Polyphase Circuits

- Nearly all electrical power generated and distributed in three phase at the operating frequency of 50 Hz (India) or 60 Hz (US).
- If one phase or two phases are required, then they are taken from the three-phase system directly rather than generating independently.
- The instantaneous power in the three-phase system is constant and not pulsating in nature like single phase system.
- As a result, the power transmission is uniform, and the vibration is less in the rotating electrical machines.
- For the same amount of power with same transmission efficiency, three phase system is more economical than single phase system.
- For the aforementioned conditions, the volume of copper used in three phase system is less than the same of single-phase system.
- Circuit diagram of single-phase system and the three phase system.

POLYPHASE VOLTAGES

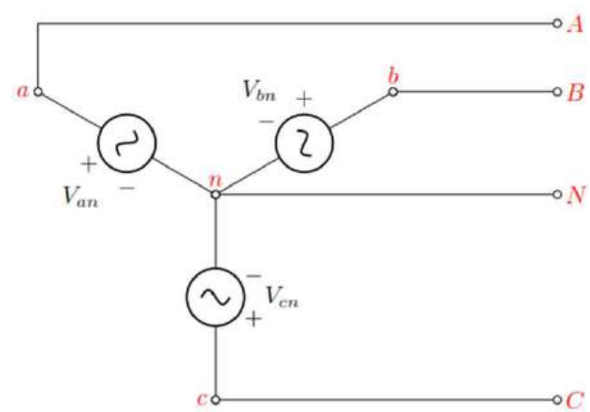
- ❖ We begin our discussion with three phase balanced voltage.
- ❖ Three phase voltages are usually generated with three phase generator.
- ❖ A typical three-phase system consists of three voltage sources connected to loads by three or four wires.
- ❖ A three-phase system is equivalent to three single-phase circuits.
- ❖ A three-phase system can be star connected or delta connected.
- ❖ In case of three-phase voltages, the voltage magnitudes are same for each of the three phases; however, the three phases are electrically 120^0 apart from each other.

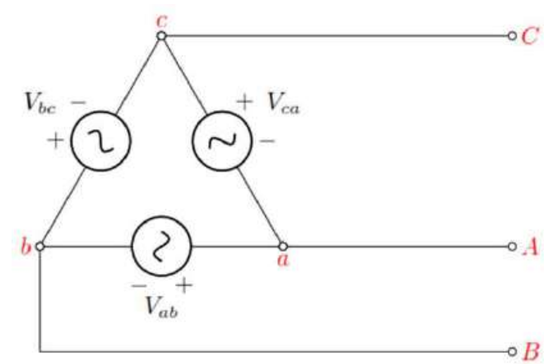
$$V_{An} = V_m\angle 0^0$$

$$V_{Bn} = V_m\angle -120^0$$

$$V_{Cn} = V_m\angle -240^0 = V_m\angle +120^0$$

Phase sequence is **A-B-C**, or **B-C-A**, or **C-A-B**





Polyphase Loads

- General Concept of load. Electrically, load is represented by current.
- Three phase load also either star or delta connected like the three-phase voltage source.
- Balanced load is one for which the phase impedances are equal in magnitude and phase. From the concept of load, electrically a balanced three-phase current should flow through the balanced three phase load.
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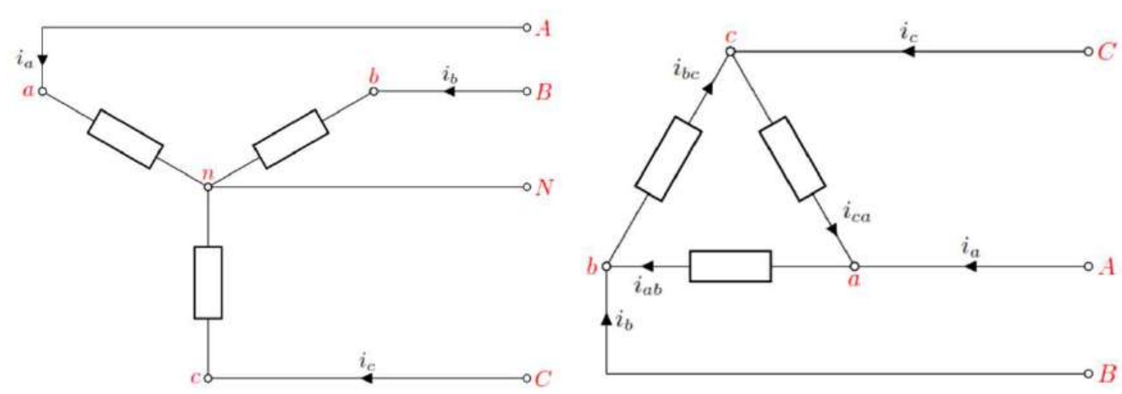
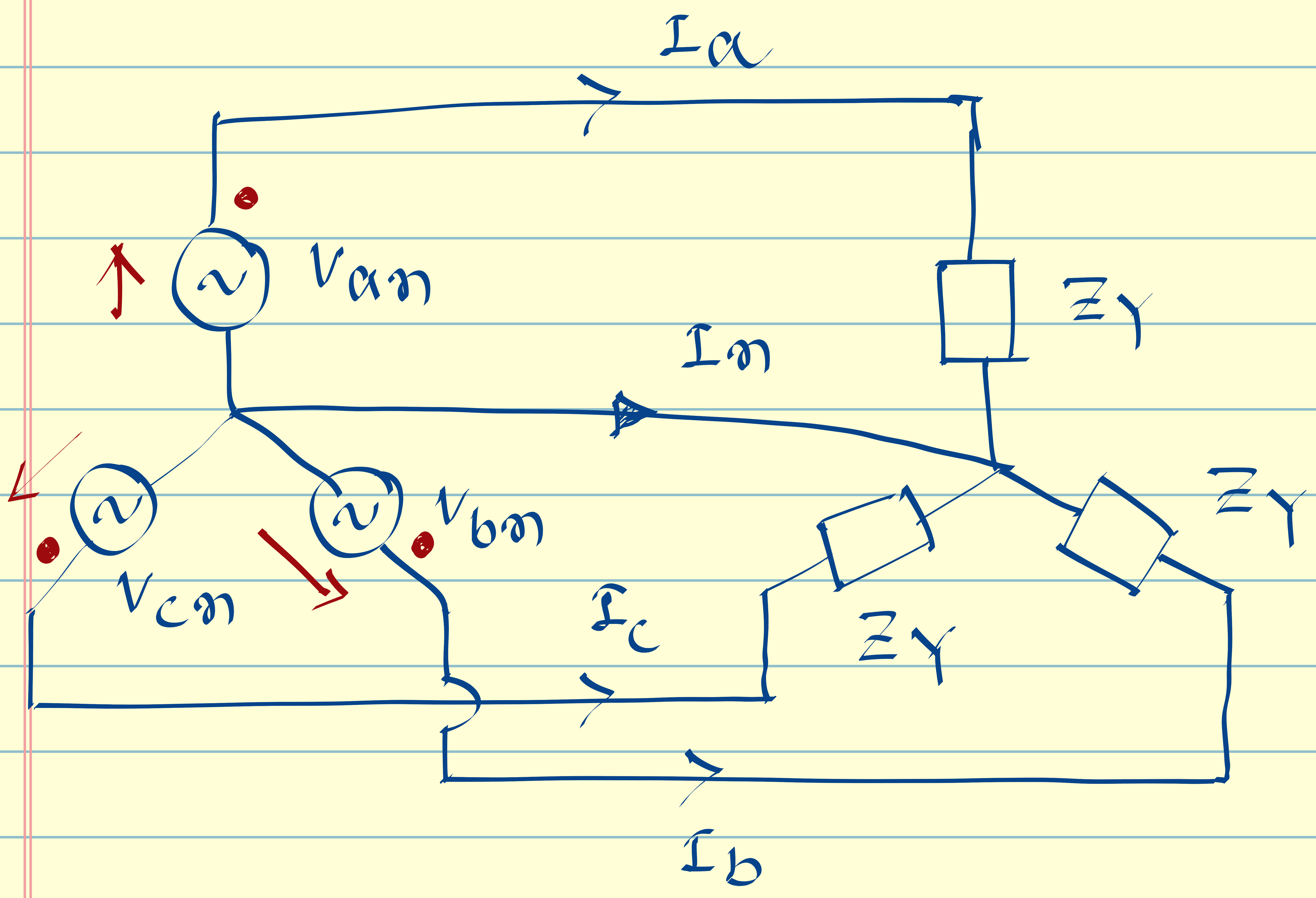


Three Phase System

Y - Y System :-



Phase voltages are

$$|I_a| = |I_b| = |I_c| = I_p$$

$$V_{an} = V_p \angle 0$$
$$V_{bn} = V_p \angle -120$$
$$V_{cn} = V_p \angle +120 = V_p \angle -240$$

For balanced system $I_n = 0$. Therefore, for balanced system one can skip 4th wire.

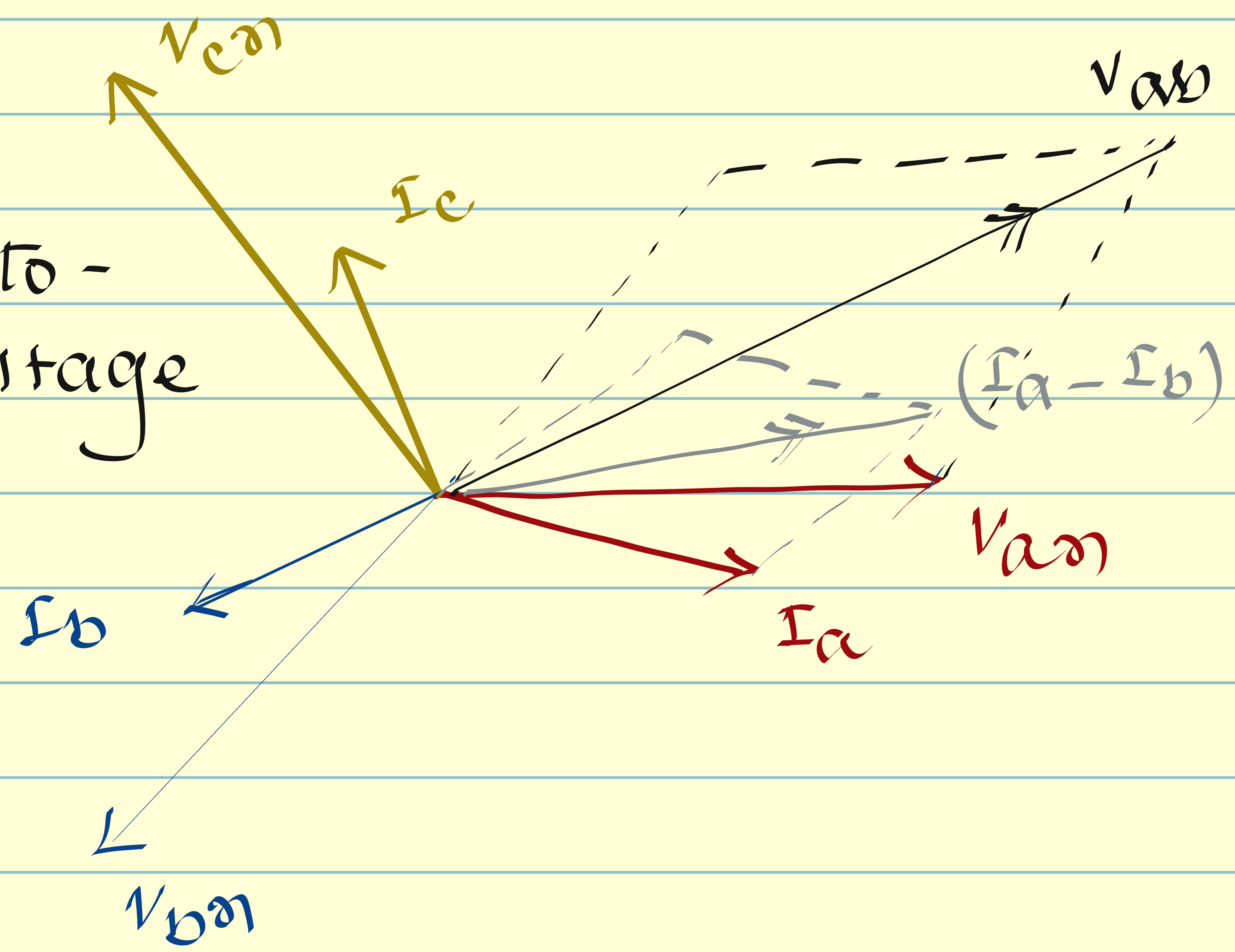
Voltage appears across any two lines

Apply KVL

$$V_{an} - I_a Z_Y + I_b Z_Y - V_{bn} = 0$$

$$V_{an} - V_{bn} = (I_a - I_b) Z_Y$$

Now, assume the Z_Y is resistive inductive



V_{ab} = line-to-line voltage

$$I_a - I_b = \frac{V_{ab}}{Z_Y} = \frac{\sqrt{3}\, V_p \angle 30^\circ}{Z_Y}$$

$$I_a - I_b = \sqrt{3}\, I_p \angle 30^\circ$$

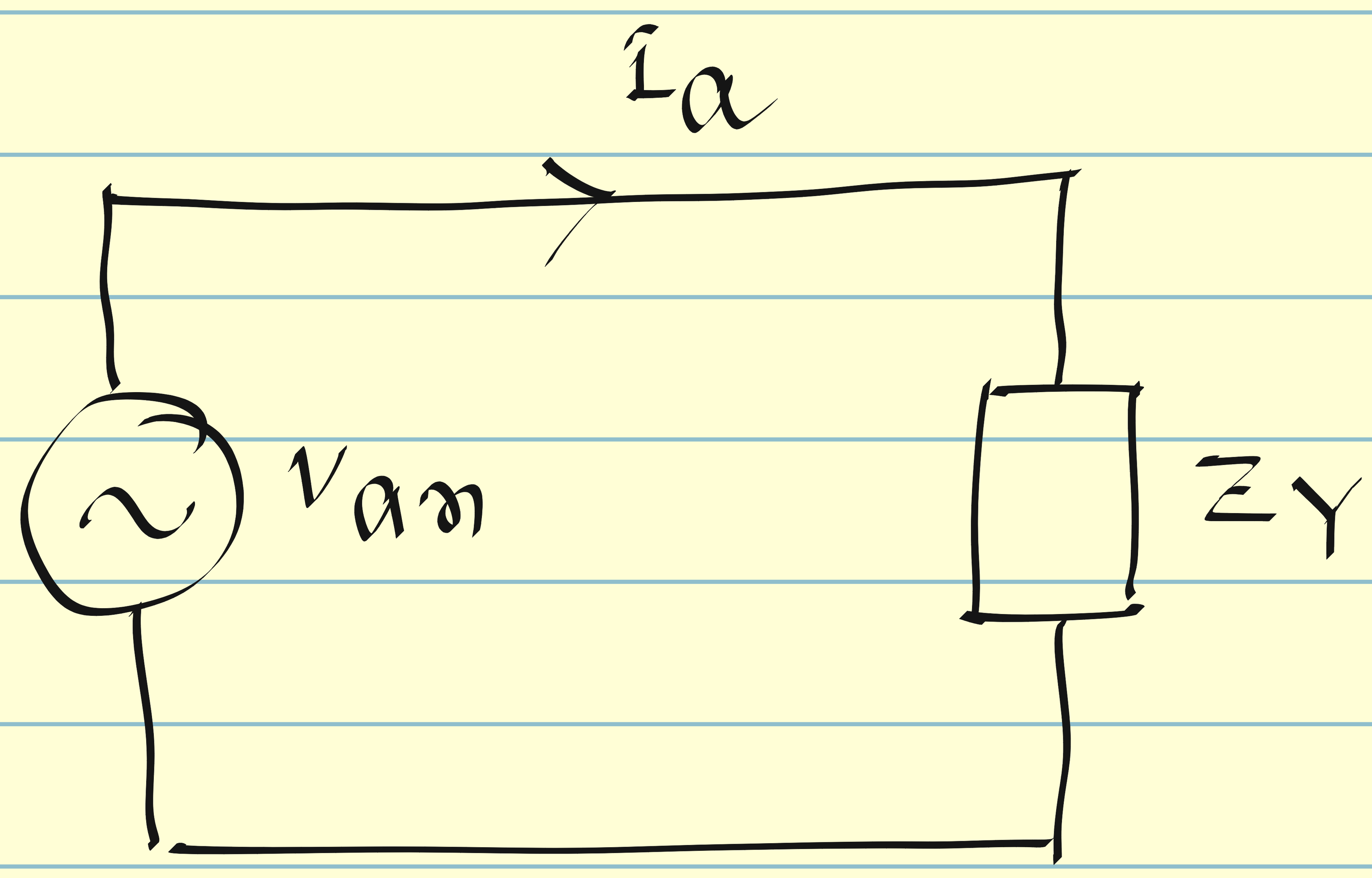
$$I_p = \frac{V_p}{Z_Y}$$

$$I_a = I_p \angle 0 = \frac{V_p \angle 0}{Z_Y} = \frac{V_{an}}{Z_Y}$$

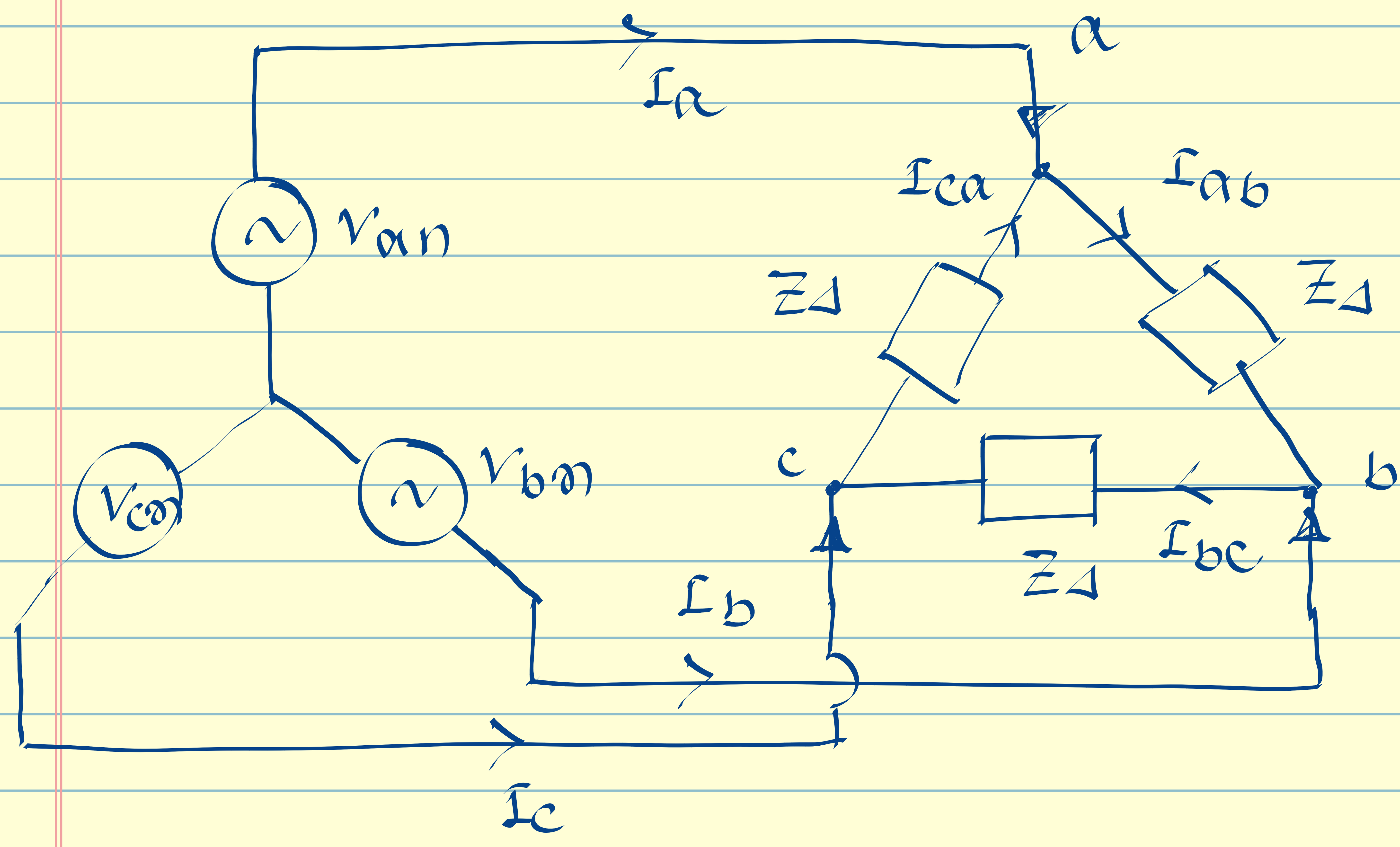
$$I_b = \frac{V_p \angle -120}{Z_Y} = \frac{V_{bn}}{Z_Y}$$

$$I_c = \frac{V_{cn}}{Z_Y}$$

Single phase equivalent



Y - Δ System :-



Apply KVL

$$V_{an} - I_{ab} Z_\Delta - V_{bn} = 0$$

$$\Rightarrow \quad I_{ab} = \frac{V_{an} - V_{bn}}{Z_\Delta} = \frac{V_{ab}}{Z_\Delta}$$

$$I_a = I_{ab} - I_{ca} \qquad\qquad I_{ab} + I_{bc} + I_{ca} = 0$$
$$I_b = I_{bc} - I_{ab}$$
$$I_c = I_{ca} - I_{bc}$$

$$I_a - I_b = I_{ab} - I_{ca} - I_{bc} + I_{ab}$$
$$= 3 I_{ab}$$

$$I_{ab} = \frac{I_a - I_b}{3}$$

$$I_a - I_b = \frac{3V_{ab}}{Z_\Delta} = \frac{V_{ab}}{(Z_\Delta/3)}$$

$$I_a - I_b = \sqrt{3}\, I_p \angle 30$$
$$V_{ab} = \sqrt{3}\, V_p \angle 30$$

$$I_p = \frac{V_p}{(Z_\Delta/3)}$$

$$I_a = \frac{V_{an}}{(Z_\Delta/3)} \qquad I_b = \frac{V_{bn}}{(Z_\Delta/3)}$$

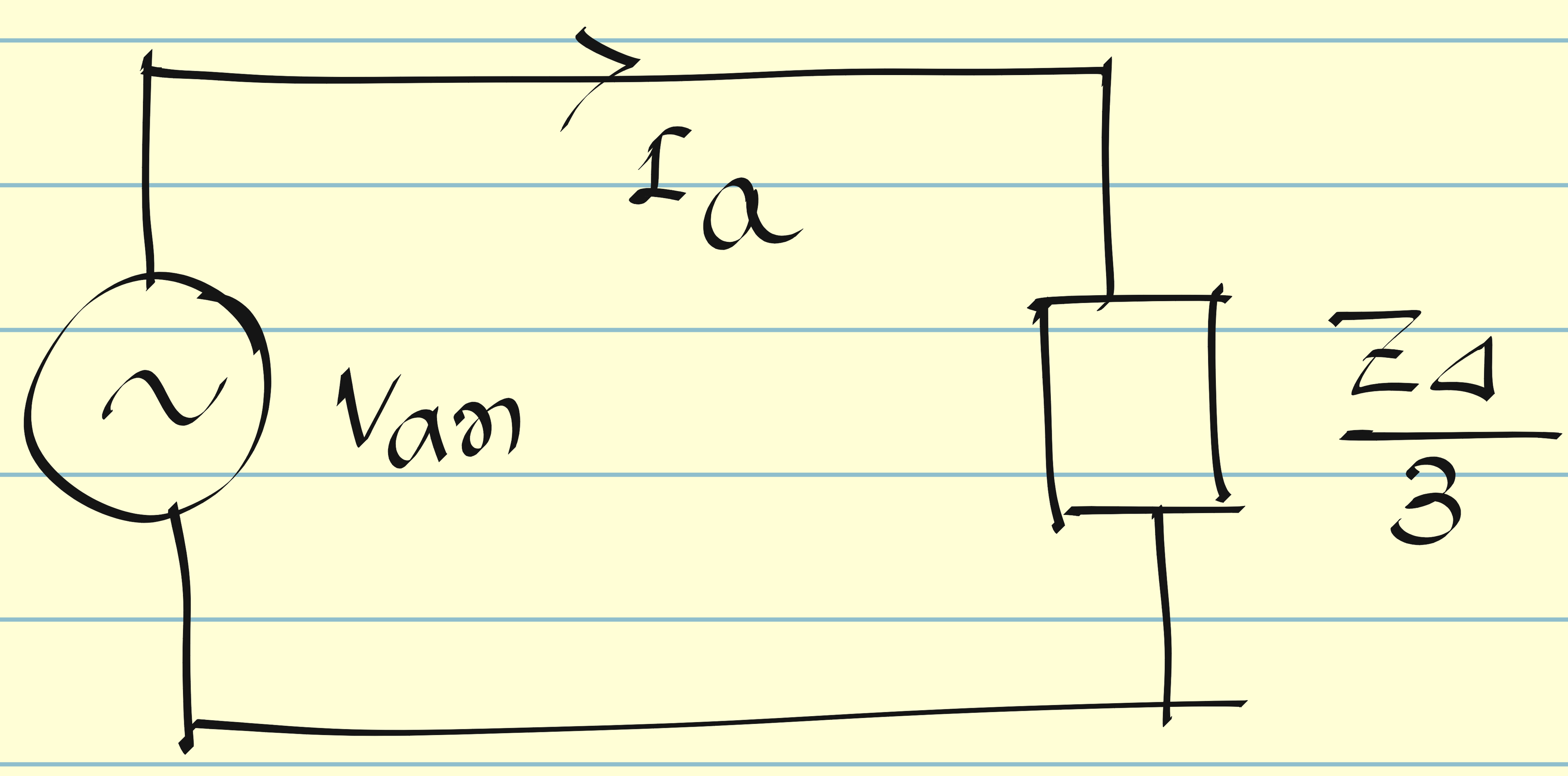
$$I_c = \frac{V_{cn}}{(Z_\Delta/3)}$$

$$I_{ab} = (I_a - I_b)/3$$
$$= \frac{\sqrt{3}\, I_p \angle 30}{3} = \frac{I_p}{\sqrt{3}} \angle 30^\circ$$

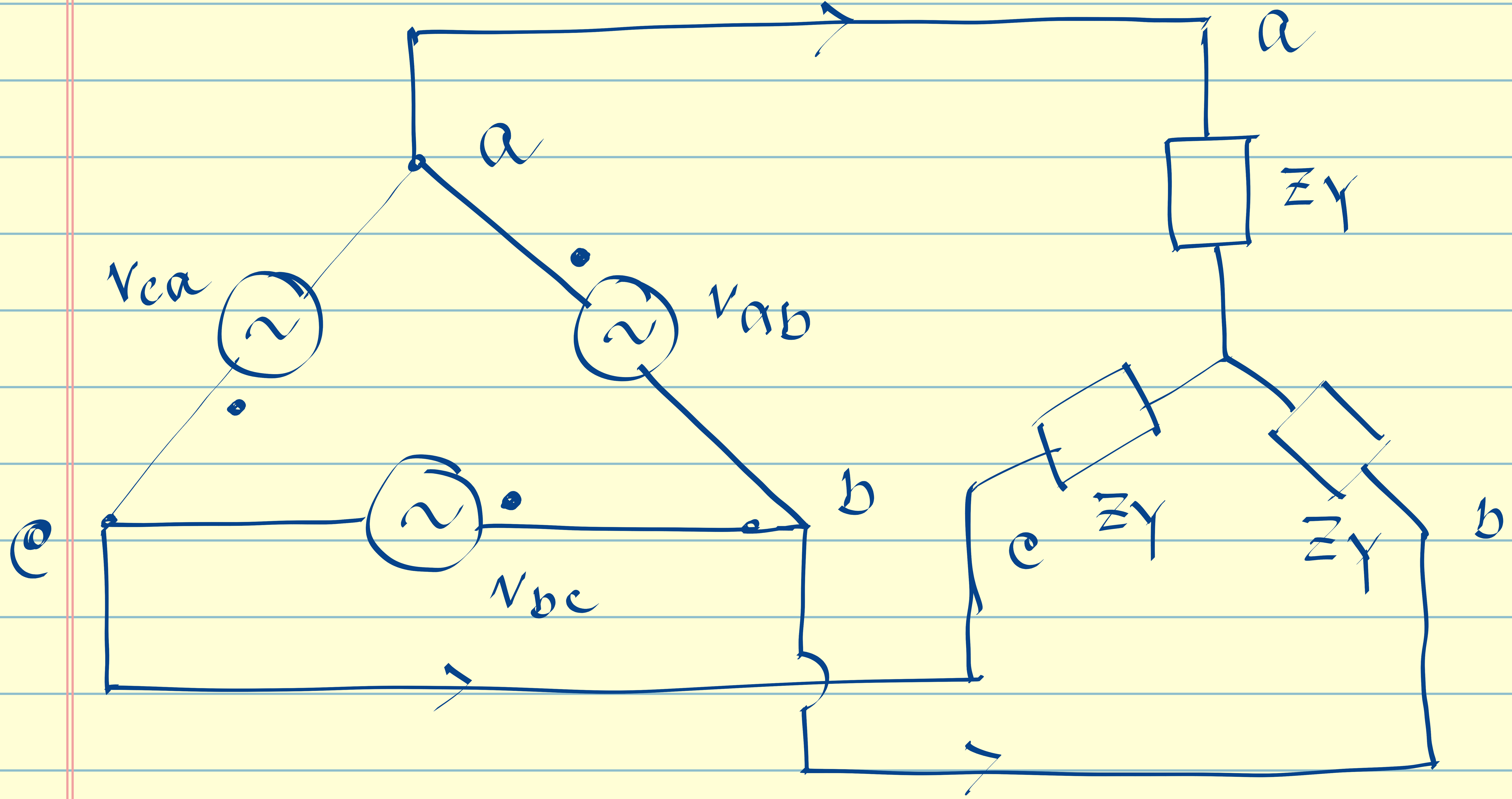
$|I_{ab}|$ = Phase current

$= \frac{\text{Line current}}{\sqrt{3}}$

Single phase equivalent ckt



Δ - Y System :-



Apply KVL

$$V_{ab} = V_p \angle 0$$
$$V_{bc} = V_p \angle -120$$
$$V_{ca} = V_p \angle +120$$

$$V_{ab} - I_a Z_Y + I_b Z_Y = 0$$

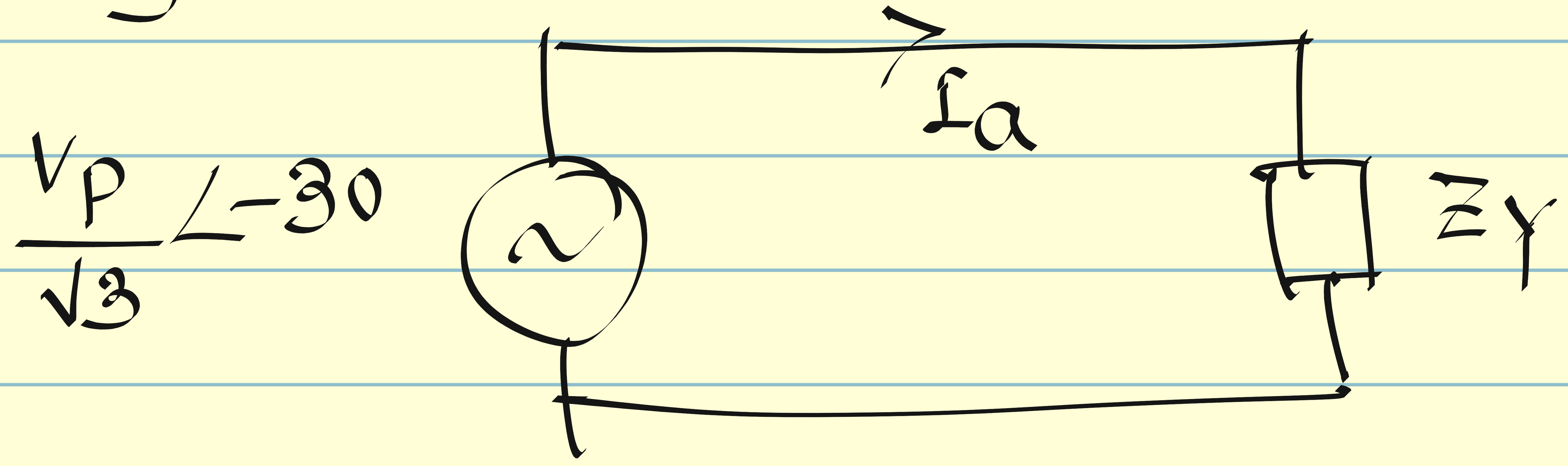
$$\Rightarrow V_{ab} = (I_a - I_b) Z_Y$$

$$\Rightarrow I_a - I_b = \frac{V_{ab}}{Z_Y}$$

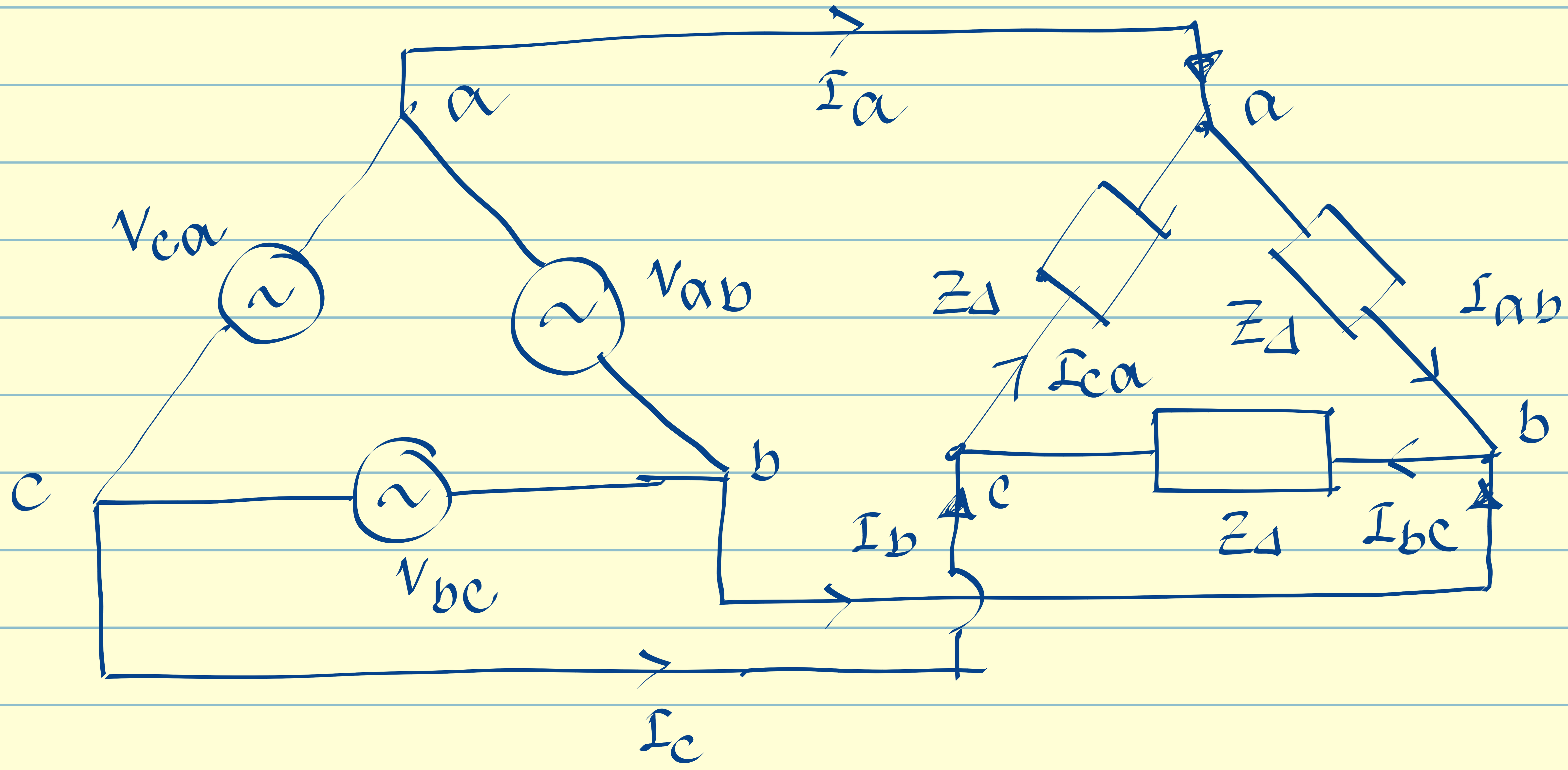
$$\Rightarrow \sqrt{3} I_p \angle 30 = \frac{V_{ab}}{Z_Y}$$

$$I_p = \left(\frac{V_p}{\sqrt{3}} \angle -30 \right) \frac{1}{Z_Y}$$

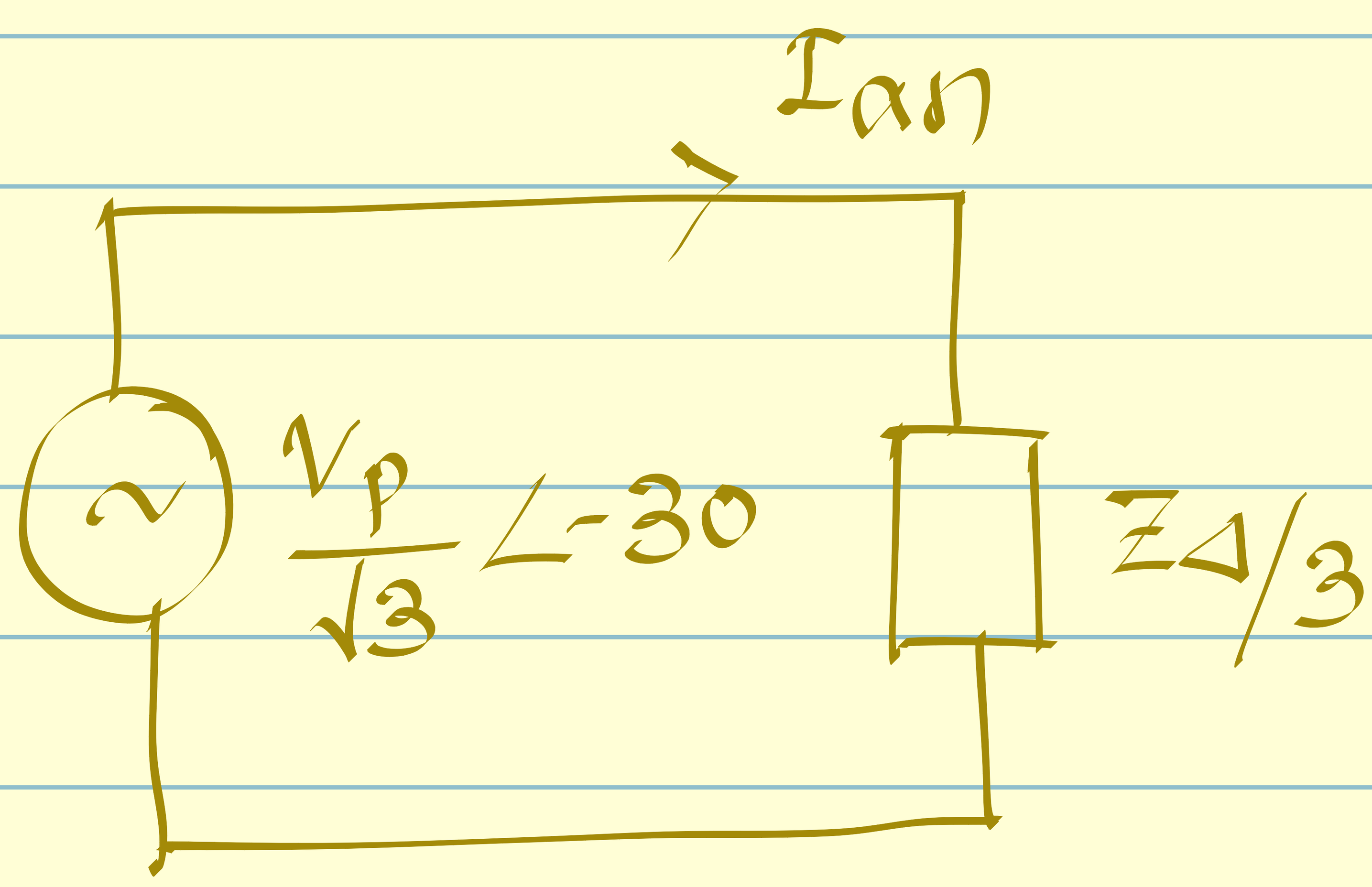
Single phase equivalent ckt



Δ - Δ System :-



$$V_{ab} = V_p \angle 0$$
$$V_{bc} = V_p \angle -120$$
$$V_{ca} = V_p \angle +120$$



Apply KVL

$$\tilde{V}_{ab} = \tilde{I}_{ab} \tilde{Z}_\Delta$$

$$\Rightarrow I_{ab} = \frac{V_{ab}}{Z_\Delta} = \frac{V_p \angle 0}{Z_\Delta}$$

$$\Rightarrow \frac{1}{3}(I_a - I_b) = \frac{V_p \angle 0}{Z_\Delta}$$

$$\Rightarrow \sqrt{3} I_p \angle 30 = \frac{V_p \angle 0}{Z_\Delta/3}$$

$$I_p = \left(\frac{V_p}{\sqrt{3}} \angle -30\right) \cdot \frac{1}{Z_\Delta/3}$$

Power in 3-ϕ balanced System:—

Let us consider a balanced Y Connected load for which, the per phase impedance is Z_Y

where $Z_Y = |Z_Y| \angle \theta$

$$v_{an} = V_m \cos \omega t$$
$$v_{bn} = V_m \cos(\omega t - 120)$$
$$v_{cn} = V_m \cos(\omega t + 120)$$

$$i_a = I_m \cos(\omega t - \theta)$$
$$i_b = I_m \cos(\omega t - \theta - 120)$$
$$i_c = I_m \cos(\omega t - \theta + 120)$$

3-ϕ instantaneous power consumed by the Y connected load is

$$p = v_{an} i_a + v_{bn} i_b + v_{cn} i_c$$

$$= \frac{1}{2} V_m I_m \Big[2\cos \omega t \cos(\omega t - \theta) + 2\cos(\omega t - 120)\cos(\omega t - \theta - 120) + 2\cos(\omega t + 120)\cos(\omega t + 120 - \theta) \Big]$$

$$= \frac{1}{2} V_m I_m \Big[\cos\theta + \cos(2\omega t - \theta) + \cos\theta + \cos(2\omega t - \theta - 240) + \cos\theta + \cos(2\omega t - \theta + 240) \Big]$$

$$= \frac{3}{2} V_m I_m \cos\theta$$

$$= 3 V_p I_p \cos\theta$$

V_p, I_p are the per phase rms voltage and current

$$V_p = V_m/\sqrt{2} \qquad I_p = I_m/\sqrt{2}$$

* 3-ϕ instantaneous power is constant w.r.t time. This is the general expression of 3ϕ power valid for both Y and Δ connected load.

* This is one of the reasons, behind the generation & transmission of 3ϕ power.

Now the per phase average power is

$$P_{1\phi} = V_p I_p \cos\theta$$

$$Q_{1\phi} = V_p I_p \sin\theta$$

$$S_{1\phi} = P_{1\phi} + j Q_{1\phi} = \tilde{V} \tilde{I}^*$$

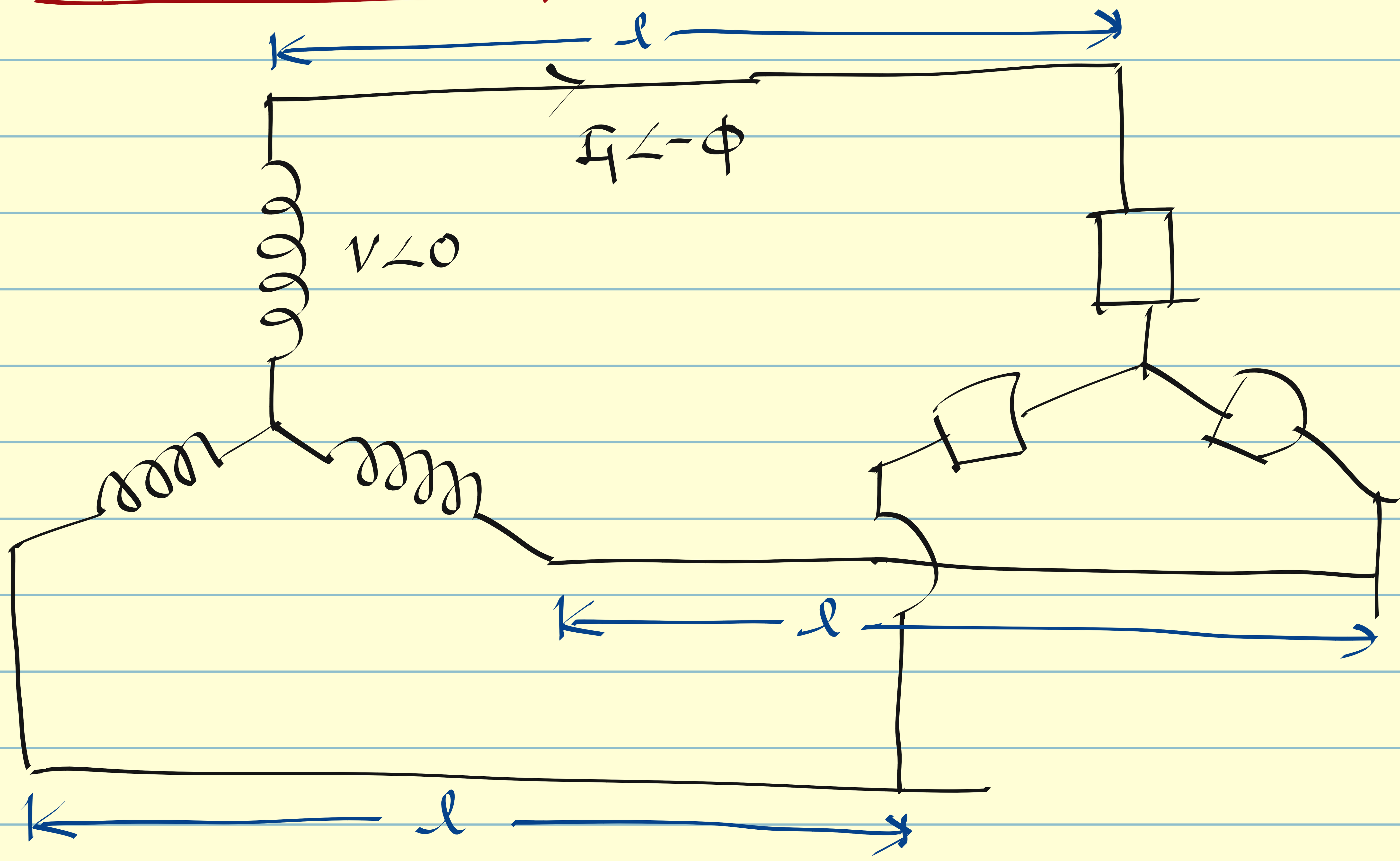
$$S_{3\phi} = P_{3\phi} + j Q_{3\phi} = 3 \tilde{V} \tilde{I}^*$$

$$P_{3\phi} = 3 V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta$$

$$Q_{3\phi} = \sqrt{3} V_L I_L \sin\theta$$

Transmitted Power Same
Efficiency Same.

3ϕ 3 wire System:-



active power $P = 3VI\cos\phi$

$$I_1 = \frac{P}{3V\cos\phi}$$

$$R_1 = \rho\frac{l}{A_1}$$

Total Power loss

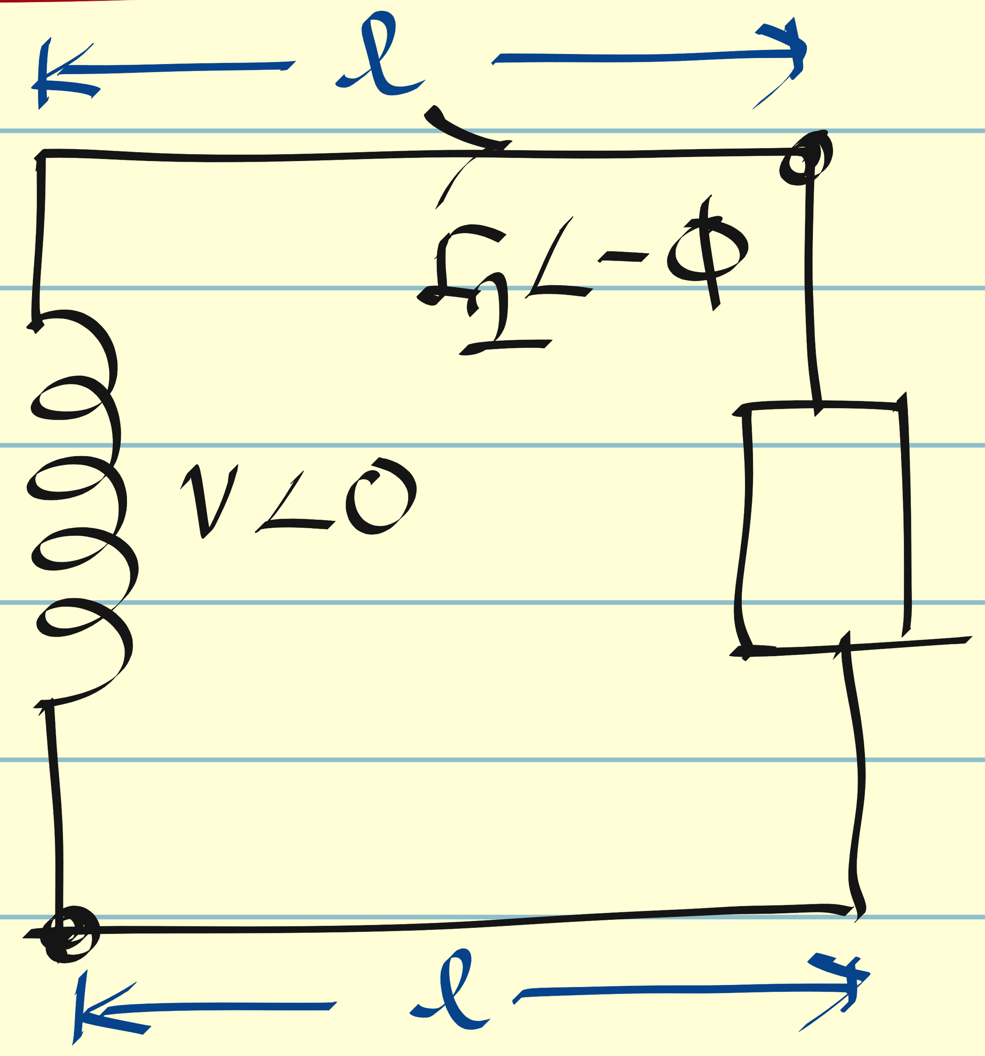
$$W_{3\phi} = 3 I_1^2 R_1$$

$$= 3\cdot\left(\frac{P}{3V\cos\phi}\right)^2 \times \rho\frac{l}{A_1}$$

$$A_1 = \frac{P^2\rho l}{3V^2\cos^2\phi\, W_{3\phi}}$$

Volume $= 3lA_1 = \dfrac{P^2\rho l^2}{V^2\cos^2\phi\, W_{3\phi}}$
(Vol 3ϕ)

1ϕ 2 wire System :-



$$P = V I_2 \cos\phi$$

$$I_2 = \frac{P}{V\cos\phi} \qquad R_2 = \rho\frac{l}{A_2}$$

$$W_{1\phi} = 2 I_2^2 \times R_2$$
$$= 2 \frac{P^2}{V^2\cos^2\phi} \cdot \rho\frac{l}{A_2}$$

$$A_2 = \frac{2P^2\rho l}{V^2\cos^2\phi \, W_{1\phi}}$$

efficiency is same for both of the cases.

Volume $= 2 l A_2$

(Vol 1ϕ) $= 2l \cdot \frac{2P^2\rho l}{V^2\cos^2\phi \, W_{1\phi}}$

$$= \frac{4P^2\rho l^2}{V^2\cos^2\phi \, W_{1\phi}}$$

$$\frac{\text{Vol } 1\phi}{4} = \text{Vol } 3\phi$$

$$\boxed{\text{Vol } 1\phi > \text{Vol } 3\phi}$$

More Cu is required for 1ϕ compared to 3 phase.