PHYSICS CRASH COURSE

WORK, ENERGY AND POWER

IN 1 SHOT

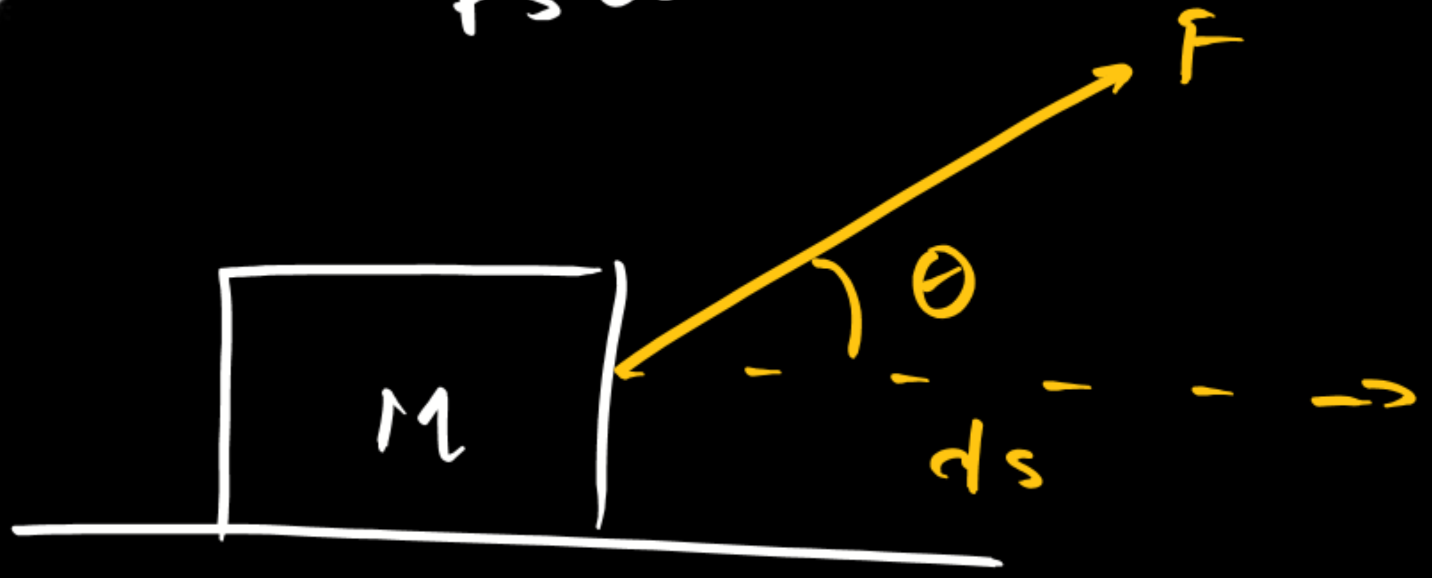
JEE Main & Advanced

Practice Sheet on PW App

Work, Energy and power

Work

$Fs \cos$



Work $= \vec{F} \cdot \vec{ds}$ {Dot Prod

Scalar Qty. $= |F||ds| \cos\theta$.

This is displacement of point of app of force in dir of force.

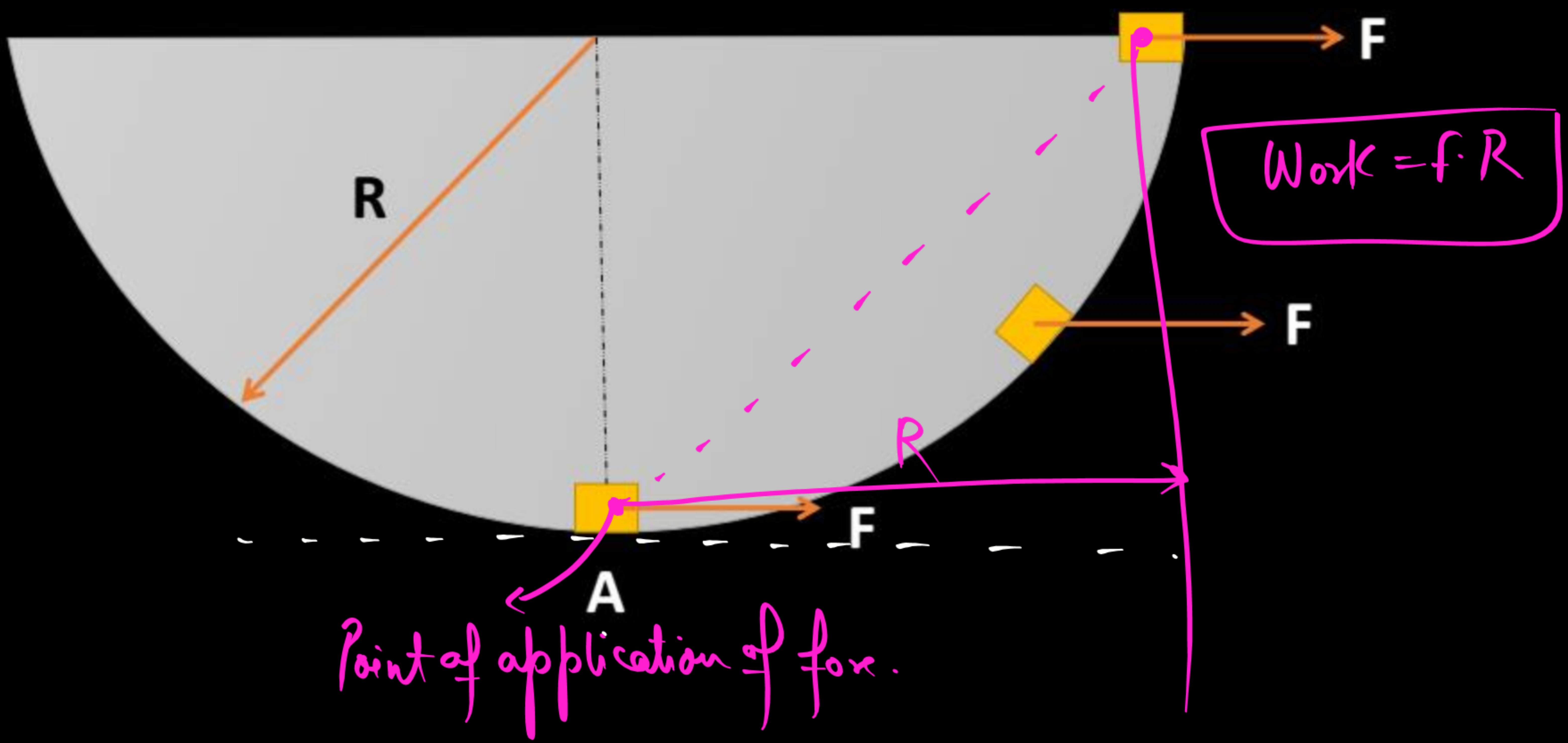
	(SI)	(CGS)
Unit :-	Joules	erg

$\boxed{1J = 10^7 \text{ erg}}$.

Dimension $[ML^2T^{-2}]$

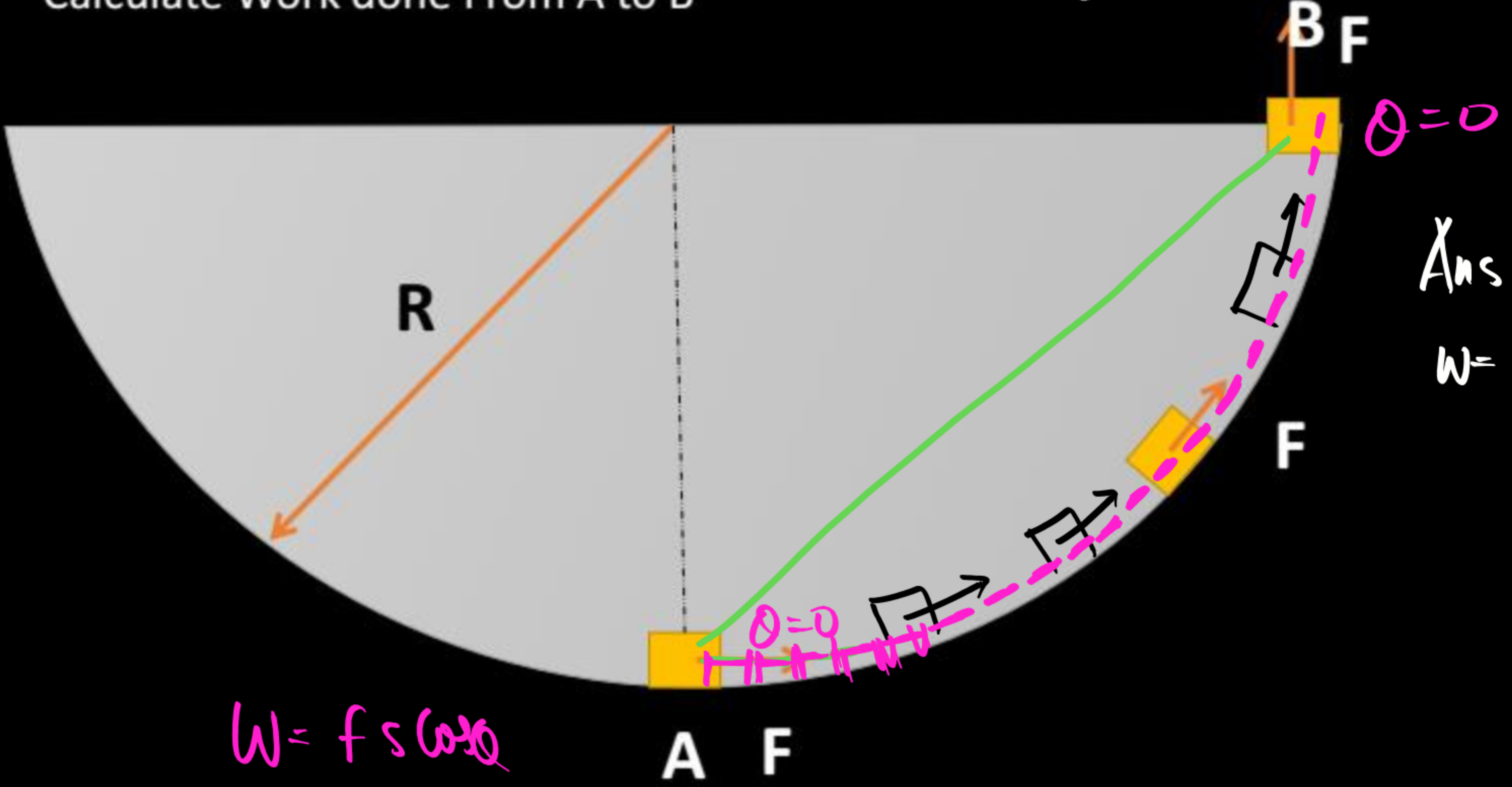
Calculate Work done From A to B

F is always ∥r to Ground



Calculate Work done From A to B

force is tangential to surface.



$W = f s \cos\theta$

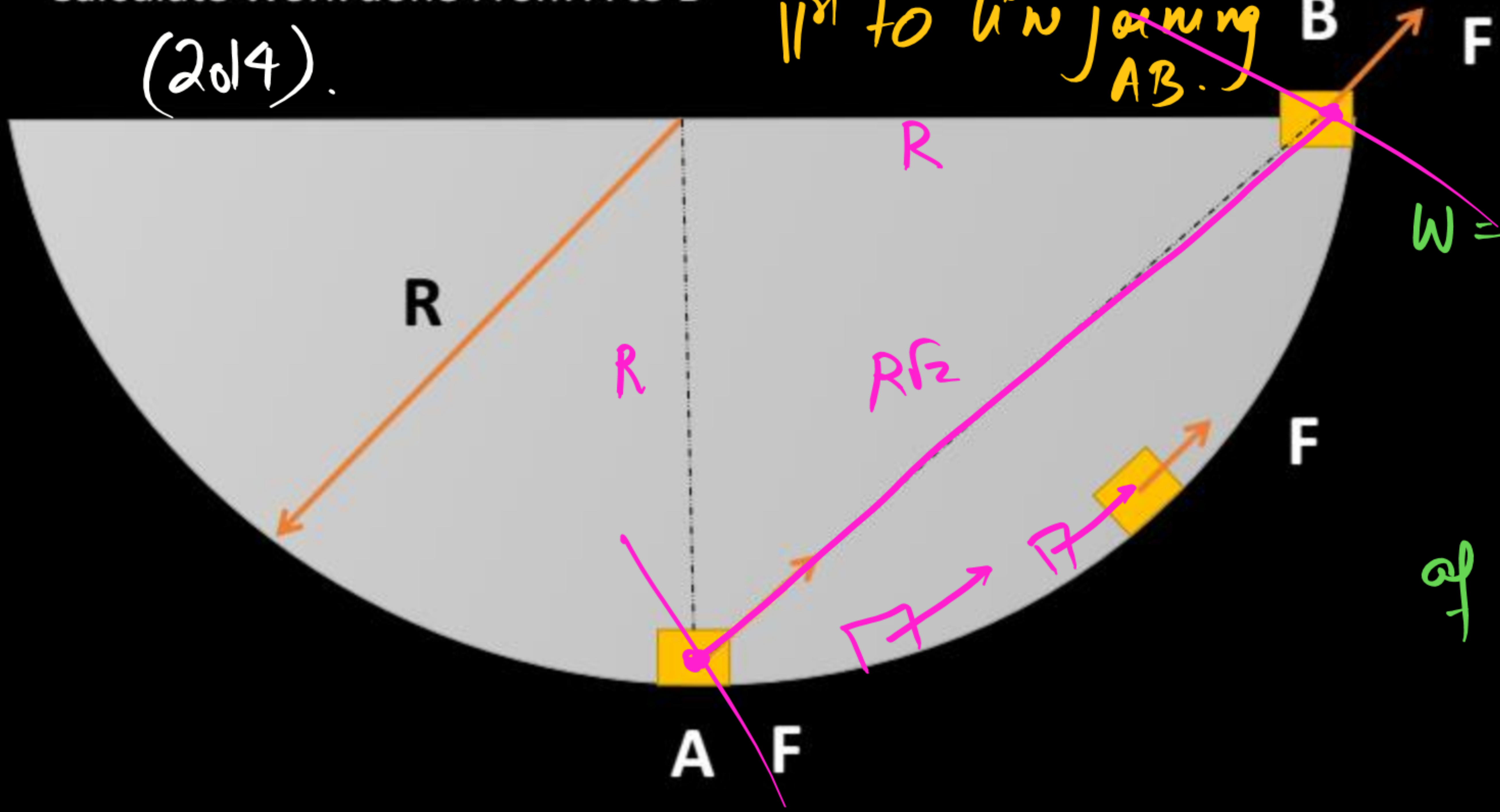
Ans

$W = F \cdot \frac{\pi R}{2}$.

Calculate Work done From A to B

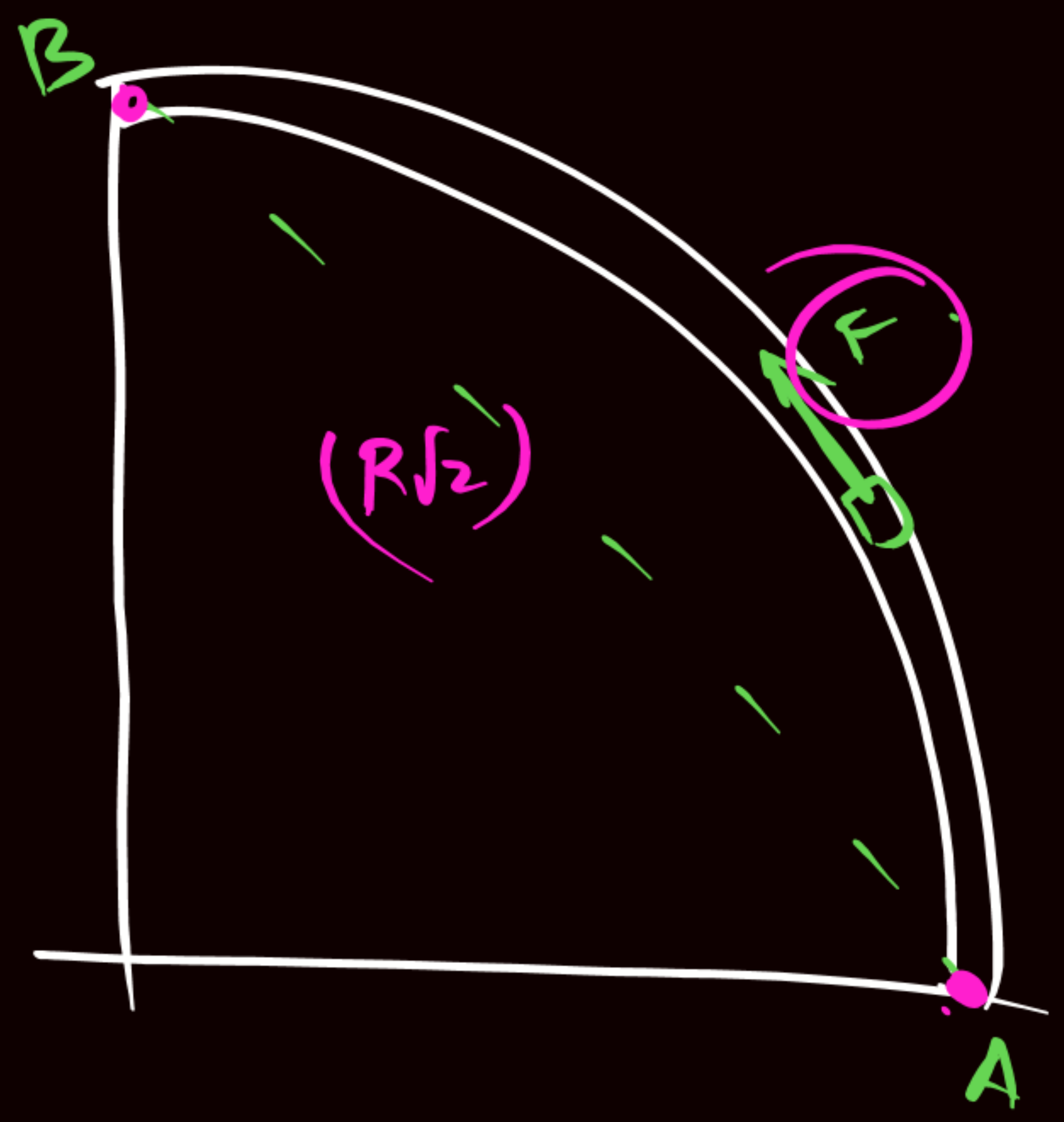
(2014).

force applied on block is always $\parallel^{el}$ to line joining AB.



$W = F \cdot S = FR\sqrt{2}$.

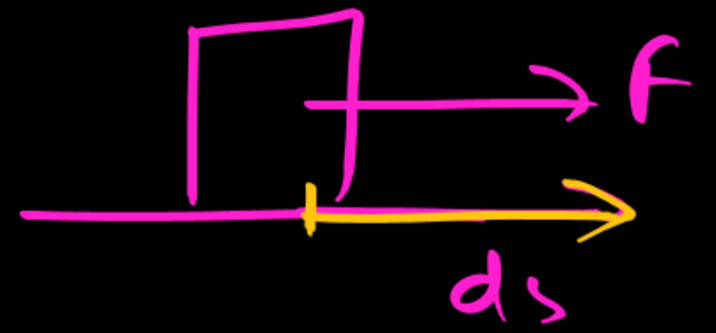
↳ of point of application of force in dir of for


B
A
F
(R√2)

Work can Be Positive, Negative and Zero

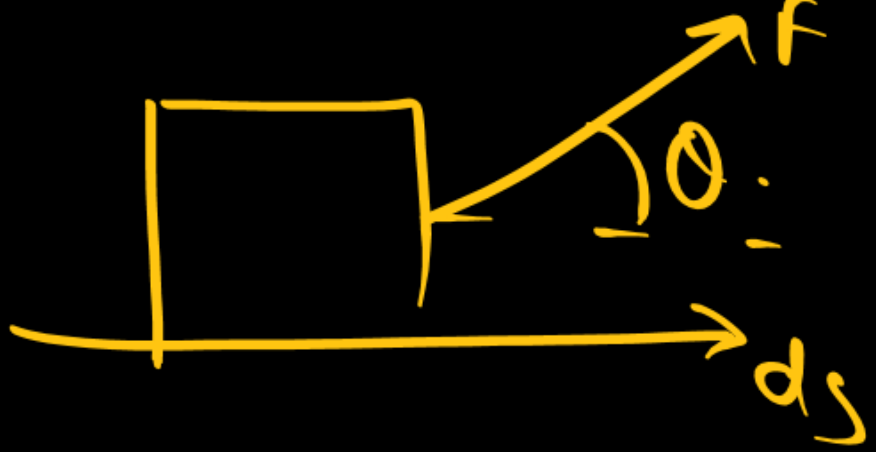
$\cos\theta$ $-ve$ | $\cos\theta$ $+ve$

$$W = \vec{F}\cdot\vec{S} = FS\cos\theta$$



"Positive Work"

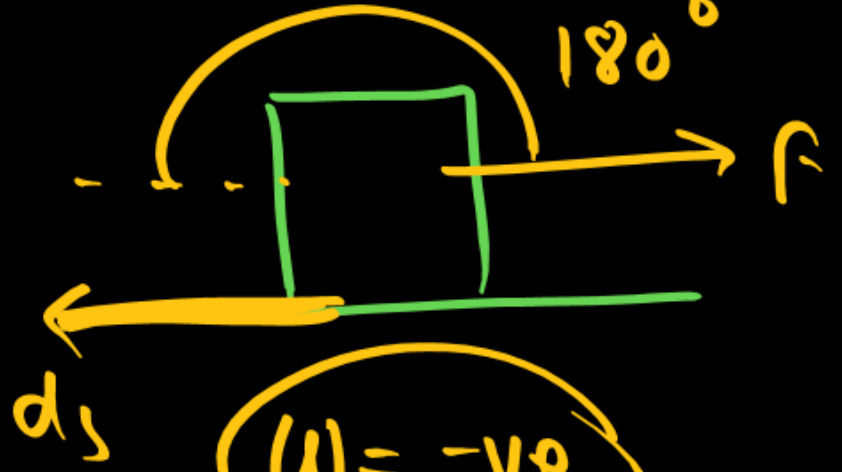
$$\left\{\begin{aligned}&\cos\theta = +ve\\&\theta = \text{Acute}\end{aligned}\right\}$$



Negative Work

$\cos\theta = -ve$

"θ = obtuse"



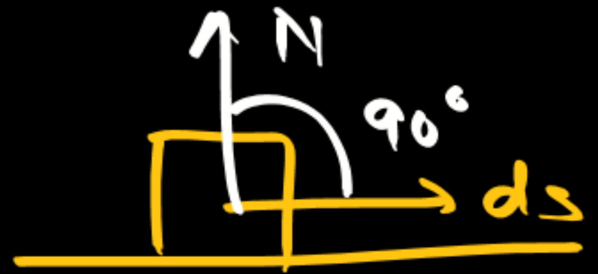
$W = -ve$

Zero Work

when $ds = 0$

$W = 0$

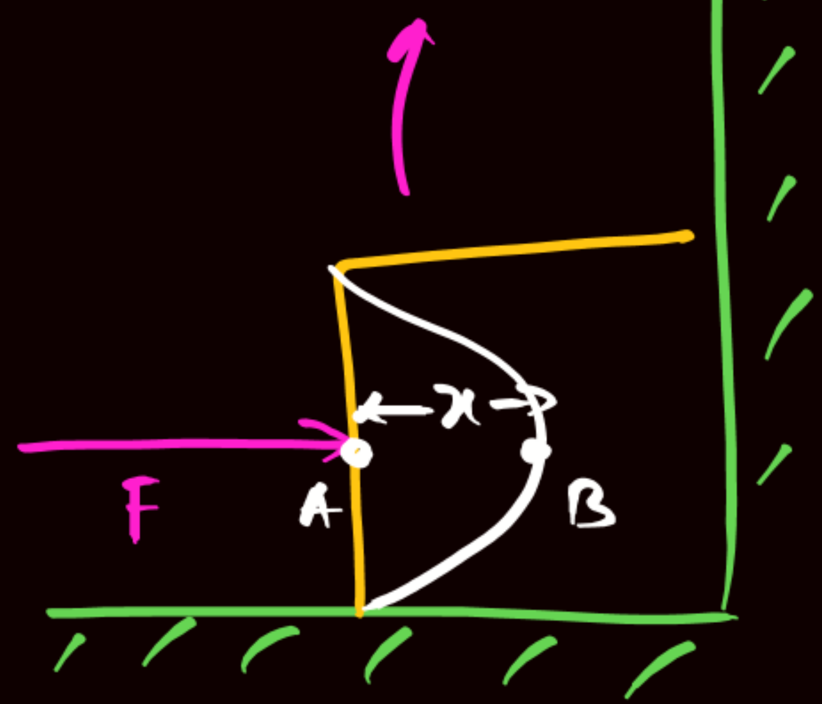
$\theta = 90^\circ$

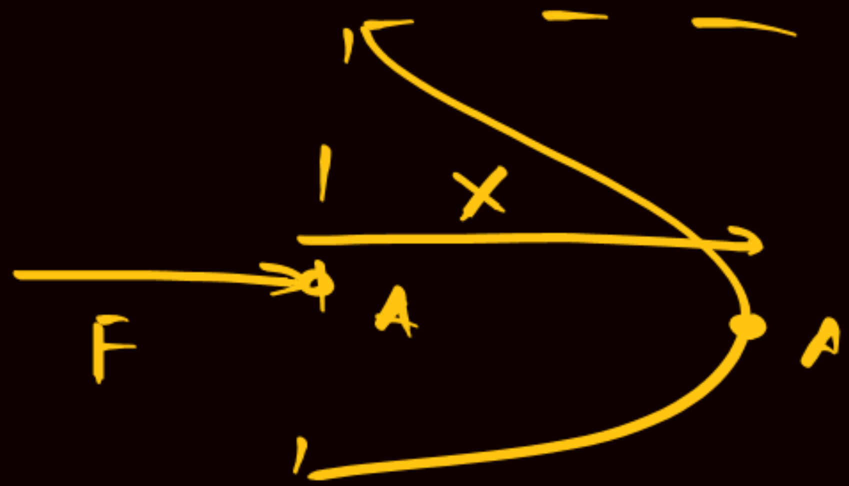


$\boxed{W_N = 0}$

This is not a rigid body.

"Sponge"





$$W = +F x$$

CONCEPTS ON CALCULATION OF WORK

Work Done By constant Force

Concept

Whenever force is Constant

$$W = \vec{F} \cdot \vec{S}$$

$$= FS\cos\theta$$

$$= F_x x + F_y y + F_z z.$$

$$F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{ds} = x\hat{i} + y\hat{j} + z\hat{k}.$$

A particle is moving along a straight line from point A to point B with position vectors $2\hat{i} + 7\hat{j} - 3\hat{k}$ and $5\hat{i} - 3\hat{j} - 6\hat{k}$ respectively. One of the force acting on the particle is $\vec{F} = 20\hat{i} - 30\hat{j} + 15\hat{k}$. Find the work done by this force.

(a) 300 units
(b) 315 units Ans
(c) 415 Units
(d) 200 units

$\vec{r_A} = 2\hat{i} + 7\hat{j} - 3\hat{k}$

$\vec{r_B} = 5\hat{i} - 3\hat{j} - 6\hat{k}$.

$\boxed{\vec{F} = 20\hat{i} - 30\hat{j} + 15\hat{k}}$.

$\vec{S} = (5-2)\hat{i} + (-3-7)\hat{j} + (-6+3)\hat{k}$

$= 3\hat{i} - 10\hat{j} - 3\hat{k}$.

$W = \vec{F} \cdot \vec{dS} = 60 + 300 - 45$

$= 360 - 45$

$= 315$ //

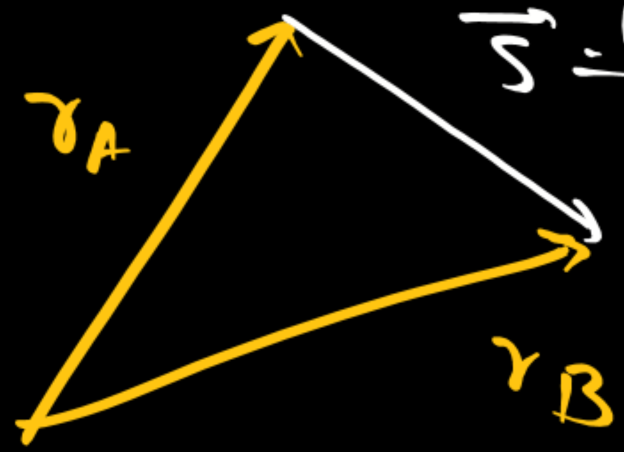
Work Done By Multiple constant Force

Concept

$\vec{F_1} =$

$\vec{F_2} =$

$\vec{F_3} =$



$\vec{S} = (\vec{r}_B - \vec{r}_A)$

W_{Total}

$W_1 = \vec{F_1} \cdot \vec{S}$

$W_2 = \vec{F_2} \cdot \vec{S}$

$W_3 = \vec{F_3} \cdot \vec{S}$

$\underline{W_T = W_1 + W_2 + W_3}$

find $\vec{F}_{net}$

$\vec{F}_T = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$\boxed{W = \vec{F}_T \cdot d\vec{S}}$

imp.

Work Done by "Variable" force

↳ which change.

"We have to Integrate".

Concept

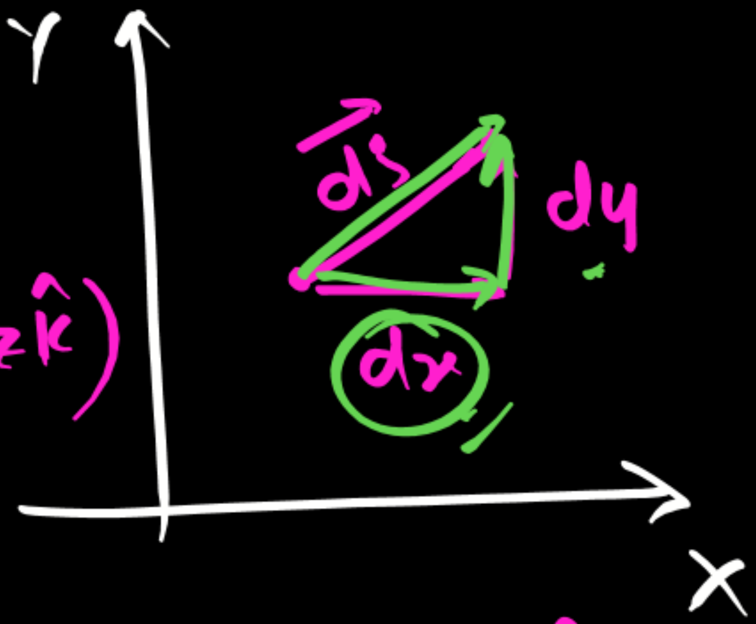
F = function of Position.

Small work done

$$dW = \vec{F} \cdot \vec{ds}$$

$$= \vec{F} \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$\boxed{W_T = \int dW}$$



$$\vec{ds} = dx\,\hat{i} + dy\,\hat{j}.$$

1. A force $F = (10 + 0.5\,x)$ N acts on a particle in x-direction, where x is in metre. Find the work done by this force during a displacement from x = 0 to x = 2. (AIEEE 2009)

(a) 21
(b) 20
(c) 35
(d) 21 Ans

$$f = (10 + 0.5x)\hat{i} \quad \begin{array}{l} \rightarrow x = 0 \\ \rightarrow x = 2 \end{array}$$

Small work done $dw = \vec{f}\,\vec{ds}$

$$= (10 + 0.5x)\hat{i}\,(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$dw = 10\,dx + 0.5\,dx$$

Variable = x

$$W_T = \int dW = \int_0^2 10\,dx + 0.5x\,dx = 10[x]_0^2 + 0.5\left[\frac{x^2}{2}\right]_0^2$$

$$= 20 + \frac{0.5}{2}4 = 21\,J.$$

$(0,0)$ $(1,1)$

2. A body is displaced from (0, 0) to (1m, 1m) along the path $x = y$ by a force $F = (x^2\hat{j} + y\hat{i})$ N. The work done by this force will be (2007)

(a) $\frac{4}{3}$ J

(b) $\frac{5}{6}$ J Ans

(c) $\frac{3}{2}$ J

(d) $\frac{7}{5}$ J

$f = x^2\hat{j} + y\hat{i}$

This is a Variable force.

$\rightarrow f = y\hat{i} + x^2\hat{j}$

$dW = \vec{F}\cdot(dx\hat{i} + dy\hat{j})$

$dW = y\,dx + x^2\,dy$

Since $x = y$

$dW = x\,dx + y^2\,dy$

$W_T = \int dW$

$= \int_0^1 x\,dx + \int_0^1 y^2\,dy$

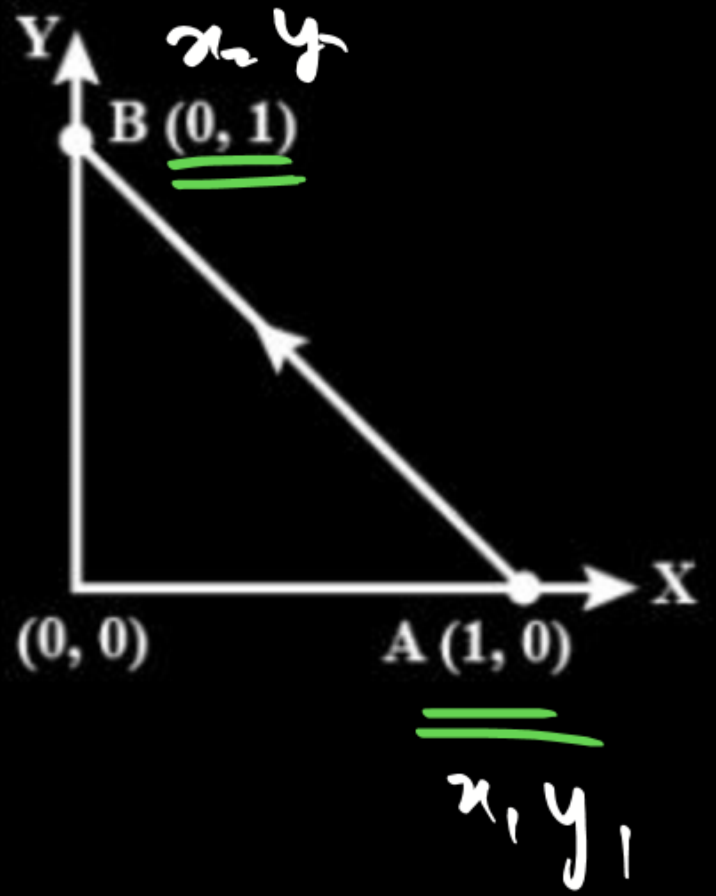
$= \left[\frac{x^2}{2}\right]_0^1 + \left[\frac{y^3}{3}\right]_0^1$

$= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

3. Consider a force $\vec{F} = -x\vec{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B (0, 1) along the line segment is : [Jee Main-2020 (January)]
(All quantities are in SI units)

(a) 1/2 (b) 3/2

(c) 2 (d) 1 Ans



$F = -x\hat{i} + y\hat{j}$

$dW = \vec{F} \cdot \vec{ds}$

$= (-x\hat{i} + y\hat{j})(dx\hat{i} + dy\hat{j})$

$= -x\,dx + y\,dy$

$W_T = \int dW = \int_1^0 -x\,dx + \int_0^1 y\,dy$

$= -\left[\frac{x^2}{2}\right]_1^0 + \left[\frac{y^2}{2}\right]_0^1$

$= -\left[0 - \frac{1}{2}\right] + \left[\frac{1}{2} - 0\right]$

$= \frac{1}{2} + \frac{1}{2} = 1.$

4. The relation between displacement x and time t for a body of mass 2Kg moving under the action of a force is given by $x = t^3/3$, where x is in meter and t in second, calculate the work done by the body in first 2 seconds. (NCERT)

(a) 16 Ans (b) 4

(c) –8 (d) 24

"time Variable"

$m = 2\,\text{Kg}$

$x = \frac{t^3}{3}$

Particle is moving along X.

$W = F\,dx$

$= m\,a\,dx$

$= m\left(v\frac{dv}{\cancel{dx}}\right)(\cancel{dx})$

$= m v\,dv$

$dW = m t^2\, 2t\,dt = 2m t^3\,dt$

$a = \frac{dv}{dt}$

$= v\frac{dv}{dx}$

$x = \frac{t^3}{3}$

$\frac{dx}{dt} = v = t^2$

$a = \frac{dv}{dt} = 2t$

$dv = 2t\,dt$

$$W_T = \int dW$$

$$= \int_0^2 2mt^3\,dt$$

$$= 2m\left[\frac{t^4}{4}\right]_0^2$$

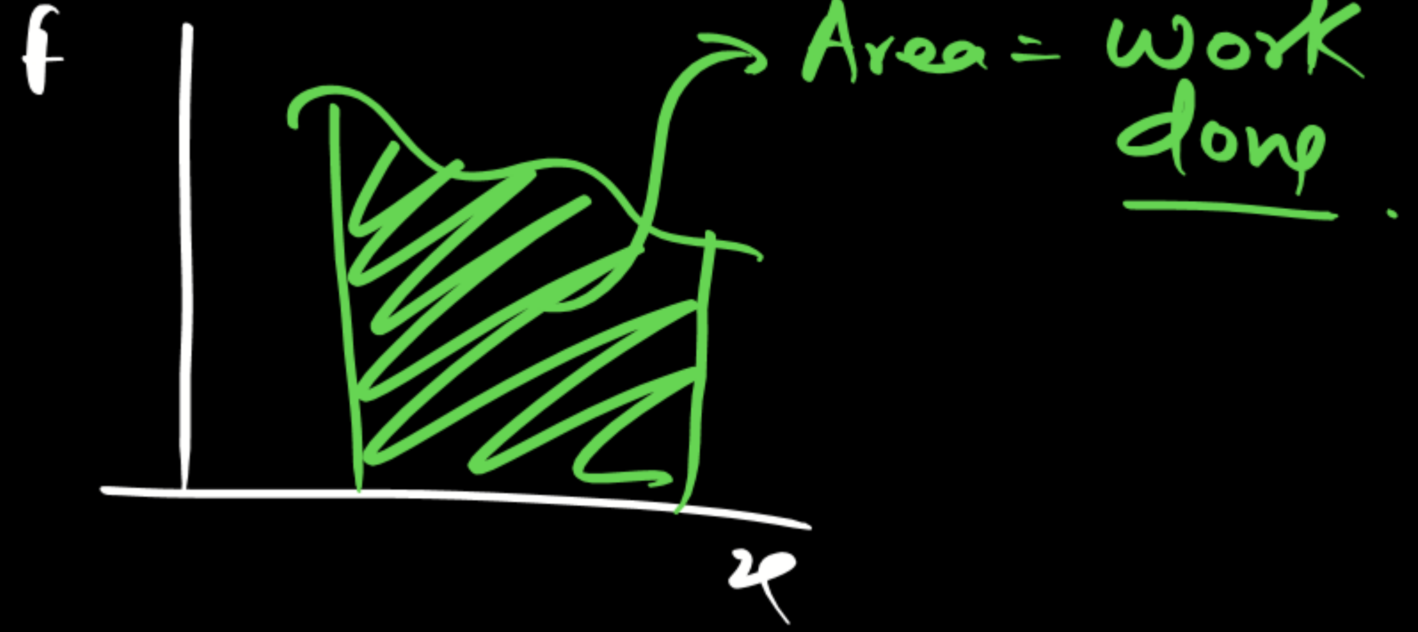
$$= \frac{m}{2}\left[2^4 - 0^4\right]$$

$$= 2^4 = 16\,J$$

Work Done from Graph

Concept : Whenever we have

f/x



Integeration → Graphically → Area under Curve.

$$W_T = \int f \cdot dx$$

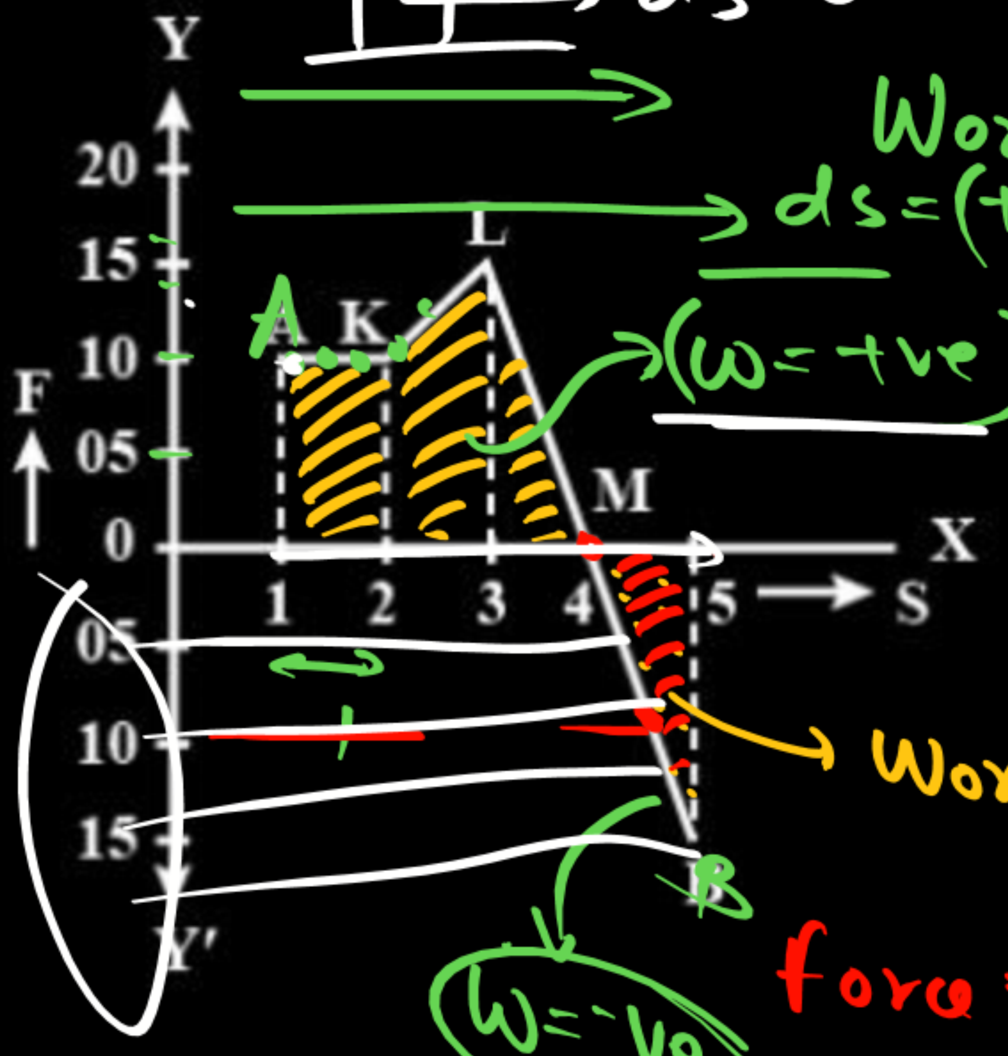
= Area under f/x graph.

5. A body moves from point A to B under the action of a force, varying in magnitude. What is the work done?

~~(a)~~ 22.5 J Ans (b) 24 J

(c) 20 J (d) 25 J

$\vec{F}$, $\vec{ds}$ $\theta = 0$ $W = +ve$



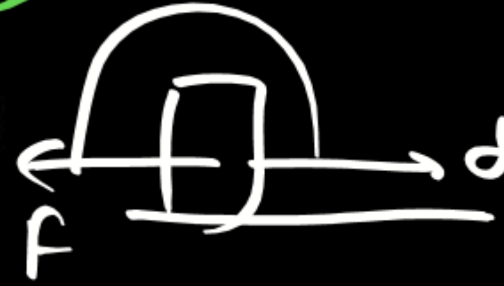
ds = (+ve)

(W = +ve)

Work done is negative.

W = -ve

force = -ve
ds = +ve $\left\{ \text{Work} = -ve \right\}$



F ds

Work = Area = $10\times1 + \frac{1}{2}(10+15)\times1 + \cancel{\frac{1}{2}\cdot1\times15}$

$\cancel{-\frac{1}{2}\cdot1\times15}$

$= 10 + \frac{25}{2}$

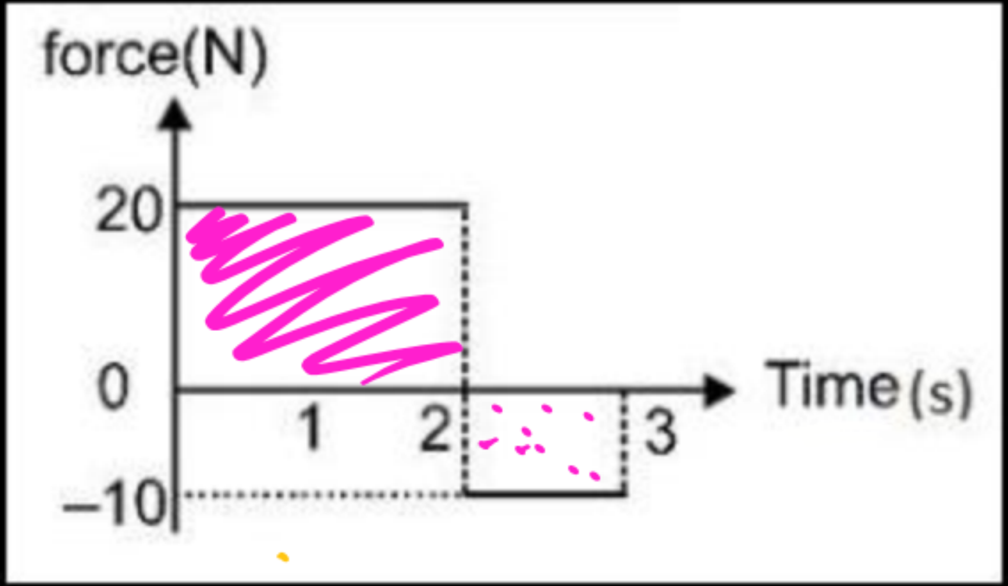
$= 10 + 12.5$

$= 22.5$ J.

6. Starting at rest, a 5 kg object is acted upon by only one force as indicated in figure. Then the total work done by the force is

(a) 90 J Ans (b) 245 J

(c) 125 J (d) 490 J



Area under F/t Graph :-

$F \times t$ = Impulse = $P_2 - P_1$

$u = 0$

$P_1 = 0$

Area under f/t = $P_2 - \cancel{P_1}$ → 0

$\boxed{P_f = 30 \text{ Kg m/s}}$

Area = $20 \times 2 - 10$

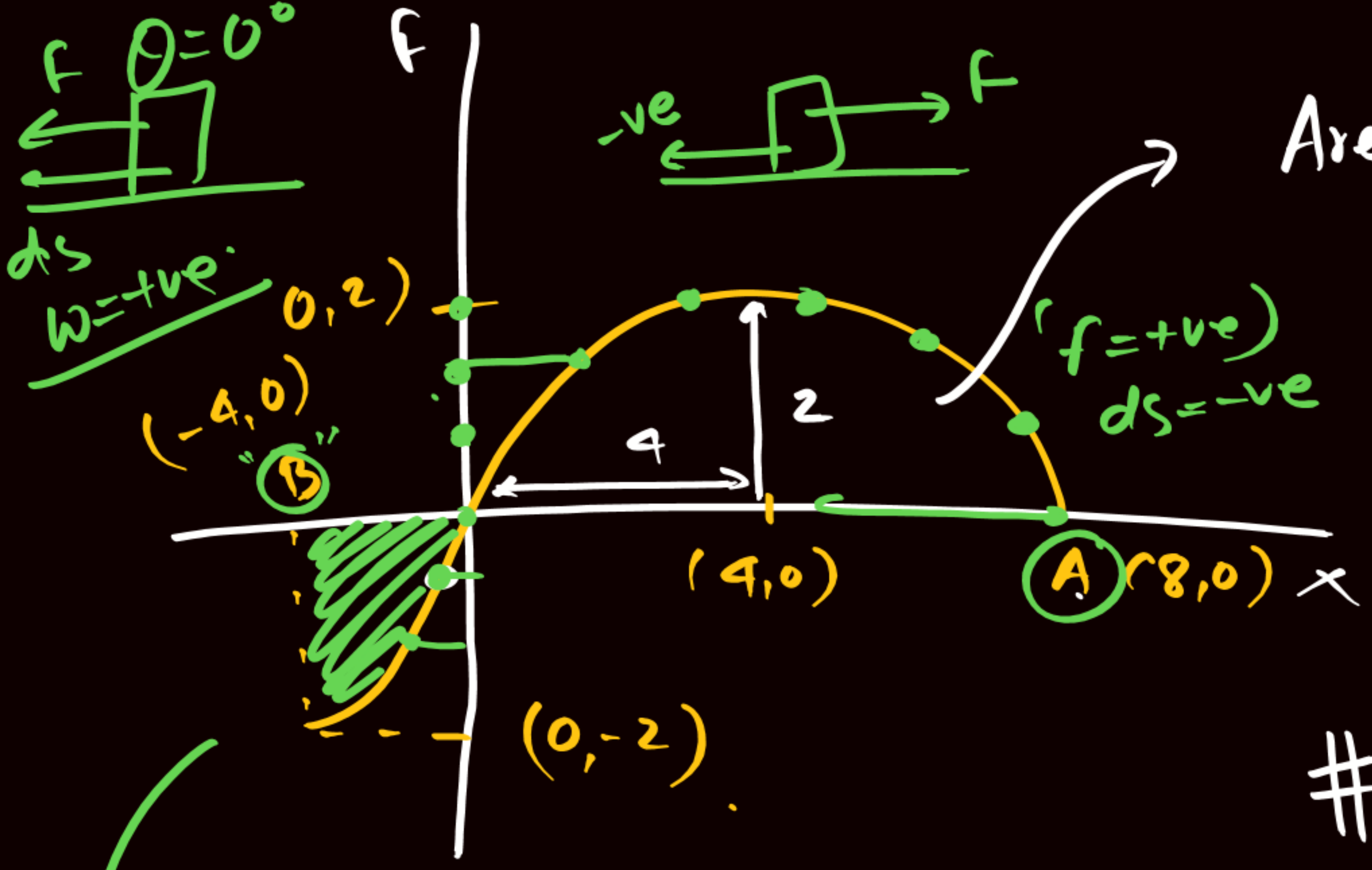
$= 40 - 10$

$= 30$.

$\boxed{\text{Work} = K_f - K_i}$

Initially Rest $K = 0$

$= \frac{P_f^2}{2m} = \frac{30 \times 30}{2 \times 5} = 90 \text{ J}$.



$$\text{Area} = \frac{\pi a b}{2} = \frac{\pi\, 4\times 2}{2} = 4\pi.$$

$$W_T = 4\pi + 2\pi = 6\pi$$
$$= 4\pi - 2\pi = 2\pi$$
$$= 2\pi - 4\pi = -2\pi$$

Work (+) Work (−)

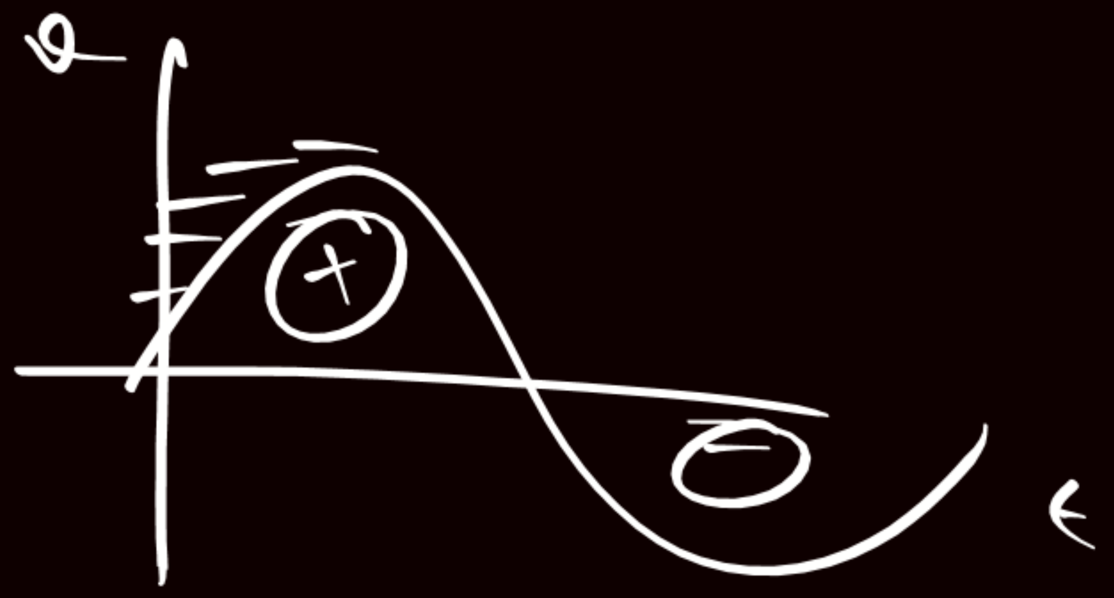
Work done from A → B.

"Quater of Ellipse" $\frac{\pi a b}{4} = \frac{\pi\, 4\times 2}{4} = 2\pi.$

$W_{upper} = -4\pi$

$W_{lower} = +2\pi$

-2π



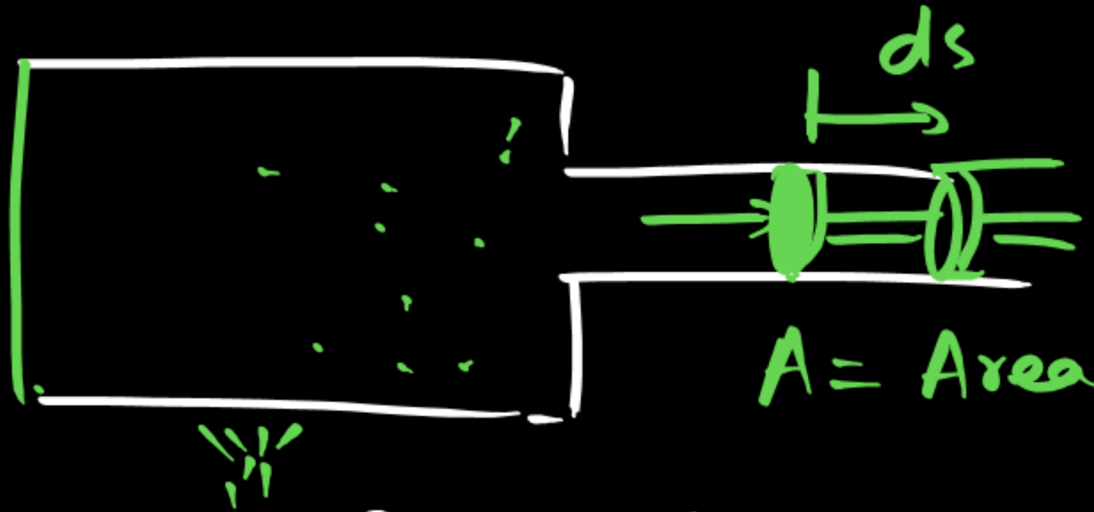
Area = disp = v × t

+ve   +ve

−ve   −ve

Work Done by Gas

Concept



$A \cdot ds = \text{Volume}$

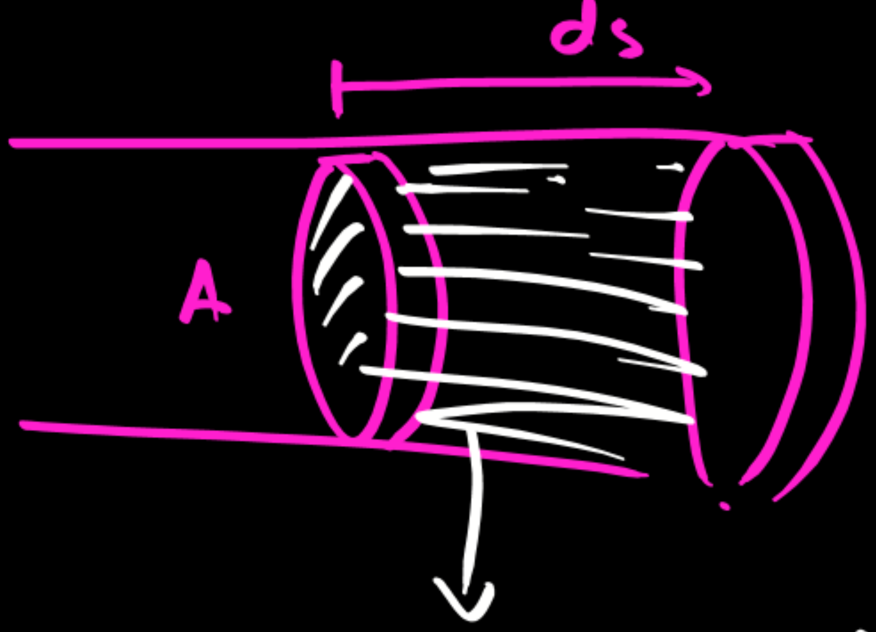
$= dV$

A = Area of Piston

$W = F \cdot dS \, \frac{A}{A}$

$= \frac{F}{A} \cdot A\,ds$

$$\boxed{W_{gas} = P_{gas} \cdot dV}$$



Volume $= A\,ds = dV$

7. Pressure and volume of a gas changes from (p_0 V_0) to ($p_0/4$, $2V_0$) in a process PV^2 = constant. Find work done by the gas in the given process. (2012)

(a) $\frac{(p_0V_0)}{2}$ Ans

(b) $\frac{(p_0V_0)}{3}$

(c) $\frac{(2p_0V_0)}{3d}$

(d) $\frac{(3p_0V_0)}{2}$

$W = f \cdot ds$

$\downarrow$ for gas

$\boxed{dW = P \cdot dV}$

$W_T = \int_{V_1}^{V_2} P\,dV$

$(P_0 V_0)$

$\downarrow$

$\left(\frac{P_0}{4}, 2V_0\right)$

$PV^2 = \text{Const.}$

$P = \frac{C}{V^2}$

$PV^2 = C$

at any point $P_0 V_0^2 = C$

$dW = \int P\,dV$

$= \int \frac{C}{V^2}\,dV$

$= C\left[\frac{-1}{V}\right]_{V_0}^{2V_0}$

$= C\left[\frac{1}{V_0} - \frac{1}{2V_0}\right]$

$= \frac{C}{2V_0} = \frac{P_0 V_0^2}{2V_0} = \frac{P_0 V_0}{2}$

BREAK

TILL

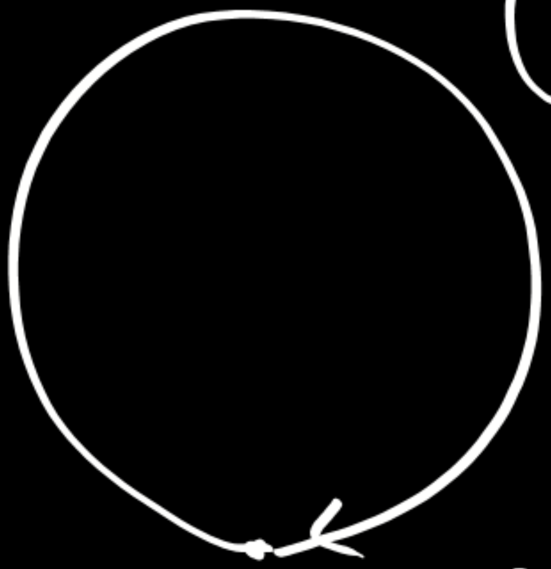
19:45 PM...

Conservative and Non Conservative Force

(magnetic)

There are two types of forces depending on work done.

(Electric force, gravity, spring)



Initial = final

Conservative

Work done independent of Path.

$$\boxed{W_{close\ Path} = 0}$$

Non-Conservative.

(friction)

Whose work done is Path dependent

$$\boxed{W_{closed\ path} \neq 0}$$

8. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3

[IIT-JEE (Screening) 2003]

(a) $W_1 > W_2 > W_3$

(b) $W_1 = W_2 = W_3$ Biceps

(c) $W_1 < W_2 < W_3$

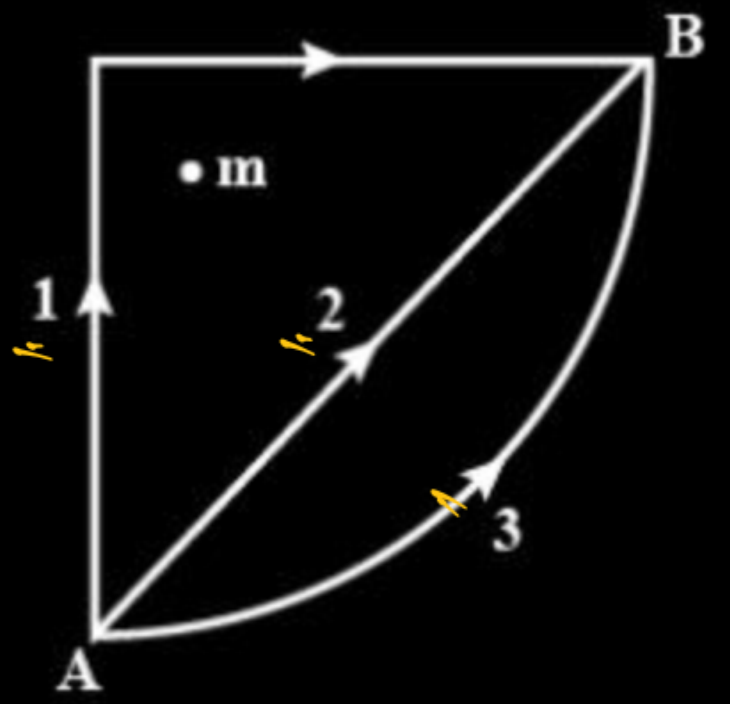
(d) $W_2 > W_1 > W_3$

Conservative

↳ Gravitational

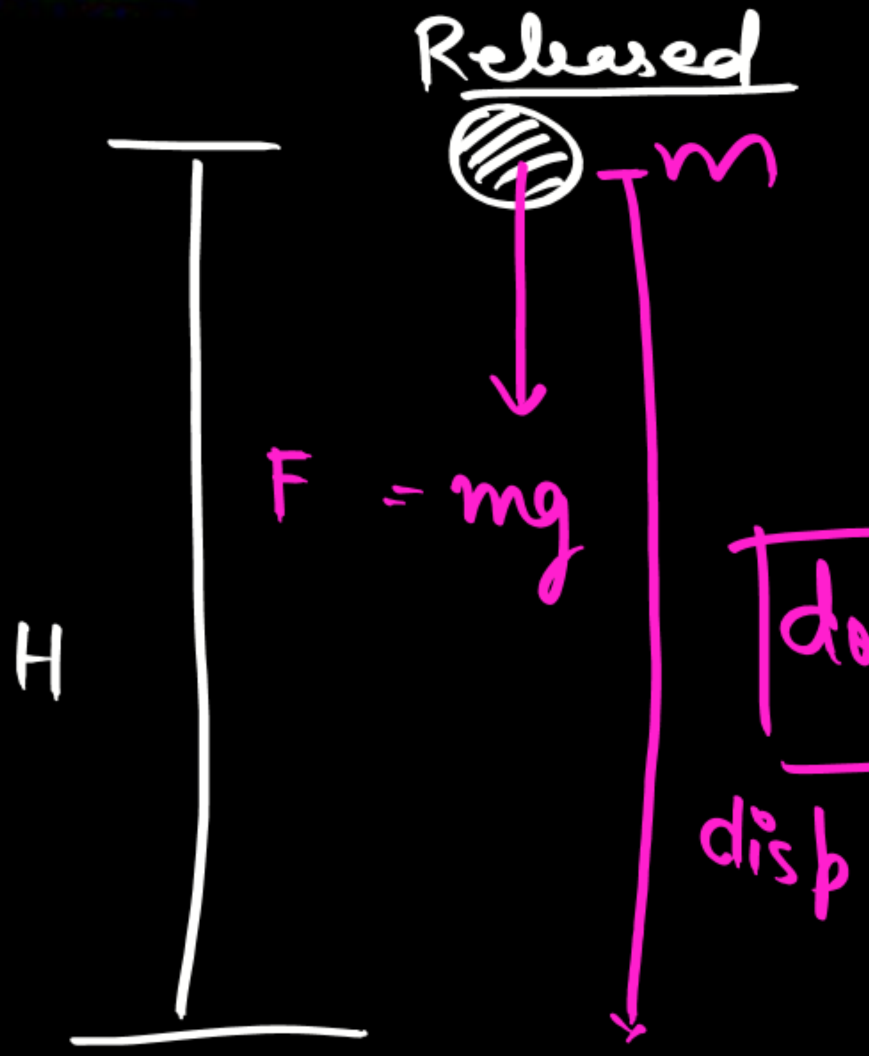
↳ Electrostatic

$$\boxed{W_1 = W_2 = W_3}$$



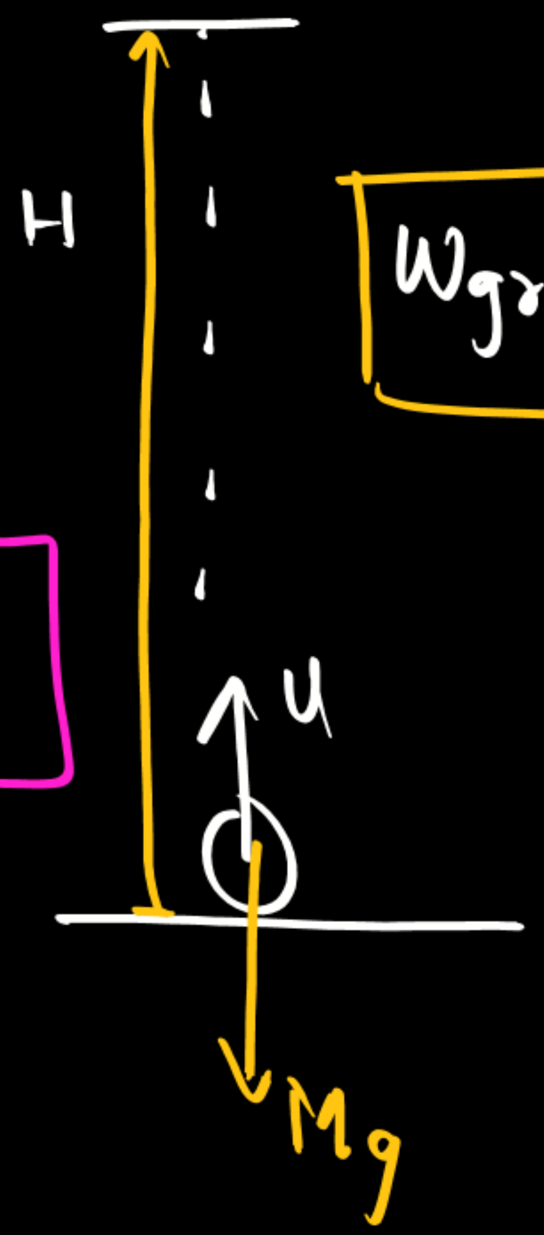
Work Done by Gravity

Concept $W = \vec{F} \cdot \vec{S}$

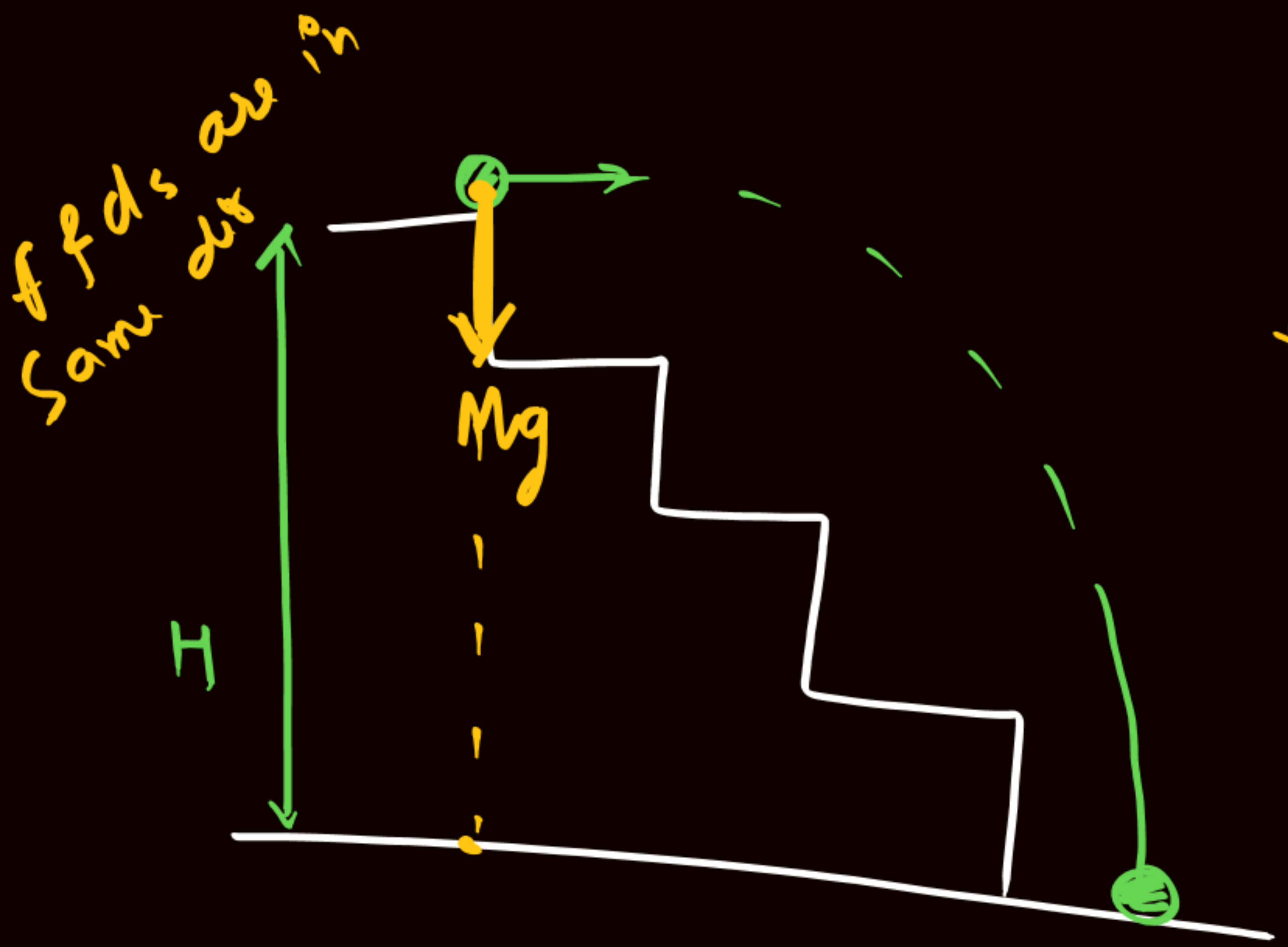


$\boxed{\text{downward} = +mgh}$

Case 2

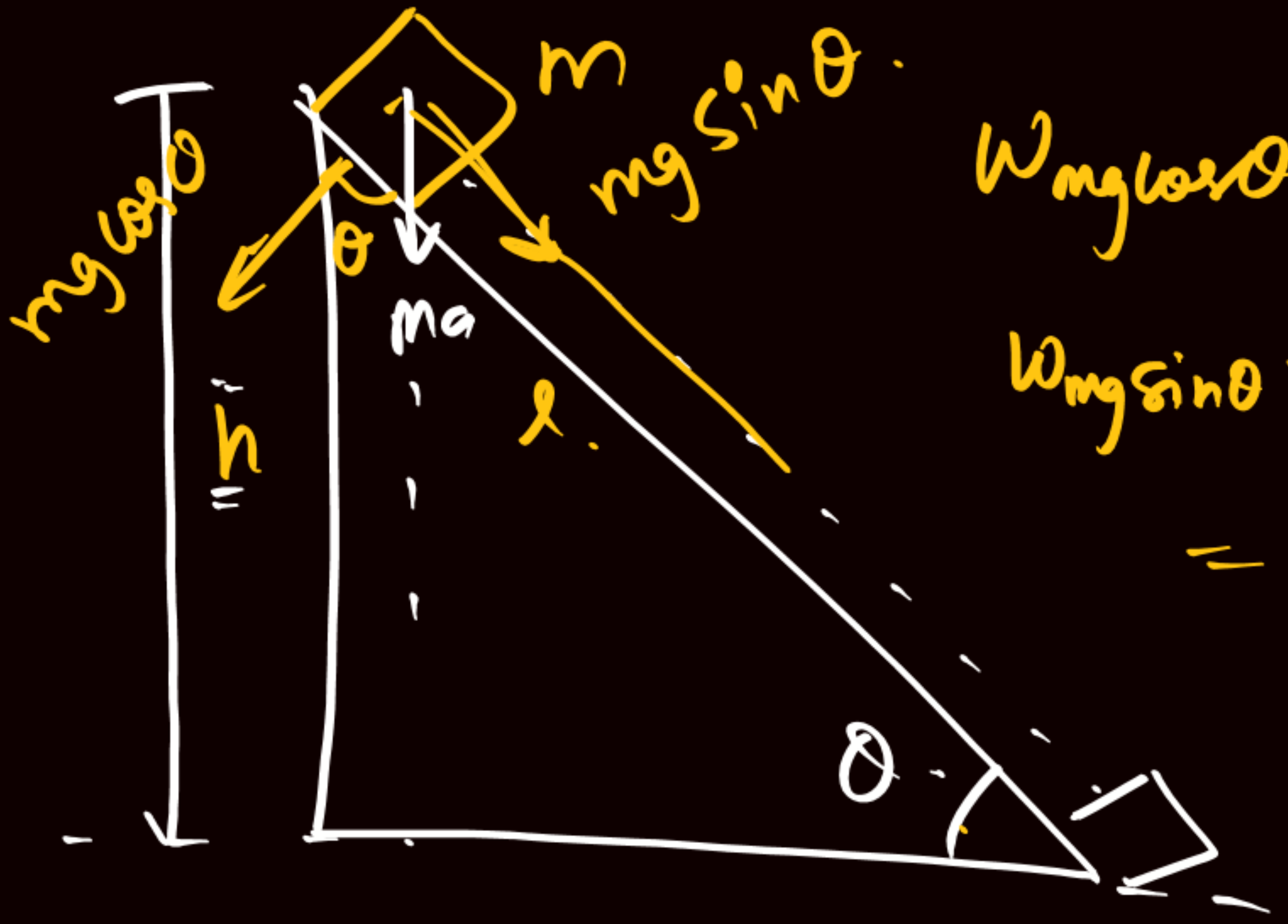


$\boxed{W_{gravity} = -mgh}$



In this Case

$$\boxed{W_{gravity} = +mgh}$$



$$W_{mg\cos\theta} = 0$$

$$W_{mg\sin\theta} = (mg\sin\theta)\,l$$

$$= mg\cancel{\sin\theta}\,\frac{h}{\cancel{\sin\theta}}$$

$$= mgh.$$

$$\boxed{W_{gravity} = +mgh}$$

$$\sin\theta = \frac{h}{l}$$

$$l = \frac{h}{\sin\theta}$$

9. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $(1/n)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be

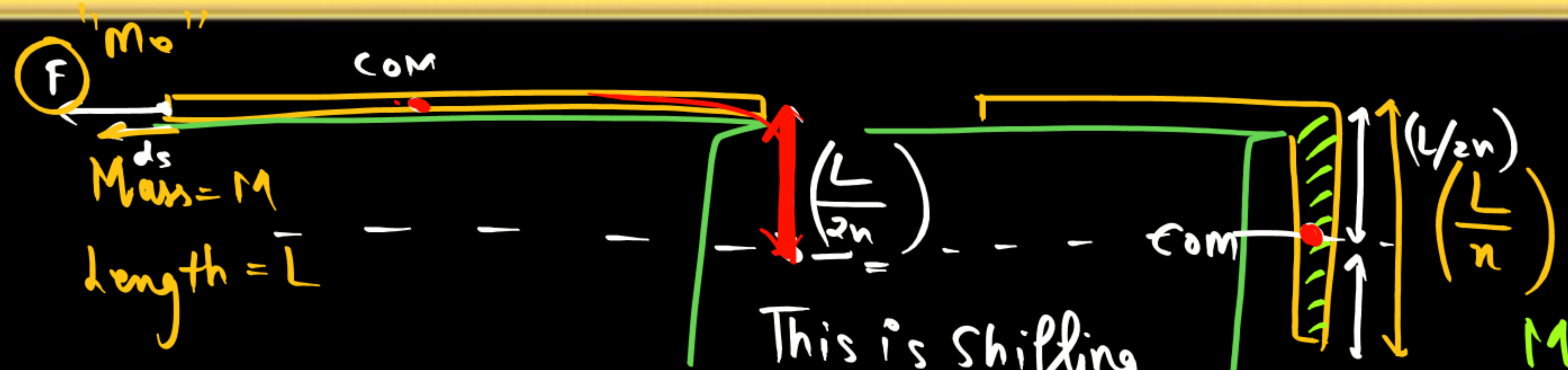
[Jee Main-2019 (April)]

(a) $\frac{MgL}{n^2}$

(b) $\frac{MgL}{2n^2}$ Ans

(c) $\frac{2MgL}{n^2}$

(d) nMgL



This is shifting of COM $L/2n$

$\frac{M}{n}$ = Mass of hanging Part.

Work done by gravity $= -\frac{M}{n} g \frac{L}{2n} = -\frac{MgL}{2n^2}$.

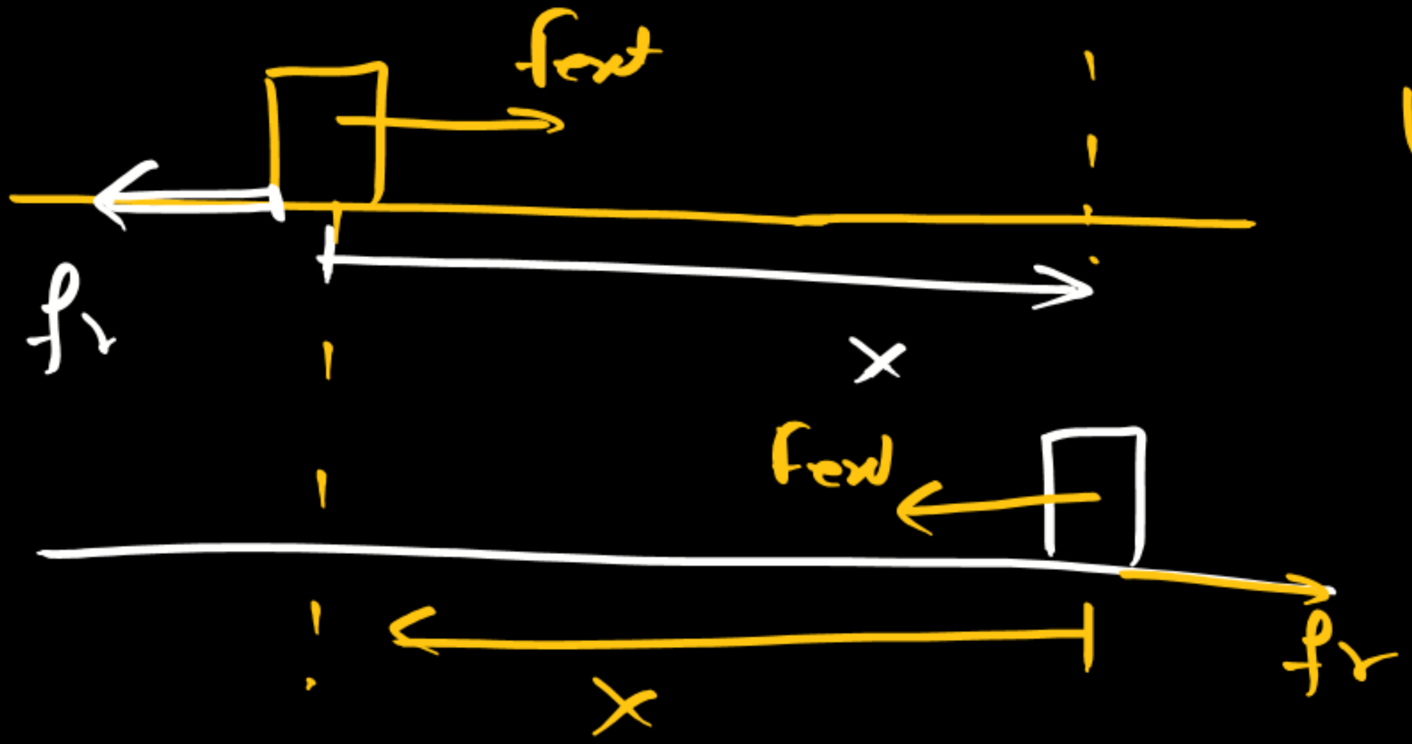
$\text{Work} = \frac{MgL}{2n^2}$

Work Done by Friction

Non-Conservative force.

Concept

#



$W_{friction} = -fx.$

$W_{friction} = -fx$

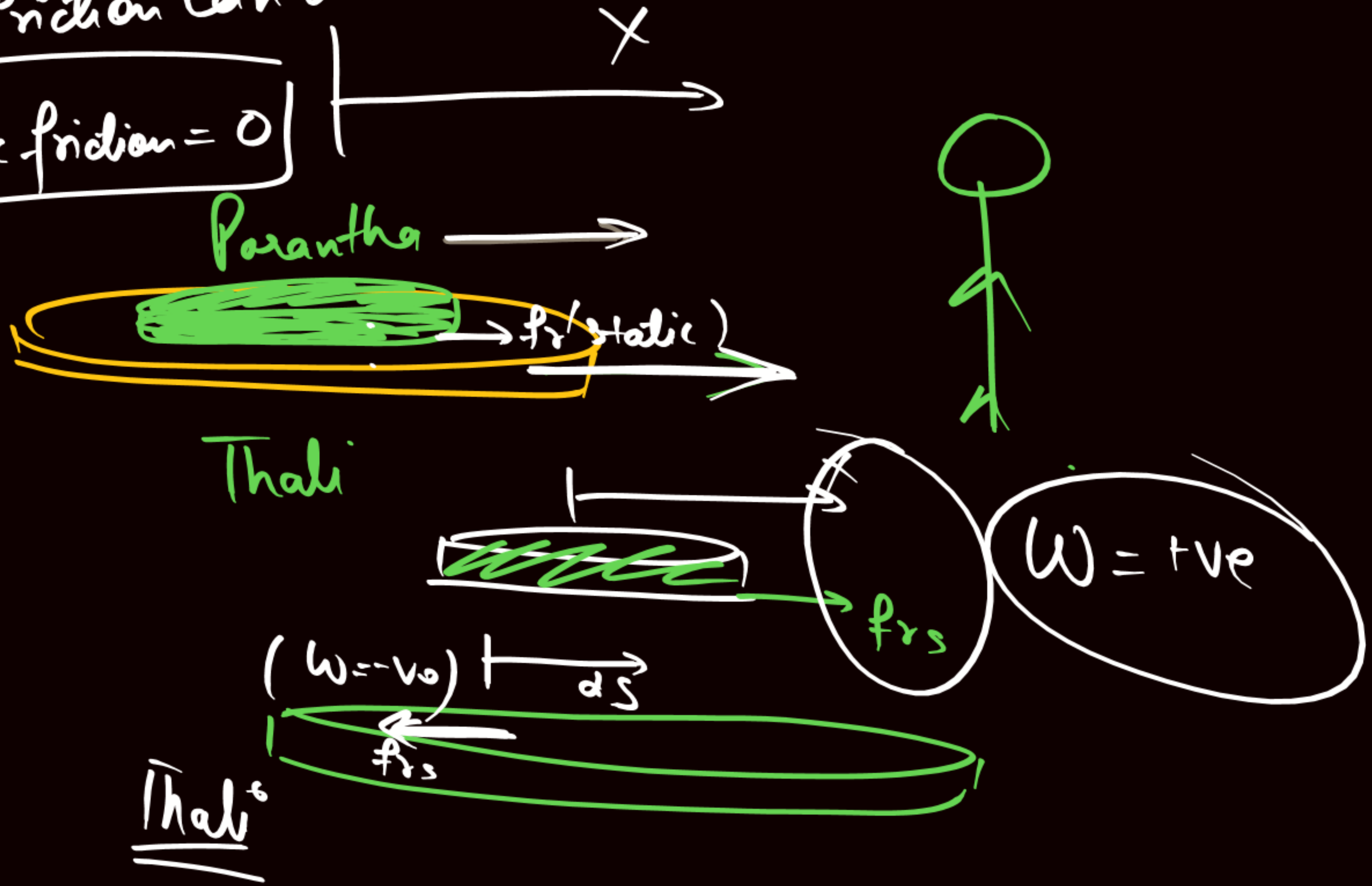
$W_{Total} = -2fx$

$\neq 0.$

Work done by friction can be +ve?

$W_{static\,friction}$ can do +ve work ✗

$\boxed{W_{Total\ static\ friction} = 0}$



Work Done by Spring

Concept of force due to spring:

$y = -mx$

$f_{spring} = -Kx$

K = Spring Constant (Stiffness Constant)

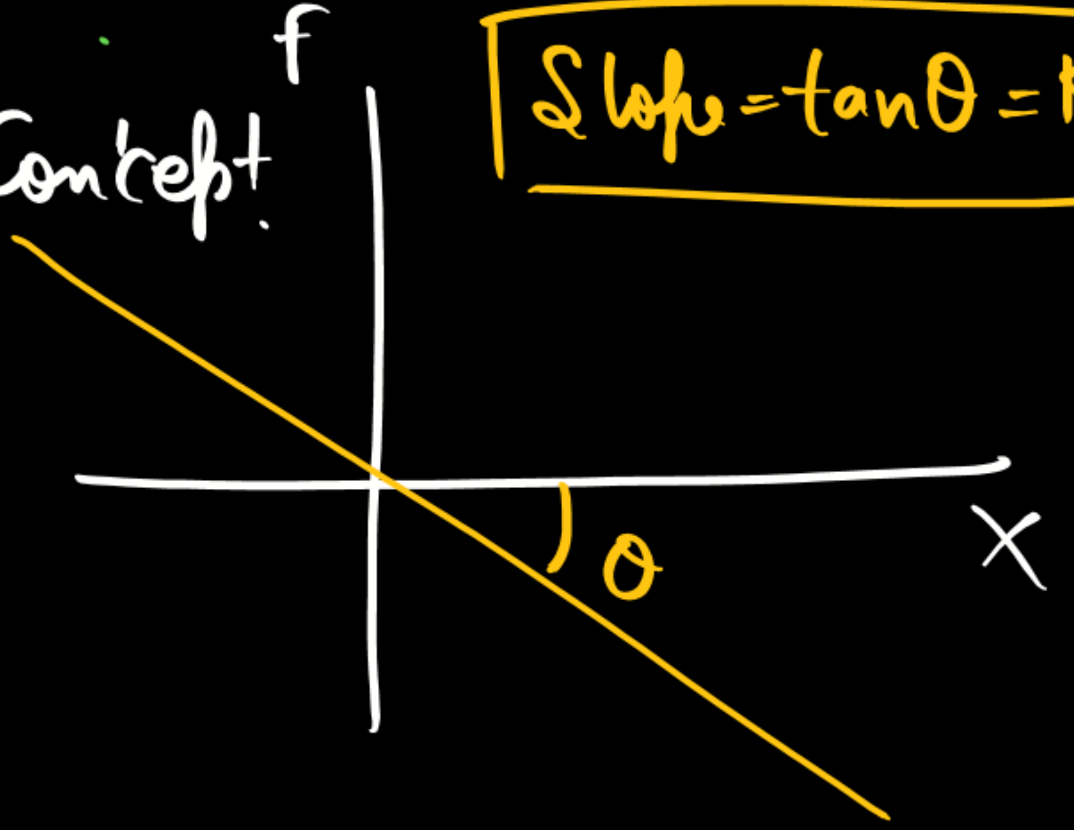
(−) → Restoring Nature

X → Change in length.

Elongation

Compression

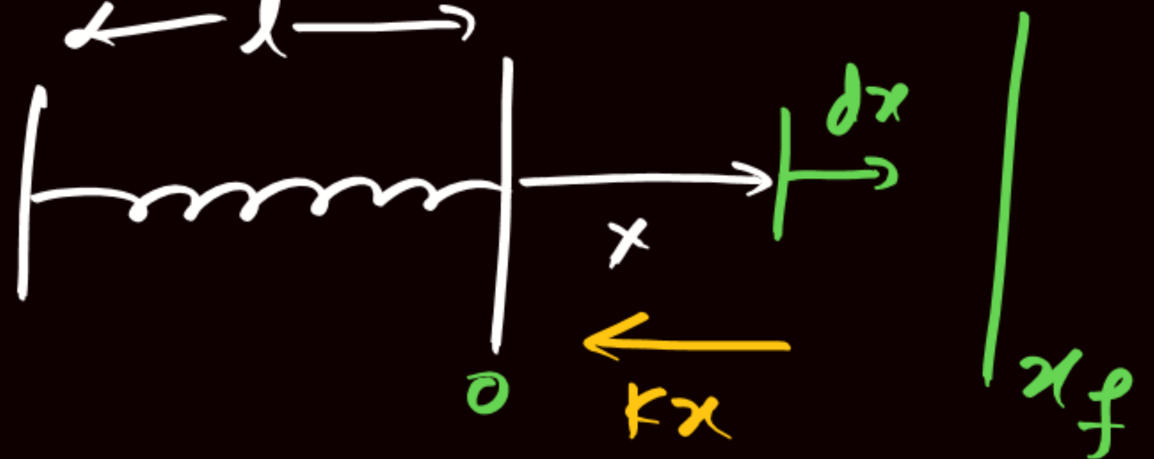
Concept



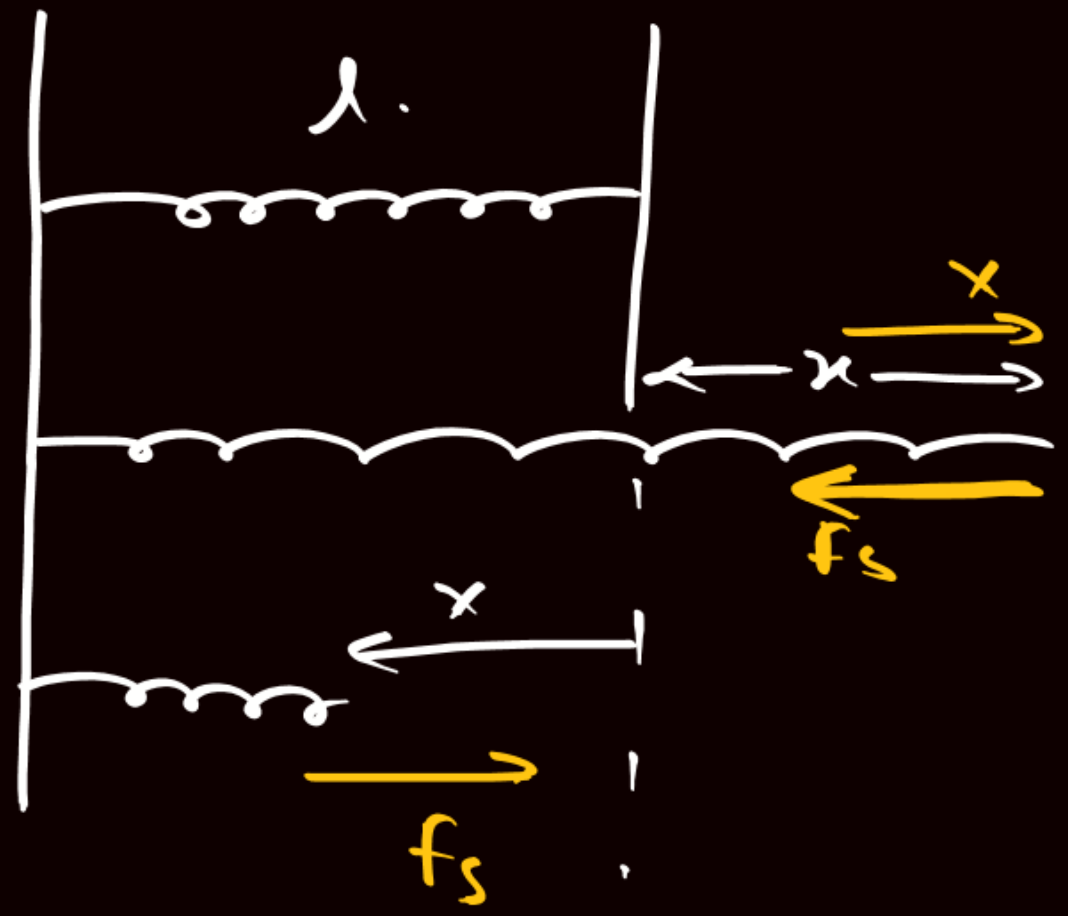
$\boxed{\text{Slope} = \tan\theta = K}$

Concept of work done by spring:

Concept of Work done by Spring



<u>Yaad Karna</u>

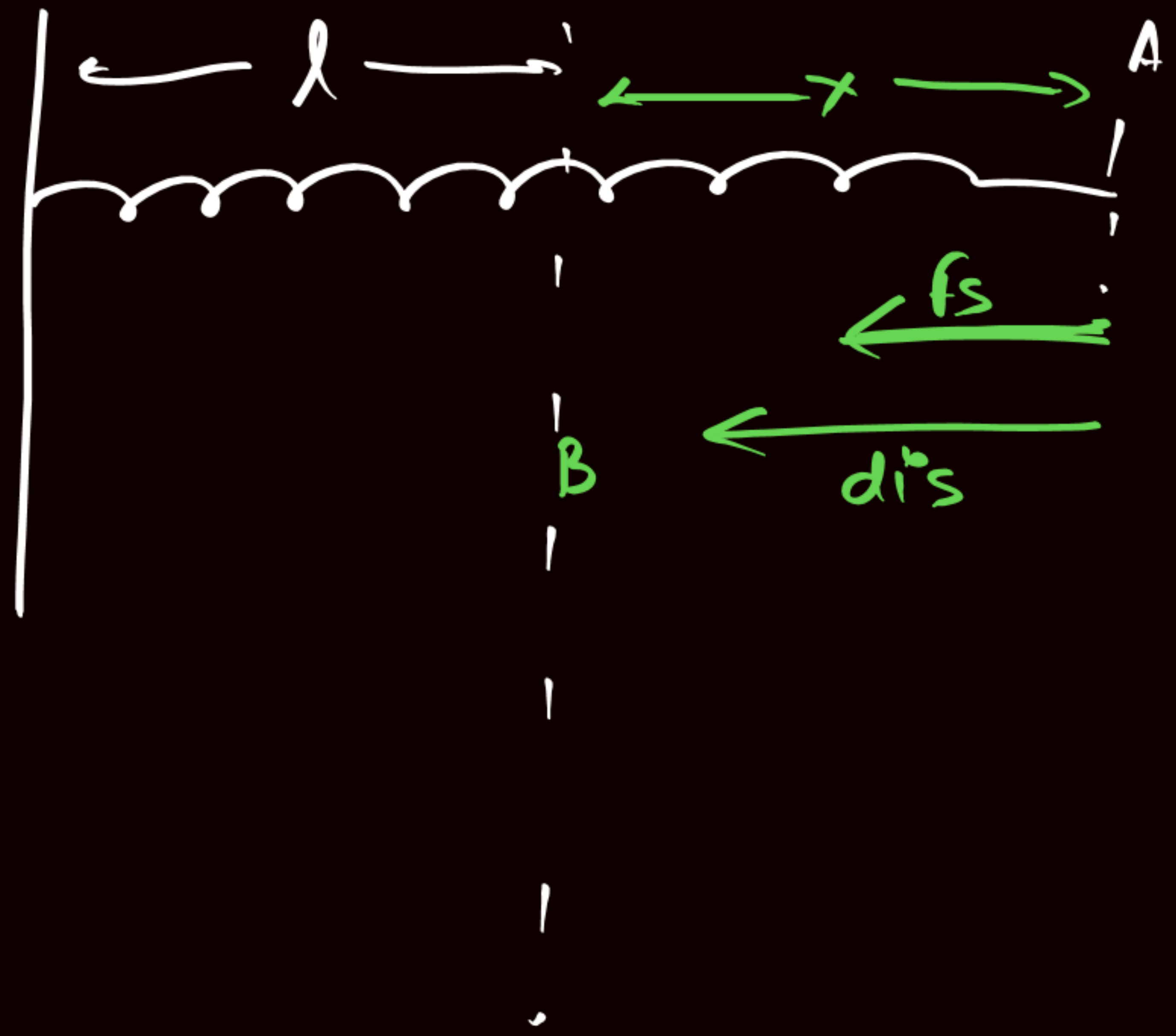


$$W_{spring} = -\frac{1}{2}kx^2$$

$$W_{spring} = -\frac{1}{2}kx^2$$

$$dW = \vec{f}\cdot\vec{dx}$$
$$= -kx\,dx$$

$$W_T = \int -kx\,dx$$
$$= -k\left(\frac{x^2}{2}\right)_0^{x_f}$$
$$= -\frac{1}{2}kx_f^2$$



Initially Elongated

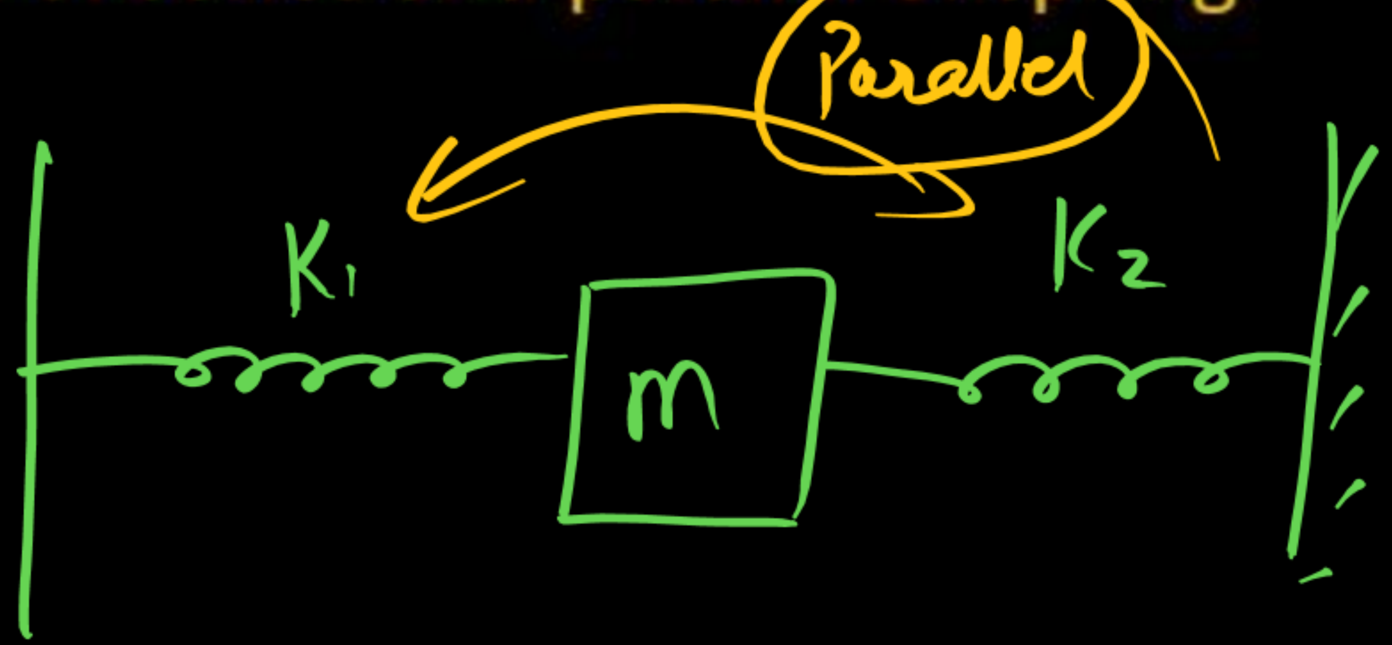
$$\boxed{W_{spring\,A\to B} = +\frac{1}{2}Kx^2}$$

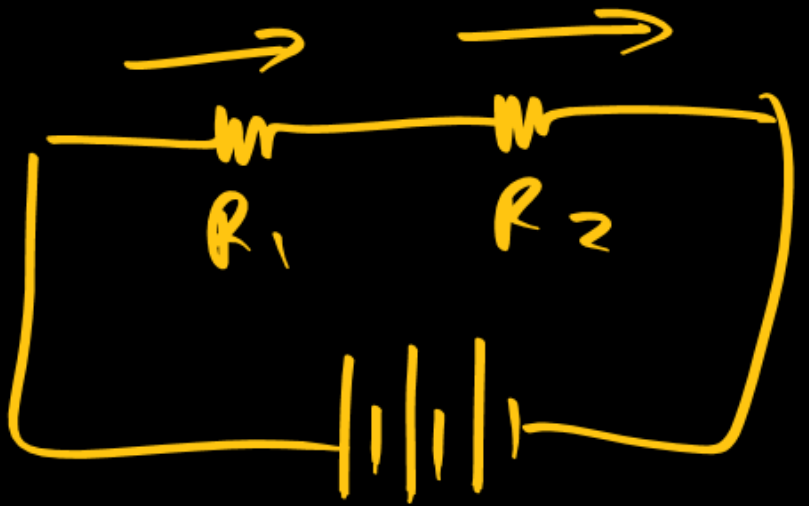
$$|W_{spring}| = \frac{1}{2}Kx^2$$

$\pm$

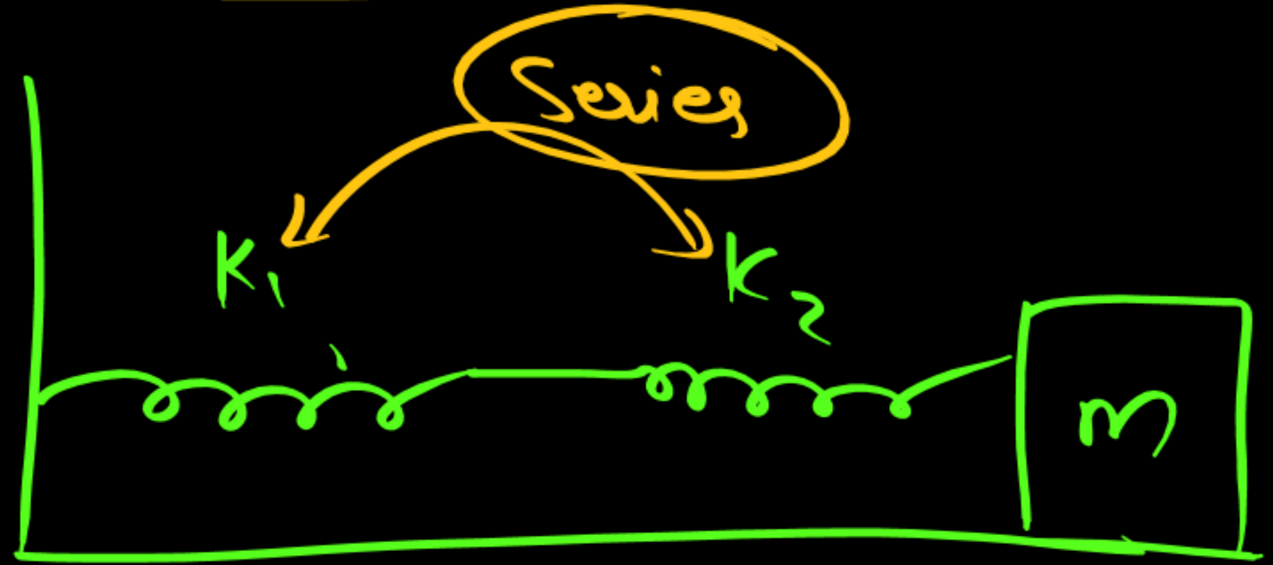
θ between f & x.

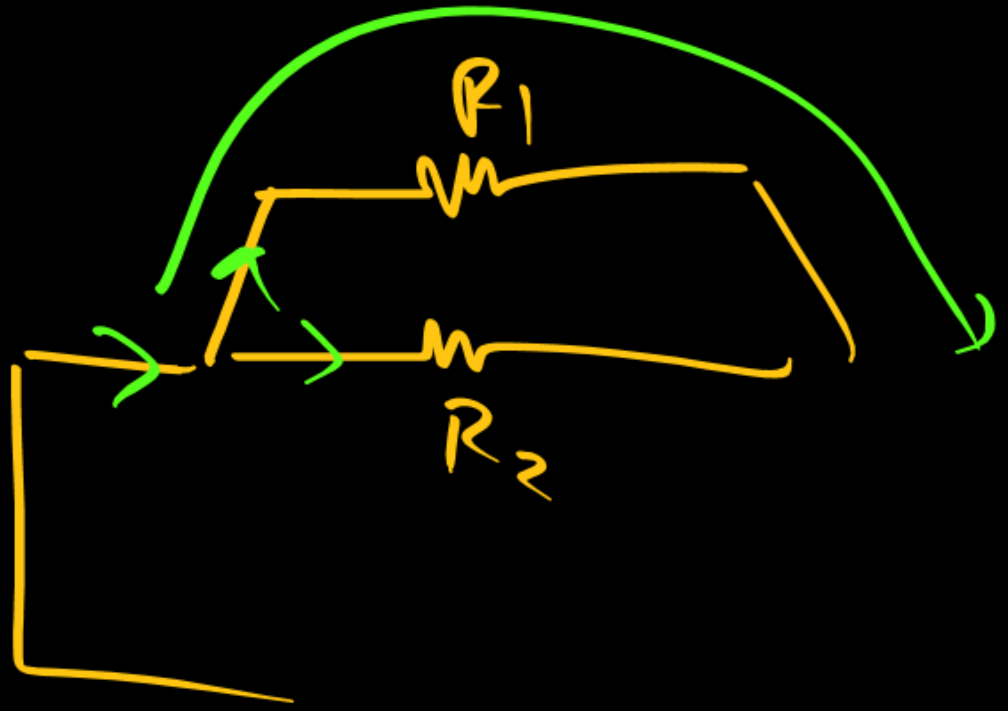
Concept of series and parallel of spring





Parallel #





Serie

When two springs are in $\parallel^{el}$ $K_{eq} = K_1 + K_2$

Series $K_{eq} = \frac{K_1 K_2}{K_1 + K_2}$

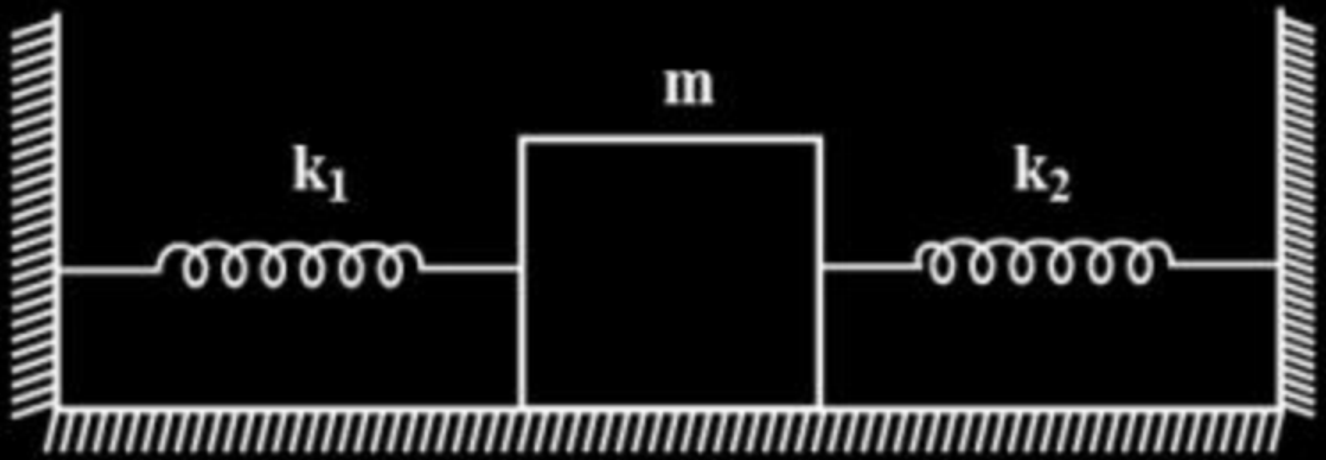
10. A block of mass m is attached to two un-stretched springs of spring constants k, and k, as shown in figure. The block is displaced towards right through a distance x and is released. Find the speed of the block as it passes through the mean position shown.

(a) $\sqrt{\frac{k_1+k_2}{m}}$

(b) $\sqrt{\frac{k_1 k_2}{m(k_1+k_2)}}$

(c) $\sqrt{\frac{k_1^2 k_2^2}{m(k_1^2+k_2^2)}}$

(d) $\sqrt{\frac{k_1^3 k_2^3}{m(k_1^3+k_2^3)}}$



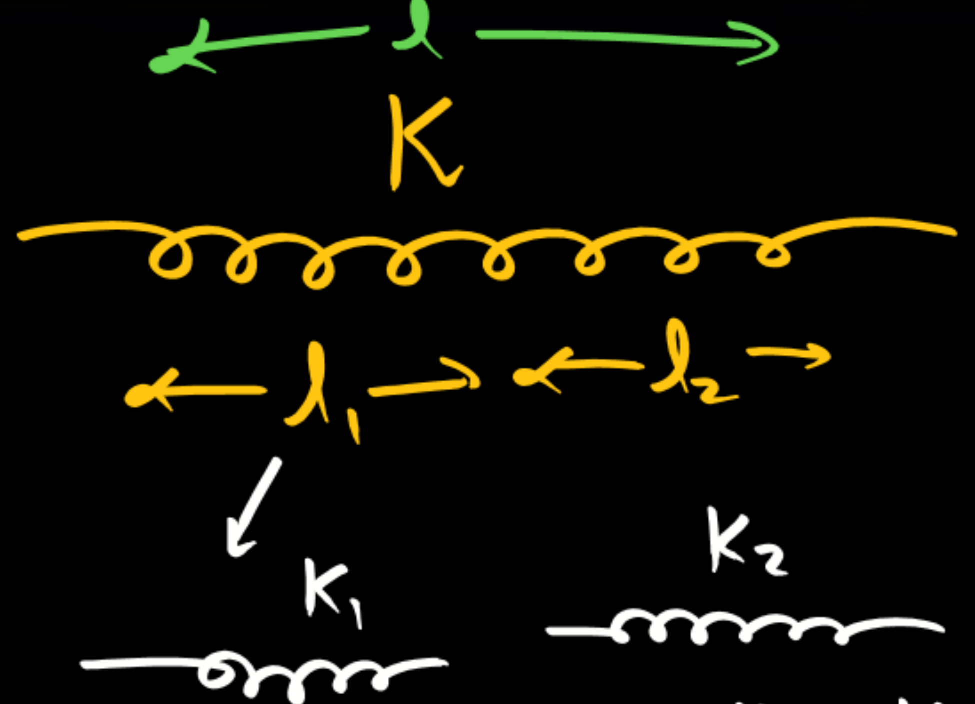
Concept of cutting of spring

11. A spring whose un-stretched length is l has a force constant k. The spring is cut into two pieces of un- stretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be:

[JEE Main 2019 (April)]

(a) $1/n^2$ (b) n^2

(c) $1/n$ Ans (d) n



$K_1 = K$

$K_1 = K\left(\frac{l}{l_1}\right)$ $K_2 = K\left(\frac{l}{l_2}\right)$

$l_1 = n l_2$

$\frac{l_2}{l_1} = \frac{1}{n}$

$\frac{K_1}{K} = \frac{\cancel{K}\left(\frac{\cancel{l}}{l_1}\right)}{\cancel{K}\left(\frac{\cancel{l}}{l_2}\right)}$

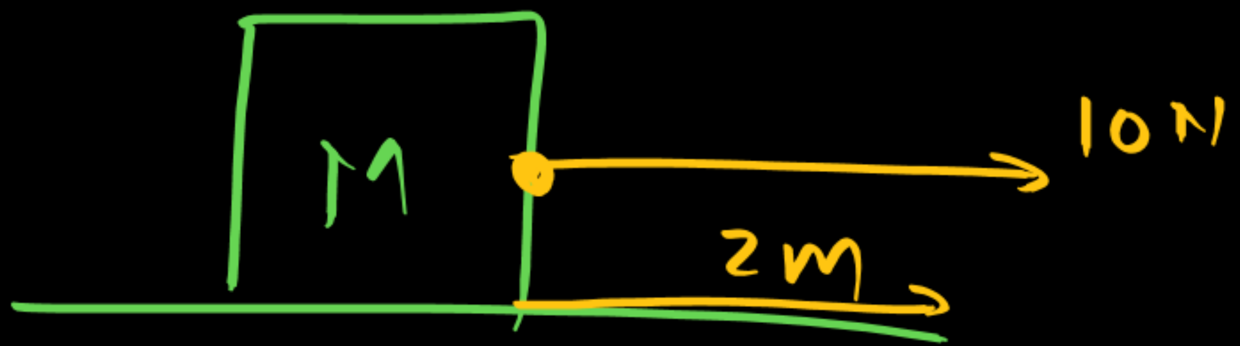
$= \frac{l_2}{l_1} = \frac{1}{n}$

Work Done by Pseudo Force

Concept

NIF

"Work done is frame of Ref dependent"



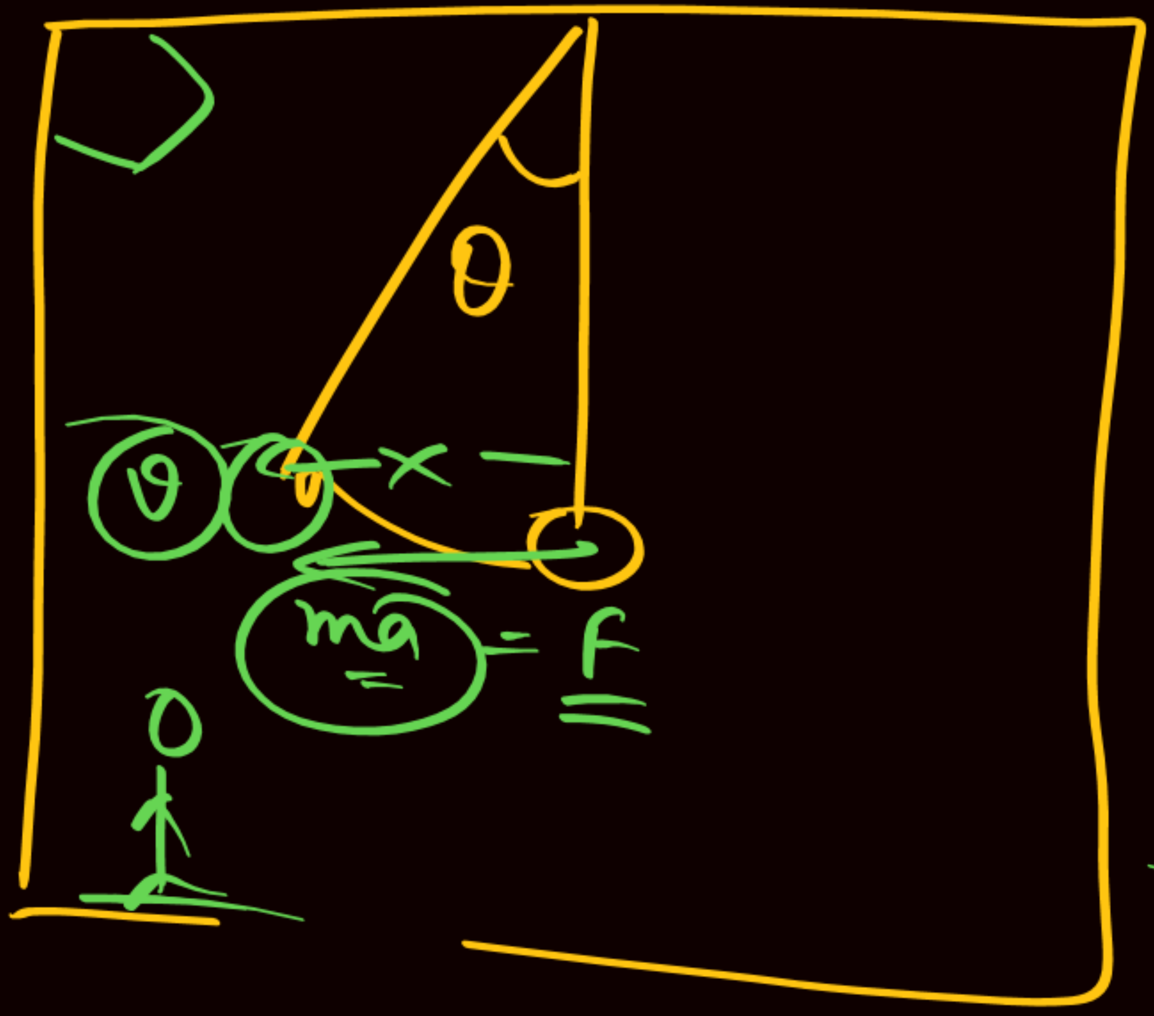
Work $= +10 \times 2$

$= 20\,J$

"O"

In Case 2 $W = 0$

Camera.



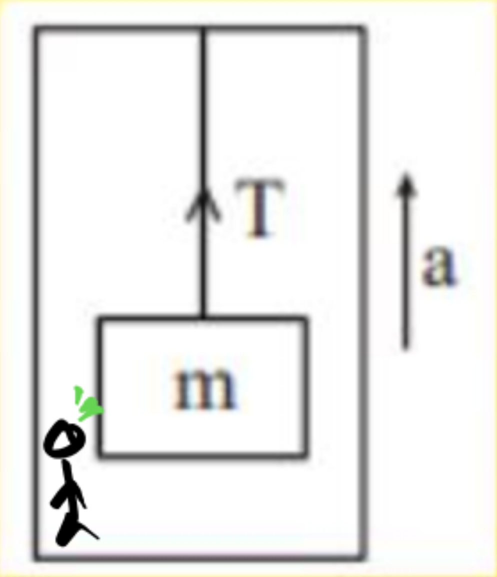
$\rightarrow a$.

"NIF"

$$\text{Work} = K_2 - K_1$$

$$W_{\text{pseudo}} = +max = K_2 - K_1$$

12. A block of mass m is suspended by a light thread from an elevator. The elevator is accelerating upward with uniform acceleration a. The work done by tension with respect to elevator on the block during t seconds is (u = 0):



→ NIIT

(a) $\frac{m}{2}(g+a)at^2$

(b) $\frac{m}{2}(g-a)at^2$

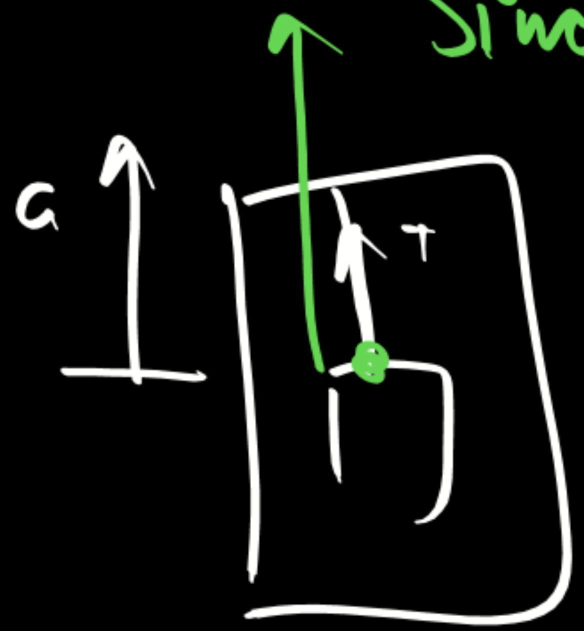
(c) $\frac{m}{2}gat^2$

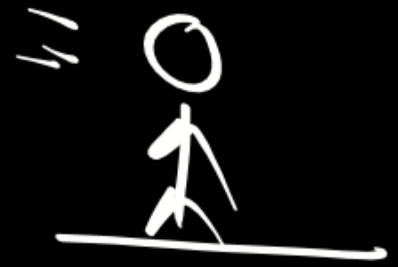
(d) 0 Ans

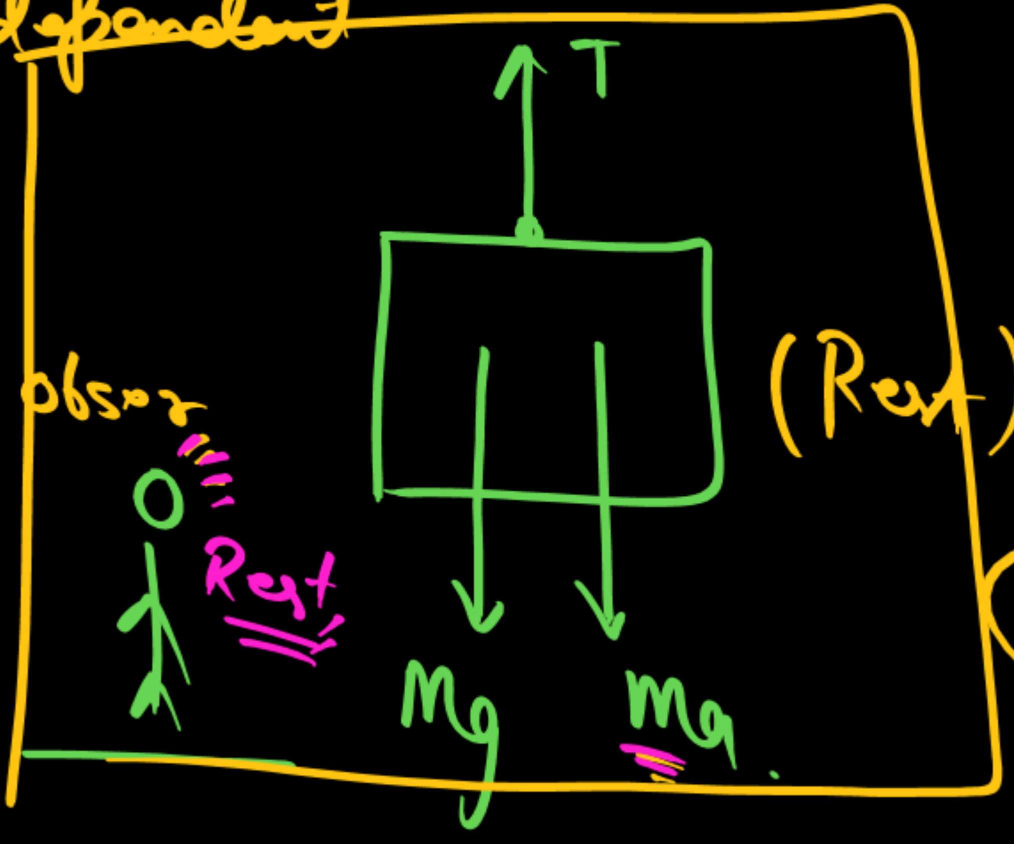
"Work is frame of Ref dependent"

Since Elevator is acc

(NIIF)







(Rest) $ds = 0$

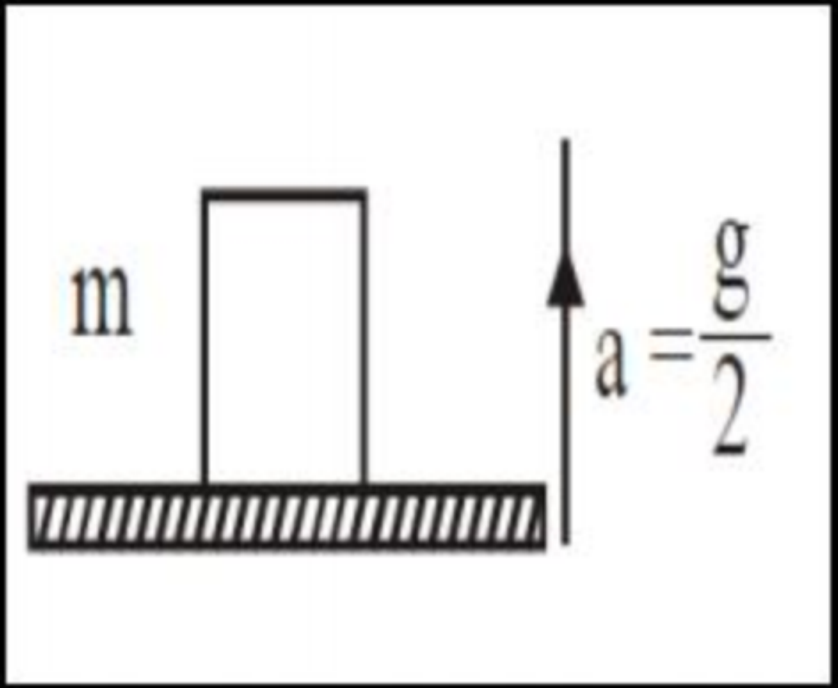
$W_T = 0$

$T = m(g+a)$

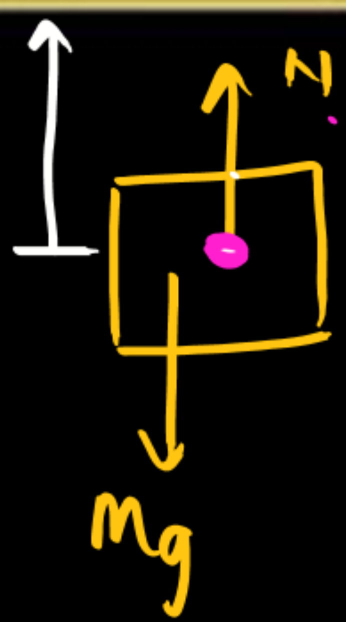
13. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in figure. Work done by normal reaction with respect to ground on block in time t is: [JEE Main - 2019 (January)]

(a) $-\frac{mg^2t^2}{8}$ (b) $\frac{mg^2t^2}{8}$

(c) 0 (d) $\frac{3mg^2t^2}{8}$ Ans



Equation by obs. $\frac{g t^2}{4} = s$



$\uparrow g/2 = a$

Motion $\Rightarrow$ $N - mg = m(g/2)$

$N = mg + \frac{mg}{2} = \frac{3mg}{2}$

In time t $s = \cancel{ut} + \frac{1}{2}at^2$

$= \frac{1}{2}\frac{g}{2}t^2$

$W = F \cdot ds$

$= +\frac{3mg}{2} \cdot \frac{g t^2}{4} = \frac{3}{8} mg^2t^2$

BREAK
TILL
21:20PM

Kinetic energy

$$\bar{A} \cdot \bar{A} = A^2$$

Concept : $KE = \frac{1}{2} m v_{rel}^2$

$\vec{v}_{rel}$ = velocity of a body wrt observer.

↳ KE is <u>frame of Ref dependent</u>.

$$KE = \frac{1}{2} m(\vec{v} \cdot \vec{v}).$$

Dependence of KE on Momentum

Concept $KE = \frac{1}{2} m v^2 = \frac{1}{2} \frac{m^2 v^2}{m} = \frac{p^2}{2m}$.

"we can find the magnitude of $|p|$." ← $\boxed{KE = \frac{p^2}{2m}} = \frac{\vec{P} \cdot \vec{P}}{2m}$

Can we Find Momentum of a body if Kinetic Energy of Body is Known?

$\vec{P} = m\vec{v}$

✓ KE = Known

✓ mass = Known

$P = ?$

"No".

14. When the momentum of a body increases by 100%, its K.E. increases by

(a) 20% (200%)　　(b) 40%

(c) 100%　　(d) 300%

→ Error Analysis is applicable % is less than 5%.

Wrong Method

$$KE = \frac{P^2}{2m}$$

$$\frac{\Delta K}{K} = 2\frac{\Delta P}{P}$$

$$\frac{\Delta K}{K} = 2\% P$$

$$= 200\%$$

$$P_i = P_0$$

$$K_0 = \frac{P_0^2}{2m}$$

$$P_f = 2P_0$$

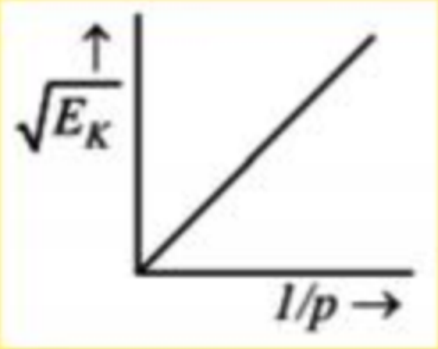
$$K_f = \frac{(2P_0)^2}{2m} = 4\frac{P_0^2}{2m}$$

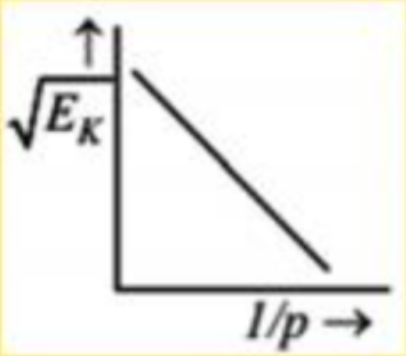
$$\boxed{K_f = 4K_i}$$

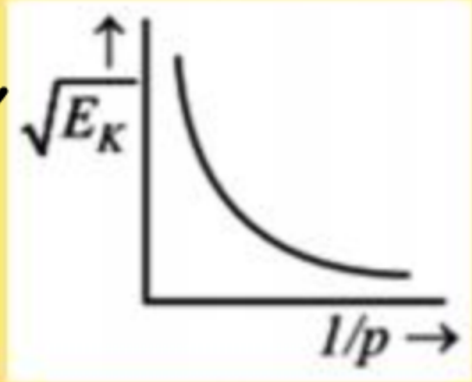
Increase = 300%

Graphs :

15. The graph between $\sqrt{E_k}$ and $1/p$ is
(E_K = kinetic energy and p = momentum) – **(2015)**

(a)


(b)


(c)


$xy = \text{Const}$

(d)


$PV = \text{const}$

$KE = \dfrac{p^2}{2m} \Rightarrow \sqrt{E_k} = \dfrac{p}{2m}$

$\dfrac{1}{p} = x$

$p = 1/x$

$y = \dfrac{1}{2mx}$

$xy = \dfrac{1}{\sqrt{2m}} = \underline{\underline{\text{Const}}}$

Work Energy Theorem

Concept : Work done by all forces = ΔK

$$\boxed{W_{con} + W_{non\,cons} + W_{ext} + W_{Pseudo} = K_f - K_i}$$

→ gravity

→ Spring

(friction)

16. Figure shows a smooth track AB joint with a rough horizontal surface at S having friction coefficient 0.2. A block of 1 kg mass released on track from a height of 1m, Find the distance it will travel on rough track before coming to rest.

~~(a)~~ 5m Ans (b) 3m

(c) 10m (d) 7.5m

Sol:- $W_{con} + W_{non} + W_{ext} = K_2 - K_i$

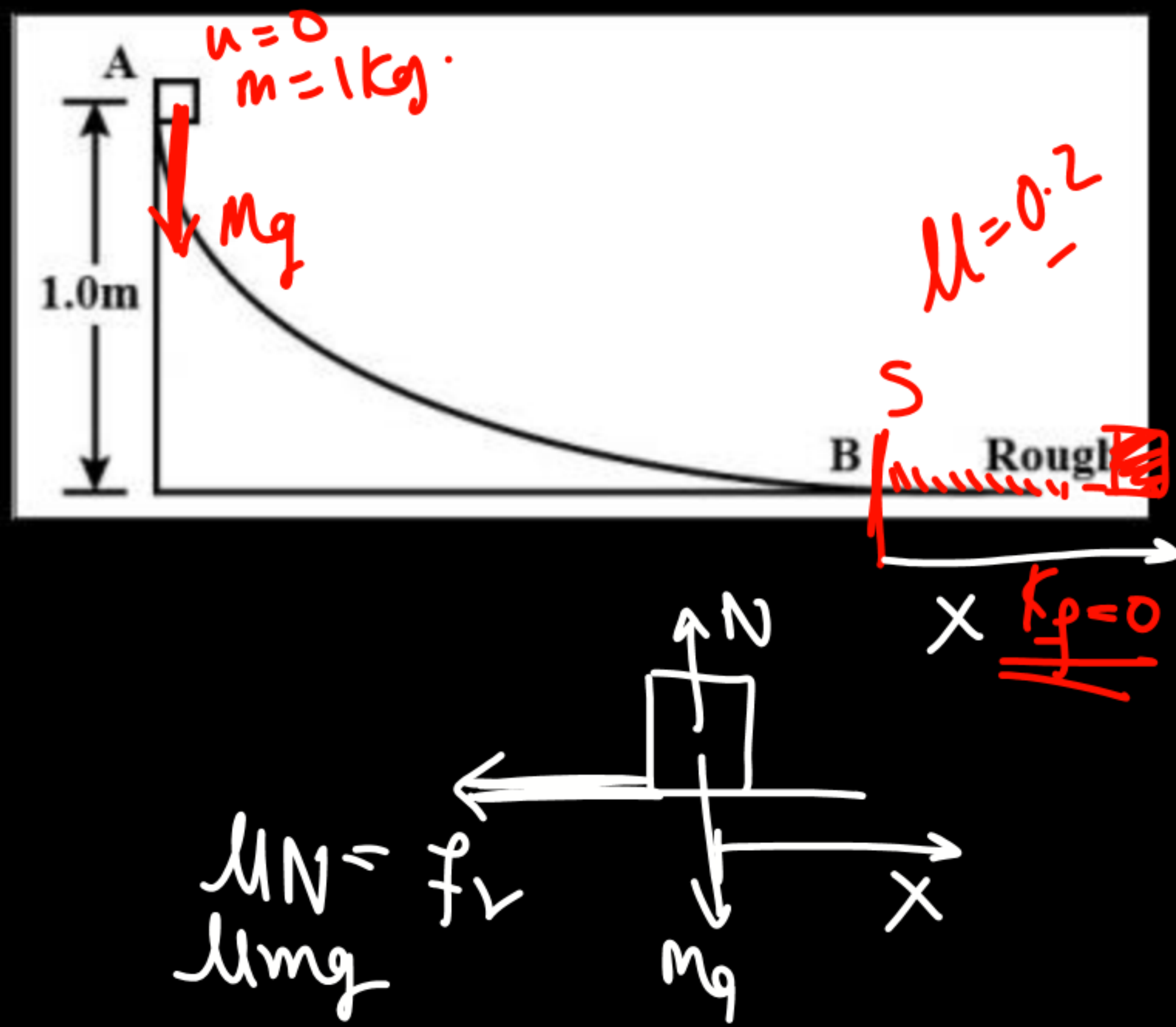
$W_{gravity} + W_{fric} = K_2 - K_1$

$h =$

$+mgh - f_r x = 0 - 0$

$\cancel{mg}h = \mu \cancel{mg} x$

$\frac{1}{\mu} = x = \frac{1}{0.2} = 5m$



17. In the figure the length of incline is $2\sqrt{3}$ m and a block is projected up the incline at speed 10 m/s. The find speed with which blocks will moves off the top of incline plane. ($\mu = 1/\sqrt{3}$, $\theta = 60^0$)

(a) $\sqrt{20}$ Ans
(b) 30
(c) $\sqrt{10}$
(d) 40

Sol $W_{con} + W_{non} = K_2 - K_1$

$-\cancel{m}g(3) - \mu \cancel{m} g \cos 60\, l = \frac{1}{2}\cancel{m}v^2 - \frac{1}{2}\cancel{m}u^2$

$2\sqrt{3}\sin 60 = h$

$2\sqrt{3} \times \frac{\sqrt{3}}{2} = h$

$h = 3m$

$(l = 2\sqrt{3})$

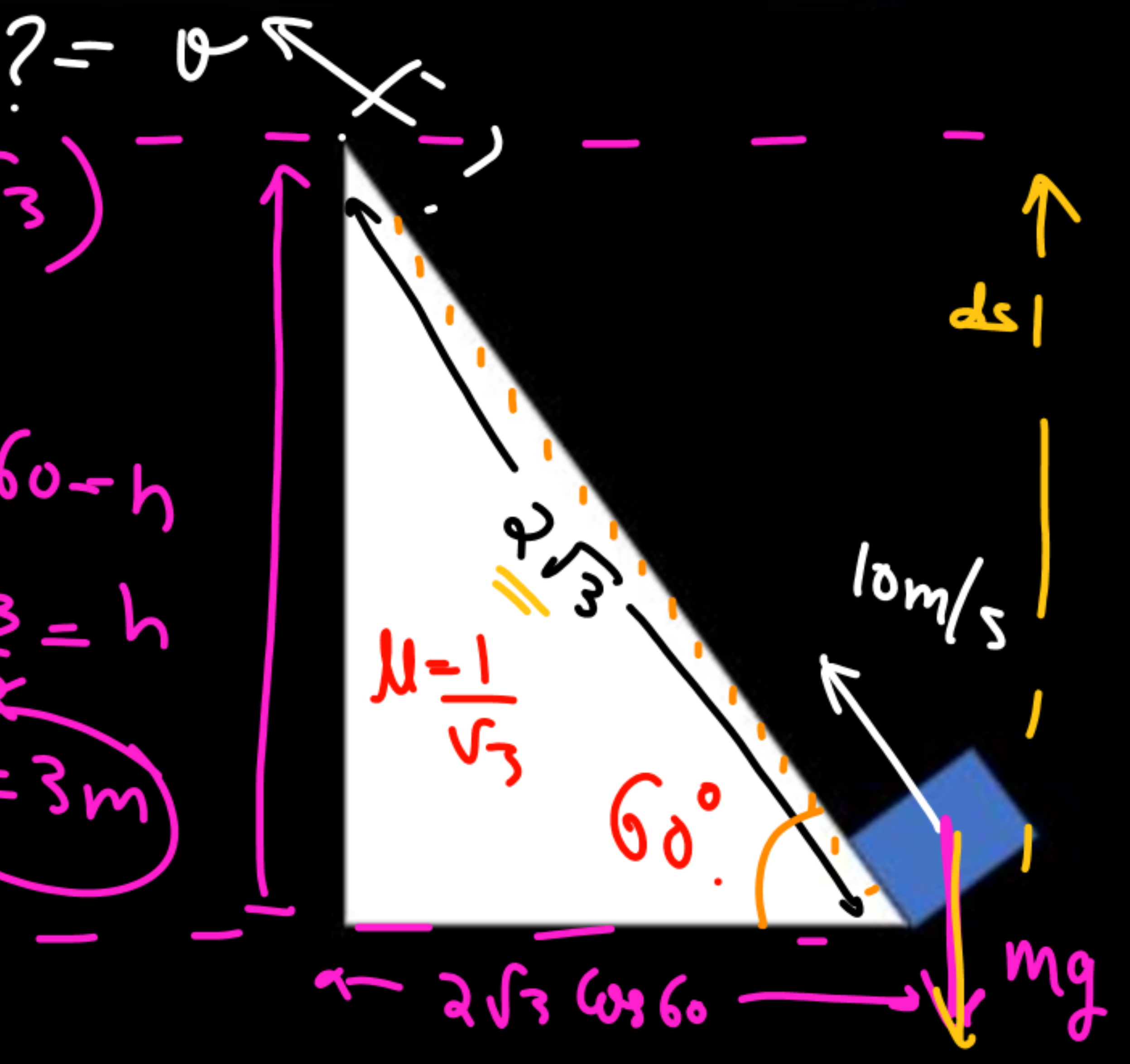
$-30 - \frac{10}{\sqrt{3}} \times \frac{1}{2} \times 2\sqrt{3} = \frac{1}{2}(v^2 - 10^2)$

$-30 - 10 = \frac{1}{2}(v^2 - 100)$

$-80 = v_f^2 - 100$

$v_f^2 = 20$

$v_f = \sqrt{20}$



$W_{or} = \underset{=}{f} \cdot d \cos\theta$

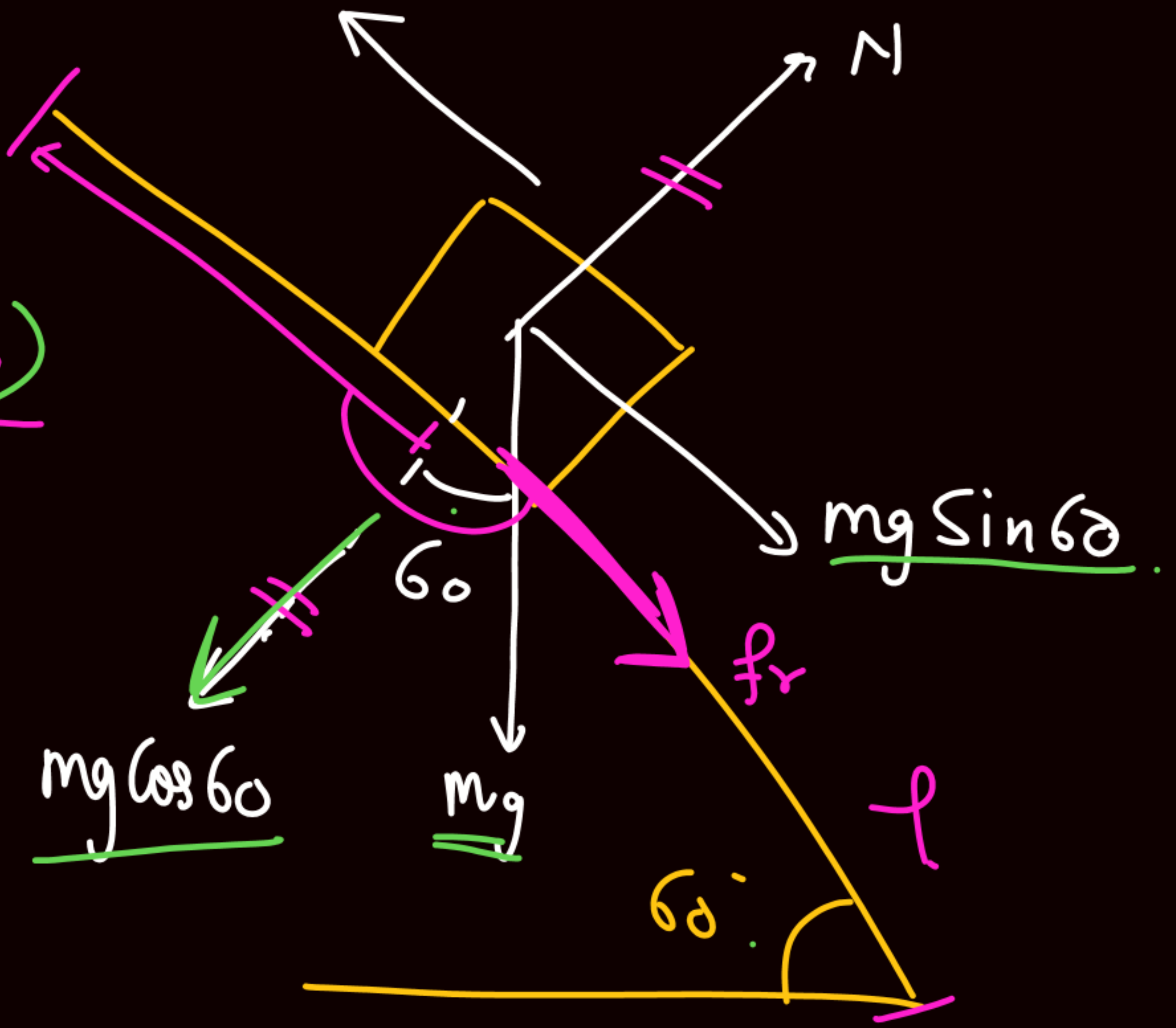
$N = mg\cos 60$

$f_r = \mu \underline{N}$

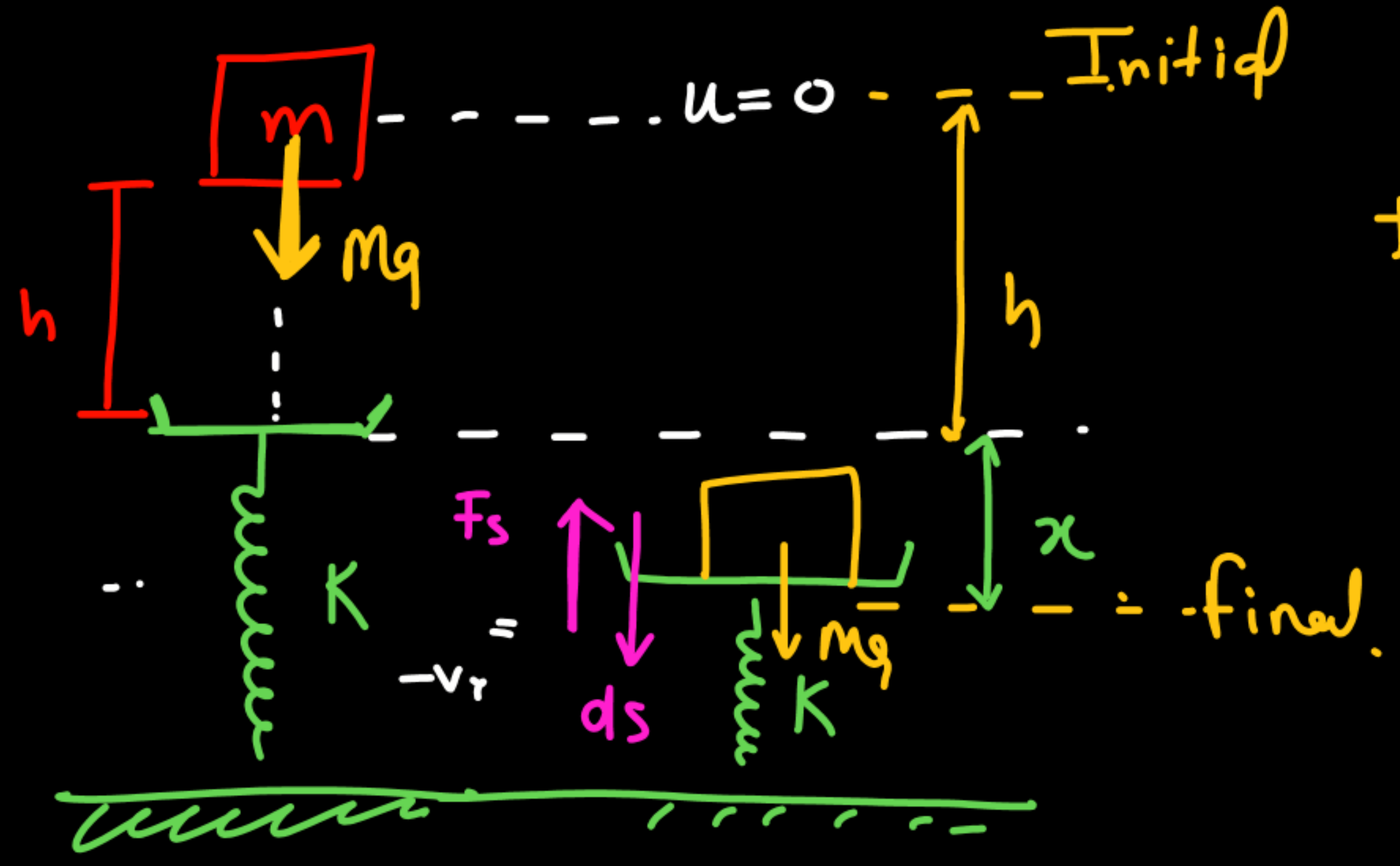
$= \underline{\mu(mg\cos 60)}$

$W_f = -\underline{f_r} \times l$

$= -\mu mg\cos 60\, \underline{l}$



18. A block of mass *m* is released from rest from a height h above the light pan attached to a vertical spring of force constant k, mounted on floor as shown. Find the maximum compression in the spring by the block. [(HC Verma]



$$W_{spring} = -\frac{1}{2}kx^2$$

$$W_{con} + W_{non} + W_{ex} = K_2 - K_1$$

$$+mg(h+x) - \frac{1}{2}kx^2 = 0 - 0$$

$$2mgh + 2mgx - kx^2 = 0$$

$$kx^2 - 2mgx - 2mgh = 0$$

$$x = \frac{2mg \pm \sqrt{4m^2g^2 + 4k\,2mgh}}{2k}$$

= ______ Ans

19. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in a equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is

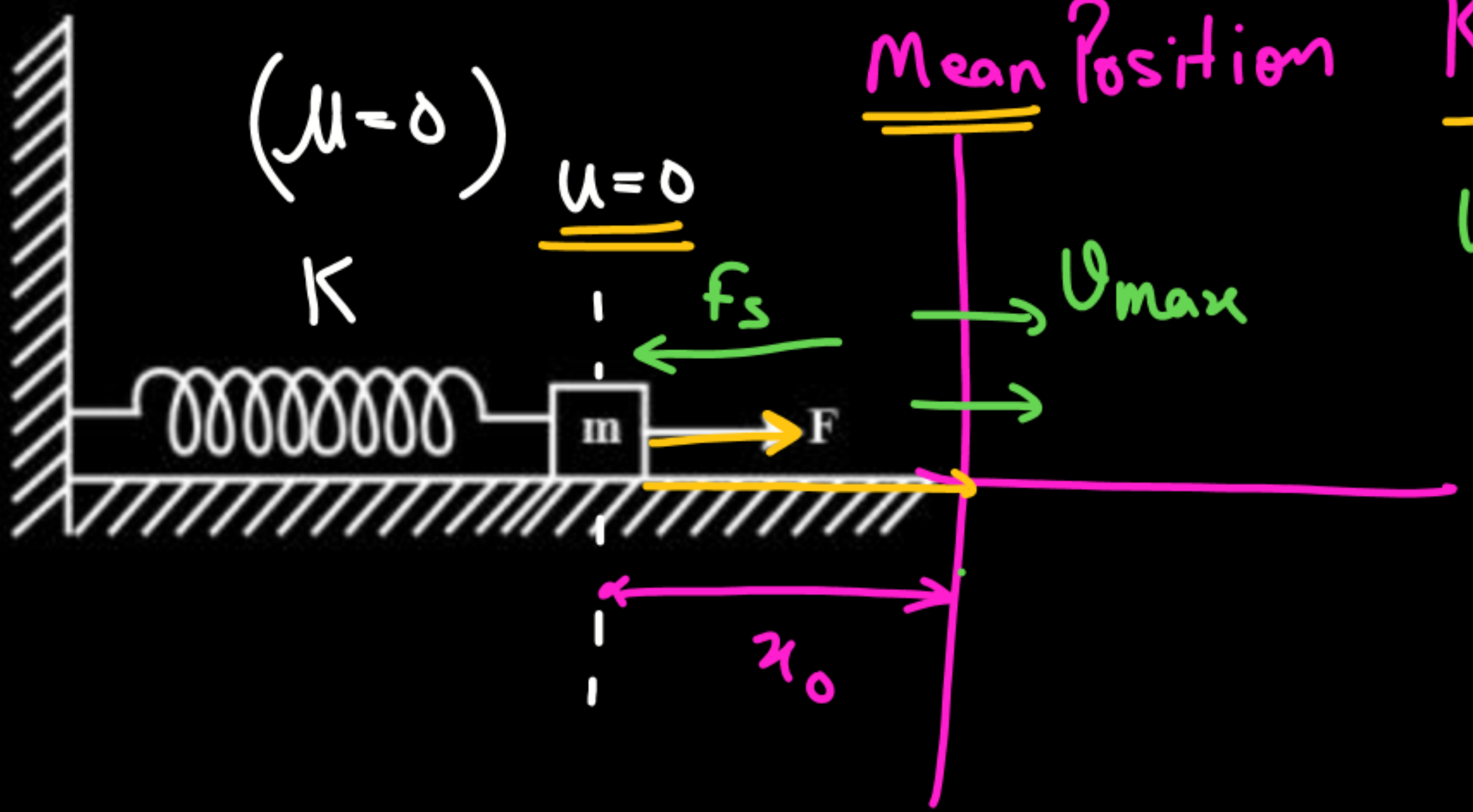
[(JEE Main - 2019 (January)]

(a) $\frac{2F}{\sqrt{mk}}$

(b) $\frac{F}{\pi\sqrt{mk}}$

(c) $\frac{\pi F}{\sqrt{mk}}$

(d) $\frac{F}{\sqrt{mk}}$



$Kx_0 = F$ $\quad x_0 = \frac{F}{K}$

$W_c + W_n + W_{ext} = K_2 - K_1$

$W_{gravity} = 0$

$W_{spring} = -\frac{1}{2}Kx_0^2$

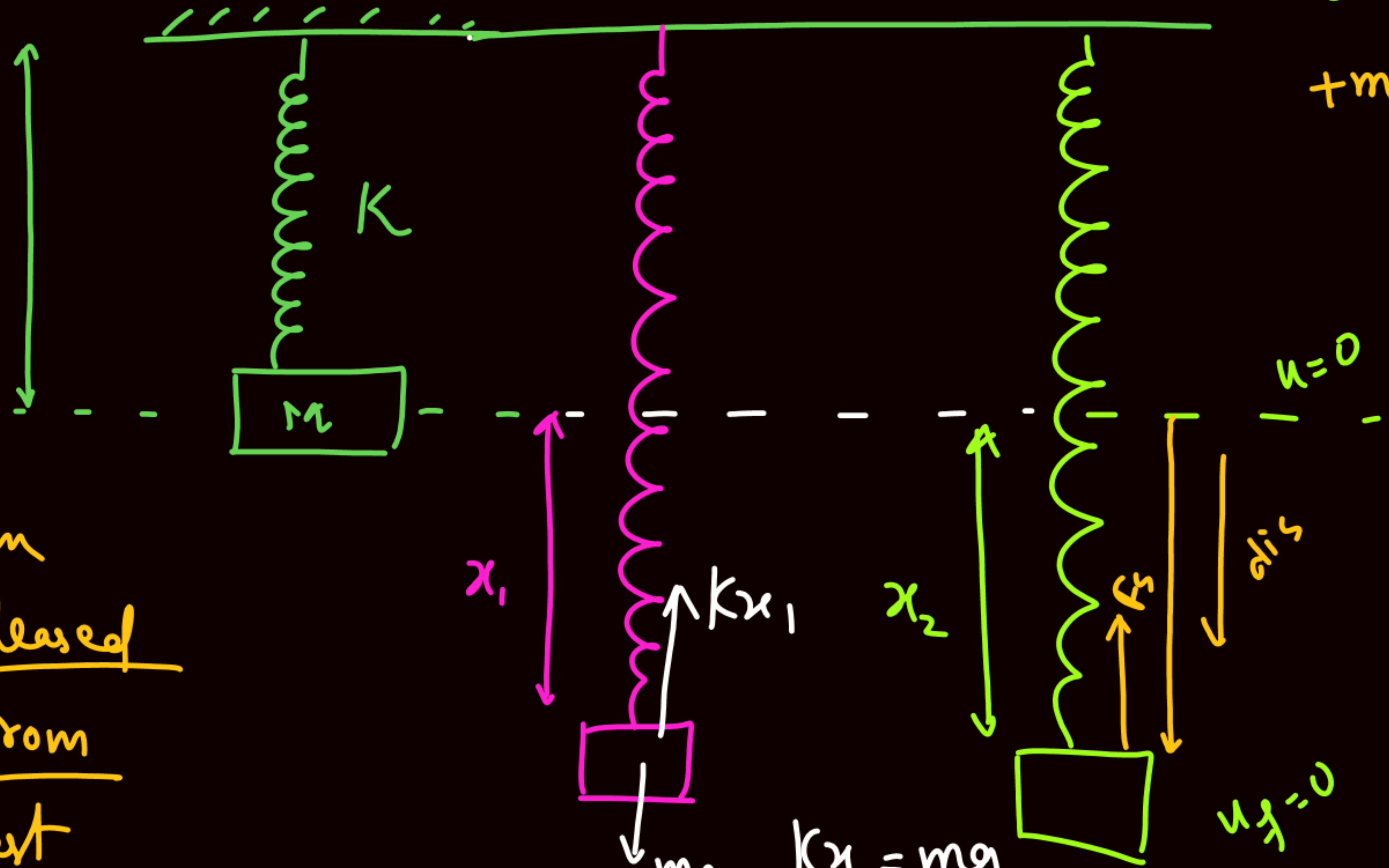
$W_{ext} = +Fx_0$

$-\frac{1}{2}Kx_0^2 + Fx_0 = \frac{1}{2}mv_{max}^2 - 0$

$-\frac{1}{2}K\frac{F^2}{K^2} + F\frac{F}{K} = \frac{1}{2}mv_{max}^2$

$-\frac{F^2}{2K} + \frac{F^2}{K} = \frac{1}{2}mv_{max}^2$

$l =$ Natural Length



$$W_c + W_n + W_e = K_2 - K_1$$

$$+mgx_2 - \frac{1}{2}kx_2^2 = 0 - 0$$

$$mg\cancel{x_2} = \frac{kx_2^{\cancel{2}}}{2}$$

$$\frac{2mg}{k}$$

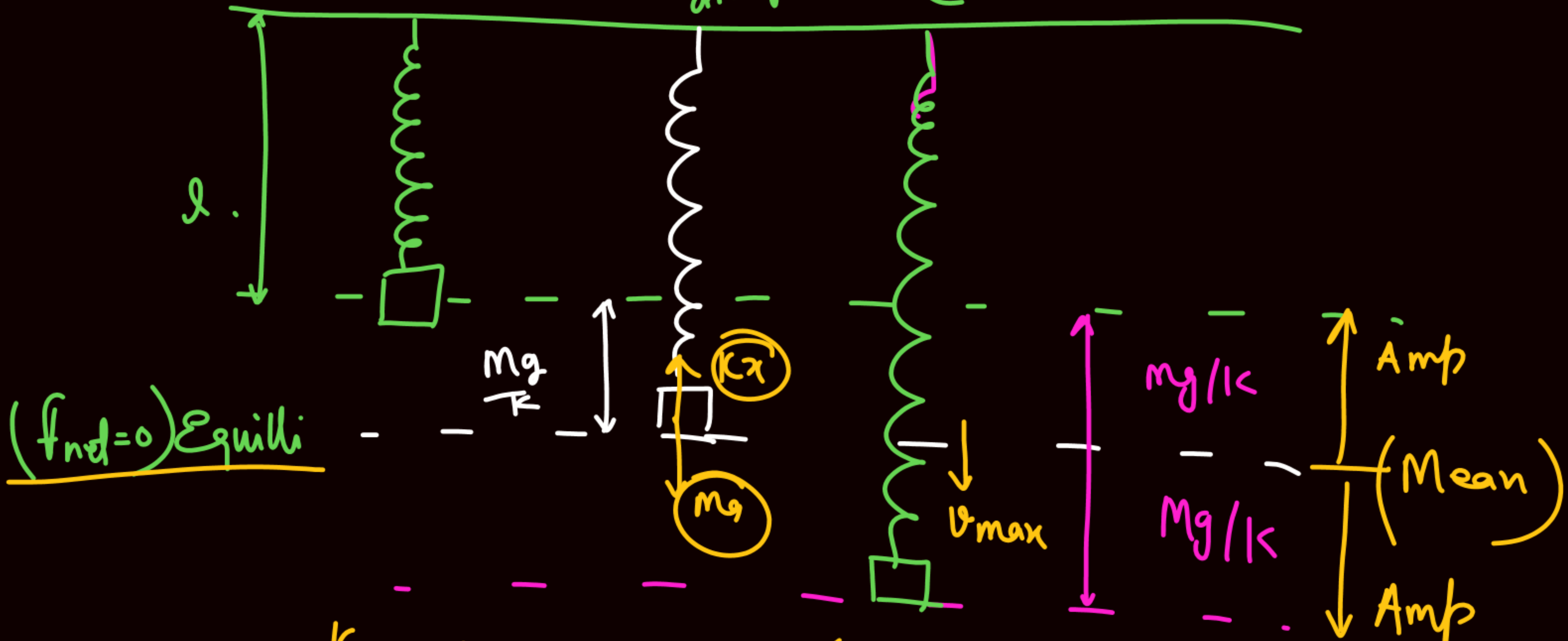
When Released from Rest

Maximum Elongation.

$$Kx_1 = mg$$

$$x_1 = \frac{mg}{k}$$

force analysis (WET).



$$Kx = mg$$

$$x = \frac{mg}{k}$$

(Released)

22. Figure shows a block A held at rest in a state when spring of force constant k is in its natural length with an attached mass m at rest Find mass of block A for which on releasing it, block of mass m will break off from ground. (2015)

(a) $\frac{m}{2}$ (b) $\frac{m}{4}$

(c) $\frac{m}{8}$ (d) m

The block will Rise when $f_{spring} = mg$

$Kx = mg$

$\boxed{x = \frac{mg}{K}}$

When Spring is Elongate $= -\frac{1}{2}Kx^2$

mass of A = m'

Spring Elongate

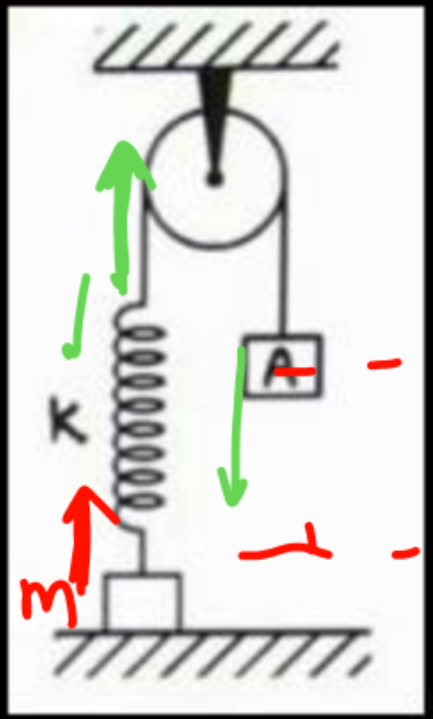
$W_c + W_n + W_e = K_2 - K_1$

$+m'gx - \frac{1}{2}Kx^2 = 0 - 0$

$m'gx = \frac{1}{2}Kx^2$

$m' = \frac{1}{2g}K.x = \frac{K}{2g}\frac{mg}{K}$

$= m/2$



m' $u = 0$ f_s Kx x $V_f = 0$

Block A has to move atleast $x = \frac{mg}{K}$ so that other block will Rise up.

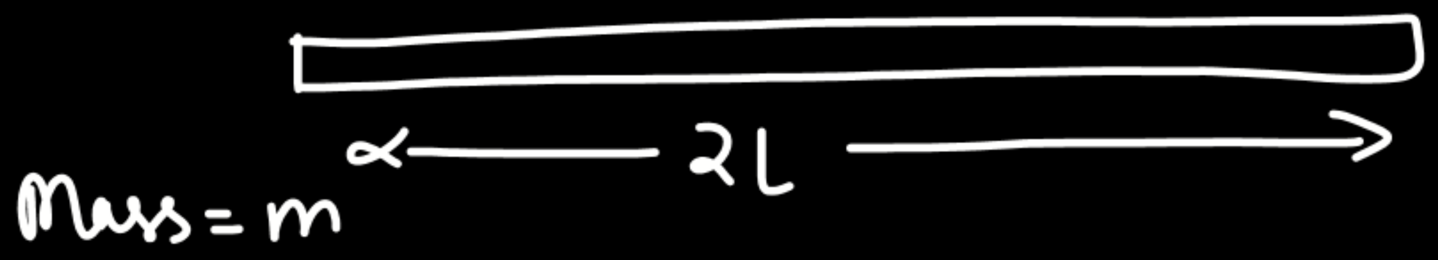
23. Figure shows a uniform smooth and flexible rope of mass m and length $2l$ is symmetrically supported on a nail A. If due to a slight jerk rope starts falling, find its speed when it leaves off the nail. (2003)

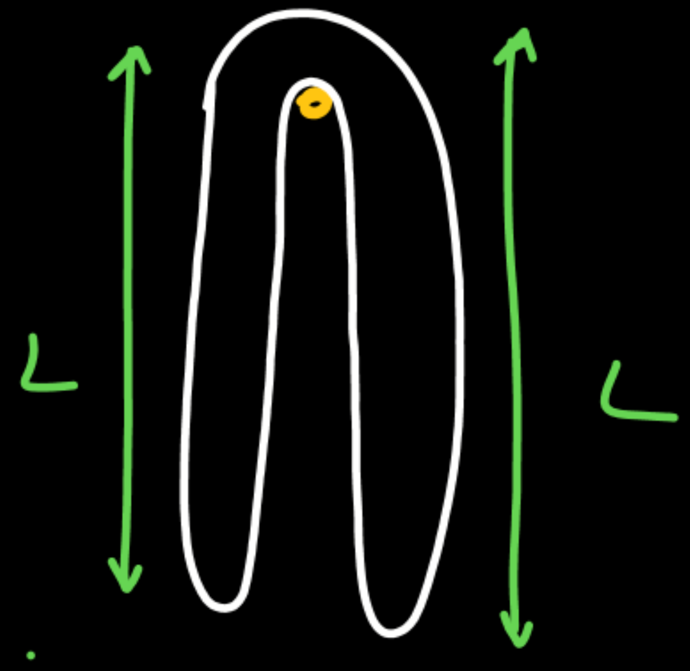
(a) $\sqrt{gl}$

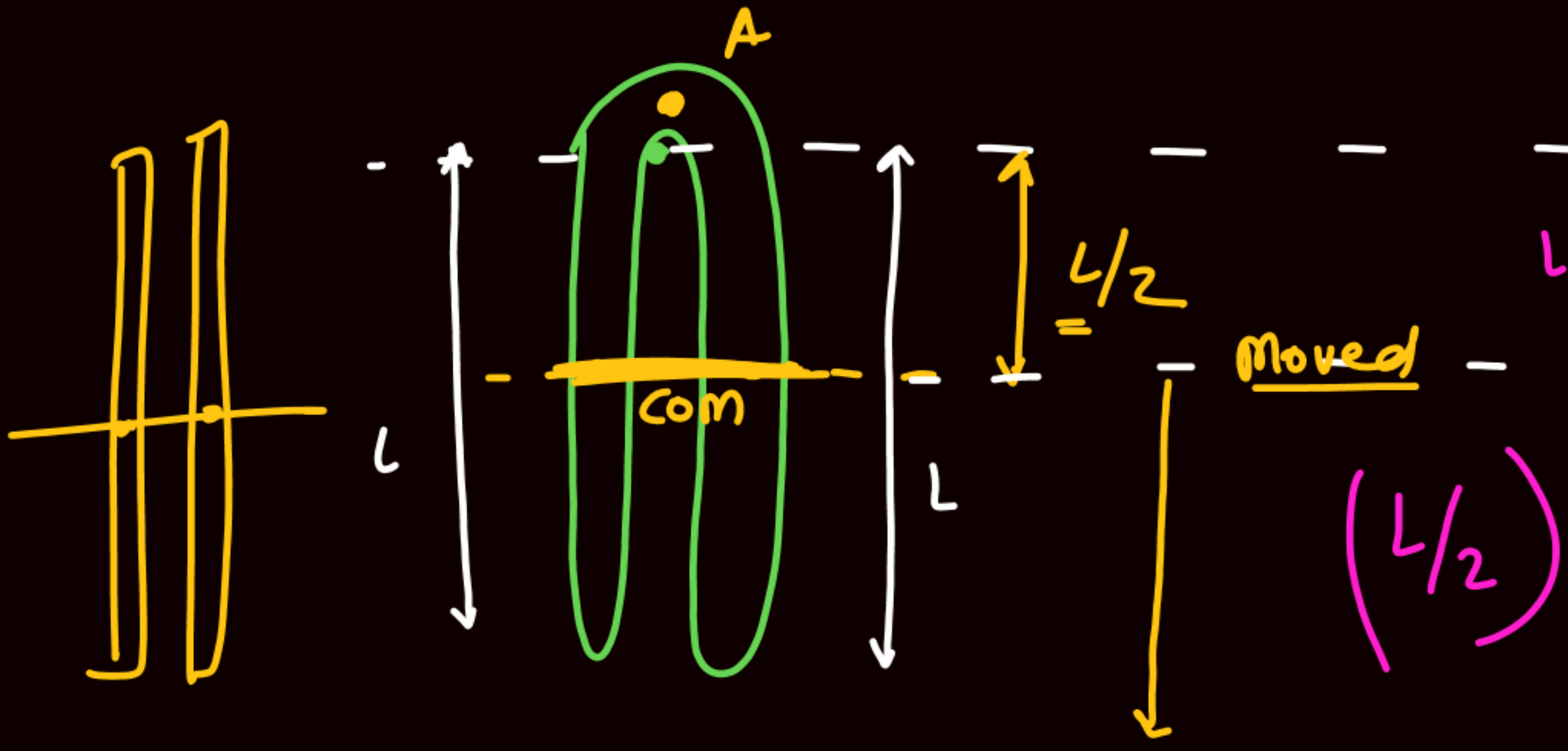
(b) $2\sqrt{gl}$

(c) $\sqrt{3gl}$

(d) none of these



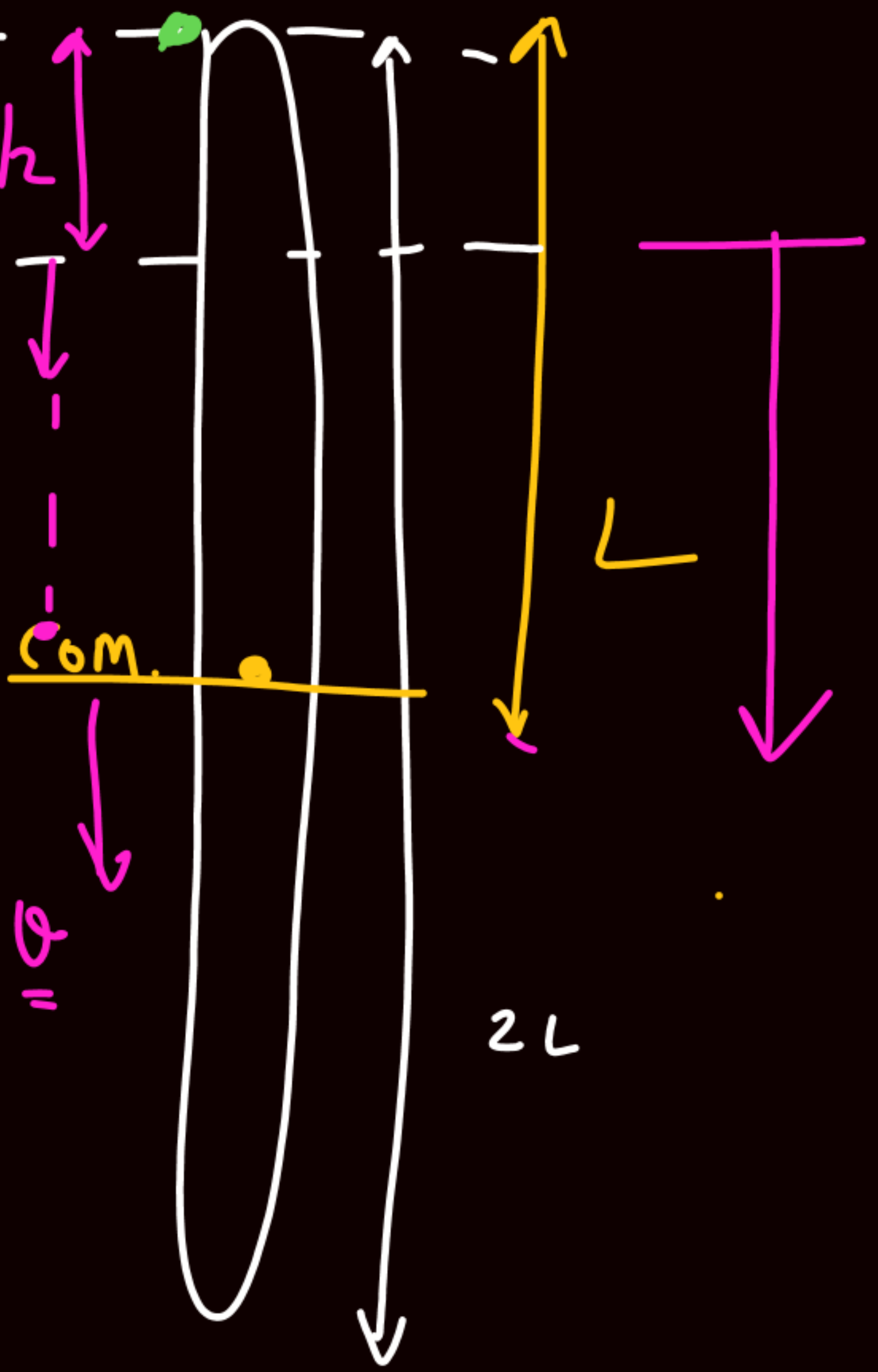


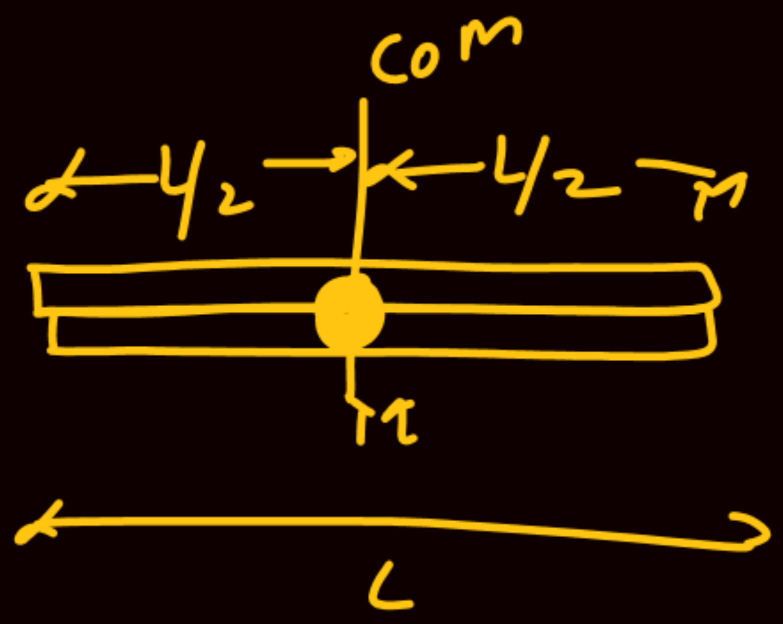


$$W_c + W_n + W_e = K_2 - K_1$$

$$+\frac{\cancel{M}gL}{\cancel{2}} = \frac{1}{\cancel{2}}\cancel{m}v^2 - 0$$

$$v = \sqrt{gl}$$





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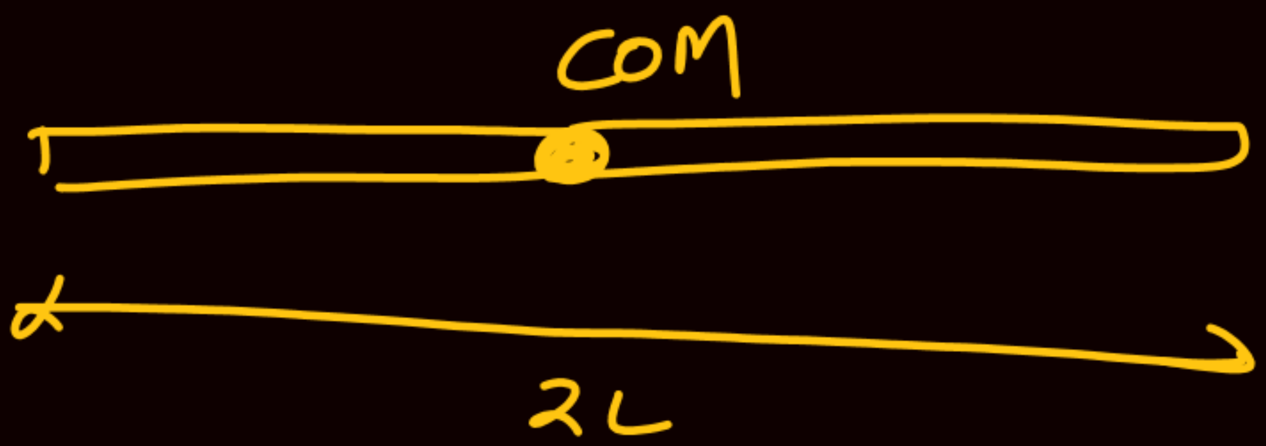

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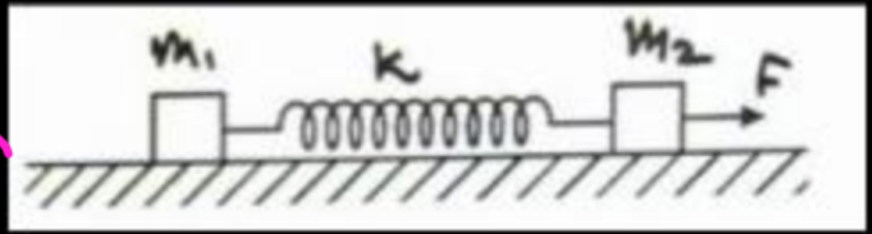
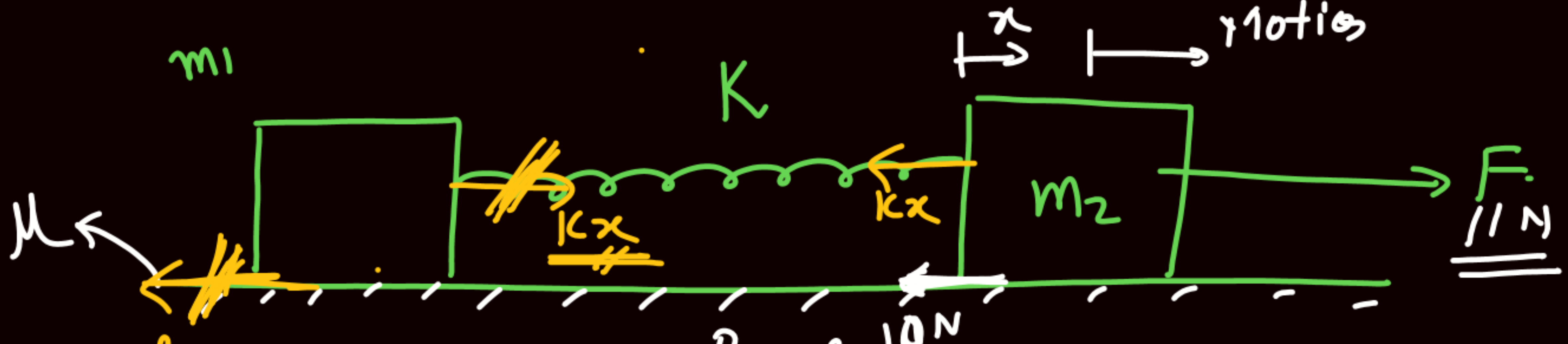
2L

27. Two blocks of masses m_1 and m_2 connected by a non-deformed light spring rest on a rough ground as shown in figure. If friction coefficient on ground is μ find the minimum constant force F to be applied on m_2 so that m_1 start sliding. (1987)

Ans: $\mu g((m2 + (m_1/2))$

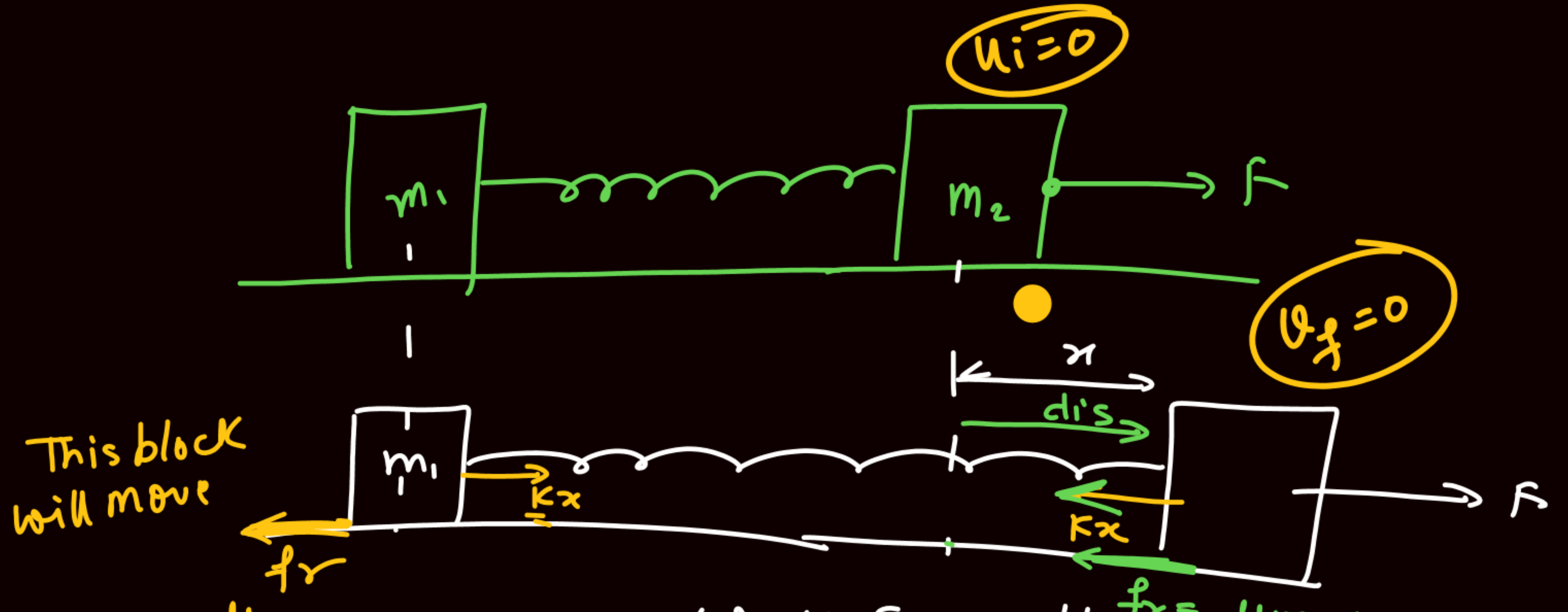
μ





$\mu = 0.1 \qquad m_2 = 10\,g$

$$f_{max} = \mu N$$
$$= 0.1 \times 10 \times 10$$
$$\underline{f_{max} = 10N}$$



$Kx = \mu m_1 g$

$x = \dfrac{\mu m_1 g}{K}$

Work Energy thm

$$W_c + W_n + W_e = K_2 - K_1$$

$$-\frac{1}{2}Kx^{\cancel{2}} - \mu m_2 g \cancel{x} + F\cancel{x} = 0 - 0$$

$$-\frac{K\mu^2 m_1^2 g^2}{2K^2} - \mu m_2 g \frac{\mu m_1 g}{K} + F\frac{\mu m_1 g}{K} = 0$$

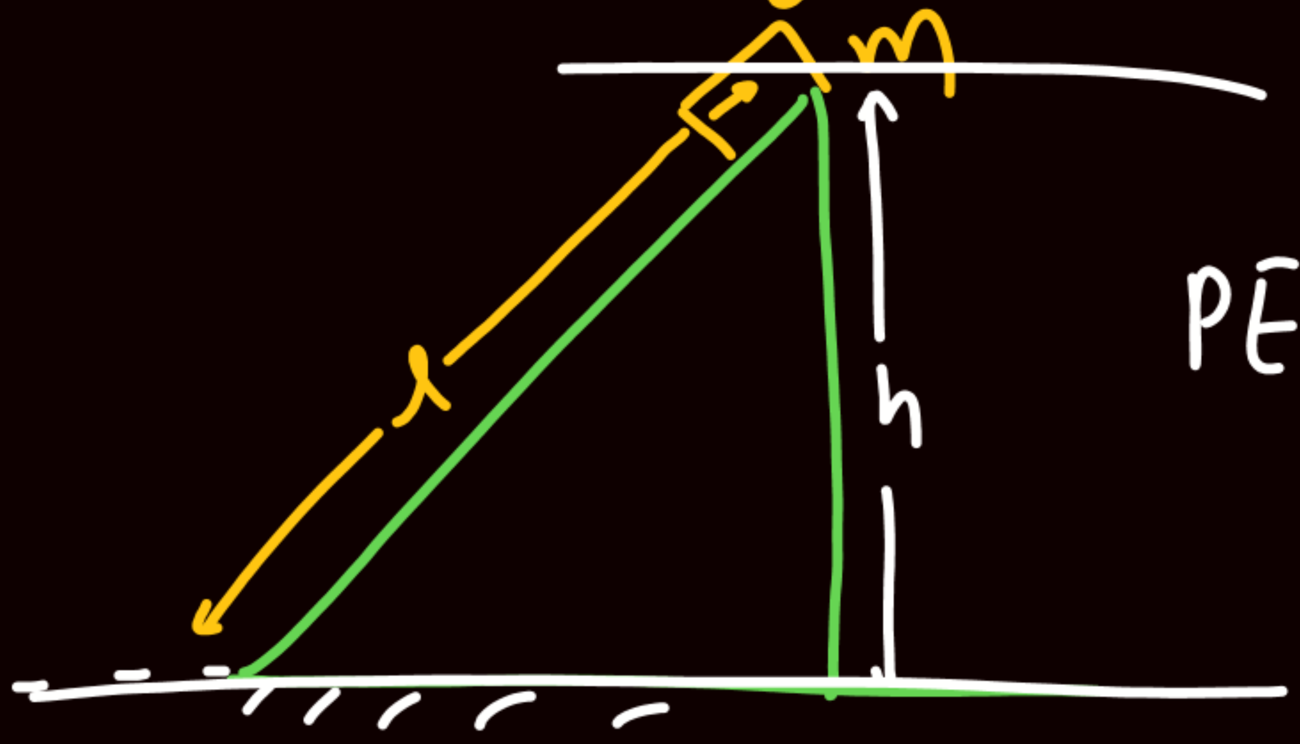
$$\boxed{F = \mu g\left(m_2 + \frac{m_1}{2}\right)}$$

Potential Energy due to gravity

Concept

Potential Energy

↳ Energy possessed by a body by virtue of Position.



PE of block = $\underline{mgh}$

$\underline{PE = 0}$

① PE of a body is Ref dependent.

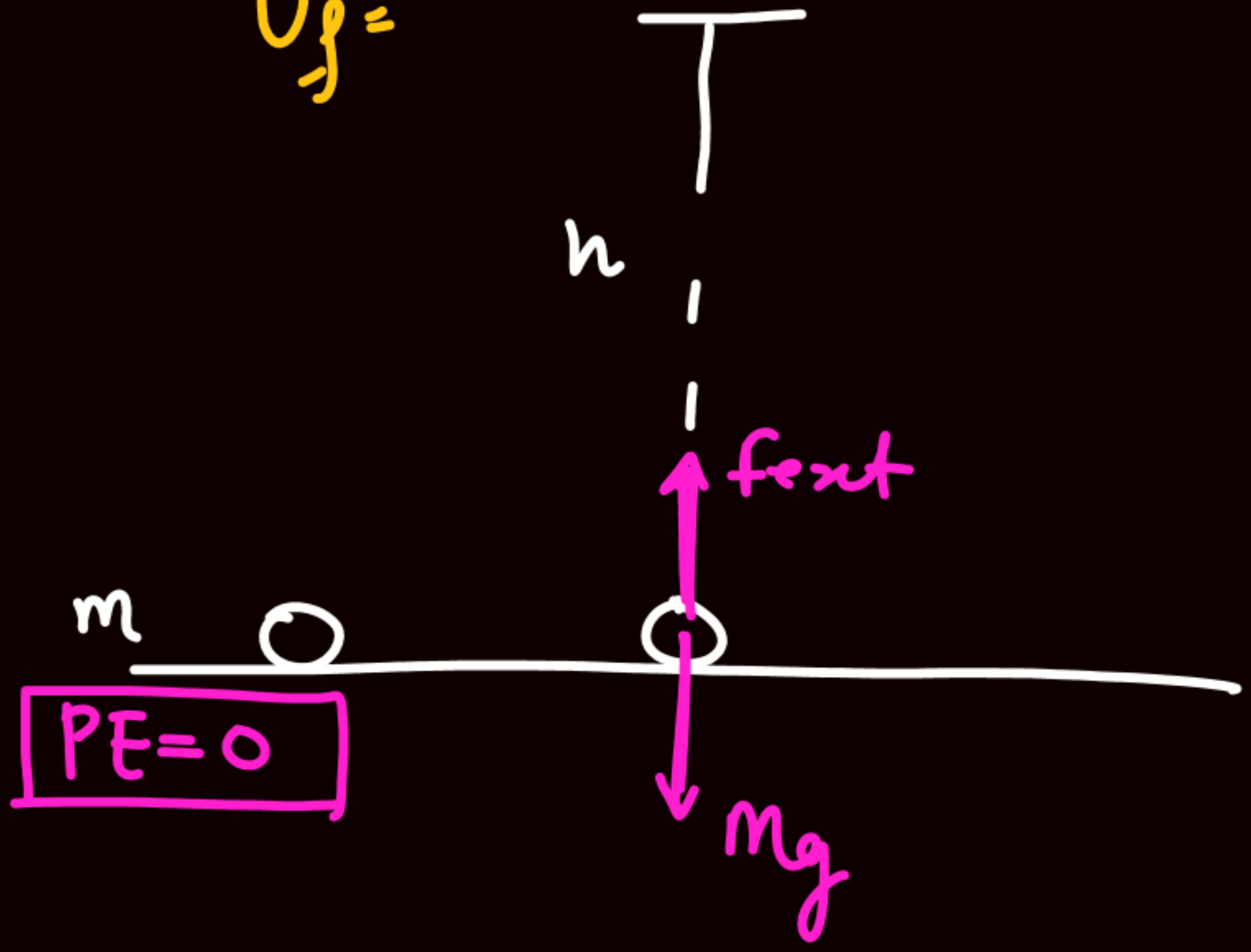
② Change in PE is independent of Ref.

V.imp

V.imp

$$\boxed{\begin{aligned} W_{\text{conservative force}} &= -\Delta U = -(U_f - U_i) \\ W_{ext} &= \Delta U = (U_f - U_i) \end{aligned}}$$

$U_f =$



$$W_{con} = -(U_f - U_i)$$

$$W_{gravity} = -mgh.$$

$$W_{con} = -(U_f - U_i)$$

$$-mgh = -(U_f - 0)$$

$$\boxed{U_f = mgh}$$

BREAK
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23:00PM.

30. In a region potential energy of a body is given as

$$U = 4x^2 + 3x - 5 \text{ J}$$

Find work done by an external agent in slowly moving the body from point (3, 2) to (–1, 4).

$PE = 4x^2 + 3x - 5$

$(3, 2)$

$(-1, 4)$

$PE(x=3)$ (i) $\quad PE = 4(3)^2 + 3\times 3 - 5$

$= 36 + 9 - 5$

$= 40\text{ J}.$

$PE(x=-1)$ f $\quad PE = 4(1) - 3 - 5$

$= 4 - 8 = -4\text{ J}.$

$W_{ext} = (U_f - U_i)$

$= -4 - 40$

$= -44\text{ J}.$

Relation between Force and PE

Concept

$$\vec{F} = -\frac{\partial U}{\partial x}$$

Vector

Potential Energy

Scalar

$$\vec{F} = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$$

$$F_x = -\frac{\partial U}{\partial x}\Big|_{y,z} \qquad F_z = -\frac{\partial U}{\partial z}\Big|_{x,y}.$$

$$F_y = -\frac{\partial U}{\partial y}\Big|_{x,y}$$

32. In a region, potential energy of a particle of mass 4 kg depends on its x-coordinates as $U = 5x^3 + 4x + 7$ Joule

Find the acceleration of particle at position (2, 4).

$PE = 5x^3 + 4x + 7$

$m = 4\,kg.$

at $x = 2$
$y = 4$

$acc = \frac{F}{m} = -\frac{64}{4}$

$= -16\hat{i}$

$F_x = -\frac{\partial U}{\partial x} = -\left(\frac{\partial}{\partial x}(5x^3 + 4x + 7)\right)$

$F_x = -(15x^2 + 4).$

at $x = 2$ $\quad F_x = -(15 \times 4 + 4) = -64\,N\hat{i}$

33. In a region potential energy of a particle of mass 2 kg is given by

$$U = 3x^2 + 4y^2 \text{ J}$$

Find the acceleration of particle when it is released from (1, 1).

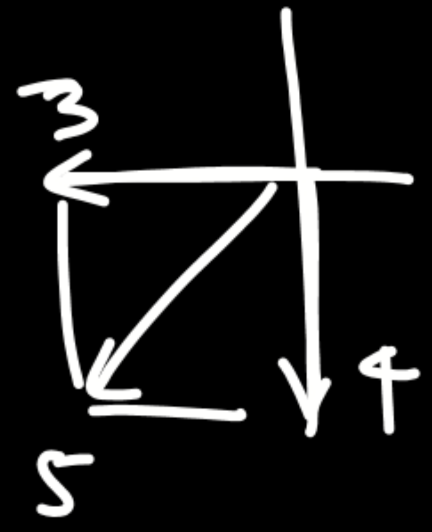
$$U = 3x^2 + 4y^2$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j}$$

$$F_x = -\frac{\partial U}{\partial x}\bigg|_{y, z = \text{const}}$$

$$F_x = -\left(\frac{\partial}{\partial x}(3x^2 + 4y^2)\right) = -(6x + 0)$$

$$= -6x$$



$$F_y = -\frac{\partial U}{\partial y}\bigg|_{x, z} = -\frac{\partial}{\partial y}(3x^2 + 4y^2)$$

$$F_y = -8y$$

$$\vec{F} = -6x\hat{i} - 8y\hat{j}.$$

$$\vec{F}_{(1,1)} = -6\hat{i} - 8\hat{j}.$$

$$\vec{a} = -3\hat{i} - 4\hat{j}$$

$$|a| = 5,$$

Concept Of Equilibrium

$$\boxed{F_{net} = 0}$$

We Know $\vec{F} = -\frac{\partial U}{\partial r}$

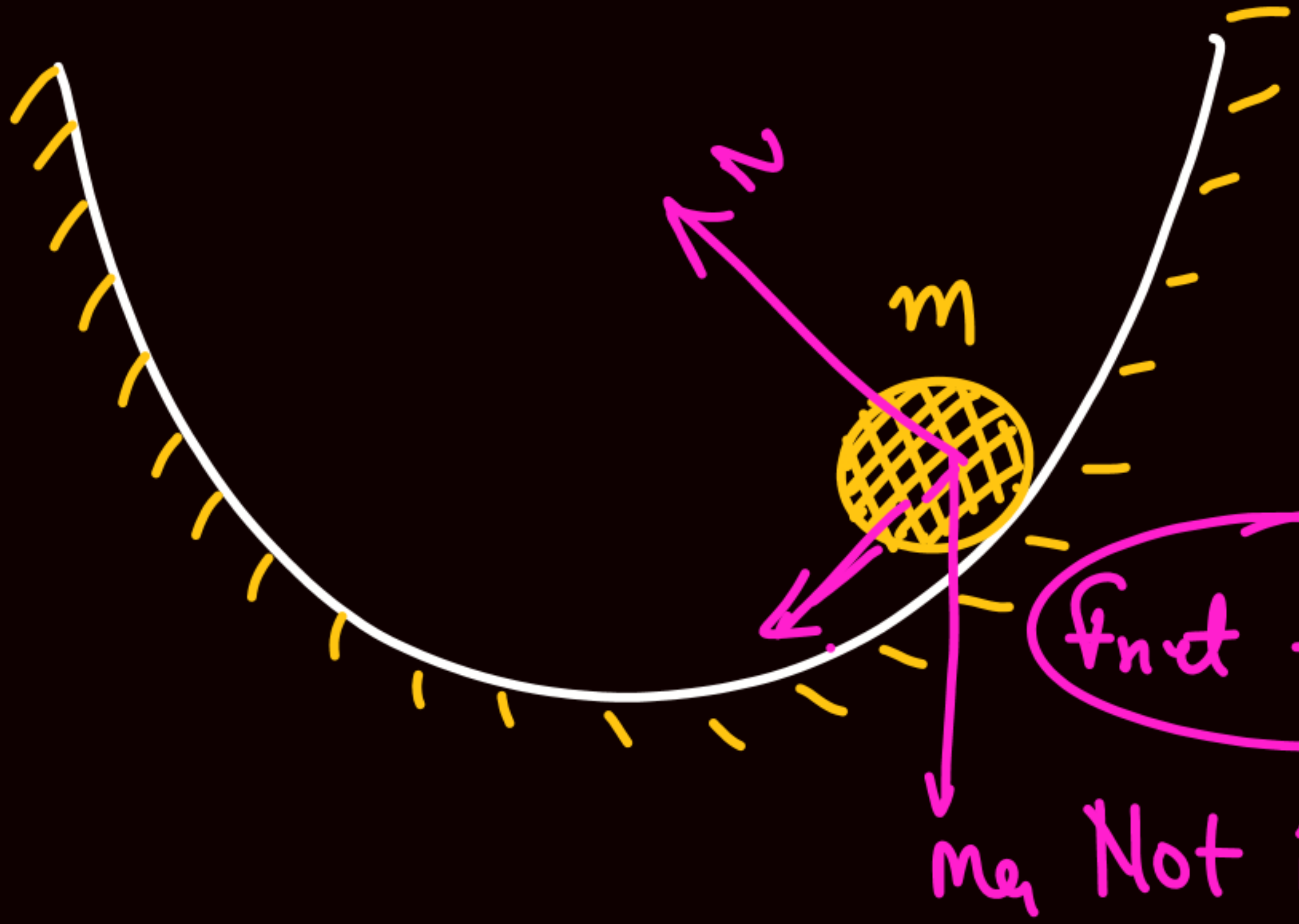
Stable Equili

Unstable Equili

Neutral Equili

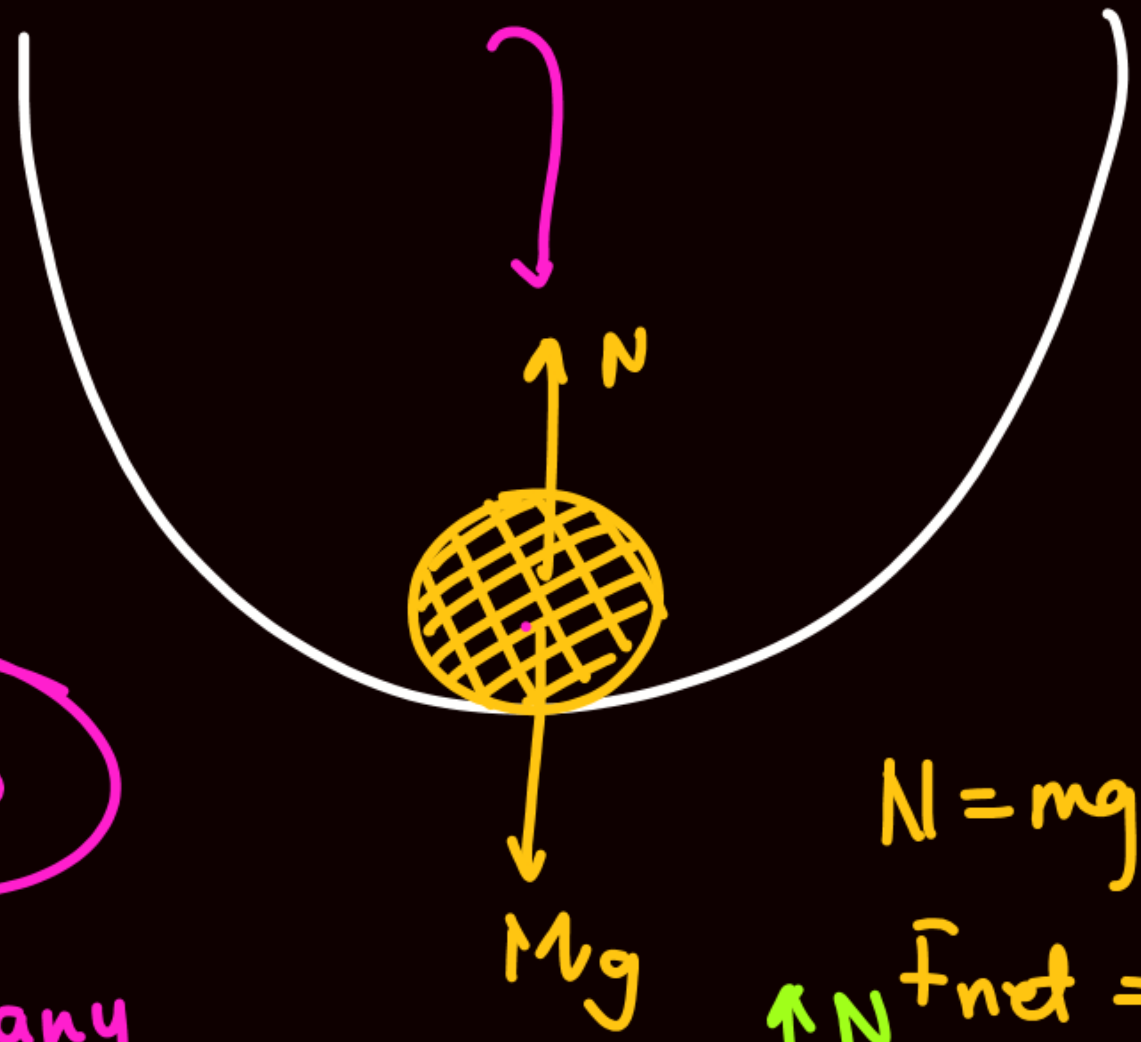
If we Know PE

① $\vec{F} = -\frac{\partial U}{\partial r} = 0$ → we will get those Points $F=0$



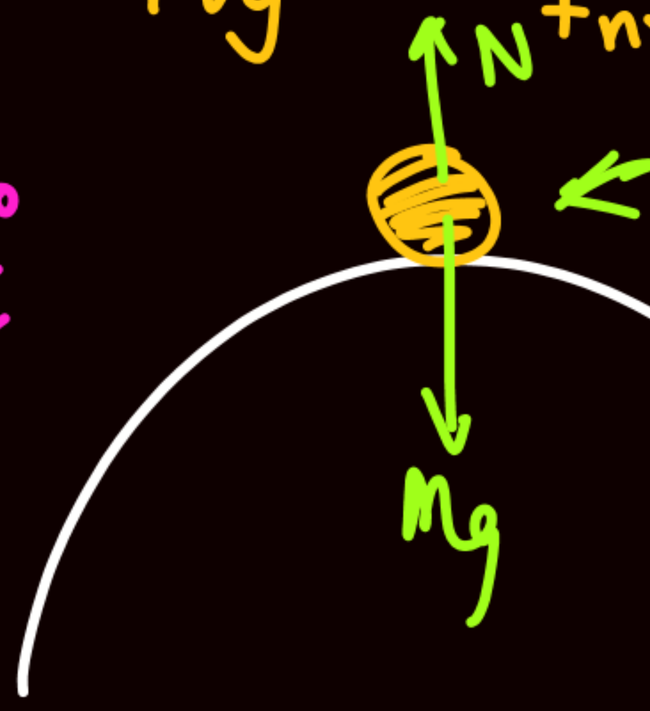
$f_{net} \neq 0$

Not in any Kind of Equili

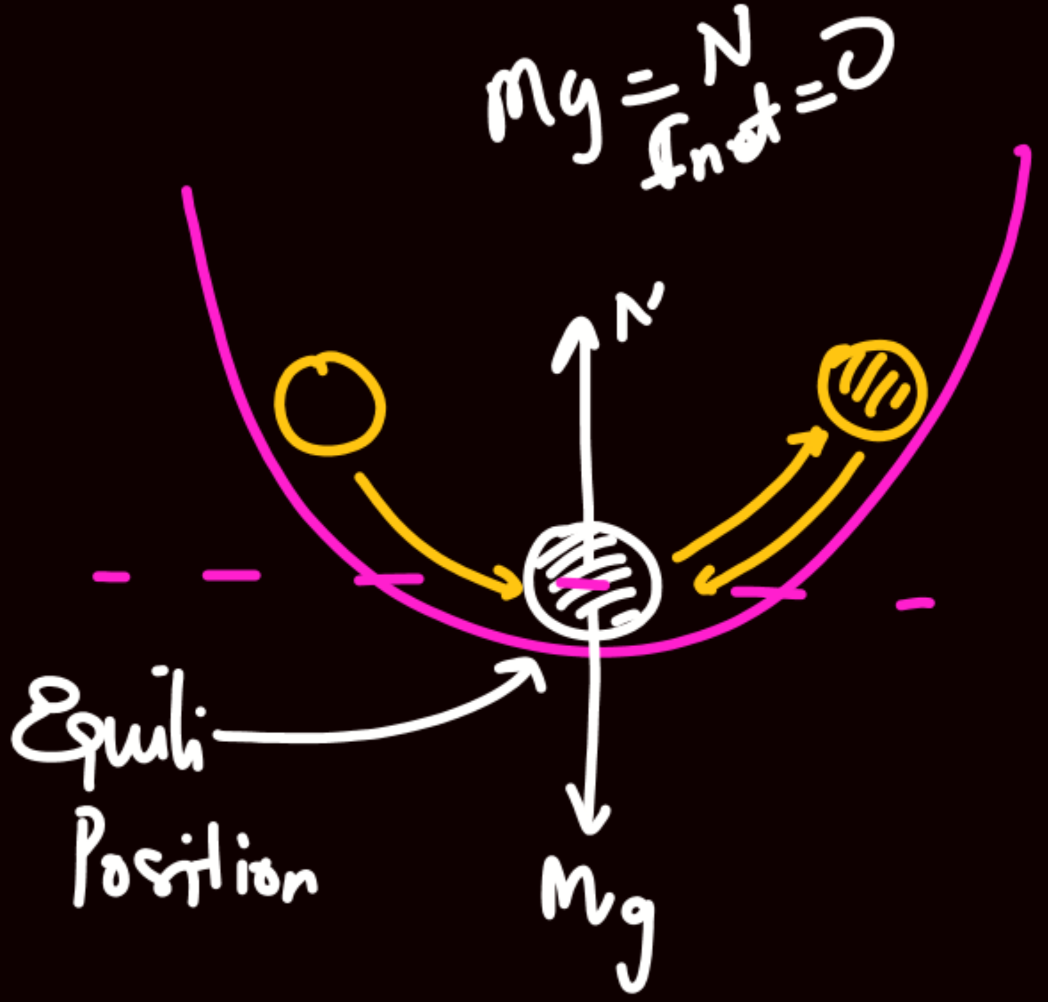


$N = mg$

$f_{net} =$ Equilli

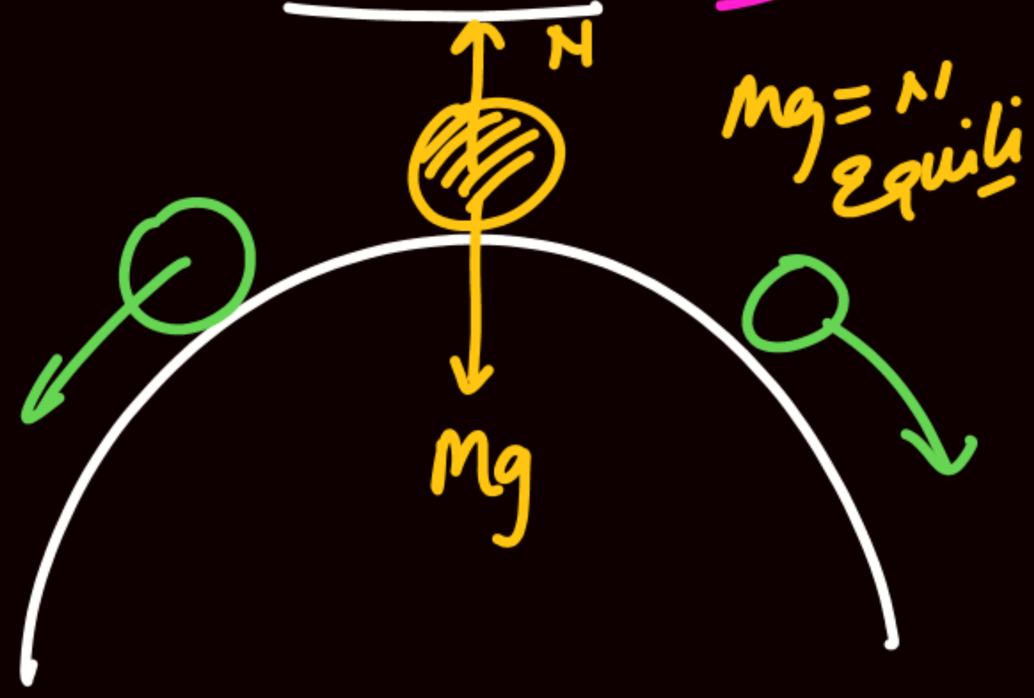


1 Stable (PE min)



When Particle tries to Reach back its original Position.

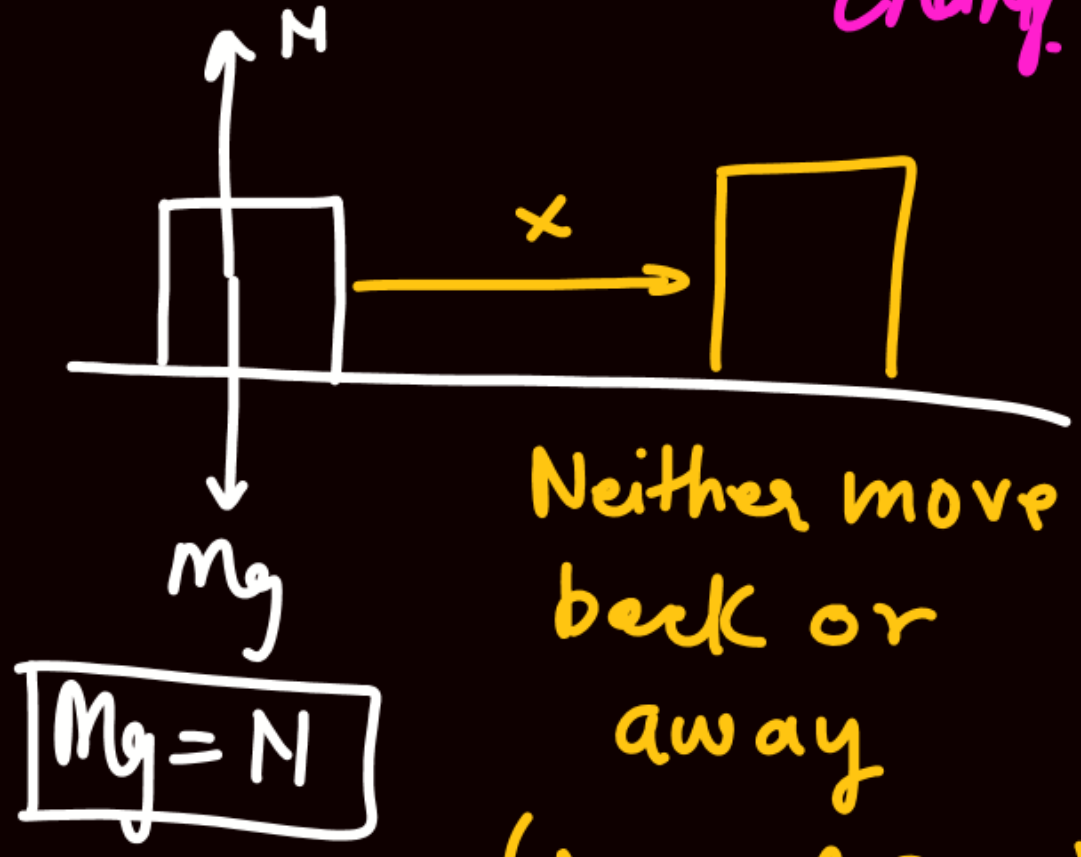
unstable = PE max



Block will move away from pt of Equili

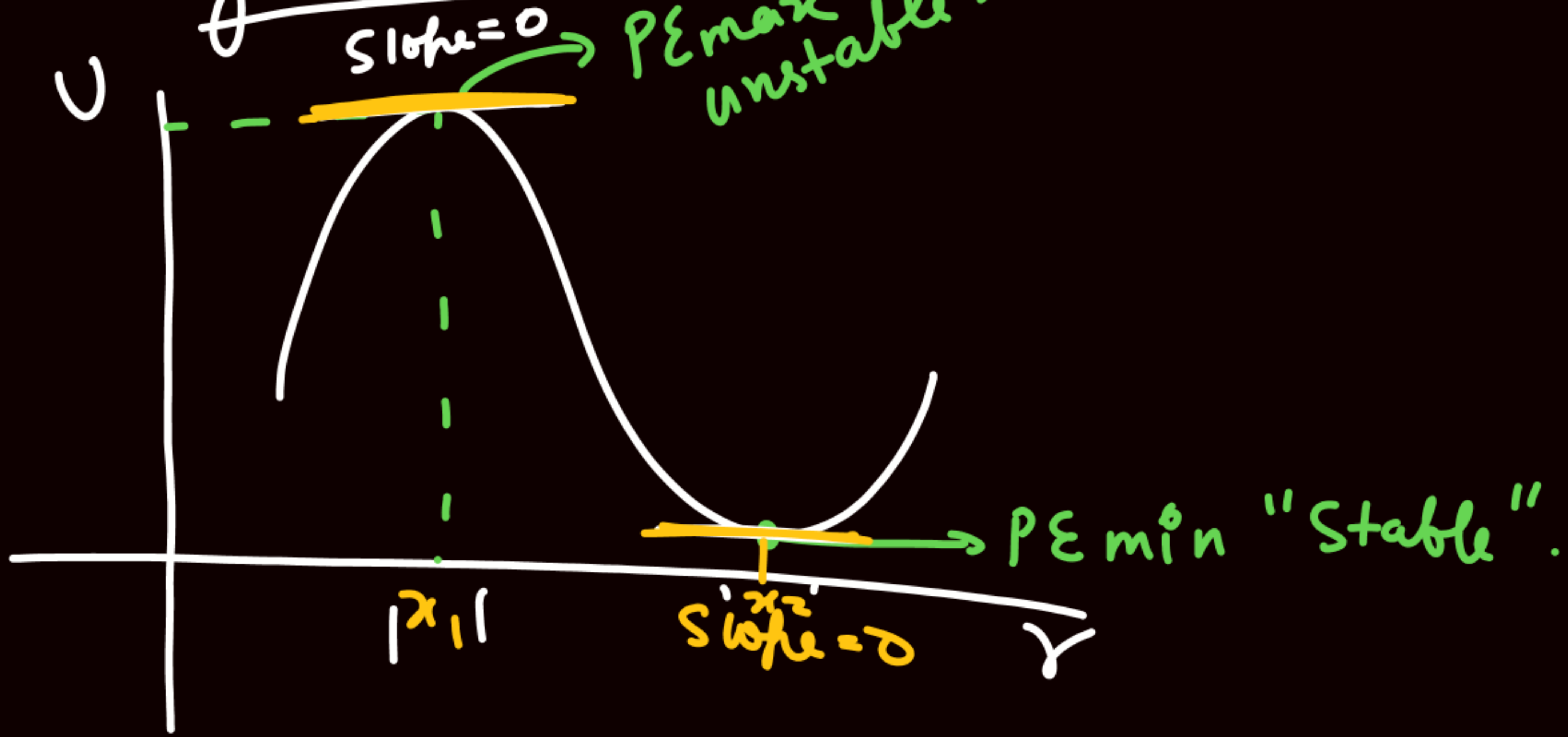
Neutral.

PE ⇒ donot chang.



Neither move back or away (Neutral Equi)

for Questions.



$$f = -\frac{\partial U}{\partial r}$$

$= -$ Slope of U/r.

$x_1, x_2 \rightarrow$ Equili points

If U_{max} at $x_1 \rightarrow$ unstable

U_{min} at $x_2 \rightarrow$ Stable.

Question

$U = f(r)$

① $f = -\frac{\partial U}{\partial r}$

② $f = 0$

$x_1, x_2 \rightarrow$ Critical Points.

③ $\frac{d^2U}{dr^2} > 0$ Min Stable

$\frac{d^2U}{dr^2} < 0$ Max unst

$\frac{d^2U}{\partial r^2} = 0$ Neutral

THANK YOU ☺

PHYSICS CRASH COURSE

WORK, ENERGY AND POWER

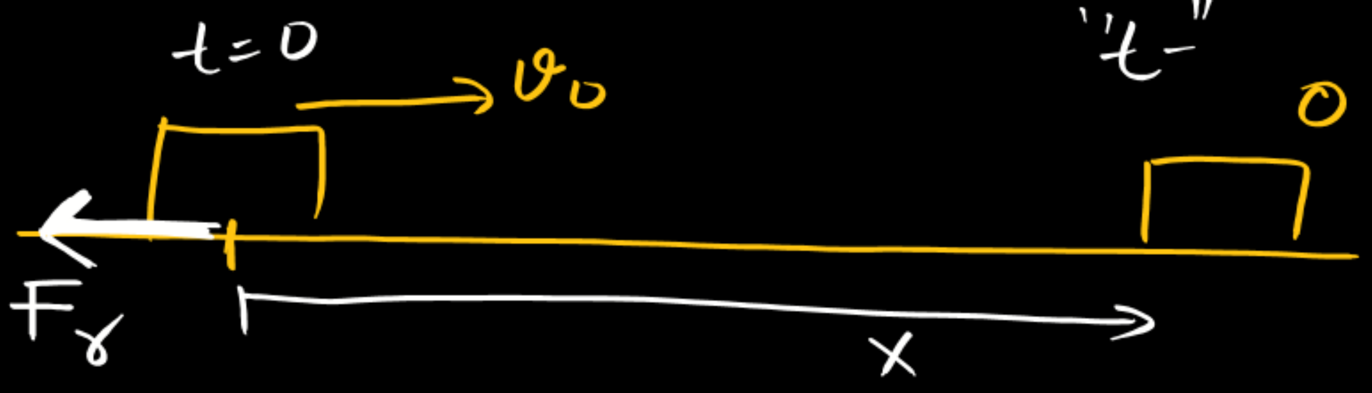
IN 1 SHOT 2

JEE Main & Advanced

Practice Sheet on PW App

Concept of Stopping Distance and Stopping Time

Concept



"WET" :-

$$W_c + W_n + W_e = K_2 - K_1$$

$$-f_r X = 0 - KE_i$$

$$\boxed{X = \frac{KE_i}{F}}$$

Impulse Momentum theorem

Stopping time

$$\boxed{\pm\sum Ft = P_2 - P_1}$$

(+) F increases velocity

(−) F dec velocity

$$-Ft = 0 - mv_0$$

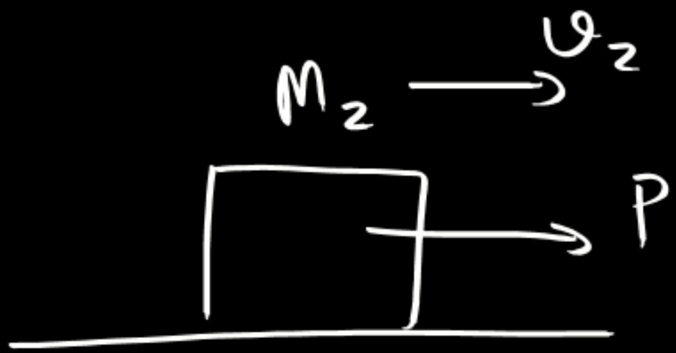
$$\boxed{t = \frac{P_i}{F_r}}$$

Ex: If two cars M_1 and M_2 having same momentum are having same Retarding Force. Find ratio of Stopping time and Stopping distance.

X

$$X = \frac{KE_i}{F} \qquad X_1 = \frac{P^2}{2m_1 F}$$

$$t_1 = \frac{P}{F} = \frac{P}{F}$$



$$X_2 = \frac{P^2}{2m_2 F}$$

$$t_2 = \frac{P}{F}$$

$$P = |m_1 v_1| = |m_2 v_2|$$

$$KE = \frac{P^2}{2m}$$

$$\boxed{\frac{X_1}{X_2} = \frac{m_2}{m_1}}$$

$$\frac{t_1}{t_2} = \frac{1}{1}$$

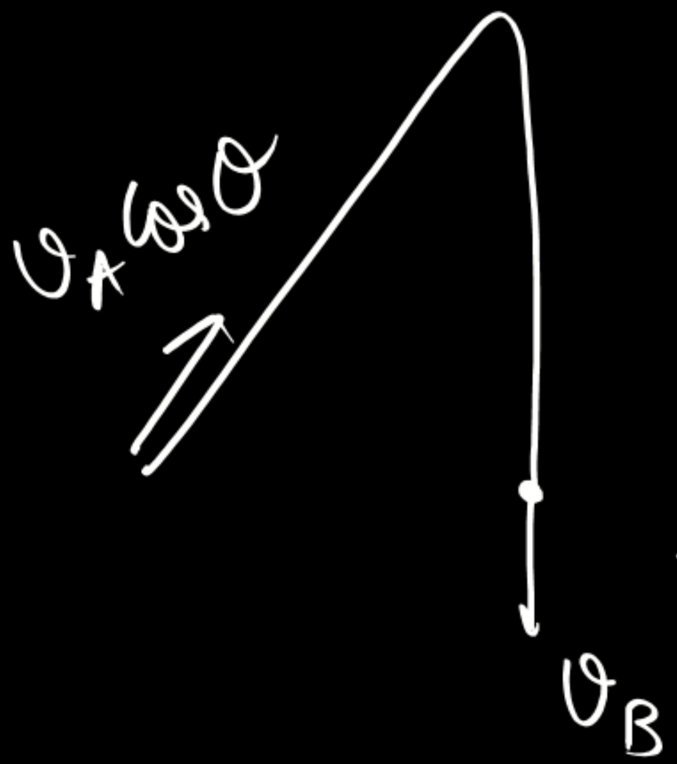
A block A of mass m is held at rest on a smooth horizontal floor. A smooth light pulley is fixed at a height 6 m above the ground over which a string of length 16 m connected to A as shown in figure. On the other end of string a block B of same mass m connected and hanging at a height of 5 m above floor. If system is released, find the speed at which block B hits the floor. (2006)

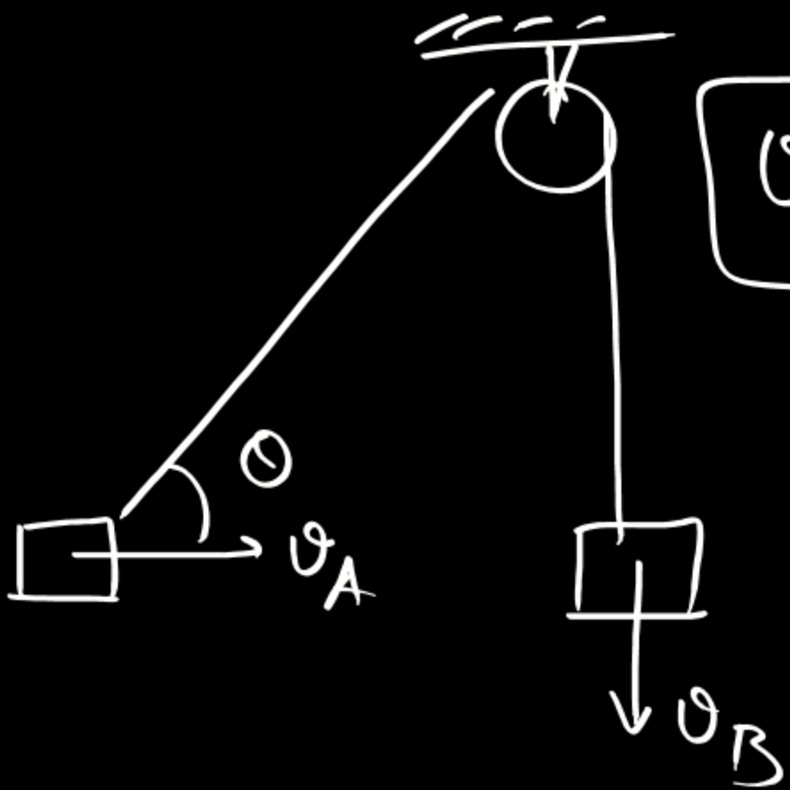
(a) $40/\sqrt{41}$ Ans

(b) $1/\sqrt{41}$

(c) $20/\sqrt{41}$

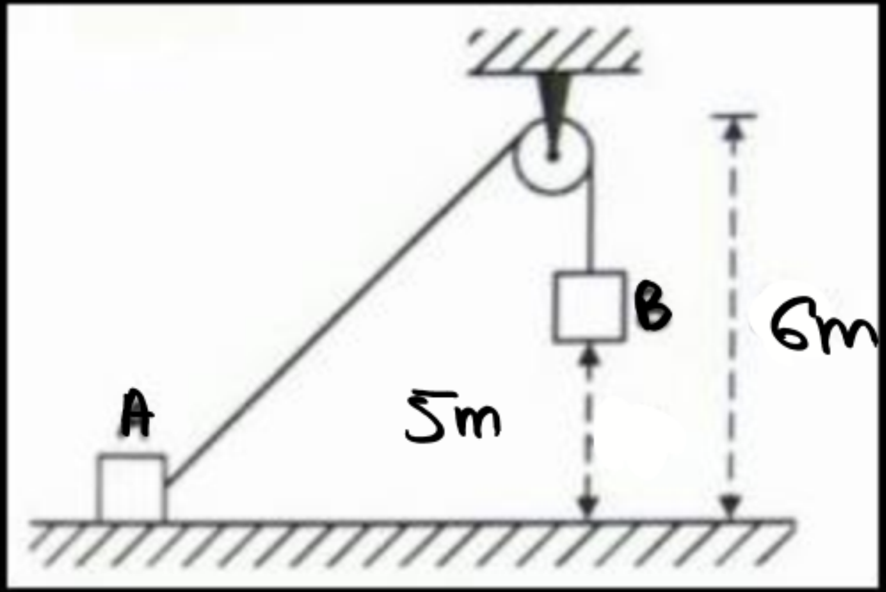
(d) none of these

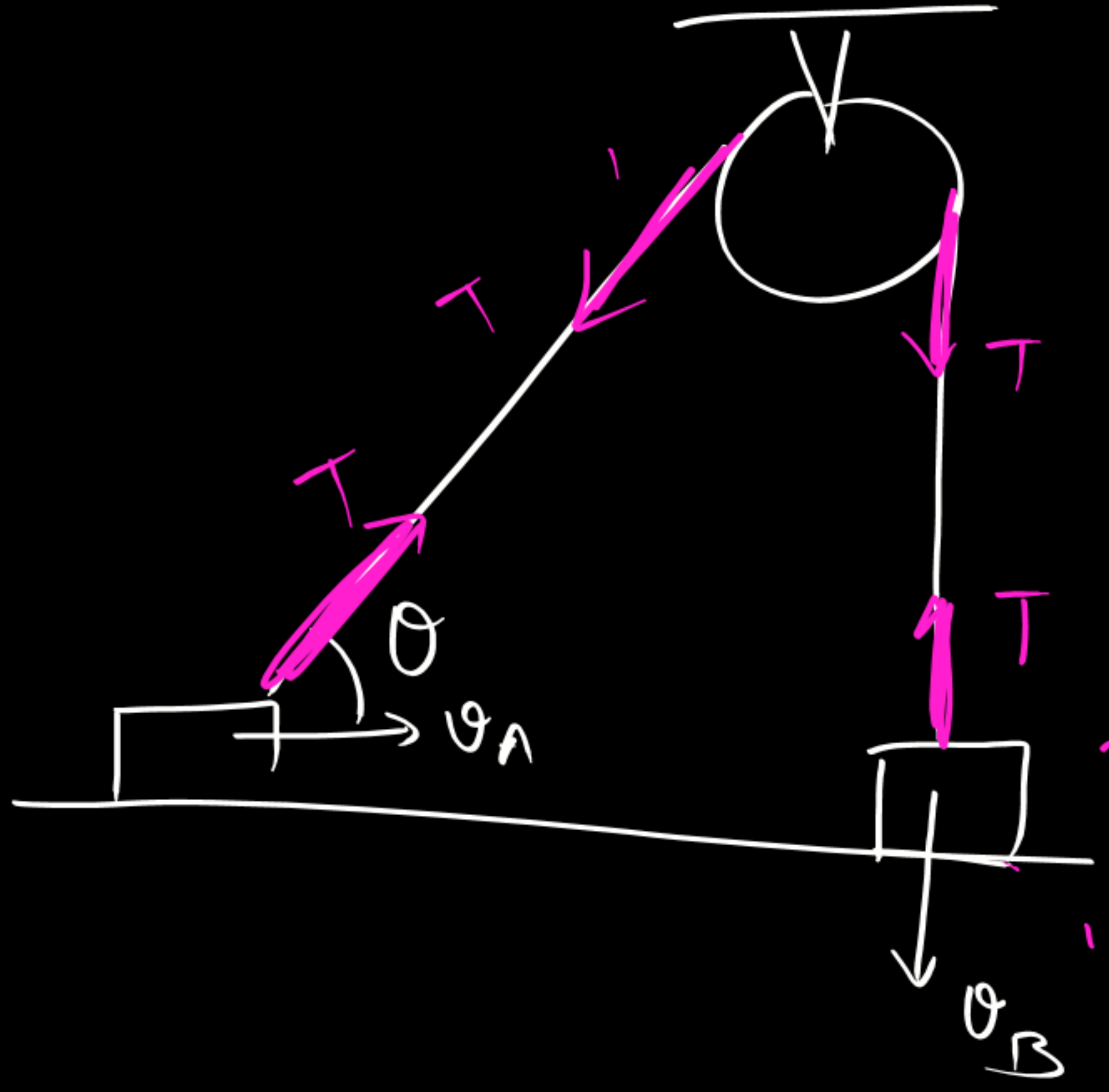




$$\boxed{v_B = v_A \cos\theta}$$

$l = 16m$





Container

$\Sigma F_{int} = 0$

$W_{int} = 0$

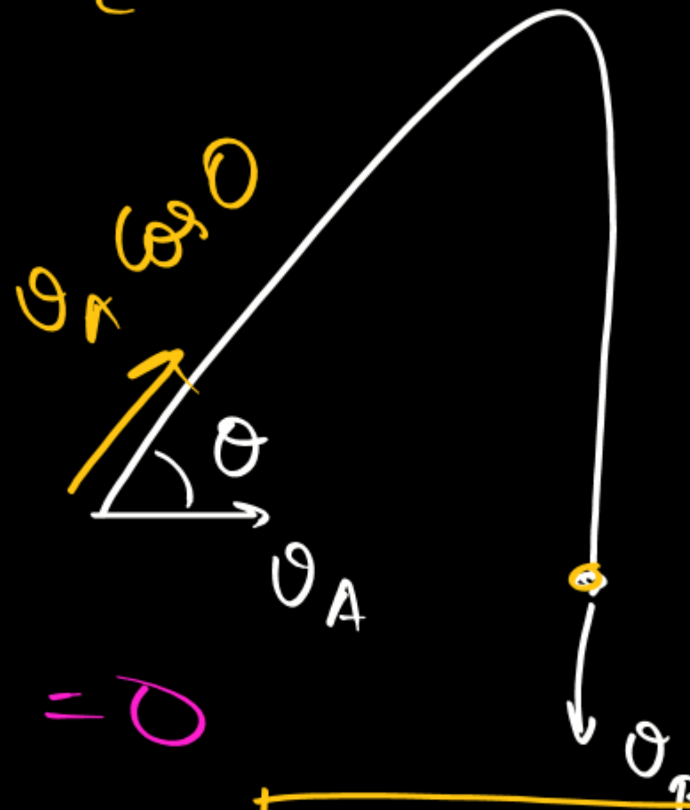
$P_{int} = \frac{W}{t} = \vec{F}\cdot\vec{v}$

$\boxed{\Sigma P_{int} = 0}$

$T v_B \cos 180 + T v_A \cos\theta = 0$

$-v_B + v_A \cos\theta = 0$

$v_B = v_A \cos\theta$



Length of string = Const

$\boxed{v_B = v_A \cos\theta}$

$$l_{string} = 16m$$

$$v_B = v_A \cos\theta$$

$$v_B = v_A \frac{\cancel{8}\,4}{\cancel{10}\,5}$$

$$v_A = \frac{5v_B}{4}$$

$$W_c + \overset{\times}{W_n} + \overset{\times}{W_e} = K_2 - K_1$$

$$+mg5 = \left(\frac{1}{2}mv_B^2 + \frac{1}{2}mv_A^2\right) - 0$$

$$\cancel{m}g5 = \frac{1}{2}\cancel{m}\left(v_B^2 + \frac{25v_B^2}{16}\right)$$

$$100 = \frac{(25+16)}{16}v_B^2$$

$$\sqrt{\frac{1600}{41}} = v_B = \frac{40}{\sqrt{41}}$$

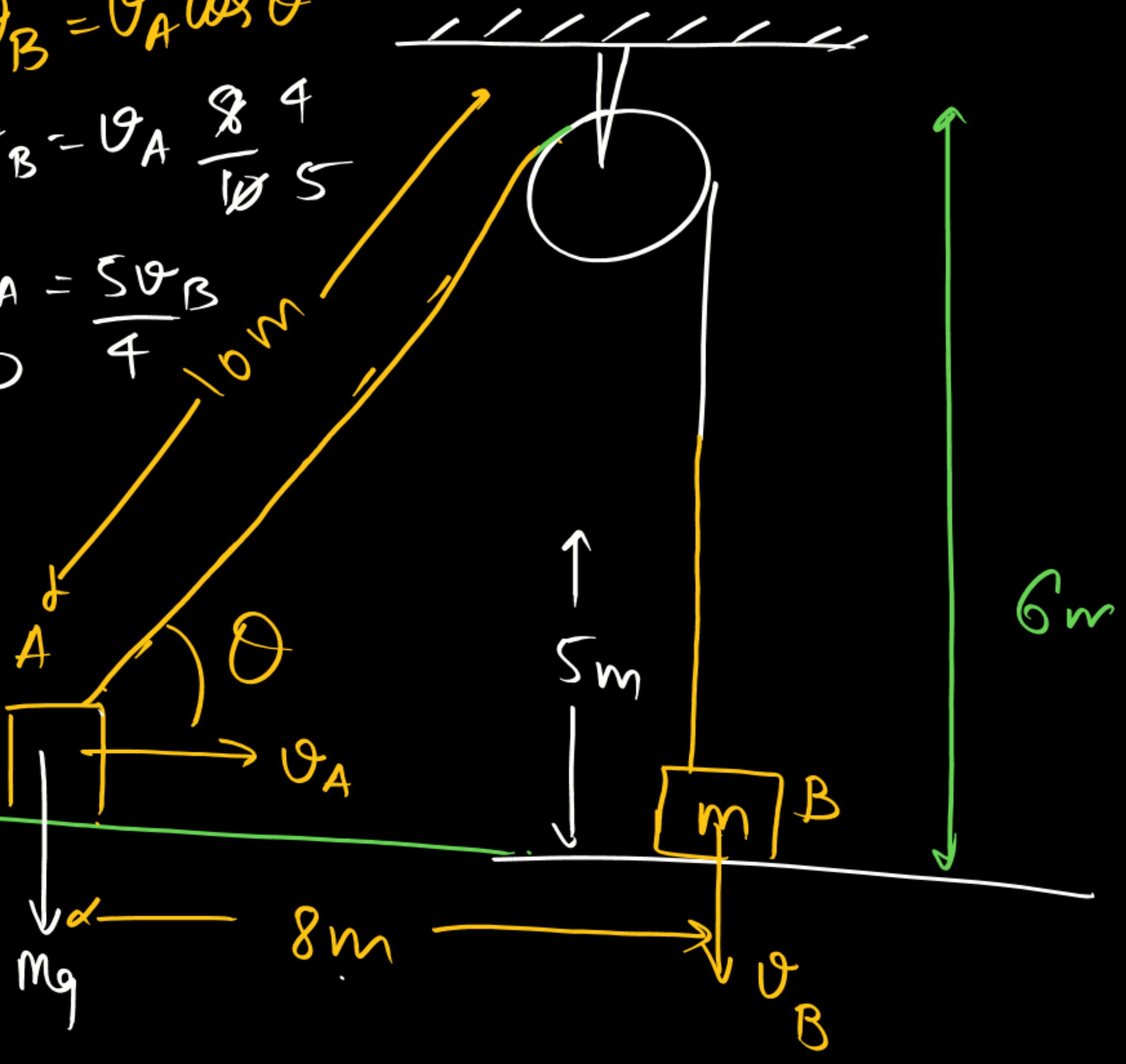
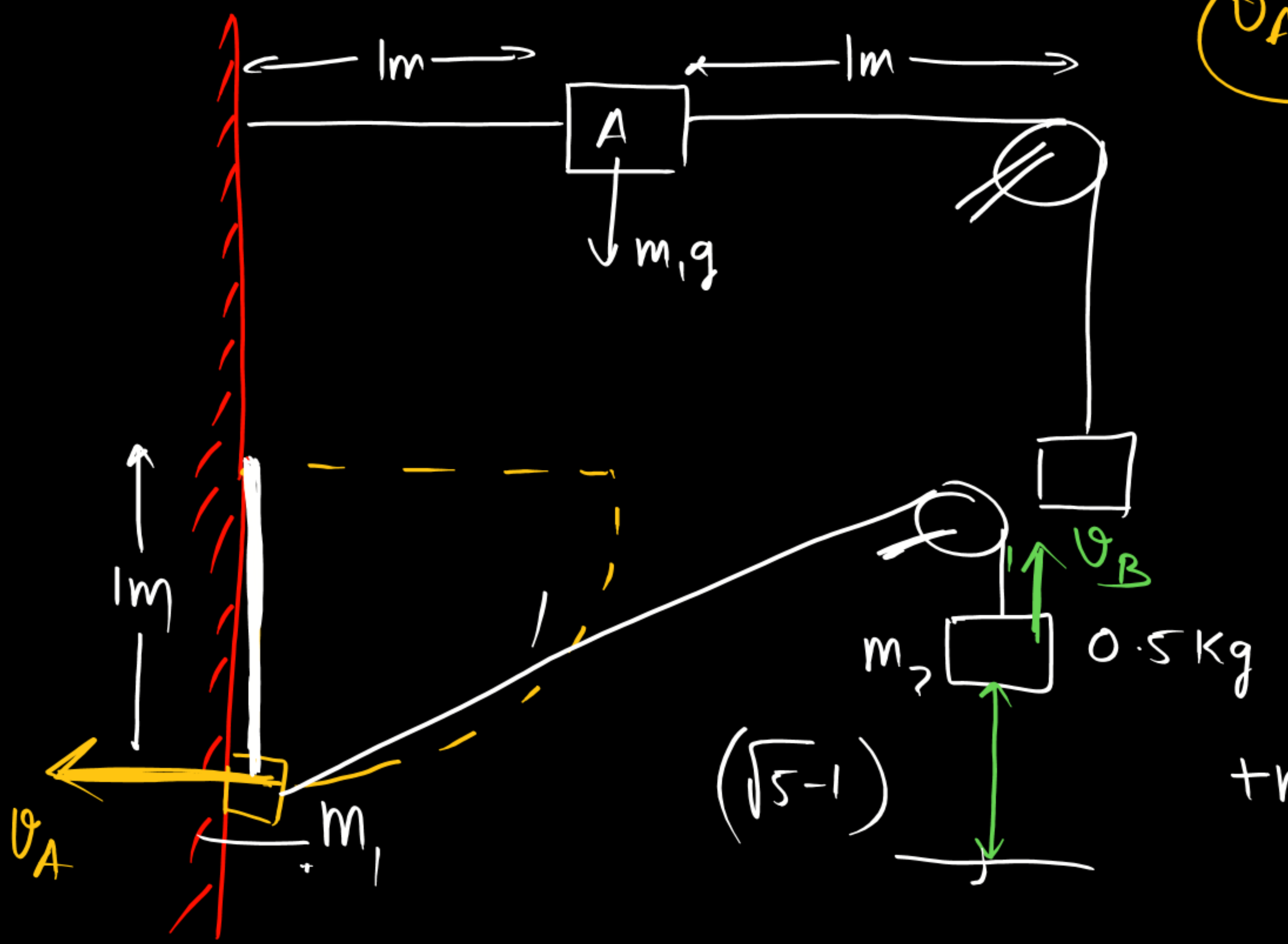
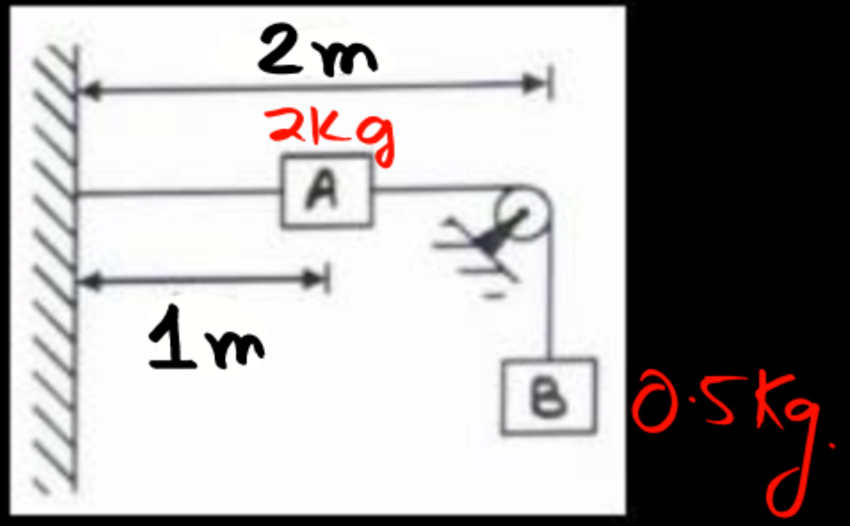


Figure shows a system held at rest. The masses of block A and Bare 2 kg and 0.5 kg. If the system is released, find with what speed block A will hit the wall. (1998)

$v_A = ?$

Ans: 3.4 m/s





WET

$$W_c + W_n + W_e = K_2 - K_1$$

$$+m_1 g(1) - m_2 g(\sqrt{5}-1) = \frac{1}{2} m_1 v_A^2 + \frac{1}{2} m_2 v_B^2 - 0$$

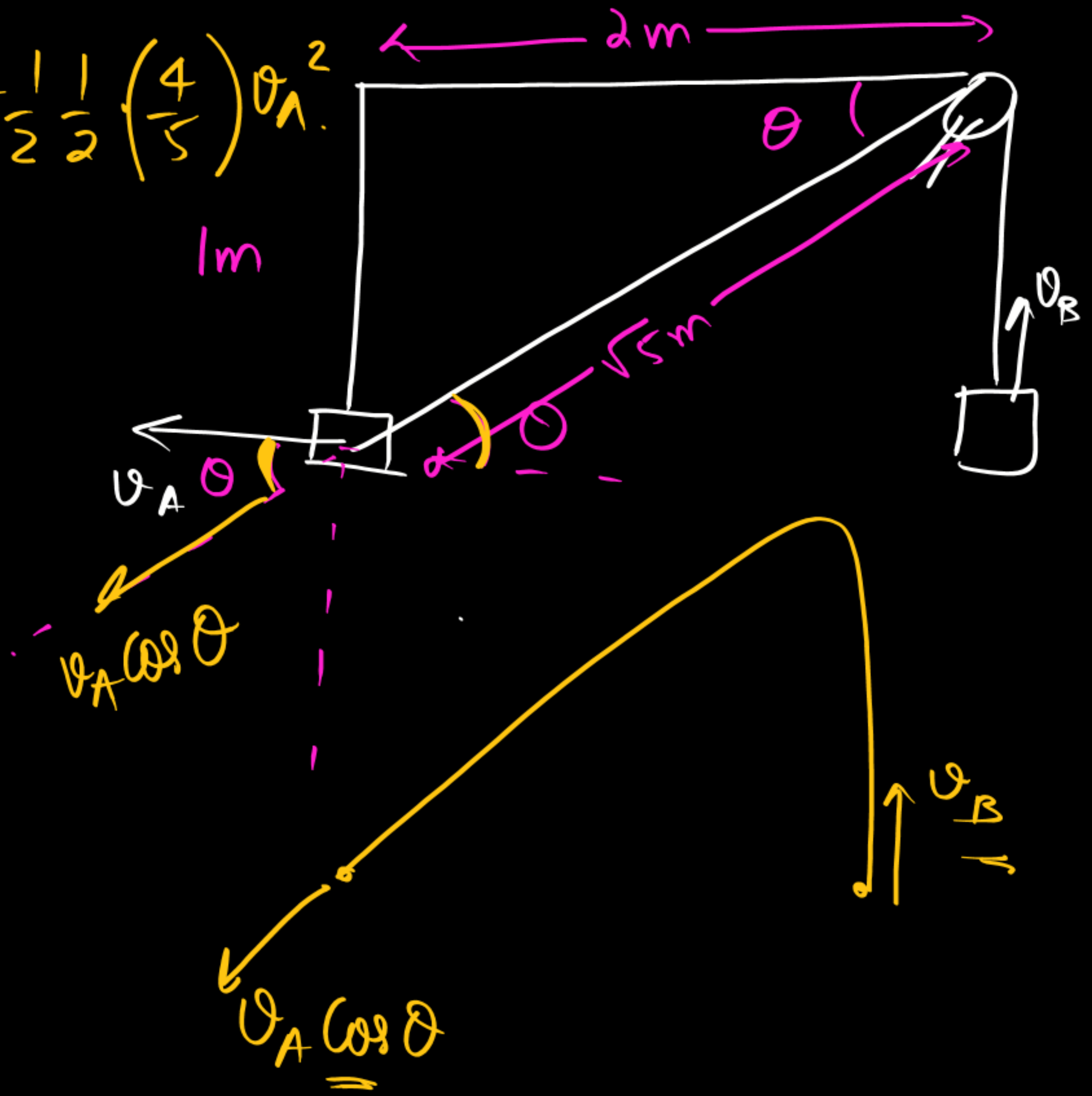
$$m_1 g(1) - m_2 g(\sqrt{5}-1) = \frac{1}{2} m_1 v_A^2 + \frac{1}{2} m_2 v_B^2$$

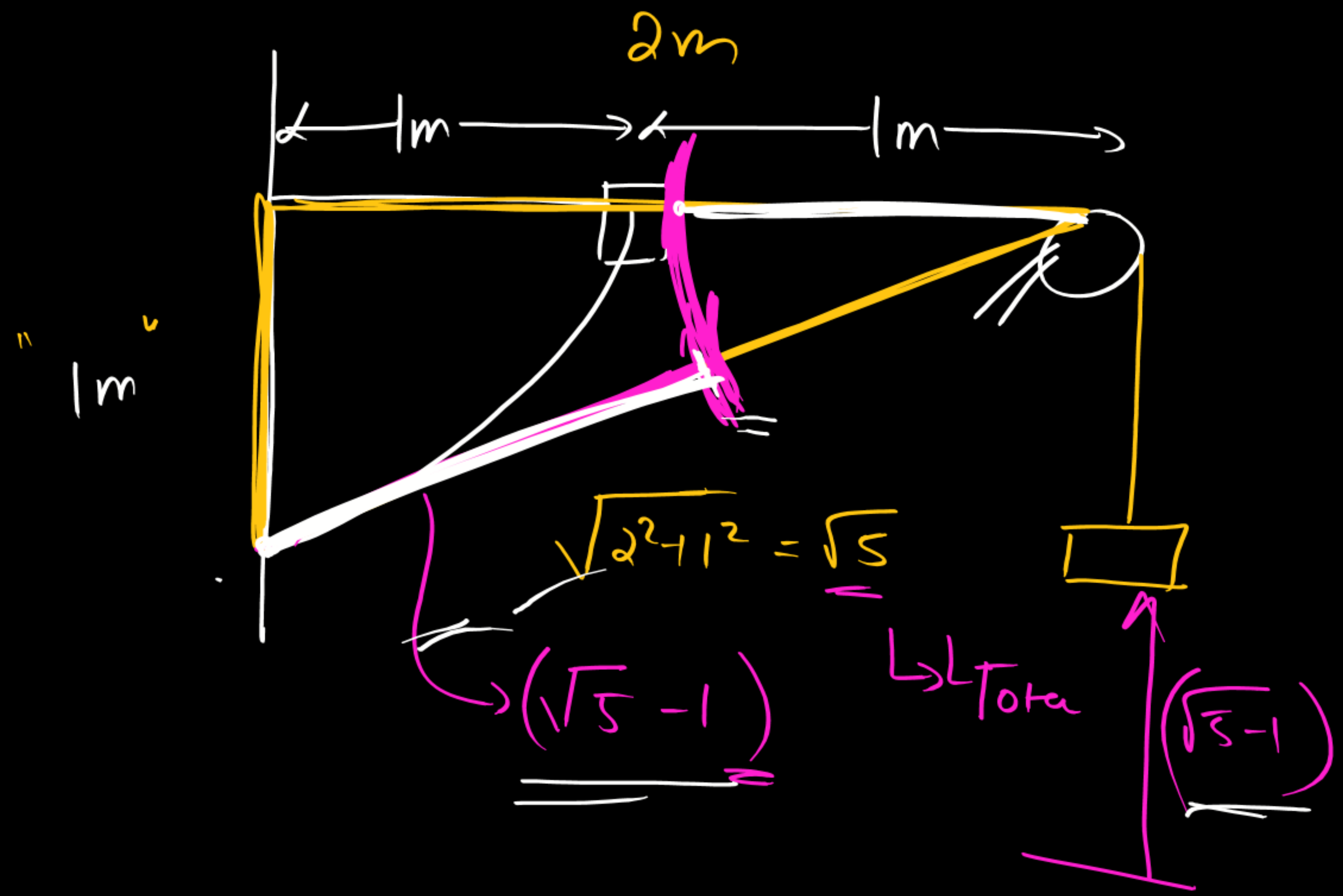
$$v_A \cos\theta = v_B$$

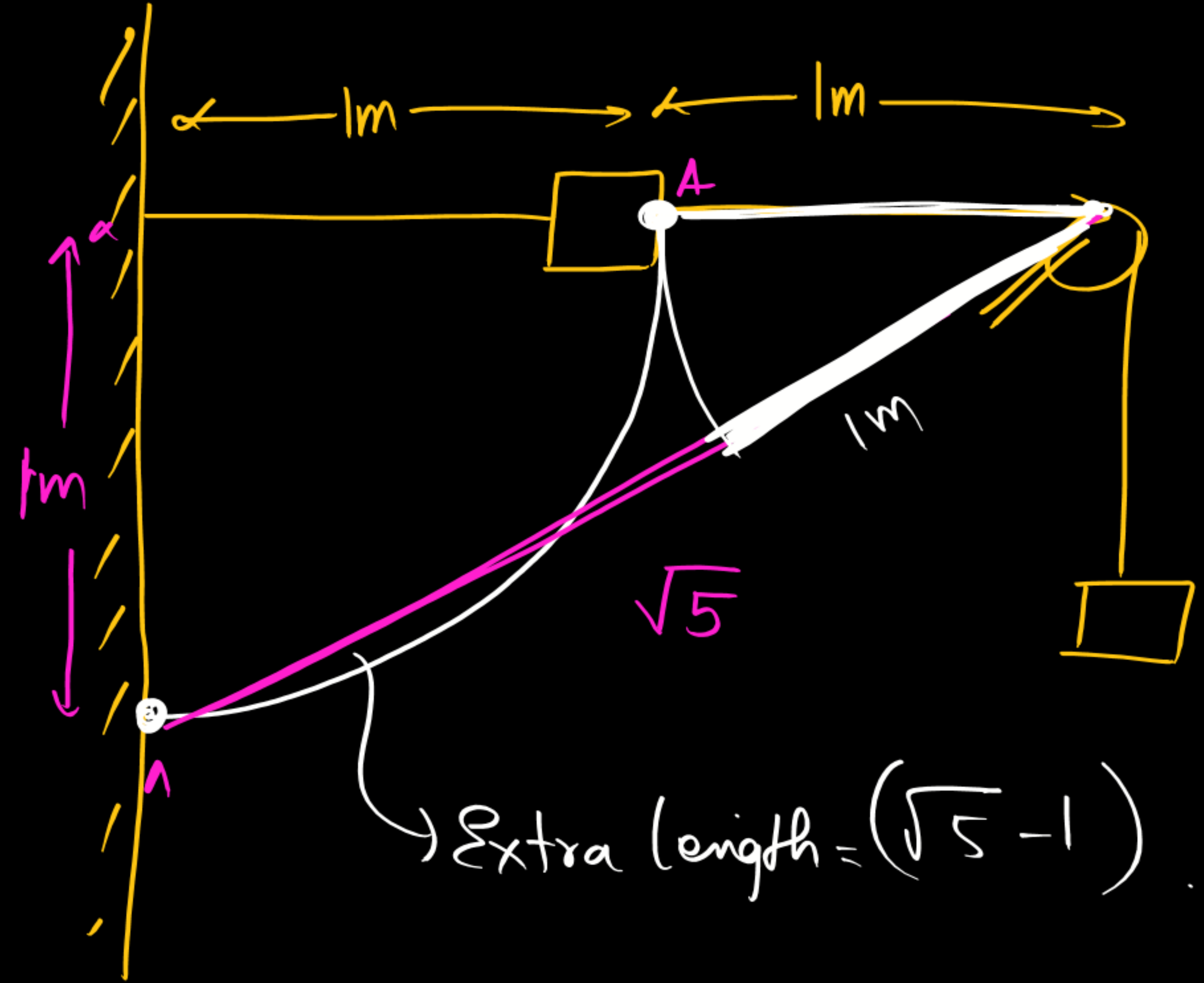
$$v_A \frac{2}{\sqrt{5}} = v_B$$

$$2\times 10 - \frac{1}{2}\times 10(\sqrt{5}-1) = \frac{1}{2} 2 v_A^2 + \frac{1}{2}\frac{1}{2}\left(\frac{4}{5}\right) v_A^2$$

$$v_A = 3.4\, m/s$$




2m
1m
1m
"1m"
$\sqrt{2^2+1^2} = \sqrt{5}$
$(\sqrt{5} - 1)$
L Total
$(\sqrt{5} - 1)$



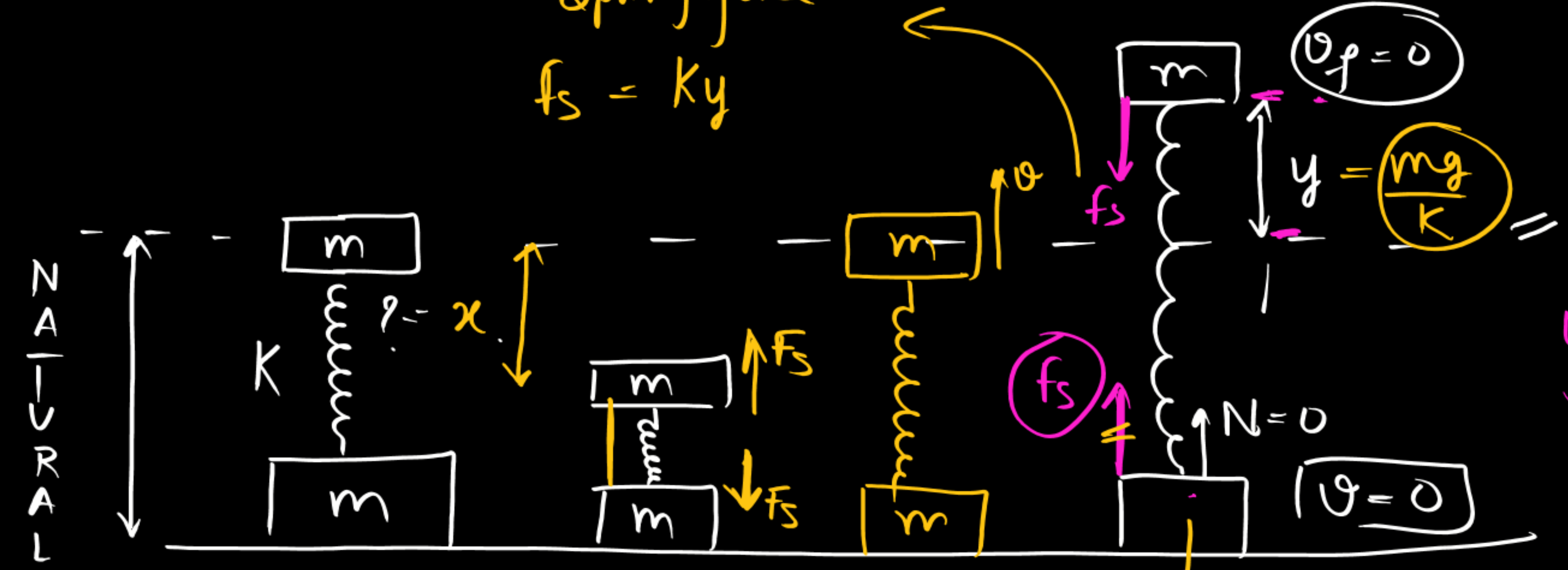
Extra length = $(\sqrt{5}-1)$

Figure shows two identical cubes each of m connected by a light spring of force constant k. The spring is initially compressed and the cubes are connected by a thread with spring compressed. Find the initial compression so that on burning the thread, finally lower cube will break off from ground. (2005)

Ans:3mg/k

Spring force

$f_s = Ky$



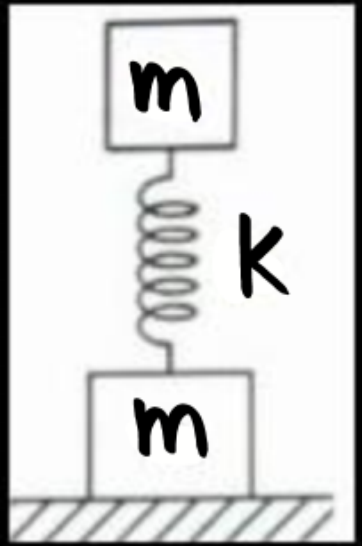
This block will loose Contact

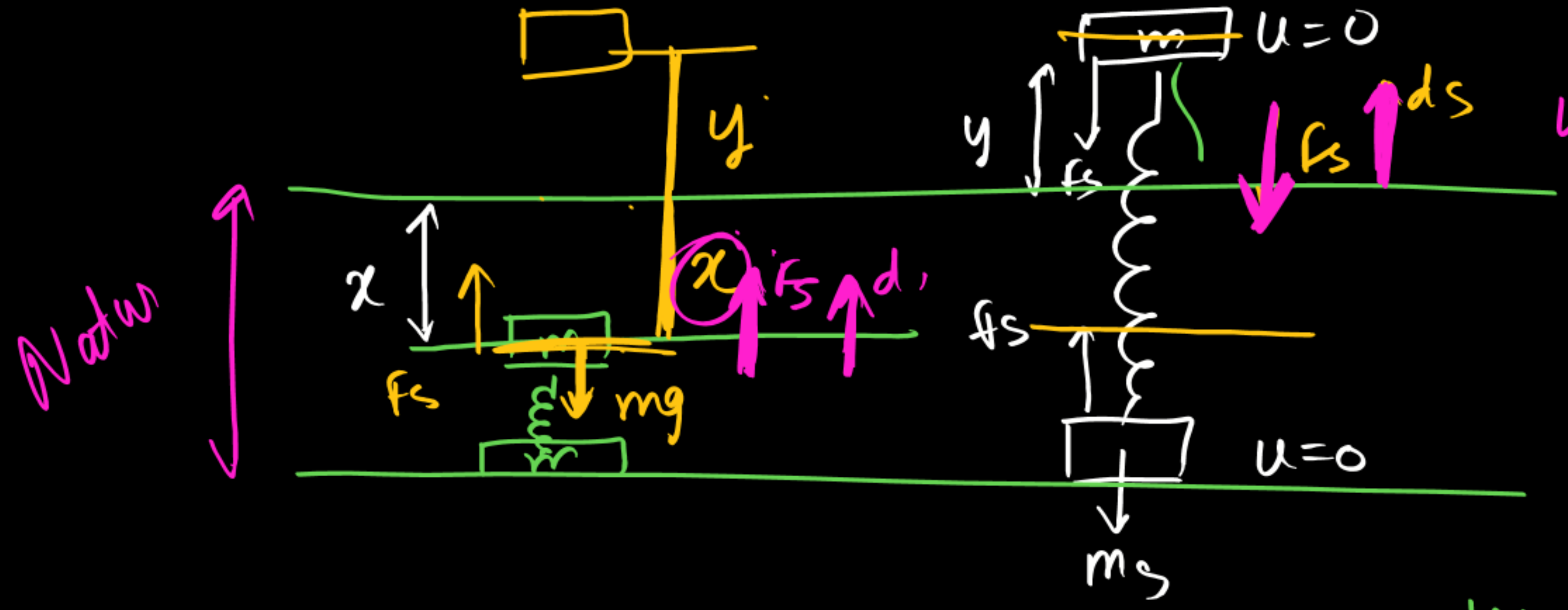
$F_s = mg = Ky$ $\boxed{y = \frac{mg}{K}}$

$y = \frac{x}{3}$

$x = 3y$

$= \frac{3mg}{K}$





$w = -\frac{1}{2}Ky^2$

$Ky = mg$

This Should Min Elongation for lower block to lift up from ground. $\rightarrow y = \frac{mg}{K}$

$W_c + W_n + W_e = K_2 - K_1$

$-mg(x+y) + \frac{1}{2}Kx^2 - \frac{1}{2}Ky^2 = 0 - 0$

we Know $y = \frac{mg}{K}$

$-mg\left(x + \frac{mg}{K}\right) + \frac{1}{2}Kx^2 - \frac{1}{2}K\frac{m^2g^2}{K^2} = 0$

$x =$ ______

$W_{gr} = mg(V_{dif})$

Potential Energy due to gravity and Spring

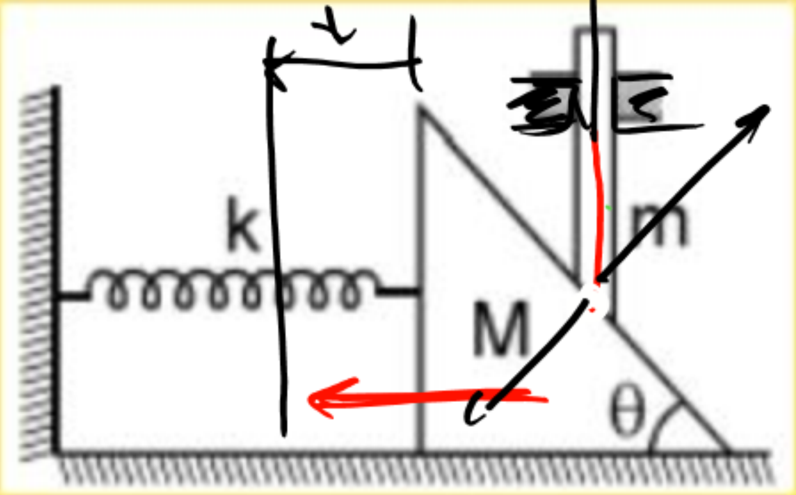
Concept

$$\boxed{\begin{aligned} W_{cons} &= -(U_f - U_i) \\ W_{ext} &= (U_f - U_i) \end{aligned}}$$

$W_{cons} = -(U_f - U_i)$ → This is always applicable.

$W_{ext} = (U_f - U_i)$ → This is only applicable whe $KE = 0$ through process.

A wedge of mass M fitted with a spring of stiffness 'K' is kept on a smooth horizontal surface. A rod of mass m is kept on the wedge as shown in the figure. System is in equilibrium. Assuming that all surfaces are smooth, the potential energy stored in the spring is : **(2019)**



(a) $\dfrac{mg^2\tan^2\theta}{2K}$

(b) $\dfrac{m^2g\tan^2\theta}{2K}$

(c) $\dfrac{m^2g^2\tan^2\theta}{2K}$ Ans

(d) $\dfrac{m^2g^2\tan^2\theta}{K}$

Bigger Wedge is at Rest

$Kx = N'\sin\theta$

$mg = N'\cos\theta$

$\dfrac{Kx}{mg} = \tan\theta$

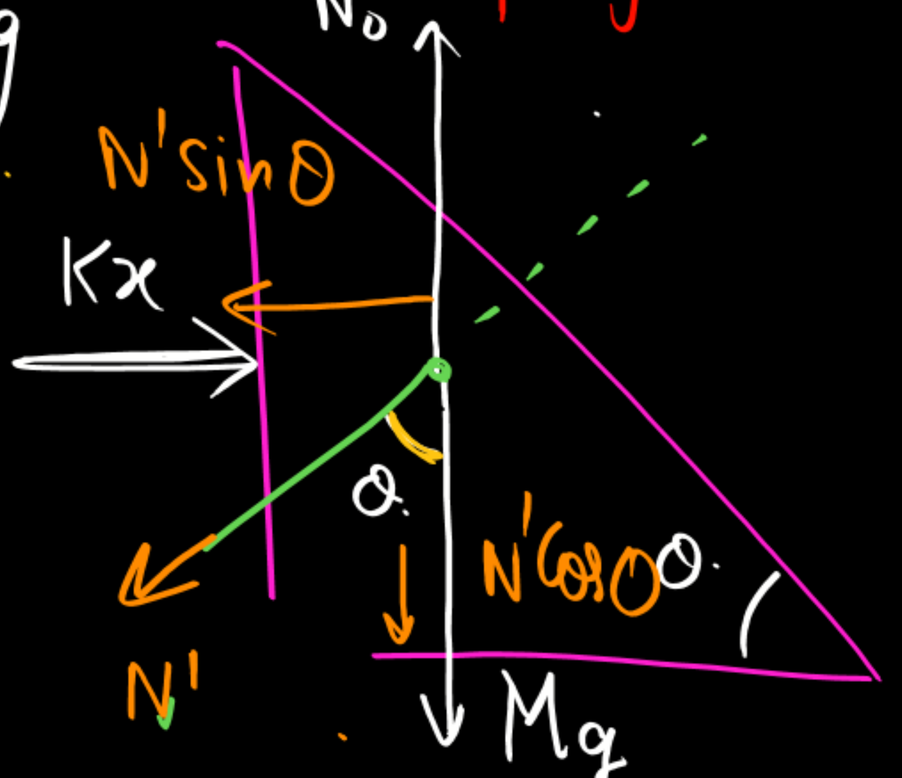
$x = \dfrac{mg\tan\theta}{K}$

Energy is Spring

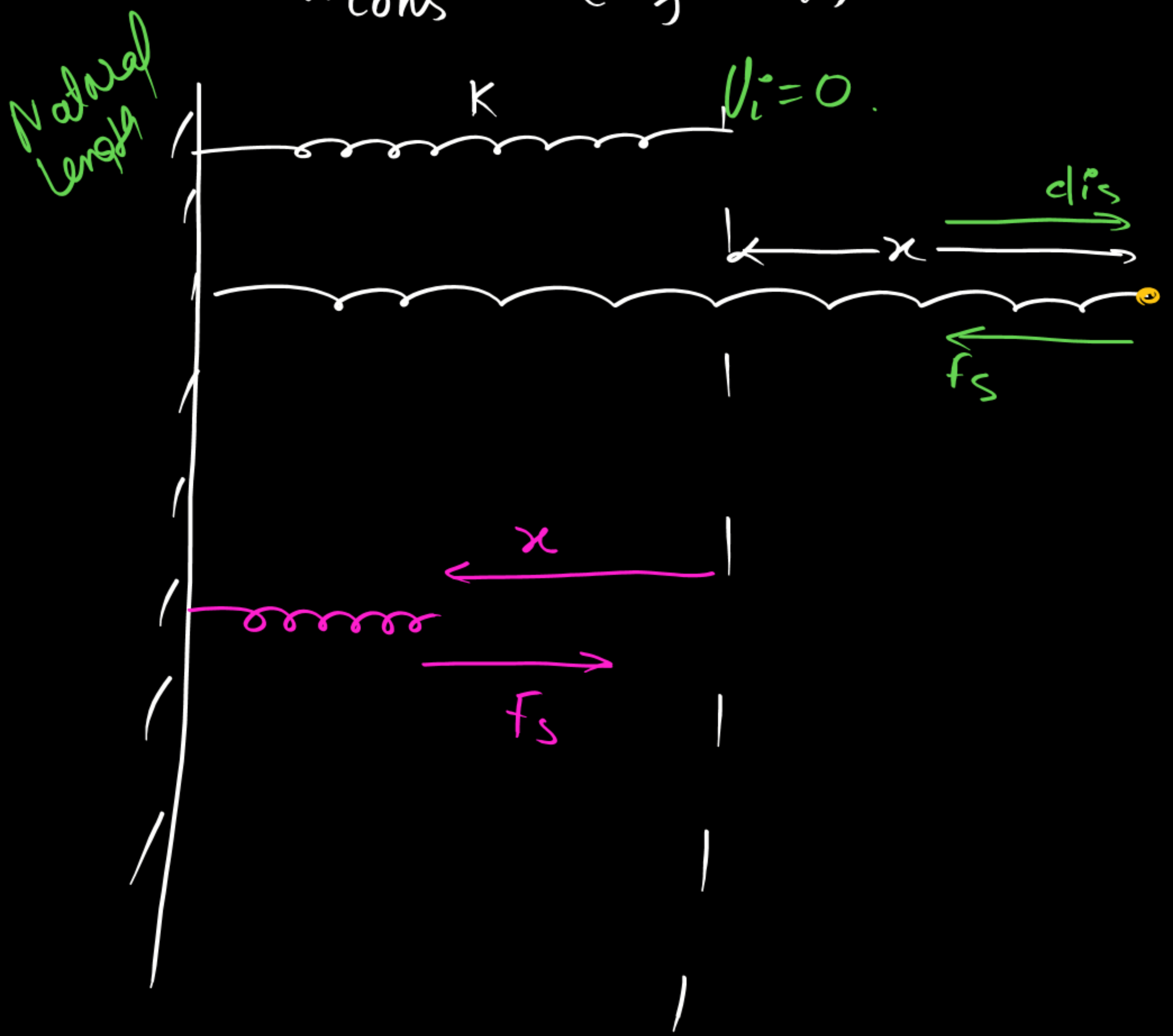
$E = \dfrac{1}{2}\dfrac{Km^2g^2\tan^2\theta}{K^2}$

$= \dfrac{m^2g^2\tan^2\theta}{2K}$

Then Spring will Compressed.



$$W_{cons} = -(U_f - \overset{0}{U_i})$$



$$W_{spring} = -\frac{1}{2}kx^2$$

$$\therefore W_c = -U_f$$

$$\boxed{PE = +\frac{1}{2}kx^2}$$

$$W_s = -\frac{1}{2}kx^2$$

$$\boxed{PE = +\frac{1}{2}kx^2}$$

Mechanical Energy Conservation

Concept

Relation between Force and PE

In Electrostatics $= \vec{E} = -\frac{\partial V}{\partial r}$

Concept Revision :- (gradient)

$\vec{F} = -\frac{\partial U}{\partial r}$

PE

Scalar

Vector

Que. PU = Given

f = asked.

$\vec{F} = F_x\hat{i} + f_y\hat{j} + f_z\hat{k}$

$f_x = -\frac{\partial U}{\partial x}\Big|_{y,z=\text{const}}$

$f_y = -\frac{\partial U}{\partial y}\Big|_{x,z=\text{Const}}$

$f_z = -\frac{\partial U}{\partial z}\Big|_{x,y=\text{Const}}$

graphically

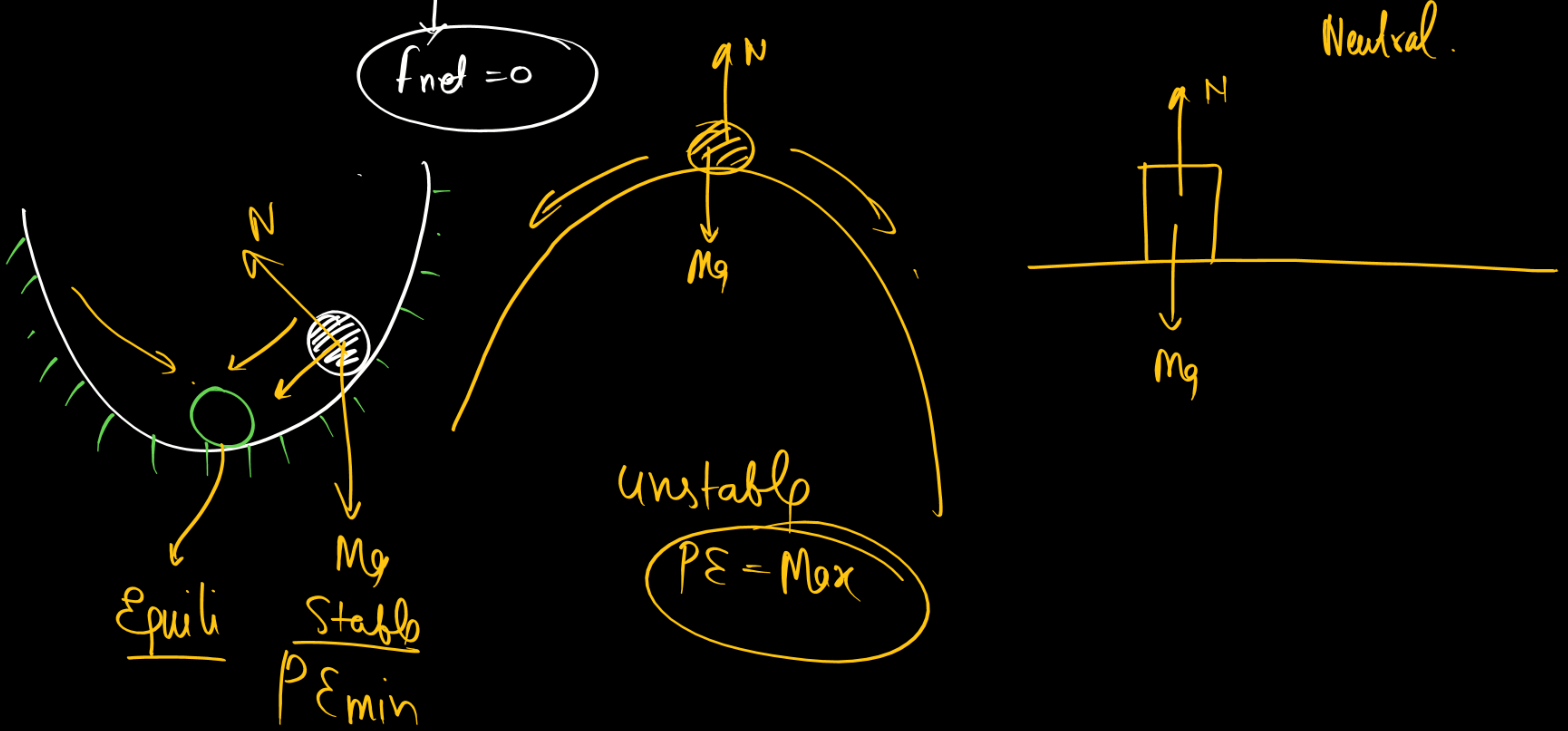
slope of U/r

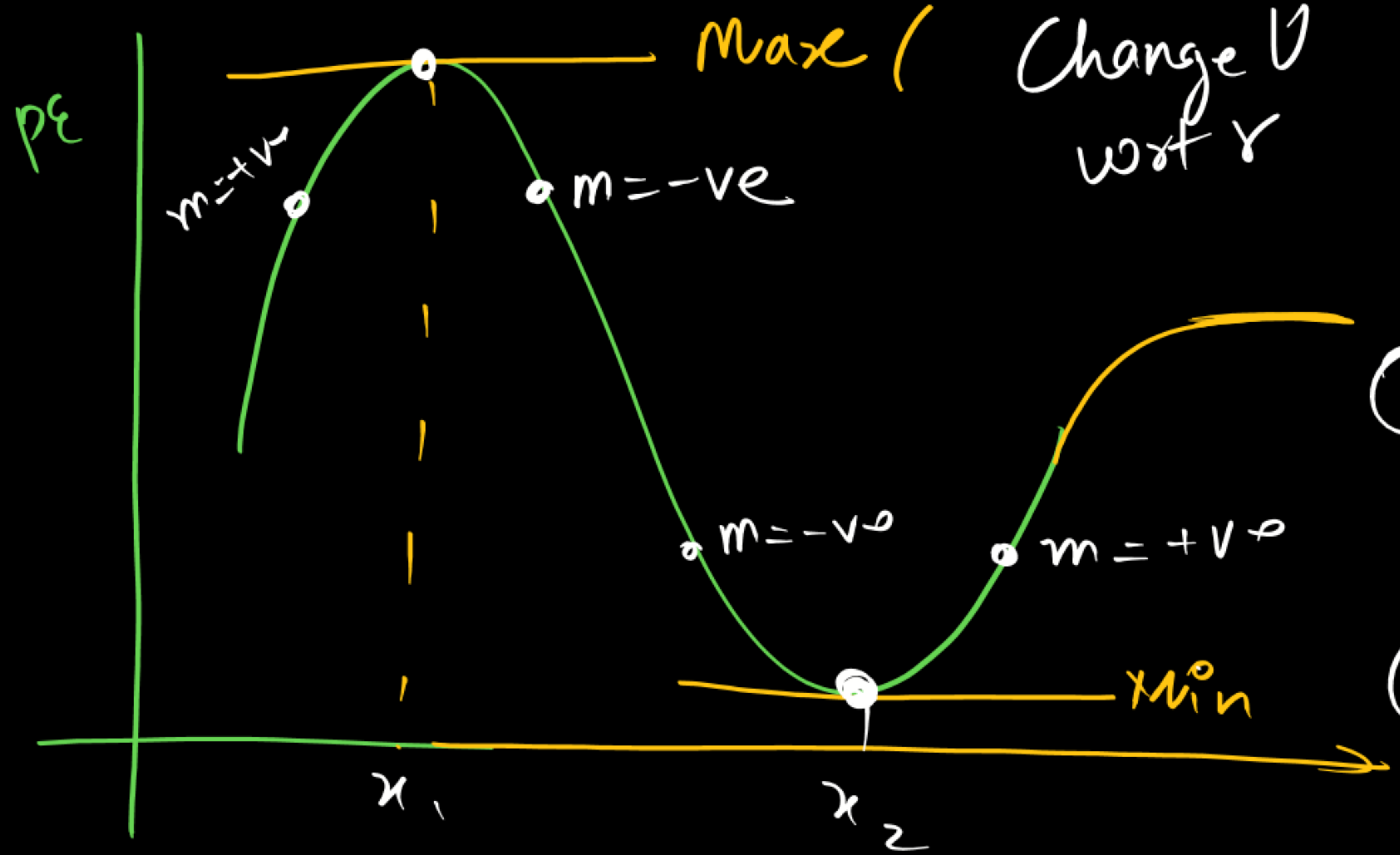
→ − force.

Que $\int \vec{F}\cdot\vec{dr} = -\int dU$?

graphically − Area under f/r $\Rightarrow -(U_2 - U_1)$

Concept Of Equilibrium





Change U wrt r

① $\frac{dU}{\partial r}$ {we have Calculated Slope

② $\frac{dU}{\partial r} = 0$ $\{x_1, x_2\}$ → Critical points

③ $\left.\frac{d^2U}{\partial r^2}\right|_{x_2} > 0$ Min (Stable)

$= \frac{d}{\partial r}\left(\frac{\partial U}{\partial r}\right)$ ← Change of Slope wrt r.

$\frac{d}{\partial r}(\text{Slope})$

$\left.\frac{d^2U}{\partial r^2}\right|_{x_1} < 0$ Max unstable

$\left.\frac{d^2U}{\partial r^2}\right| = 0$ Neutral

$-\frac{dU}{\partial r}$ = Slope.

Equil = f = 0

$-\frac{\partial U}{\partial r} = 0$

BREAK
TILL
20:47 PM

The potential energy of a conservative system is given by $U = ax^2 - bx$ where a and b are positive constants. Find the equilibrium position and discuss whether the equilibrium is stable, unstable or neutral.

$U = ax^2 - bx.$

Equilibrium $f = -\frac{\partial U}{\partial x} = 0$

$\frac{\partial U}{\partial x} = 2ax - b$

$f = 0 = -\frac{\partial U}{\partial x} = 0 = 2ax - b$

$x = \frac{b}{2a} = ?$

$x = \frac{b}{2a}$

$\frac{d^2U}{\partial x^2} = 2a > 0$

Min → Stable.

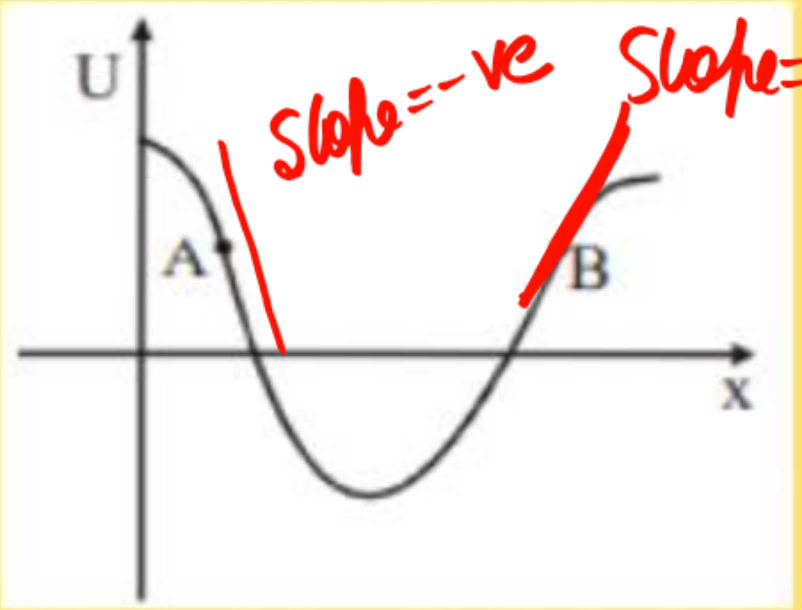
Potential energy v/s displacement curve for one dimensional conservative field is shown. Force at A and B is respectively.

(a) Positive, Positive

(b) Positive, Negative Ans (Repulsive, Attractive)

(c) Negative, Positive

(d) Negative, Negative



$$\text{force} = -\frac{\partial U}{dx}$$

$$= -\text{Slope}.$$

$\text{force} = +ve$

A

$\text{force}_B = -ve$

$+ \quad -$

$$f = -\frac{Kq_1q_2}{r^2}$$

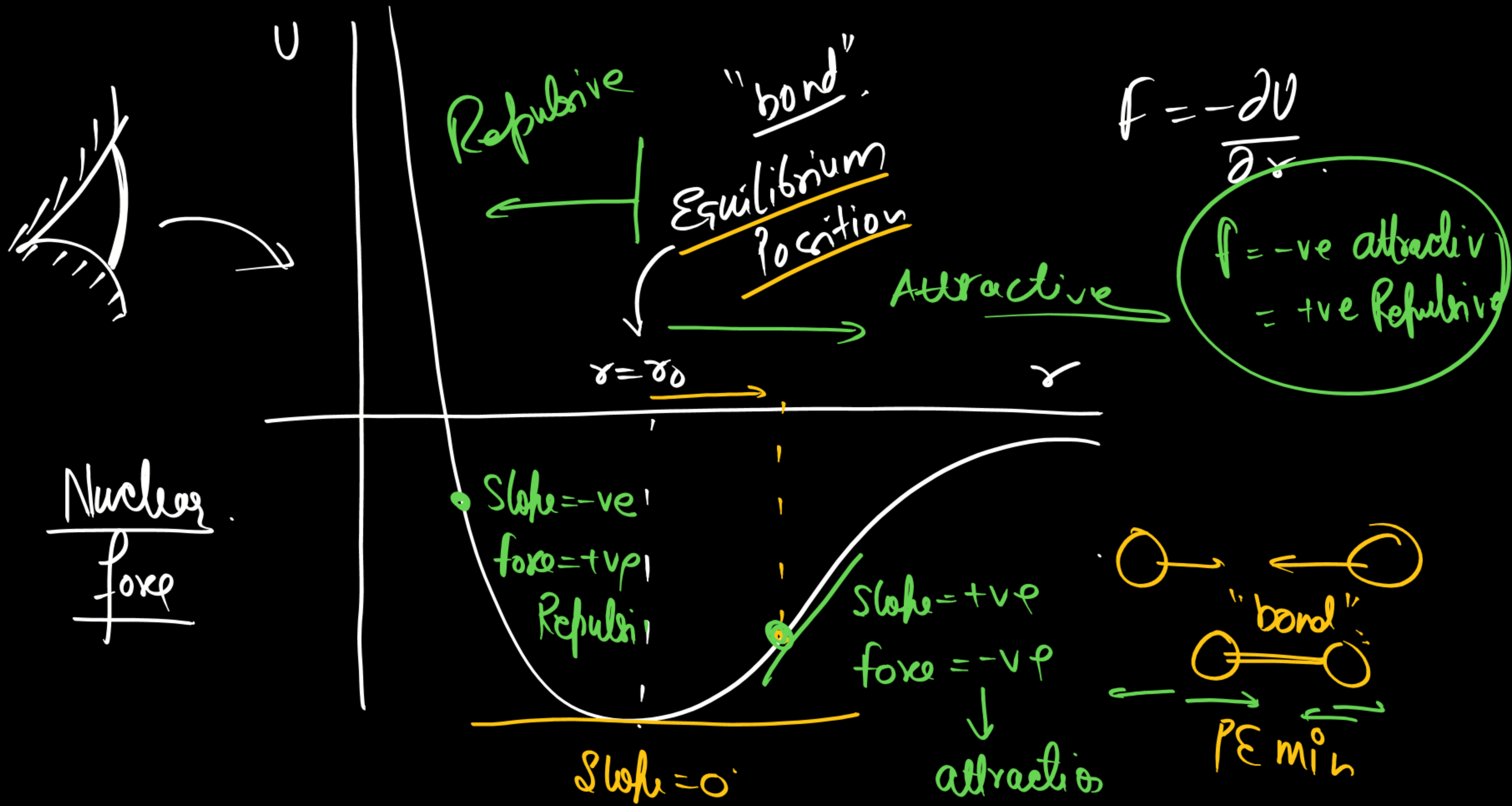
attraction

$+ \quad +$

$- \quad -$

$$F = +\frac{Kq_1q_2}{r^2}$$

Repulsive



U
Repulsive
"bond"
Equilibrium Position
$F = -\frac{\partial U}{\partial r}$
F = -ve attractive
= +ve Repulsive
Attractive
$r = r_0$
r
Nuclear force
Slope = -ve
force = +ve
Repulsi
Slope = +ve
force = -ve
attraction
"bond"
PE min
Slope = 0

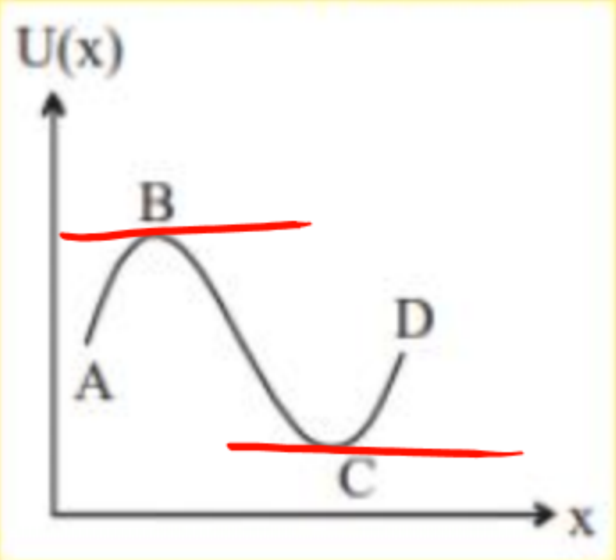
The potential energy of a particle varies with distance x as shown in the graph. The force acting on the particle is zero at **(AIEEE 2008)**

(a) C

(b) B

(c) B and C   Ans

(d) A and D



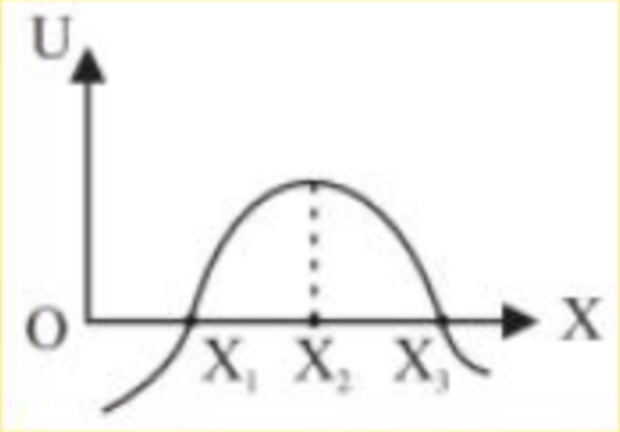
force = 0 = Equilibrium

from U/x graph

$$F = -\frac{\partial U}{\partial x} = -\text{Slope}.$$

$$\text{Slope} = 0 = F$$

In the figure shown the potential energy (U) of a particle is plotted against its position 'x' from origin. Then which of the following statement is correct. A particle at: **(2009)**



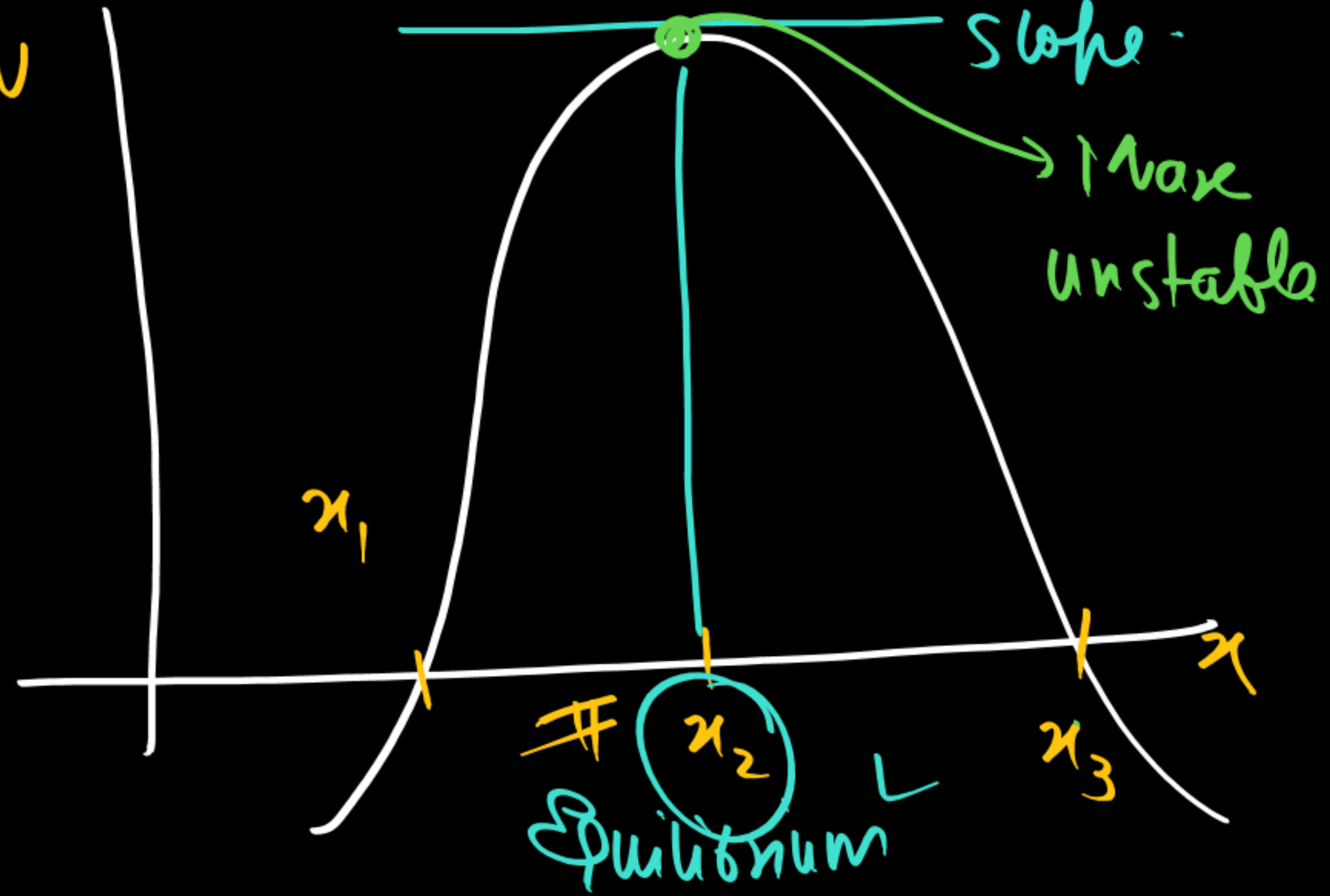
(a) x_1 is in stable equilibrium

(b) x_2 is in stable equilibrium

(c) x_3 is in stable equilibrium

(d) None of these   Ans

$$\boxed{\text{Slope of } U/x = -\text{force}}$$

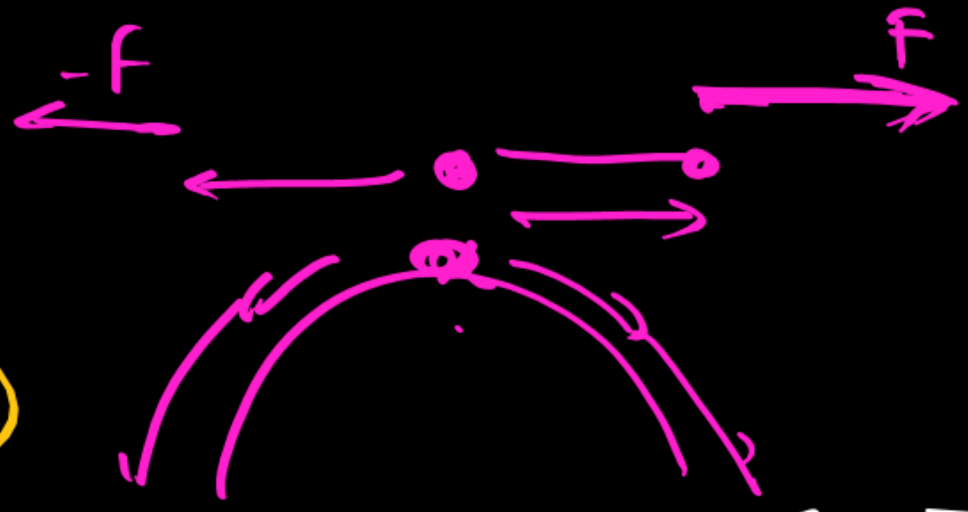


No of Equil points

2

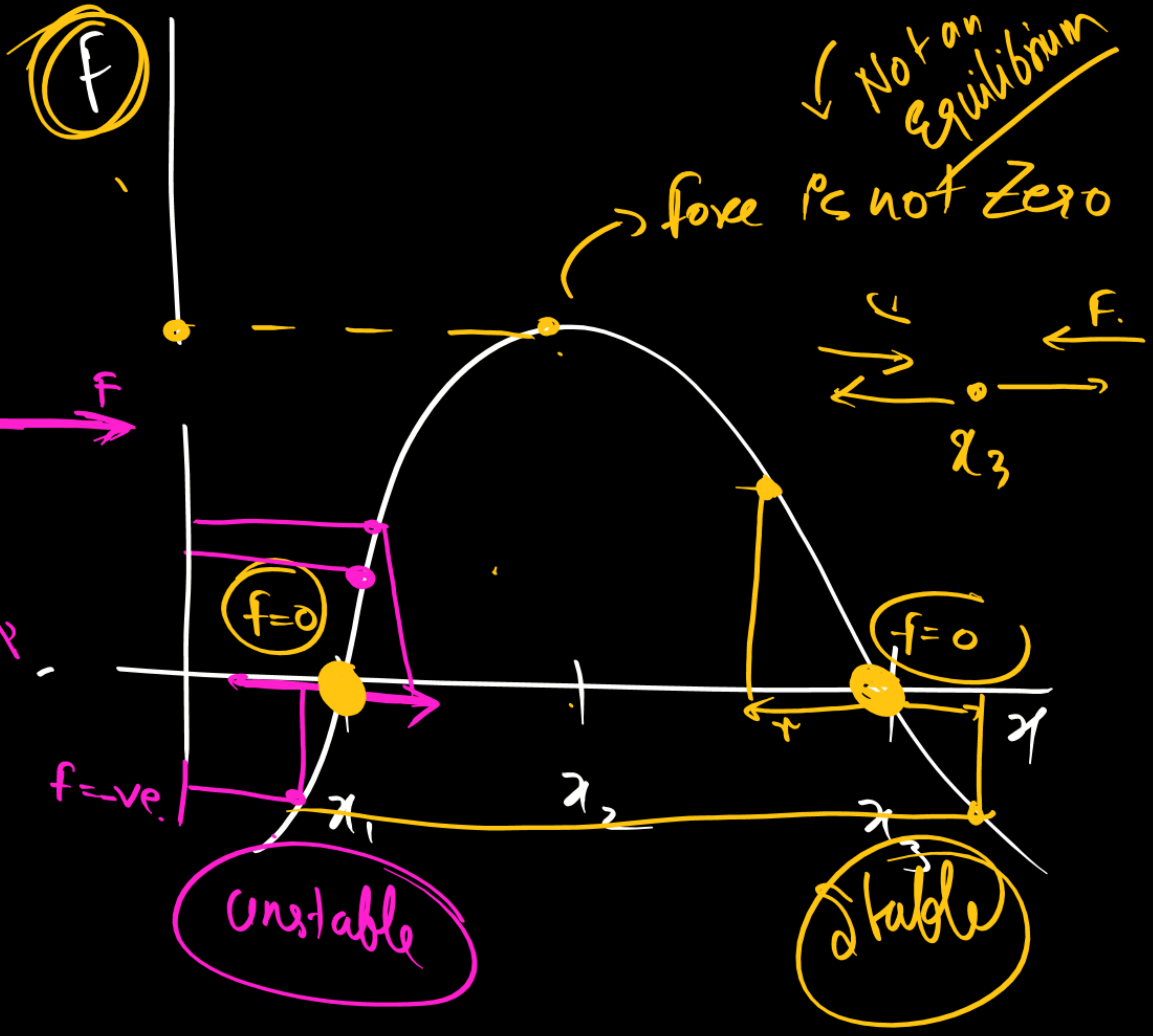
Stable

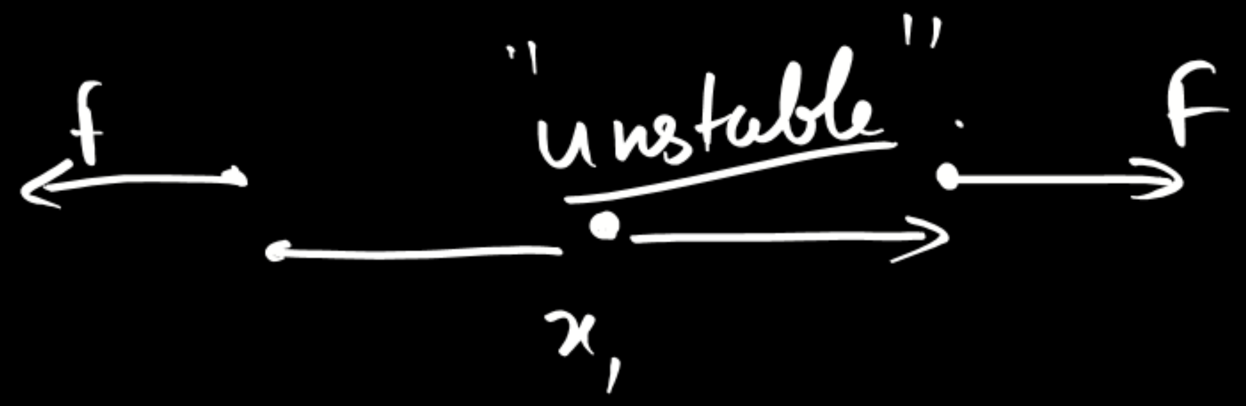
x_3

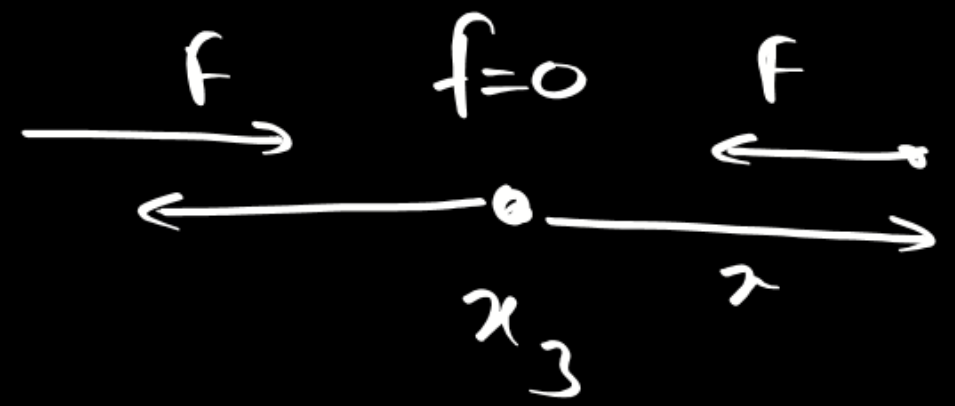


Unstable :-

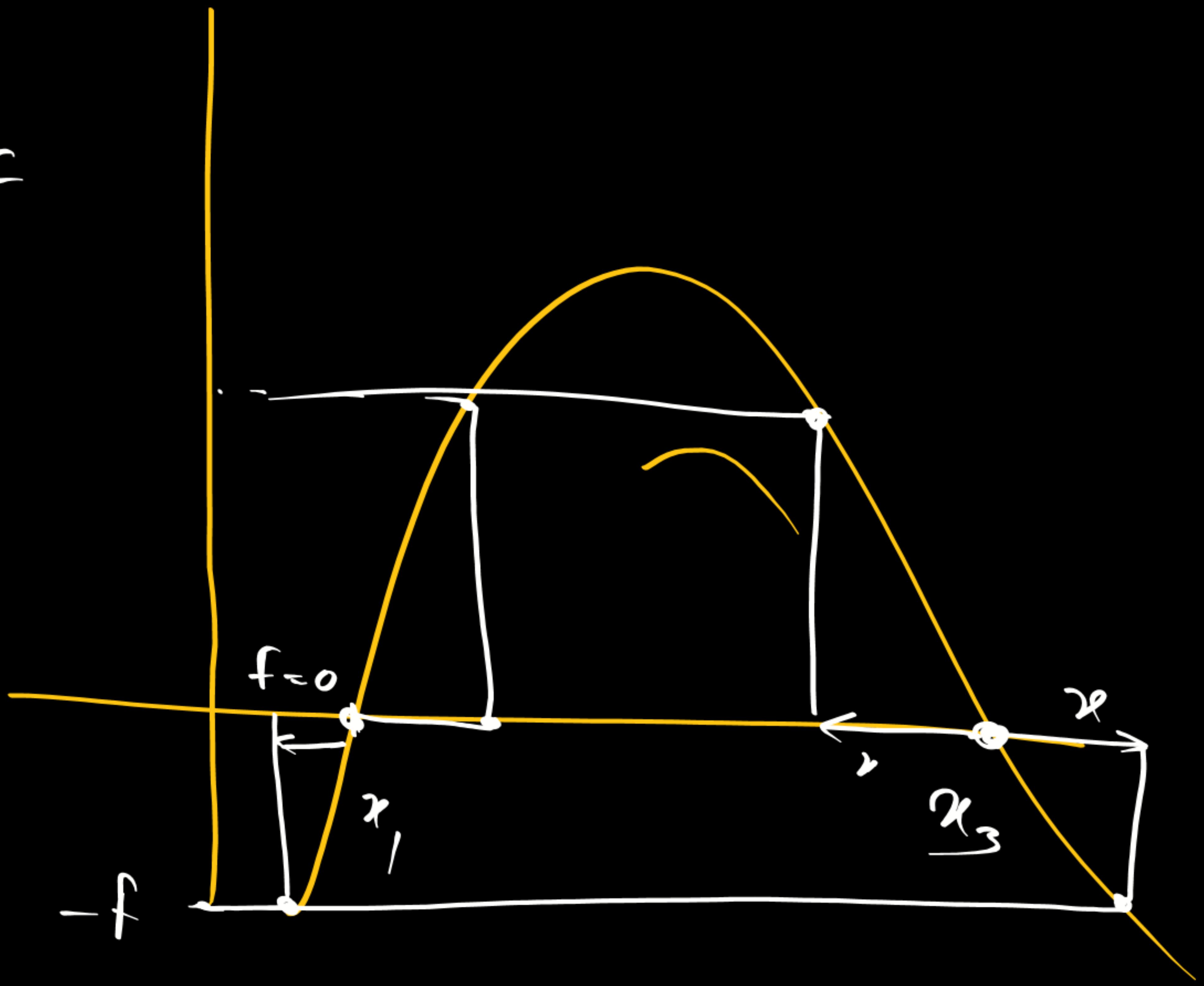
x_1







Stable



If the potential energy of two molecules is given by $U = \frac{A}{r^{12}} - \frac{B}{r^6}$, then at equilibrium position, its potential energy is equal to :

(a) $\frac{A^2}{4B}$

(b) $-\frac{B^2}{4A}$ Ans

(c) $\frac{2B}{A}$

(d) $3A$

$$U = \frac{A}{r^{12}} - \frac{B}{r^6} = Ar^{-12} - Br^{-6}$$

At Equilibrium $f = 0$

$$f = -\frac{\partial U}{\partial r} = -\left[\frac{-12A}{r^{13}} + \frac{6B}{r^7}\right] = 0$$

$$\frac{6B}{r^7} - \frac{12A}{r^{13}}$$

$$\frac{1}{r^7}\left[6B - \frac{12A}{r^6}\right] = 0$$

$r \to \infty$

$$6B = \frac{12A}{r^6}$$

$$r^6 = \frac{2A}{B}$$

$$r = \left(\frac{2A}{B}\right)^{1/6}$$

$$PE = \frac{A}{r^{12}} - \frac{B}{r^{6}} \Rightarrow \frac{A}{\left(\frac{2A}{B}\right)^2} - \frac{B}{\frac{2A}{B}}$$

$$r = \left(\frac{2A}{B}\right)^{1/6}$$

$$= \frac{B^2}{4A} - \frac{B^2}{2A}$$

$$PE \text{ at } r = \left(\frac{2A}{B}\right)^{1/6}$$

$$= -\frac{B^2}{4A}$$

The potential energy for a force field $\vec{F}$ is given by U(x, y) = sin (x + y). The force acting on the particle of mass m at $\left(0, \frac{\pi}{4}\right)$ is (2020)

(a) 1 Ans
(b) $\sqrt{2}$
(c) $1/\sqrt{2}$
(d) 0

$$|F| = \sqrt{\frac{1}{2}+\frac{1}{2}} = \sqrt{1} = 1$$

$$U(x,y) = \sin(x+y).$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j}$$

$$F_x = -\frac{\partial U}{\partial x}\bigg|_{y=\text{Const}} = -\frac{\partial}{\partial x}(\sin(x+y))$$

$$= -\cos(x+y)$$

$$F_y = -\frac{\partial U}{\partial y}\bigg|_{x=\text{con}} = -\frac{\partial}{\partial y}\sin(x+y)$$

$$= -\cos(x+y).$$

$$\vec{F} = -\cos(x+y)\hat{i} - \cos(x+y)\hat{j}.$$

$$= -\cos(\pi/4)\hat{i} - \cos(\pi/4)\hat{j}.$$

$$= -\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} =$$

$F = 2x^2 - 3x - 2$. Choose correct option

(a) $x = -1/2$ is position of stable equilibrium

(b) $x = 2$ is position of stable equilibrium

(c) $x = -1/2$ is position of unstable equilibrium

(d) $x = 2$ is position of neutral equilibrium

Sol:- $f = 2x^2 - 3x - 2 =$

Equilibrium $f = 0 = 2x^2 - 3x - 2$

$f = -\frac{\partial U}{\partial x}$

$0 = 2x^2 - 4x + x - 2$

$0 = 2x(x-2) + 1(x-2)$

$0 = (2x+1)(x-2)$

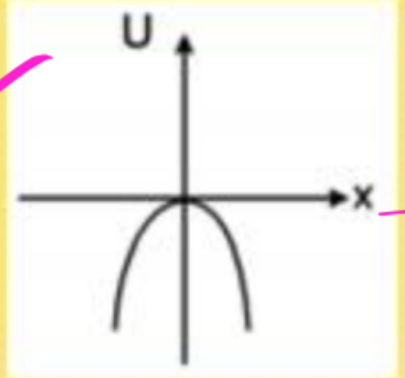
$x = -1/2 \qquad x = 2$

$\frac{d^2U}{\partial x^2} = \frac{dF}{\partial x} = 4x - 3$

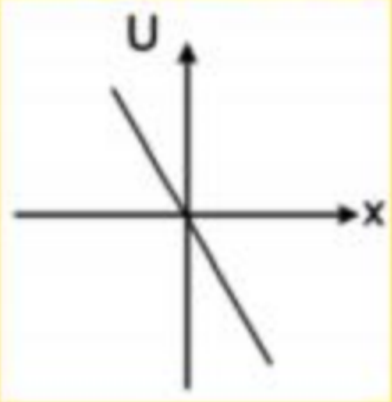
at $x = 2$ $\quad \frac{dF}{dx} > 0$ Min Stable.

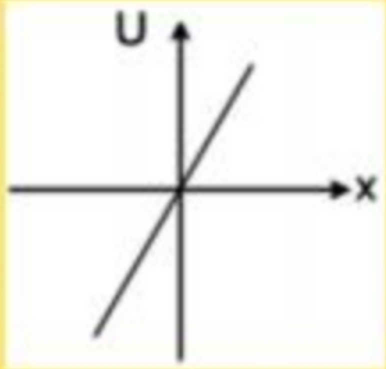
$x = -1/2 \quad \frac{dF}{dx} < 0$ Max unstable.

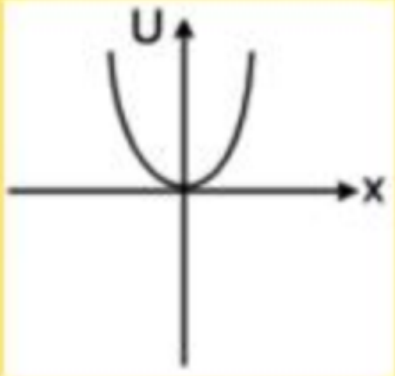
A particle moves under the influence of a force F = kx in one dimensions (k is a positive constant and x is the distance of the particle from the origin). Assume that the potential energy of the particle at the origin is zero, the schematic diagram of the potential energy U as a function of x is given by [JEE-2004]

(a) 

Ans

(b) 

(c) 

(d) 

$F = Kx$

$x = 0 \quad PE = 0$

$F = -\frac{\partial U}{\partial x}$

$F \cdot dx = -\partial U$

$\int_{x=0}^{x} kx\,dx = -\int_{0}^{U} dU$

$\frac{kx^2}{2} = -(U - 0)$

$U = -\frac{1}{2}kx^2$

A particle is moving in a circular path of radius a under the action of an attractive potential $U = -k/2r^2$. Its total energy is **[JEE Main-2018]**

(a) $\frac{k}{2a^2}$

(b) Zero Ans.

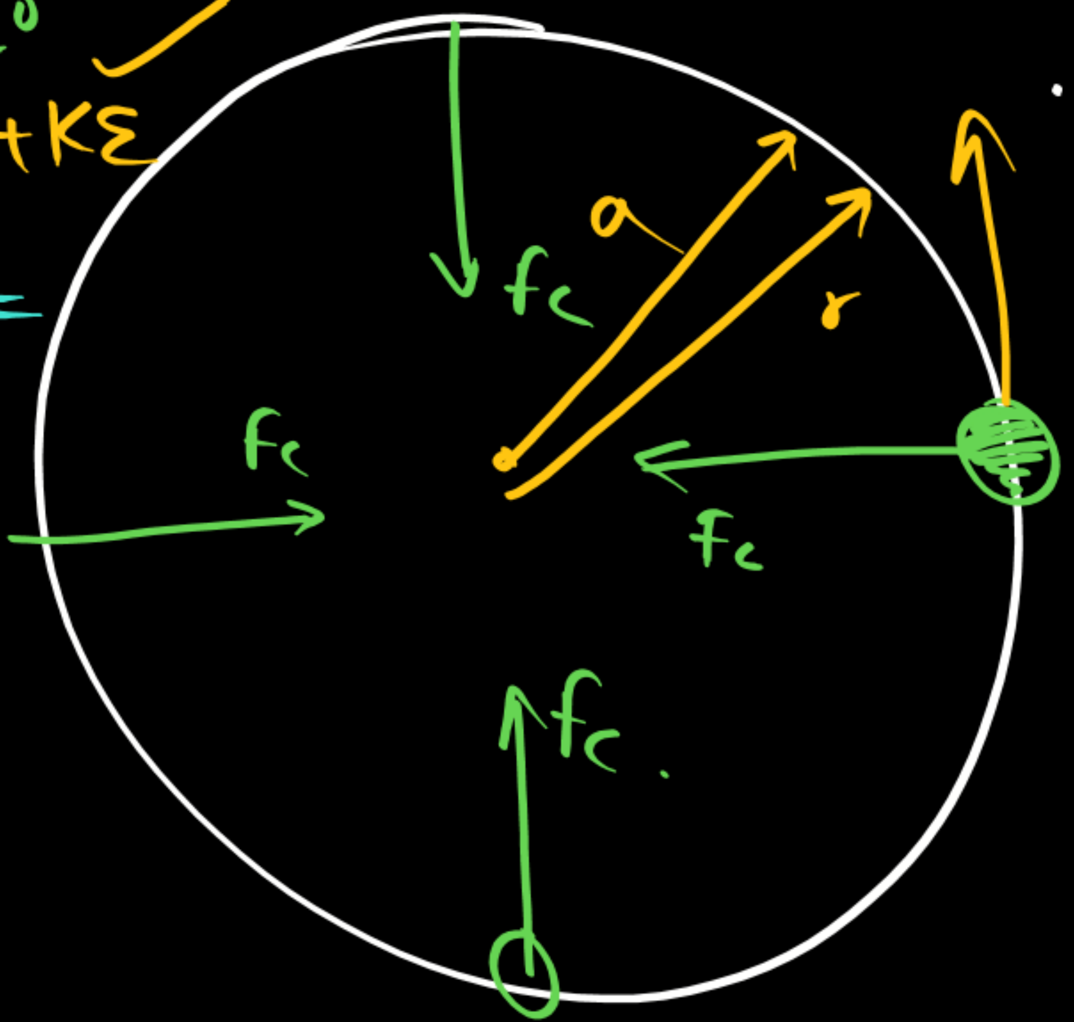
(c) $-\frac{3}{2}\frac{k}{a^2}$

(d) $-\frac{k}{4a^2}$

Sol:

TE = PE + KE

=



$PE = \frac{-K}{2r^2}$

$= \frac{-K}{2}(r^{-2})$

$F = -\frac{\partial U}{\partial r} = -\left(\frac{-K}{2}\right)\frac{\partial}{\partial r} r^{-2}$

$= \frac{K}{2}(-2r^{-3})$

$\boxed{F = \frac{-K}{r^3}}$

$\left|\frac{K}{r^3}\right| = \frac{mv^2}{r}$

$\frac{1}{2}\frac{K}{r^2} = \frac{1}{2}mv^2 = KE$

$$T.E = P.E + K.E$$

$$T.E = -\frac{K}{2r^2} + \frac{K}{2r^2}$$

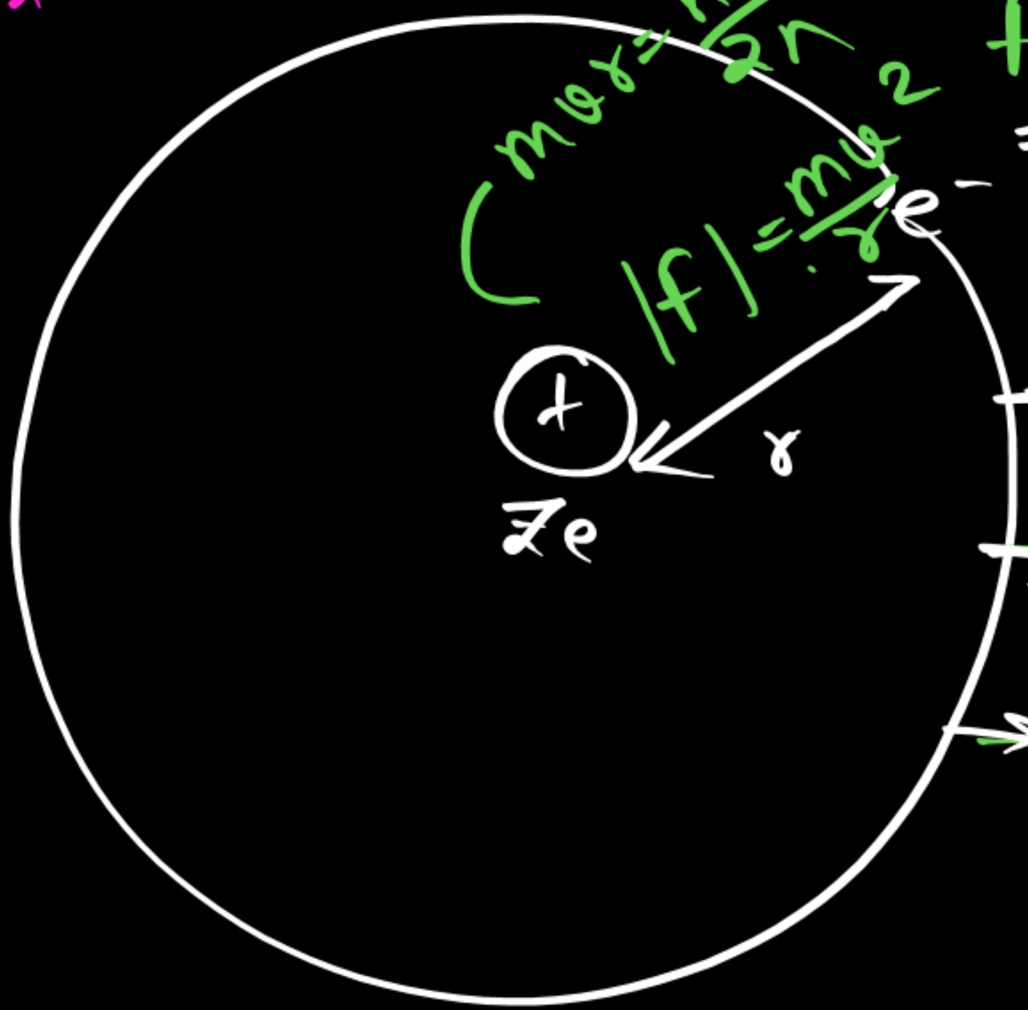
$r = a$

$$\boxed{T.E = 0}$$

On Nuclear Phy (NCERT Exemplar).

Hypothetical Ques

Atom



$$PE = \frac{-KZe^2}{2r^3}$$

$$f = -\frac{\partial U}{\partial r}$$

$$PE = \frac{-KZe^2}{r}$$

$$F = -\frac{\partial U}{\partial r}$$

$$f = \frac{-Kq_1q_2}{r^2} = \frac{-KZe^2}{r^2}$$

$$v_n = v_0 \frac{Z}{n}$$

$$r_n = a_0 \frac{n^2}{Z}$$

$$E_n = -13.6 \frac{Z^2}{n^2}$$

Bohr's Pos

1. $\frac{KZe^2}{r^2} = \frac{mv^2}{r}$
2. $mvr = \frac{nh}{2\pi}$

Mechanical energy conservation

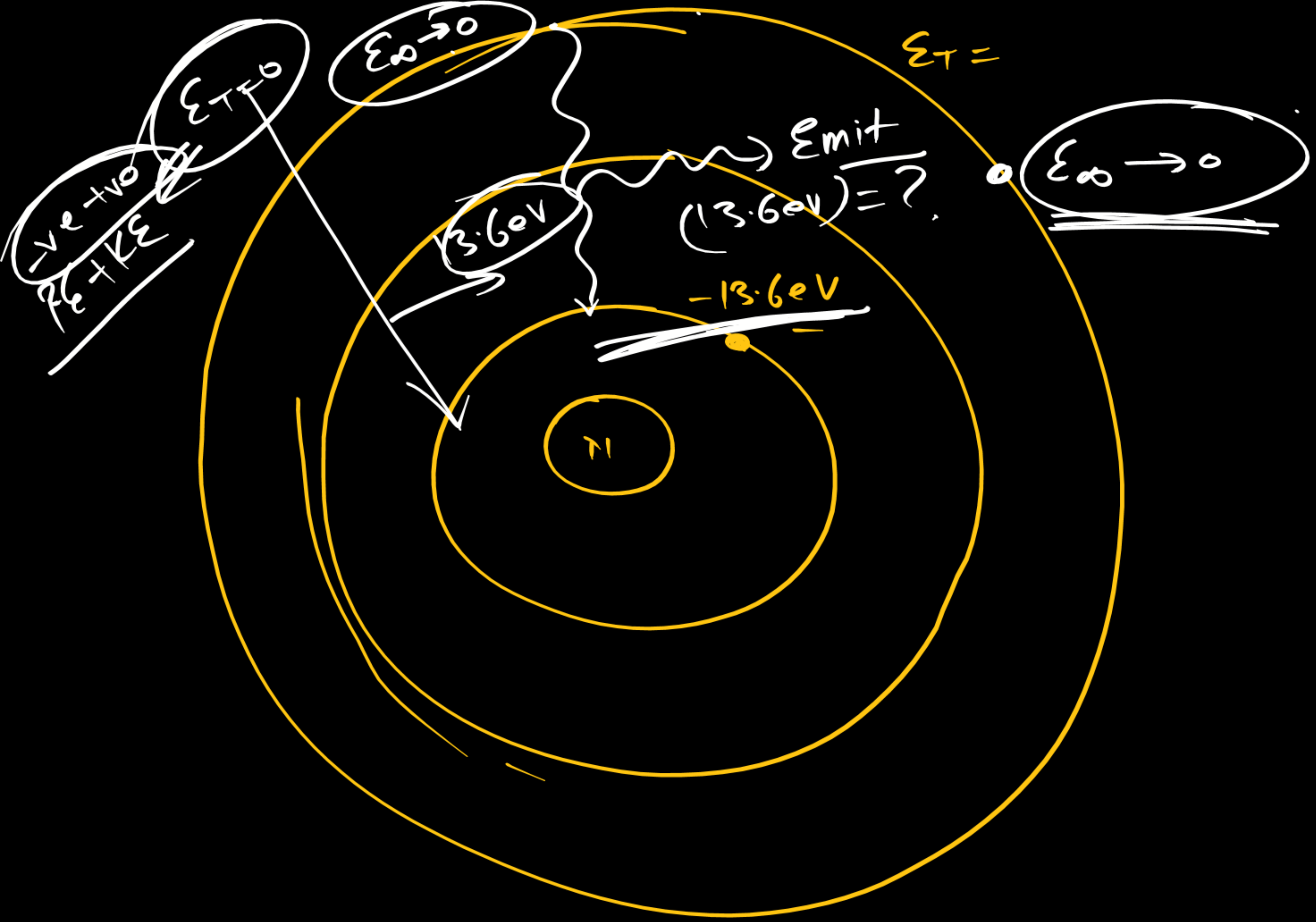
Sum of $PE + K.E$

When dissipative forces in System $= 0$

(Non-Conservative $= 0$)

$$E_i = E_f$$

$$\boxed{K_i + P_i = K_f + P_F}$$

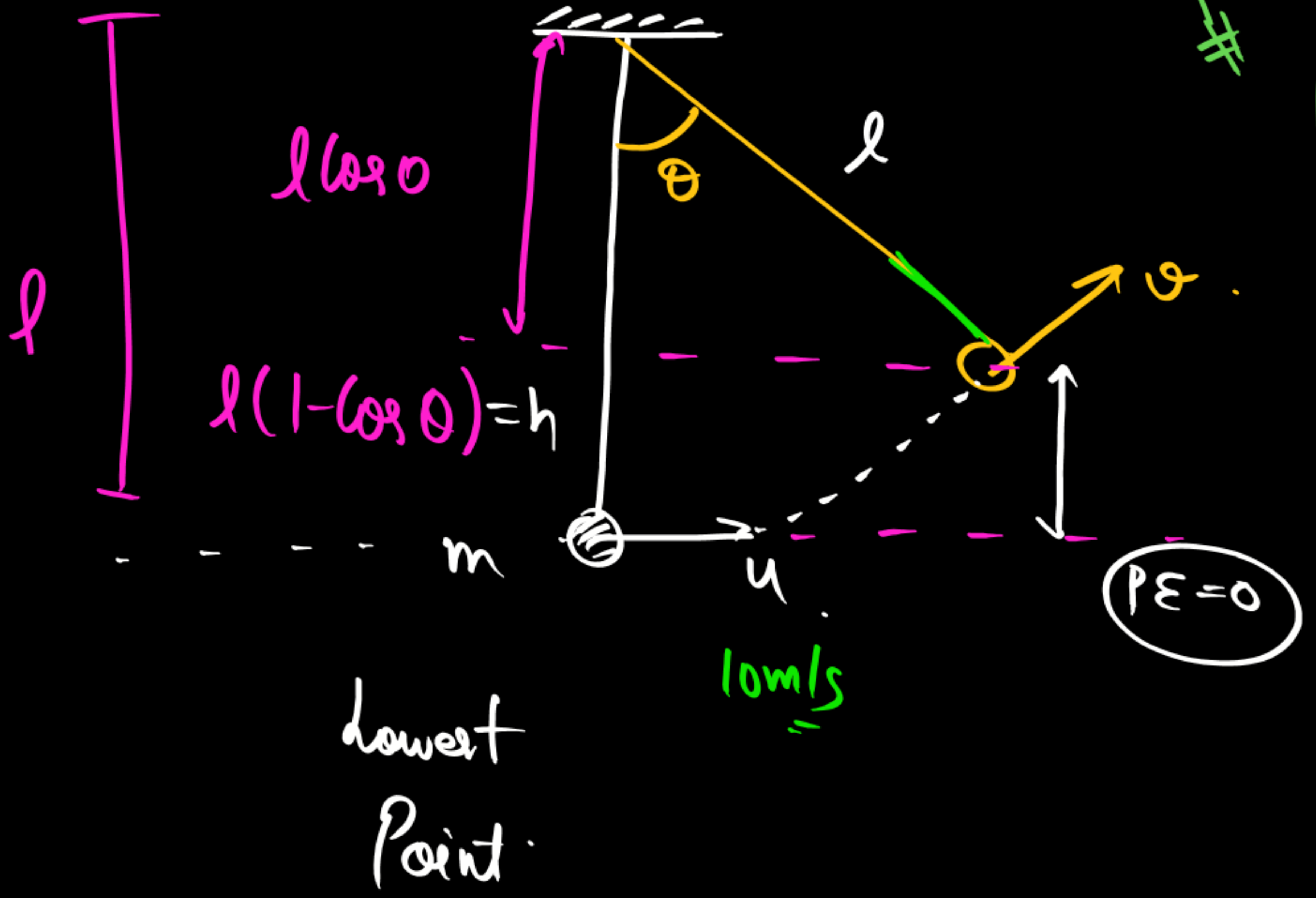


$E_\infty \to 0$
$E_T = 0$
$-ve + ve$
$PE + KE$
$E_T =$
E_{mit}
$(13.6eV) = ?$
$E_\infty \to 0$
$13.6eV$
$-13.6eV$
n_1

Vertical Circular Motion

Case 1: VCM with Thread

Velocity at any angle θ.



$$E_i = E_f$$

$$(K+P)_i = (K+P)_f$$

$$\frac{1}{2}\cancel{m}u^2 + 0 = \frac{1}{2}\cancel{m}v^2 + \cancel{m}gl(1-\cos\theta)$$

$$\boxed{v = \sqrt{u^2 - 2gl(1-\cos\theta)}}$$

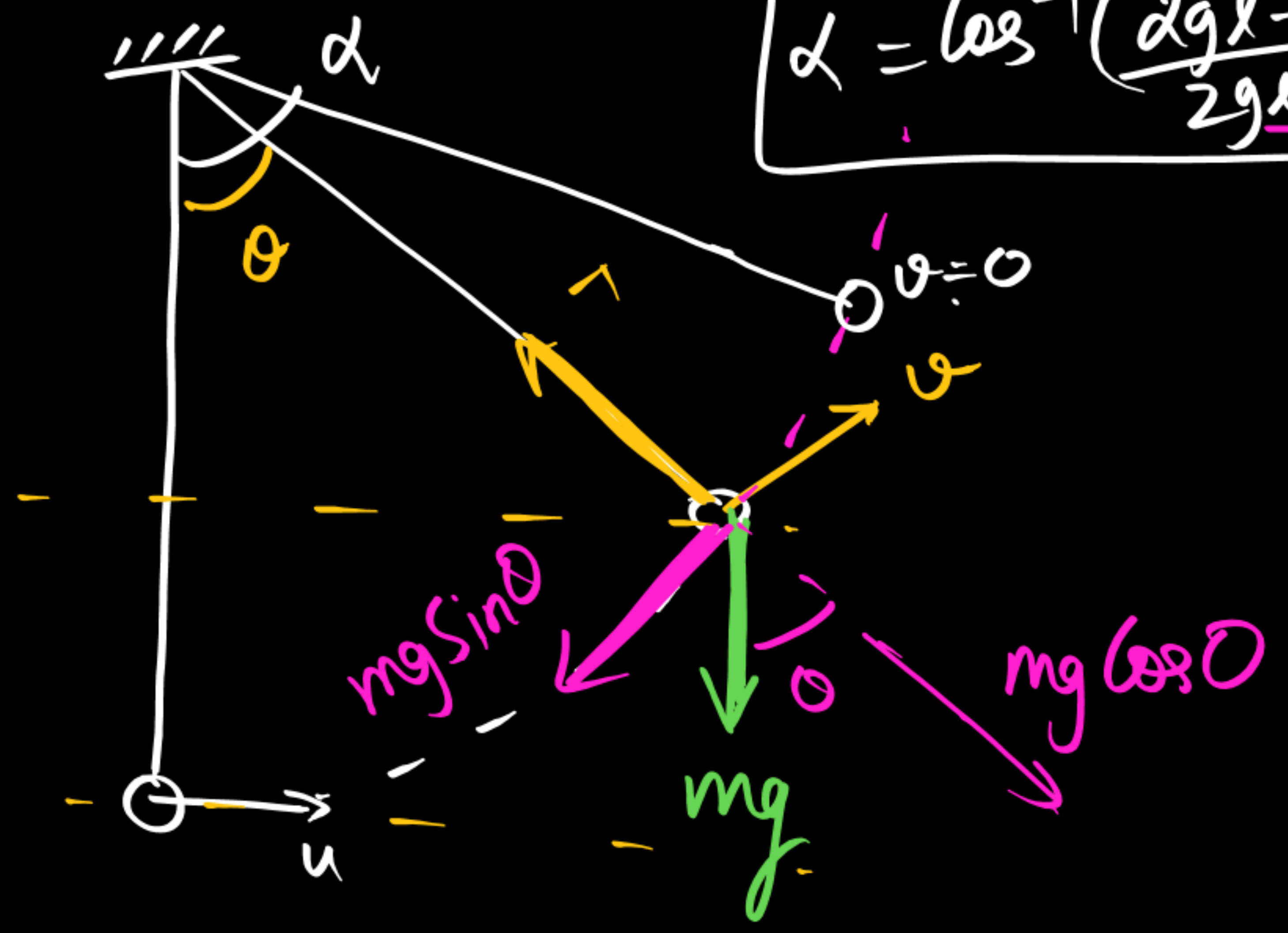
Angular Amplitude $(\theta = \alpha)$

$v = 0$

$$u^2 = 2gl(1-\cos\alpha)$$

$$\frac{u^2}{2gl} = 1 - \cos\alpha$$

$$\boxed{\alpha = \cos^{-1}\left(\frac{2gl - u^2}{2gl}\right)}$$



"String"

$$\boxed{\alpha = \cos^{-1}\left(\frac{2gl - u^2}{2gl}\right)}$$

$$T - mg\cos\theta = \frac{mv^2}{l}$$ → Centripetal.

v at any θ

$$\boxed{v = \sqrt{u^2 - 2gl(1-\cos\theta)}}$$

$$T - mg\cos\theta = \frac{m}{l}\left(u^2 - 2gl(1-\cos\theta)\right)$$

$$T - mg\cos\theta = \frac{mu^2}{l} - 2mg + 2mg\cos\theta$$

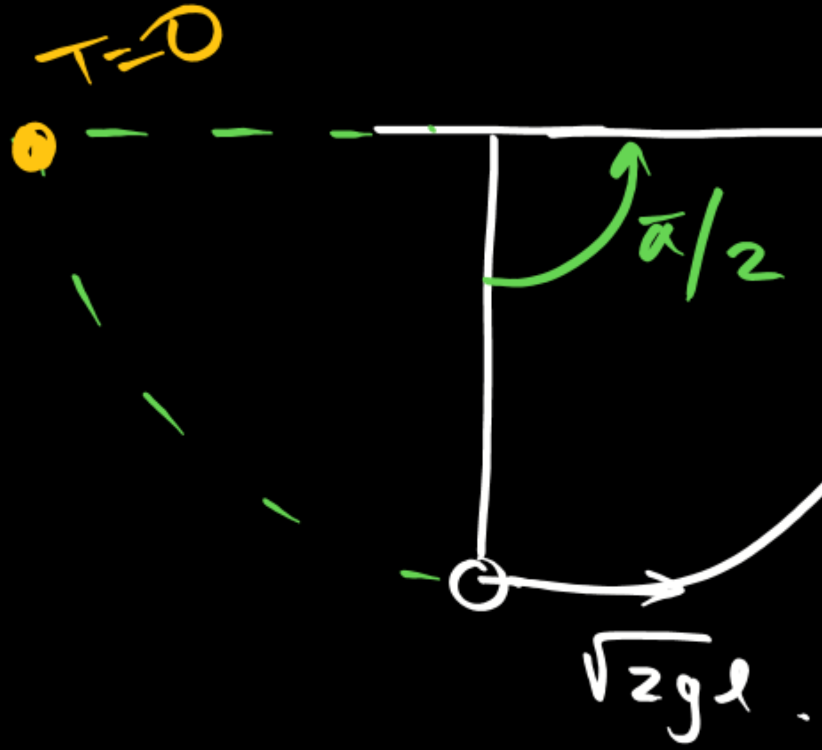
$$\boxed{T = \frac{mu^2}{l} - 2mg + 3mg\cos\theta}$$

In CM we
Resolve forces radially
&
tangentially

String

$$v=\sqrt{u^2-2gl(1-\cos\theta)} \qquad \alpha=\cos^{-1}\left(\frac{2gl-u^2}{2gl}\right)$$

$$T=\frac{mu^2}{l}-2mg+3mg\cos\theta$$



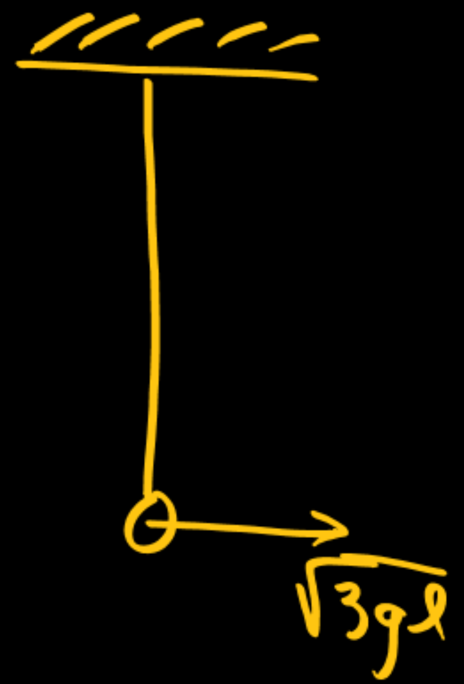
$$\alpha=\cos^{-1}\left(\frac{2gl-2gl}{2gl}\right)$$

$$\cos^{-1}0$$

$$\alpha=\pi/2$$

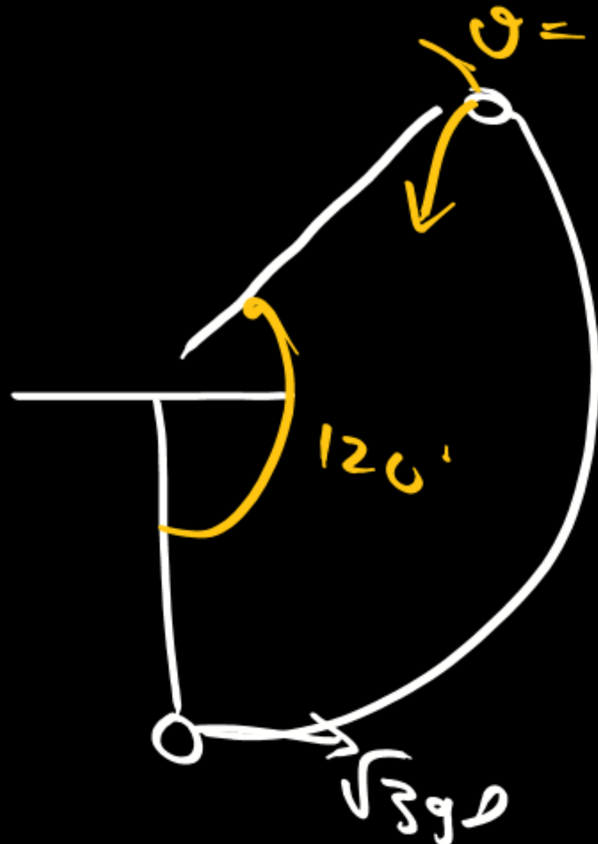
$$T=\frac{m2gl}{l}-2mg+\cancel{3mg\cos\pi/2}$$

$$T=0$$

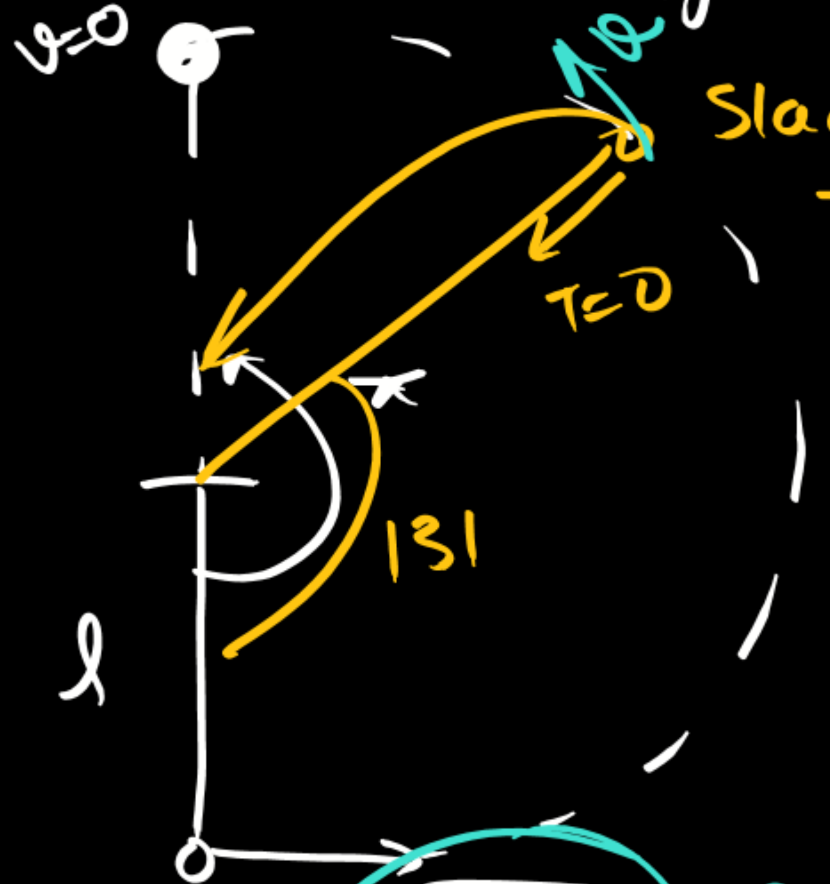


$$\alpha=\cos^{-1}\left(\frac{2gl-3gl}{2gl}\right)$$

$$=\cos^{-1}\left(-\frac{1}{2}\right) \qquad \alpha=120$$



① When $v=\sqrt{4gl}$.



$\sqrt{4gl}$. only for Rod.

$$\alpha = \cos^{-1}\left(\frac{2gl-4gl}{2gl}\right).$$

when v=0

$\cos^{-1}(-1)$

$\alpha=\pi$.

$$T=\frac{mu^2}{l}-2mg+3mg\cos\theta.$$

at what ∠ T=0

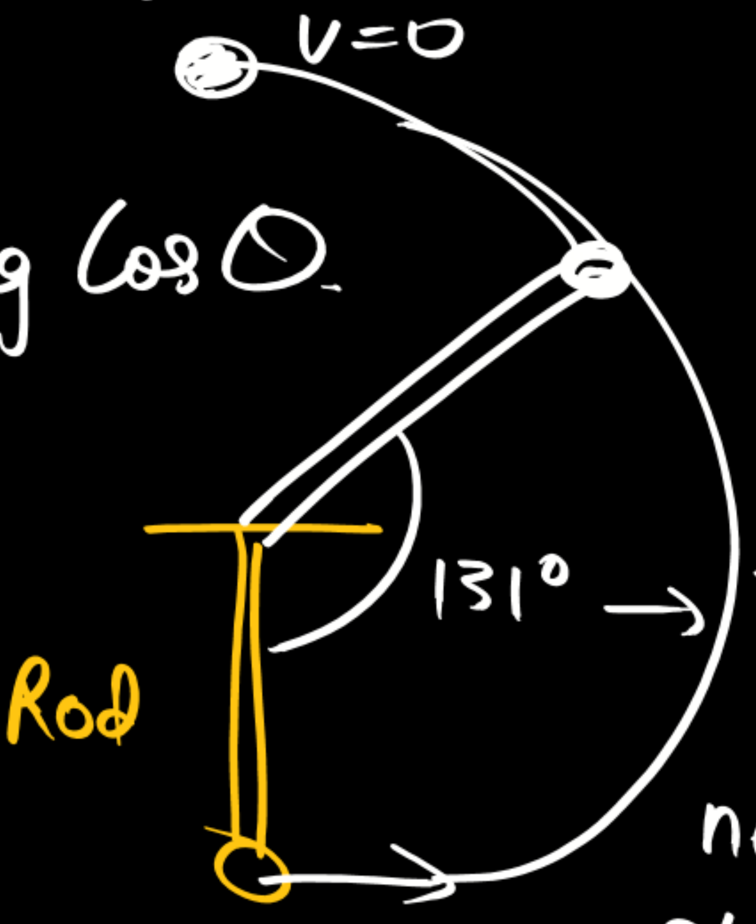
$$0=\frac{mu^2}{l}-2mg+3mg\cos\theta$$

$$0=\frac{m4g\cancel{l}}{\cancel{l}}-2mg+3mg\cos\theta.$$

$$-2\cancel{m}g=3\cancel{m}g\cos C$$

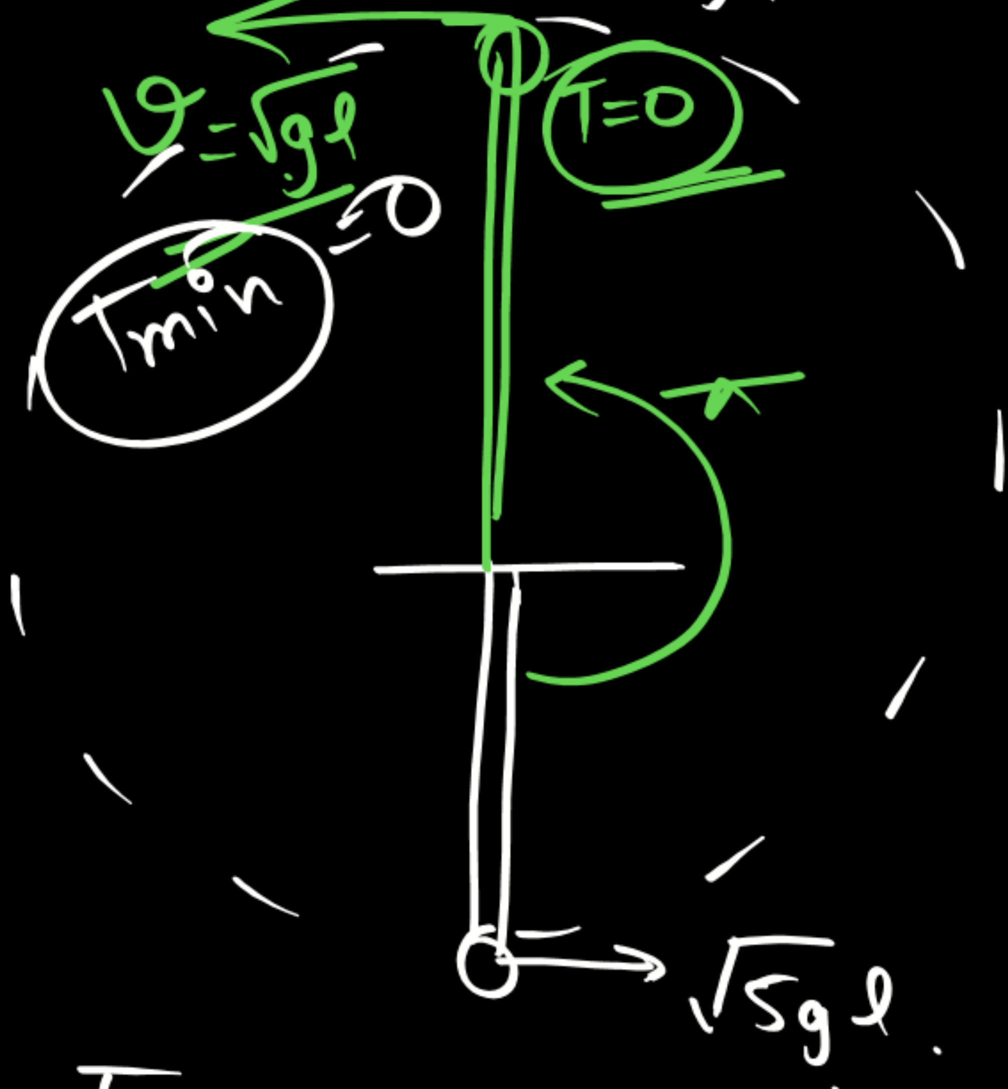
$$-\frac{2}{3}=\cos\theta$$

$$\theta\approx 131^\circ$$



→ There is no Slackin

When $v=\sqrt{5gl}$



$T_{max} = 6mg$

lower

$T_{max} - T_{min} = 6mg$

$$\alpha = \cos^{-1}\left(\frac{2gl-5gl}{2gl}\right)$$

$$= \cos^{-1}\left(-\frac{3}{2}\right)$$

$$= \cos^{-1}(-1.5)$$

$=$

There is no angle where $v=0$

$$T = 0 = \frac{mu^2}{l} - 2mg + 3mg\cos\theta \qquad u \to \sqrt{5gl}$$

$$\theta = \pi$$

* for String $\sqrt{5gl}$
* for Rod $= \sqrt{4gl}$

$$v = \sqrt{u^2 - 2gl(1-\cos\theta)}$$

$$= \sqrt{5gl - 2gl(1-\cos\pi)} = \sqrt{gl}$$

BREAK
TILL
22:45 PM.

Vertical Circular Motion

Case 2: VCM with Rod

Vertical Circular Motion

Case 3: VCM on spherical Surface

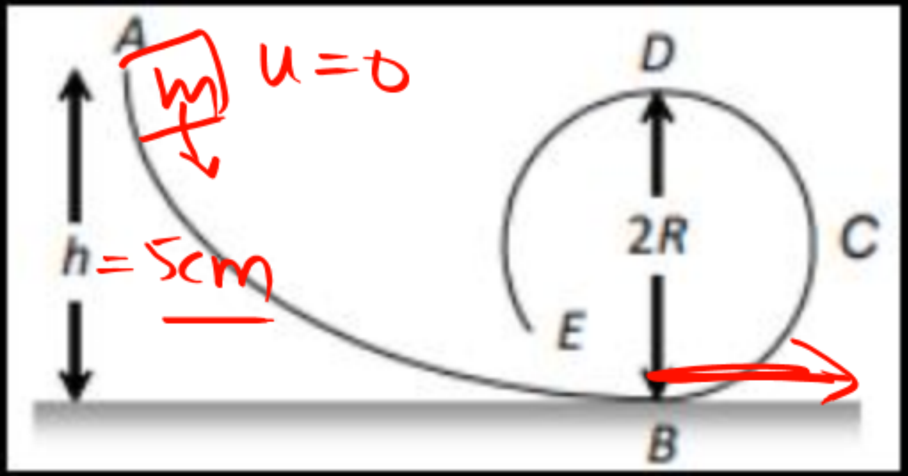
A frictionless track ABCDE ends in a circular loop of radius R. A body slides down the track from point A which is at a height h = 5 cm. Maximum value of R for the body to successfully complete the loop is

(a) 5 cm

(b) $\frac{15}{4}$ cm

(c) $\frac{10}{3}$ cm

~~(d)~~ 2 cm Ans



$v_{min} = \sqrt{5gR}$

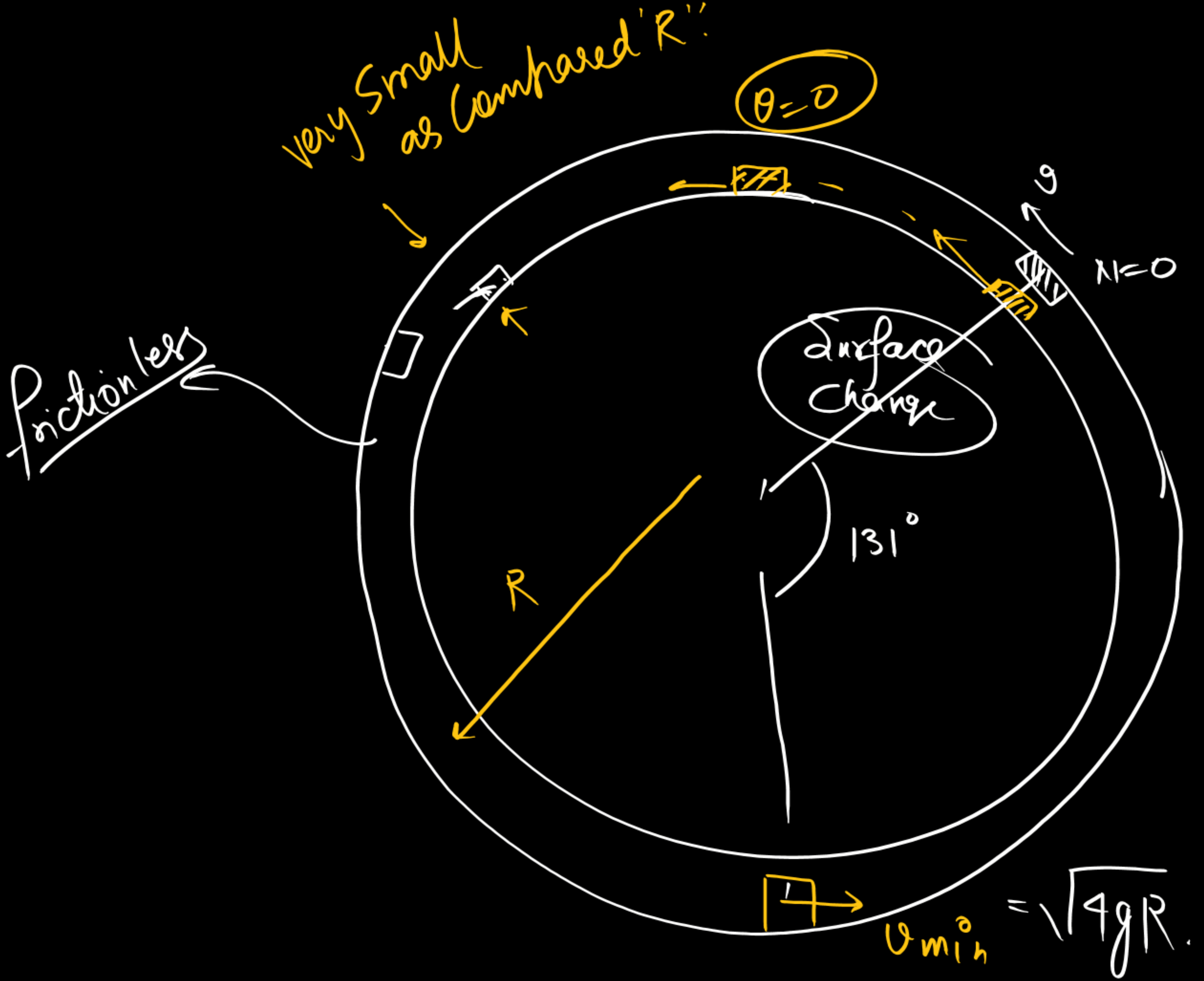
$\cancel{m}gh = \frac{1}{2}\cancel{m}v_B^2$

$\sqrt{2gh} = v_B = \sqrt{5gR}$

$2\cancel{g}h = 5\cancel{g}R$

$10\text{ cm} = 5R$

$2\text{ cm} = R$



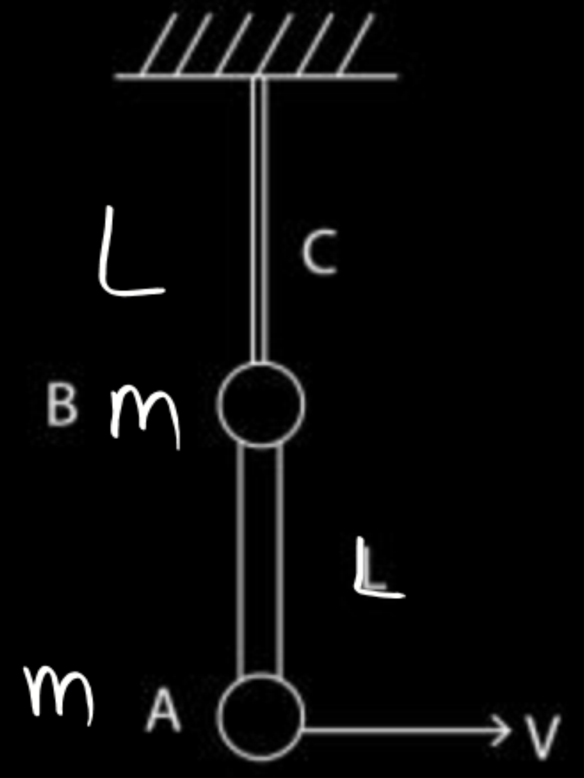
When a block can leave circular track

$v_{min} = \sqrt{5gl}$

When it cannot leave VCM

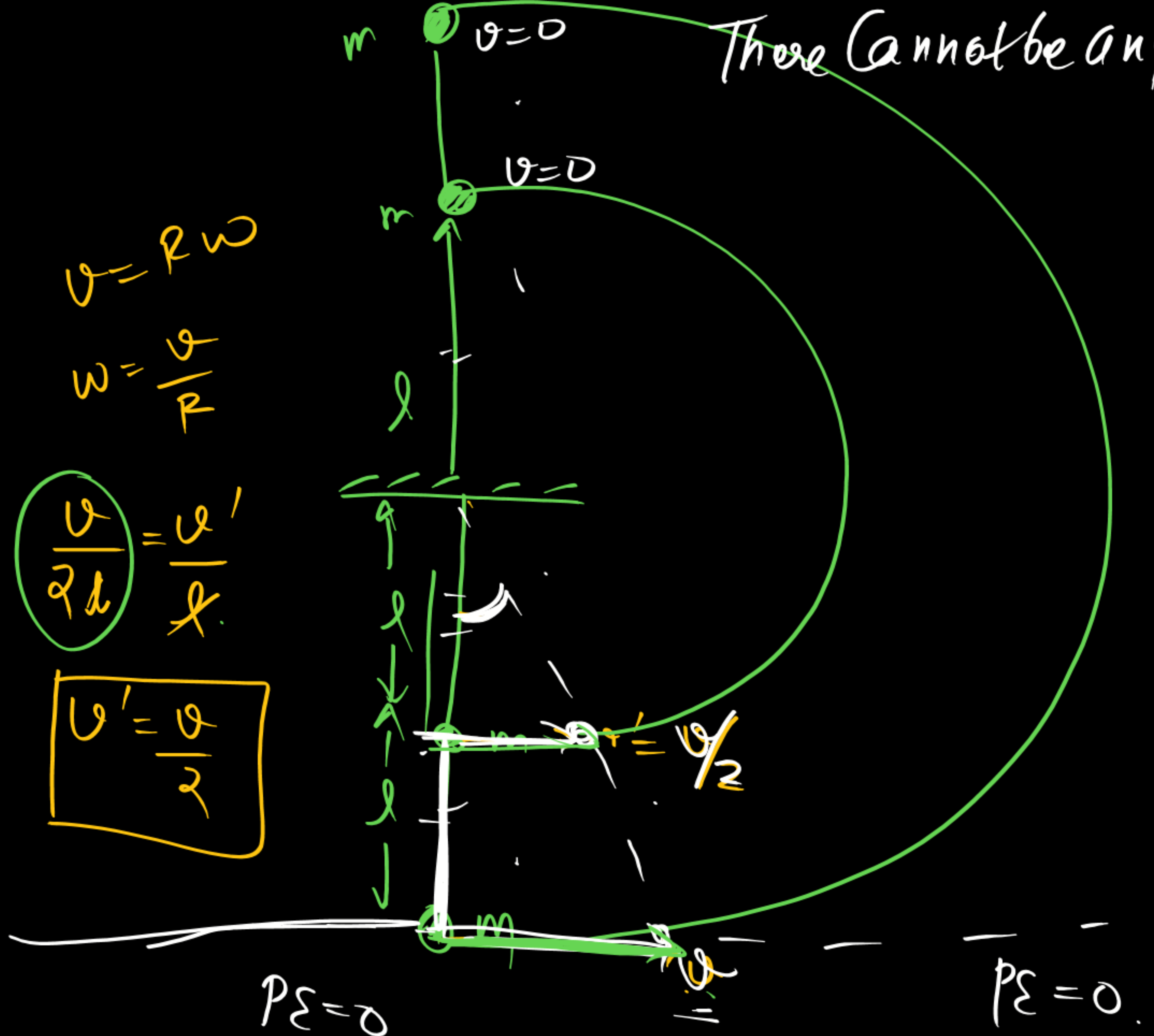
$v_{min} = \sqrt{4gl}$

A dumbell consist of two masses A and B each of mass m joined by a light rod of length l with mass B attached to a suspension point O by a light rod of length l as shown in figure. Find the minimum required speed with which mass A to be projected horizontally so that the system completes the vertical circular motion.



There Cannot be any Slack even

if this Position is attained.



$v = R\omega$

$\omega = \frac{v}{R}$

$\frac{v}{2l} = \frac{v'}{l}$

$\boxed{v' = \frac{v}{2}}$

$\mathcal{E}_i = \mathcal{E}_f$

$(K+P)_i = (K+P)_f$

$\frac{1}{2}mv^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + mgl = mg(3l) + mg4l$

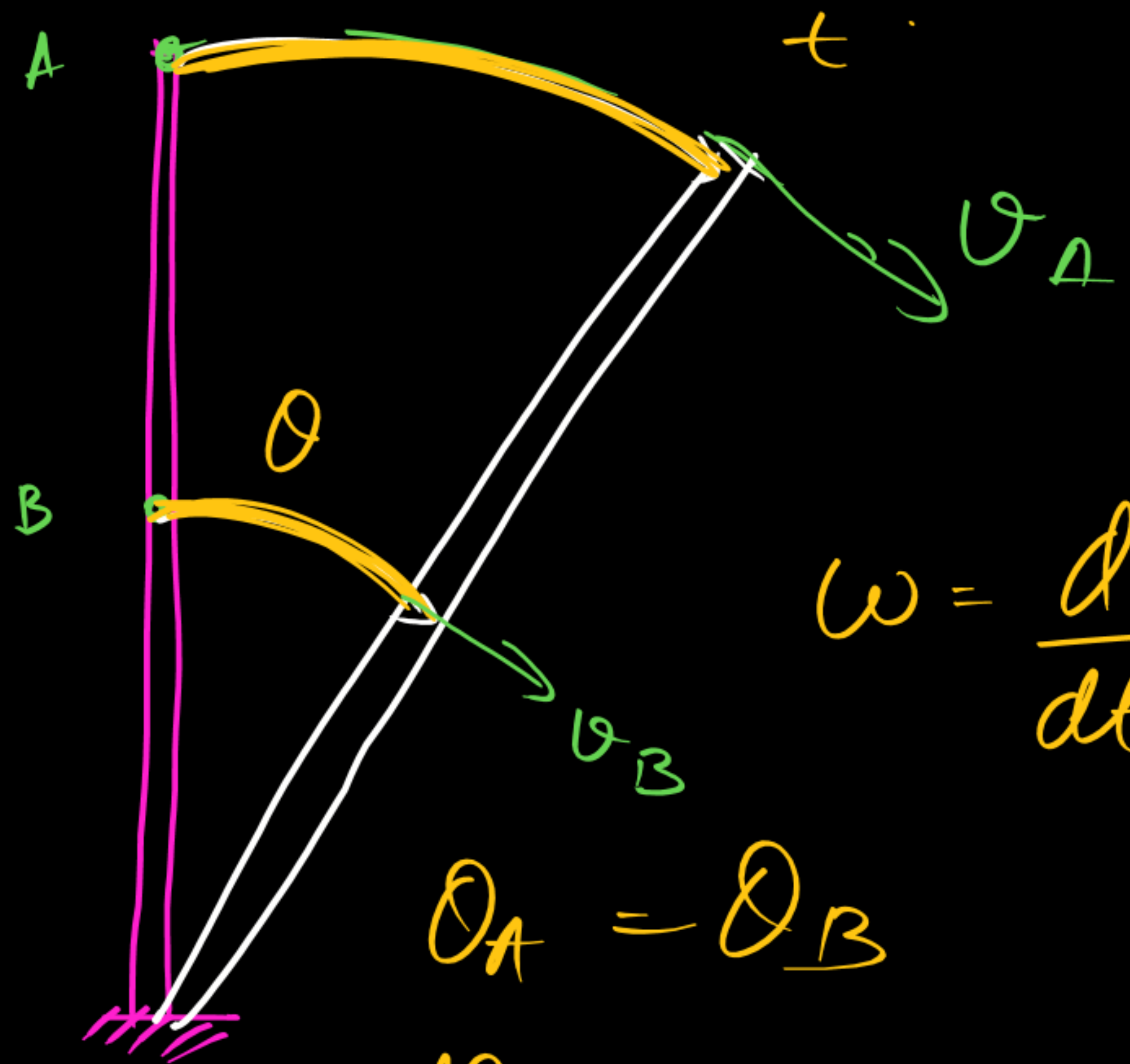
$\frac{v^2}{2} + \frac{v^2}{8} + gl = 7gl$

$\frac{v^2}{2}\left[1+\frac{1}{4}\right] = 6gl$

$v^2 \frac{5}{8} = 6gl$

$v = \sqrt{\frac{48gl}{5}}$

Rigid body



$$v = R\omega$$

$$\omega = \frac{v}{R}$$

$$\omega = \frac{d\theta}{dt}$$

$$\theta_A = \theta_B$$

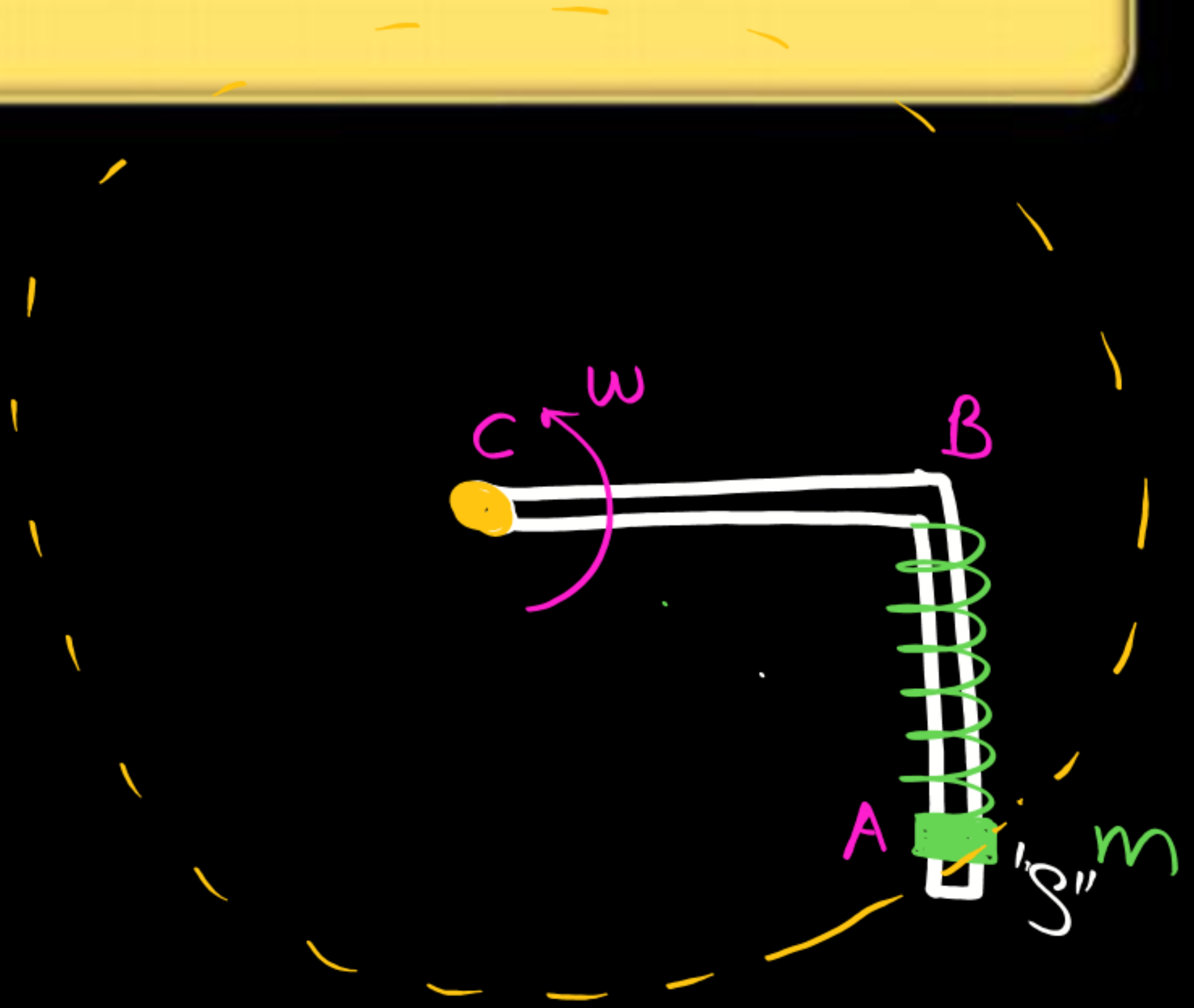
$$\frac{d\theta_A}{dt} = \frac{d\theta_B}{dt}$$

$$\omega_A = \omega_B$$

A simple pendulum of mass m is projected from its lowest point at speed so that the string will slack at a position where it makes an angle 60° with upward vertical. Find the tension in string at the lowest position.

A bent rod *ABC* is pivoted freely at point *C* about which it rotates at uniform angular speed ω in horizontal plane as shown in figure. On arm *AB* a spring of natural length *l* and force constants is attached at point *B* and on its other end a movable sleeve *S* is attached which can smoothly slide on rod. Find the elongation of spring when sleeve is in equilibrium on rod.

Natural length = l.



Where x is Elongation in Spring

$$r\cos\theta = l + x$$

At Equili

$$Kx = mr\omega^2\cos\theta$$

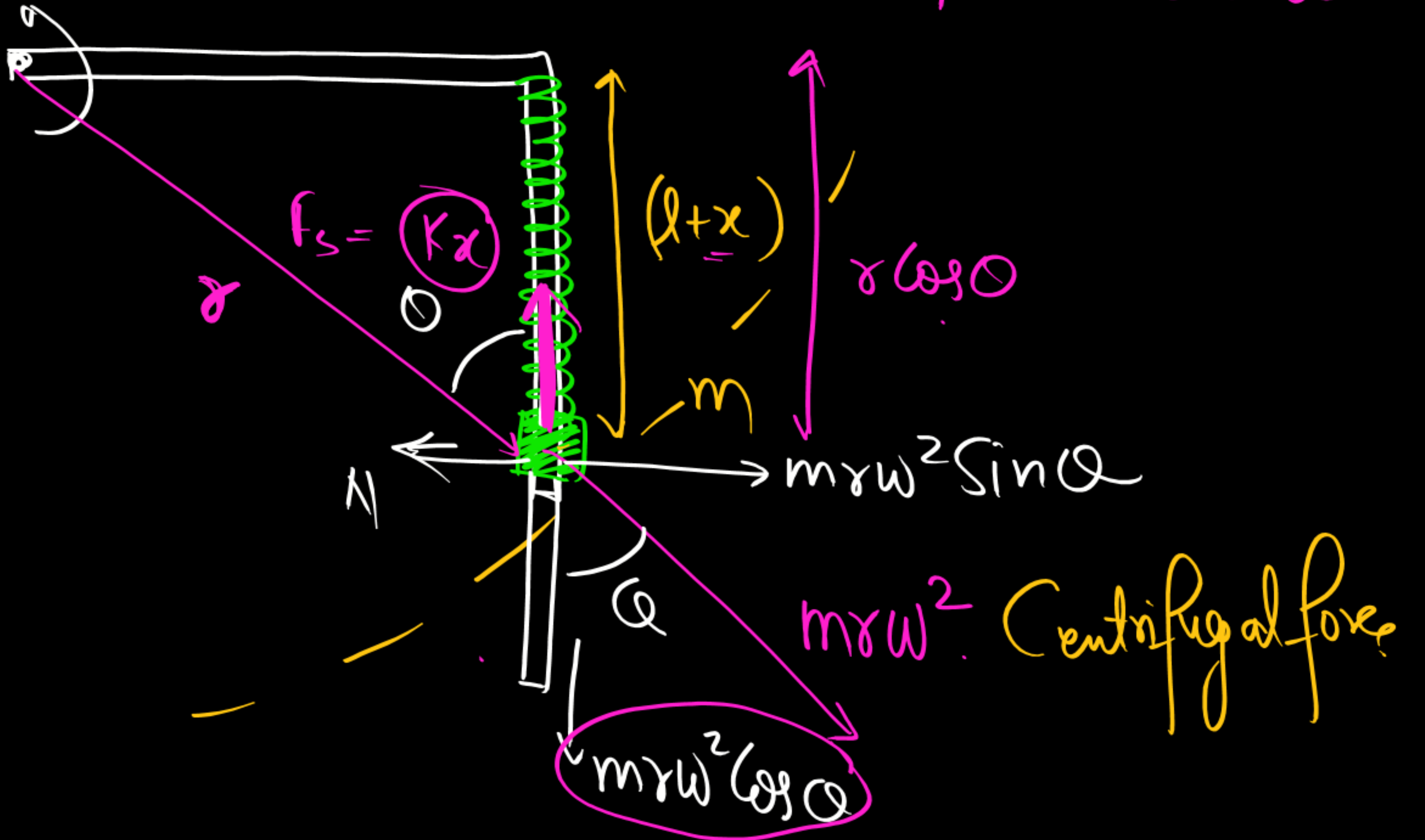
$$Kx = m\omega^2(r\cos\theta)$$

$$Kx = m\omega^2(l+x)$$

$$Kx = m\omega^2 l + m\omega^2 x$$

$$(K - m\omega^2)x = m\omega^2 l$$

$$x = \frac{m\omega^2 l}{K - m\omega^2}$$



Power

Rate of doing work:

$P = \dfrac{mgh}{t}$

Concept:

$$P = \frac{dW}{dt} \quad \left(\frac{J}{s}\right) = \text{watt}.$$

Slope of W/t graph

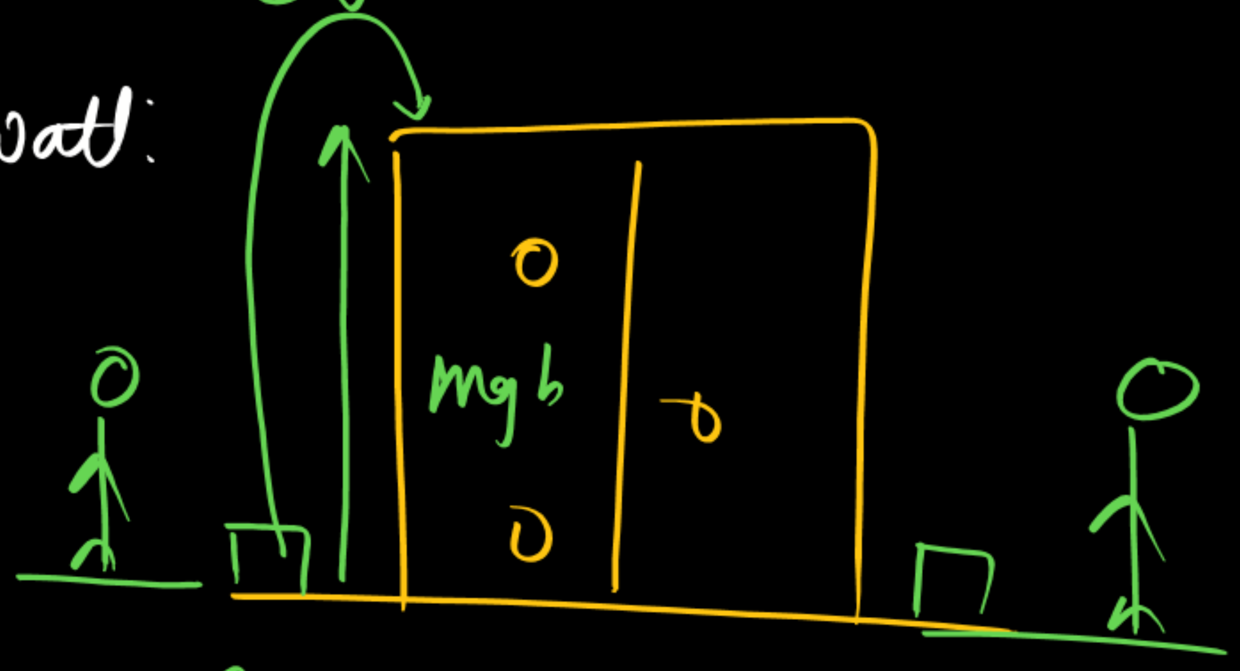
Power.

diff of work = Power

Area under P/t graph = Work.

$$\int P\,dt = \int dW$$

Integration of Power wrt time = Work



Power = +ve

Work is done on system $K_f > K_i$

Power = -ve

$K_f < K_i$.

$$\boxed{P = \vec{F} \cdot \vec{v}}$$

$\{$Contraint Relations$\}$

A box is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to :

(a) $t^{1/2}$

(b) $t^{3/4}$

(c) $t^{3/2}$ Ans

(d) t^2

$\rightarrow \boxed{x = f(t)}$

$P = \text{Constant}$

$Fv = \text{Constant}$

$mav = \text{Constant}$

$\frac{v\,dv}{dt} = c'$

$v\,dv = c'\,dt$

$\frac{v^2}{2} = c't + C$

$v^2 \propto t$

$v \propto t^{1/2}$

$\frac{dx}{dt} \propto t^{1/2}$

$\int dx \propto \int t^{1/2}\,dt$

$x \propto t^{3/2}$

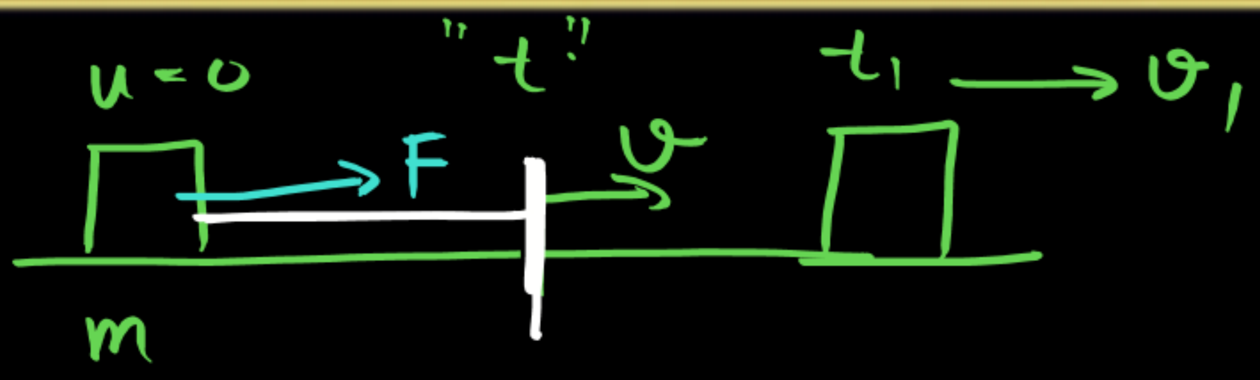
A body of mass m accelerates uniformly from rest to v_1 in time t_1. As a function of time t, the instantaneous power delivered to the body is **(AIEEE 2005)**

(a) $\frac{mv_1t}{t_1}$

(b) $\frac{mv_1^2t}{t_1}$

(c) $\frac{mv_1t_1^2}{t}$

(d) $\frac{mv_1^2t}{t_1^2}$ Ans



Impulse Momentum theo

$+Ft_1 = mv_1 - 0$

$f = \frac{mv_1}{t_1}$

velocity at any time

$F \times t = mv - 0$

$v = \frac{Ft}{m}$

$P = F \cdot v$

$= \frac{\cancel{m}v_1}{t_1} \times \frac{Ft}{\cancel{m}}$

$= \frac{v_1}{t_1} \times F \times t$

$P = \frac{v_1}{t_1} t \cdot \frac{mv_1}{t_1}$

$= \frac{mv_1^2 t}{t_1^2}$

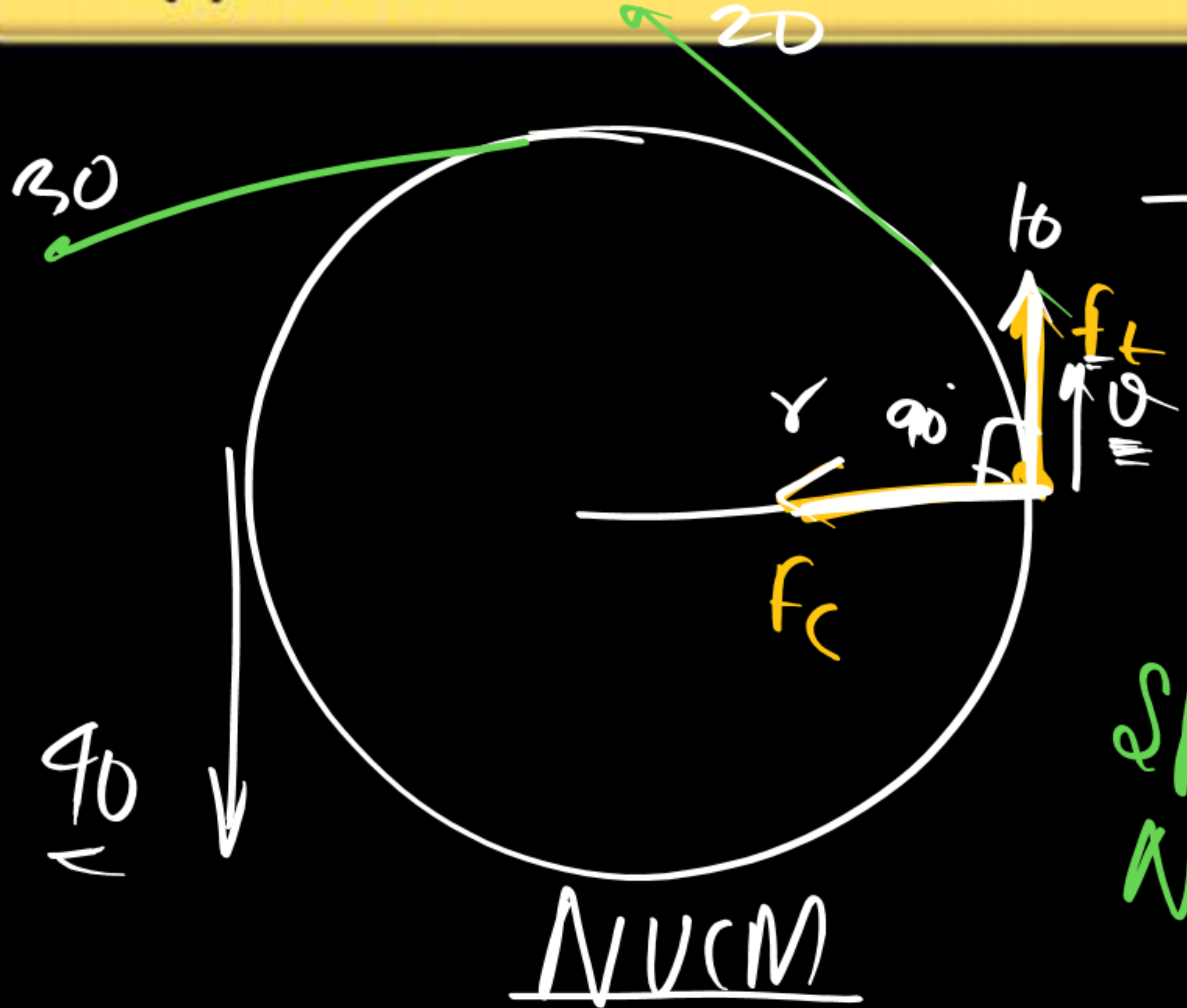
A particle of mass m is moving in a circular path of constant radius r such that centripetal acceleration, a_c varying with time is $a_c = k^2 rt^2$, where k is a constant. What is the power delivered to the particle by the force acting on it? [2008]

(a) $2\,mkr^2t$

(b) mkr^2t^2

(c) mk^2rt^3

(d) mk^2rt $= mk^2r^2t$ Ans



$$a_c = k^2 r t^2$$

$$\frac{v^2}{r} = k^2 r t^2$$

$$\boxed{v = krt}$$

Speed is fn of time.
NUCM

$$a_t = \left|\frac{dv}{dt}\right|$$

$$= a_t = kr. \quad f_T = m a_t$$

$P = f v \cos 90$ for Centripetal

$P_{cent} = 0$

$$P_T = F_T \cdot v \cos 0 = (mkr)\,krt.$$

An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator ($g = 10\ m/s^2$) must be at least: [JEE Main-2020 (January)]

($g = 10\ m/s^2$)

(a) 62360 W

(b) 56300 W

(c) 48000 W

(d) 66000 W

A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5s is: [JEE Advanced-2013]

A body of mass m is projected on a rough horizontal surface with velocity v_0 at $x = 0$. The friction coefficient on surface is varying with distance at $\mu = bx$. Find the maximum instantaneous power of friction force during motion of body and instantaneous power for friction force as a function of x. [JEE Advanced-2013]

THANK YOU ☺