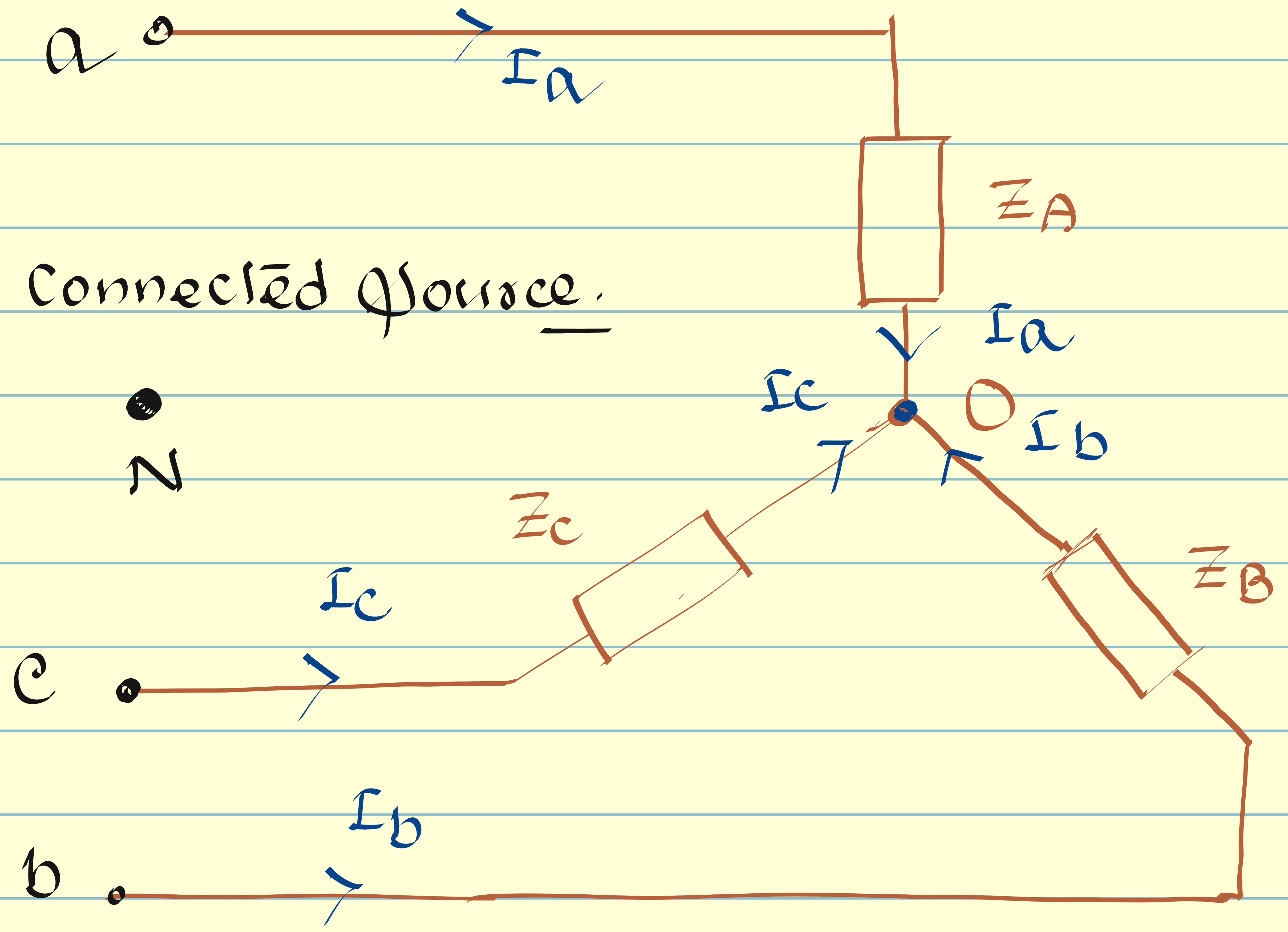
Unbalanced 3ϕ System:-

Y Connected Source.



Here we are assuming that the unbalancing is coming into the picture due to the difference in the per phase impedance.

3ϕ Source is balanced

Here $$\tilde{I}_a = \frac{\tilde{V}_{aO}}{Z_A} = \frac{\tilde{V}_{aN} - \tilde{V}_{ON}}{Z_A}$$

$$\tilde{I}_b = \frac{\tilde{V}_{bO}}{Z_B} = \frac{\tilde{V}_{bN} - \tilde{V}_{ON}}{Z_B}$$

$$\tilde{I}_c = \frac{\tilde{V}_{cN} - \tilde{V}_{ON}}{Z_C}$$

$$\tilde{I}_a + \tilde{I}_b + \tilde{I}_c = \frac{\tilde{V}_{aN}}{Z_A} + \frac{\tilde{V}_{bN}}{Z_B} + \frac{\tilde{V}_{cN}}{Z_C} -$$

$$- \tilde{V}_{ON}\left(\frac{1}{Z_A}+\frac{1}{Z_B}+\frac{1}{Z_C}\right)$$

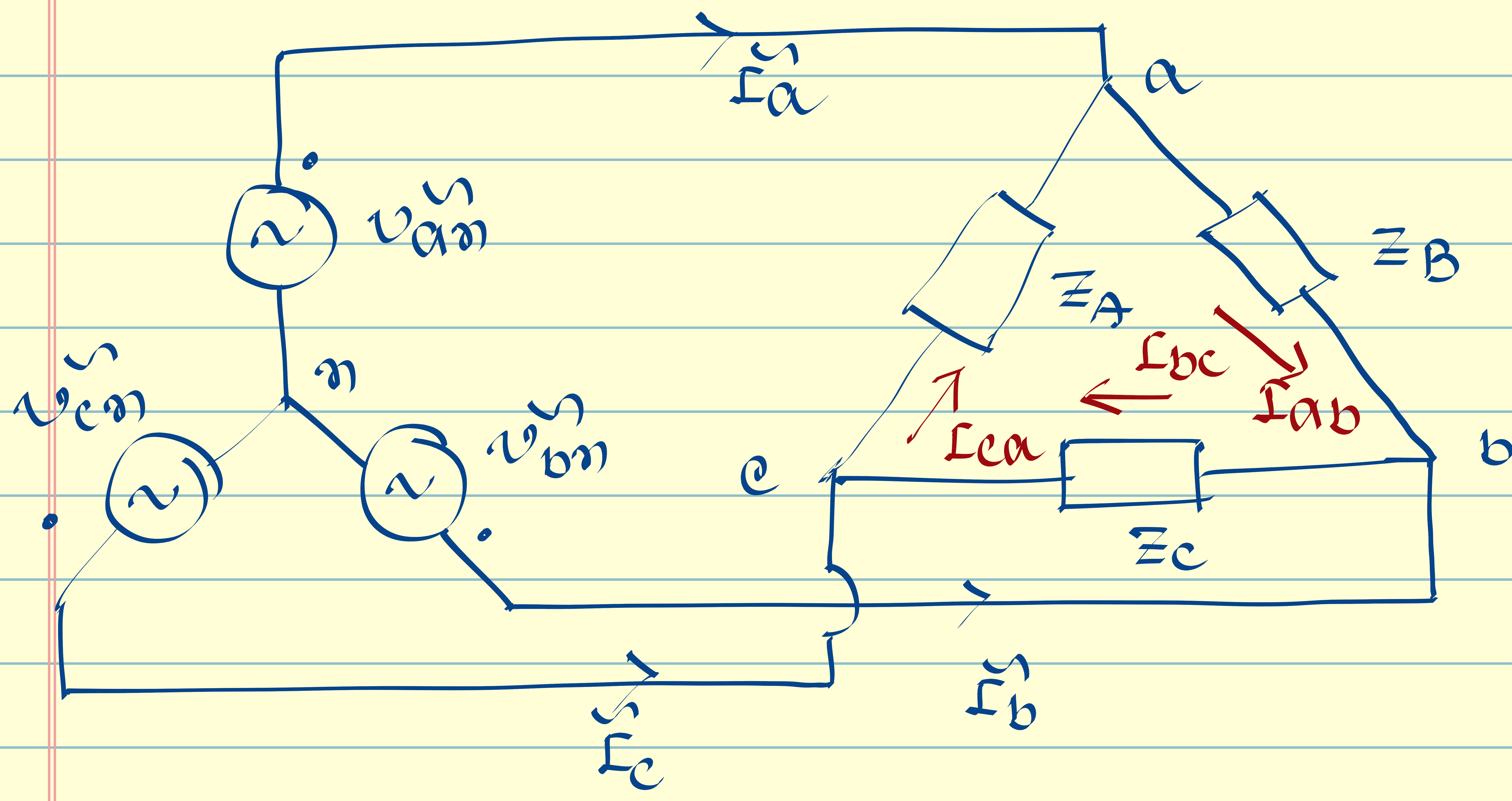
@ point O apply KCL

$$\tilde{I}_a+\tilde{I}_b+\tilde{I}_c=0$$

$$\tilde{V}_{ON}=\frac{\dfrac{\tilde{V}_{aN}}{Z_A}+\dfrac{\tilde{V}_{bN}}{Z_B}+\dfrac{\tilde{V}_{cN}}{Z_C}}{\dfrac{1}{Z_A}+\dfrac{1}{Z_B}+\dfrac{1}{Z_C}}$$

Now we can find out the values of $\tilde{I}_a$, $\tilde{I}_b$, $\tilde{I}_c$.

Let us now consider a system



$\tilde{I}_a$? $\tilde{I}_b$? $\tilde{I}_c$?

$$\tilde{I}_a = \tilde{I}_{ab} - \tilde{I}_{ca}$$

$$= \frac{v_{ab}}{Z_B} - \frac{v_{ca}}{Z_A}$$

$$= \frac{\tilde{v}_{an} - \tilde{v}_{bn}}{Z_B} - \frac{\tilde{v}_{cn} - \tilde{v}_{an}}{Z_A}$$

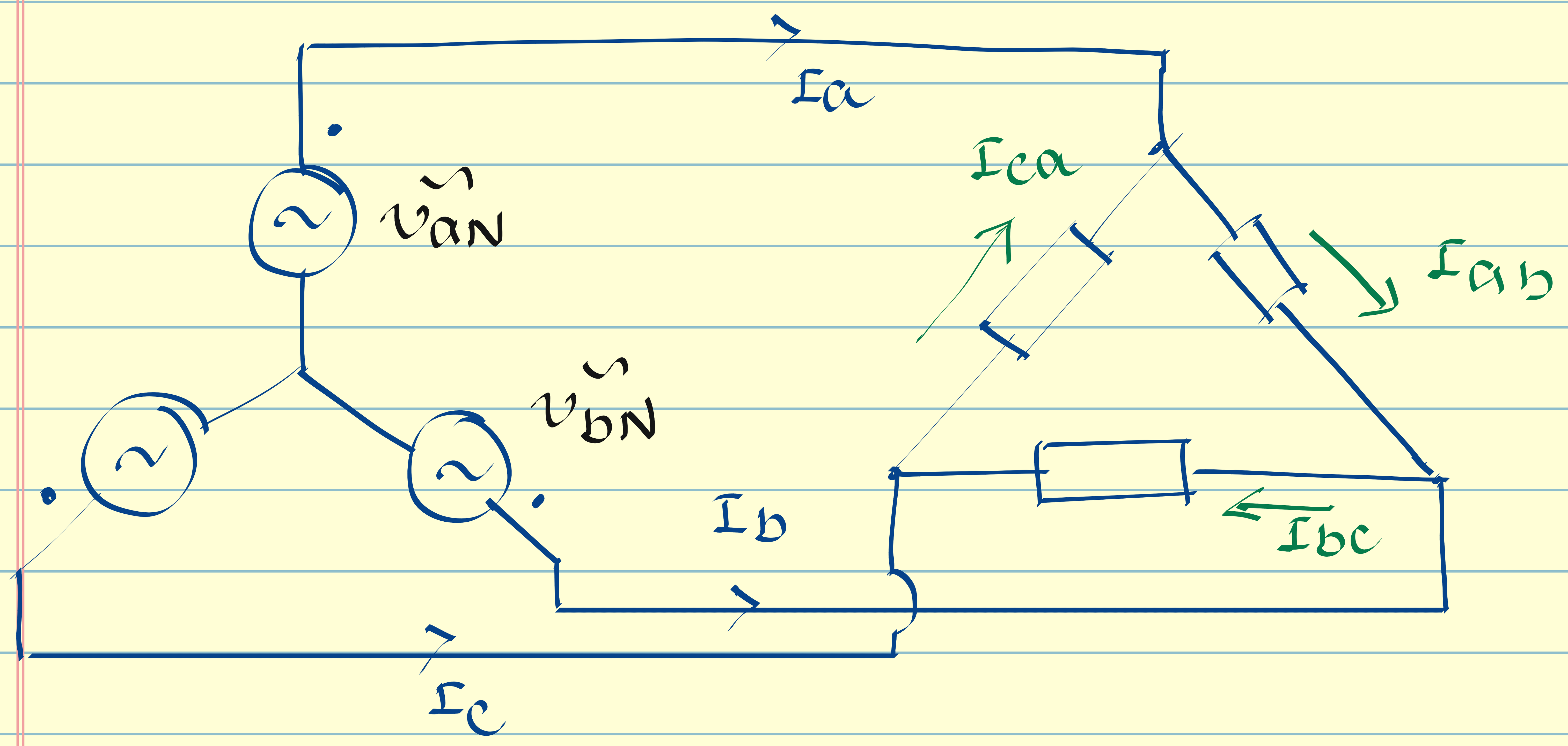
$$= \tilde{v}_{an}\left(\frac{1}{Z_A} + \frac{1}{Z_B}\right) + \tilde{v}_{bn}\left(-\frac{1}{Z_B}\right) + \tilde{v}_{cn}\left(-\frac{1}{Z_A}\right)$$

Similarly you can do the calculations for $\tilde{I}_b$ and $\tilde{I}_c$

Example-0 :-

A balanced Y connected source $V_{aN} = 100\angle 10^\circ$ is connected to a balanced Δ connected load with per phase load impedance of $Z_{ph} = 8+j4\,\Omega$

Determine the phase and line current.



using KVL in the mesh

$$\tilde{V}_{aN} - I_{ab} Z_\Delta - \tilde{V}_{bN} = 0$$

$$I_{ab} = \frac{100\angle 10 - 100\angle -110}{8+j4}$$

$$= 19.3649 \angle 13.4349 \text{ A}$$

for balanced system

$$I_{bc} = 19.3649\angle -106.5651 \text{ A}$$

$$I_{ca} = 19.3649\angle +133.4349 \text{ A}$$

phase current

$$I_a = I_{ab} - I_{ca}$$

$$= 19.3649\angle 13.4349 - 19.3649\angle 133.4349$$

$$\underbrace{= 33.541\angle -16.5651}_{\text{Line current}}$$

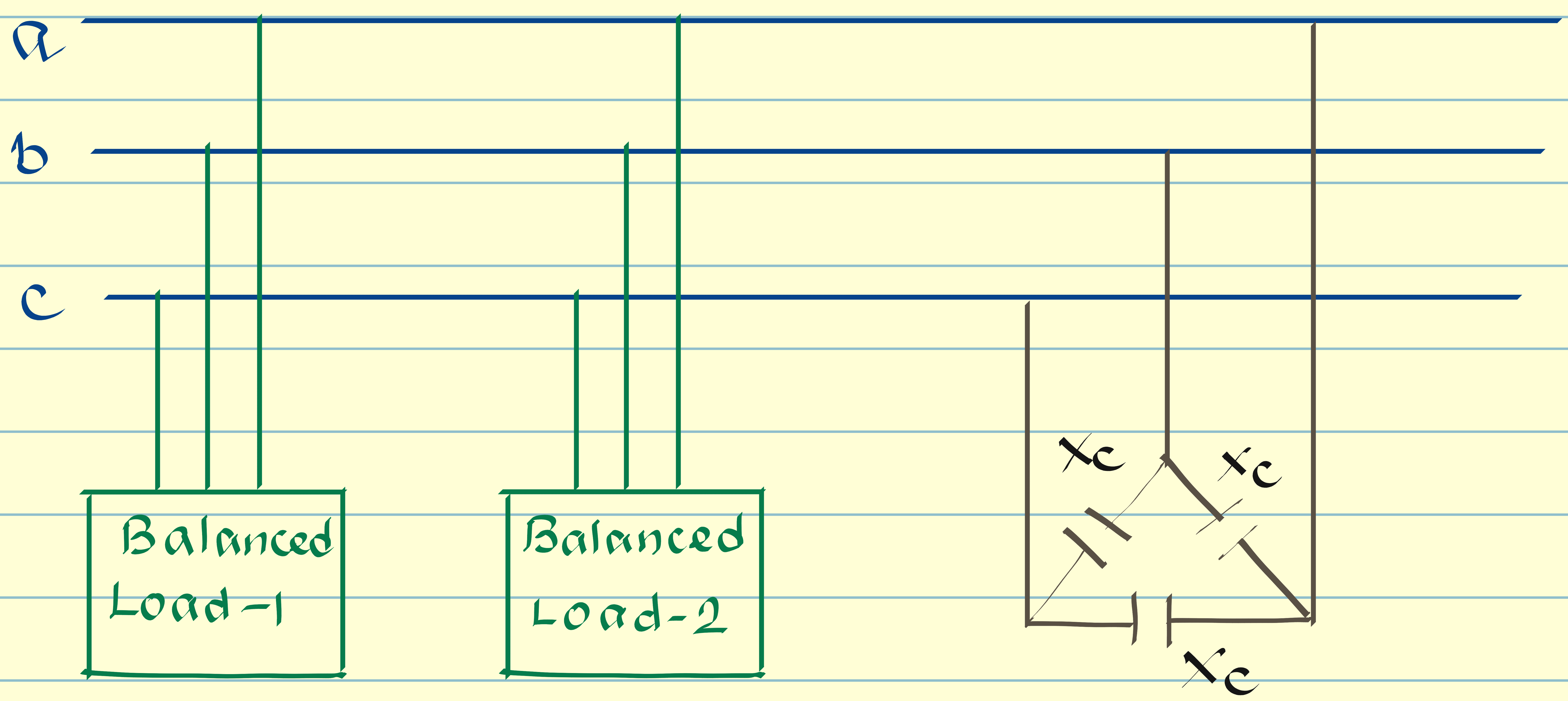
(b) Single phase ckt analysis

$$I_a = \frac{V_{aN}}{Z_\Delta/3}$$

$$\underbrace{= \frac{(100\angle 10)\times 3}{8+j4} = 33.541\angle -16.5650}$$

answer is matching by both of the methods.

Example-1 :-



This is a 240 V, 60 Hz 3 ϕ balanced system with phase seq a-b-c

Balanced load-1 : 30 kW 0.6 pf lag

Balanced load-2 : 60 kW 0.8 pf lag

* Determine the complex real & reactive power of the combined load.

S. $P_{L1} = 30\text{ kW}$ $Q_{L1} = 30 \times \tan\cos^{-1}(0.6)$ kVAr

$= 40\text{ kVAr}$

$P_{L2} = 60\text{ kW}$ $Q_{L2} = 60 \times \tan\cos^{-1}(0.8)$

$= 45\text{ kVAr}$

For combined load

$$P_t = P_{L1} + P_{L2} = 90 \text{ kW}$$
$$Q_t = Q_{L1} + Q_{L2} = 85 \text{ kVAr}$$

* kVAr rating of the capacitor bank to raise the power factor to 0.9 lagging

As the required pf is 0.9 (lag), so the reactive power generated by the capacitor bank should be ⇒

$$Q_c = 85 - 90 \times \tan(\cos^{-1} 0.9)$$
$$= 41.4110 \text{ kVAr}$$

* In that case, determine the capacitance c

$$\frac{Q_c}{3} = \frac{V^2}{X_c}$$

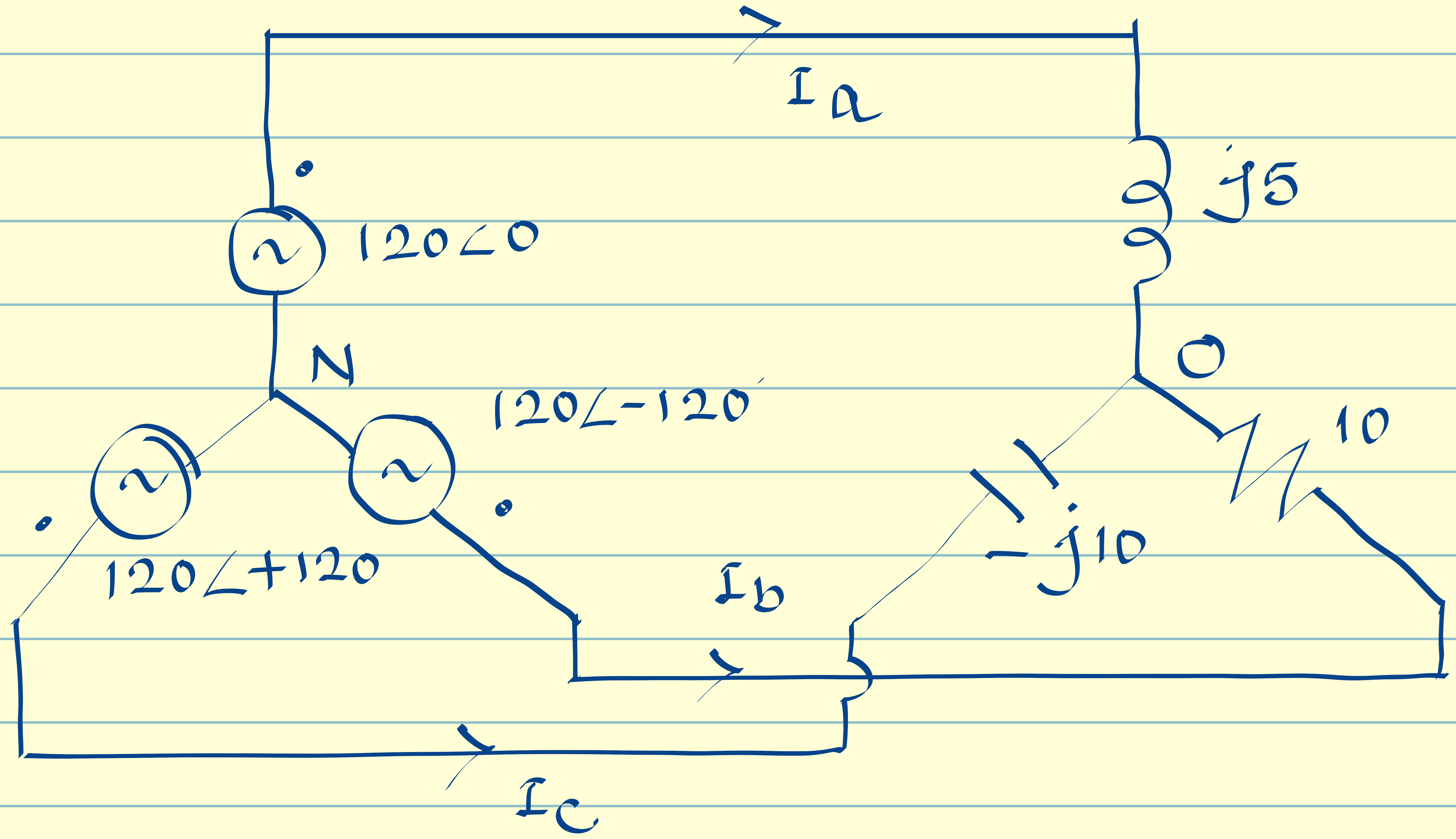
$$X_c = \frac{(240 \times 10^3)^2 \times 3}{41.4110 \times 10^3} = 4.1728 \times 10^6$$

$$C = \frac{1}{2\pi \times 60 \times 4.1728 \times 10^6} \text{ F}$$

$$= 635.6840 \text{ pF}$$

Example-2 :-

Consider a 3ϕ Y connected unbalanced load which is powered from 3ϕ balanced supply voltage.



* Determine the line currents.

$$\boxed{\tilde{V}_{on} = \frac{\dfrac{\tilde{V}_{An}}{Z_1} + \dfrac{\tilde{V}_{Bn}}{Z_2} + \dfrac{\tilde{V}_{cn}}{Z_3}}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \dfrac{1}{Z_3}}}$$

Current that will flow depends on the connection.

$$V_{on} = 308.2406 \angle -67.088 \ V$$

$$\tilde{I}_a = \frac{V_{aN} - V_{oN}}{Z_A}$$

$$= \frac{120\angle 0 - 308.2406\angle -67.088}{j5}$$

$$= (56.784 \angle 0.000569) \ A.$$

$$\tilde{I}_b = \frac{V_{bN} - V_{oN}}{Z_B}$$

$$= \frac{120\angle -120 - 308.2406\angle -67.088}{10}$$

$$= 25.4559 \angle 135.0007 \ A$$

$$\tilde{I}_c = \frac{V_{cN} - V_{oN}}{Z_C}$$

$$= \frac{120\angle +120 - 308.2406\angle -67.088}{-j10}$$

$$= 42.758 \angle -155.1034^\circ \ A$$

* Determine the complex power consumed by the load.

$$S_A = P_A + jQ_A$$

$$P_A = (56.784)^2 \times 0 = 0$$

$$Q_A = (56.784)^2 \times 5 = 16.1221 \text{ KVAR}$$

$$S_B = P_B + jQ_B$$

$$P_B = (25.4559)^2 \times 10 = 6.480 \text{ KW}$$

$$Q_B = 0$$

$$S_C = P_C + jQ_C$$

$$P_C = 0$$

$$Q_C = -(42.758)^2 \times 10$$

$$= -18.2824 \text{ KVAR}$$

$$S = S_A + S_B + S_C$$

$$= (6.480 + j16.1221 - j18.2824) \text{ KVA}$$

$$= \boxed{6.480 - j2.1603 \text{ KVA}}$$

Ans