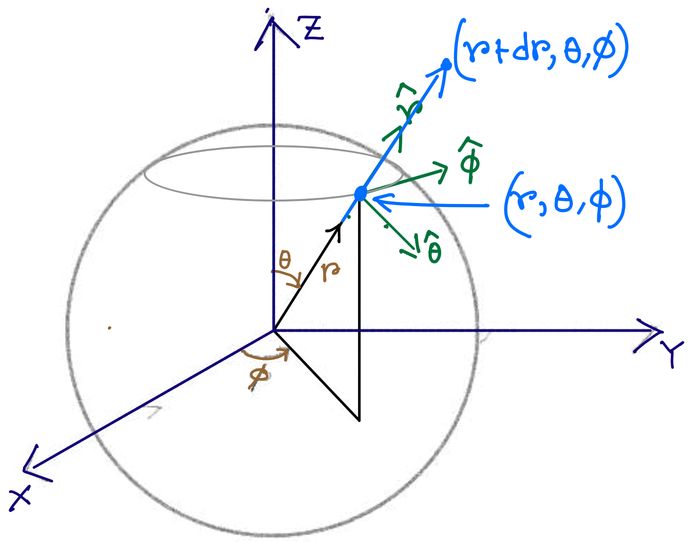
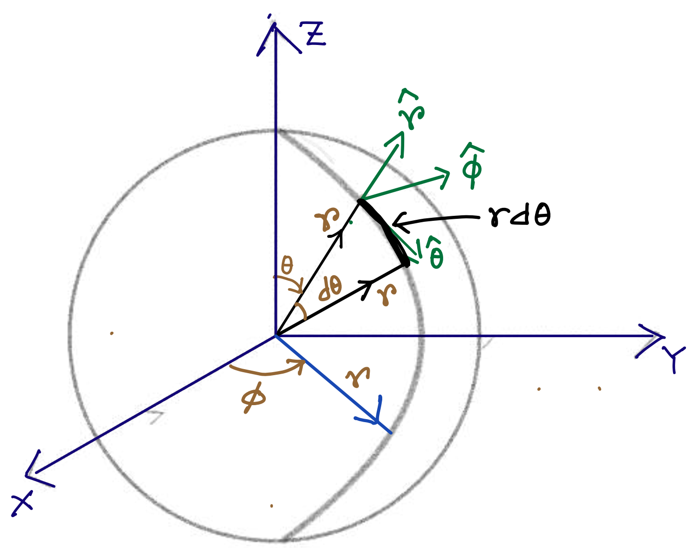
Useful Coordinate Systems

1. SPHERICAL POLAR COORDINATES

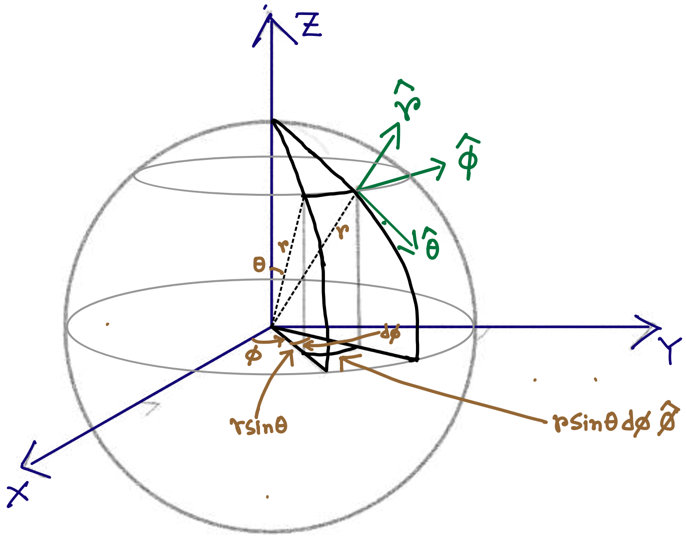


$$\vec{dl_r} = \hat{r}\, dr$$



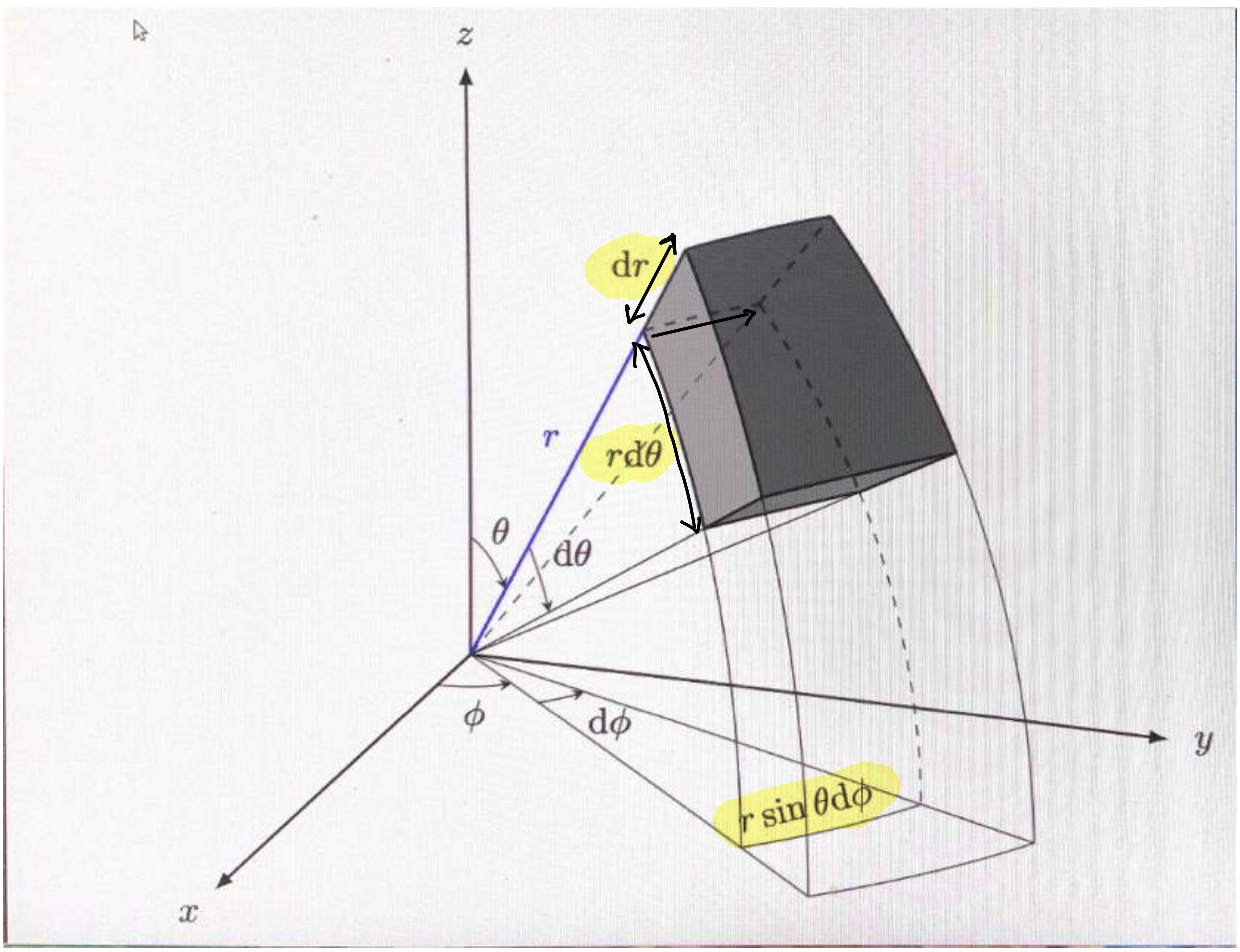
$$\vec{dl_\theta} = r\, d\theta\, \hat{\theta}$$

$$\vec{dl_\phi} = r\sin\theta\, d\phi\, \hat{\phi}$$

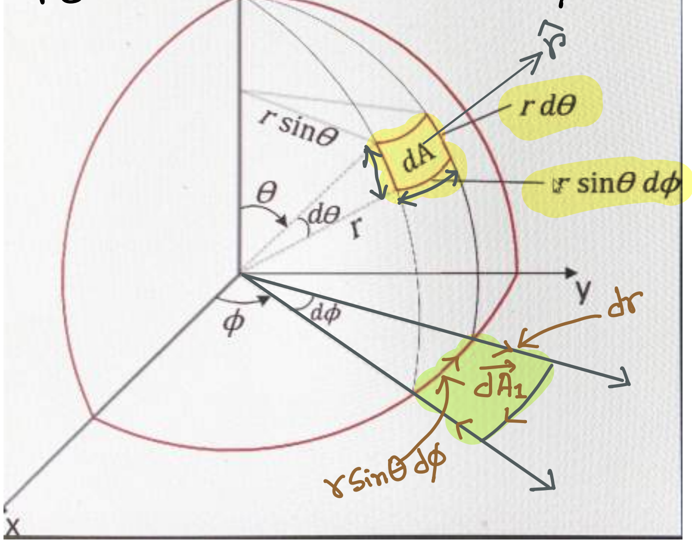


$$\vec{dl} = \hat{r}\, dr + \hat{\theta}\, r\, d\theta + \hat{\phi}\, r\sin\theta\, d\phi$$

Infinitesimal Vector displacement.



$$d\tau = dr\; r\,d\theta\; r\sin\theta\, d\phi = r^2 \sin\theta\, dr\, d\theta\, d\phi$$



$$\vec{dA} = r\,d\theta\; r\sin\theta\, d\phi\; \hat{r}$$

$$\vec{dA_1} = r\sin\theta\, dr\, d\phi\; \hat{\theta}$$
$$= r\,dr\,d\phi\; \hat{\theta} \quad (\theta = \pi/2).$$

In spherical polar coordinates:

$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{r}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta}$$
$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\phi}$$