NPHI101: Engineering Physics

Mid-Semester Examination 2024-25 (Monsoon Semester)

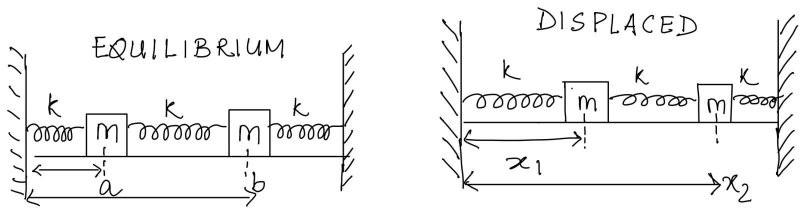
Total Marks: 60 **Time: 2 hours**

Instructions:

1. **Use separate answer sheets for Part 1, Part 2, and, Part 3. Mention the Part No. at the top.**
2. **Some important values are provided at the end.**

Part 1 (Classical Mechanics and Electrodynamics)

1. Consider the coupled oscillator shown below; k is the spring constant for all the springs; m is the mass of both masses, $(x_1 - a)$ is the displacement of the left mass from its equilibrium position, a, and $(x_2 - b)$ is the displacement of the right mass from its equilibrium position, b. The displacement grows positively moving from left to right.



(a) Write down the generalised coordinates for this system. (b) Write down the Lagrangian for this system. (c) Show that the Euler-Lagrange equations of motion can be written as: $\ddot{x}_1 = -\frac{k}{m}(2x_1 - x_2)$, $\quad \ddot{x}_2 = -\frac{k}{m}(2x_2 - x_1)$. **Marks: 2+2+4=8**

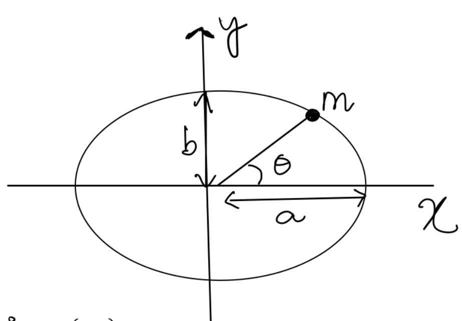


Fig.2(L): BEAD ON ELLIPSE

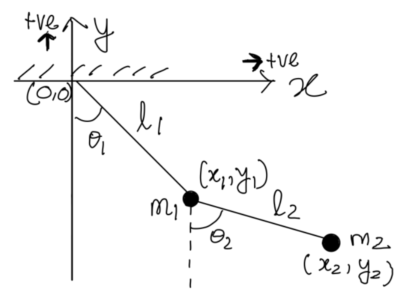


Fig.2(R): DOUBLE PENDULUM

2. (a) Write down the generalised coordinates for the two systems shown above in Fig.2(L), and Fig.2(R). Also, write down the transformation equations (relation between x, y, and generalised coordinates in Fig. 2(L); x_1, y_1, x_2, y_2, and generalised coordinates in Fig. 2(R)).

   (b) Given a Lagrangian, $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2)$, write down the cyclic coordinates. (Note that r, θ, and z are the generalised coordinates in this problem.) **Marks: (2+3)+2=7**

3. Given a scalar potential, $V = 3x^2z^2 - xy^2z^3 + \text{constant}$, find:

   (a) the respective force, $\vec{F}$, (the force will be conservative), and, (b) the work done by $\vec{F}$ in moving a particle from the point, $A(-2, 1, 3)$ to $B(1, -2, -1)$. **Marks: 2+3=5**

Part 2 (Thermal and Statistical Physics)

4. (a) A real gas follows the Van der Waals equation of state $[(P+\frac{a}{V^2})(V-b)=Nk_\mathrm{B}T]$, where a and b are constants and k_B is the Boltzmann constant. Calculate the work done when the gas is isothermally compressed from volume V_0 to $V_0/2$.

(b) From the combined first and second law of thermodynamics, deduce the thermodynamic relation: $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$

(c) The P-V diagram of a Carnot cycle was discussed in the class. Draw the T-S diagram of a Carnot cycle and mark the isothermal and adiabatic parts of the cycle. **Marks: 4+3+3=10**

5. (a) The temperature of a black body increased from $T_1 = 500\,\mathrm{K}$ to $T_2 = 1227°\mathrm{C}$. Qualitatively show the spectral energy density as a function of wavelength (use appropriate units on the axis) for both temperatures. Find the wavelength of the radiation corresponding to the maximum spectral energy density at $T_1 = 500\,\mathrm{K}$. Find the ratio of final to initial value of total emissive power (P_2/P_1) of the black body.

(b) Write down Planck's expression for the spectral energy density in black body radiation. Show that it reduces to Wien's law at short wavelength limit. **Marks: (3+2+2)+3=10**

Part 3 (Modern Physics)

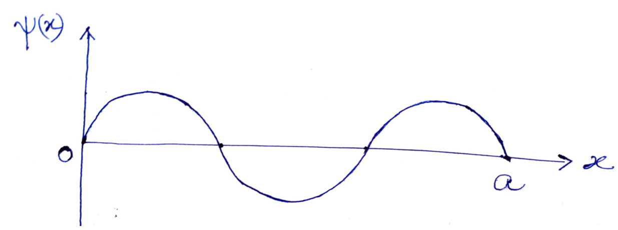
6. (a) Considering the wave-particle duality of light, calculate light particle energy in eV if the corresponding wavelength is 800 nm. Using pictorial demonstration explain the atomic stimulated transitions.

(b) The lifetime of an upper energy level E_2 of an atom due to spontaneous decay is $\tau_2 = 30$ ns. If the initial population at E_2 is 5×10^{15}, then calculate the population at that level at time $t = 60$ ns.

(c) Write down the axial mode resonance condition in a standing wave laser cavity. Assume that the cavity length of a laser system is 60 cm. Derive the expression of the axial mode spacing and then calculate its value for this laser system. **Marks: (2+2)+4+(2+2)=12**

7. (a) Explain the physical interpretation of the wave function $\psi(x)$ of a quantum particle.

(b) The wave function of a quantum particle is defined between $x=0$ to $x=a$ as shown below. Qualitatively plot the probability density.



(c) Consider a quantum particle in one dimension (x-axis). The wave function is given by: $\psi(x) = Ae^{-a|x|}$, where a and A are positive constants. Calculate the expectation value $<x>$ of the position of the particle in this state. **Marks: 2+2+4=8**

Some important data:

$1\,\mathrm{MeV} = 1.6\times10^{-13}$Joule $= 1.6\times10^{-6}$erg, $\hbar = 1.05\times10^{-25}$erg s, Wien constant $b = 2.898$ mm-K. $k_\mathrm{B} = 1.38\times10^{-23}$ J/K, $h = 6.626\times10^{-34}$ J.s, $\sigma = 5.67\times10^{-8}$ W.m^{-2}.K^{-4}

4.(a) $\left(P+\frac{a}{V^2}\right)(V-b) = Nk_BT \Rightarrow P = \frac{Nk_BT}{V-b} - \frac{a}{V^2}$

Work done during compression,

$$W = -\int_{V_0}^{V_0/2} P\,dV = -\int_{V_0}^{V_0/2}\left(\frac{Nk_BT}{V-b} - \frac{a}{V^2}\right)dV$$

$$= -Nk_BT\ln(V-b)\Big|_{V_0}^{V_0/2} - \frac{a}{V}\Big|_{V_0}^{V_0/2} = Nk_BT\ln\left(\frac{V_0-b}{V_0/2-b}\right) - \left(\frac{a}{V_0/2} - \frac{a}{V_0}\right)$$

$$= Nk_BT\ln\left(\frac{2V_0-2b}{V_0-2b}\right) - \frac{a}{V_0} \quad \underline{\text{Ans}}.$$

4.(b) $dU = TdS - PdV$

$$\Rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P \quad \text{......(i)}$$

We know $dF = -SdT - PdV$ ($F \rightarrow$ Helmholtz free energy) $F = F(V,T)$.

$$\Rightarrow P = -\left(\frac{\partial F}{\partial V}\right)_T \quad \text{and} \quad S = -\left(\frac{\partial F}{\partial T}\right)_V$$

Since dF is an exact differential,

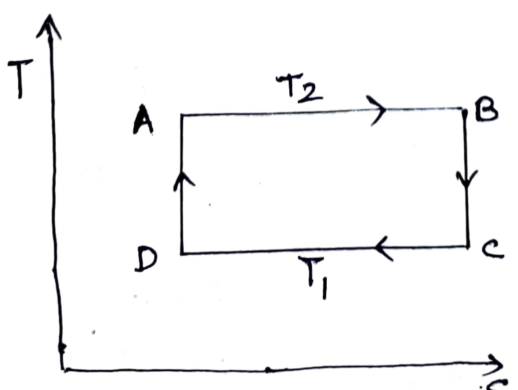
$$\frac{\partial^2 F}{\partial V \partial T} = \frac{\partial^2 F}{\partial T \partial V} \Rightarrow \left[\frac{\partial}{\partial V}\left(\frac{\partial F}{\partial T}\right)_V\right]_T = \left[\frac{\partial}{\partial T}\left(\frac{\partial F}{\partial V}\right)_T\right]_V$$

$$\Rightarrow -\left(\frac{\partial S}{\partial V}\right)_T = -\left(\frac{\partial P}{\partial T}\right)_V \Rightarrow \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

Replacing $\left(\frac{\partial S}{\partial V}\right)_T$ in equation (i) we get,

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

4.(c)

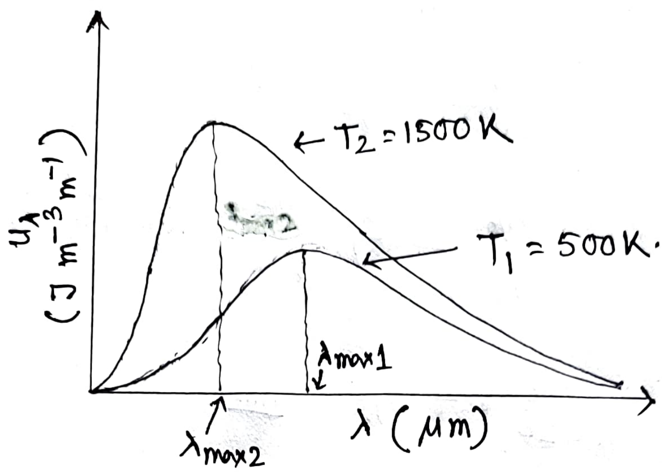


- AB and CD are isothermal part
- BC and DA are adiabatic part.

Temp $T_2 > T_1$.

5 (a) $T_1 = 500\,K$

$T_2 = 1227^\circ C = 1500\,K$



Wien's Displacement law, $\lambda_{max} T = b = 2.898\ mm\,K$

~~For~~ Here λ_{max} is the wavelength corresponding to maximum spectral energy density.

For $T_1 = 500\,K$,

$$\lambda_{max1} \times 500K = 2.898\ mm.K$$

$$\Rightarrow \lambda_{max1} = \frac{2.898}{500}\ mm = \underline{5796\ nm}$$

According to Stefan Boltzman law total emissive power of a black body $P \propto T^4$ where T is the temperature.

Here, $$\frac{P_2}{P_1} = \frac{T_2^4}{T_1^4} = \left(\frac{1500}{500}\right)^4 = 3^4 = 81$$

The final total emissive power (P_2) is 81 times the initial value.

5. (b) Planck's expression, $$u(\lambda) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

For short wavelength, $\lambda \to 0$

Then $e^{\frac{hc}{\lambda kT}} \gg 1$ and 1 can be ignored.

We get $$u(\lambda) = \frac{8\pi hc}{\lambda^5} e^{-\frac{hc}{\lambda kT}} \equiv \frac{A}{\lambda^5} e^{-\frac{a}{\lambda T}} \quad \text{(Wien's law)}$$

Taking $A = 8\pi hc$, $a = \frac{hc}{k}$