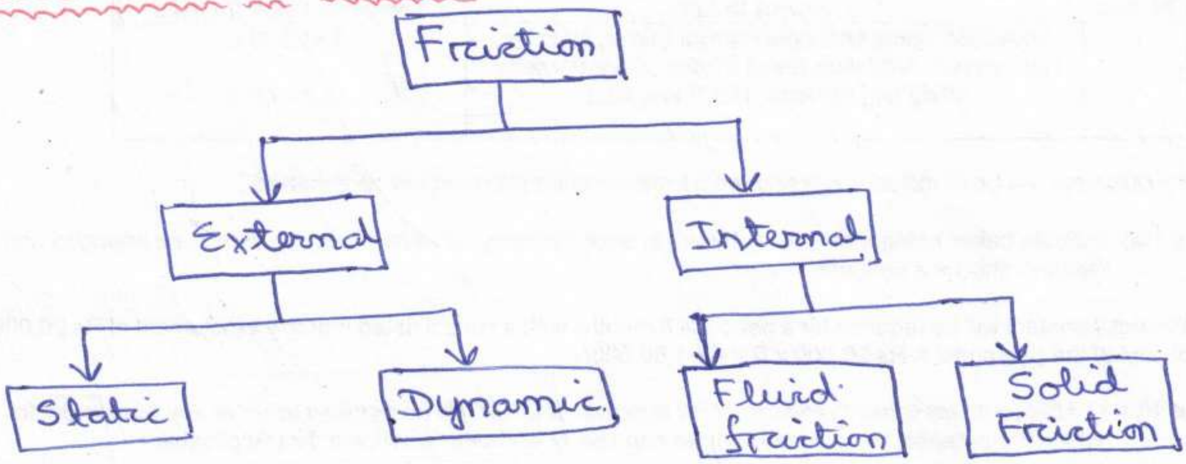
Friction

- Friction plays an important role in our day-to-day activities.
- Beneficial:- To walk, belt drives, brakes.
- Undesirable: Bearings and many other situations (Lubrication of the moving components are required for smooth operation).

Classification of Friction



External Friction

- The interaction of two surfaces of solid bodies in contact

Static Friction

- When the two bodies are at rest, but there is a tendency for relative motion.

Dynamic Friction

- When the surfaces are in relative motion.

Fluid Friction

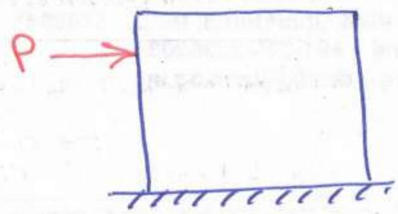
- Developed between fluid elements when adjacent layers in a fluid are moving at different velocities.
- This is also called as viscous friction.
- Useful in problems of flow through pipes or orifices, bodies immersed in fluids, lubricated surfaces, etc.
- Appears as frictional drag, viscous damping in aerodynamics & vibration studies, respectively.

Solid Friction

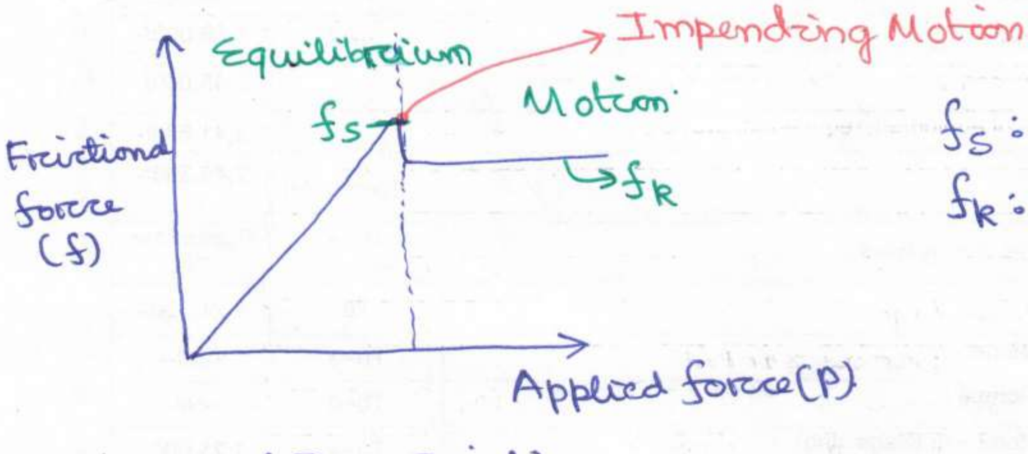
- Found in all solid materials subjected to cyclic loading.
- Energy is dissipated internally within the material.
- In vibration problems, material damping is usually considered as equivalent viscous or Coulomb friction.

Dry Friction

- No frictional force in the absence of an external force to cause a relative motion.
- When a pushing force 'P' is applied, frictional force starts developing at the interface of the objects.



- 'P' is balanced by frictional force and the object remains in eqlb^m. and at rest.
- When the force 'P' reaches the maximum frictional force, the object will be on the verge of sliding.



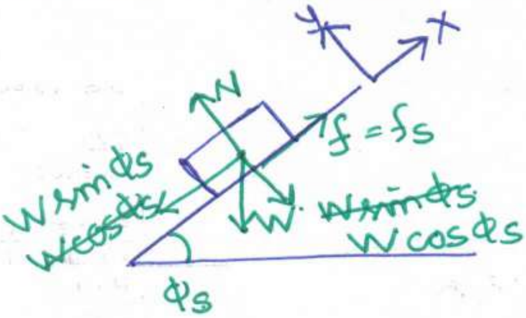
f_s : static friction

f_k : kinetic friction

Laws of Dry Friction

- Magnitude of the frictional force (max. static frictional force) is directly proportional to the normal load b/w the surfaces for a given pair of materials $\Rightarrow \boxed{f \leq \mu_s N}$
- Magnitude of the frictional force (max. static frictional force) is independent of the area of contact surfaces (apparent area) for a given load condition.

Experimental determination of co-efficient of friction



For impending motion,

$$f_s = \mu_s N$$

$$\Sigma F_y = 0$$

$$\Rightarrow N = W \cos \phi_s$$

$$\Sigma F_x = 0$$

$$\Rightarrow f_s = W \sin \phi_s$$

$$\Rightarrow \mu_s N = W \sin \phi_s$$

$$\Rightarrow \mu_s = \frac{W \sin \phi_s}{W \cos \phi_s}$$

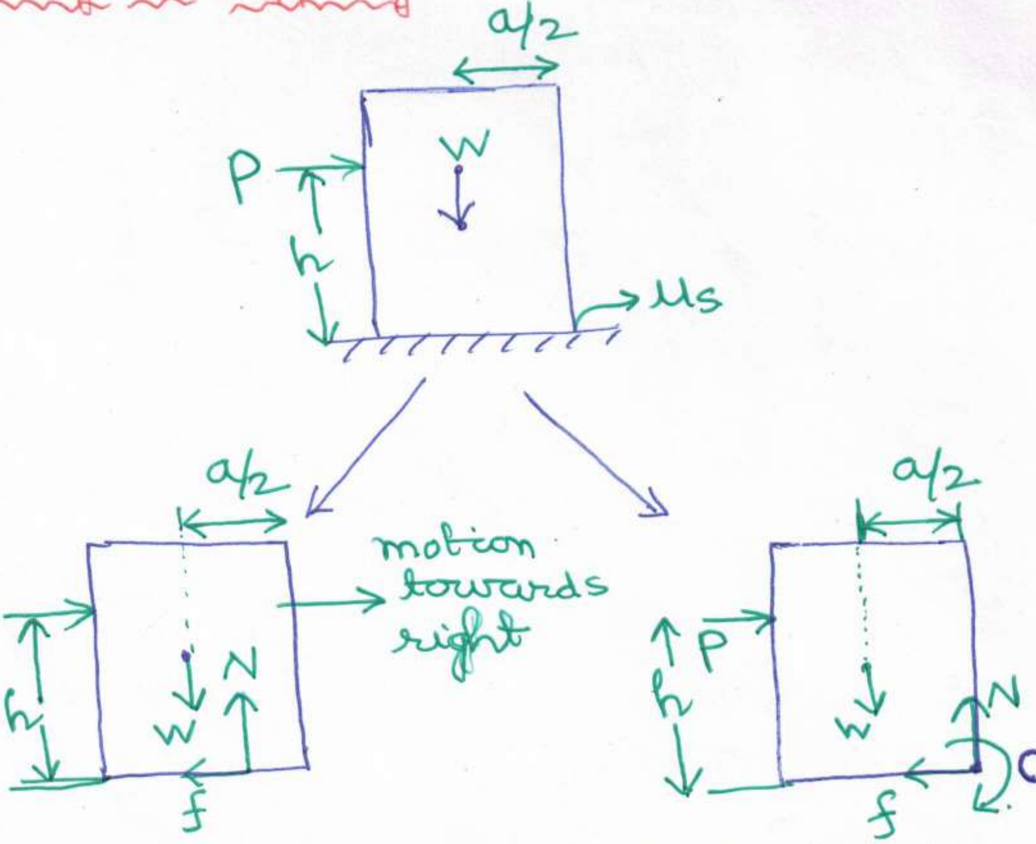
$$\Rightarrow \boxed{\mu_s = \tan \phi_s}$$ Independent of 'm'

→ Depends only on the nature of the two contact surfaces.

Mechanisms of Static & Dynamic Friction

- On the atomic scale, all the material surfaces are rough. Surfaces have microscopic projections, depressions, and other irregularities.
- When the two bodies come in contact, micro-projections and depression mesh and impedes relative motion.
- Surface adhesion also impedes relative motion.
- Sliding friction is due to plastic deformation. However pushing force less than max^m static friction may lead to elastic deformation of micro-projections.
- Frictional force reduces as motion of the body/object begins because of the reduced meshing & adhesion of the surfaces.
- Friction depends on the quality of surface finish. For very high class finish, the bodies adhere resulting in high friction coefficient in such cases.

Sliding vs. Tipping



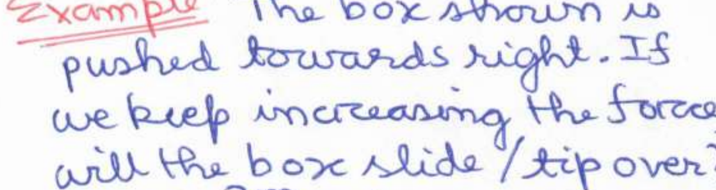
* If $P > f = \mu_s N$
  $\Rightarrow$ the block will slide to the right.

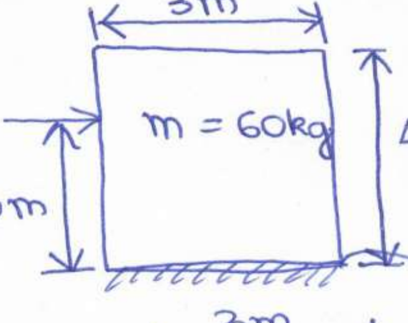
* For tipping over condition of the block, the right corner will be in contact with the ground before the block completely tips over.

$\therefore$ $(M_C)_1$ due to force P ↻
$= Ph$
$(M_C)_2$ due to weight W ↺
$= W\left(\frac{a}{2}\right)$.

If $Ph > \frac{Wa}{2}$, then the block will tip over at point 'C'.

Example: The box shown is pushed towards right. If we keep increasing the force, will the box slide / tip over?

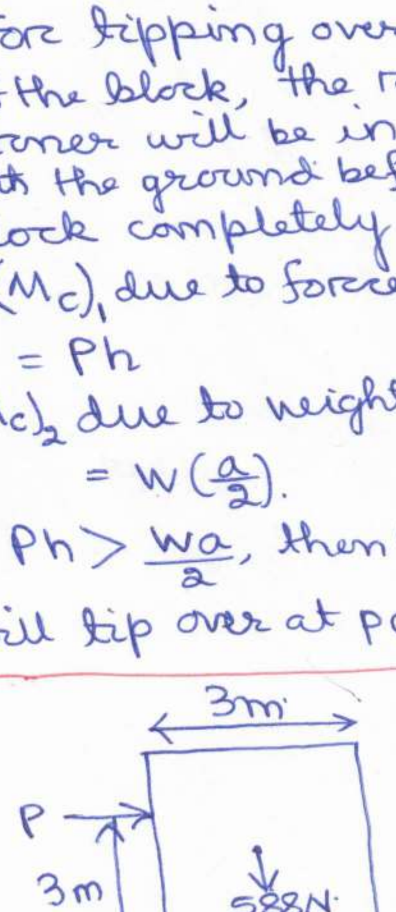




$W = mg = 60 \times 9.81$
$= 588N$

$P = f_s$

$f_s = \mu_s N$
$= \mu_s W$
$= (0.62)(588)$
$\simeq 365N$

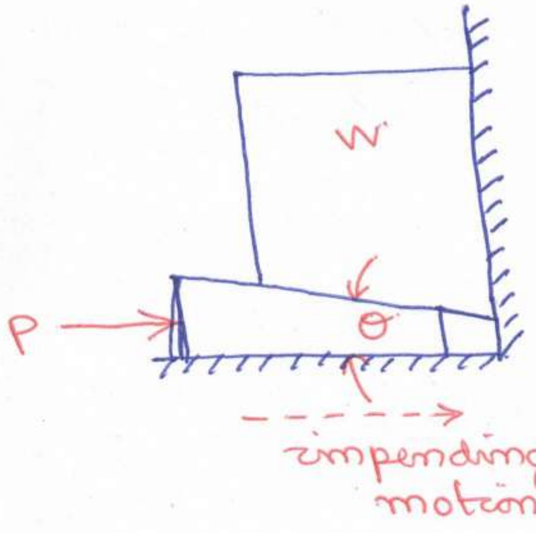


$\therefore$ Block will tip over if the force is kept increasing

$P(3) = 588\left(\frac{3}{2}\right)$

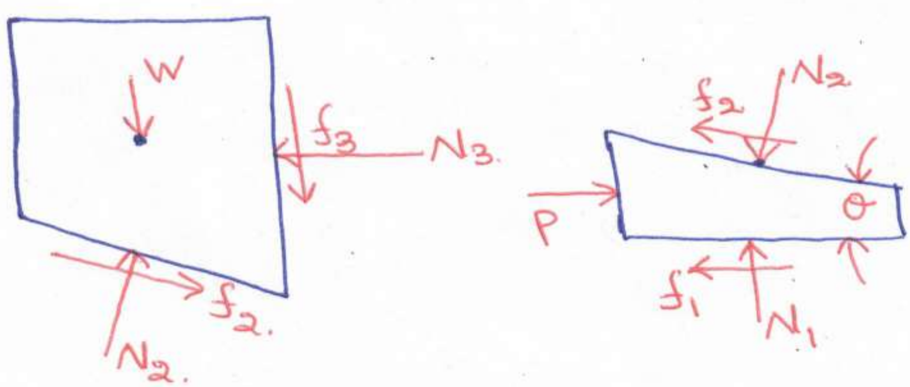
$\Rightarrow \underline{P = 294N}$

Application of friction in machines: Wedges.



- A Wedge is a simple machine that is often used to transform an applied force into much larger forces.

→ Minimum force 'P' required to raise the block.



- The weight of the wedge is excluded as ~~the~~ its weight is small compared to the weight 'W' of the block.
- The location of resultant normal forces are not important as neither the block nor the wedge will "tip over".

⇒ Moment equilibrium equations will not be considered (Moment eqlb. eqn. is needed to find the positi[on] of the normal reactio[n])

$\boxed{\text{Unknowns}}$:- 7 unknown forces.

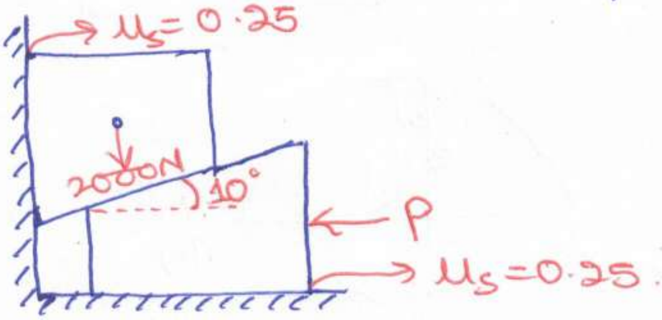
- Applied force 'P'
- 3 normal forces (N)
- 3 reaction forces / 3 frictional forces (f)

$\boxed{\text{No. of Equations}}$

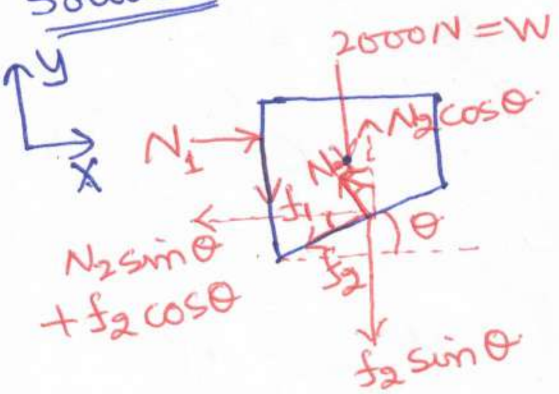
- Four force eqlb^n. eqns. ($\Sigma F_x=0, \Sigma F_y=$
  ↳ 2 for each block and wedge.
- Three frictional equations.

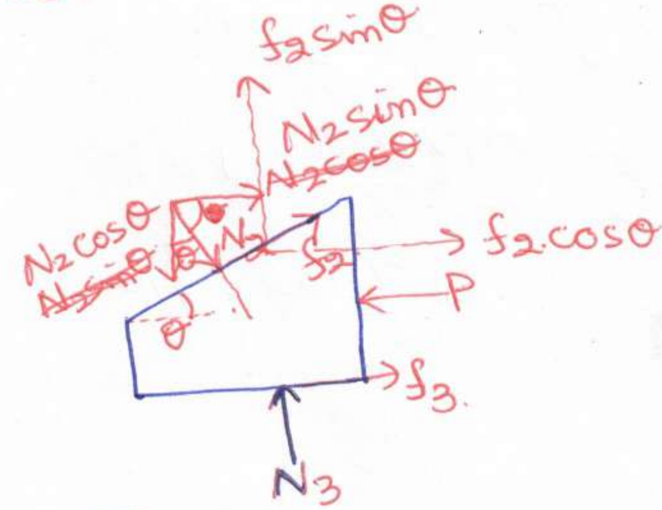
$$\underline{f = \mu N}$$

Que) A block overlaying a 10° wedge on a horizontal floor, leaning against a vertical wall, and weighing 2000N is to be raised by applying a horizontal force to the wedge. Assuming the frictional co-efficient for all contact surfaces as 0.25, determine the minimum horizontal force to be applied to raise the block.



Solution.





- $\sum \vec{F_x} = 0$

$\Rightarrow N_1 = N_2 \sin\theta + f_2\cos\theta$

$\Rightarrow N_1 = N_2(\sin\theta + \mu\cos\theta)$

- $\sum \vec{F_y} = 0$

$\Rightarrow N_2\cos\theta = f_1 + f_2\sin\theta + W$

$\Rightarrow N_2\cos\theta = \mu N_1 + \mu N_2 \sin\theta + W$

$\Rightarrow N_2\cos\theta = \mu N_2(\sin\theta + \mu\cos\theta) + \mu N_2\sin\theta + W$

$$\Rightarrow \boxed{N_2 = \frac{W}{(\cos\theta - 2\mu\sin\theta - \mu^2\cos\theta)}}$$

Substituting $\theta = 10°$.

$N_2 = 2391.1N$.

- $\sum \vec{F_y} = 0$

$\Rightarrow N_3 + f_2\sin\theta = N_2\cos\theta$

$\Rightarrow N_3 + \mu N_2\sin\theta = N_2\cos\theta$

$\Rightarrow N_3 = N_2(\cos\theta - \mu\sin\theta)$.

- $\sum \vec{F_x} = 0$

$\Rightarrow P = f_3 + f_2\cos\theta + N_2\sin\theta$

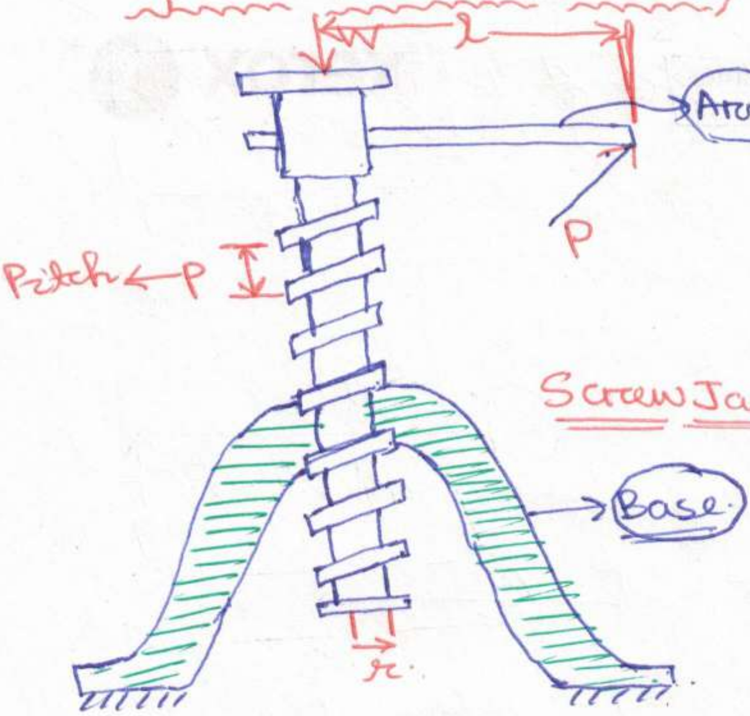
$\Rightarrow P = \mu N_3 + \mu N_2\cos\theta + N_2\sin\theta$

$\Rightarrow P = \mu N_2(\cos\theta - \mu\sin\theta) + \mu N_2\cos\theta + N_2\sin\theta$

$\Rightarrow P = N_2(\sin\theta + 2\mu\cos\theta - \mu^2\sin\theta)$

$\Rightarrow P = 1566.65\ N$

Square-Threaded Screw / Screw Jack



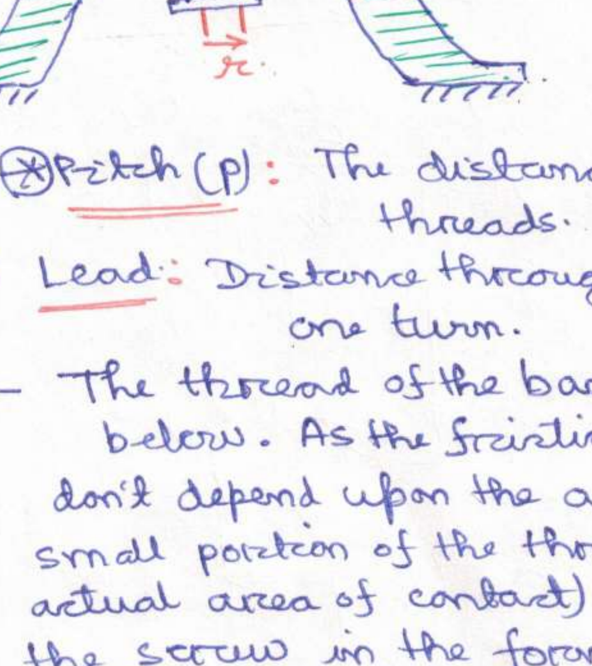
- The figure shown is for a screw jack which is used for lifting the heavy loads.
- The screws of the screw jack carries the load 'W' and is supported by the base of the jack.
- By applying a load 'P' at the arm, the screw can be made to turn and raise the load 'W'.

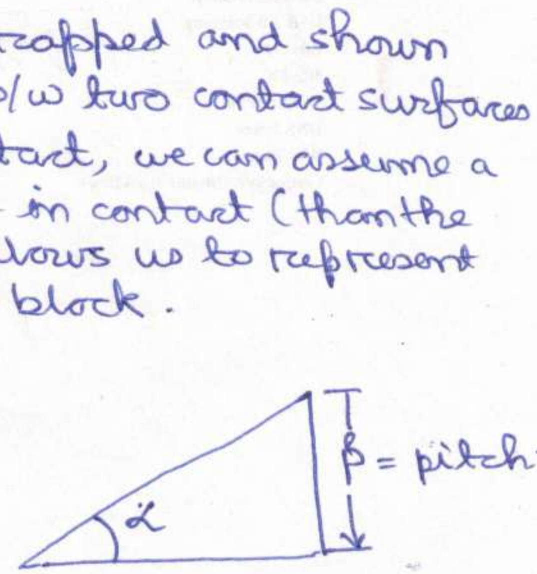
⊛ Pitch (p): The distance measured b/w two sucessive threads.

⊛ Lead: Distance through which the screw advances in one turn.

- The thread of the base is unwrapped and shown below. As the frictional force b/w two contact surfaces don't depend upon the area of contact, we can assume a small portion of the threads to be in contact (than the actual area of contact), which allows us to represent the screw in the form of small block.

(Raising the Load)





α : Helix angle of the screw.

- $\vec{Q}$ has the same effect as the force $\vec{P}$ exerted on the arm. $\vec{Q}$ should have the same moment as $\vec{P}$ about the axis of the screw.

$$\therefore Qr = Pl \Rightarrow Q = \left(\frac{Pl}{r}\right)$$

- Once, $\vec{Q}$ is evaluated from the FBD, the value of $\vec{P}$ can be evaluated using the above moment equation.

- $\sum F_x = 0$

$\Rightarrow Q\cos\alpha = W\sin\alpha + f = W\sin\alpha + \mu_s N$

- $\sum F_y = 0$

$\Rightarrow N = W\cos\alpha + Q\sin\alpha.$

$\therefore Q\cos\alpha = W\sin\alpha + \mu_s (W\cos\alpha + Q\sin\alpha)$

$\Rightarrow Q.(\cos\alpha - \mu_s \sin\alpha) = W(\sin\alpha + \mu_s\cos\alpha).$

$$\Rightarrow Q = W\frac{(\sin\alpha + \mu_s\cos\alpha)}{(\cos\alpha - \mu_s\sin\alpha)} = W\frac{(\sin\alpha + \tan\phi_s\cos\alpha)}{(\cos\alpha - \tan\phi_s\sin\alpha)}$$

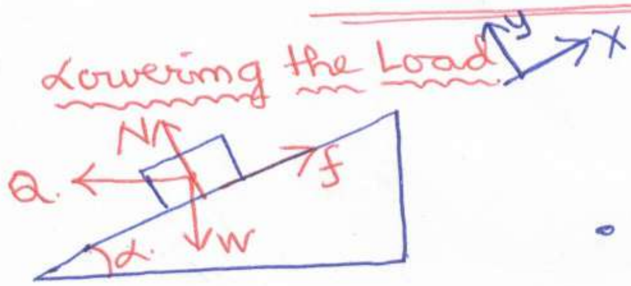
$$\Rightarrow Q = W\left(\frac{\sin\alpha\cos\phi_s + \cos\alpha\sin\phi_s}{\cos\phi_s\cos\alpha - \sin\phi_s\sin\alpha}\right) = W\frac{\sin(\alpha+\phi_s)}{\cos(\alpha+\phi_s)}$$

$$\Rightarrow \boxed{Q = W\tan(\alpha+\phi_s)}$$

$\therefore P\ell = Qr$

$$\Rightarrow P = Q\left(\frac{r}{\ell}\right) = W\tan(\alpha+\phi_s)\left(\frac{r}{\ell}\right)$$

Lowering the Load



- $\sum F_x = 0$

$\Rightarrow Q\cos\alpha + W\sin\alpha = f = \mu_s N$

- $\sum F_y = 0$

$\Rightarrow N = W\cos\alpha - Q\sin\alpha$

$\Rightarrow Q\cos\alpha + W\sin\alpha = \mu_s(W\cos\alpha - Q\sin\alpha).$

$$\Rightarrow Q = W\frac{\mu_s\cos\alpha - \sin\alpha}{\cos\alpha + \mu_s\sin\alpha} = \frac{\tan\phi_s\cos\alpha - \sin\alpha}{\cos\alpha + \tan\phi_s\sin\alpha}$$

$$\Rightarrow \boxed{Q = W\tan(\phi_s - \alpha)}$$

Again, $P\ell = Qr.$

$$\Rightarrow P = Q\left(\frac{r}{\ell}\right) = W\tan(\phi_s-\alpha)\left(\frac{r}{\ell}\right)$$

(*) For the lowering of the load 'W', the effort/force required was calculated as-

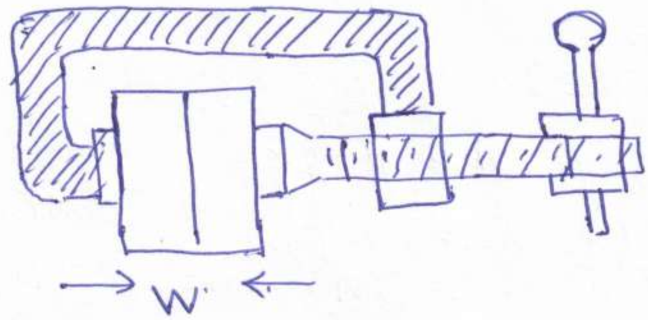
$$Q = W\tan(\phi_s - \alpha).$$

- In the above expression if $\phi_s < \alpha$, then 'Q' = -ve.

  ⇒ Load will start to move downwards without application of any force/effort. Such a condition is called as "Over-Hauling of Screws"

- If $\phi_s > \alpha \Rightarrow Q = +ve \Rightarrow$ Effort is required to lower the load. Such a screw is called "Self-Locking Screw"

Que) A clamp is used to hold two pieces of wood together as shown. The mean diameter of the square thread is 10mm and pitch is 2mm. The co-efficient of friction between the threads is 0.3. If a max^m. couple of 40N-m is applied in tightening the clamp, then calculate the force required to hold the blocks. Assume the lead to be same as pitch.



Solution: Let W' be the force required to hold the blocks together.

This is equivalent to the situation where a screw jack was used to raise the load W.

$d = 10mm$

$\Rightarrow r = 5mm$

$\mu_s = 0.3$

$\Rightarrow \phi_s = \tan^{-1}(0.3) = 16.7°$

$T_{max} = 40N\text{-}m$

$\Rightarrow Qr = 40$

$\Rightarrow Q = \frac{40}{5\times10^{-3}} = 8000N$

$\tan\alpha = \frac{p}{2\pi r}$

$\Rightarrow \alpha = 3.64°$

$\therefore Q = W\tan(\alpha + \phi_s)$

$\Rightarrow W = \frac{8000}{\tan(16.7 + 3.64°)}$

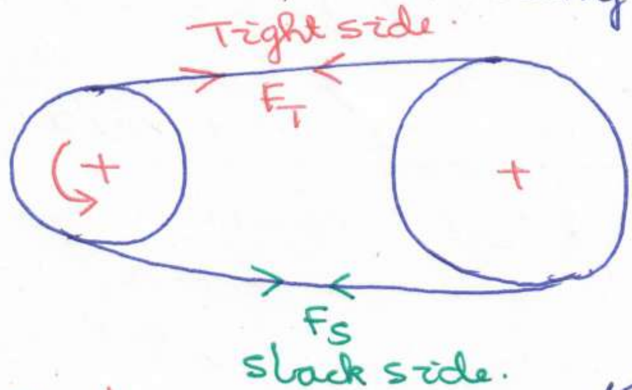
$\Rightarrow W = 21580.5\ N$

$\Rightarrow \underline{W = 21.58kN}$

Belt Friction

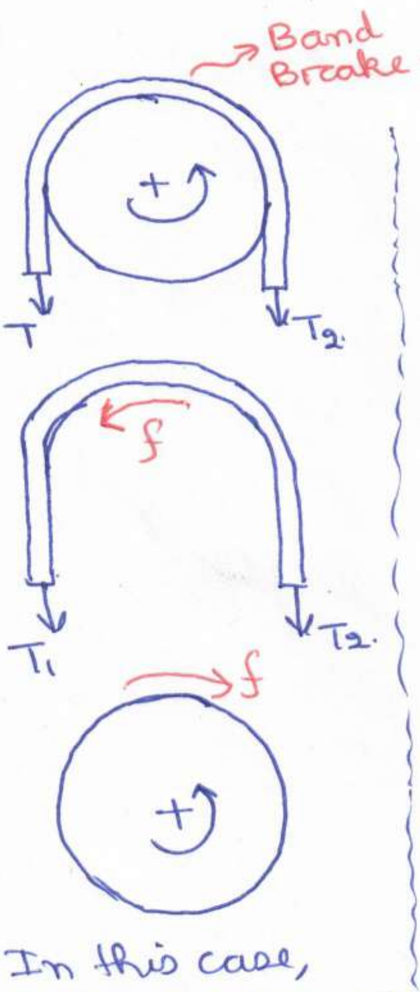
⊛ Why to study belt friction?

— Whenever belt drives or band brakes are designed, it is necessary to determine frictional forces developed b/w the belt and its contacting surface.

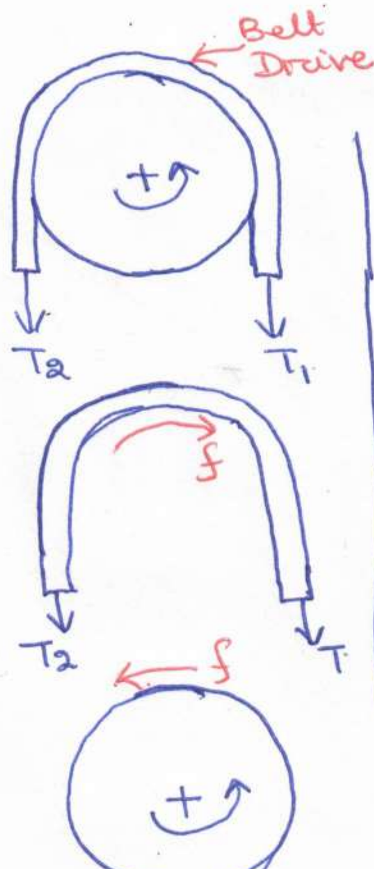


⊛ Normal & frictional forces, acting at different points vary both in magnitude & direction

⊛ Due to these unknown distribution, the analysis of the problem requires the study of forces acting on the differential element of the belt.

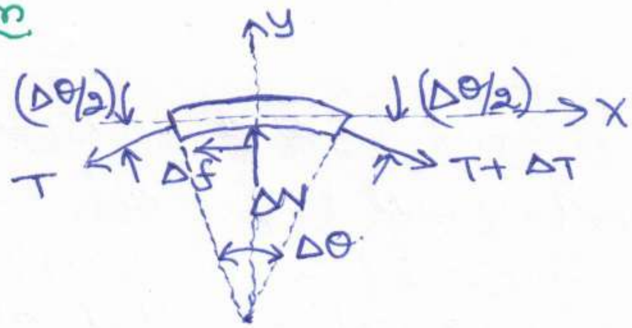


→ In this case, the drum provides friction to the belt, which in turn restricts the motion/rotation of the drum.



→ In this case, the belt drives the drum. Hence, the friction of the belt is transfered to the drum, which makes it rotate.

Derivation



- $\sum F_x = 0 \Rightarrow (T+\Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - \Delta f = 0$

  For impending motion $= \Delta f = \mu_s \Delta N$.

  $\therefore\ \Delta T \cos\frac{\Delta\theta}{2} = \mu_s \Delta N$

- $\sum F_y = 0 \Rightarrow \Delta N = T\sin\frac{\Delta\theta}{2} + (T+\Delta T)\sin\frac{\Delta\theta}{2}$

  $\therefore\ \Delta T\cos\frac{\Delta\theta}{2} = \mu_s\left(T\sin\frac{\Delta\theta}{2} + (T+\Delta T)\sin\frac{\Delta\theta}{2}\right)$

  For $\Delta\theta$ to be small, $\sin\frac{\Delta\theta}{2} \simeq \frac{\Delta\theta}{2}$ and $\cos\frac{\Delta\theta}{2} \simeq 1$.

  $\therefore\ \Delta T = \mu_s\left(2T\cdot\frac{\Delta\theta}{2} + \cancel{\Delta T\cdot\frac{\Delta\theta}{2}}\right)$ neglected

  $\Rightarrow \Delta T = \mu_s(T\Delta\theta)$.

  $\Rightarrow \frac{\Delta T}{\Delta\theta} = \mu_s T$

  lim $\Delta\theta \to 0$, we have, $\frac{dT}{d\theta} = \mu_s T$

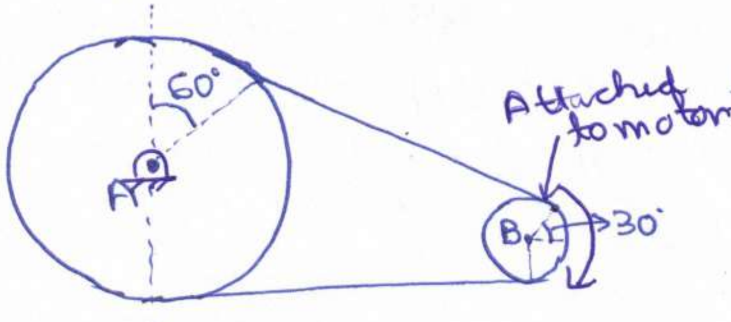
$$\Rightarrow \int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} \mu_s\, d\theta$$

$$\Rightarrow \boxed{\frac{T_2}{T_1} = e^{\mu_s \beta}}$$

$\boxed{T_2 > T_1}$

- T_2 represents the tension in that part of the band breake or belt which pulls
- T_1 is the tension in that part which resists.
- $\beta \rightarrow$ angle of belt-to-surface contact, measured in radians.

Que) If the maximum allowable tension in the belt is 3kN, determine the largest torque that the belt can exert on pulley A.



$r_A = 200\text{ mm}$

$r_B = 25\text{ mm}$

$(\mu_A)_s = (\mu_B)_s = 0.25$

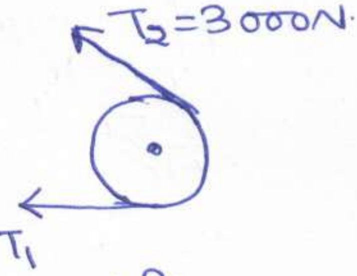
$(\mu_A)_k = (\mu_B)_k = 0.2$

Solution: In this problem, we will have to identify the pulley where the slippage will take place first. The resistance to slippage depends upon the angle of contact (β) b/w the belt and pulley, as well as upon the coefficient of static friction (μ_s).
As 'μ_s' is same for both the pulleys, slippage will occur first in pulley 'B' due to smaller value of 'β'.

$$\beta_A = 180° + 60° = 240° = \frac{4\pi}{3}$$

$$\beta_B = 90° + 30° = 120° = \frac{2\pi}{3}$$

Pulley B:



$$T_2 = T_1 e^{\mu\beta}$$

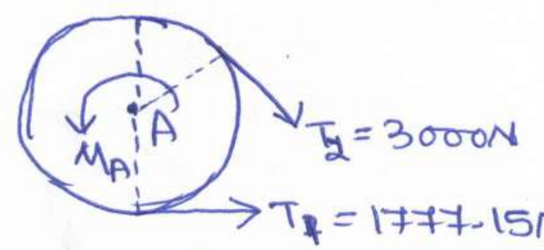
$$\Rightarrow T_1 = \frac{3000}{e^{\frac{1}{4}\left(\frac{2\pi}{3}\right)}} = 1777.15\text{ N}$$

Taking $\sum \vec{M}_A = 0$ (+↺)

$$\Rightarrow M_A - (3000)(0.2) + (1777.15)(0.2) = 0$$

$$\Rightarrow M_A = 244.57\text{ N-m}$$

Pulley A



$$\frac{T_2}{T_1} = e^{\mu\beta}$$

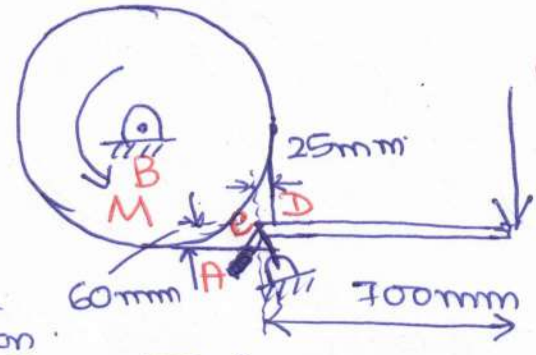
$$\Rightarrow \mu = \frac{1}{\beta}\ln\left(\frac{T_2}{T_1}\right)$$

$$\Rightarrow \mu = \frac{3}{4\pi}\ln\left(\frac{3000}{1777.15}\right)$$

$$\Rightarrow \mu = 0.125 < 0.25$$

$\Rightarrow$ The belt doesn't slip on pulley A.

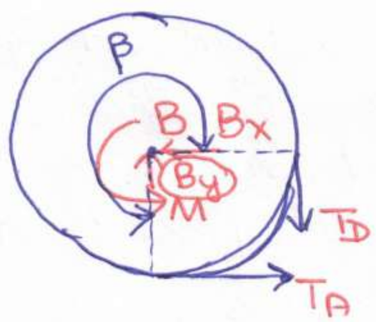
Que) Determine the force P that must be applied to the handle of lever so that the wheel is on the verge of turning if $M = 300\,N\text{-}m$. $\mu_s = 0.3$.



$r = 300mm$

Solution

$T_D > T_A \;,\; \beta = \frac{3\pi}{2}$



- We know that,

$T_D = T_A e^{\mu\beta} = T_A e^{(0.3)\left(\frac{3\pi}{2}\right)}$

$\Rightarrow \underline{T_D = 4.11\,T_A}$

- $\sum \overrightarrow{M_B} = 0$ $+\circlearrowleft$

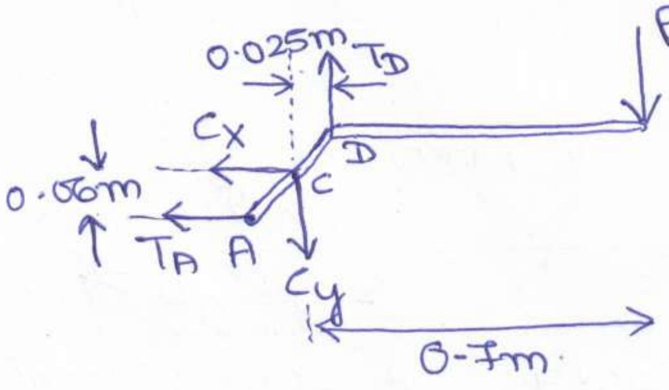
$\Rightarrow M + T_A(0.3) - T_D(0.3) = 0$

$\Rightarrow \frac{M}{0.3} = T_D - T_A$

$\Rightarrow \frac{300}{0.3} = (4.11 - 1)\,T_A$

$\Rightarrow \boxed{T_A = 321.5N}$

$\Rightarrow \boxed{T_D = 1321.365N}$



$\sum M_c = 0$ $+\circlearrowleft$

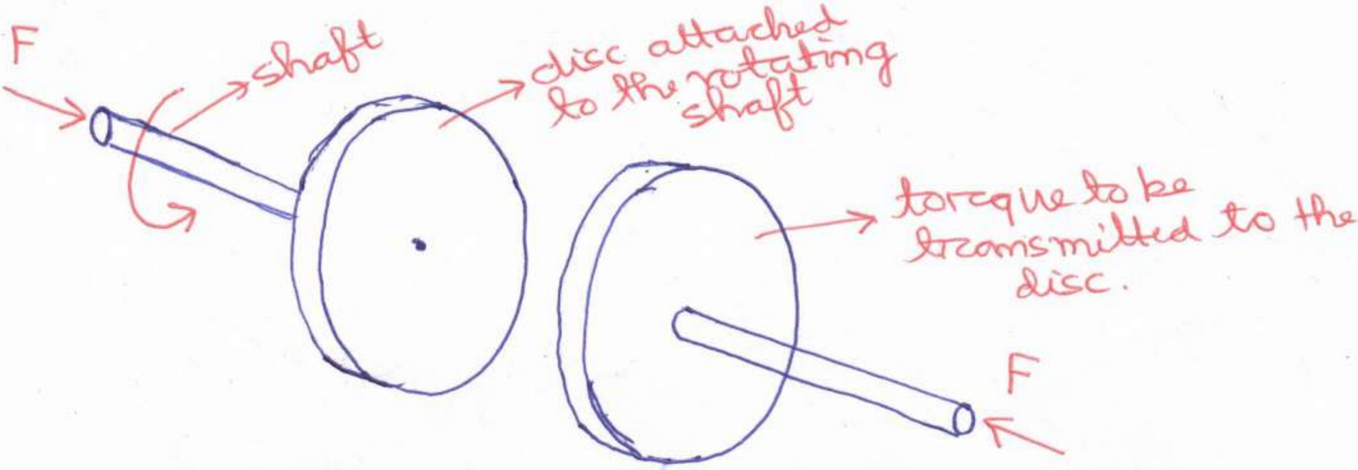
$\Rightarrow -P(0.7) - T_A(0.06) + T_D(0.025) = 0$

$\Rightarrow P(0.7) = 1321.365(0.025) - 321.5(0.06)$

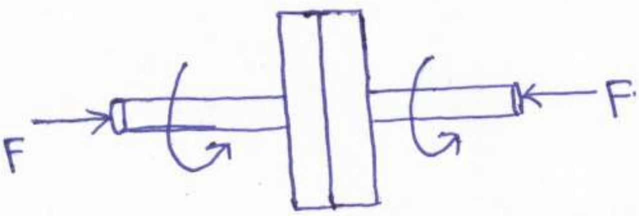
$\Rightarrow \boxed{P = 19.6N}$

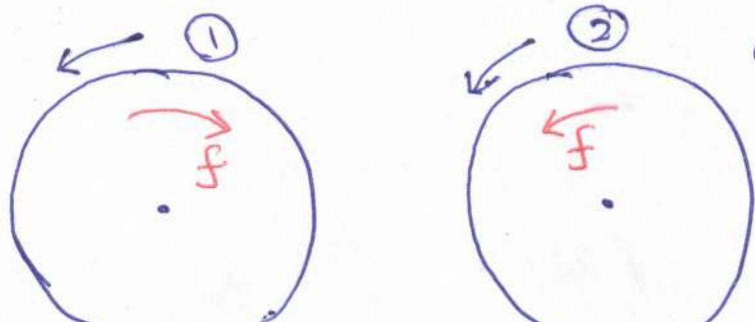
Disc Friction:

- Transfer of torque between two friction discs.
  Eg. Clutch plate in an automobile.



→ During engagement of both the discs.



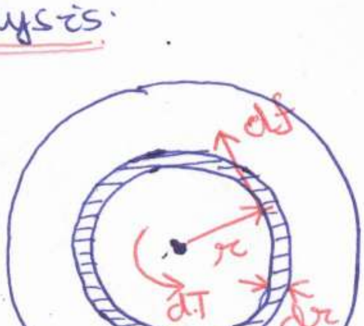


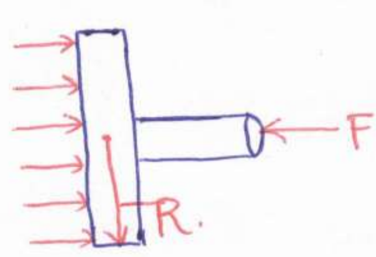
(*) The friction in the disc 2 helps the disc in rotating.
⇒ The disc-2 rotates as a result of friction force produced on it.

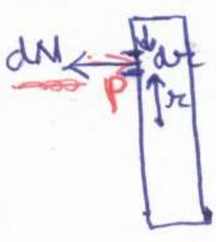
Rotating disc

Disc on which torque is to be transmitted.

Analysis:







The torque due to the frictional force acting on the elemental ring is given by-

$dT = r\,df = r\,\mu\,dN$

Let 'P' be the uniform pressure acting on the disc. So to have the force balance.

$P(\pi R^2) = F$

$\Rightarrow P = \dfrac{F}{\pi R^2}$

$dN = P\,dA$

$\Rightarrow P(2\pi r)\,dr$

$dN = \dfrac{F}{\pi R^2}(2\pi r)\,dr$

$\therefore$ The total torque can be given as -

$$\int_0^T dT = \int_0^R \mu r \frac{F}{R^2}(2r)\,dr$$

$$\Rightarrow T = \frac{2\mu F}{R^2}\left[\frac{r^3}{3}\right]_0^R$$

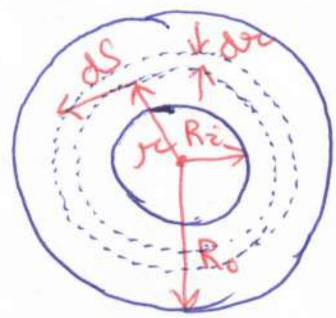
$$\Rightarrow \boxed{T = \frac{2}{3}\mu F R}$$

- Analysis for an annular disc

$$P\left[\pi(R_o^2 - R_i^2)\right] = F$$

where R_o = outer radius

R_i = inner radius



$$dN = P\,dA$$
$$= P(2\pi r)\,dr$$
$$\Rightarrow dN = \frac{F}{\pi(R_o^2 - R_i^2)}(2\pi r)\,dr$$

Now, $dT = r\,df$

$$= r\mu\,dN$$

$$dT = \mu r \frac{2F}{(R_o^2 - R_i^2)} r\,dr$$

$\therefore$ The total torque is given as -

$$\int_0^T dT = \frac{2\mu F}{(R_o^2 - R_i^2)}\int_{R_i}^{R_o} r^2\,dr$$

$$\Rightarrow \boxed{T = \frac{2}{3}\mu F\frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)}}$$