NPHI101: Engineering Physics
Mid-Semester Examination 2025-26 (Monsoon Semester)

Total Marks: 30 **Time: 1 hour**

Use separate answer sheets for different parts. Mention the Part No. at the top.
All the symbols have their usual meaning.

Part 1 (Classical Mechanics and Electrodynamics)

1. A bead of mass m slides without friction on a frictionless wire in the shape of a cycloid, with equations, $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, $0 \leq \theta \leq 2\pi$, and a is a constant. Find **(a)** the Lagrangian function L (in terms of θ, $\dot{\theta}$), **(b)** the Euler-Lagrange equation of motion. Find **(c)** an expression for the generalized momentum p_θ, and **(d)** the Hamiltonian of the system in terms of p_θ and L.

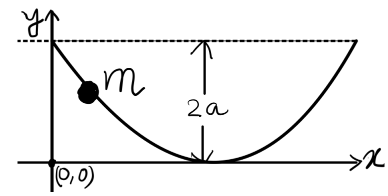


Figure: Bead sliding on a cycloid

Marks: 2+2+1+2=7

2. Show that the force field given by, $\vec{F} = x^2yz\hat{i} - xyz^2\hat{k}$, is non-conservative. **Marks: 3**

Part 2 (Thermal and Statistical Physics)

1. (a) Consider a gas that obeys the modified equation of state: $P(V - b) = Nk_{\mathrm{B}}T$, where b is a constant, N is the number of particles, k_{B} is Boltzmann's constant. The gas is expanded isothermally where the volume increased from V_0 to $4V_0$. Find the work done (in terms of b, N, k_{B}, V_0 and T) by the gas. Sketch the P-V diagram for this process indicating the initial and final states and shade the region that represents the work done during the expansion. **Marks: 2+2**

   (b) The Helmholtz free energy of N number of molecules is given by $F = N\epsilon_0 - N\beta V - N\alpha T\ln(T) - Nk_BT\ln(V/N)$. Here ϵ_0, β, and α are constants. Calculate the pressure of the system. **Marks: 2**

2. The radiation emitted by cavity (black body) oscillators has discrete energy values: 0, hc/λ, $2hc/\lambda$, $3hc/\lambda$, and so on (here λ is the wavelength). Calculate the average energy per oscillator. **Marks: 4**

Part 3 (Modern Physics)

1. The lifetimes associated with the atomic spontaneous decays from E_3 to E_2 and from E_3 to E_1 are 70 ns and 200 ns, respectively. Calculate the lifetime of E_3 for the overall spontaneous decay from that level? **Marks: 3**

2. Explain the pumping model for a four-level laser and derive an expression for a steady-state population inversion between the lasing levels. **Marks: 3**

3. From the probabilistic interpretation, show why the wavefunction Ψ of a quantum particle should be square-integrable. **Marks: 2**

4. Assume that $\Psi(x) = Ax^4$, where A is a suitable constant. What kind of quantum particle does it represent? Explain. **Marks: 2**

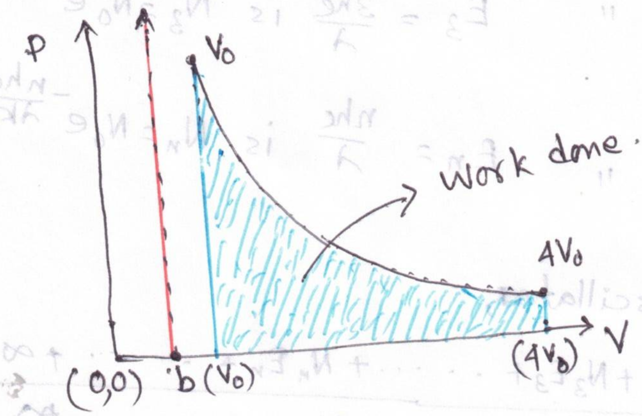
Part 2 Midsem 2025-26 Monsoon

1. (a) Equation of state $P(V-b) = Nk_BT$.

$$\Rightarrow P = \frac{Nk_BT}{V-b}.$$

Work done $W = -\int PdV = -\int_{V_0}^{4V_0} \frac{Nk_BT}{V-b} dV$

$$\Rightarrow W = Nk_BT \ln\left(\frac{V_0-b}{4V_0-b}\right)$$



2. $F = N\epsilon_0 - N\beta V - N\alpha T\ln(T) - Nk_BT\ln\left(\frac{V}{N}\right)$

$dF = -PdV - SdT$.

$$\Rightarrow P = -\left(\frac{\partial F}{\partial V}\right)_T.$$

$$\Rightarrow P = -\left[-N\beta - Nk_BT\cdot\frac{1}{V}\right] = N\left(\beta + \frac{k_BT}{V}\right)$$

3.

Part 2

3. Cavity oscillators has discrete energy values $0, \frac{hc}{\lambda}, \frac{2hc}{\lambda}, \frac{3hc}{\lambda}, \ldots\ldots$

Lets consider No of oscillators in the ground state is N_0 $(E_0 = 0)$

No of oscillators with energy $E_1 = \frac{hc}{\lambda}$, is $N_1 = N_0 e^{-\frac{hc}{\lambda kT}} = N_0 x$

(here $e^{-hc/\lambda kT} = x$)

Here a Boltzmann distribution is considered.

No of oscillators with energy $E_2 = \frac{2hc}{\lambda}$ is $N_2 = N_0 e^{-\frac{2hc}{\lambda kT}} = N_0 x^2$

" " " " " $E_3 = \frac{3hc}{\lambda}$ is $N_3 = N_0 e^{-3hc/\lambda kT} = N_0 x^3$

⋮

" " " " $E_n = \frac{nhc}{\lambda}$ is $N_n = N_0 e^{-\frac{nhc}{\lambda kT}} = N_0 x^n$.

Average energy per oscillators,

$$\langle E \rangle = \frac{N_0 E_0 + N_1 E_1 + N_2 E_2 + N_3 E_3 + \ldots\ldots + N_n E_n + \ldots\ldots + \infty}{N_0 + N_1 + N_2 + N_3 + \ldots\ldots + N_n + \ldots\ldots \infty}$$

$$= \frac{N_0 \cdot 0 + N_0 x \cdot \frac{hc}{\lambda} + N_0 x^2 \cdot \frac{hc2}{\lambda} + N_0 x^3 \cdot \frac{3hc}{\lambda} + \ldots\ldots}{N_0 + N_0 x + N_0 x^2 + N_0 x^3 + \ldots\ldots}$$

$$= \frac{xhc}{\lambda}\left[\frac{1 + 2x + 3x^2 + \ldots\ldots}{1 + x + x^2 + x^3 + \ldots\ldots}\right] = \frac{xhc}{\lambda}\left[\frac{S_1}{S_0}\right]$$

$$S_0 = 1 + x + x^2 + x^3 + \ldots\ldots = \frac{1}{1-x} \quad (\text{as } x \ll 1)$$

$$S_1 = \frac{d(S_0)}{dx} = \frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{1}{(1-x)^2}$$

Hence $\langle E \rangle = \frac{xhc}{\lambda} \cdot \frac{1}{1-x} = \frac{hc}{\lambda} \cdot \frac{1}{\frac{1}{x} - 1} = \frac{hc}{\lambda} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$