UNIT-01 Mathematical Reasoning

**Statement :-** A statement is declarative sentence. i.e. either true/false.

| | declaration | Truth value |
|---|---|---|
| | $\{x;\ x^2 = 36\},\ x \in (6, -6)$ | True |
| | $3 < 9$ | True |

# Statement is denoted by Symbol (∴). p, q, r, s, ...... (∴).

**Logical connectives :-** Logical connectivity are the words or symbols used to combine two statement.

| Connective words | Name of the connective | Symbols |
|---|---|---|
| NOT | Negation | $\sim$ or $\neg$ |
| AND | conjunction | $\wedge$ meet |
| OR | Disjunction | $\vee$ join |
| If .... then | conditional statement | $\rightarrow$, or $\Rightarrow$ |
| If and only if | Biconditional statement | $\Leftrightarrow$ or $\longleftrightarrow$ |

# $B(B, +, \cdot, ', 0, 1)$

Types of Sentence -

1) **Simple sentences :-** A sentences has no connective words is called simple sentences.

ii) **Compound Sentences :-** A compound sentence is composed various connective words.

eg - π is greater than 3 $\underbrace{\text{and}}_{\text{connective words.}}$ π is less than 3.2

$p \equiv$ π is greater than 3

$q \equiv$ π is less than 3.2

eg - a is equal to 4 or b is equal to 4

$p \equiv$ a is equal to 4
$q \equiv$ b is equal to 4.

Note :- ( ∪ or ∨ : disjunction )

( ∩ AND ∧ . Conjunction )

iii) Principle connectivity :-

Logical connectives-

1) Negation (NOT)(∼) :- If p is a statement then negation of p is a statement not p denoted by ∼p

i.e. If p is true then ∼p is false.

* Truth table for Negation.

| p | ∼p |
|---|---|
| T | F |
| F | T |

$p \equiv 2+3>1$ T
$\sim p \equiv 2+3<1$ F

Conjunction (AND) ($\wedge$) :- If p and q are two statement then conjunction of p and q is compound sentence denoted by. $p \wedge q$

| p | q | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

Note :- Formula $2^n$ where n is the number of statements.
If compound sentences have two components p and q then truth table has $2^2 = 4$ Rows or pair.

Note :- 1) Conjunction, $\cdot$, $\wedge$, T T $\rightarrow$ T otherwise F

ii) Disjunction, +, $\vee$, F F $\rightarrow$ F otherwise T

Disjunction 'OR' '$\vee$' :- If p and q are two Statement then disjunction of p and q is the compound sentence ~~do~~ $p \vee q$.
i.e. p or q.

# The compound Statement $p \vee q$ are false only when p & q both are false otherwise it is true.

Truth table :-

| p | q | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

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oated

"If .. than" ($\Rightarrow$)

**Conditional Statement :-** If p and q are the statement then the compound statement 'If p then q denoted by $p \Rightarrow q$, is called conditional statement or (implication).

eg – p : I am hungry
q : I will eat

$p \Rightarrow q$

Truth table :-

| p | q | $p \Rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

**Note :-** The conditional statement is false only when first statement is true and second statement state is false.

i.e. If p is true and q is false then $p \Rightarrow q$ is false.

**Biconditional Statement ($\Leftrightarrow$) :-** If p and q are the statement then the compound statement p if and only if q denoted by $p \Leftrightarrow q$ is called biconditional statement.

eg – p : $3 > 2$
q : $3-2 > 0$
$p \Leftrightarrow q$

**Note :-** Biconditiona[l] ...
Component hav[e] ...

T T → T
F F → T

Truth table :-

| p | q | ... |
|---|---|---|
| T | T | |
| T | F | |
| F | T | |
| F | F | |

**Note :** $\wedge$

$\vee$

$\Rightarrow$

$\Leftrightarrow$

"If .. than" ($\Rightarrow$).

**Conditional Statement :-** If p and q are the statement then the compound statement 'If p then q denoted by $p \Rightarrow q$, is called conditional statement or (implication).

eg — $p$ : I am hungry
$q$ : I will eat

$$p \Rightarrow q$$

**Truth table :-**

| $p$ | $q$ | $p \Rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

**Note :-** The conditional statement is false only when first statement is true and second statement state is false.

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**Biconditional Statement ($\Leftrightarrow$) :-** If p and q are the statement then the compound statement p if and only if q denoted by $p \Leftrightarrow q$ is called biconditional statement.

eg — $p : 3 > 2$
$q : 3 - 2 > 0$
$p \Leftrightarrow q$

**Note :-** Bicond
Component

T T →
F F →

**Truth table :-**

| $p$ | $q$ |
|---|---|
| T | T |
| T | F |
| F | T |
| F | F |

**Note :**

Note :- Biconditional statement is true only when both component have same value.

T T → T } Here all conditions have same value then,
F F → T } condition is also true, otherwise false.

Truth table :-

| p | q | p ⇔ q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

Note:

| | | p q | |
|---|---|---|---|
| ∧ | conjunction | T T → T | otherwise false |
| ∨ | disjunction | F F → F | otherwise True. |
| ⇒ | conditional | T F → F | otherwise true |
| ⇔ | Biconditional | T T → T<br>F F → T | otherwise false. |

Q.1. $(p \Leftrightarrow q \wedge r) \Rightarrow (\sim r \Rightarrow \sim p)$ find the truth table for the given statement.

Solve-

| $p$ | $q$ | $r$ | $\sim r$ | $q \wedge r$ | $\sim p$ | $p \Leftrightarrow q \wedge r$ | $\sim r \Rightarrow \sim p$ | $(p \Leftrightarrow q \wedge r) \Rightarrow (\sim r \Rightarrow \sim p)$ |
|---|---|---|---|---|---|---|---|---|
| T | T | T | F | T | F | T | T | T |
| T | T | F | T | F | F | F | F | T |
| T | F | T | F | F | F | F | T | T |
| T | F | F | T | F | F | F | F | T |
| F | T | T | F | T | T | F | T | T |
| F | T | F | T | F | T | T | T | T |
| F | F | T | F | F | T | T | T | T |
| F | F | F | T | F | T | T | T | T |

Q.2- $[\sim p \vee (p \Leftrightarrow r)] \wedge (r \Rightarrow q)$

Solution-

| $p$ | $q$ | $r$ | $\sim p$ | $p \Leftrightarrow r$ | $r \Rightarrow q$ | $\sim p \vee p \Leftrightarrow r$ | $[\sim p \vee (p \Leftrightarrow r)] \wedge (r \Rightarrow q)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | T | T | T | T |
| T | T | F | F | F | T | F | F |
| T | F | T | F | T | F | T | F |
| T | F | F | F | F | T | F | F |
| F | T | T | T | F | T | T | T |
| F | T | F | T | T | T | T | T |
| F | F | T | T | F | F | T | F |
| F | F | F | T | T | T | T | T |

Tautology :-

eg- $p \vee \sim p$

| $p$ |
|---|
| T |
| F |

Contigency :-

eg- $p \Rightarrow \sim$

| $p$ |
|---|
| T |
| F |

Contraduct

eg- $p \Leftrightarrow \sim$

| $p$ |
|---|
| T |
| F |

Q.1. Prove t

$[(p \rightarrow$

Solution-

| $p$ | $q$ |
|---|---|
| T | T |
| T | T |
| T | F |
| T | F |
| F | T |
| F | T |
| F | F |
| F | F |

**Tautalogy :-** Tautalogy is a statement which is true for all possible value of its components.

eg- $p \vee \sim p$

| $p$ | $\sim p$ | $p \vee \sim p$ |
|---|---|---|
| T | F | T |
| F | T | T |

**Contigency :-** A contigency is a statement that can be either true or false.

eg- $p \Rightarrow \sim p$ is a contigency.

| $p$ | $\sim p$ | $p \Rightarrow \sim p$ |
|---|---|---|
| T | F | F |
| F | T | T |

**Contraduction :-** A contraduction is a statement which is false for all possible values of it components.

eg- $p \Leftrightarrow \sim p$

| $p$ | $\sim p$ | $p \Leftrightarrow \sim p$ |
|---|---|---|
| T | F | F |
| F | T | F |

$p \wedge \sim p$

| $p$ | $\sim p$ | $p \wedge \sim p$ |
|---|---|---|
| T | F | F |
| F | T | F |

Q.1. Prove that the given compound statement is a tautalogy.
$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Solution-

| $p$ | $q$ | $r$ | $p \rightarrow q$ | $q \rightarrow r$ | $p \rightarrow r$ | $(p \rightarrow q) \wedge (q \rightarrow r)$ | $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | T | F | T |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T |

**Tautology :-** Tautology is a statement which is true for all possible value of its components.

eg- $p \vee \sim p$

| $p$ | $\sim p$ | $p \vee \sim p$ |
|---|---|---|
| T | F | T |
| F | T | T |

**Contigency :-** A contigency is a statement that can be either true or false.

eg- $p \Rightarrow \sim p$ is a contigency.

| $p$ | $\sim p$ | $p \Rightarrow \sim p$ |
|---|---|---|
| T | F | F |
| F | T | T |

**Contraduction :-** A contraduction is a statement which is false for all possible values of it components.

eg- $p \Leftrightarrow \sim p$

| $p$ | $\sim p$ | $p \Leftrightarrow \sim p$ |
|---|---|---|
| T | F | F |
| F | T | F |

$p \wedge \sim p$

| $p$ | $\sim p$ | $p \wedge \sim p$ |
|---|---|---|
| T | F | F |
| F | T | F |

Q.1. Prove that the given compound statement is a tautology.
$[(p \to q) \wedge (q \to r)] \to (p \to r)$

Solution-

| $p$ | $q$ | $r$ | $p \to q$ | $q \to r$ | $p \to r$ | $(p\to q) \wedge (q \to r)$ | $[(p \to q) \wedge (q \to r)] \to (p \to r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | T | F | T |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T |

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A statement

| Always true | either true or false | always false |
|---|---|---|
| T | T/F | F |
| Tautology | contigency | contraduction |
| | | unsatisfiable |
| | | logicaly invalid |

Satisfiable.
i.e logicalyvalid

Satisfiablity :- A compound statement is a Satisfiability if there is atleast one true result in its truth table.

Q. Prove that the compound statement is a ~~contigency~~ tautology $(p \Rightarrow q) \vee r \Leftrightarrow [(p \vee r) \Rightarrow (q \vee r)]$

| $p$ | $q$ | $r$ | $p \Rightarrow q$ | $p \vee r$ | $q \vee r$ | $(p \Rightarrow q) \vee r$ | $(p \vee r) \Rightarrow (q \vee r)$ | All |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T | T |
| T | T | F | T | T | T | T | T | T |
| T | F | T | F | T | T | T | T | T |
| T | F | F | F | T | F | F | F | T |
| F | T | T | T | T | T | T | T | T |
| F | T | F | T | F | T | T | T | T |
| F | F | T | T | T | T | T | T | T |
| F | F | F | T | F | F | T | T | T |

Since, the last coloum of the truth table is always true.
Therefore given compound statement is a tautology.

Q. Prove that the compound statement is a contigens Contraduction.

$$[(p \vee q) \wedge (\sim p)] \wedge (\sim q)$$

Solution:-

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \vee q$ | $(p \vee q) \wedge \sim p$ | $[(p \vee q) \wedge (\sim p)] \wedge (\sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F |
| T | F | F | T | T | F | F |
| F | T | T | F | T | T | F |
| F | F | T | T | F | F | F |

Logicaly equivalence :- Two statements are called logicaly equlvalence if the truth values of both the statement are always Identical.

Two statement $p$ & $q$ are equivalent then it is denoted by $p \cong q$.