The lifetime of R levels in ruby crystal
~ 4 msec

Simple apparatus using pulsed stroboscopic light source $\longrightarrow$ lifetime (fluorescence) can be measured.

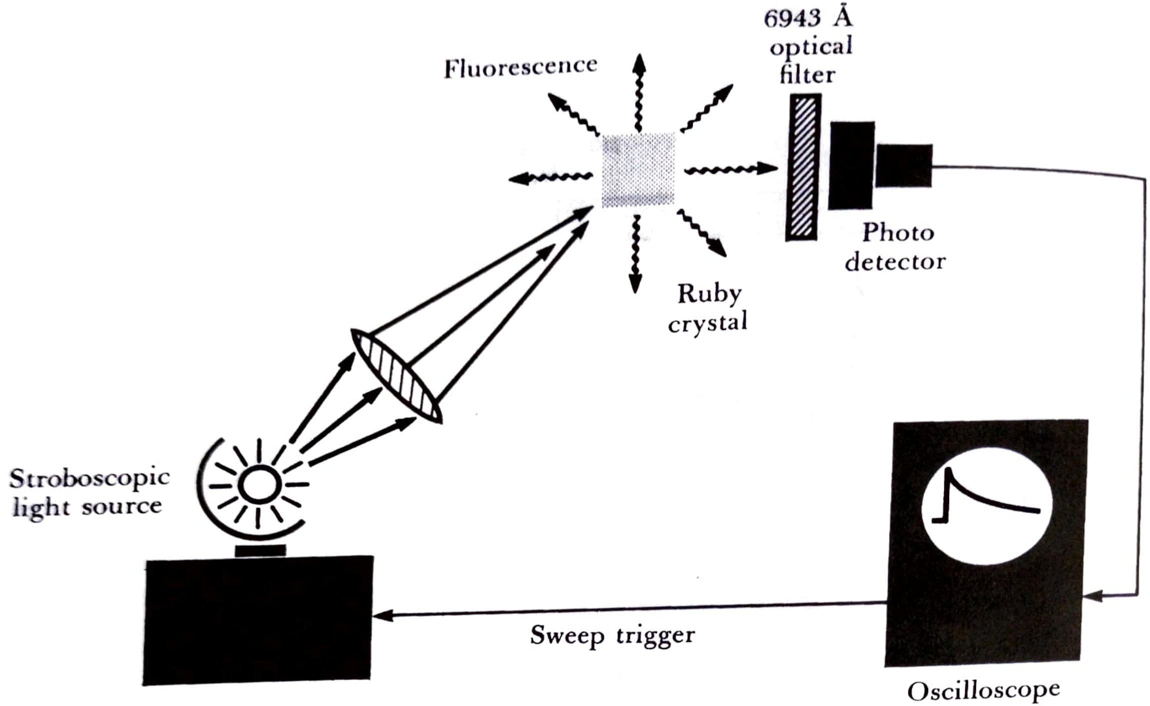


FIGURE 1.15
Measurement of ruby fluorescent lifetime.

2. Stimulated atomic transitions

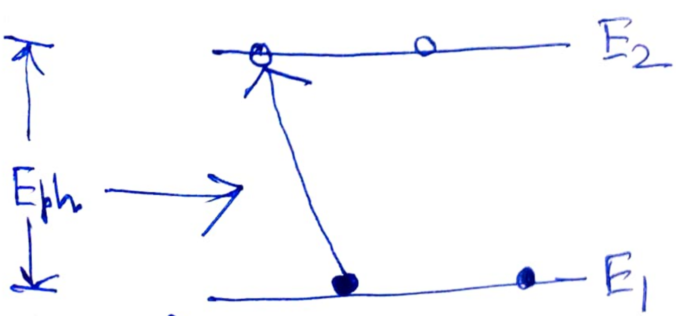
spontaneous ⟶ always downward transition

stimulated ⟶ upward & downward transition.

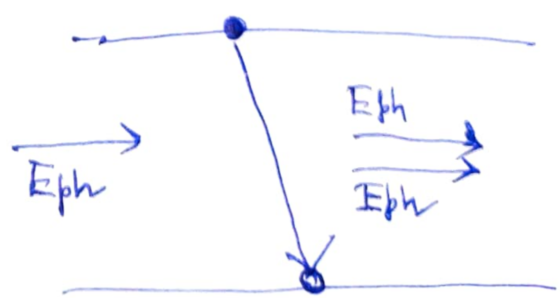
absorption   emission

Essential process for maser & laser action

stimulated transition occur when an external radiation signal is applied to an atom/molecule.



stimulated absorption



Stimulated emission

Additional photon is emitted by 'stimulation' (by incident photon)

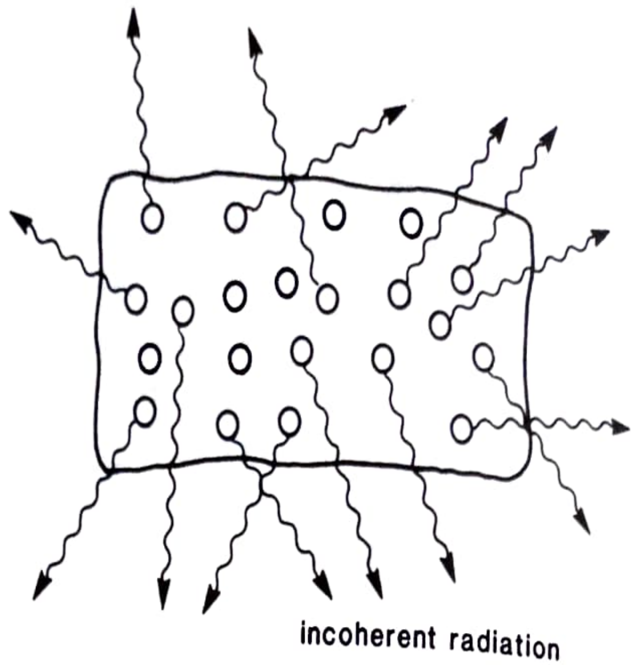


FIGURE 1.21
Spontaneous emission is incoherent or noise-like, emerging randomly in all directions.

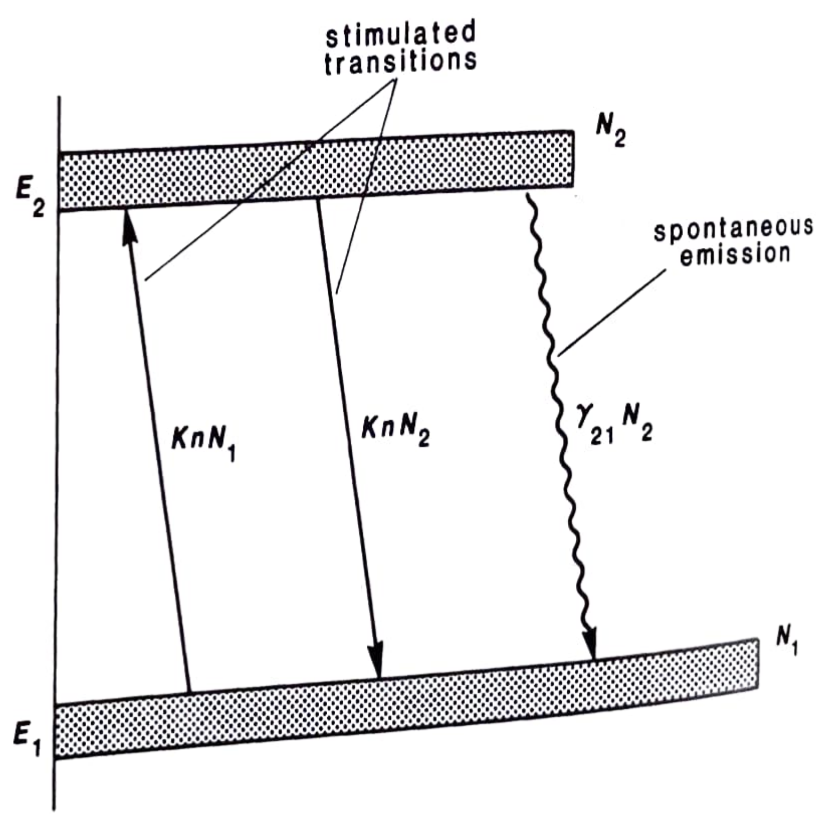


FIGURE 1.22
An energy-level population diagram, showing spontaneous emission plus stimulated transitions.

Atomic Rate Equation

Follow the figure above (1.22) Ignore any other level for this moment.

$$-\left.\frac{dN_2(t)}{dt}\right|_{spon} = +\left.\frac{dN_1(t)}{dt}\right|_{spon} = +\gamma_{21} N_2(t)$$

Now consider stimulated transition

(by applying optical signal)

Condition $\hbar\omega_{ph} \cong E_2 - E_1$

$\omega \approx \omega_{21} \pm \Delta\omega_a$ $\Delta\omega_a$ is linewidth of atomic transition.

The rate will depend on intensity of light which is proportional to photon density n

$$\left.\frac{dN_2(t)}{dt}\right|_{\text{stimu up (absorption)}} = K\, n(t)\, N_1(t) = -\left(\frac{dN_1}{dt}\right)_{st.\,abs}$$

$K\, n(t) \propto B_{12}$ Einsten coff. for stimulated transition from level 1 to level 2

$= B_{12}\,\rho(\nu)\, N_1$

$\rho(\nu)$ spectral energy density

Also $$\left.\frac{dN_2(t)}{dt}\right|_{\text{stimu down (emission)}} = -B_{21}\, N_2(t)\, \rho(\nu)$$

B_{21}: Einstein coeff. for stimulated transition from level 2 to 1

If both the levels are not degenerate

then $B_{21} = B_{12} = \boxed{K\, n(t) / \rho(\nu)}$

So, $$\left.\frac{dN_2}{dt}\right|_{st.\ emiss} = -K\, n(t)\, N_2(t)$$

E_2

E_1

Total rate equation considering spontaneous and stimulated transitions

$$\left.\frac{dN_2(t)}{dt}\right|_{total} = \left.\frac{dN_2}{dt}\right|_{stim\ up} + \left.\frac{dN_2(t)}{dt}\right|_{stim\ down} + \left.\frac{dN_2}{dt}\right|_{spon}.$$

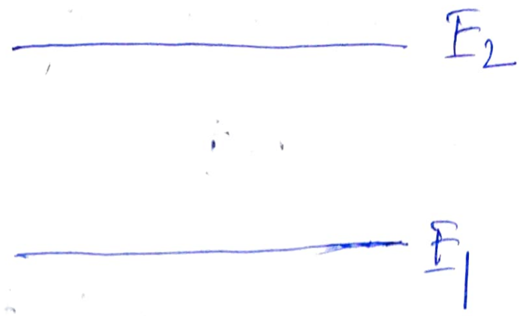
$$= K\, n(t)\, N_1(t) - K\, n(t)\, N_2(t) - \gamma_{21}\, N_2(t)$$

$$= K\, n(t) \times [N_1(t) - N_2(t)] - \gamma_{21} N_2(t) = -\left.\frac{dN_1}{dt}\right|_{total}$$

(For two level system)

K can be derived by semiclassical quantum analysis (atom as quantum and radiation as classical wave) (E.M. wave).

γ can be derived only by full quantum analysis (~~both~~ quantum treatment for both atom & electromagnetic field)



Stimulated transitions and laser amplification

Only consider the stimulated transitions

$$\left.\frac{dN_2(t)}{dt}\right|_{total} = K\, n(t)\,(N_1 - N_2) = -\frac{dN_1}{dt}$$

In order to get laser beam we need decrease in N_2 ($\Rightarrow$ increase in N_1)

~~So net signal energy density U_{sig} can be written~~

(*) $$\frac{dU_{sig}}{dt} = -\left.\frac{dN_2}{dt}\right|_{total} \times \hbar\omega$$

[$U_{sig}(t) = n(t) \times \hbar\omega$ ~~applied~~ signal energy density]

$$\frac{dU_{sig}}{dt} = -K[N_1(t) - N_2(t)]\,n(t) \times \hbar\omega$$

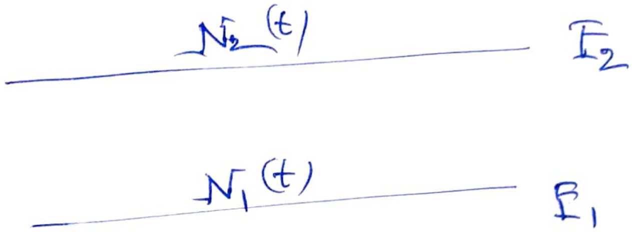
$$= -K[N_1(t) - N_2(t)]\,U_{sig}$$

In terms of photon density

$$\frac{dn}{dt} = -K\left[N_1(t) - N_2(t)\right] n(t)$$

$U_{sig}(t)$ will grow if $N_2 > N_1$

This condition is called population inversion → Net energy will be given up by atoms and taken up by the applied signal.

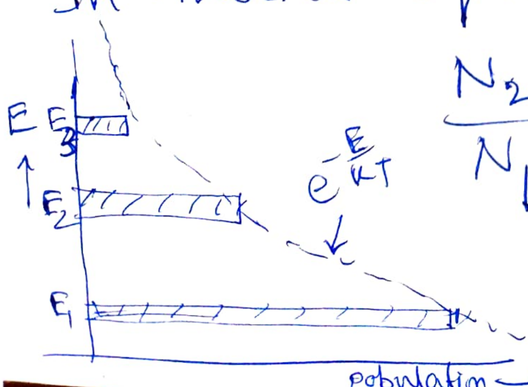


$$\boxed{\text{Population inversion is NOT possible in a system at equilibrium.}}$$

At thermodynamic equilibrium, population at lower levels are always higher than population at higher levels.

Boltzmann's Principle

In thermal equilibrium



$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{k_B T}\right)$$

$$= e^{-\frac{\hbar\omega}{k_B T}}$$

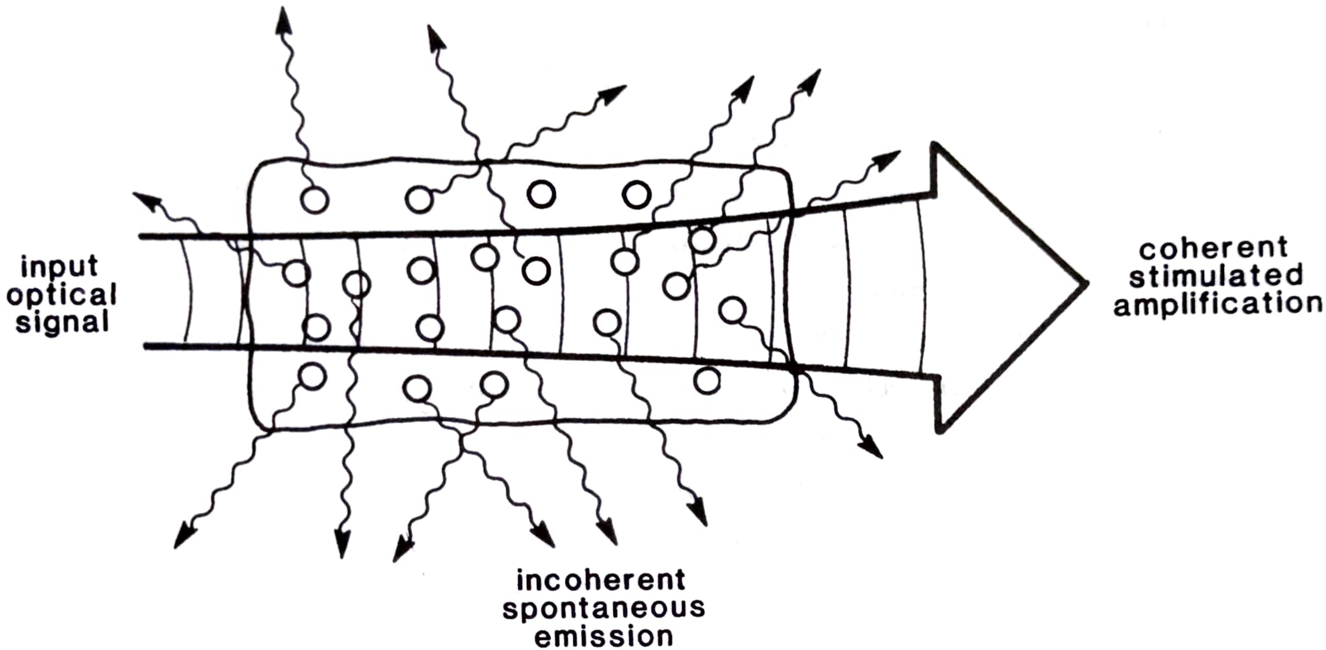


FIGURE 1.24
Incoherent spontaneous emission and coherent stimulated amplification occur simultaneously and in parallel in the laser medium.