Lecture - 9

Section B : 23/09/2025

Section C : 25/09/2025

Section A : 25/09/2025

(Some part taught in Lecture-10)

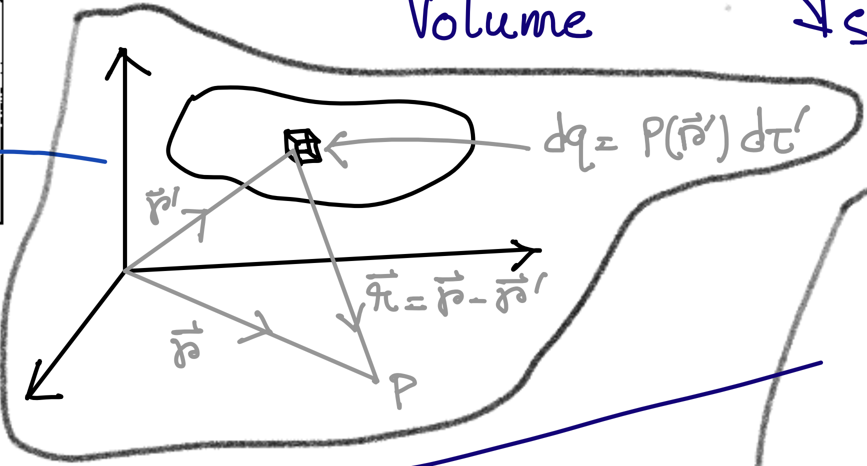
Electric field due to charge distributions

Volume

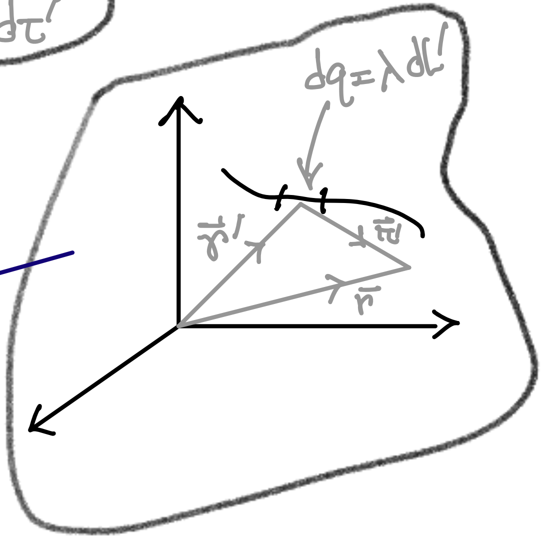
surface

line

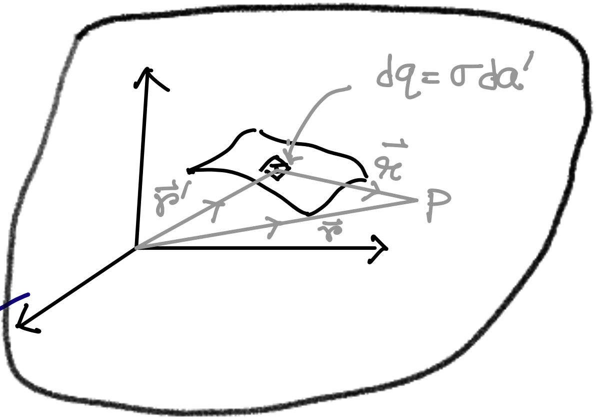
$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho(\mathbf{r}')}{\mathscr{r}^2}\hat{\mathscr{r}}\, d\tau'.$$



$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{P}} \frac{\lambda(\mathbf{r}')}{\mathscr{r}^2}\hat{\mathscr{r}}\, dl'$$



$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{S}} \frac{\sigma(\mathbf{r}')}{\mathscr{r}^2}\hat{\mathscr{r}}\, da'$$



Find the electric field a distance z above the midpoint of a straight line segment of length $2L$, which carries a uniform line charge λ Griffiths Ex-2.1.

$$d\mathbf{E} = 2\frac{1}{4\pi\epsilon_0}\left(\frac{\lambda\, dx}{\mathcal{r}^2}\right)\cos\theta\,\hat{\mathbf{z}}.$$

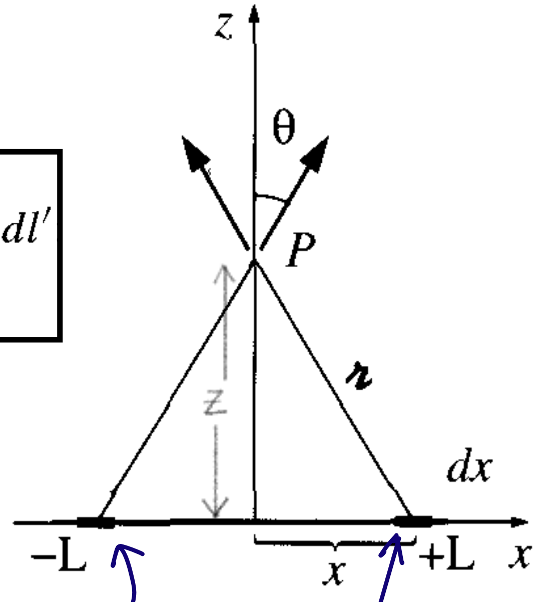
$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{P}} \frac{\lambda(\mathbf{r}')}{\mathcal{r}^2}\hat{\boldsymbol{\mathcal{r}}}\, dl'}$$

Here $\cos\theta = z/\mathcal{r}$, $\mathcal{r} = \sqrt{z^2+x^2}$, and x runs from 0 to L:

$$\vec{E} = \int_0^{+L} d\vec{E}$$

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0}\int_0^L \frac{2\lambda z}{(z^2+x^2)^{3/2}}dx \\ &= \frac{2\lambda z}{4\pi\epsilon_0}\left[\frac{x}{z^2\sqrt{z^2+x^2}}\right]\Bigg|_0^L \\ \vec{E} &= \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}},\ \hat{z} \end{aligned}$$

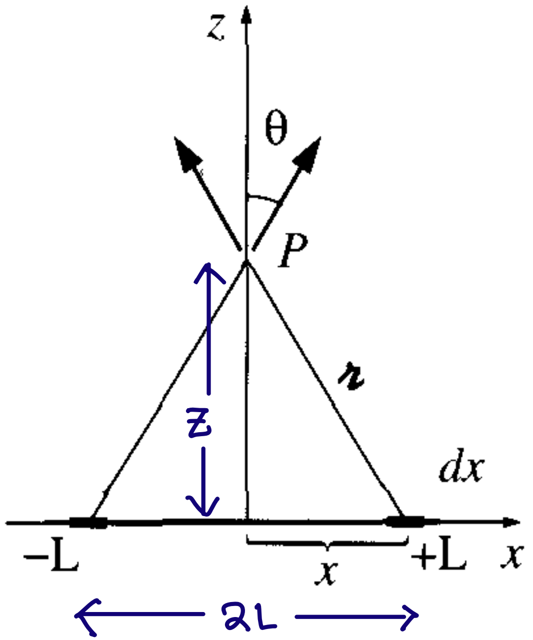
and it aims in the z-direction.



$\hat{\mathcal{r}} = \cos\theta\,\hat{z} + \sin\theta\,\hat{x}$

$\hat{\mathcal{r}} = \cos\theta\,\hat{z} - \sin\theta\,\hat{x}$

Find the electric field a distance z above the midpoint of a straight line segment of length $2L$, which carries a uniform line charge λ Griffiths Ex-2.1.



$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}},}$$

take limits : $z \gg L$

very far from the line

$$z^2 + L^2 \simeq z^2$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z^2} \Rightarrow \text{total charge} = 2\lambda L$$

seems like a point charge!

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}$$

Another limit

$L \to \infty \to$ infinitely long wire

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda \cancel{L}}{z \cdot \cancel{L}} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z} = \frac{\lambda}{2\pi\varepsilon_0 z}$$

$z =$ perpendicular distance from line.

Repeat the above problem using Gauss' law.

[Taken from Griffiths]

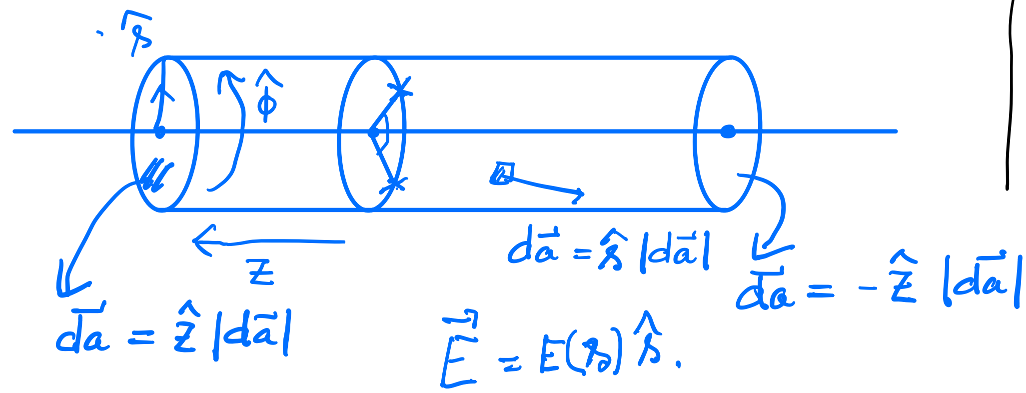
Gauss's law is always *true*, but it is not always *useful*. If ρ had not been uniform (or, at any rate, not spherically symmetrical), or if I had chosen some other shape for my Gaussian surface, it would still have been true that the flux of $\mathbf{E}$ is $(1/\epsilon_0)q$, but I would not have been certain that $\mathbf{E}$ was in the same direction as $d\mathbf{a}$ and constant in magnitude over the surface, and without that I could not pull $|\mathbf{E}|$ out of the integral. *Symmetry is crucial* to this application of Gauss's law. As far as I know, there are only three kinds of symmetry that work:

$\vec{E}\cdot d\vec{a} \leftrightarrow \vec{\nabla}\cdot\vec{E}$

1. *Spherical symmetry.* Make your Gaussian surface a concentric sphere.
2. *Cylindrical symmetry.* Make your Gaussian surface a coaxial cylinder (Fig. 2.19).
3. *Plane symmetry.* Use a Gaussian "pillbox," which straddles the surface (Fig. 2.20).

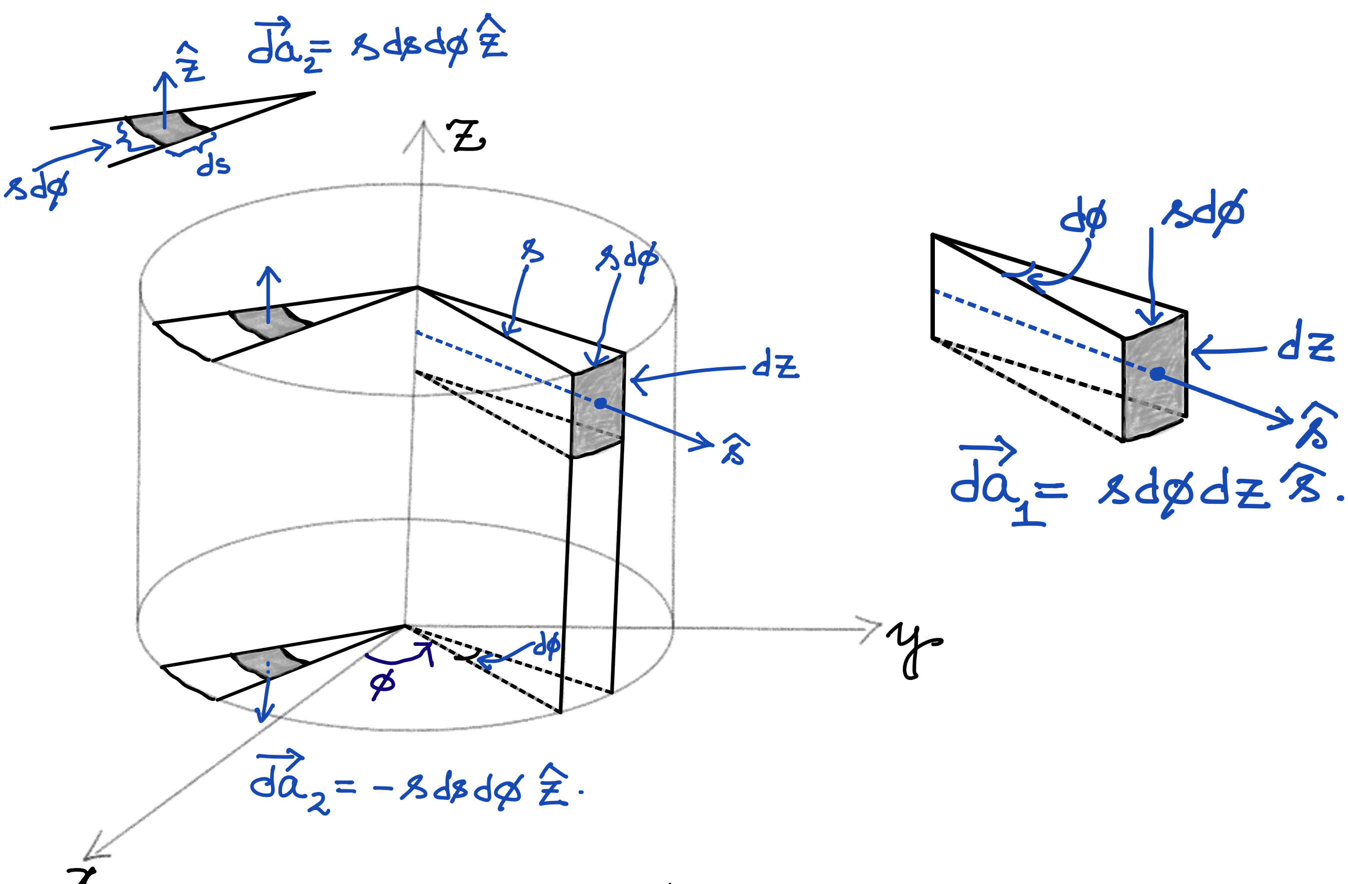
Although (2) and (3) technically require infinitely long cylinders, and planes extending to infinity in all directions, we shall often use them to get approximate answers for "long" cylinders or "large" plane surfaces, at points far from the edges.

Need to understand how to deal with symmetric surfaces like the sphere or cylinder.

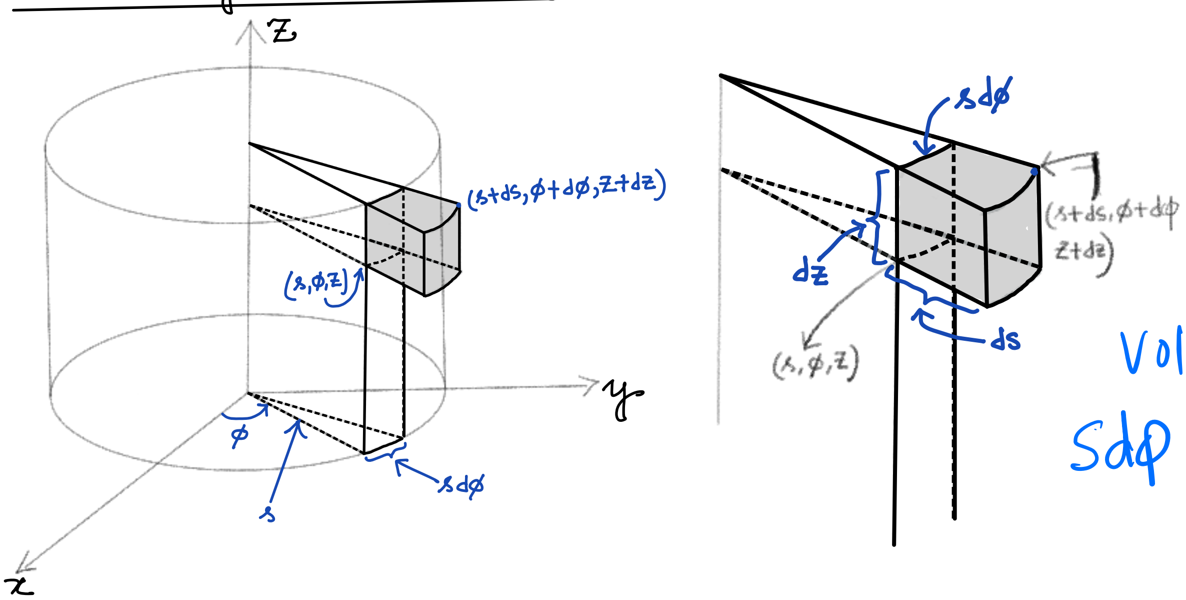


Here $\hat{z}$ is the axial direction So be aware of the notation.

Elementary Area surfaces on cylinder:



Elementary Volume in cylinder



Vol. element

$s\,d\phi\,dz\,ds$

Repeat the above problem using Gauss' law.

[Taken from Griffiths]

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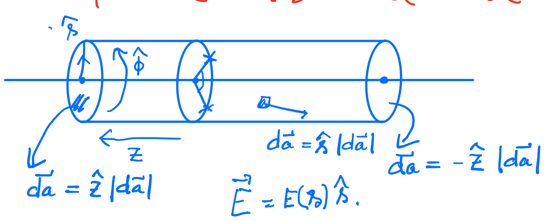
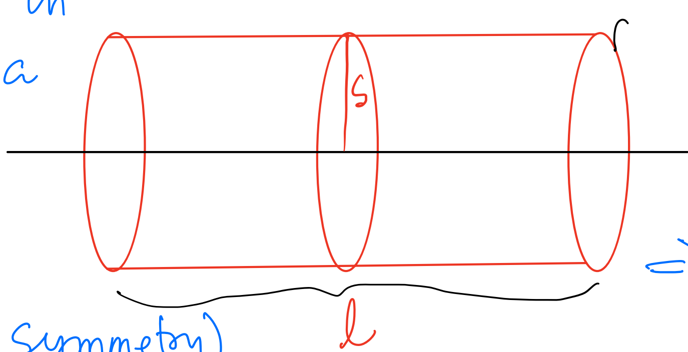
Although (2) and (3) technically require infinitely long cylinders, and planes extending to infinity in all directions, we shall often use them to get approximate answers for "long" cylinders or "large" plane surfaces, at points far from the edges.

For the above problem, we shall use a Cylindrical Gaussian surface:



$\vec{E}$ is constant in magnitude at a given 's' and is in $\hat{s}$ direction (by symmetry)



$$\oint \vec{E}\cdot d\vec{a} = Q_{enc}/\epsilon_0$$

$$\Rightarrow E \times 2\pi s\, l = \frac{\lambda l}{\epsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 s}$$

$$\Rightarrow \vec{E} = \frac{\lambda}{2\pi\epsilon_0 s}\,\hat{s}$$