NPHI101: Engineering Physics

Mid-Semester Examination 2024-25 (Winter Semester)

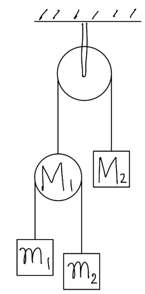
Total Marks: 60 **Time: 2 hours**

Instructions:

1. **Use separate answer sheets for Part 1, Part 2, and, Part 3. Mention the Part No. at the top.**
2. **Some important values are provided at the end.**

Part 1 (Classical Mechanics and Electrodynamics)

1. A mass M_2 hangs at one end of a string which passes over a fixed frictionless non-rotating pulley. At the other end of this string there is a non-rotating pulley of mass M_1 over which there is a string carrying masses m_1 and m_2. (a) Write down the constraint equations. (b) Set up the Lagrangian of the system. (c) Find the acceleration of mass M_2 using the Euler-Lagrange equations of motion. **Marks: 2+3+4=9**



2. (a) The Lagrangian for a particle of mass m moving in an orbit, under the influence of a central force is given by, $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + V(r)$, where V is the potential due to the central force, θ is the angular coordinate and r is the radial coordinate. What is the cyclic coordinate here? Find the conjugate momentum corresponding to the cyclic coordinate.

   (b) Given, $\vec{F} = (2xy + z^3)\,\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$, compute $\int_A^B \vec{F}\cdot\vec{dl}$ between the two points $A(0,0,0)$ and $B(1,1,1)$ for **two** different paths as follows, (a) **Path 1**: $A(0,0,0)$ to $C(1,0,0)$ along the x-axis; then from $C(1,0,0)$ to $D(1,1,0)$ along the line parallel to the y-axis and finally from $D(1,1,0)$ to $B(1,1,1)$ along the line parallel to the z-axis. **Path 2** is given as: From $A(0,0,0)$ and $B(1,1,1)$ along a straight line $x = y = z$. What is the significance of your result? **Marks: (2+2)+(6+1)=11**

Part 2 (Thermal and Statistical Physics)

3. (a) One mole of oxygen, initially kept at 47°C, is adiabatically compressed so that the pressure becomes 8 times of its initial value. Find the final temperature. (Consider ideal gas law and ignore vibrational degree of freedom).

   (b) Prove the difference between heat capacities, $C_P - C_V = \left\{\left(\frac{\partial U}{\partial V}\right)_T + P\right\}\left(\frac{\partial V}{\partial T}\right)_P$

   (c) The Helmholtz free energy of a system with N number of molecules is given by $F = N\epsilon_0 - N\beta T - N\alpha T ln(T) - Nk_B T ln(V/N)$. Here the symbols have their usual meanings, and ϵ_0, β, and α are constants. Find the entropy of the system.

   (d) Draw P-V diagram of an isochoric, and isobaric process. **Marks: 4+3+2+1=10**

4. (a) Qualitatively show (in a graph) how the Rayleigh-Jeans law, Wien's law and Planck's law describe the experimental data for spectral energy density of a black body at T=500K. What is ultra-violet catastrophe? Find the wavelength (λ_{max}) corresponding to the maximum spectral energy density.

   (b) Calculate the average energy per oscillator inside a cavity black body where the energy emitted by the cavity oscillators can have discrete energy values 0, hf, $2hf$, $3hf$ and so on.

   Marks: (4+1+1)+4=10

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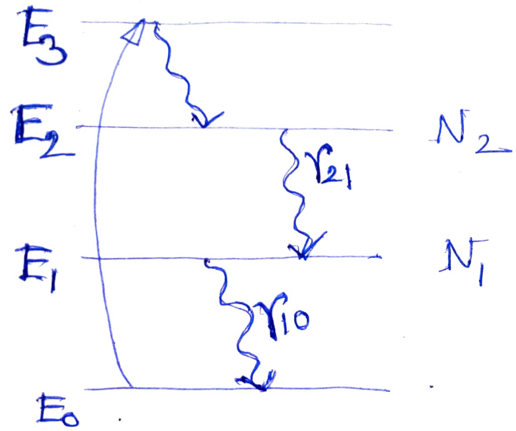
5. (a) Consider a four-level pumping model of a laser system (see the schematic below). Assume the pumping rate from E_0 to E_3 is R_{p0}. The effective pumping efficiency (to populate E_2) is η_p. (i) Write down the rate equations for the populations N_2 and N_1 at the levels E_2 and E_1 respectively. (ii) Assume that continuous pumping is applied to achieve a steady state. At steady state, find the expression for $N_2 - N_1$ in terms of the lifetimes. Do not consider lasing action (stimulated transitions) in your calculation.



(b) The lifetime of an upper energy level E_2 of an atom due to spontaneous decay is $\tau_2 = 60$ ns. If the population is 5×10^{17} at that level at time $t = 30$ ns, what is the initial population at $t = 0$?

Marks: (4+3)+3=10

6. (a) Consider a quantum particle in one dimension (x-axis). The wave function is given by: $\psi(x) = Aie^{-ax^2}$, where a and A are positive constants and $i = \sqrt{-1}$. Calculate the expectation value of the position, $< x >$ of the particle in this state.

(b) Mathematically and physically explain that the wave function of a quantum particle should be square integrable.

(c) What are the stationary states of a quantum particle under a suitable potential V?

Marks: 4+4+2=10

Some important data:

Wien constant $b = 2.898$ mm-K, $k_\mathrm{B} = 1.38 \times 10^{-23}$ J/K

3.(a) Oxygen is a diatomic molecule.

Ignoring vibrational deg. of freedom, total deg. of freedom $f = 5$.

Initial temp. is $T_1 = 47^\circ C = 320\,K$

Lets consider initial pressure P_1 and final pressure P_2 and final temperature T_2.

Here, $P_2 = 8 P_1$

For an adiabatic process, $V T^{f/2} = \text{const.}$ ....(i)

Considering ideal gas law i.e. $\frac{PV}{T} = \text{const} \Rightarrow V = \frac{T}{P} \times \text{const.}$

Replacing V in eq^n (i),

$$\frac{T}{P} \times T^{f/2} = \text{const.} \Rightarrow \frac{T^{f/2+1}}{P} = \text{cont.}$$

$f = 5$

$\frac{f}{2} + 1 = \frac{7}{2}$

$$\Rightarrow \frac{T_2^{7/2}}{P_2} = \frac{T_1^{7/2}}{P_1} \Rightarrow \left(\frac{T_2}{T_1}\right)^{7/2} = \frac{P_2}{P_1} = 8$$

$$\Rightarrow T_2 = 8^{2/7} \times T_1 = 8^{2/7} \times 320\,K \approx 579.7\,K.$$

The final temperature is $579.7\,K \approx 580\,K$.

Taking $8^{2/7} \approx 1.8$

$T_2 \approx 576\,K$.

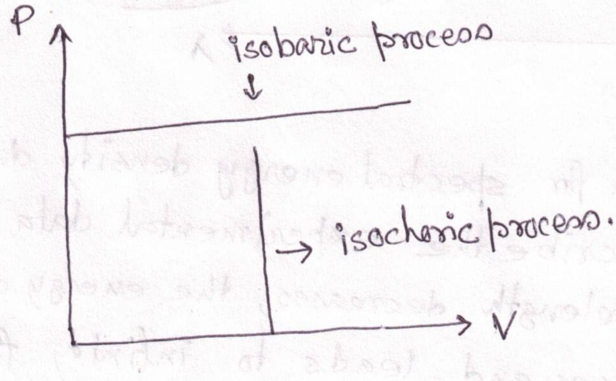
3.(c) $F = N\epsilon_0 - N\beta T - N\alpha T \ln T - N k_B T \ln\left(\frac{V}{N}\right)$

Entropy $S = -\left(\frac{\partial F}{\partial T}\right)_V = -\left[-N\beta - N\alpha \ln T - N\alpha - N k_B \ln \frac{V}{N}\right]$

$$= N\beta + N\alpha + N\alpha \ln T + N k_B \ln \frac{V}{N}$$

$$= N\left(\alpha + \beta + \alpha \ln T + k_B \ln \frac{V}{N}\right).$$

3. (d)



3.(b) First law of thermodynamics $đQ = dU + PdV$ ---- (i)

Considering $U = U(V, T)$ we get

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \quad \text{---- (ii)}$$

Substituting in eqn (i) we get,

$$đQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left\{\left(\frac{\partial U}{\partial V}\right)_T + P\right\} dV \quad \text{---- (iii)}$$

Now $C_V = \left(\frac{\partial Q}{\partial T}\right)_V$ and $C_P = \left(\frac{\partial Q}{\partial T}\right)_P$.

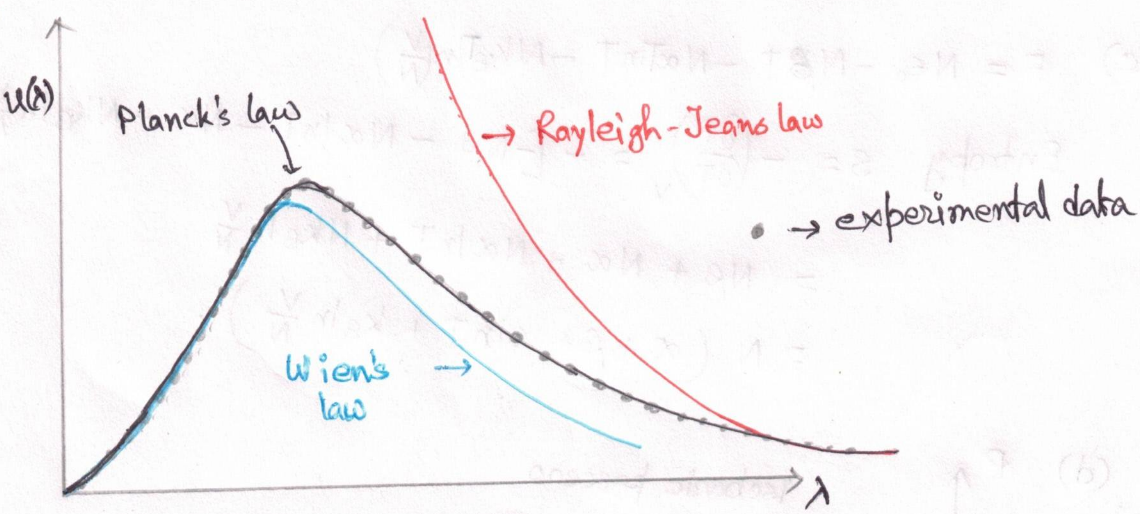
Hence $C_V = \left(\frac{\partial Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$

$$C_P = \left(\frac{\partial Q}{\partial T}\right)_P = \left(\frac{\partial U}{\partial T}\right)_V + \left\{\left(\frac{\partial U}{\partial V}\right)_T + P\right\}\left(\frac{\partial V}{\partial T}\right)_P$$

$$\Rightarrow C_P = C_V + \left\{\left(\frac{\partial U}{\partial V}\right)_T + P\right\}\left(\frac{\partial V}{\partial T}\right)_P$$

$$\Rightarrow C_P - C_V = \left\{\left(\frac{\partial U}{\partial V}\right)_T + P\right\}\left(\frac{\partial V}{\partial T}\right)_P$$

4. (a)



The classical formula for spectral energy density derived by Rayleigh-Jeans describe the experimental data only for high wavelength. As wavelength decreases, the energy density for RJ-formula increases and leads to infinity for $\lambda \to 0$. This is known as ultraviolet catastrophe.

4.(a) From Wien's displacement law $\lambda_{max} T = b$

Here b is Wien constant = 2.898 mm-K.

Hence $\lambda_{max} = \frac{2.898 \text{ mm.K}}{T} = \frac{2.898 \text{ mm K}}{500 \text{ K}}$

$\Rightarrow \lambda_{max} = 5.796 \times 10^{-6} \text{ m} \approx 5.8 \,\mu\text{m}.$

4.(b) The energy levels are $E_0 = 0$, $E_1 = hf$, $E_2 = 2hf$, $E_3 = 3hf$ ......

The no of oscillators in a energy level E at temperature T is

$$N(E) = N_0 e^{-E/kT}$$

Therefore the no of oscillators at energy levels

$E_0 = 0$, $\quad N = N_0$

$E_1 = hf$, $\quad N_1 = N_0 e^{-hf/kT} = N_0 x$

$E_2 = 2hf$, $\quad N_2 = N_0 e^{-2hf/kT} = N_0 x^2$

$N_3 = 3hf$, $\quad N_3 = N_0 e^{-3hf/kT} = N_0 x^3$ and so. on.

Here $e^{-hf/kT} = x$.

The average energy $\langle E \rangle = \frac{\text{Total energy}}{\text{Total no of oscillators}}$

$$\Rightarrow \langle E \rangle = \frac{N_0 E_0 + N_1 E_1 + N_2 E_2 + N_3 E_3 + \cdots \infty}{N_0 + N_1 + N_2 + N_3 + \cdots \infty}$$

$$= \frac{0 + N_0 x \cdot hf + N_0 x^2 . 2hf + N_0 x^3 . 3hf + \cdots}{N_0 + N_0 x + N_0 x^2 + N_0 x^3 + \cdots}$$

$$= xhf \frac{1 + 2x + 3x^2 + \cdots}{1 + x + x^2 + x^3 + \cdots} = xhf \left[\frac{S_1}{S_0}\right]$$

$S_0 = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$ $\quad \Big|$ as $x = e^{-hf/kT} \ll 1$

$S_1 = \frac{dS_0}{dx} = \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \cdots$

$$\Rightarrow \langle E \rangle = xhf \cdot \frac{1}{(1-x)^2} \Big/ \frac{1}{1-x} = \frac{xhf}{1-x} = \frac{hf}{\frac{1}{x} - 1} = \frac{hf}{e^{hf/kT} - 1}$$