NONLINEAR WAVELET DENOISING OF DATA SIGNALS

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ABSTRACT

The purpose of this work is to develop a new and efficiently implementable method for filtering transmitted (or received) noisy data signals with minimum loss of information. This method consists of a wavelet domain nonlinear filtering that combines a discrete wavelet transform based decomposition phase and a wavelet coefficient shrinkage phase. Data-like signals are generated, white noise is added, the results obtained after filtering are evaluated through signal to noise ratios and obtained gains. The proposed nonlinear denoising procedure performs a decomposition of identified noise and tries to find and cancel the correlation between the data signal and noise. The denoising method is carried out in Matlab environment test signal and added gaussian white noise is used. The results for different filtering techniques using different wavelet decompositons and different levels, are compared through signal to noise ratio and obtained gain.

Keywords: discrete wavelet transform, wavelet domain denoising.

1 INTRODUCTION

In almost every industrial application, as well as in data communications, measured signals, received data signals contain noise that originates from various sources such as different electrical and electronic measuring devices and circuits or the process itself. If the signal to noise ratio (SNR) is too small, then in stages of data processing, one may encounter misleading or erroneous results. The process of noise canceling with the purpose of extracting relevant information from measurement data has proven to be one of the most important steps in all digital signal processing methods. If noise owns the same frequency bands as the original signal, conventional filtering approaches encounter serious difficulties. In recent years, the field of signal processing has witnessed the introduction of a new powerful theory coming from time-frequency analysis, namely the wavelet analysis. The discrete wavelet transform is a powerful tool for the processing of nonstationary signals and is closely related to subband decomposition. The discrete wavelet transform based filtering uses thresholding techniques, there are several methods of choosing the analyzing function and the threshold values for different subbands. To present this type of de-noising, first the wavelet transform is briefly introduced as a background. Wavelets are functions defined as translations and re-scales of a single function, called the mother wavelet. By wavelet transform, the signal is decomposed into a number of frequency bands (scales), the transform coefficients are interpreted and processed scale by scale. This paper is divided as follows: Section 2 introduces the definition of the continuous and discrete wavelet transforms. In section 3 the proposed wavelet transform based nonlinear filtering method is presented. The results obtained in signal denoising are shown in section 4. Finally, the main conclusions concerning the wavelet transform based nonlinear denoising are summarized in section 5.

2 USING WAVELETS

2.1 The Continuous Wavelet Transform

During recent years wavelet methods have proved their use in various signal processing tasks. For any square integrable function f(t), the continuous wavelet transform (CWT) is defined as a function of two variables, the inner product of a time-varying signal f(t) and the set of wavelets $\psi_{s,\tau}(t)$ given by [3]:

$$Wf(s,\tau)=\langle f, \psi_{s,\tau}\rangle= s^{-1/2}\int[f(t)\ \psi^*((t-\tau)/s)]dt \quad (1)$$

where ψ^* denotes the complex conjugate of a complex valued function with zero mean and satisfying certain mathematical conditions [3]. The scaling and shifting the mother wavelet (ψ) with a factor of s and τ (with s>0), respectively, generate a family of functions called wavelets, given by:

$$\psi_{s,\tau}(t)= s^{-1/2}\psi((t-\tau)/s) \quad (2)$$

Figure 1 presents the interpretation of CWT of a data signal.

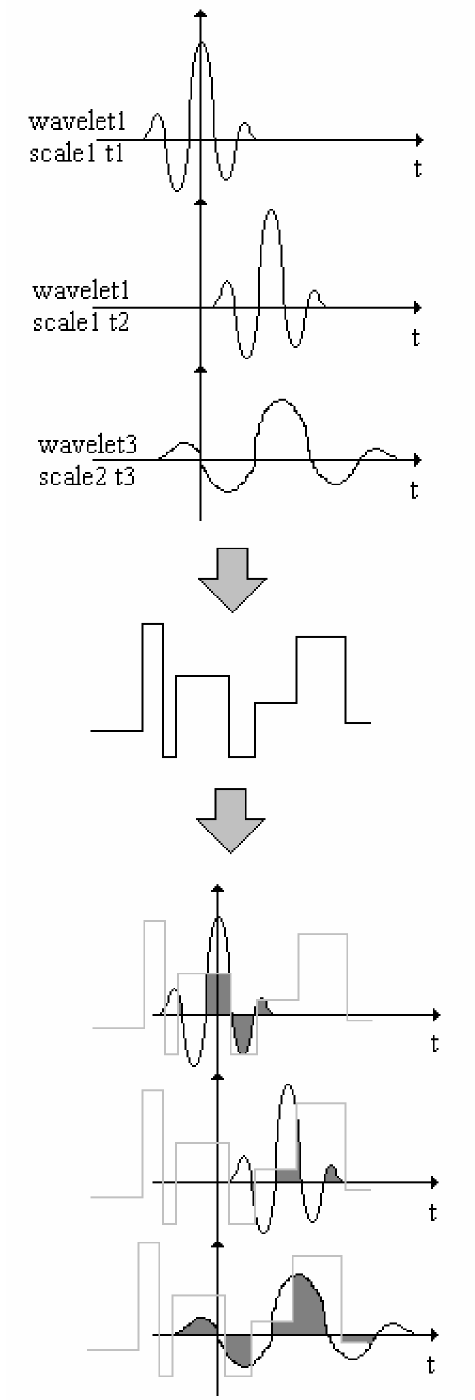

Figure 1 The Continuous Wavelet Transform as a convolution between data signal and scaled and shifted versions of the mother wavelet

The Continuous Wavelet Transform can be used to obtain spectrograms which show the frequency contents of signals as a function of time, but in signal processing the discrete transform is more practical.

2.2 The Discrete Wavelet Transform

The continuous wavelet transform of a signal of one variable is a function of two variables. Clearly, this transform is redundant. To optimize this transform one can introduce discrete values s and l and still have a lossless transform. That means:

$$s=2^{-j},\ \tau=k2^{-j},\ j,k\in Z \tag{3}$$

That will help to achieve the minimal but complete basis. Any coarser sampling will not produce a unique inverse transform. Dilations and translations of the "mother function," or "analyzing wavelet" $\psi(x)$ defines an orthogonal basis, the wavelet basis [3],[5]:

$$\psi_{s,l}(t)=2^{-s/2}\psi(2^{-s}t-l) \tag{4}$$

The variables l and s are integers that scale and dilate the mother function $\psi(x)$ to generate wavelets (analyzing func-tions). The scale index s indicates the wavelet's width, and the location index l gives its position. The mother wavelets are rescaled, or "dilated" by powers of two, and translated by integers. The discrete wavelet transform (DWT) represents a one-dimensional signal f(t) in terms of shifted versions of a low-pass scaling function of multi resolution analysis (MRA), $\varphi(t)$ and shifted and dilated versions of a prototype band-pass wavelet function of MRA, $\psi(t)$. A wavelet decomposition (or transform) simply re-expresses a function (a data signal) in term of the wavelet basis ($\psi_{s,l}(t)$). This amounts to decomposing the function space L2 into a direct sum of orthogonal subspaces (Wj) and choosing the combination of the orthonormal bases for L2. In the case of finite data with information up to a certain resolution level J, a wavelet transform performs a decomposition of the spaces VJ into a direct sum of orthogonal subspaces[3]:

$$V_J=W_{J-1}\oplus V_{J-1}=W_{J-1}\oplus W_{J-2}\oplus V_{J-2}=.\cdots. \tag{5}$$

and the union of the bases of these subspaces forms a basis for the wavelet decomposition. It is preferable to approach the DWT from the filter banks point of view. Given a signal, a low-pass and high-pass (using two quadrature mirror filters), half-band filtering is performed on it, obtaining two sequences. Having already done a half-band filtering, one can subsample the new sequences by a factor of two, following the Nyquist theorem. The process is iterated on the low pass signal as long as needed. The corresponding filter bank has a tree structure giving an octave decomposition of the frequency axis. Figure 4 shows a third level decomposition structure for a given data signal with the corresponding bandwidths [3]:

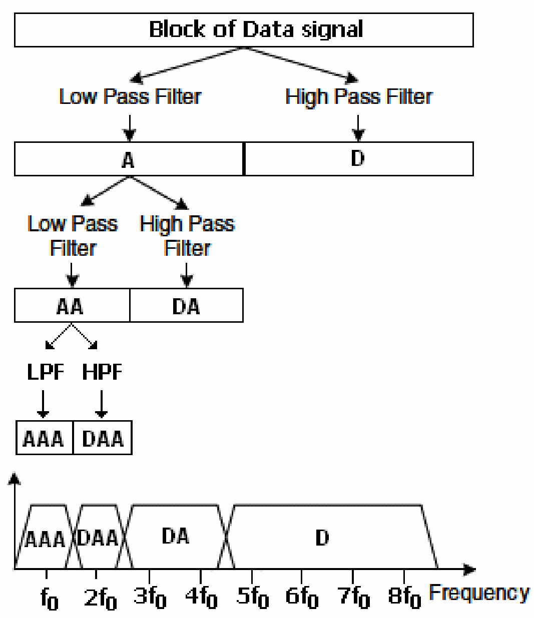

Figure 2 The Discrete Wavelet Transform based decomposition structure and the corresponding filter bank structure

As we can see this dyadic division of the bandwidth could be the key of subband filtering techniques which allow independent processing in these subbands (scales) [8],[9],[10].

3 THE PROPOSED NONLINEAR METHOD

The wavelet based signal denoising is performed using a technique called wavelet shrinkage and thresholding. This paper focuses on wavelet shrinkage denoising techniques. As developed originally by Donoho et al [1],[4], wavelet shrinkage denoising means non-linear thresholding of coefficients in wavelet transform domain. The technique works in the following way. When we decompose a data set using wavelets, we use filters that act as averaging filters and others that produce details. Some of the resulting wavelet coefficients correspond to details in the data set. If the details are small, they might be omitted without substantially affecting the main features of the data set. The idea of thresholding, then, is to set to zero all coefficients that are less than a particular threshold. These coefficients are used in an inverse wavelet transformation to reconstruct the data set. The signal is transformed, thresholded and inverse-transformed. The thresholding procedure is applied only to detail coefficients because we assume that the major part of noise is contained in this components.

Figure 3 The basic denoising procedure

In this section, the denoising of data signal corrupted by white Gaussian noise will be considered as:

$$x = d+n. \quad (6)$$

The underlying model for the noisy signal is the superposition of the signal and a zero mean gaussian white noise with a variance of σ^2. The power spectral density of a white noise is theoretically constant with amplitude of σ^2 along the whole frequency domain. To eliminate the noise, an algorithm will determine which wavelet coefficients should be thresholded. The simplest nonlinear thresholding rules for wavelet-based denoising assume that the wavelet coefficients are independent. Let W(•) and W−1(•) denote the forward and inverse wavelet transform operators. Let D(•, λ) denote the denoising operator with soft threshold λ. We intend to wavelet shrinkage denoise x[n] in order to recover d[n] as an estimate of x[n]. Then the main three steps are

$$y = W(x) \quad (7)$$

the wavelet transform,

$$y_\lambda = T(y, \lambda) \quad (8)$$

the thresholding procedure

$$d' = W^{-1}(y_\lambda) \quad (9)$$

and the reconstruction of signal from thersholded components.

Our approach applies the soft thresholding method and uses the so-called universal threshold given by [1]

$$Thr=\sigma(2\log(N))^{-1/2} \quad (10)$$

Where the standard deviation in the case of white noise will be estimated from the median value of its detail coefficients, calculated as follows [4] :

$$\sigma=(1/0.6745)\mathrm{median}(dj) \quad (11)$$

The chosen test signal has a rectangular shape as most of transmitted data signals and it is well suited for testing the proposed filtering algorithms. The test signal is presented on figure 3.

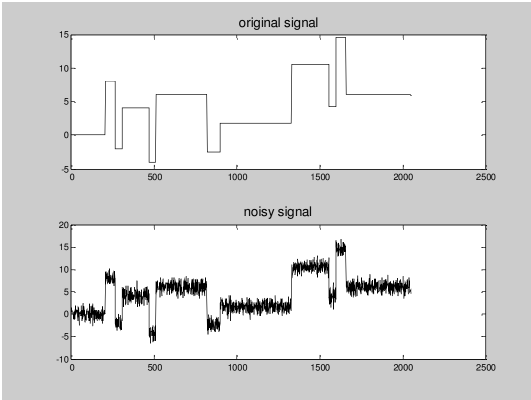

Figure 4 The test signal without and with added noise

The analysis of the different DWT levels shows that the first level detail sequence of the noisy data signal is highly influenced by the noise energy and it can be see as a superposition of the noise and a set of peaks which are related to the main variations from the data signal (figure5).

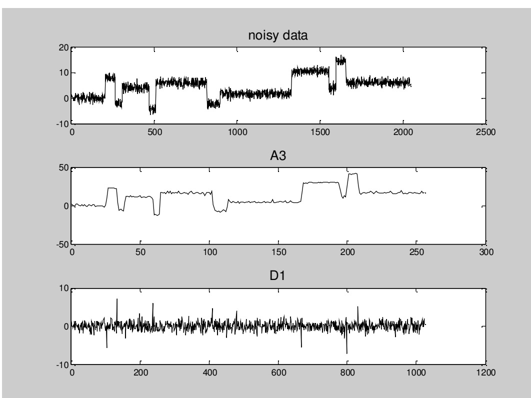

Figure 5 The representation of third level approximation and first level decomposition of the noisy data signal

The thresholding is based on a value λ that is used to compare with all the detailed coefficients. There are two types of thresholding, the hard and the soft method, as presented in Figure 6. Hard thresholding is the process of setting to zero the coefficients whose absolute values are lower than the threshold λ. Soft thresholding is another method by first setting to zero coefficients whose absolute values are lower than the threshold λ and then shrinking the nonzero coefficients toward zero. Hard thresholding provides better edge preservation in comparison with the soft thresholding but soft thresholding provides smoother results. Based on these properties, the soft thresholding technique is applied in this work.

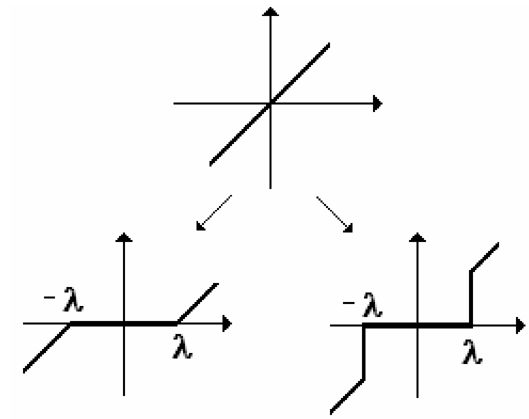

Figure 6 The soft and hard thresholding methods

The proposed algorithm performs a third level discrete wavelet transform based decomposition, the noise is estimated from the first level detail coefficients. After that the first detail component is also decomposed in detail coefficients to find the correlation between the data signal and the added noise. After that the detail coefficients are thresholded in a particular way, as follows:

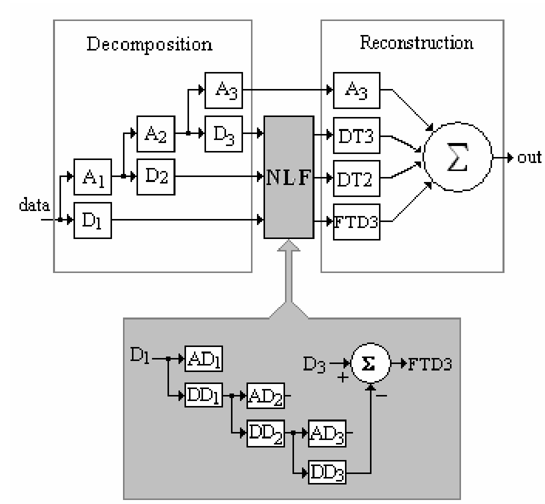

Figure 7 The proposed nonlinear filtering procedure

The proposed filtering process consists of these steps:
1) third level DWT decomposition of the noisy data signal and noise level estimation from the first detail coefficients (D1);
2) identifying the resulting three detail sequences (D1, D2, D3,) and the approximation sequence (A3);
3) third level DWT decomposition of the estimated

noise signal (D1) and identification of the 3rd level detail sequence (DD3);
4) generation of the 3rd level filtered detail sequence given by: FDT3=D3 –DD3;
5) setting the used denoising threshold (Thr), given by (10)
6) thresholding the set of the detail sequences (D1, D2, and FDT3), with respect to the computed threshold (T), which results the set of the sequences (DT1, DT2, and FDT3);
7) reconstruction of the denoised data signal using the inverse wavelet tarnsform (IDWT) giving the 3 detail sequences (DT1, DT2, and FDT3) and the approximation (non-thresholded) sequence (A3).

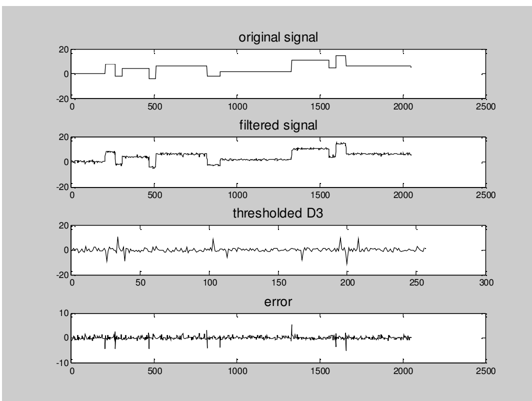

Figure 8 Subband filtering results

The DWT based decomposition is performed using Matlab (Wavelet Toolbox) environment [6], the used wavelet function was the 'Daubechies4' (db4), some of subband filtering results are presented on figure 8. Both of soft and hard thresholding were applied, but the obtained results are better if soft thresholding method is applied.

4 RESULTS

The parameters used to evaluate filtering performances are: the signal to noise ratio, the absolute mean error and the obtained gain defined as follows:

$$SNR=10\log((P_{\text{original signal}})/(P_{\text{noise}}) \quad (12)$$

$$SNR1=10\log((P_{\text{filtered signal}})/(P_{\text{estimated noise}}) \quad (13)$$

Where the estimated noise is calculated as:

$$\text{estimated noise}= \text{noisy signal}-\text{filtered signal} \quad (14)$$

$$\text{error}=\text{mean}(\text{abs}(\text{original signal}-\text{filtered signal})) \quad (15)$$

$$\text{Gain}= SNR1- SNR \quad (16)$$

The experimental results obtained by soft thresholding method (signal to noise ratio, obtained gain and reconstruction error) are presented below.

Table 1: The obtained values in nonlinear filtering procedure using soft thresholding

SNR	SNR1	Gain	Error
35.6235	25.4709	-10.1526	0.0720
29.4174	25.0709	-4.3466	0.1109
25.8944	24.4420	-1.4524	0.1542
23.6440	24.0865	0.4426	0.1856
21.3432	23.2987	1.955	0.2316
19.9354	22.6418	2.7064	0.2813
18.3491	22.1792	3.7943	0.3069
17.7096	21.5905	3.8809	0.3464
16.4619	20.7054	4.2436	0.3881
15.7319	19.7694	4.0295	0.4417

The graphical representation of these results is presented in figure 9.

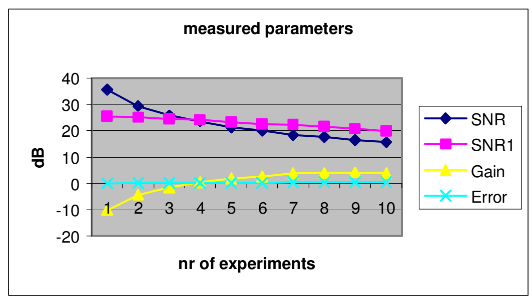

Figure 9 Experimental results

5 CONCLUSIONS

The presented data signal denoising method uses the same procedures as the ordinary discrete wavelet transform or the wavelet packet decomposition, but introduces a noise-discarded detail component in the signal reconstruction which in most of cases leads to a smoother data signal. The results show that the nonlinear filtering efficiency is greater if the noise is greater too. This algorithm was carried out on a test signal in Matlab environment and gave better results (obtained gain) for lower signal to noise ratios. As further work, this algorithm may be implemented in a field programmable gate array (FPGA) based reconfigurable hardware structure in order to perform real-time filtering of data signals.

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