Quantum Mechanics :

(Q. M.)

Book
Introduction to
Quantum Mechanics
David J. Griffiths

Classical Mechanics.

Consider a particle of mass m, moves only in x-direction

(1 dimensional problem)
(1-D)

Force acting $\vec{F}(x,t)$

~~M~~ Newton's second law

$$\frac{d\vec{p}}{dt} = \vec{F} \qquad \vec{p} = m\frac{d\vec{x}}{dt}$$

In 1-D vector sign can be avoided

$$m\frac{d^2x}{dt^2} = F$$

We can solve for $x(t)$ [The location of the particle]

Then, $v = \frac{dx}{dt}$ $\qquad T = \frac{1}{2}mv^2$

$$p = mv$$

All these physical quantities can be calculated deterministically (No uncertainty) when appropriate initial conditions are provided. (typically position & velocity at $t=0$)

* Q. Mech. approches this problem very differently. Particularly for microscopic particles, the classical description can't explain many physical phenomena And this New approach of "Q. Mech." is necessary.

Microscopic particle (e.g. electron)

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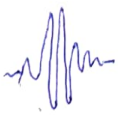
point

classical picture



wave packet

True quantum picture

Similar to light (EM wave & photon) material particle also has dual ~~na~~ nature

~~The ~~ electron is ref~~

The microscopic particle is represnted by a wave packet (is a "wave function" ψ) having wave properties λ & $\vec{k}$ and particle property E & $\vec{p}$

~~If~~ The wave function $\psi(x,t)$ for a particle under the influence of a potential energy V, follow the fundamental equation → Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi$$

In quantum mechanics can we ask about exact location of the particle with as much precision as we want, along with the precise momentum?

→ NO ←

NO

Q. Mech. is not deterministic (indeterminacy)

rather probabilities of finding any physical observable (position, momentum etc.) are to be discussed in the framework of Q. Mech.

What is wave function ?

Born's statistical interpretation

$|\psi|^2$ is probability density

In mathematical language

$$\int_a^b |\psi(x,t)|^2 dx = \left\{ \begin{array}{l} \text{probability of finding} \\ \text{the particle between} \\ a \;\&\; b, \text{ at time } t \end{array} \right\}$$

Probability of finding the particle between a & b = Area under the graph of $|\psi|^2$

Q. Mech. offers statistical information about possible results.

↳ This is a fact of nature and not a defect in the theory.

$$\boxed{\int_{-\infty}^{+\infty} |\psi|^2 dx = 1}$$

Global conservation of prob.

Probability

Measuring a quantity x (Energy or momentum or position...)

Discrete possible results $x_1, x_2, x_3, x_4, \dots\dots$

Corresponding probabilities $p(x_1), p(x_2), \dots\dots$

Statistical interpretation of probability

Performed the experiment to find x large N number of times.

Let say we get $x = x_n$ r_n times out of N

$$\boxed{p(x_n) = \frac{r_n}{N}}$$

probability of getting result x_n

Simple example Only three possible results

x_1 x_2 x_3

r_1 r_2 r_3 $N = r_1 + r_2 + r_3$

$$p(x_1) = \frac{r_1}{N} \quad p(x_2) = \frac{r_2}{N} \quad p(x_3) = \frac{r_3}{N}$$

$$P_{\text{any } x} = p(x_1) + p(x_2) + p(x_3) = \frac{r_1}{N} + \frac{r_2}{N} + \frac{r_3}{N} = \frac{N}{N} = 1$$

Mean value / Average / expectation value

↳ language in Q.M.

$$\langle x \rangle = \frac{(x_1 + x_1 + x_1 + \cdots) + (x_2 + x_2 + x_2 + \cdots) + (x_3 + x_3 + x_3 + \cdots)}{N}$$

$$= \frac{n_1 x_1}{N} + \frac{n_2 x_2}{N} + \frac{n_3 x_3}{N} = x_1\left(\frac{n_1}{N}\right) + x_2\left(\frac{n_2}{N}\right) + x_3\left(\frac{n_3}{N}\right)$$

$$= x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3)$$

$$= \sum_{i=1}^{3} x_i\, p(x_i)$$

<u>Generalize</u>

$$\langle x \rangle = \sum_{i=1}^{n} x_i\, p(x_i)$$

For continuous result $\xleftrightarrow{\quad x \quad}$
instead of discrete possible results

$$\langle x \rangle = \int x\, p(x)\, dx$$

$p(x)\,dx$ → probability

Mean value or expectation value

$$\boxed{\langle x \rangle = \int_{-\infty}^{\infty} x\, |\psi|^2\, dx}$$

Quantum particle with ψ

$p(x)$ → probability density.