comes to rest and becomes non-oscillatory, then the motion is called over damping motion.

Condition:- $\beta^2 > \omega_0^2$

Example:- An oscillatory pendulum dipped into a thick oil bath.

2. Critical Damped motion:- When the amplitude of oscillation of an oscillator decreases rapidly or suddenly or instantly and the body comes to rest and becomes non-oscillatory, then the motion is called critical damping motion.

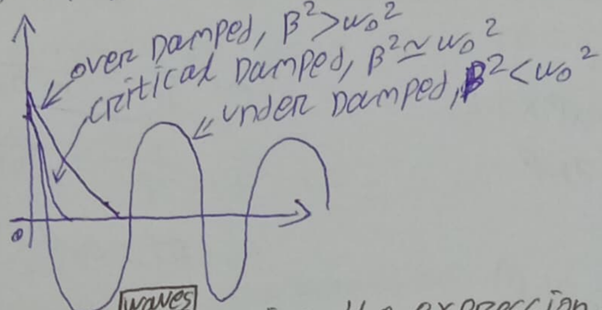
Condition:- $\beta^2 \simeq \omega_0^2$

Example:- All pointer device like ammeter, voltmeter, galvanometer, speedometer, etc.

3. Under Damping motion:- It is also known as Real damping motion. When the amplitude of oscillation decreases gradually and a body comes to rest after making few number of oscillations but till the end the body is oscillatory. This motion is called under damping motion.

Condition:- $\beta^2 < \omega_0^2$

Example:- oscillation of pendulum in viscous medium.



(26) What is superposition of waves? Derive the expression for the resultant amplitude of the superposition of two waves having equal frequencies and different amplitudes.

Ans:- When number of waves are traveling in a medium and combined together then a resultant wave is produced which is different from the individual wave, this phenomenon is called superposition of waves.

Two wave superposition:-

Let us consider two waves having different amplitudes but equal frequency and wave length, superpose (combine) to produce a resultant wave.

$y_1 = A_1 \sin(\omega t - kx)$ --- (1) and $y_2 = A_2 \sin(\omega t - kx + \phi)$ --- (2)

∴ according to principle of superposition resultant wave function

$y = y_1 + y_2$

$y = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx + \phi)$

$\Rightarrow y = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx)\cos\phi + A_2 \cos(\omega t - kx)\sin\phi$

$\Rightarrow y = \sin(\omega t - kx)\left[A_1 + A_2\cos\phi\right] + \cos(\omega t - kx)\left[A_2 \sin\phi\right]$

Let $A_1 + A_2\cos\phi = A\cos\phi$ --- (3) and $A_2 \sin\phi = A\sin\phi$ --- (4)

$\Rightarrow y = \sin(\omega t - kx) A\cos\phi + A\cos(\omega t - kx)\sin\phi$

$\Rightarrow y = A\left[\sin(\omega t - kx)\cos\phi + \cos(\omega t - kx)\sin\phi\right]$

$\Rightarrow y = A\sin(\omega t - kx + \phi)$

squaring eqn (3) and (4), we get

$$A^2 = (A_1 + A_2\cos\phi)^2 + A_2^2\sin^2\phi$$
$$A^2 = A_1^2 + 2A_1A_2\cos\phi + A_2^2\cos^2\phi + A_2^2\sin^2\phi$$
$$A^2 = A_1^2 + A_2^2(\cos^2\phi + \sin^2\phi) + 2A_1A_2\cos\phi$$
$$\boxed{A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

Now dividing eqn (3) and (4), we get

$$\boxed{\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}}$$

(27) Find the velocity of a longitudinal wave in a medium having bulk modulus 7×10^{11} dyne/cm^2 and density 2×10^3 gm/cc.

Ans:- $V_L = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{7\times10^{11}}{2\times10^3}} = 18708.28 = 1.87\times10^4$

(28) Find the velocity of transverse wave in a wire stretched by 2kg load if the length of the wire is 100 cm and mass 20 gm.

Ans:- $V_T = \sqrt{\frac{T}{m}} = \sqrt{\frac{mg}{mass/length}} = \sqrt{\frac{2\times9.8\times10^6}{20/100}} = 3.13\times10^3$

(29) A wave function is given by $y = 16\sin[10\pi t - 0.5x]$

Find (i) Amplitude $= 16$ cm

(ii) Frequency $= f = \frac{\omega}{2\pi} = \frac{10\pi}{2\pi} = 5$ Hz

(iii) wave length $k = 0.5 = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{0.5} = 4\pi$

(iv) wave vector $k = 0.5$

~~(v) wave velocity $v = \omega\times A = 16\times10\pi = 160\pi$~~

(v) wave velocity $v = n\lambda = 5\times12.56 = 62.83$ cm/sec

(vi) maximum particle velocity $V = \omega\times A = 10\pi\times16 = 160\pi$

(vii) Direction of wave

Direction of wave is opposite of the sign in +ve, as the direction of wave is opposite of the sign in $\sin(\omega t \pm \phi x)$

(30) A wave function is given by $Y = 100\cos[25\pi t + 0.8x]$

Find (i) Amplitude $= 100$ cm

(ii) Frequency $= f = \frac{\omega}{2\pi} = \frac{25\pi}{2\pi} = 12.5$ Hz

(iii) wave length $= k = 0.8 = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{0.8} = 7.853$ cm

(iv) wave vector $= k = 0.8$

(v) wave velocity $= V = n\lambda = 12.5\times7.853 = 98.174$ cm/sec

(vi) maximum particle velocity $= V = \omega\times A = 25\pi\times100 = 2500\pi = 7853.98$

(vii) Direction of wave $=$ -ve (z-Axis)

31) Write the classical wave equation for the following waves.
(i) Longitudenal waves in a medium.
Ans:- $\frac{\partial^2 y}{\partial t^2} = \frac{E}{S} \times \frac{\partial^2 y}{\partial x^2}$
(ii) Transverse wave in a string or wire.
Ans:- $\frac{\partial^2 y}{\partial t^2} = \frac{T}{m} \frac{\partial^2 y}{\partial x^2}$
(iii) Electromagnetic wave in vaccume or free space.
Ans:- $\frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 y}{\partial x^2}$ (or) $c^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$

32) Write the maximum and minimum value of the resultant amplitude.
Ans:- $A_{max} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2(1)} = A_1 + A_2$
$A_{min} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2(-1)} = A_1 - A_2$

33) If $\theta = \frac{\pi}{2}$, then find amplitude.
Ans:- $A = \sqrt{A_1^2 + A_2^2}$

34) The resultant amplitude due to 2 equal waves is equal to the magnitude of either wave. Find the phase difference between the 2 waves.
Ans:- $A = A_1 = A_2$
$\Rightarrow A^2 = A^2 + A^2 + 2A^2 \cos\phi$
$\Rightarrow A^2 = 2A^2 + 2A^2 \cos\phi$
$\Rightarrow \cancel{A^2} = 2\cancel{A^2}(1 + \cos\phi)$
$\Rightarrow \frac{1}{2} = 1 + \cos\theta$
$\Rightarrow \cos\theta = \frac{1}{2} - 1$
$\Rightarrow \cos\theta = -\frac{1}{2}$
$\Rightarrow \theta = \cos^{-1}\left(\frac{-1}{2}\right)$
$\Rightarrow \theta = 120$

35) Differentiate two types of superpositions.
Ans:-

Coherent superpositions	In-coherent superpositions
(i) If the phase difference between the waves are constant, then the superpositon is called coherent superpositions.	(i) If the phase difference between the waves are not constant, then the superposition is called incoherent superpositions.
(ii) $A^2 = I = A_1^2 + A_2^2 + 2A_1A_2 \cos\phi$ $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$ Here $\cos\phi = \pm 1$	(ii) Here $\cos\phi = 0$
(iii) $I_{max} = (A_1 + A_2)^2$ $I_{min} = (A_1 - A_2)^2$	(iii) $I = A_1^2 + A_2^2$
(iv) $I \neq I_1 + I_2$ (As $\phi = \pm 1$)	(iv) $I = I_1 + I_2$ (As $\phi = 0$)
(v) $I_{max} = N^2 a^2$	(v) Na^2

(35) 3 waves having amplitudes 2cm, 3cm and 4cm respectively superpose in a medium coherently and incoherently. Find there resultant intensity.

Ans:- $I_c = (2+3+4)^2 = 9^2 = 81\,cm^2$

$I_{inc} = 2^2+3^2+4^2 = 29\,cm^2$

(36) 4 waves having amplitudes 5, 6, 7, 8 respectively superpose in a medium coherently and incoherently. Find there resultant intensity.

Ans:- $I_c = (5+6+7+8)^2 = 676\,cm^2$

$I_{inc} = 5^2+6^2+7^2+8^2 = 174\,cm^2$

(37) $A_1 = 10, A_2 = 15$. Find the ratio between intensity due to maximum and minimum coherent superposition.

Ans:- $I_{max} = (A_1+A_2)^2 = (10+15)^2 = 625$

$I_{min} = (A_1-A_2)^2 = (10-15)^2 = 25$

$\frac{I_{max}}{I_{min}} = \frac{625}{25} = 25\,cm^2$

(38) 25 number of equal waves superpose coherently and incoherently. If their amplitude is 3cm then find there resultant intensities.

Ans:- coherently $= N^2a^2 = 25^2a^2 = 5625\,cm^2$

incoherently $= Na^2 = 25a^2 = 225\,cm^2$

(39) 20 number of equal waves superpose incoherently to produce resultant intensity 3200 unit. If they will superpose coherently then find the resultant intensity and also the individual amplitude.

Ans:- $Na^2 = 3200$

$N^2a^2 = N \cdot Na^2$

$= 20 \times 3200$

$= 64{,}000\,cm^2$

$Na^2 = 3200$

$20a^2 = 3200$

$a = \sqrt{\frac{3200}{20}} = 12.64\,cm$

Interference of waves

(40) Write the conditions for maximum intensity and minimum intensity for interference.

Ans:- For maximum intensity, $x = 2n\frac{\lambda}{2}$

$\Rightarrow$ Path Diff. = Even multiple of $\frac{\lambda}{2}$

For minimum intensity, $x = (2n+1)\frac{\lambda}{2}$

$\Rightarrow$ Path Diff. = Odd multiple of $\frac{\lambda}{2}$

(41) The path difference between two waves is $\Delta x = 15 \times 10^{-5}$ cm. Find wheather it produce maxima or minima. If the wavelength of the light is 6000 Å.

Ans:- $\frac{\lambda}{2} = \frac{6000}{2} \times 10^{-8} = 3 \times 10^{-5}$ cm

Path diff $= N \times \frac{\lambda}{2}$

$\Rightarrow N = \frac{\text{Path diff}}{(\lambda/2)} = \frac{15 \times 10^{-5}}{3 \times 10^{-5}} = \cancel{5.32} = 5$

As N is 5 which is odd.

$\therefore$ It is minima and produce minimum light.

(42) if $\Delta x = 24 \times 10^{-5}$. Find whether it produce maxima or minima if the wavelength of the light is 6000 Å.

Ans :- $N = \frac{24 \times 10^{-5}}{3 \times 10^{-5}} = 8$

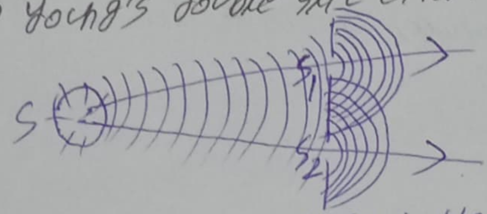
∴ it is maximum and produce maximum light.

(43) what is wave front ?

Ans:- wave front is a locus of all distrubances having equal phase.

(44) Explain young's double slit experiment.

Ans :-

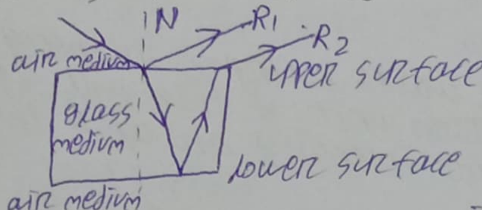


young double slit

when a source wave front (S) passes through a double slit then it produces two coherent sources S_1 and S_2. If the two coherent sources will superpose, then interference occurs.

(45) Explain coherent sources formation by division of amplitude.

Ans :-

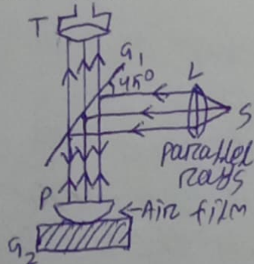


when a source of light passes through a medium it produces two reflected light. ① one from upper surface of the medium (R_1)
② From lower surface of the medium (R_2).

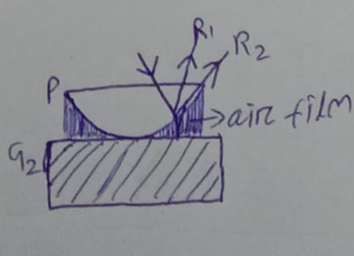
when these two coherent sources (R_1 and R_2) will superpose, then interference occurs,

(46) Define Newton's Ring experiment and explain.

Ans:-



S = monochromatic source
L = convex lens
G_1 = glass plate held at 45° with incedent
P = Plano-convex lens
G_2 = glas plate
T = Travelling microscope.



Experyment:-

→A monochromatic source of light 'S' is in incident at a convex lens(L) to produce parallel rays.

→ The parallel rays incident on a glass plate (G_1) which is kept at 45° with the incident light to produce normal rays.

→ The normal rays incident on a plano-convex lens (P) which is placed over a glass plate (G_2).

→ Two reflected lights R_1 and R_2 are produced from:-

(i) The upper surface of the medium (R_1)

(ii) the lower surface of the medium (R_2).

→ Since R_1 and R_2 are coherent and superpose to produce interference patter.

→ when the interference pattern is viewed through the travelling microscope (T), then alternet dark and bright concentric rings are visible which are called Newton's ring.

(17) Derive an expression for the diameters of dark and bright rings.

Ans:- The path difference between the two reflected lights R_1 and R_2 is given by. $\Delta_{R_1R_2} = 2\mu t \cos \gamma + \frac{\lambda}{2}$

where, μ = Refractive index of the medium

t = thickness of the medium

γ = Angle of Refraction

λ = wavelength of light

For normal incidence, $\gamma = 0 \Rightarrow \cos \gamma = 1$

$$\Delta_{R_1R_2} = 2\mu t + \frac{\lambda}{2}$$

For dark condition (minimum intensity or minima):-

path diff. = odd multiple of $\frac{\lambda}{2}$

$$\Rightarrow 2\mu t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow 2\mu t = (2n+1)\frac{\lambda}{2} - \frac{\lambda}{2}$$

$$\Rightarrow 2\mu t = \frac{\lambda}{2}(2n+1-1)$$

$$\Rightarrow 2\mu t = \frac{\lambda}{\cancel{2}} \times \cancel{2}n$$

$$\Rightarrow \boxed{2\mu t = n\lambda}$$

For bright condition (maximum intensity or maxima):-

path diff. = even multiple of $\frac{\lambda}{2}$

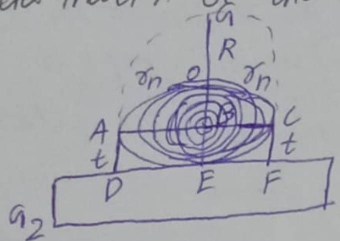
$$\Rightarrow 2\mu t + \frac{\lambda}{2} = 2n\frac{\lambda}{2}$$

$$\Rightarrow 2\mu t = 2n\frac{\lambda}{2} - \frac{\lambda}{2}$$

$$\boxed{\Rightarrow 2\mu t = \frac{\lambda}{2}(2n-1)}$$

(48) Determination of the thickness of the medium.

Ans :-



where $GE = 2R$

$GB = (2R - t)$

γ_n = Redius of n^{th} ring

R = Radius of curvature of Plano-convex lens

$AD = BE = CF = t$

From Geometry: $GB \times BE = BA \times BC$

$\Rightarrow (2R-t).t = \gamma_n \times \gamma_n$

$\Rightarrow (2R-t).t = \gamma_n^2$

$\Rightarrow 2Rt - t^2 = \gamma_n^2$

It is very small. So it can be neglated

$\Rightarrow 2Rt = \gamma_n^2$

$\Rightarrow \boxed{t = \frac{\gamma_n^2}{2R}}$

(49) Determination of diameter of n^{th} dark ring.

Ans :- we have dark condition i.e.

$2\mu t = n\lambda$, $t = \frac{\gamma_n^2}{2R}$

$\Rightarrow 2\mu \frac{\gamma_n^2}{2R} = n\lambda$

$\Rightarrow \gamma_n^2 = \frac{nR\lambda}{\mu}$ $\{\because \gamma = \frac{D}{2} \Rightarrow \gamma^2 = \frac{D^2}{4}\}$

$\Rightarrow \frac{D_n^2}{4} = \frac{nR\lambda}{\mu}$

$\boxed{\Rightarrow D_n^2 = \frac{4nR\lambda}{\mu}}$

$\boxed{\Rightarrow D_n = \sqrt{\frac{4nR\lambda}{\mu}}}$

For air medium $\mu = 1$

$\boxed{\Rightarrow D_n = \sqrt{4nR\lambda}}$

$\boxed{\Rightarrow D_n^2 = 4nR\lambda}$

This is the Required expression for diameter of n^{th} dark ring.

(50) Determination of diameter of n^{th} bright ring.

Ans :- we know, bright condition i.e.

$2\mu t = \frac{\lambda}{2}(2n-1)$

$\Rightarrow 2\mu \frac{\gamma_n^2}{2R} = \frac{\lambda}{2}(2n-1)$ $\{\because t = \frac{\gamma_n^2}{2R}\}$

$\Rightarrow \gamma_n^2 = \frac{\lambda}{2}(2n-1)\frac{R}{\mu}$

$\Rightarrow \frac{D_n^2}{4} = \frac{\lambda R(2n-1)}{2\mu}$

$\boxed{\Rightarrow D_n^2 = \frac{2\lambda R(2n-1)}{\mu}}$ $\boxed{\Rightarrow D_n = \sqrt{\frac{2\lambda R(2n-1)}{\mu}}}$

For air medium, $\mu = 1$

$$\boxed{\Rightarrow D_n^2 = 2\lambda R(2n-1)}$$

$$\boxed{\Rightarrow D_n = \sqrt{2\lambda R(2n-1)}}$$

(51) Find the diameters of the following ring with $R = 100\,cm$ and wavelength $6000\,Å$. Find (a) $2^{nd}, 4^{th}, 6^{th}, 8^{th}, 10^{th}$ dark ring
(b) $3^{rd}, 5^{th}, 7^{th}, 9^{th}, 11^{th}$ Bright ring.

Ans:- The expression for nth Bright ring is $D_n = \sqrt{2\lambda R(2n-1)}$
The expression of nth dark ring is $D_n = \sqrt{4n\lambda R}$

(a) $2^{nd} = \sqrt{4 \times 2 \times 6000 \times 10^{-8} \times 100} = 0.219\,cm$
$4^{th} = 0.3098\,cm$
$6^{th} = 0.379\,cm$
$8^{th} = 0.438\,cm$
$10^{th} = 0.489\,cm$

(b) $3^{rd} = 0.024\,cm$
$5^{th} = 0.0328\,cm$
$7^{th} = 0.039\,cm$
$9^{th} = 0.045\,cm$
$11^{th} = 0.050\,cm$

(52) The diameters of thick 5^{th} dark ring and 9^{th} dark ring are $0.45\,cm$ and $0.85\,cm$ respectively. If the radius of curvature of the plano-convex lens is 100cm the wavelength of the light.

Ans:- $D_5 = 0.45\,cm$, $D_9 = 0.85\,cm$, $R = 100\,cm$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR} = \frac{0.85^2 - 0.45^2}{4 \times 4 \times 100} = 3.25 \times 10^{-4}\,cm$$

(53) The diameter of 7^{th} dark ring in air medium is given as $1.73\,cm$ when a liquid is introduced the same diameter shrinks to $1.41\,cm$. Find the refractive index of the liquid.

Ans:- $$\mu = \frac{D_n^2(air)}{D_n^2(liquid)} = \frac{1.73^2}{1.41^2} = 1.505\,cm$$

(54) In air medium $D_5 = 5\,mm$, $D_9 = 9\,mm$ and in liquid medium $D_5 = 3\,mm$, $D_9 = 7\,mm$.

Ans:- $$\mu = \frac{D_n^2(air)}{D_n^2(liquid)} = \frac{D_{(n+p)}^2 - D_n^2(air)}{D_{(n+p)}^2 - D_n^2(liquid)} = \frac{9^2 - 5^2}{7^2 - 3^2} = 1.4\,mm$$

(55) The diameter of nth order dark ring for wave length $5000\,Å$ coinsides with $(n+1)$th order for wavelength $4000\,Å$. If $R = 100\,cm$, then find diameter of $n+5$ order for wavelength $6000\,Å$.

Ans:- $4nR\lambda_1 = 4(n+1)R\lambda_2$
$\cancel{4}n\cancel{R}(5000) = \cancel{4}(n+1)\cancel{R}(4000)$
$5n = 4n + 4$
$n = 4$

$n+5 = 9$
$D_9 = \sqrt{4n\lambda R} = \sqrt{4 \times 9 \times 6 \times 10^{-5} \times 100} = 0.464$

(56) The diameter of n+5 dark ring is 5mm and diameter of n+9 dark ring is 9mm. Find the diameter of n+13.

Ans:- $D_{n+5} = 5mm$ $D_{n+9} = 9mm$ $D_{n+13} = ?$

$\lambda_1 = \frac{D^2_{n+9} - D^2_{n+5}}{4.4.R}$ $\lambda_2 = \frac{D^2_{n+13} - D^2_{n+9}}{4.4.R}$

$D^2_{n+9} - D^2_{n+5} = D^2_{n+13} - D^2_{n+9}$

$\Rightarrow 9^2 - 5^2 + 9^2 = D^2_{n+13}$

$\Rightarrow D^2_{n+13} = 137$

$\Rightarrow D_{n+13} = 11.70 cm$

(57) If $D_{10} = 2.56 cm$, $D_{20} = 3.5cm$. Find wavelength. $R = 100 cm$

Ans:- $\lambda = \frac{D^2_{20} - D^2_{10}}{4.P.R} = \frac{3.5^2 - 2.56^2}{4.10.100} = 1.42 \times 10^{-3} cm$

(58) $D_{n+11} = 11mm$, $D_{n+22} = 22mm$, $D_{n+33} = ?$

Ans:- $D^2_{n+22} - D^2_{n+11} = D^2_{n+33} - D^2_{n+22}$

$\Rightarrow 2 D^2_{n+22} - D^2_{n+11} = D^2_{n+33}$

$\Rightarrow D_{n+33} = \sqrt{2 \times 22^2 - 11^2}$

$\Rightarrow D_{n+33} = 29.103 mm$

(59) $D_5 = 2cm$ in air $D'_5 = 1.2$ in liquid. Find refractive index.

Ans:- $\mu = \frac{(D_5)^2 (air)}{(D_5)^2 (liquid)} = \frac{2^2}{1.2^2} = 2.78$

(60) Why the central part of the Netwon's ring pattern is dark?

Ans:- Because the interference is by two reflected lights which are coherent.

$\boxed{\text{Electromagnetic Theory}}$

(61) Find the gradient of a scalar quantity $s = x^2 + y^2 + z^2$

Ans:- Grad $s = \vec{\nabla} s = \hat{i}\frac{\partial s}{\partial x} + \hat{j}\frac{\partial s}{\partial y} + \hat{k}\frac{\partial s}{\partial z}$

$\vec{\nabla} s = \hat{i}\frac{\partial}{\partial x}(x^2+y^2+z^2) + \hat{j}\frac{\partial}{\partial y}(x^2+y^2+z^2) + \hat{k}\frac{\partial}{\partial z}(x^2+y^2+z^2)$

$\vec{\nabla} s = \hat{i}(2x) + \hat{j}(2y) + \hat{k}(2z)$

(62) $g = 2x^3 + 3y^3 + 4z^3$. Find grad g.

Ans:- grad $g = \vec{\nabla} g = \hat{i}\frac{\partial g}{\partial x} + \hat{j}\frac{\partial g}{\partial y} + \hat{k}\frac{\partial g}{\partial z}$

$\vec{\nabla} g = \hat{i}\frac{\partial}{\partial x}(2x^3+3y^3+4z^3) + \hat{j}\frac{\partial}{\partial y}(2x^3+3y^3+4z^3) + \hat{k}\frac{\partial}{\partial z}(2x^3+3y^3+4z^3)$

$= \hat{i}(6x^2) + \hat{j}(9y^2) + \hat{k}(12z^2)$

(63) $f = xy + yz + zx$. Find grad f.

Ans:- grad $f = \vec{\nabla} f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$

$\vec{\nabla} f = \hat{i}\frac{\partial}{\partial x}(xy+yz+zx) + \hat{j}\frac{\partial}{\partial y}(xy+yz+zx) + \hat{k}\frac{\partial}{\partial z}(xy+yz+zx)$

$\vec{\nabla} f = \hat{i}(y+z) + \hat{j}(x+z) + \hat{k}(y+x)$

(64) Find the grad $|\vec{r}|$, where $\vec{r}$ is position vector.

Ans:- $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$

$|\vec{r}| = [x^2+y^2+z^2]^{1/2}$

Grad $|\vec{r}| = \hat{i}\frac{\partial|\vec{r}|}{\partial x} + \hat{j}\frac{\partial|\vec{r}|}{\partial y} + \hat{k}\frac{\partial|\vec{r}|}{\partial z}$

$= \hat{i}\frac{\partial}{\partial x}[x^2+y^2+z^2]^{1/2} + \hat{j}\frac{\partial}{\partial y}[x^2+y^2+z^2]^{1/2} + \hat{k}\frac{\partial}{\partial z}[x^2+y^2+z^2]^{1/2}$

$= \hat{i}\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-1/2}\cancel{2}x + \hat{j}\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-1/2}\cancel{2}y + \hat{k}\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-1/2}\cancel{2}z$

$= \frac{\hat{i}x}{(x^2+y^2+z^2)^{1/2}} + \frac{\hat{j}y}{(x^2+y^2+z^2)^{1/2}} + \frac{\hat{k}z}{(x^2+y^2+z^2)^{1/2}}$

$= \frac{\hat{i}x + \hat{j}y + \hat{k}z}{(x^2+y^2+z^2)^{1/2}}$

$= \frac{\vec{r}}{|\vec{r}|} = \hat{r}$

$\therefore$ grad $|\vec{r}| = \hat{r}$ [Hence proved]

(65) $\vec{A} = \hat{i}2x^2y + \hat{j}3y^2z + \hat{k}4z^2x$. Find divergence at $(1,1,1)$.

Ans:- Div $\vec{A} = \vec{\nabla}\cdot\vec{A} = \frac{\partial}{\partial x}(2x^2y) + \frac{\partial}{\partial y}(3y^2z) + \frac{\partial}{\partial z}(4z^2x)$

$= 4xy + 6yz + 8zx$

at $(1,1,1) = 4+6+8 = 18$

(66) $\vec{B} = \hat{i}3xy^2z + \hat{j}4x^2yz + \hat{k}5xyz^2$. Find divergence at $(1,1,1)$.

Ans:- Div $\vec{B} = \vec{\nabla}\cdot\vec{B} = \frac{\partial}{\partial x}(3xy^2z) + \frac{\partial}{\partial y}(4x^2yz) + \frac{\partial}{\partial z}(5xyz^2)$

$= 3y^2z + 4x^2z + 10xyz$

at $(1,1,1) = 3+4+10 = 17$

(67) $\vec{E} = \hat{i}xy + \hat{j}yz + \hat{k}zx$. Find Div $\vec{E}$ at $(1,1,-2)$.

Ans:- Div $\vec{E} = \vec{\nabla}\cdot\vec{E}$

$= \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(yz) + \frac{\partial}{\partial z}(zx)$

$= y + z + x$

at $(1,1,-2) = 1-2+1$

$= 0$

(68) What is solenoidal vector?

Ans:- If the divergence of a vector is zero, then the vector is called solenoidal vector i.e. Div $\vec{A}=0 \Leftrightarrow \vec{A}$ is solenoidal vector.

(69) $\vec{A}=\hat{i}\,2x^2y+\hat{j}\,b3y^2z+\hat{k}\,4yz^2x$. Find 'b' if $\vec{A}$ solenoidal at (1,1,1).

Ans:- $\text{Div}\,\vec{A}=\vec{\nabla}\cdot\vec{A}=0$

$$=\frac{\partial}{\partial x}(2x^2y)+\frac{\partial}{\partial y}b(3y^2z)+\frac{\partial}{\partial z}(4z^2x)$$

$$=4xy+6byz+8zx$$

at (1,1,1) $=4+6b+8=0$

$$\Rightarrow 12+6b=0$$

$$\Rightarrow b=\frac{-12}{6}=-2$$

(70) $\vec{E}=\hat{i}\,x^3y+\hat{j}\,y^3z+\hat{k}\,cz^3x$. Find 'c' if $\vec{E}$ is solenoidal at (1,2,3).

Ans:- $\text{Div}\,\vec{E}=\vec{\nabla}\cdot\vec{E}=0$

$$=\frac{\partial}{\partial x}(x^3y)+\frac{\partial}{\partial y}(y^3z)+\frac{\partial}{\partial z}(cz^3x)=0$$

$$=3x^2y+3y^2z+3cz^2x=0$$

at (1,2,3) $=3.1.2+3.4.3+3.c.9.1=0$

$$\Rightarrow 6+36+27c=0$$

$$\Rightarrow c=\frac{-42}{27}$$

(71) $\vec{A}=\hat{i}\,xy+\hat{j}\,yz+\hat{k}\,zx$. Find curl $\vec{A}$.

Ans:- $\text{curl}\,\vec{A}=\vec{\nabla}\times\vec{A}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ xy & yz & zx\end{vmatrix}$

$$=\hat{i}\left[\frac{\partial}{\partial y}zx-\frac{\partial}{\partial z}yz\right]-\hat{j}\left[\frac{\partial}{\partial x}zx-\frac{\partial}{\partial z}xy\right]+\hat{k}\left[\frac{\partial}{\partial x}yz-\frac{\partial}{\partial y}xy\right]$$

$$=\hat{i}(0-y)-\hat{j}(z-0)+\hat{k}(0-x)$$

$$\Rightarrow -\hat{i}y-\hat{j}z-\hat{k}z$$

(72) Find Div $\vec{r}$ where $\vec{r}$ = position vector.

Ans:- $\vec{r}=\hat{i}x+\hat{j}y+\hat{k}z$

$$\text{Div}\,\vec{r}=\vec{\nabla}\cdot\vec{r}=\frac{\partial}{\partial x}r_x+\frac{\partial}{\partial y}r_y+\frac{\partial}{\partial z}r_z$$

$$=\frac{\partial}{\partial x}x+\frac{\partial}{\partial y}y+\frac{\partial}{\partial z}z$$

$$=1+1+1$$

$$=3$$

(73) Find curl of position vector.

Ans:- $\vec{\nabla}\times\vec{r}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x & y & z\end{vmatrix}=\hat{i}\left[\frac{\partial}{\partial y}z-\frac{\partial}{\partial z}y\right]-\hat{j}\left[\frac{\partial}{\partial x}z-\frac{\partial}{\partial z}x\right]+\hat{k}\left[\frac{\partial}{\partial x}y-\frac{\partial}{\partial y}x\right]$

$$=\hat{i}0-\hat{j}0-\hat{k}0$$

$$=\hat{0}$$

(74) What is irrotational vector?

Ans:- If the curl of a vector is 0 then the vector is called irrotational vector. curl $\vec{A} = 0 \Rightarrow \vec{A}$ is irrotational.

(75) $\vec{A} = \hat{i}\, 2x^2y + \hat{j}\, 3y^2z + \hat{k}\, 4z^2x$. Find line integral of $\vec{A}$ along x, y, z axis.

Ans:- $I_x = \int \vec{A} \cdot \hat{i}\, dx = \int 2x^2y\, dx = 2y \frac{x^3}{3} = \frac{2x^3y}{3}$

$I_y = \int \vec{A} \cdot \hat{j}\, dy = \int 3y^2z\, dy = \frac{3zy^3}{3} = y^3z$

$I_z = \int \vec{A} \cdot \hat{k}\, dz = \int 4z^2x\, dz = \frac{4xz^3}{3}$

(76) $\vec{A} = \hat{i}\, 2x^2y + \hat{j}\, 3y^2z + \hat{k}\, 4z^2x$. Find the surface Integral of $\vec{A}$ along xy, yz and zx plane.

Ans:- Along xy-plane

$\iint \vec{A} \cdot \hat{k}\, dx\, dy$

$= \iint 4z^2x\, dx\, dy$

$= 4z^2 \int x\, dx \int dy$

$= 4z^2 \frac{x^2}{2} y$

$= 2z^2x^2y$

Along yz-Plane

$\iint \vec{A} \cdot \hat{i}\, dy\, dz$

$= \iint 2x^2y\, dy\, dz$

$= 2x^2 \int y\, dy \int dz$

$= 2x^2 \frac{y^2}{2} z$

$= x^2y^2z$

Along zx-Plane

$\iint \vec{A} \cdot \hat{j}\, dz\, dx$

$= \iint 3y^2z\, dz\, dx$

$= 3y^2 \int z\, dz \int dx$

$= 3y^2 \frac{z^2}{2} x$

$= \frac{3y^2z^2x}{2}$

(77) State gauss divergence theorem and stoke's theorem?

Ans:- Gauss divergence theorem:- It converts a surface integral into a volume integral. It states that "the surface integral of a vector is equal to the volume integral of the divergence of that vector enclosed by the surface". mathematically:-

$$\iint_S \vec{A} \cdot \vec{ds} = \iiint_V \text{Div}\, \vec{A} \cdot dv$$

$$\Rightarrow \iint_S \vec{A} \cdot \vec{ds} = \iiint_V (\vec{\nabla} \cdot \vec{A})\, dv$$

Stroke's theorem:- It converts a line integral into a surface integral. It states that, "the line integral of a vector is equal to the surface integral of curl of that vector inclosed by the line". Mathematically:-

$$\int_l \vec{A} \cdot \vec{dl} = \iint_S (\text{curl} \times \vec{A})\, \vec{ds}$$

$$\int_l \vec{A} \cdot \vec{dl} = \iint_S (\vec{\nabla} \times \vec{A}) \cdot \vec{ds}$$

(78) Using gauss divergence theorem prove that the volume of a sphere is $\frac{4}{3}\pi r^3$ where r is the radius or position.

Ans:- $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$

According to gauss divergence theorem,

$$\iint_S \vec{r} \cdot \vec{ds} = \iiint_V (\vec{\nabla} \cdot \vec{r})\, dv \qquad \{\because \vec{\nabla} \cdot \vec{r} = 3\}$$

$$\Rightarrow \iint_S \vec{r} \cdot \vec{ds} = 3 \iiint_V dv$$

$$\Rightarrow \iint_S \vec{r} \cdot \vec{ds} = 3V$$

$$\Rightarrow \vec{r} \iint_S \vec{ds} = 3V$$

$$\Rightarrow \vec{r}\, S = 3V \qquad \{\because S = 4\pi r^2\}$$

$$\Rightarrow 4\pi r^3 = 3V$$

$$\Rightarrow V = \frac{4}{3}\pi r^3$$

(Hence Proved)

(79) What is conservative vector and non-conservative vector?

Ans:- Conservative vector:- It is a vector which is path independent. Mathematically if a vector conservative, then its line integral is zero. $\oint_L \vec{A} \cdot \vec{dl} = 0 \Rightarrow \vec{A}$ is conservative.

Non-conservative vector:- It is a vector which is path dependent. Mathematically if a vector non-conservative, then its line integral is non-zero. $\oint_L \vec{A} \cdot \vec{dl} \neq 0$.

(80) If A is a conservative vector, then prove that A is also irrotational vector.

Ans:- As A is a conservative vector i.e. $\oint_L \vec{A} \cdot \vec{dl} = 0$ and by using stoke's theorem.

$$\oint_L \vec{A} \cdot \vec{dl} = \iint_S (\text{curl} \times \vec{A}) \cdot \vec{ds} = 0$$

$$\Rightarrow \vec{\nabla} \times \vec{A} = 0$$

$\Rightarrow \vec{A}$ is irrotational vector.

(81) Prove that Gradient of any scalar value is an irrotational vector.

Ans:- Grad $S = \hat{i}\frac{\partial S}{\partial x} + \hat{j}\frac{\partial S}{\partial y} + \hat{k}\frac{\partial S}{\partial z}$

$$\text{Curl}(\text{Grad } S) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial S}{\partial x} & \frac{\partial S}{\partial y} & \frac{\partial S}{\partial z} \end{vmatrix} = \hat{i}\left(\frac{\partial^2 S}{\partial y \partial z} - \frac{\partial^2 S}{\partial y \partial z}\right) - \hat{j}\left(\frac{\partial^2 S}{\partial x \partial z} - \frac{\partial^2 S}{\partial x \partial z}\right) + \hat{k}\left(\frac{\partial^2 S}{\partial x \partial y} - \frac{\partial^2 S}{\partial x \partial y}\right)$$

$= \text{Null vector or } \hat{0}$

From the above relation it is seen that Gradient of any scalar value is a vector which is irrotational vector.

(82) Prove that curl of any vector is solenoidal vector.

Ans:- $$\text{curl } \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \hat{j}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\text{Div}(\text{curl } \vec{A}) = \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$= 0$

From the above relation it is seen that curl of any vector is solenoidal vector.

(83) If $\vec{A}$ and $\vec{B}$ are irrotational vectors, then prove that $\vec{A} \times \vec{B}$ is a solenoidal vector.

Ans:- We know that $\text{Div}(\vec{A} \times \vec{B})$

$= \vec{B} \text{ curl } \vec{A} - \vec{A} \text{ curl } \vec{B}$

$= \vec{B} \times 0 - \vec{A} \times 0$ (As $\vec{A}$ and $\vec{B}$ are irrotational vector)

$= 0$

As $\text{Div}(\vec{A} \times \vec{B}) = 0$

$\therefore \vec{A} \times \vec{B}$ is a solenoidal vector.

(84) $\nabla^2 g = 0$ then show $\vec{\nabla} g$ is irrotational as well as solenoidal.

Ans:- $\nabla^2 g = 0$

$\Rightarrow \vec{\nabla}(\vec{\nabla} \cdot g) = 0$

$\Rightarrow \vec{\nabla} \cdot g$ is solenoidal vector

As gradient of any scalar value is an irrotational vector i.e. curl $(\vec{\nabla} \cdot g) = 0$.

(85) Write the mathematical defination of gauss law of Electrostatics and also write it's integral form and differential form.

Ans:- $\phi_{\vec{E}} = \oint_S \vec{E} \cdot \vec{dS} = \frac{1}{\epsilon_0} q_{Net}$

Integral form:-

$$\oint_S \vec{E} \cdot \vec{dS} = \frac{1}{\epsilon_0} \iiint_V \rho \, dV$$

Differential form:-

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

(86) using gauss law of electrostatics find electric field.

Ans:- $\oint\oint_S \vec{E}\cdot\vec{ds} = \frac{1}{\epsilon_0} q_{net}$

$\Rightarrow E \oint\oint_S \vec{ds} = \frac{1}{\epsilon_0} q_{net}$

$\Rightarrow E.S = \frac{q}{\epsilon_0}$

$\Rightarrow E.4\pi r^2 = \frac{q}{\epsilon_0}$ $\{\because S = 4\pi r^2\}$

$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$

(87) write the mathematical defination of Gauss law of magnetostatics and also write the integral form and differential form of maxwell's 2nd equation.

Ans:- $\phi_{\vec{B}} = \oint\oint_S \vec{B}\cdot\vec{ds} = 0$

Integral form:-

$\oint\oint_S \vec{B}\cdot\vec{ds} = 0$

Differential form:-

$\vec{\nabla}\cdot\vec{B} = 0$

(88) Show that magnetic field is a solenoidal vector.

Ans:- From maxwell's 2nd equation we know that $\vec{\nabla}\cdot\vec{B} = 0$.

Hence $\vec{B}$ is solenoidal vector.

(89) Show that monopole does not exist.

Ans:- From maxwell's 2nd equation we know that

$\vec{\nabla}\cdot\vec{B} = 0$ (or) $\oint\oint_S \vec{B}\cdot\vec{ds} = 0$

Hence this shows that monopole does not exist as the total flux is zero. (poles exist in pairs)

(90) write the mathematical defination of Faraday's law and also write the integral form and differential form of maxwell's 3rd eqn.

Ans:- $\varepsilon = -\frac{\partial\phi_{\vec{B}}}{\partial t}$

Integral form:-

$\oint_L \vec{E}\cdot\vec{dl} = -\frac{\partial}{\partial t}\oint\oint_S \vec{B}\cdot\vec{ds}$

Differential form:-

$\vec{\nabla}\cdot\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

(91) The magnetic flux linked with a circuit is given by $\phi = 4t^3 + 3t^2 + 5t$. Find the induced emf at $t = 1$ sec.

Ans:- $\varepsilon = +\frac{\partial\vec{B}}{\partial t} = \frac{\partial}{\partial t}(4t^3 + 3t^2 + 5t)$

$= (12t^2 + 6t + 5)$

$= 12 + 6 + 5$

$= 23\,V$

(92) write the mathematical defination of Ampere's circuital law and also write its integral form and differential form.

Ans:- $\oint_L \vec{B}\cdot\vec{dl} = \mu_0 I_{net}$

Integral form:- $\oint_L \vec{B}\cdot\vec{dl} = \mu_0 \iint_S \vec{J}\cdot\vec{ds}$

Differential form:- $\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$

(93) using Ampere's circuital law derive the expression for magnetic field.

Ans:- we know, $\oint_L \vec{B}\cdot\vec{dl} = \mu_0 I_{net}$

$\Rightarrow \vec{B}\oint_L \vec{dl} = \mu_0 I_{net}$

$\Rightarrow B l = \mu_0 I$ $\{\because l = 2\pi r\}$

$\Rightarrow B(2\pi r) = \mu_0 I$ $\{\because$ where r is the radius of loop$\}$

$\Rightarrow B = \dfrac{\mu_0 I}{2\pi r}$

(94) A parallel plate capacitor ~~cont~~ contains circular plates of radius 1m is charged by a varying electric field of 5×10^{12} unit. Find the displacement current.

Ans:- $\dfrac{dE}{dt} = 5\times10^{12}$ unit $\qquad A = \pi r^2 = \pi\, m^2$

$I_d = \epsilon_0 A \dfrac{dE}{dt} = 8.85\times10^{-12}\times\pi\times5\times10^{12} = 139.01$ amp

(95) A parallel plate capacitor is charged by a varying electric flux of 7×10^{12} unit. Find displacement current.

Ans:- $I_d = \epsilon_0 \dfrac{d\phi}{dt} = 8.85\times10^{-12}\times7\times10^{12} = 61.95$ amp

(96) The capacitance of a parallel plate capacitor is $C = 24\mu F$ is charged by a varying potential of 4×10^{6} volt/sec.

Ans:- $C = 24\mu F = 24\times10^{-6}$ F

$\dfrac{dV}{dt} = 4\times10^{6}$ volt/sec

$I_d = C\dfrac{dV}{dt} = 24\times10^{-6}\times4\times10^{6} = 96$ amp

(97) Difference between conduction current and displacement current.

Ans:-

Conduction current (I_c)	Displacement current (I_d)
→ It is real current	→ It is fictitious/virtual.
→ It is due to the flow of electrons or charges.	→ It is due to varying electric field.
→ It obeys ohm's law	→ It does not obeys ohm's law.
→ $I_c = \dfrac{V}{R}$	→ $I_d = \epsilon_0 A \dfrac{dE}{dt}$
→ It depends on (a) P.D (V) (b) Resistance (R)	→ It depend on (a) permitivity (ϵ_0) (b) Area of plate (A) (c) rate of change of $\vec{E}$
→ It flows through a metallic conductor.	→ It can flow through any medium even in vaccum.

(48) Write the mathematical defination of modified Ampere's circuital law and also write its integral and differential form.

Ans:- $\oint_L \vec{B}\cdot\vec{dl} = \mu_0 (I_c + I_d)$

Integral form:-

$$\oint_L \vec{B}\cdot\vec{dl} = \mu_0 \iint_S \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)\vec{ds}$$

Differential form:-

$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$