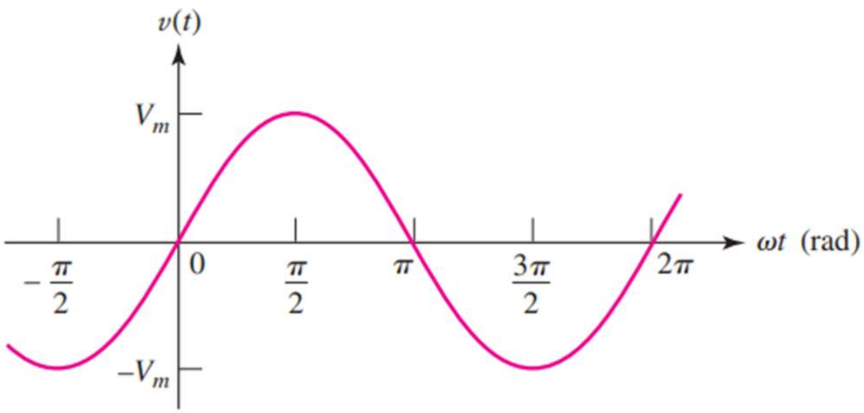
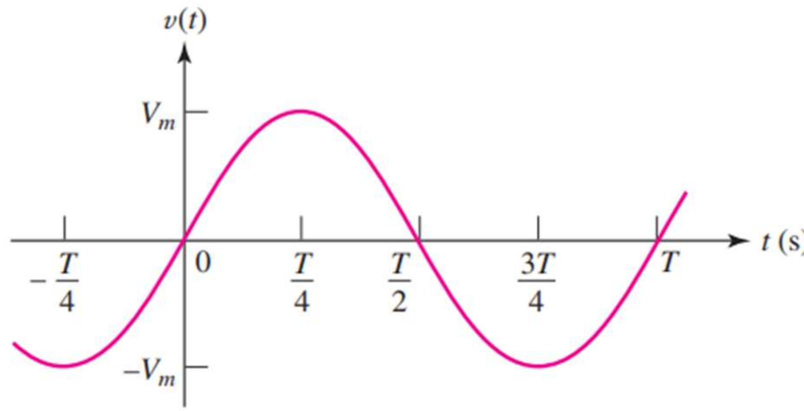
CHARACTERISTICS OF SINUSOIDS





- A sinusoidally varying voltage function can be represented by $v(t) = V_m \sin(\omega t)$
- The function repeats itself in every 2π radians. Therefore, period is 2π radians.
- Frequency $f = 1/T$
- We know $\omega T = 2\pi$

Ref. William H. Hayt Jr, Jack E. Kemmerly and Steven M. Durbin, “Engineering Circuits Analysis”, McGraw Hill publishers

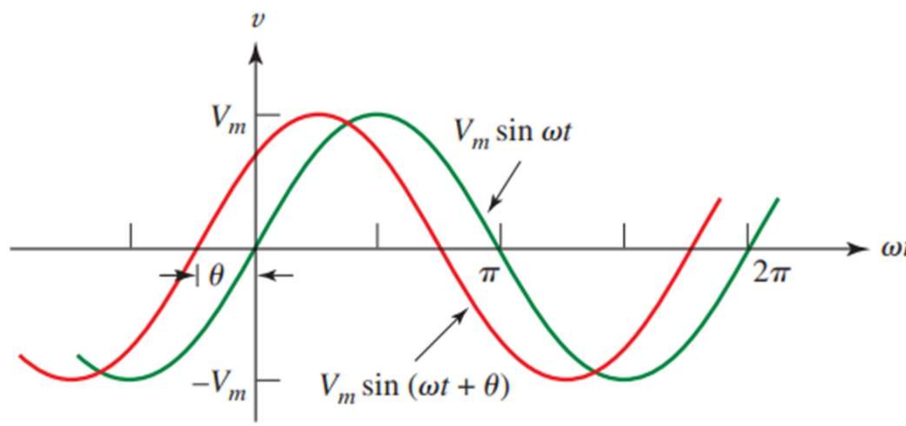
CONCEPTS OF LAGGING & LEADING

- More general form of sinusoids is $v(t) = V_m \sin(\omega t + \theta)$

- Here θ is the phase angle measured in radian. However, for representation purpose sometimes we use to express θ in degree.

- At $t = 0$, $v(t = 0) = V_m \sin\theta$

- Therefore, $V_m \sin(\omega t + \theta)$ sinusoid leads $V_m \sin(\omega t)$ sinusoid.



Ref. William H. Hayt Jr, Jack E. Kemmerly and Steven M. Durbin, “Engineering Circuits Analysis”, McGraw Hill publishers

COMMON TERMS IN AC CIRCUITS

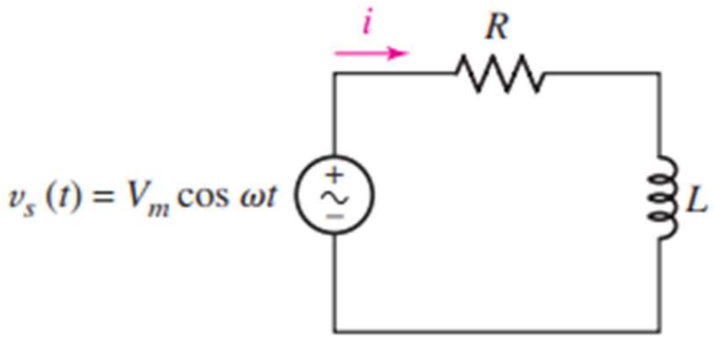
RMS and the average value of a signal.

Ref. William H. Hayt Jr, Jack E. Kemmerly and Steven M. Durbin, “Engineering Circuits Analysis”, McGraw Hill publishers

SINUSOIDAL FORCED RESPONSE

- A series R-L circuit is excited by a sinusoidal voltage waveform.
- The corresponding differential equation is

$$\boldsymbol{Ri(t) + L\frac{di(t)}{dt} = V_m \cos \omega t}$$



- The response $i(t)$=Natural Response+ Forced Response
- Natural Response

$$\boldsymbol{i_{nat}(t) = ke^{-\frac{R}{L}t}}$$

- The forced Response is

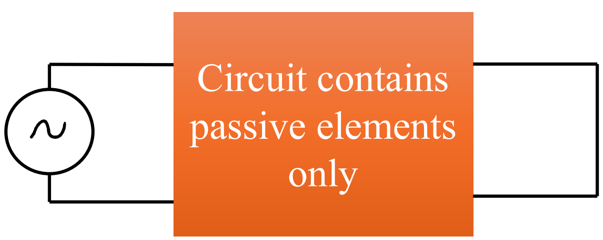
$$\boldsymbol{i_{for}(t) = C_1 \cos \omega t + C_2 \sin \omega t}$$

- Apply $i_{for}(t)$ in the differential equation and determine the values of C_1, C_2
- The overall response $\boldsymbol{i(t) = i_{for}(t) + ke^{-\frac{R}{L}t}}$

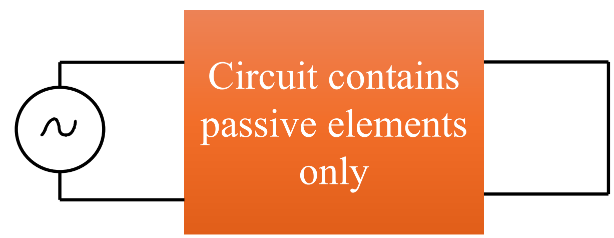
Ref. William H. Hayt Jr, Jack E. Kemmerly and Steven M. Durbin, "Engineering Circuits Analysis", McGraw Hill publishers

COMPLEX FORCING FUNCTION

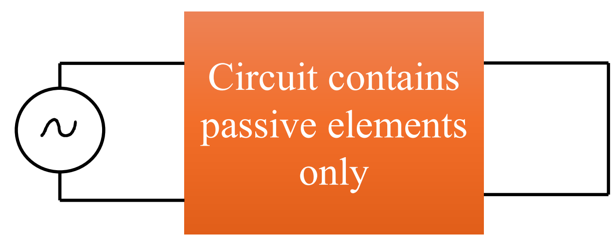
- In this module we will focus only on the sinusoidal steady state response of the circuit.
- Using Mesh analysis/Nodal analysis we can solve the problem, however, the approach will be cumbersome.
- We use an alternative approach, where, $v - i$ relationship will be a simple algebraic expression.
- We start with Euler's identity $e^{j\theta} = \cos\theta + j\sin\theta$.



$\boldsymbol{v(t) = V_m e^{j\omega t}}$ $\quad$ $\boldsymbol{i(t) = I_m e^{j(\omega t+\theta)}}$



$\boldsymbol{v(t) = V_m \cos\omega t}$ $\quad$ $\boldsymbol{i(t) = I_m \cos(\omega t+\theta)}$



$\boldsymbol{v(t)' = jV_m \sin\omega t}$ $\quad$ $\boldsymbol{i(t)' = jI_m \sin(\omega t+\theta)}$

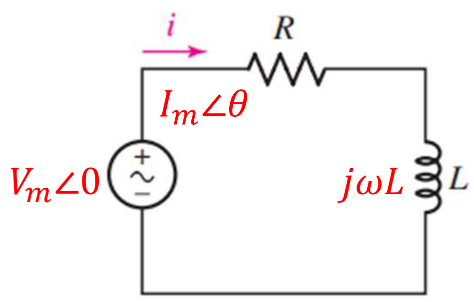
COMPLEX FORCING FUNCTION

- Put $v(t) = V_m e^{j\omega t}, i(t) = I_m e^{j(\omega t+\theta)}$ in the differential equation of R-L circuit.

$$L\frac{d}{dt}\left(I_m e^{j(\omega t+\theta)}\right) + RI_m e^{j(\omega t+\theta)} = V_m e^{j\omega t}$$

$$I_m e^{j\theta} = \frac{V_m}{R + j\omega L}$$

$$I_m\angle\theta = \frac{V_m\angle 0}{R + j\omega L}$$



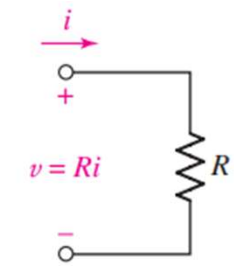
- Find out I_m, θ in terms of R, L, ω, V_m.

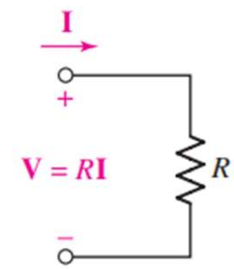
$$I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\theta = -\tan^{-1}\frac{\omega L}{R}$$

- This abbreviated complex representation is also known as phasor.

THE RESISTOR (PHASOR DIAG.)

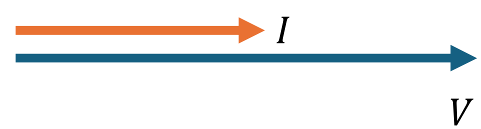




- For the resistance $v(t) = Ri(t)$
- Let us apply a voltage $v(t) = V_m e^{j(\omega t+\phi)}$
- The current flowing through the resistance

$$i(t) = \frac{V_m}{R} e^{j(\omega t+\ \ )} = I_m e^{j(\omega t+\phi)}$$

- Which says that the voltage phasor and the current phasor are in the same direction and phase lag/lead is zero.
- For resistive load $V = RI$, where V, I are the phasor representation of voltage and current, respectively.



Ref. William H. Hayt Jr, Jack E. Kemmerly and Steven M. Durbin, "Engineering Circuits Analysis", McGraw Hill publishers

THE INDUCTOR (PHASOR DIAG.)

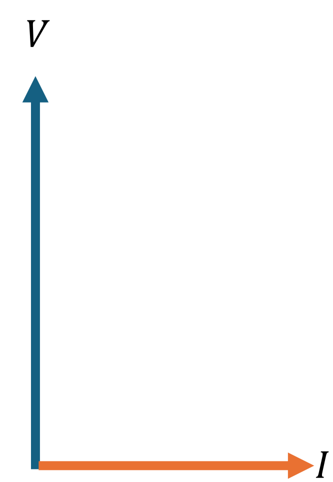
- For the inductor $v(t) = L\frac{di(t)}{dt}$
- Let us apply a voltage $i(t) = I_m e^{j(\omega t+\phi)}$
- The current flowing through the resistance

$$v(t) = j\omega L I_m e^{j(\omega t+\phi)} = j\omega L i(t)$$

- Which says that the voltage phasor leads the current phasor by an angle of 90^0.
- For sinusoidal input, the effective impedance of an inductor is

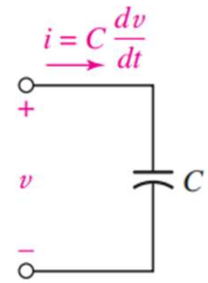
$$X_L = \omega L$$

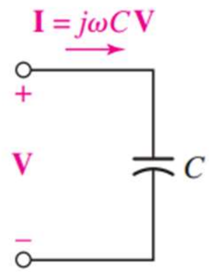
- For inductive load $V = jX_L I$, where V, I are the phasor representation of voltage and current, respectively.



Ref. William H. Hayt Jr, Jack E. Kemmerly and Steven M. Durbin, "Engineering Circuits Analysis", McGraw Hill publishers

THE CAPACITOR (PHASOR DIAG.)





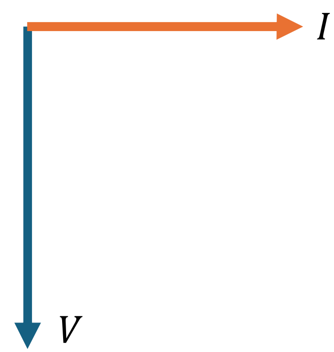
- For the capacitor $i(t) = C\frac{dv(t)}{dt}$
- Let us apply a voltage $v(t) = V_m e^{j(\omega t+\phi)}$
- The current flowing through the resistance

$$i(t) = j\omega C V_m e^{j(\omega t+\phi)} = j\omega C v(t)$$

- Which says that the voltage phasor lags the current phasor by an angle of 90^0.
- For sinusoidal input, the effective impedance of a capacitor is

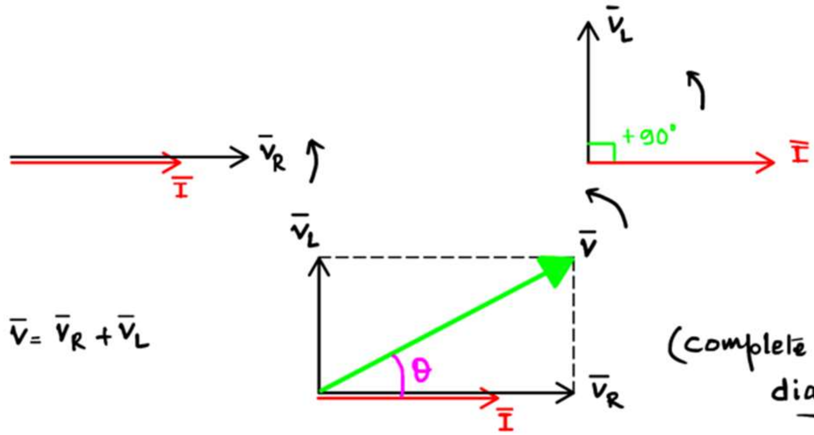
$$X_C = \frac{1}{\omega C}$$

- For capacitive load $V = -jX_C I$, where V, I are the phasor representation of voltage and current, respectively.



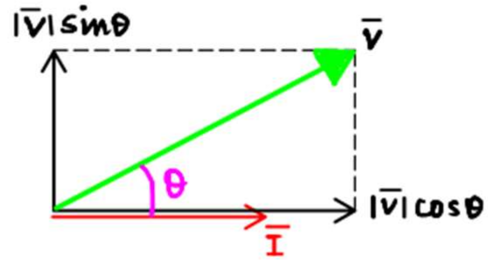
Ref. William H. Hayt Jr, Jack E. Kemmerly and Steven M. Durbin, "Engineering Circuits Analysis", McGraw Hill publishers

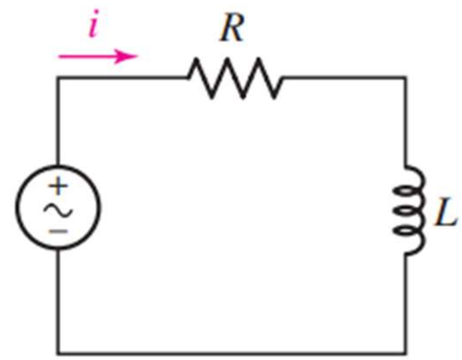
SERIES R-L CIRCUIT



$\bar{V} = \bar{V}_R + \bar{V}_L$

(complete phasor diagram)

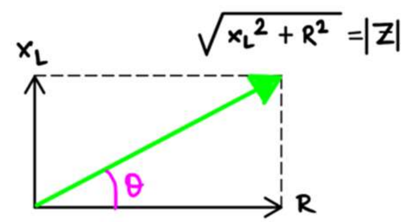




❖ Instantaneous power

$$p(t) = v(t)i(t)$$
$$= V_r I_r \cos\theta(1 - \cos 2\omega t) - V_r I_r \sin\theta \sin 2\omega t$$

as the current is a common phasor, subdivide each voltage phasor by current phasor



(Impedance Triangle)

❖ Average Power

$$P_{avg} = V_r I_r \cos\theta$$

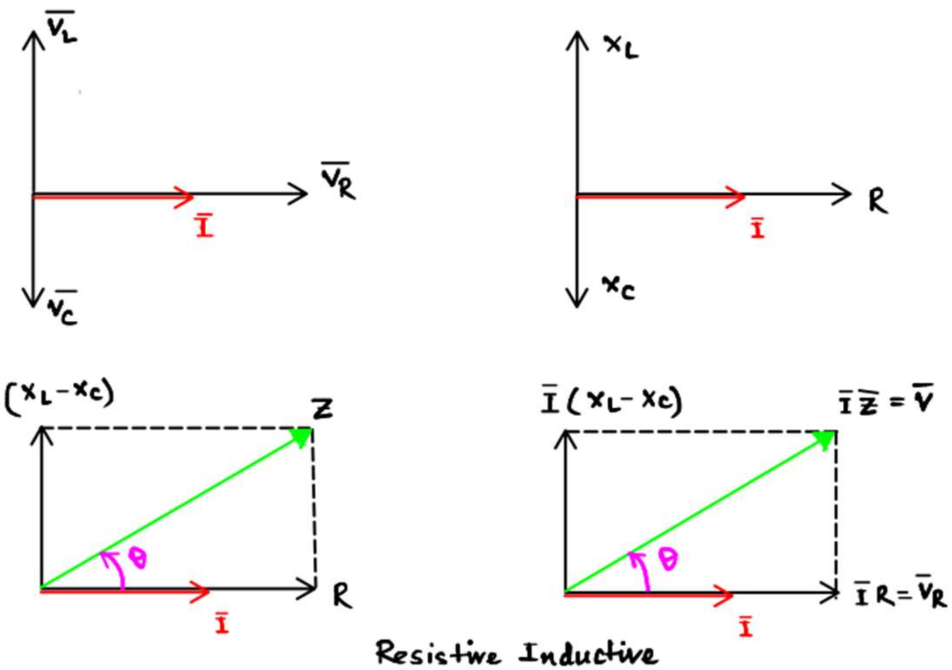
$|Z| = \sqrt{x_L^2 + R^2} = \sqrt{(\omega L)^2 + R^2}$

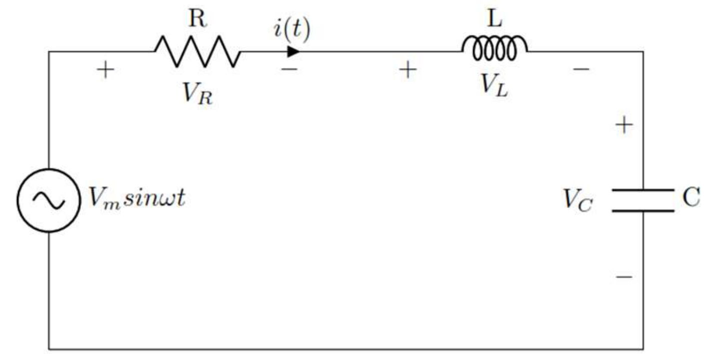
$R = |Z|\cos\theta \quad x_L = |Z|\sin\theta$

$\bar{Z} = R + jx_L = |Z|e^{j\theta} = |Z|\angle\theta$

❖ Draw the power triangle from the phasor diagram.

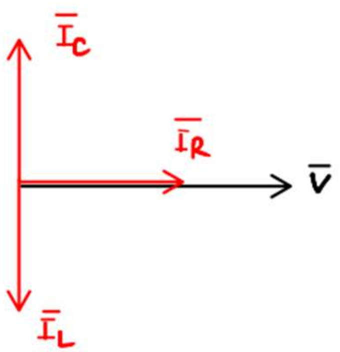
SERIES R-L-C CIRCUIT

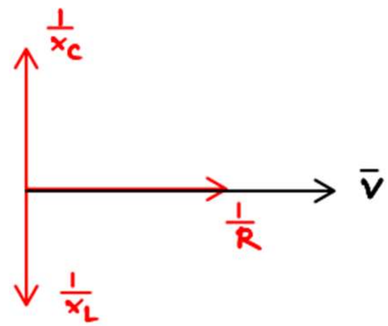




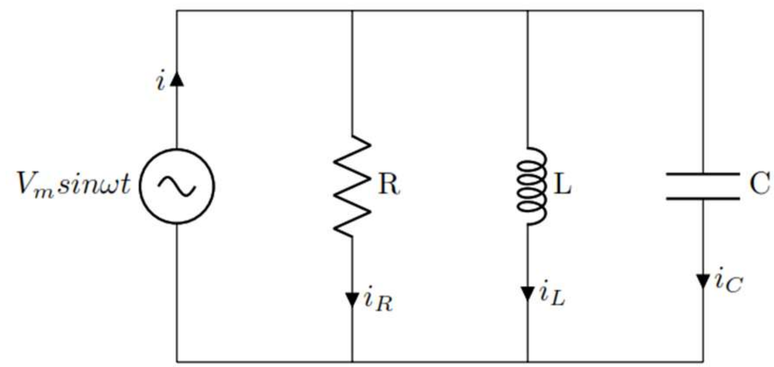
- For series R-L-C circuit assume $X_L > X_C$.
- The impedance $Z = R + j(X_L - X_C)$
- The power factor is $pf = \cos\theta = \frac{R}{|Z|}$

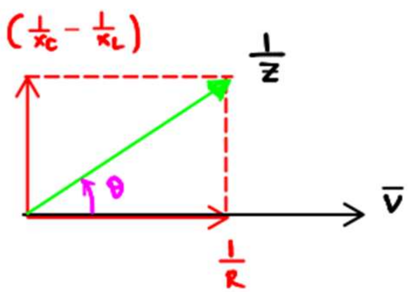
PARALLEL R-L-C CIRCUIT

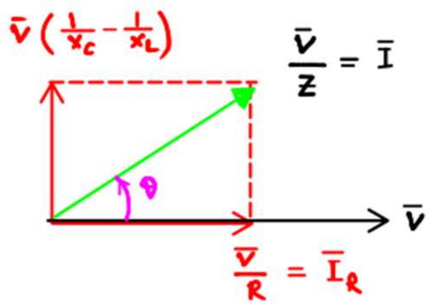




$X_L > X_C$







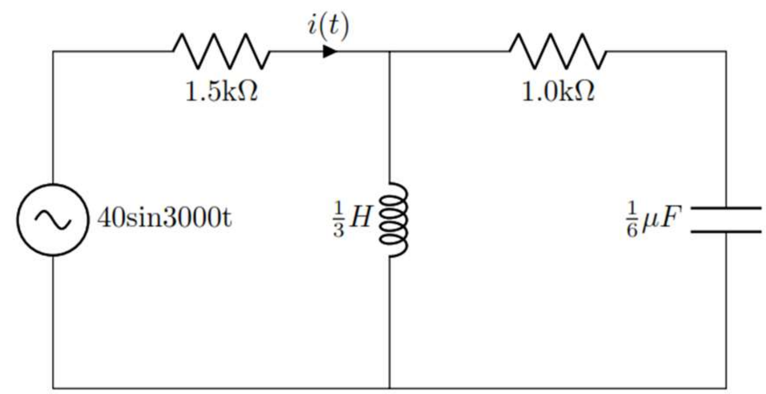
Resistive – capacitive

- ❖ For series R-L-C circuit assume $X_L > X_C$.
- ❖ The admittance $\frac{1}{Z} = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$
- ❖ The power factor is $pf = \cos\theta = \frac{|Z|}{R}$

PROBLEMS

Problem-2:

Consider the following circuit is at steady state. Determine the expression $i(t)$

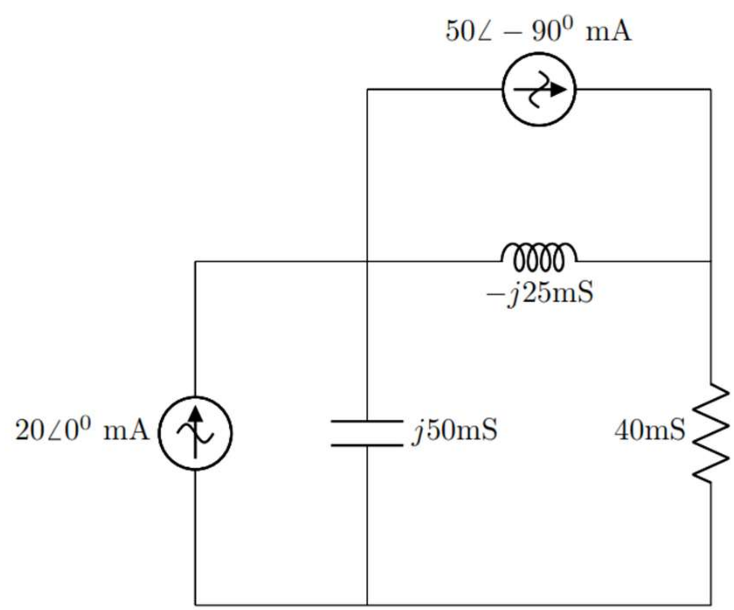


Ans:
$\boldsymbol{i(t) = 16\cos(3000t - 126.9^0)}$

PROBLEMS

Problem-3:

Using the Nodal Analysis, determine the node voltages in phasor form.



Ans:

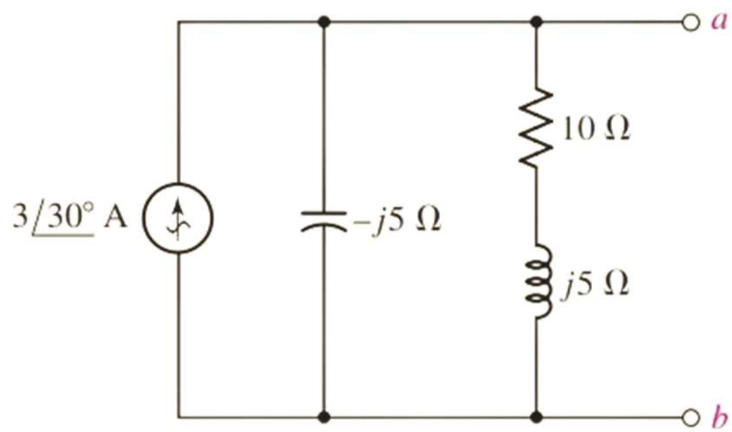
$$V_1 = 1.062\angle 23.3^0\ V$$

$$V_2 = 1.593\angle -50^0\ V$$

PROBLEMS

Problem-4:

Determine the Thevenin's and Norton's equivalent circuits of the given network.



Ans:

$$\boldsymbol{V_{th} = 16.77\angle -33.4^{0}\ V}$$
$$\boldsymbol{I_N = 2.6 + j1.5\ A}$$
$$\boldsymbol{R_{th} = R_N = 2.5 - j5\ \Omega}$$

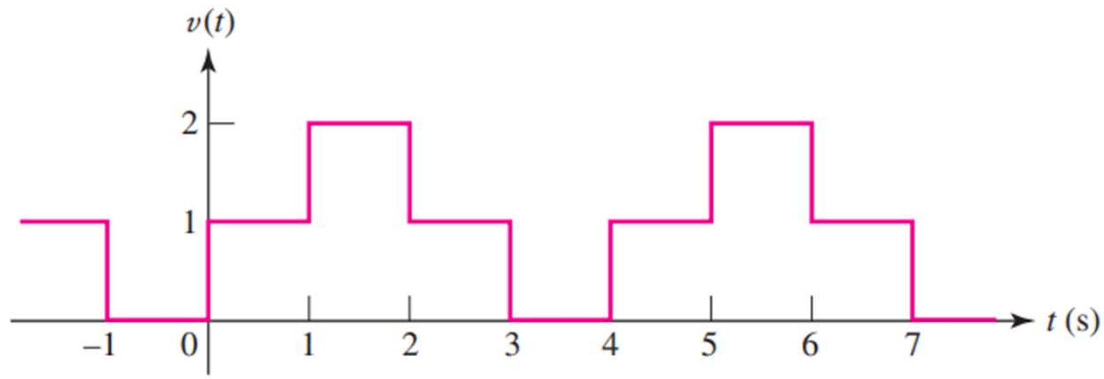
Ref. William H. Hayt Jr, Jack E. Kemmerly and Steven M. Durbin, "Engineering Circuits Analysis", McGraw Hill publishers

PROBLEMS

Problem-5:

Determine the rms and average of the given voltage waveform.

Ans:

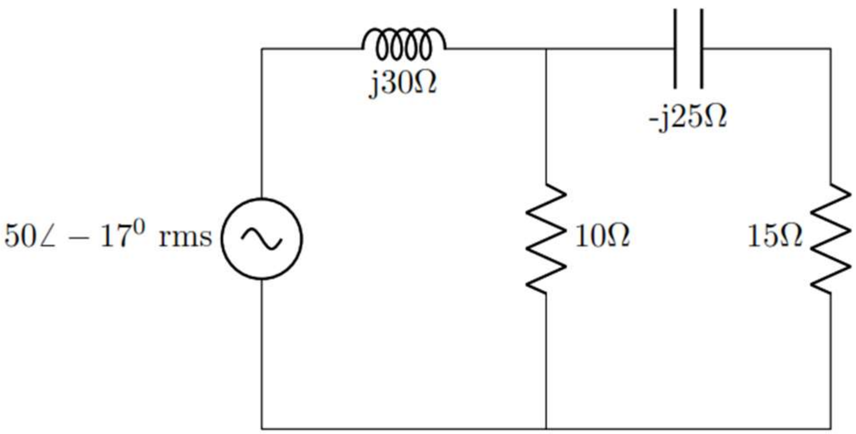


Ref. William H. Hayt Jr, Jack E. Kemmerly and Steven M. Durbin, "Engineering Circuits Analysis", McGraw Hill publishers

PROBLEMS

Problem-6:

Calculate the complex power delivered to each passive components of the given circuit. The voltage magnitude is given in rms.



Ans: