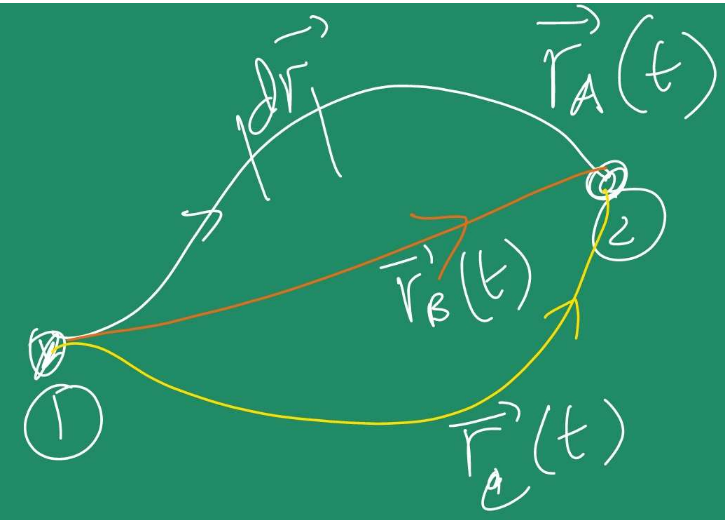
OTP : 3460

Work-Energy Theorem:

$$W_{12} = \int_{1}^{2} \vec{F} \cdot d\vec{r}$$

$\underbrace{\vec{F} \cdot d\vec{r}}$ → Line Integral

(Functional Integral)



$$\frac{d}{dt}(\vec{v} \cdot \vec{v}) = \vec{v} \cdot \frac{d\vec{v}}{dt} + \frac{d\vec{v}}{dt} \cdot \vec{v} = 2\left(\frac{d\vec{v}}{dt} \cdot \vec{v}\right)$$

$$d\vec{r} = \underbrace{\hat{x}\, dx + \hat{y}\, dy + \hat{z}\, dz}_{} = \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} dx/dt \\ dy/dt \\ dz/dt \end{pmatrix} dt = \vec{v}\, dt$$

$$\int_{1}^{2} \vec{F} \cdot d\vec{r} = m \int_{1}^{2} \frac{d\vec{v}}{dt} \cdot \vec{v}\, dt$$

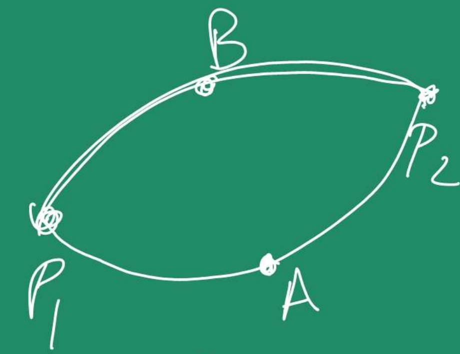
$$= \frac{1}{2} m \int_{1}^{2} \left[\frac{d}{dt}(\vec{v} \cdot \vec{v})\right] dt$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$W_{12} = (KE)_2 - (KE)_1$$

$$= T_2 - T_1$$

Prove that if $\int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$ is independent of the path joining P_1 & P_2 in given region, then $\oint \vec{F} \cdot d\vec{r} = 0$ for all closed paths in this region.



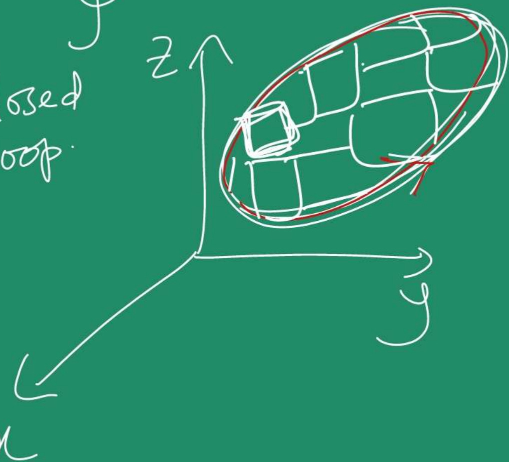
$$\oint \vec{F} \cdot d\vec{r} = \int_{P_1AP_2BP_1} \vec{F} \cdot d\vec{r} = \int_{P_1AP_2} \vec{F} \cdot d\vec{r} + \int_{P_2BP_1} \vec{F} \cdot d\vec{r} = \int_{P_1AP_2} \vec{F} \cdot d\vec{r} - \int_{P_1BP_2} \vec{F} \cdot d\vec{r}$$

$$= 0$$

If $\int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$ is independent, then $\oint \vec{F} \cdot d\vec{r} = 0$

$$\oint \vec{F} \cdot d\vec{r}$$

Closed loop.



Stokes' Theorem

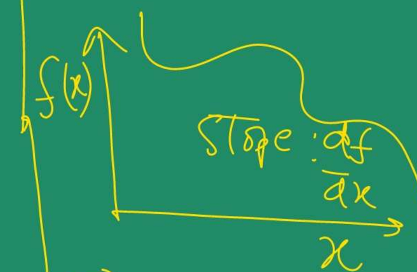
$$\vec{\nabla}(\ ) = \hat{x}\frac{\partial}{\partial x}(\ ) + \hat{y}\frac{\partial}{\partial y}(\ ) + \hat{z}\frac{\partial}{\partial z}(\ )$$

$$\vec{\nabla} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$$

$\vec{\nabla} f(x,y,z) \equiv$ Gradient

$\vec{\nabla} \cdot \vec{A}(x,y,z)$: Divergence

$\vec{\nabla} \times \vec{A}(x,y,z)$: Curl



$$\vec{\nabla} f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}; \quad \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Stokes' Theorem:

$$\oint_{\text{Closed loop}} \vec{F} \cdot d\vec{r} = \int_{\text{Area enclosed.}} (\vec{\nabla} \times \vec{F}) \cdot d\vec{A}$$

$$\oint \vec{F}\cdot d\vec{r} = 0$$

For forces due to which W_{12} is independent of path.

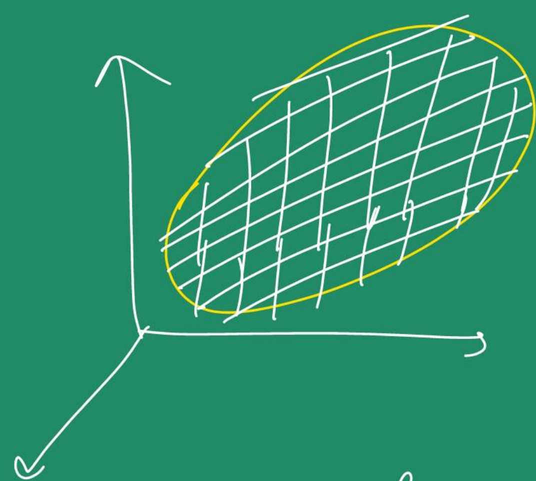
Closed loop

Yellow boundary

Stokes' Theorem

$$\int (\vec{\nabla}\times\vec{F})\cdot d\vec{A} = 0$$

Area enclosed



$$\vec{\nabla}\times\vec{\nabla}V(x,y,z) = 0$$

Mathematical Identity

$$\vec{\nabla}\times\vec{F} = 0 \Rightarrow \text{Conservative force}$$

$$\frac{\partial f_y}{\partial z} - \frac{\partial f_z}{\partial y} = 0 \; ; \quad \frac{\partial f_z}{\partial x} - \frac{\partial f_x}{\partial z} = 0 \; ; \quad \frac{\partial f_x}{\partial y} - \frac{\partial f_y}{\partial x} = 0$$

Conservative force $\vec{F} = -\vec{\nabla}V(x,y,z)$