polynomial

Taylor's $\rightarrow$ $f(x) = f(a + (x-a)) = f(a) + (x-a)f'(a) \dots + \frac{(x-a)^n}{n!} f^n(a)$

$+ R_n(x)$

$$R_n(x) = \frac{(x-a)^{n+1}}{(n+1)} f^{n+1}(a + \theta(x-a))$$

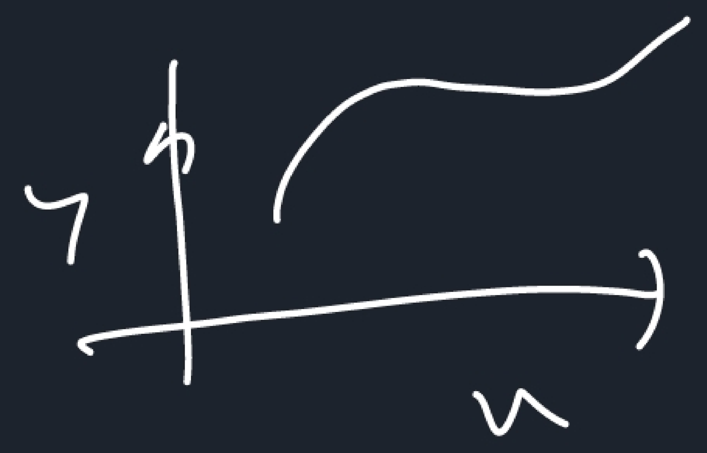
if $R_n(x) \rightarrow 0$ as $n \rightarrow \infty$ $f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f^2(a) + \dots \infty$

Taylor's series

Functions of several variables

Function of single variables

$$y = f(x)$$



$f: X \to Y$

$x \to$ independent variable

$y \to$ dependent variable

Function of several variables

$\to$ more than one independent variables

$\to$ one dependent variables

$$f: \mathbb{R}^n \to \mathbb{R}$$

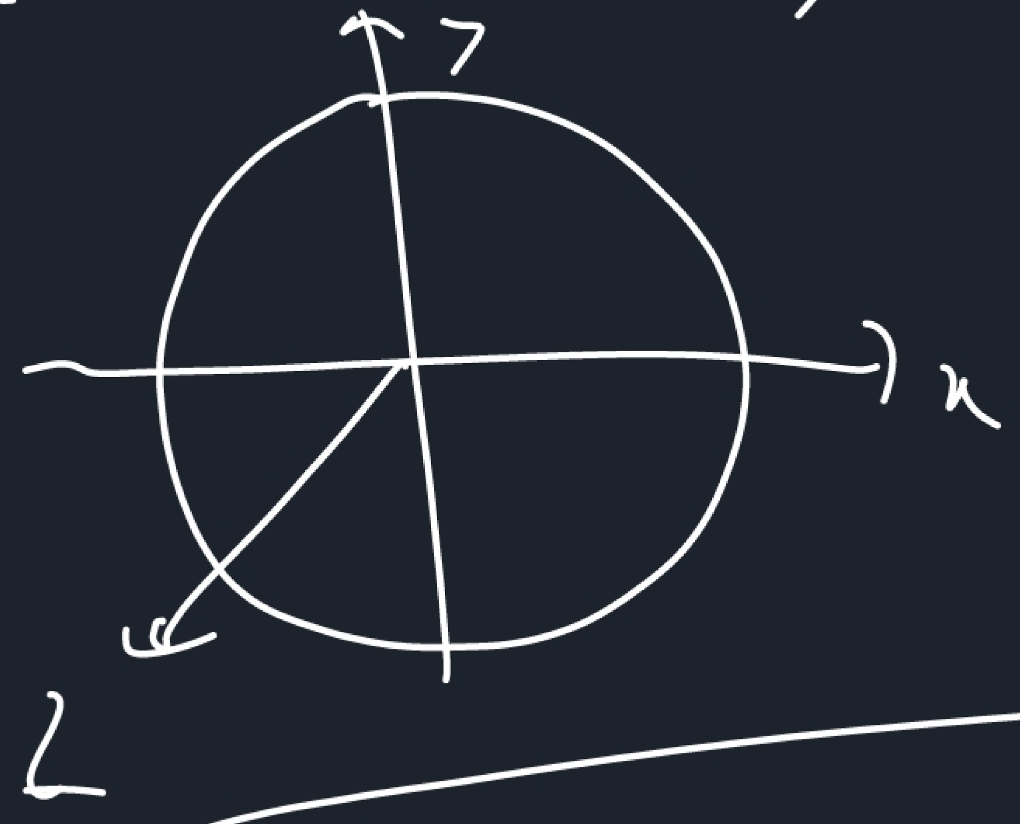
$$f(x_1, x_2, \dots x_n) = y$$

$$x^2+y^2+z^2=a^2$$

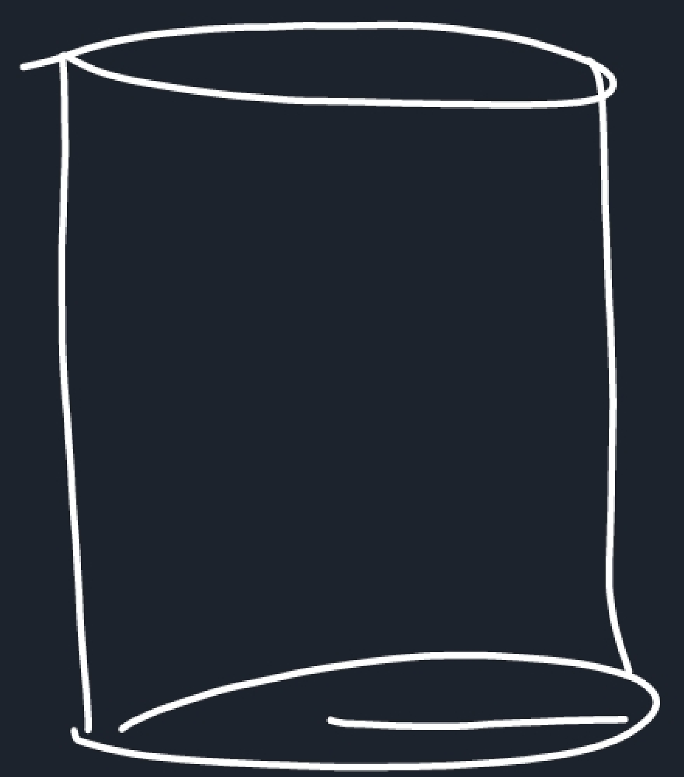
$$f: R^2 \to R$$

$$z=\sqrt{a^2-x^2-y^2}$$

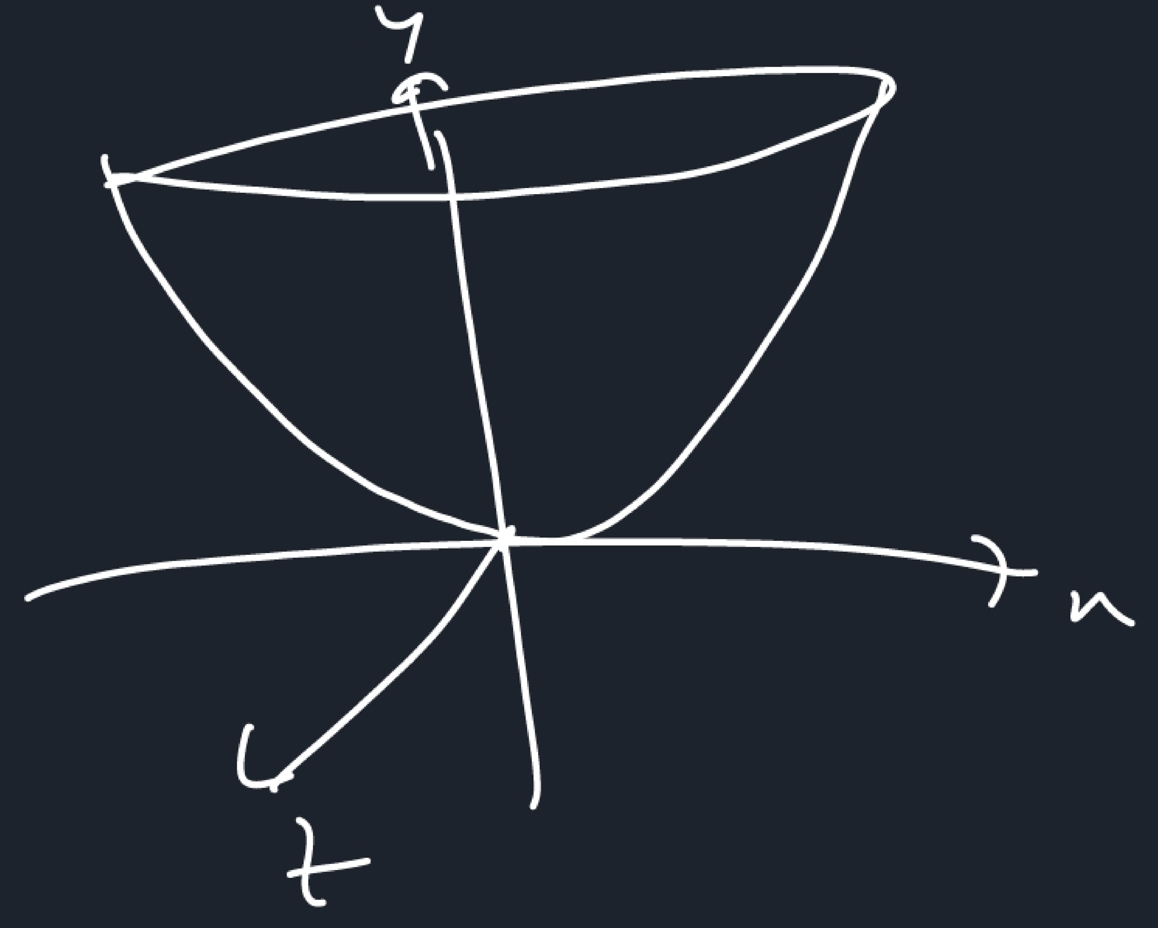
$$z=f(x,y)$$



$$x^2+y^2=a^2$$



$$\frac{x^2}{a^2}+\frac{y^2}{B^2}=\frac{2z}{c}$$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

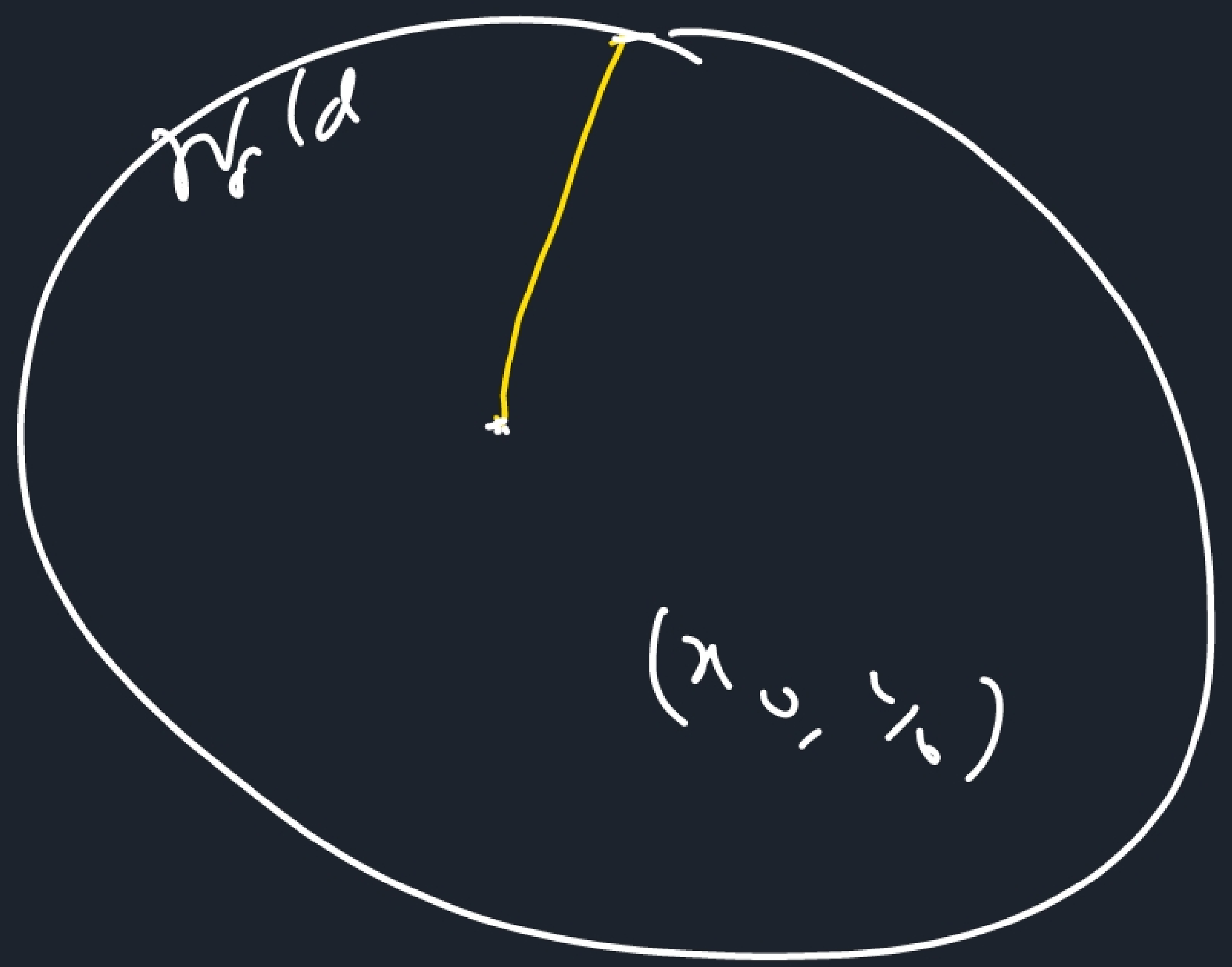
$z = f(x, y)$

$$d = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$

$$d = \sqrt{(x_1 - x_0)^2 + (x_2 - x_{20})^2 \; - - - \; (x_n - x_{n0})^2}$$

δ - Neighbourhood

$$N_\delta(d) = \sqrt{(x - x_0)^2 + (y - y_0)^2} \leq \delta$$



$$0 < N_\delta(d) \leq \delta$$

Deleted Neighbourhood

$y = f(x)$

$|x - a| \leq \delta$

$x \in (a - \delta, a + \delta)$

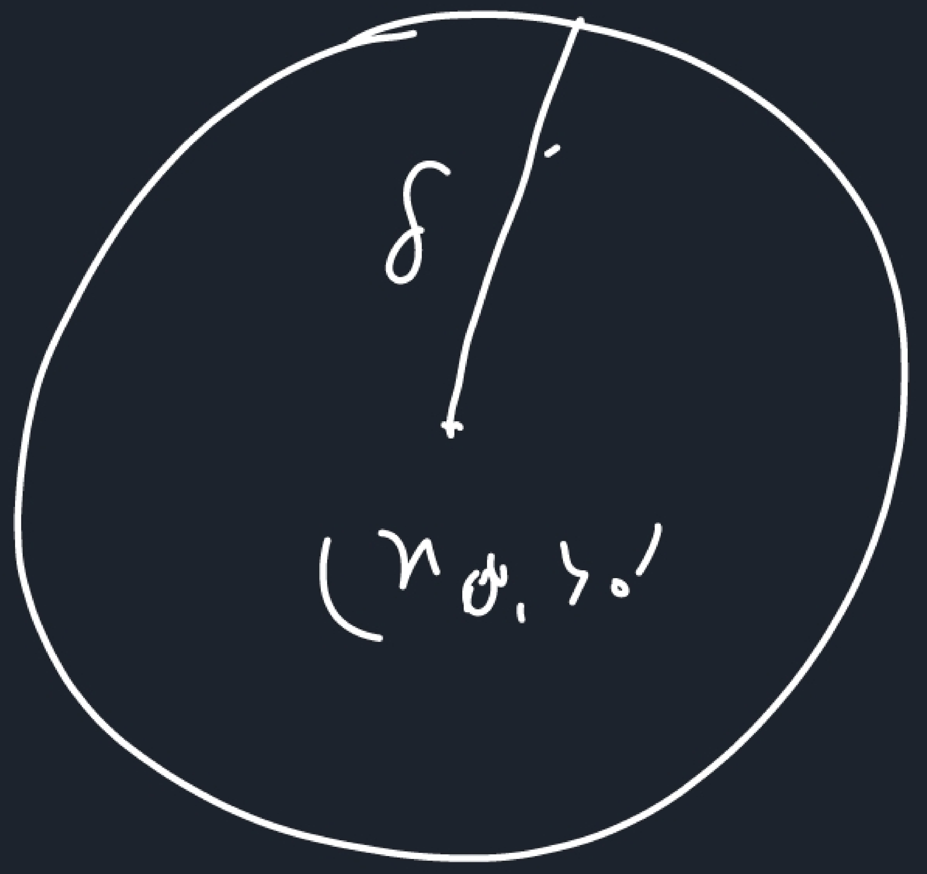
$x \in [a - \delta, a + \delta]$ include boundary

$x \in (a - \delta, a + \delta)$ exclude boundary

$z = f(x, y)$

point (x_0, y_0)

$\sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta$



$N_\delta(P)$

$0 < \sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta$

Deleted Neighbourhood.

$$|x - x_0| < \sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta$$

Neighbourhood can be defined as $|x - x_0| < \delta$ and $|y - y_0| < \delta$

$$x_0 - \delta < x < x_0 + \delta$$

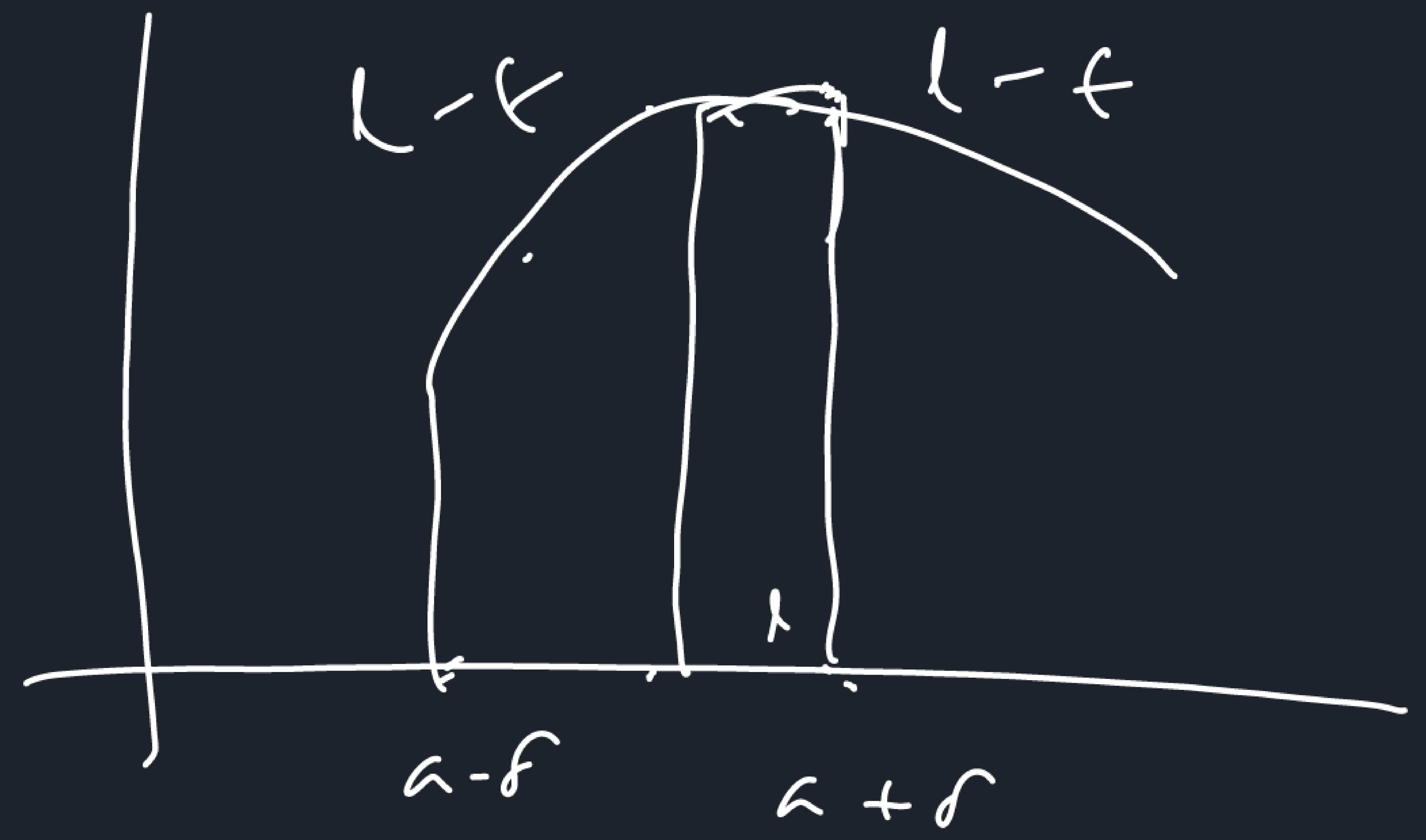
$$y_0 - \delta < y < y_0 + \delta$$

$$\lim_{x \to a} f(x) = l$$

$|x+4-6| < \epsilon$

$|x-2| < \epsilon = \delta$

$|x-2| < \delta$



prove that $\lim_{x \to 2} (x+4) = 6$

$|x+4-6| < \epsilon \Rightarrow |x-2| < \delta$

For any $\epsilon > 0$ there exist a $\delta > 0$ such that

$|f(x) - l| < \epsilon$ whenever $x \in D$ and $|x-a| < \delta$

Domain

Then we can say $\lim_{x \to a} f(x) = l$

prove that $\lim_{(x,y)\to(2,1)} (3x+4y) = 10$

We can say that $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = l$

if any $\epsilon > 0$ there exist a $\delta > 0$ for which

$$|f(x,y) - l| < \epsilon \quad \text{whenever } (x,y) \in D \text{ and } \sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta$$

$$\lim_{(x,y)\to(2,1)} (3x+4y) = 10$$

$$|3x+4y-10| < \epsilon$$

$$|3x-6+4y-4| < \epsilon$$

$$|3(x-2)+4(y-1)| < \epsilon$$

$$|x-2| < \sqrt{(x-2)^2+(y-1)^2} < \delta$$

$$\sqrt{(x-2)^2+(y-1)^2} < \delta$$

or $|x-2| < \delta$ or $|y-1| < \delta$

prove that by using $\epsilon - \delta$ approach that $\lim_{(x,y)\to(2,1)} (3x+4y) = 10$

$|3(x-2) + 4(y-1)| < \epsilon$

$|3(x-2) + 4(y-1)| \leq 3|x-2| + 4|y-1| < \epsilon$.

$3|x-2| + 4|y-1| < \epsilon$

$3|x-2| + 4|y-1| < 3\delta + 4\delta < \epsilon$

$7\delta < \epsilon$

$\delta < \epsilon/7$

$|a+b| < \epsilon$

$|a+b| \leq |a| + |b| < \epsilon$

$|x-2| < \delta$ and $|y-1| < \delta$

our aim is to find a δ interms of ϵ

$|a+b| \leq |a| + |b|$

$|a+b| < \epsilon < |a| + |b|$

① By using defn ($\delta - \epsilon$ approach)

prove that $\lim_{(x,y)\to(1,1)} (x^2 + 2y) = 3$

For any $\epsilon > 0$, there exist a $\delta > 0$.

$|x-1| < \delta$

and $|y-1| < \delta$

$|x^2+2y-3| < \epsilon$

To find δ interms of ϵ

$|x^2 - 2x + 1 + 2x - 2 + 2y - 2| < \epsilon$

$|(x-1)^2 + (2(x-1)) + 2(y-1)| < \epsilon$

$|x-1|^2 + 2|x-1| + 2|y-1| < \epsilon$

$\delta^2 + 4\delta + 4 < \epsilon + 4$

$\delta < \sqrt{\epsilon+4} - 2$