QM 3 | Note 8 | Lec 8

[1] $\hat{H}\chi = E\chi$ (TISE)

~~$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$~~ $\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$

$\hat{H}$ is Hamiltonian operator which corresponds to total energy of the quantum particle.

Another usual notation of $\chi(x)$ is $\psi(x)$

(Same thing different notation)

$$\hat{H} = \underbrace{-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}}_{\text{Kinetic}} + \underbrace{V(x)}_{\text{Potential}}$$

Kinetic energy operator $= -\dfrac{\hbar^2}{2m}\dfrac{d^2}{dx^2}$

$$= \frac{1}{2m}\left(\frac{\hbar}{i}\frac{d}{dx}\right)\left(\frac{\hbar}{i}\frac{d}{dx}\right)$$

$$= \frac{\hat{p}^2}{2m}$$

$\hat{p}$ is momentum operator

$\rightarrow \left(\dfrac{\hbar}{i}\dfrac{d}{dx}\right)$

$\boxed{\text{Generalize}}$

Expectation value of any dynamical variable $Q(x,p)$

$$\langle Q(x,p)\rangle = \int \psi^* Q\left(x, \frac{\hbar}{i}\frac{d}{dx}\right)\psi \, dx$$

Expectation value of position

$$\langle x\rangle = \int_{-\infty}^{\infty} \psi^* x \psi \, dx = \int x |\psi|^2 dx$$

postulates of Quantum Mechanics

① Quantum state of a particle is represented by wave function $\psi(x,t)$

② Every measurable physical quantity $\cancel{A}$ is described by an operator A which is called observable

For example: Energy $\longrightarrow \hat{H}$ ~~ma~~ Hamiltonian

position $\longrightarrow \hat{X} = x$

Momentum $\longrightarrow \hat{p} = \frac{\hbar}{i}\frac{d}{dx}$

Kinetic energy $\longrightarrow \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

etc. "Hermitian operators"

③ Possible results when any of the physical quantity is measured are eigen values of the corresponding observable A.

$\hat{A}\psi_i = a_i\psi_i \longrightarrow$ (I) possible result a_i when $\cancel{A}$ is measured.

(ψ_i → eigenfunction, a_i → eigenvalue)

For example: $\hat{H}\psi_i = E_i\psi_i$

If H is measured $\longrightarrow$ possible results E_i (Total energy ~~value~~)

please remember eigenstates are functions and eigenvalues are scalar numbers.

.... There are a few more very important postulates in Q.M. but you will learn in a separate Q.M. course if you take in future.

Expectation value of any operator $\hat{A}$

$\langle \hat{A} \rangle = \int \psi^* \hat{A} \psi \, dx$ will give a mean value of the results (a_i) if measured on ψ. (see eqn. (I))

Example $\langle \hat{H} \rangle$ will give average energy of the particle in state ψ_i ← one of the stationary states

$\hat{H}\psi_i = E\psi_i$

$$\langle H \rangle = \int \psi_i^* (\hat{H}\psi_i) \, dx$$

$$= \int \psi_i^* (E\psi_i) \, dx$$

$$= E_i \int \psi_i^* \psi_i \, dx = E \times 1$$

$$= E_i$$

General solution of ~~$\hat{H}\psi = E\psi$~~

Infinite number of solutions maybe possible $(\psi_1(x), \psi_2(x), \psi_3(x), \ldots\ldots)$ of $H\psi = E\psi$ with corresponding energies (= eigenvalues)

$E_1, E_2, E_3, \ldots\ldots$

General solution of time dependent Schrödinger equation.

$$\underbrace{\psi(x,t) = \sum_{n=1}^{\infty} C_n \psi_n(x)\, e^{-i\frac{E_n t}{\hbar}}}_{}$$

C_n are constants

Linear combination

Superposition of stationary states

↓

A real quantum state