k-means-and-pca

June 12, 2023

1 Unsupervised Learning

1.1 K-Means Clustering

K-Means Clustering is an unsupervised learning algorithm, which works by grouping similar datapoints together. The algorithm is: 1. Assign 'k' cluster centroids randomly 2. Update cluster labels- assign points to the nearest centroid 3. Update centroids- set center to the mean of each cluster Steps 3 and 4 are repeated until either centroids reach convergence, i.e., they cannot be moved furtherm, the maximum number of iterations has been reached.

```
[]: import pandas as pd
import numpy as np
import random as rand
import matplotlib.pyplot as plt
from scipy.optimize import linear_sum_assignment
```

```
def rotate(input_list):
    curr, prev = 0, 0
    if len(input_list) == 0:
        return input_list
    for index, el in enumerate(input_list):
        if index == 0:
            prev = el
        else:
            curr = el
            input_list[index] = prev
            prev = curr
    input_list[0] = curr
    return input_list
```

```
[]: list1 = [1, 2, 3, 4]
    rotate(list1)
    print(f"list1 = {list1}")

list1 = [4, 1, 2, 3]

[]: class KMeans():
```

```
"""Unsupervised machine learning algorithm which groups 'k' similar_{f \sqcup}
⇔datapoints."""
  def __init__(self, k = 3, max_iter = 1000):
       Constructor
       # Parameters
           k: default is 3
           max_iter: default is 1000
       11 11 11
       self.k = k
       self.max_iter = max_iter
       self.centroids = []
       self.clusters = [[] for _ in range(k)]
       self.x_size, self.num_features = 0, 0
  def predict(self, x_test):
       This method predicts the clusters based on similarity.
       # Parameters
           x test: Test data
       # Returns
           List of list of rows rows, where the index of lists at the second_{\sqcup}
⇔level indicates the cluster group.
           So, for return value [[row1, row2, row3], [row4, row5, row6]]],
           rows 1, 2, and 3 belong to cluster 0, and rows 4, 5, and 6 belong
\hookrightarrow to cluster 1.
       self.X = x_test
       converged = False
       #converting into appropriate format
       x_np = x_test.to_numpy()
       self.x_size, self.num_features = np.shape(x_np)
       #create random centroids
       random_sample_idxs = np.random.choice(self.x_size, self.k, replace = u
→False)
       self.centroids = np.asarray([x_np[i] for i in random_sample_idxs])
       i = 1
       while (not converged and i < self.max_iter):</pre>
           # update cluster labels
           self.clusters = [[] for _ in range(self.k)]
           for j in range(len(x_np)):
               dist_arr = self.get_distance(x_np[j])
               self.clusters[np.argmin(dist_arr)].append(x_np[j])
```

```
# update centroids
          old_centroids = self.centroids
          centroids = np.zeros((self.k, self.num_features))
          for idx, cluster in enumerate(self.clusters):
              cluster_mean = np.mean(x_np[idx], axis = 0)
              centroids[idx] = cluster_mean
          converged = (old_centroids == centroids.all).all()
          i += 1
      return self.clusters
  def get_distance(self, point):
      ⇔nearest centroid."""
      to return = []
      dist = lambda x, y: np.sqrt(np.sum((x - y)**2))
      for centroid in self.centroids:
          to_return.append(dist(centroid, point))
      # return np.ndarray(to return)
      return np.asarray(to_return, dtype = np.float32)
  def plot(self):
      """Plots the clusters. Plotting is done between the index of the rows_{\sqcup}
⇔of each row in a cluster and the cluster number.
      Can be compared with a plot between index number of row and label to_{\sqcup}
⇔check accuracy."""
      x = \prod
      y = []
      for i, cluster in enumerate(self.clusters):
          for j, datapoint in enumerate(cluster):
              matching_rows_index = np.where(np.all(self.X.values ==_
⇔datapoint, axis = 1))
              x.append(matching_rows_index[0][0])
              y.append(int(i))
      plt.scatter(x, y)
      plt.xlabel("Index")
      plt.ylabel("Cluster")
      plt.show()
```

1.2 Principal Component Analysis

1.2.1 Theory

Principal Component Analysis (PCA) is an unsupervised machine learning algorithm, which has applications in dimensionality reduction of data. PCA is used for feature extraction by condensing the data into lesser dimensions while preserving as much information of the original data as possible. Such reduction helps to prevent the model from overfitting on the training data, and can also enable

us to visualize it.

Math PCA works by first projecting the datapoints onto a unit vector, and then choosing whichever vector captures the highest variance as the principal component.

$$\frac{Proj_p u = P.u}{||u||}$$

Since u is a unit vector, ||u|| is equal to 1. So, the equation becomes,

$$Proj_p u = P.u$$

Hence, to get the projection of a point on a unit vector, we just calculate the dot product between the two. #### Working We first find the covariance matrix between all attributes, and then find eigenvectors and eigenvalues from this vector. The eigenvectors with the maximum eigenvalues will be the principal components. Then, to project all datapoints on the principal components, we take the dot product of the datapoints with the components.

1.2.2 Implementation

```
[]: import pandas as pd import numpy as np import matplotlib.pyplot as plt
```

```
[]: class PCA:
         def __init__(self, num_components):
             self.num_components = num_components
             self.eigenvalues = None
             self.eigenvectors = None
             self.X = None
         def fit(self, x_train):
             self.X = x_train
             self.mean = np.mean(x_train, axis = 0)
             self.X -= self.mean
             cov = np.cov(self.X.T)
             eigenvectors, eigenvalues = np.linalg.eig(cov)
             eigenvectors = eigenvectors.T
             idxs = np.argsort(eigenvalues)[::-1]
             eigenvalues = eigenvalues[idxs]
             eigenvectors = eigenvectors[idxs]
             self.components = eigenvectors[:self.num_components]
         def transform(self):
             return np.dot(self.X, self.components.T)
```

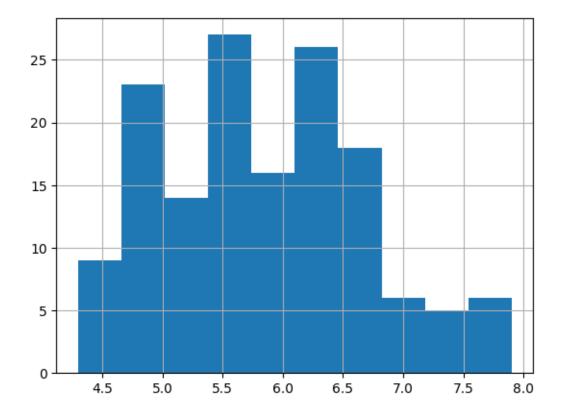
1.3 Example Dataset

```
[]: df = pd.read_csv('./Iris Dataset.csv')
    df.head()
```

[]:	Id	${\tt SepalLengthCm}$	${\tt SepalWidthCm}$	${\tt PetalLengthCm}$	${\tt PetalWidthCm}$	Species
0	1	5.1	3.5	1.4	0.2	Iris-setosa
1	2	4.9	3.0	1.4	0.2	Iris-setosa
2	3	4.7	3.2	1.3	0.2	Iris-setosa
3	4	4.6	3.1	1.5	0.2	Iris-setosa
4	5	5.0	3.6	1.4	0.2	Iris-setosa

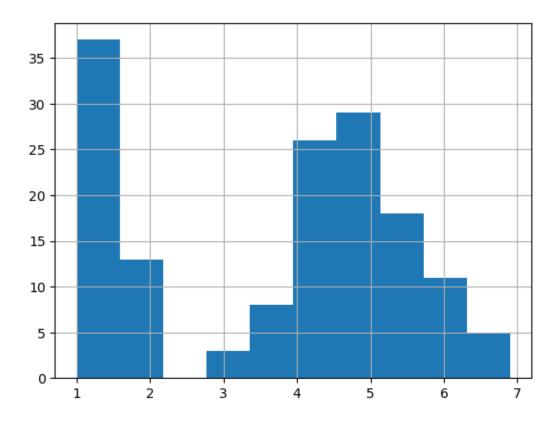
```
[]: df['SepalLengthCm'].hist()
```

[]: <Axes: >



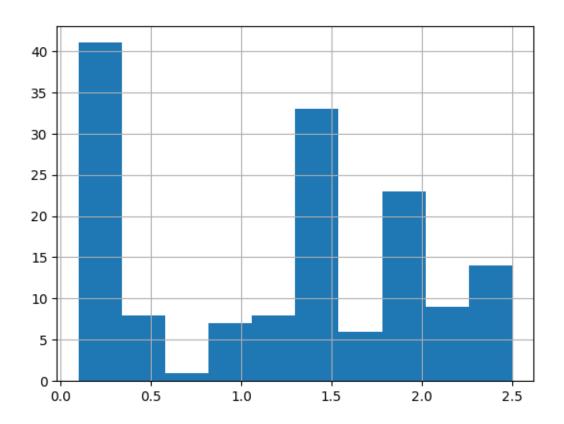
```
[]: df['PetalLengthCm'].hist()
```

[]: <Axes: >



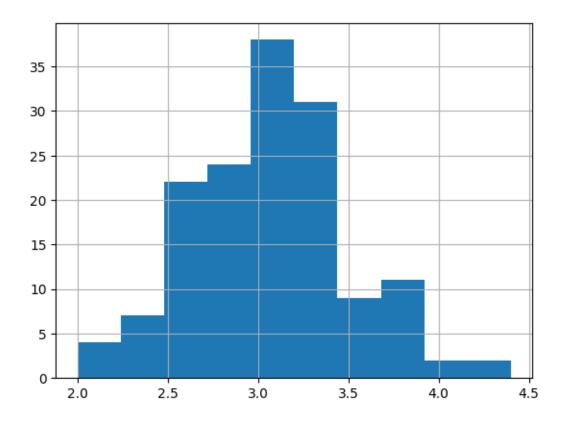
```
[]: df['PetalWidthCm'].hist()
```

[]: <Axes: >



```
[]: df['SepalWidthCm'].hist()
```

[]: <Axes: >

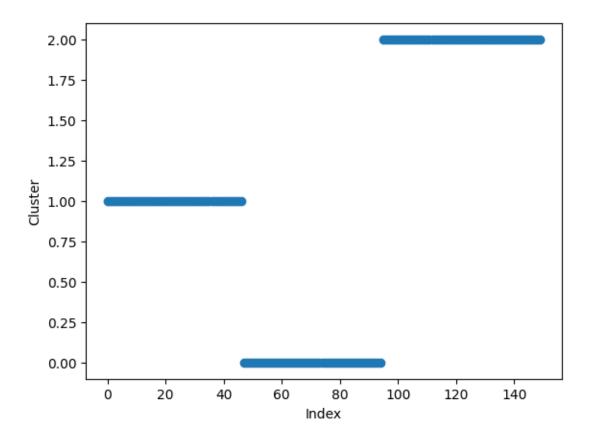


Removing the 'Species' column and storing it for later comparison.

```
[ ]: y_test = df['Species']
x_train = df.drop(['Species'], axis = 1)
```

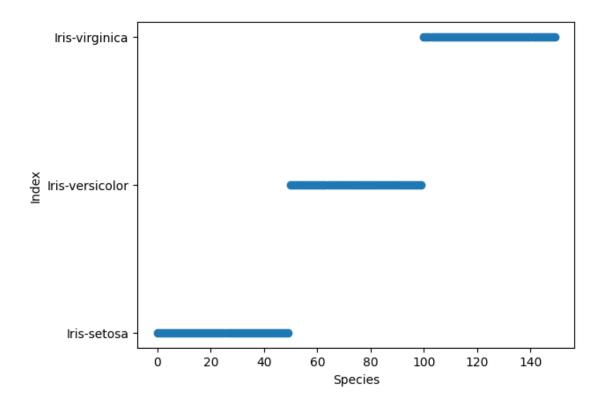
1.3.1 Testing K-Means

```
[]: x_train.head()
[]:
            SepalLengthCm SepalWidthCm PetalLengthCm PetalWidthCm
                                                                  0.2
     0
         1
                      5.1
                                     3.5
                                                    1.4
         2
                      4.9
                                                                  0.2
                                     3.0
                                                    1.4
     1
     2
         3
                      4.7
                                     3.2
                                                    1.3
                                                                  0.2
     3
                      4.6
                                                                  0.2
         4
                                     3.1
                                                    1.5
         5
                      5.0
                                     3.6
                                                    1.4
                                                                  0.2
[]: k_means = KMeans()
     k_means.predict(x_train);
[]: k_means.plot()
```



```
[]: plt.scatter(x_train.index, y_test)
   plt.xlabel("Species")
   plt.ylabel("Index")
```

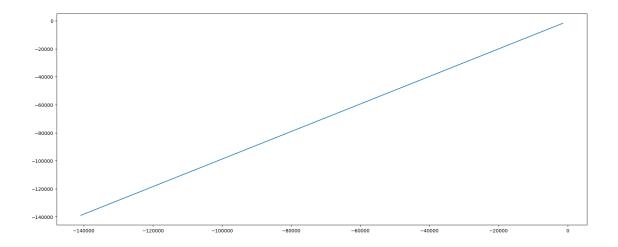
[]: Text(0, 0.5, 'Index')



Here, we can clearly see that our model clusters the data well, as most point belonging to a particular species are clustered in the same group. In this case, cluster 0 corresponds to Iris-versicolor, cluster 1 corresponds to Iris-setosa, and cluster 2 corresponds to Iris-virginica.

1.3.2 Testing PCA

Creating a new PCA object and training and transforming the data.



We can see that the original dataset had a sample size of 150 samples and 5 attributes, whereas after the PCA, the dataset has the same number of samples but only 2 attributes. Hence, using PCA, we have successfully reduced the number of dimensions of the dataset.

[]: