MTH 201 Probability and Statistics: Mid Semester Exam

Total: 100 points

1) A biased coin is tossed and the first outcome is noted. The tossing is continued until the outcome is the complement of the first outcome, thus completing the first run. Let X denote the length of the first run. (Thus the event $\{X = 1\}$ is the disjoint union of HT and TH , the event $\{X = 2\}$ is the disjoint union of "2 Hs followed by T" and "2 Ts followed by H"). If p is the probability of Head in a single toss, find the PMF of X and calculate the expectation of X.

[15 points]

2) In an effort to boost welfare measures for poor people in an election year, the city council of West Lafayette decided to introduce Food Stamp Programme. Using the stamp, a poor person can buy food from any shop of the city on the day he receives the stamp The city created a list of people below the poverty line and realized that they can not provide food stamps to every body on the list every day. Thus the city came up with the following lottery scheme. If the total number of people on the list is n, then everyday they will write those n names on n food stamps and mix thoroughly in a container. Then m food stamps will be chosen from it without replacement. The chosen m people will get food stamps on that day.

(a) Find the probability that any person on the list is chosen for a food stamo for the day.

(b) Find the distribution (PMF) of the number of days a person must wait to get his first food stamp.

(c) Find the distribution (PMF) of the number of days a person must wait to get his k-th food stamp ($k > 1$).

(d) Find the distribution (PMF) of the number of food stamps a person obtains over a period of 45 days.

In order to understand the effectiveness of the programme, the city monitors the allocation of food stamps to two randomly chosen people on the list. Let X_i and Y_i, respectively, denote the number of food stamps received by the two individuals on day i of a 45 day period ($1 \leq i \leq 45$). let $Z_i = X_i Y_i$ for any i and define $Z = \sum_{i=1}^{45} Z_i$ Also define $X = \sum_{i=1}^{45} X_i$ and $Y = \sum_{i=1}^{45} Y_i$

(e) Find $P[Z_i = 1]$, for any i.

(f) Find the conditional PMF of X given $Z = 5$.

(g) Find the conditional PMF of X given $Z = 5$ and $Y = 10$.

(h) Find the conditional PMF of X_{15} given $Z = 5$.

(i) Find $E[X_{15} | Z_{15} = 1]$.

(j) Find $E[X_{15} | Z_{15} = 0]$.

3) A game of Badminton has two players on the opposite sides of the net. A game may be thought of as multiple rallies between the players. A rally is a sequence of alternate hits by the players that ends when a player misses it (which means either the player is unable to hit the shuttlecock or hits it in the net or outside the court). Suppose that player 1 hits with probability 0.7 and misses with probability 0.3. Player 2 on the other hand hits with probability 0.5. Assume a rally always starts with player 1.

(a) Calculate the probability that a rally has less than three hits.

(b) Calculate the probability that a rally ends with a miss by player 1.

(c) Calculate the expectation of the length of a rally, where the length includes all hits and the final miss. (you can leave your answer as a sum of series)

[25 points]