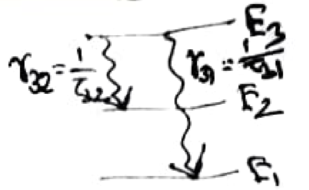
1) Spontaneous decay from E_3 to E_2 :



$$\gamma_{32} = \frac{1}{\tau_{32}} \qquad \tau_{32} = 70 \text{ ns}$$

Spontaneous decay from E_3 to E_1 :

$$\cancel{\tau_{31}}\ \gamma_{31} = \frac{1}{\tau_{31}} \qquad \tau_{31} = 200 \text{ ns}$$

Overall decay rate constant, $\gamma_3 = \gamma_{32} + \gamma_{31} = \frac{1}{\tau_{32}} + \frac{1}{\tau_{31}}$

$= \frac{\tau_{31} + \tau_{32}}{\tau_{32}\tau_{31}}$

The overall lifetime of E_3, $\tau_3 = \frac{1}{\gamma_3}$

$$\tau_3 = \frac{\tau_{32}\tau_{31}}{\tau_{31} + \tau_{32}} = \frac{70 \times 200}{70 + 200} \text{ ns} = \frac{70 \times 200}{270} \text{ ns}$$

$= 51.85$ ns (Ans.)

3) If ψ is wavefunction corresponding to a quantum particle, $|\psi|^2$ is probability density. Therefore, the probability to find the particle between x & $x+dx$ is $|\psi|^2 dx$.
The probability to find the particle between $-\infty$ to $+\infty$ must always be 1, which mathematically can be written as $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$. So, the wavefunction ψ must be square-integrable.

4) $\psi(x) = Ax^4$, A is a constant.

$$\int_{-\infty}^{\infty} |\psi|^2 dx = \int_{-\infty}^{\infty} |A|^2 x^8 dx \longrightarrow \text{diverges}$$

Therefore, ψ is not square-integrable.
Hence, ψ can not represent any quantum particle.

Four-level pumping model

pumping rate R_{p0} (atoms/second) from E_0 to E_3.

Certain ~~from~~ fraction η_p of the atoms excited upward (to E_3) will relax down to intended upper level of laser action (E_2 for this case).

So η_p is pumping efficiency for the laser system. $\therefore$ Effective pumping (to E_2) rate $\boxed{R_p = \eta_p R_{p0}}$

$$\frac{dN_2}{dt} \approx R_p - \gamma_{21} N_2$$ (Let us not consider laser action for the moment)

$$\frac{dN_1}{dt} \approx \gamma_{21} N_2 - \gamma_{10} N_1$$

Let us assume, a continuous pumping is applied and steady state is achieved

at steady state $\frac{dN_1}{dt} = 0$ $\frac{dN_2}{dt} = 0$

$$N_{2,ss} = \frac{R_p}{\gamma_{21}} \qquad N_{1,ss} = \frac{\gamma_{21}}{\gamma_{10}} N_{2,ss}$$

$$(N_2 - N_1)_{ss} = \frac{R_p(\gamma_{10} - \gamma_{21})}{\gamma_{10}\gamma_{21}} = R_p \tau_{21}\left(1 - \frac{\tau_{10}}{\tau_{21}}\right)$$ required $\tau_{10} < \tau_{21}$

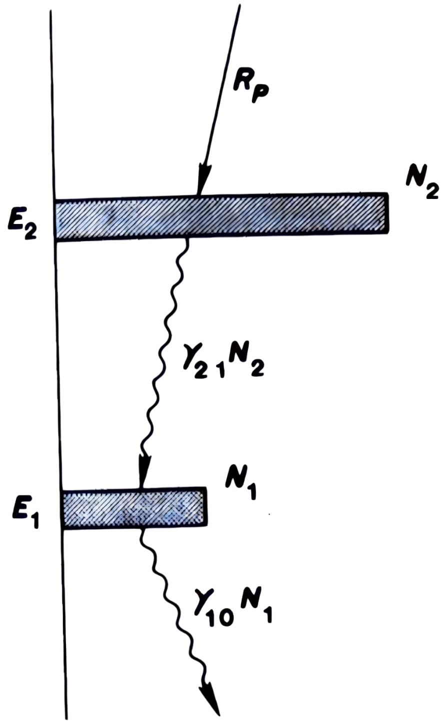


FIGURE 1.30
Rates of flow between atomic energy levels in an ideal four-level laser system.