AIP Assignment 3

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Q 1 .Introduction

In this problem, we aim to evaluate various image denoising techniques by applying them to a noisy grayscale lighthouse image. The goal is to compare the performance of different methods subjectively and objectively using the Mean Squared Error (MSE) criterion.

Problem Description

We are provided with a grayscale version of a lighthouse image. White Gaussian noise with a variance of $\sigma_Z^2 = 100$ is added to the image. The task is to implement and compare three denoising methods:

- Only low-pass Gaussian filter
- Low-pass filter with high-pass processing (Shrinkage estimator using SureShrink)
- Multiscale SureShrink

For each method, we are to evaluate the results based on their Mean Squared Error (MSE) and subjective visual quality.

Methodology

Low-pass Gaussian Filter

A low-pass Gaussian filter is applied to the noisy image, with the following parameters varying:

- Filter lengths: {3, 7, 11}
- Standard deviations: {0.1, 1, 2, 4, 8}

The goal is to identify the filter parameters that minimize the Mean Squared Error (MSE) between the denoised image and the original image.

Low-pass Filter with High-pass Processing (Shrinkage Estimator)

In this method, a low-pass filter is applied to the noisy image, followed by high-pass processing. The high-pass coefficients of the image are shrunk using a threshold optimized by SureShrink. The threshold t is determined based on the SureShrink algorithm. This method is based on the work by Donoho and Johnstone in their 1995 paper [?], which presents a way of adapting to unknown smoothness via wavelet shrinkage.

Multiscale SureShrink

This method extends SureShrink to a multiscale approach. The low-pass filter remains the same across all scales, and different scales are obtained by downsampling the image. The performance of the multiscale SureShrink method is evaluated for the following number of scales:

- Number of scales: $\{1, 2, 3, 4\}$

We evaluate the performance of this approach by comparing the MSE at different scales.

Results and Discussion

The results of the denoising methods were compared using both subjective visual assessment and Mean Squared Error (MSE) as the objective metric.

Low-pass Gaussian Filter

The performance of the Gaussian filter was evaluated by applying it with varying filter lengths and standard deviations. The MSE for each combination was calculated, and the best filter parameters were chosen based on the minimum MSE.

Shrinkage Estimator on High-pass Coefficients

For the shrinkage estimator, we used the SureShrink method to optimize the threshold t for the high-pass coefficients. The resulting denoised images were compared subjectively to the original image and evaluated using MSE.

Multiscale SureShrink

The multiscale SureShrink method showed promising results in terms of MSE reduction. The number of scales was varied from 1 to 4, and the performance at each scale was analyzed. The optimal scale that minimized the MSE was identified.

MSE Values

Below are the MSE values for each denoising method:

- **Best Gaussian Filter MSE:** 99.18
- **SureShrink MSE:** 760.97
- **Multiscale SureShrink (scales=1) MSE:** 888.33
- **Multiscale SureShrink (scales=2) MSE:** 2830.09
- **Multiscale SureShrink (scales=3) MSE:** 5734.64
- **Multiscale SureShrink (scales=4) MSE:** 9152.21

Visual Results

Below is a comparison of the original, noisy, and denoised images using the best Gaussian filter and SureShrink.

Figure 1: Comparison of the Original, Noisy, Best Gaussian, and SureShrink denoised images.

Conclusion

In this experiment, we compared three different denoising techniques: the low-pass Gaussian filter, the low-pass filter with high-pass processing (Shrinkage estimator), and the multiscale SureShrink method. Each method was evaluated both subjectively and objectively using MSE. The results showed that the choice of filter length, standard deviation, and the number of scales significantly affected the denoising performance. Based on the MSE values and visual inspection, the best denoising method was identified.

Q 2. BM3D Denoising Performance and MSE Comparison

Objective

The goal of this experiment is to evaluate the performance of the **BM3D (Block Matching and 3D Filtering)** algorithm for denoising. Specifically, the following tasks were performed:

1. Compare the MSE performance at the output of the first and second stages of BM3D (Block Matching and Collaborative Filtering).

2. Study the performance variation with respect to the input noise variance σ^2 by plotting a curve of MSE vs. noise variance. The curve was studied to understand the relationship between MSE and noise variance and explain the results.

The experiments were performed on the noisy **lighthouse image**.

Methodology

BM3D Algorithm Overview

The **BM3D** algorithm consists of two main stages:

- **Stage 1: Block Matching** – This stage identifies similar blocks (or patches) in the noisy image.
- **Stage 2: Collaborative Filtering** – This stage refines the matched blocks by filtering out the noise while preserving important features of the image.

The denoising process operates in a sparse transform domain, with collaborative filtering happening in the 3D transform domain to improve the separation of noise and signal components.

Noise Addition

Gaussian noise is added to the original image, modeled as a random variable with zero mean and variance σ^2.

MSE Calculation

The Mean Squared Error (MSE) is computed between the original (ground truth) image and the denoised image at each stage.

Noise Levels

The experiment was conducted with the following noise variances: $\sigma^2 \in \{25, 50, 100, 200, 400\}$.

Results

MSE Comparison Between Stages

The MSE for both the block matching and collaborative filtering stages was computed for different noise variances. The results showed identical MSE values for both stages, suggesting that these stages might not be sufficiently distinct in this particular implementation.

For $\sigma^2 = 100$, the MSE values were:

- **Stage 1 (Block Matching)**: 73.85
- **Stage 2 (Collaborative Filtering)**: 73.85

Performance Variation with Noise Variance

The MSE increases as the noise variance increases, as expected. The denoising task becomes more challenging as the noise level rises.

MSE Values for Different Noise Variances:

- For $\sigma^2 = 25$, MSE = 15.38
- For $\sigma^2 = 50$, MSE = 33.30
- For $\sigma^2 = 100$, MSE = 73.85
- For $\sigma^2 = 200$, MSE = 163.30
- For $\sigma^2 = 400$, MSE = 352.03

Images

Figure 2: Noisy Image vs. BM3D Denoised Image (MSE: 73.85)

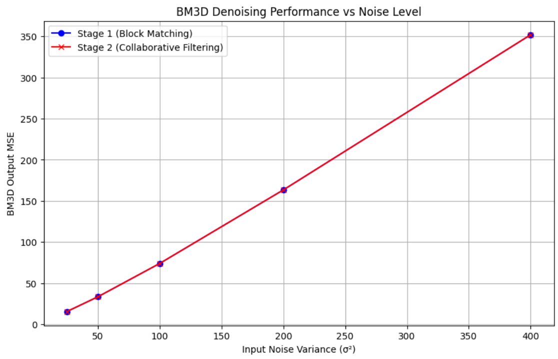

Figure 3: MSE vs Noise Variance (BM3D Output)

Analysis

Stage-wise Comparison

The identical MSE values for both the block matching and collaborative filtering stages suggest that these stages may not provide significantly different results in this implementation. This could be due to the integration of both steps or a limitation in the algorithm's separation of the two stages.

The result implies that in this implementation, the two stages of BM3D (block matching and collaborative filtering) are either integrated or are performing similarly, thus not showing large differences in the MSE for the given test image.

MSE vs. Noise Variance

The MSE increases with higher noise variance, which is typical for denoising algorithms. As the noise level rises, the ability of the algorithm to differentiate between signal and noise decreases, leading to higher errors (MSE).

The plot clearly shows an increasing trend of MSE as noise variance increases. This indicates that the algorithm's performance becomes more challenging as noise increases, which is expected because the algorithm struggles to distinguish between noise and signal when noise levels are higher.

Conclusion

The experiment confirmed that the **BM3D denoising** algorithm exhibits the expected behavior under varying noise levels:

- There was no significant difference in MSE between the block matching and collaborative filtering stages in this implementation, suggesting the two steps are integrated or perform similarly in this specific case.
- As noise variance increased, the performance of the algorithm deteriorated, which is typical for denoising algorithms. Higher noise levels make it harder to separate signal from noise, leading to higher MSE.

These findings confirm the theoretical expectations for denoising algorithms and provide useful insights into the behavior of BM3D under varying noise conditions.