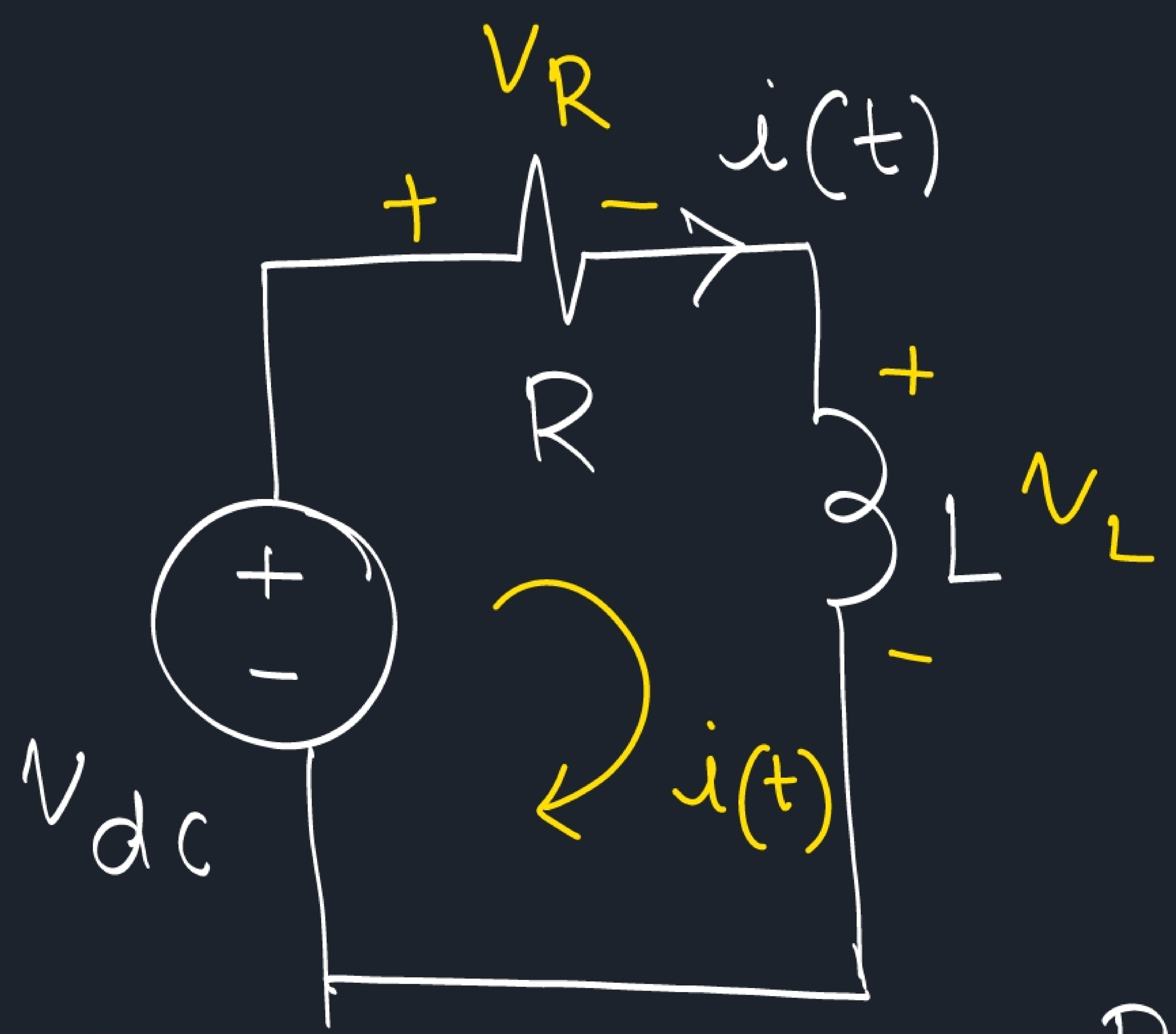
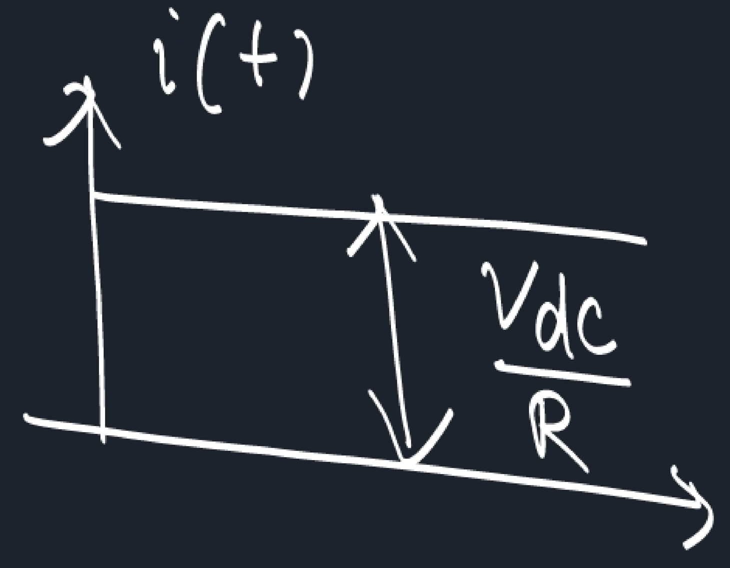
Basics:-





$$V_{dc} = V_R + V_L$$

$$\Rightarrow L\frac{di(t)}{dt} + R\,i(t) = V_{dc}$$

$$\Rightarrow \frac{di(t)}{dt} + \underbrace{\frac{R}{L}}_{P}\,i(t) = \underbrace{\frac{V_{dc}}{L}}_{Q}$$

$$e^{R/L\,t}\frac{di(t)}{dt} + e^{\frac{R}{L}t}\frac{R}{L}\,i(t) = e^{R/L\,t}\frac{V_{dc}}{L}$$

$$\Rightarrow \frac{d}{dt}\left(e^{R/L\,t}\,i(t)\right) = e^{R/L\,t}\cdot\frac{V_{dc}}{L}$$

$$e^{R/L\,t}\,i(t) = A + \frac{V_{dc}}{L}\int e^{R/L\,t}\,dt$$

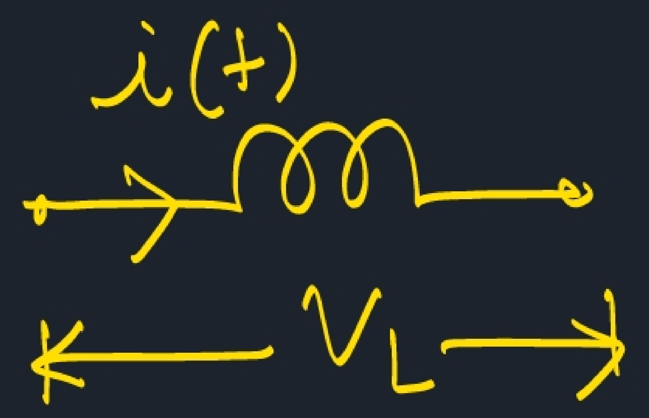
$$i(t) = A e^{-R/L\,t} + \frac{V_{dc}}{L}\,e^{-R/L\,t}\underbrace{\int e^{R/L\,t}\,dt}$$

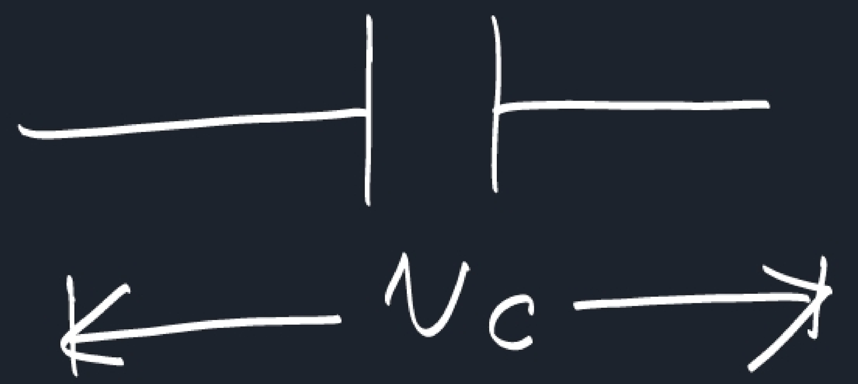
$$\boxed{i(t) = A e^{-R/L\,t} + \frac{V_{dc}}{R}}$$

@ $t = 0$ $\quad i(t) = I_0$

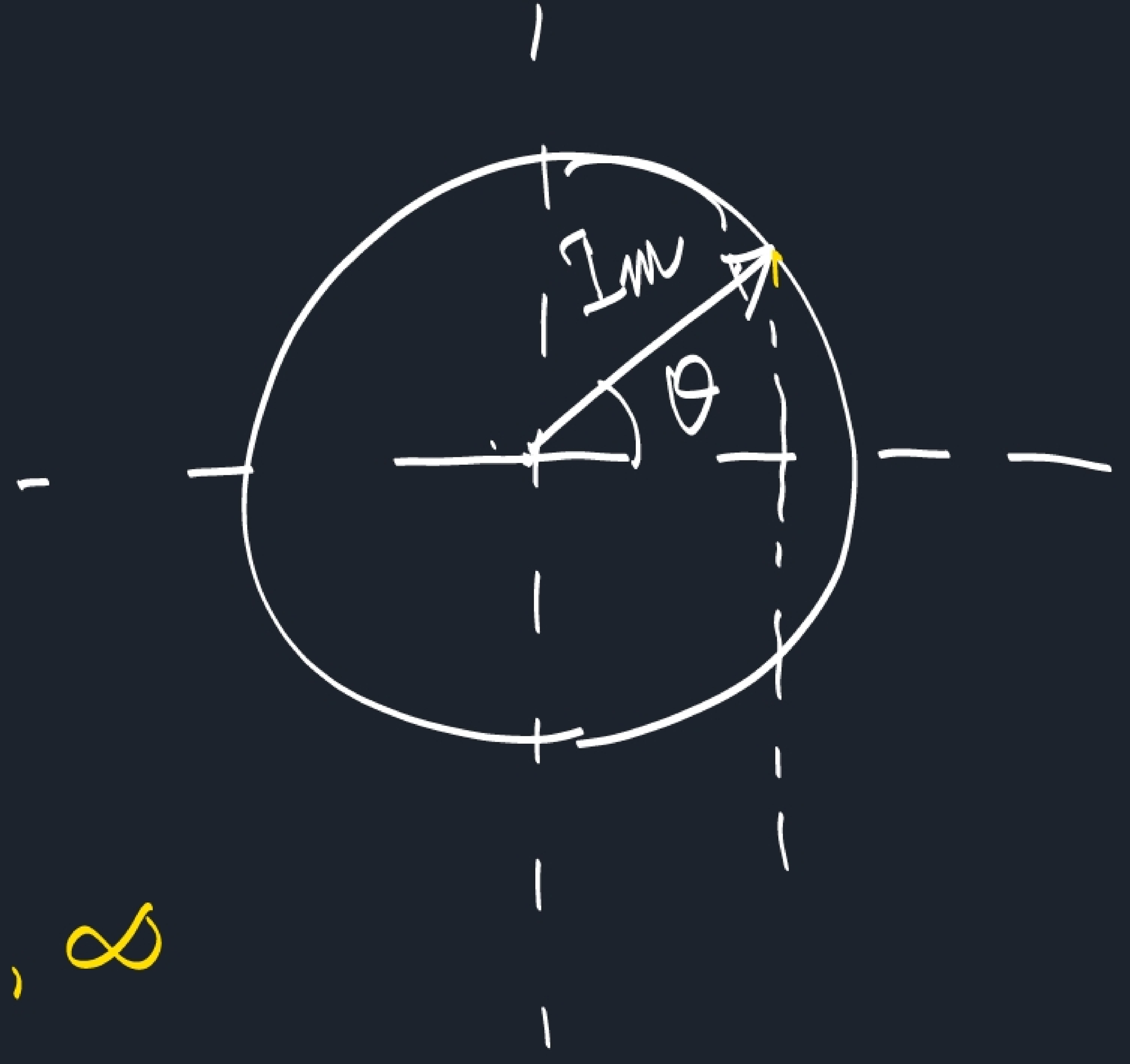
$$I_0 = A + \frac{V_{dc}}{R} \Rightarrow A = I_0 - \frac{V_{dc}}{R}$$

$$\boxed{i(t) = \frac{V_{dc}}{R}\left(1 - e^{-R/L\,t}\right) + I_0 e^{-R/L\,t}}$$





$f \propto -x$



$$V_L(t) = L\frac{di(t)}{dt}$$

$$i_c(t) = C\frac{dv_c(t)}{dt}$$

$$v_c(t) = \frac{1}{C}\int i_c(t)\,dt$$

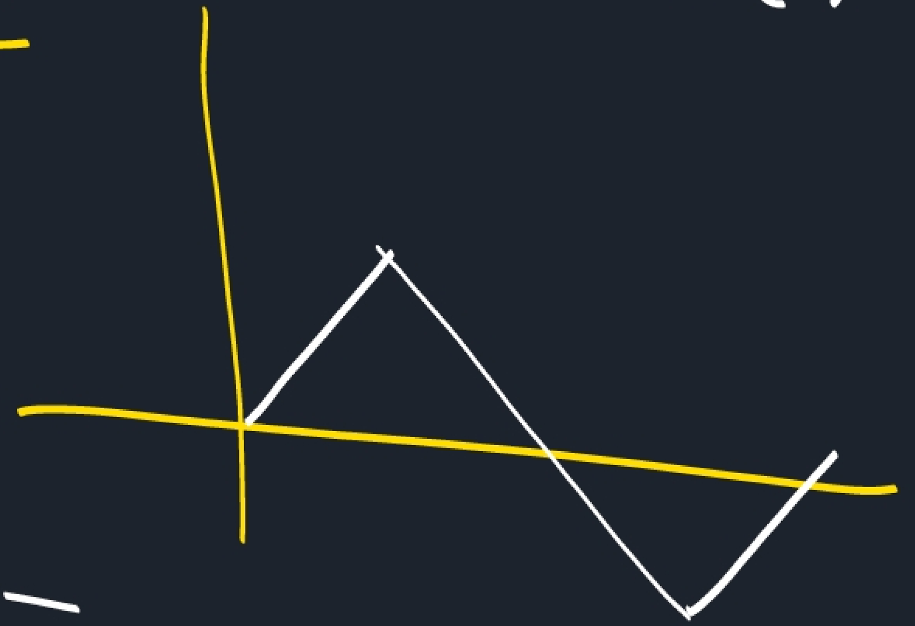
$n = 1, 2, \ldots, \infty$

$i(t) = e^{\pm \alpha t}$ α is integer.

$i(t) = \sin\omega t$

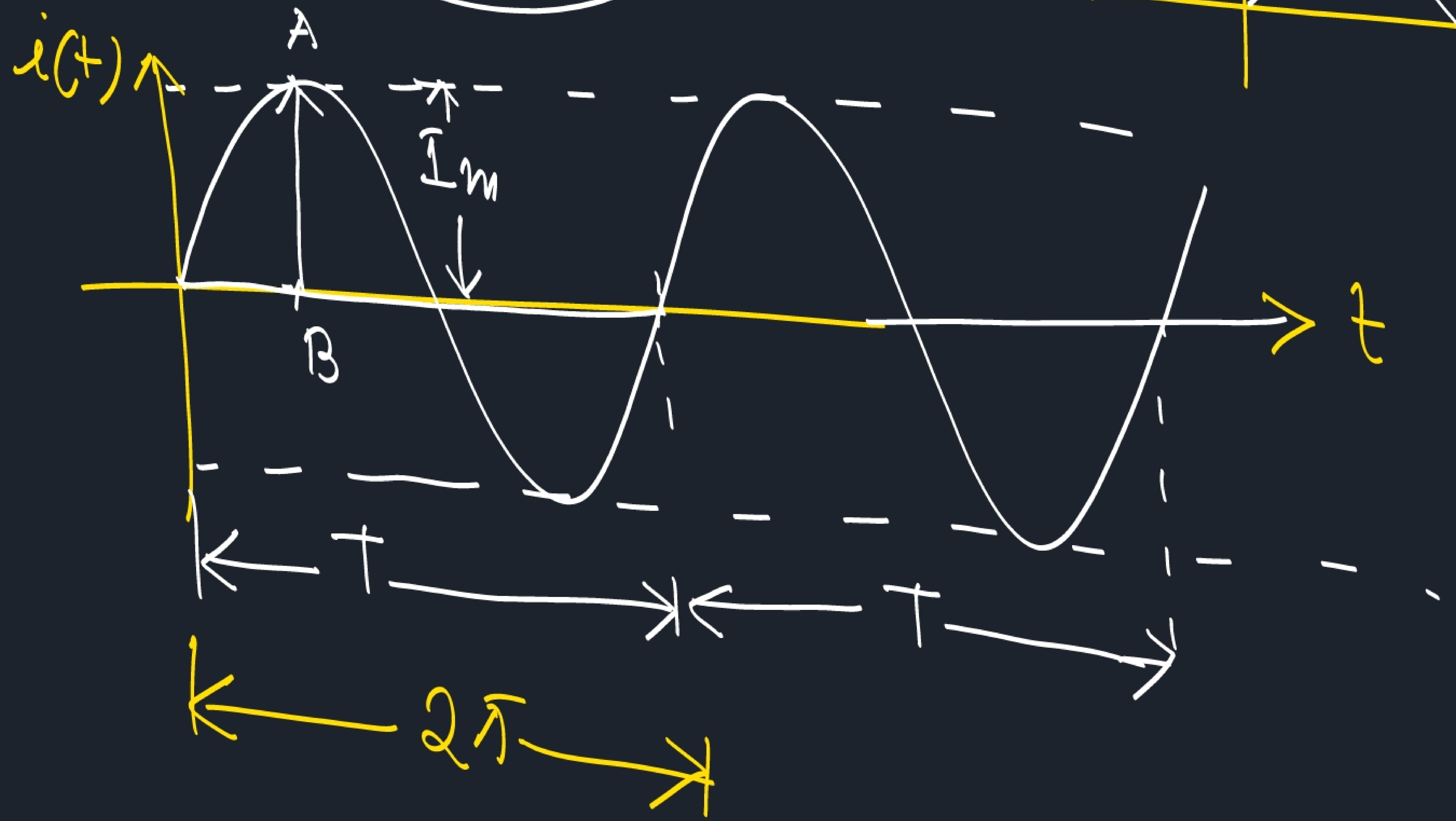
$I_m \sin\theta$

$I_m \sin\omega t$



$f(t)$ $\boxed{f(t \pm nT) = f(t)}$

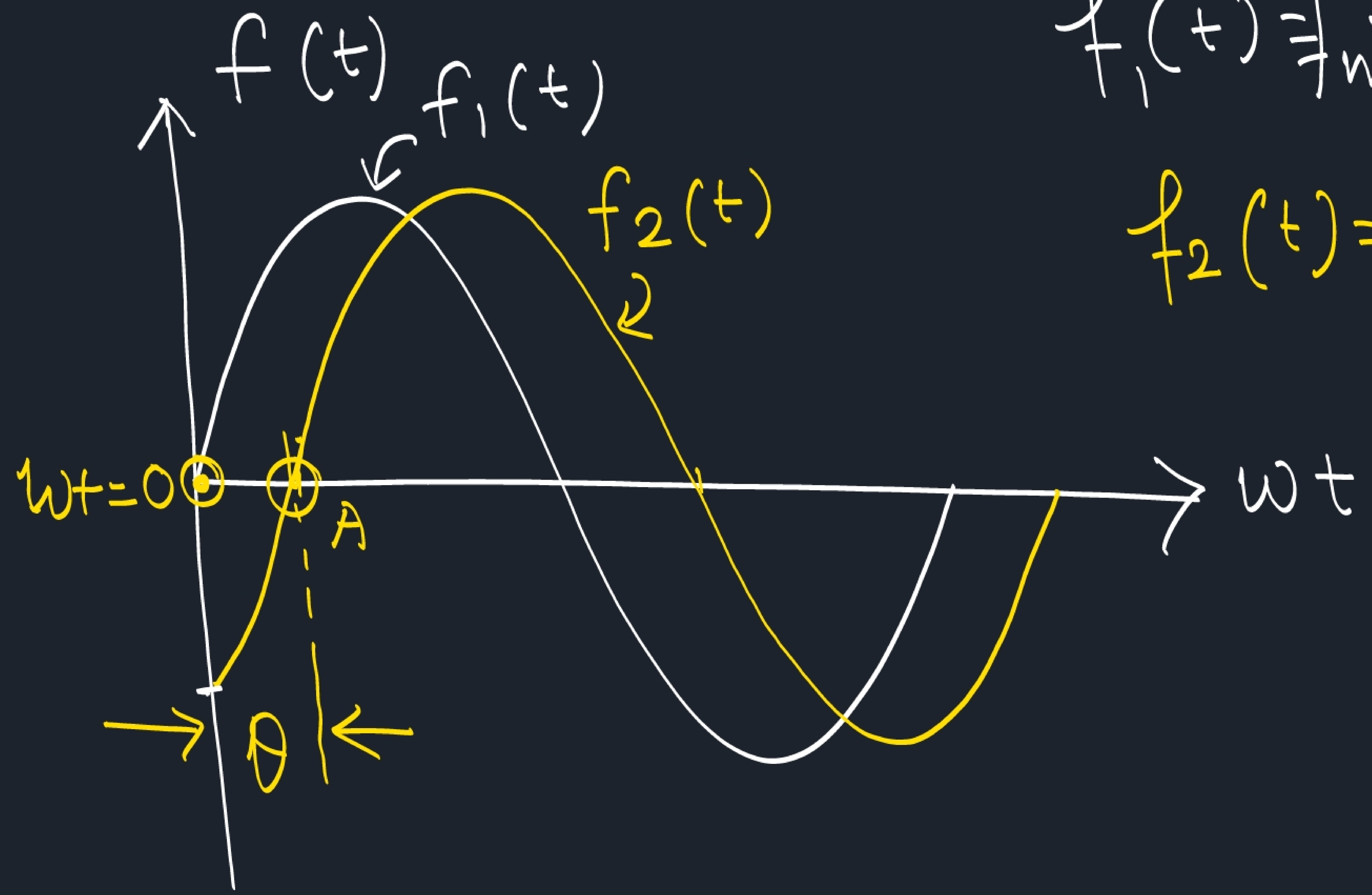
periodic waveform.



$$\text{freq} = \frac{1}{T} = 50\text{ Hz}.$$

$$\omega t\,(t = T) = 2\pi$$

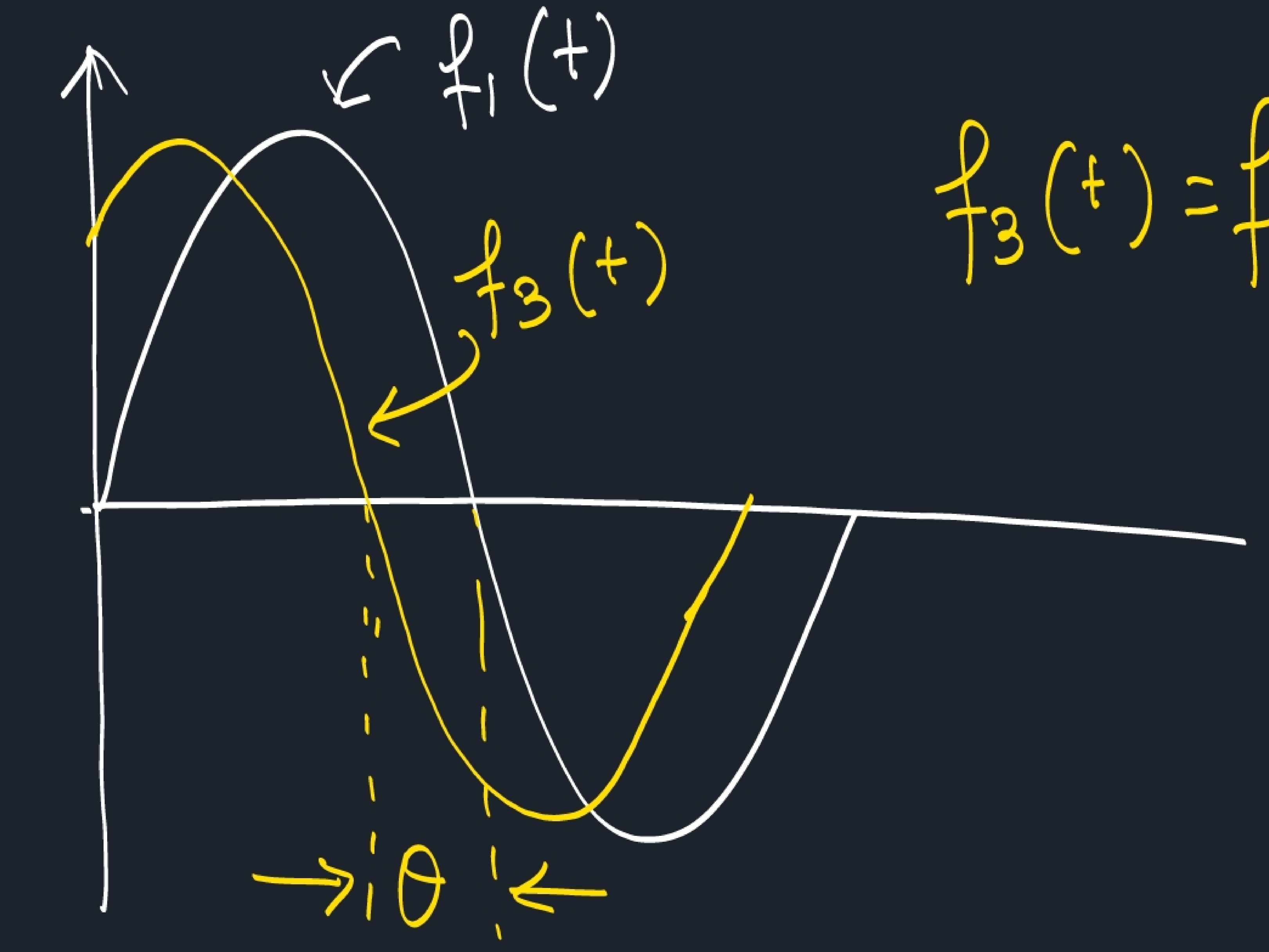
Concept of Lagging & Leadings :-



$$f_1(t) = f_m \sin(\omega t)$$

$$f_2(t) = f_m \sin(\omega t - \theta)$$

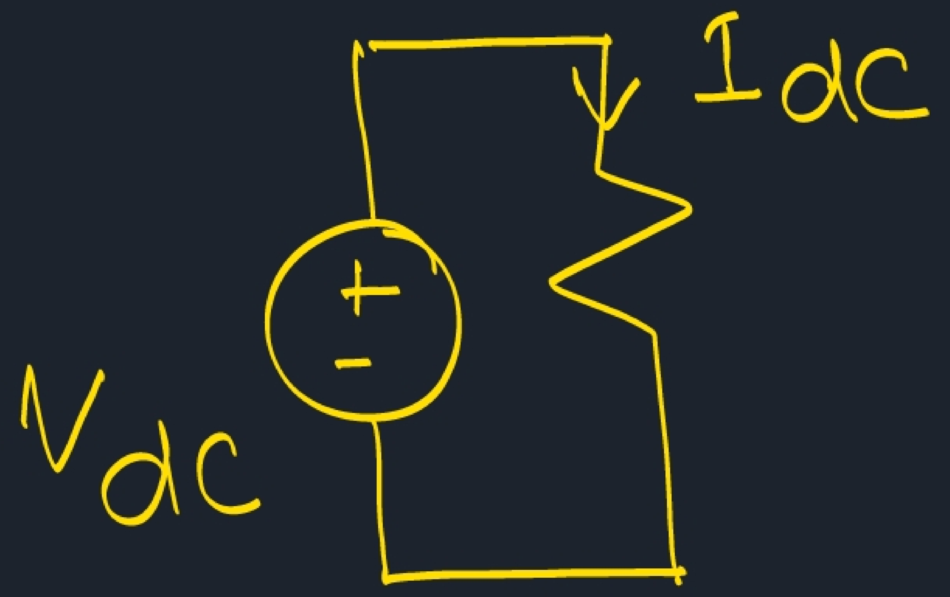
$f_2(t)$ is lagging behind $f_1(t)$ by an angle of θ.

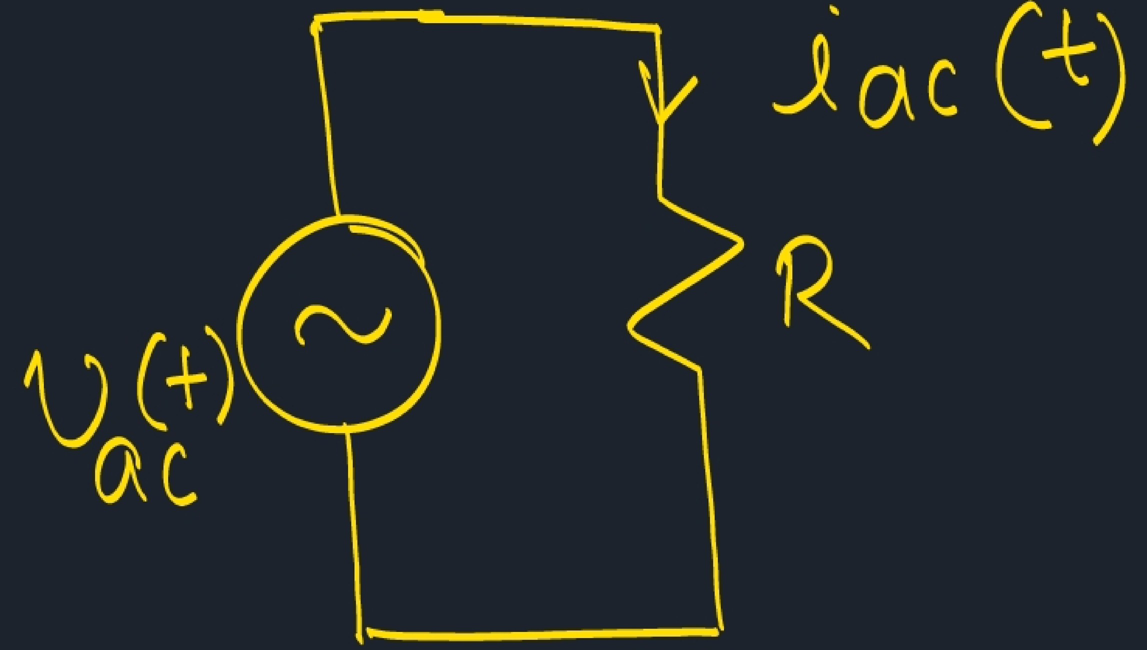


$$f_3(t) = f_m \sin(\omega t + \theta)$$

$f_3(t)$ is leading $f_1(t)$ by an angle of θ.

Rms value:-





$$E_{dc} = I_{dc}^2 \cdot R \cdot T$$

$$E_{AC} = \int_0^T i_{ac}^2 R \, dt$$

$$I_{dc}^2 R \cdot T = \int_0^T i_{ac}^2(t) R \, dt$$

$$\boxed{I_{dc} = \left(\frac{1}{T}\int_0^T i_{ac}^2(t)\, dt\right)^{1/2}} = \underline{I_{ac}^{rms.}}$$

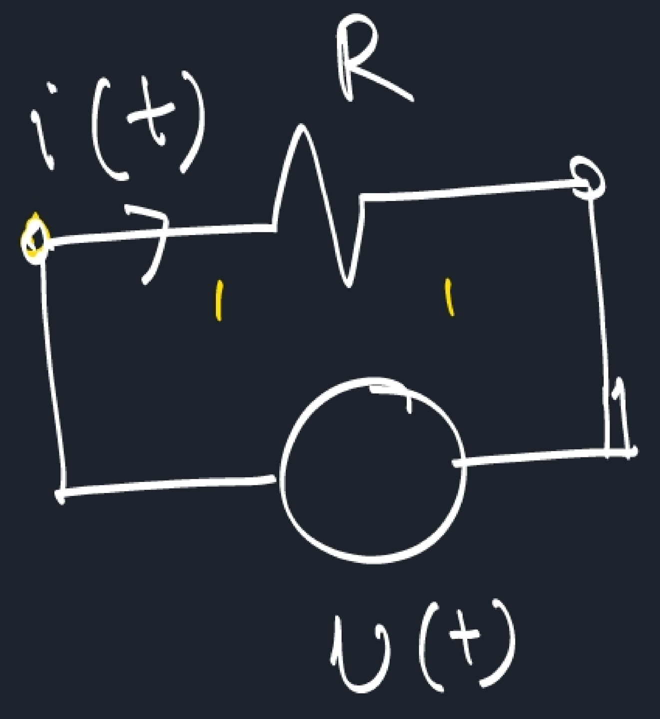
$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T \{i(t)\}^2 dt}$$

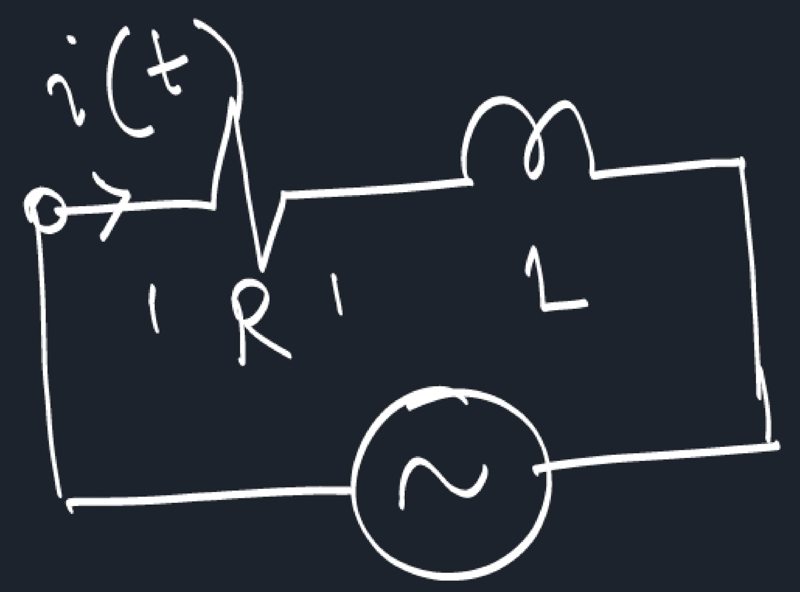
→ Effective value.

AVERAGE VALUE:-

$$I_{avg} = \frac{1}{T}\int_t^{t+T} i(t)\, dt$$

Sinusoidal Forced Response:-





$v(t) = V_m \cos \omega t$

$$L\frac{di}{dt} + Ri = v(t)$$
$$= V_m \cos \omega t$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V_m}{L} \cos \omega t$$

$$e^{R/L\, t} \cdot \frac{di}{dt} + \frac{R}{L} e^{R/L\, t} \cdot i = \frac{V_m}{L} \cos \omega t \cdot e^{R/L\, t}$$

$$\Rightarrow \frac{d}{dt}\left(e^{R/L\, t}\, i(t)\right) = \frac{V_m}{L} e^{R/L\, t} \cdot \cos \omega t$$

$$e^{R/L\, t}\, i(t) = A + \int_0^t \frac{V_m}{L} e^{R/L\, t} \cos \omega t \, dt$$

$$i(t) = A e^{-R/L\, t} + e^{-R/L\, t} \int \frac{V_m}{L} e^{R/L\, t} \cos \omega t \, dt$$

$$\int \frac{V_m}{L} e^{R/L\,t} e^{j\omega t} = \int \frac{V_m}{L} e^{(R/L + j\omega)t}$$

$$= \frac{V_m}{L} \cdot \frac{e^{R/L\,t} \cdot e^{j\omega t}}{\frac{R}{L} + j\omega}$$

$$= V_m \cdot \frac{e^{R/L\,t} \cdot e^{j\omega t}}{R + j\omega L}$$

$$a + jb = |z| \angle \theta$$

$$|z| = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$a + jb = |z| e^{j\theta}$$

$$\cos\omega t = \mathrm{Re}\{e^{j\omega t}\}$$

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$\frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\mathrm{Re}\left\{ V_m \cdot \frac{e^{j\omega t}}{R + j\omega L} \right\} = \frac{V_m \cdot e^{j\omega t}}{|z| e^{j\theta}}$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$= \frac{V_m}{|z|} e^{j(\omega t - \theta)}$$

$$\frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t - \theta)$$

$$i(t) = Ae^{-R/L\, t} + \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t - \theta)$$

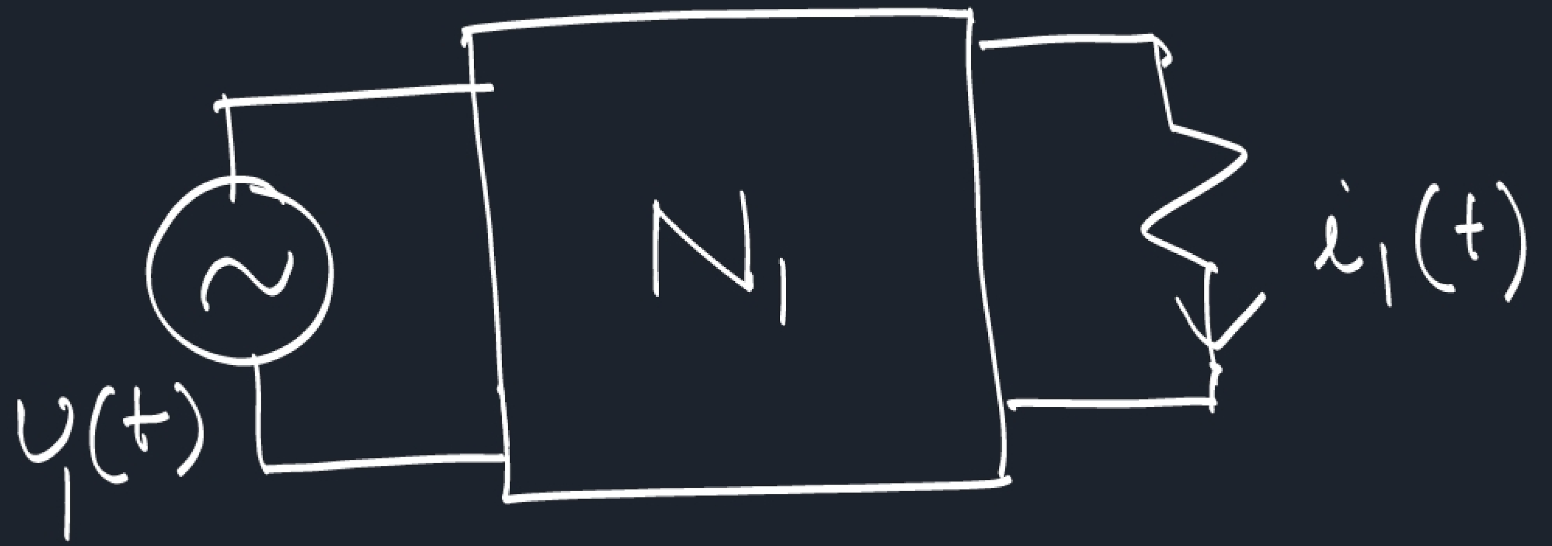
Initial Cond^n:

@ $t = 0$

$$0 = A + \frac{V_m}{\sqrt{R^2+(\omega L)^2}}\cos\theta$$

$$A = -\frac{V_m}{\sqrt{R^2+(\omega L)^2}}\cos\theta$$

$$i(t) = -\frac{V_m}{\sqrt{R^2+(\omega L)^2}}e^{-\frac{R}{L}t} + \underbrace{\frac{V_m}{\sqrt{R^2+(\omega L)^2}}\cos(\omega t - \theta)}_{\text{Steady State}}$$

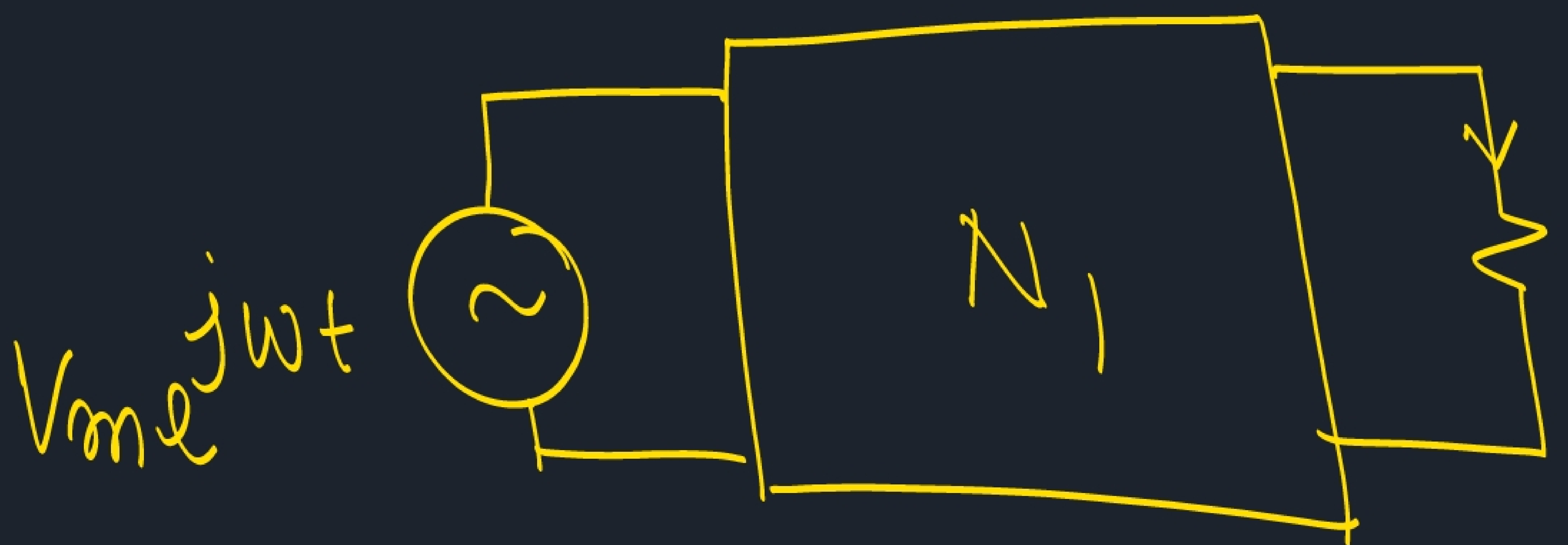


$v_2(t) = V_m \sin \omega(t)$

$i_2(t) = I_m \sin(\omega t \pm \theta)$

$v_1(t) = V_m \cos \omega t$

$i_1(t) = I_m \cos(\omega t \pm \theta)$



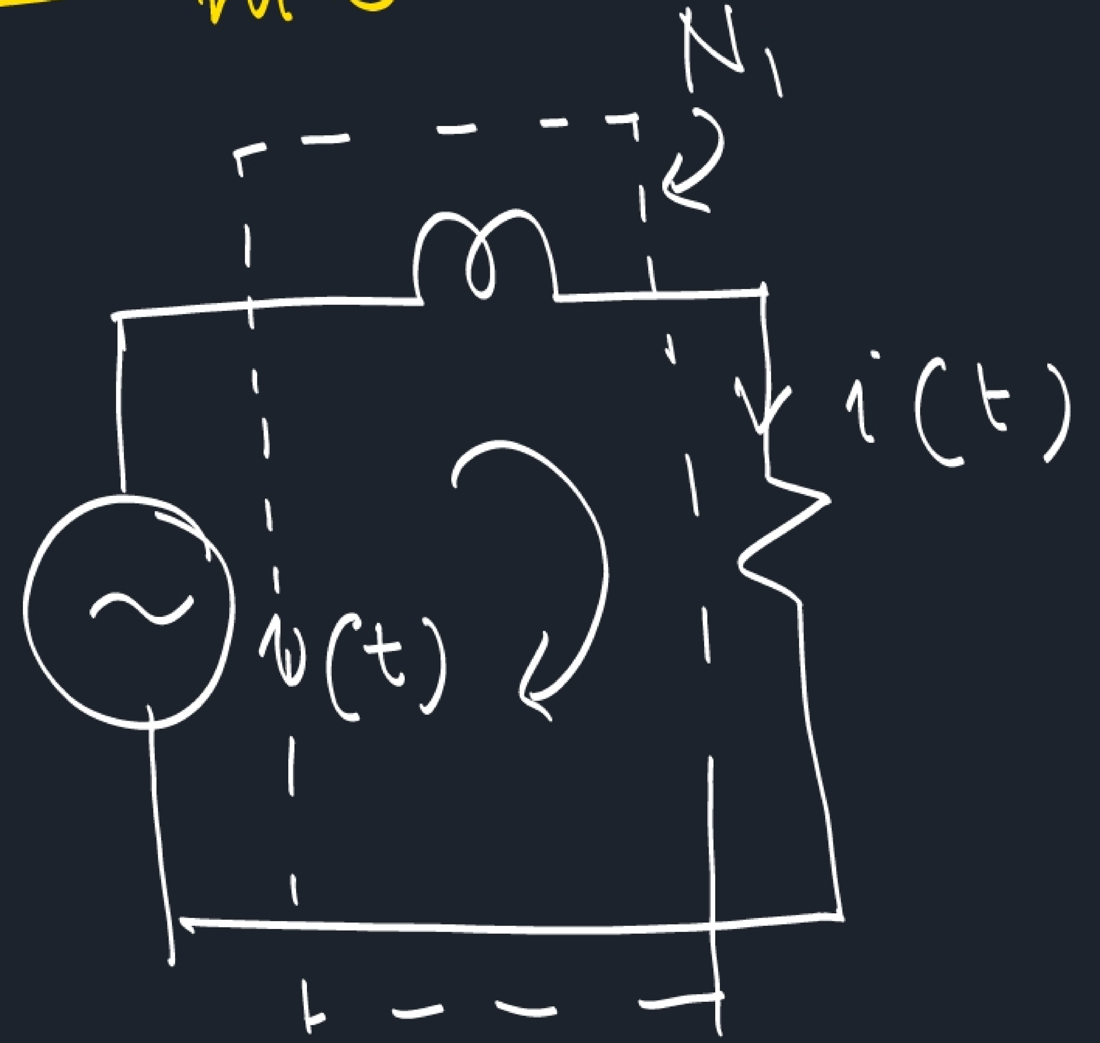
$i(t) = I_m e^{j(\omega t \pm \theta)}$

$= V_m \cos \omega t + j V_m \sin \omega t$

$= V_m e^{j\omega t}.$

$v(t) = V_m e^{j\omega t}$

$i(t) = I_m e^{j(\omega t+\theta)}$



$$v(t) = R\,i(t) + L\frac{di(t)}{dt}$$

$$\Rightarrow j\omega L\, I_m e^{j(\omega t+\theta)} + R I_m e^{j(\omega t+\theta)} = V_m e^{j\omega t}$$

$$\Rightarrow j\omega L\, I_m e^{j\theta} + R I_m e^{j\theta} = V_m$$

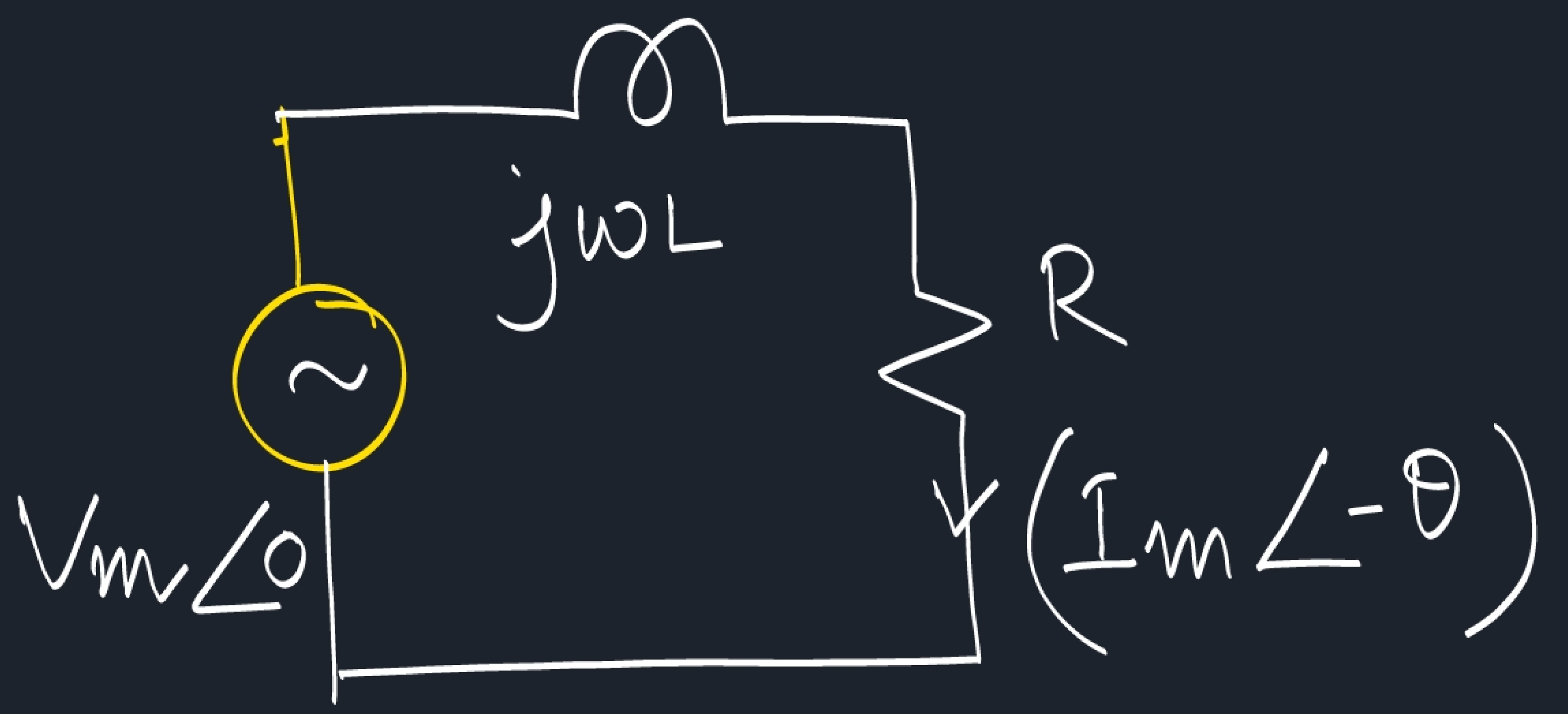
$$\Rightarrow I_m e^{j\theta}(R+j\omega L) = V_m$$

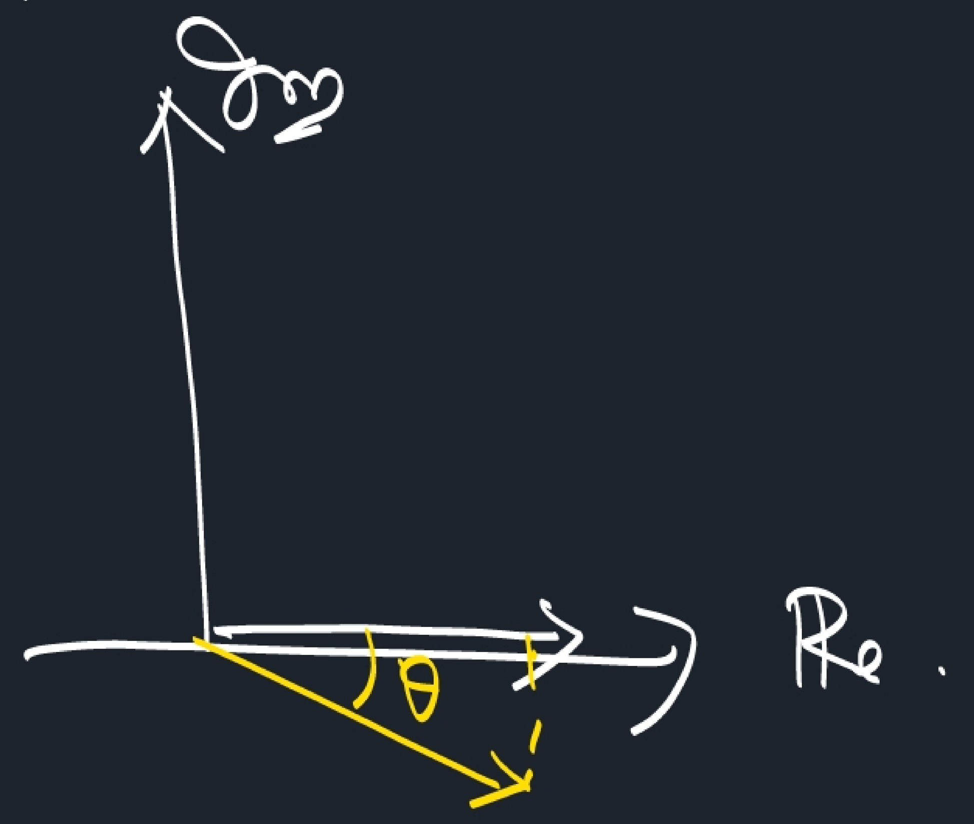
$$\boxed{I_m e^{j\theta} = \frac{V_m}{R+j\omega L}}$$

$$I_m = \left|I_m e^{j\theta}\right| = \left|\frac{V_m}{R+j\omega L}\right| = \frac{V_m}{\sqrt{R^2+(\omega L)^2}}$$

$$\theta = \angle I_m e^{j\theta} = \angle \frac{V_m}{R+j\omega L} = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

⇒ Phasor Representation.



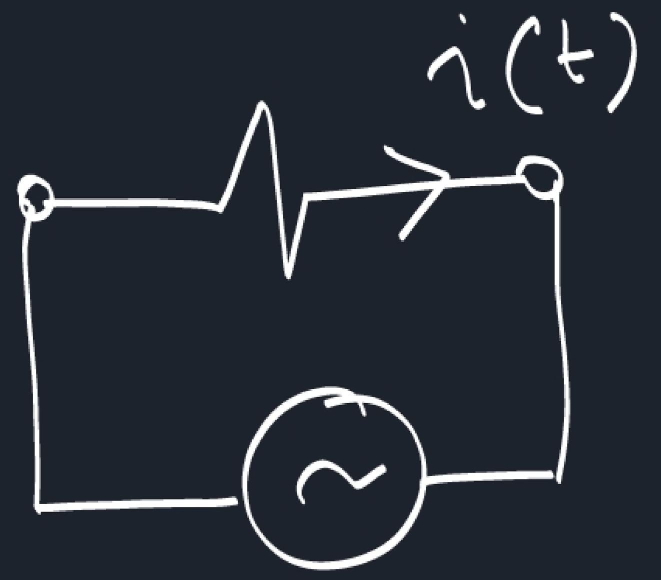


$$I_m \angle -\theta = \frac{V_m \angle 0}{R + j\omega L} \qquad \left(\theta = \tan^{-1}\frac{\omega L}{R}\right)$$

$\vec{V}$ $\quad$ $\tilde{V}$

$V_m \sin \omega t$

$I_m \sin(\omega t - \theta)$



$$v(t) = V_m \cos(\omega t + \phi)$$

$$\Downarrow$$

$$v(t) = R\, i(t)$$

$$i(t) = \frac{V_m}{R} \cos(\omega t + \phi)$$

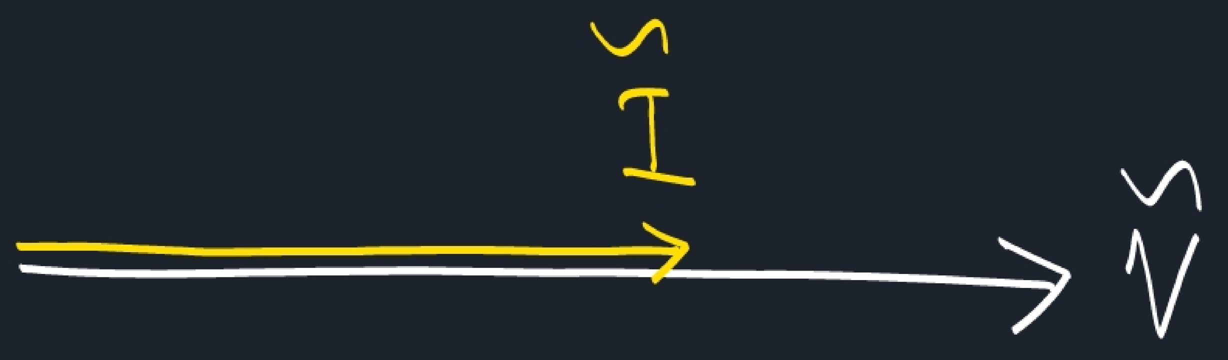
$$v(t) = V_m e^{j(\omega t + \phi)}$$

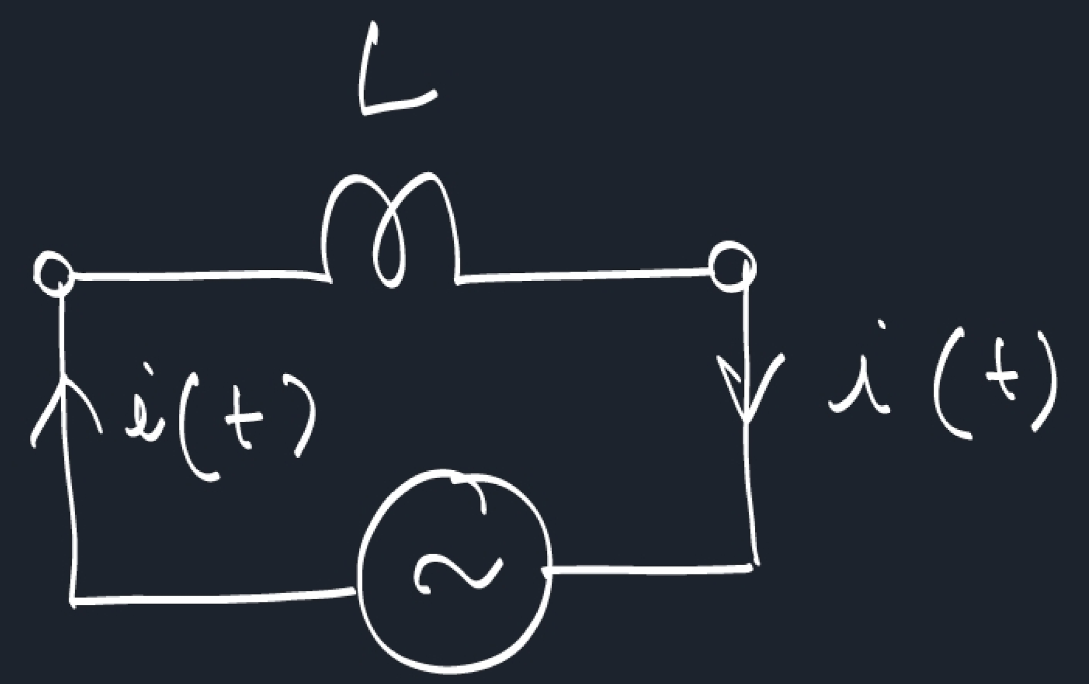
$$i(t) = I_m e^{j(\omega t + \phi)}$$

$$\hat{V} = V_m \angle +\phi = V_m e^{j\phi}$$

$$\hat{I} = I_m \angle +\phi = I_m e^{j\phi}$$

$(R > 1)$





$$v(t) = V_m \cos(\omega t + \phi)$$

$$i(t) = \frac{1}{L}\int v(t)\,dt$$

$$= \frac{1}{L}\int V_m \cos(\omega t + \phi)$$

$$= \frac{1}{\omega L} V_m \sin(\omega t + \phi)$$

$$v(t) = L\frac{di(t)}{dt}$$

$$\hat{V} = j\omega L \hat{I}$$

$\frac{d}{dt}$ is replaced by $(j\omega)$

$$v(t) = V_m e^{j(\omega t + \phi)}$$

$$i(t) = \frac{1}{L}\int V_m e^{j(\omega t + \phi)}\,dt$$

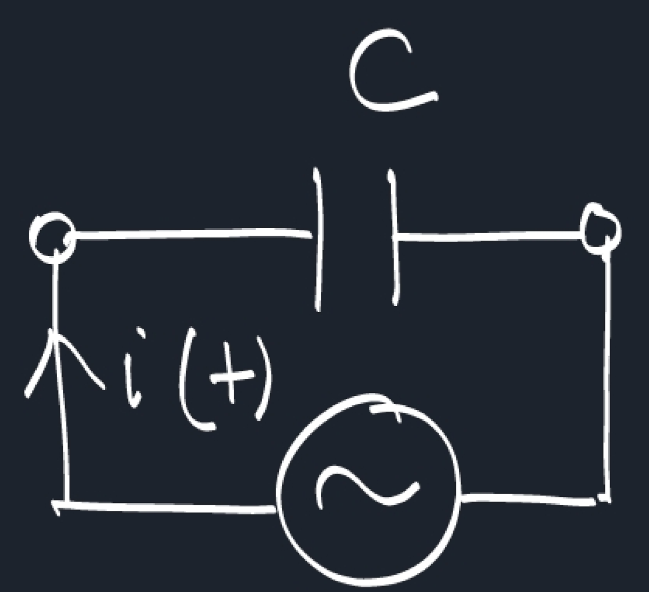
$$= \frac{1}{j\omega L} V_m e^{j(\omega t + \phi)}.$$

$$\hat{V} = V_m \angle +\phi = V_m e^{j\phi}$$

$$\hat{I} = \frac{V_m}{j\omega L} \cdot e^{j\phi} = \frac{V_m}{j X_L} e^{j\phi}$$

$X_L = \omega L =$ Inductive Resistance.

$$\hat{V} = (j\omega L)\hat{I}$$



$$v(t) = V_m \cos(\omega t + \phi) \Leftrightarrow V_m e^{j(\omega t + \phi)}$$

$$i(t) = C \frac{dv(t)}{dt}$$

$$= -C\omega \sin(\omega t + \phi)$$

$$i(t) = j\omega C V_m e^{j(\omega t + \phi)}$$

$$\hat{V} = \hat{I} \cdot \left(\frac{1}{j\omega C}\right)$$

$$= -j I (X_C)$$

$$\hat{V} = V_m \angle + \phi$$

$$\hat{I} = j\omega C V_m e^{j\phi}$$

$$= j\omega C \hat{V}$$

X_C = Capacitive Resistance.

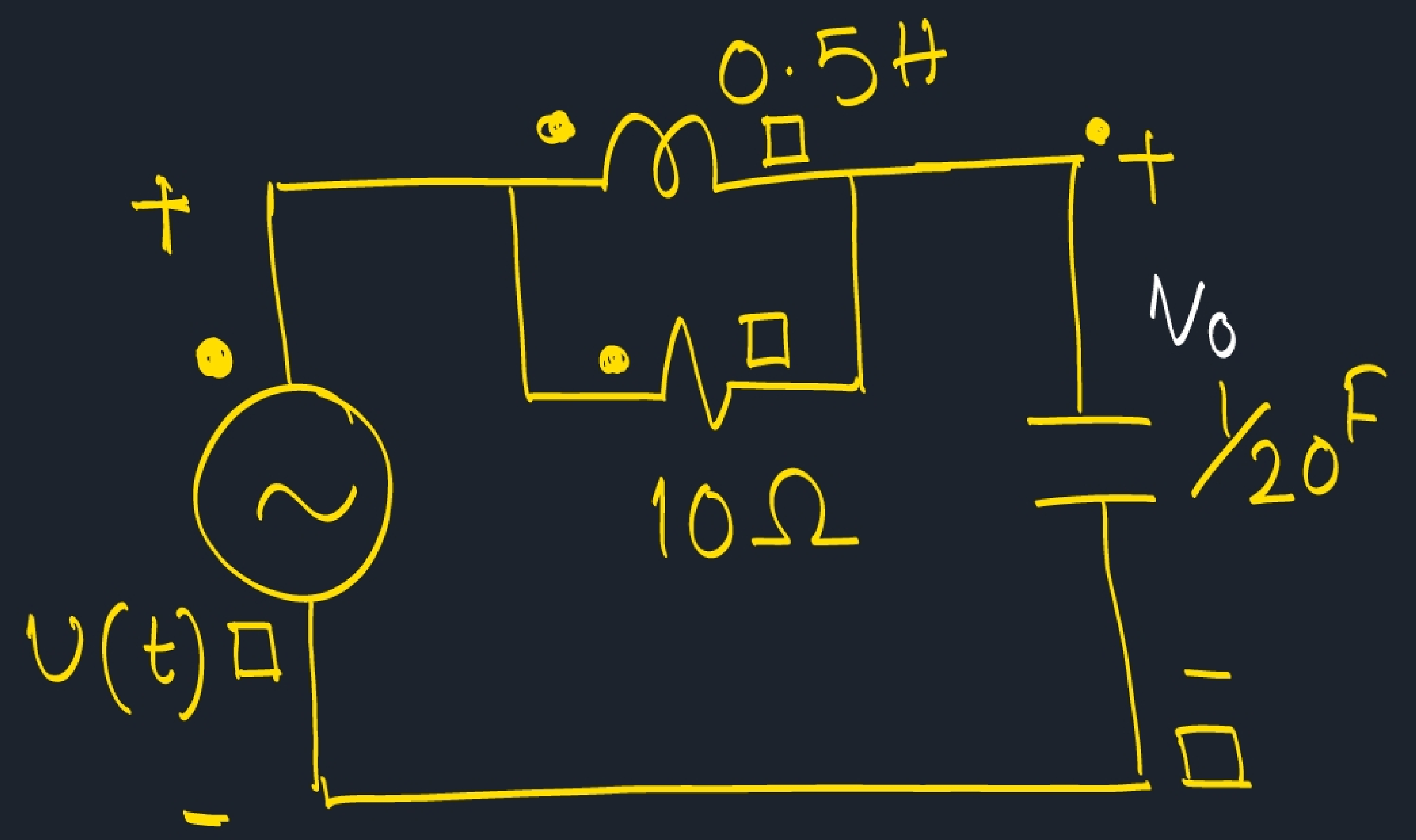
$$= \left(\frac{1}{\omega C}\right)$$

$$a_2 \frac{d^2 i(t)}{dt^2} + a_1 \frac{d i(t)}{dt} + b_1 \int i(t)\,dt + a_0 = C$$

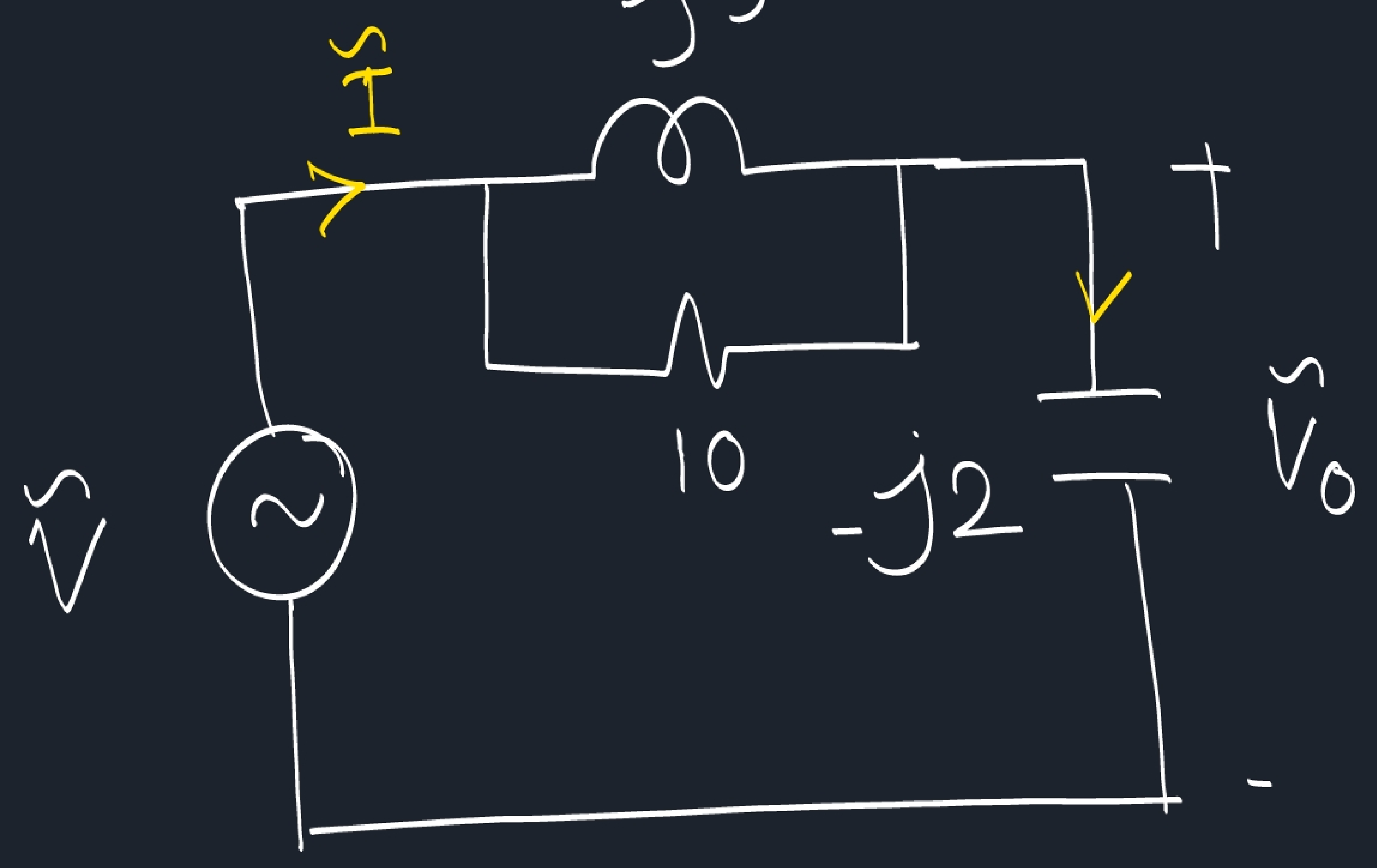
$$a_2 (jw)^2 \hat{I} + a_1 (jw) \hat{I} + \frac{b_1}{jw} \cdot \hat{I} + a_0 = \hat{C}$$

$$\hat{I} = \frac{\hat{C}}{a_2 \cdot (jw)^2 + a_1 \cdot (jw) + \frac{b_1}{jw} + a_0}$$

Prob:-



Phasor Rep. $\Longrightarrow$



$$\frac{1}{j\omega C} = -j\cdot\frac{1}{\omega C}$$

$$\frac{1\times 20}{10\times}$$

$$-j2$$

$$v(t) = 50\cos(10t + 30^\circ)\ \text{Volt}$$

$$\hat{V} = 50\angle 30^\circ\ V$$

$$\hat{V}_o = \frac{50\angle 30^\circ}{\underbrace{(10 \| j5) + (-j2)}_{\hat{I}}} \times (-j2)$$

$$= 35.35\angle -105^\circ\ V$$

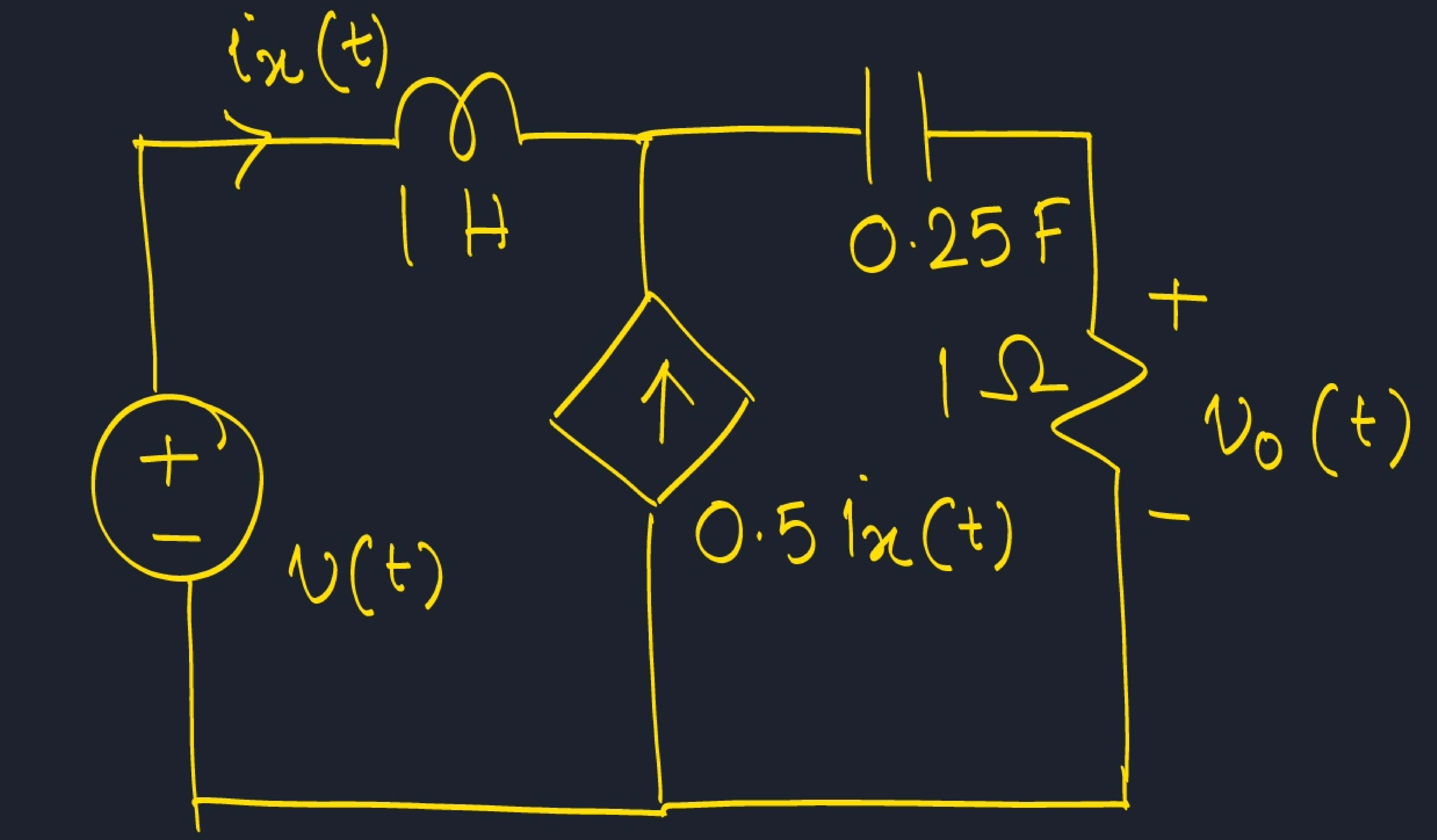
$$v_o(t) = ?$$

$$\frac{10\times 5i}{10+5i} - 2i$$

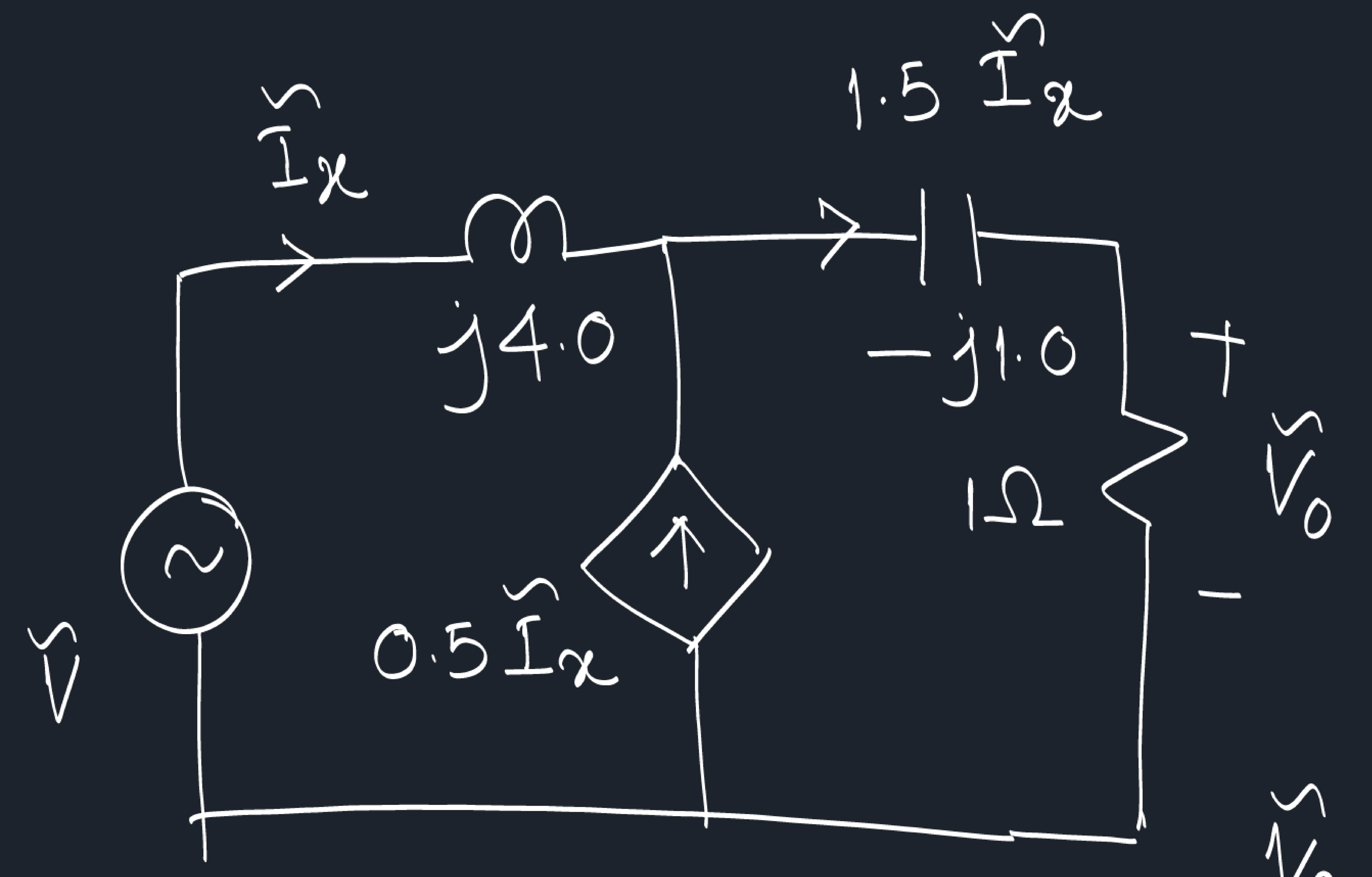
$$\frac{10\times j5}{10+j5} + (-j2)$$

$$v_o(t) = 35.35\cos(10t - 105^\circ)\ \text{Volt}$$

Prob:-



Phasor Repres. $\Longrightarrow$



$$-j\frac{1}{\omega C} = -j\frac{1}{4\times 0.25}$$

$$\hat{V}_o = (1.5\hat{I}_x)\times 1$$

$\underline{\hat{I}_x = ?}$

$$\hat{V} = (16\angle -10^\circ)\ \text{Volt}$$

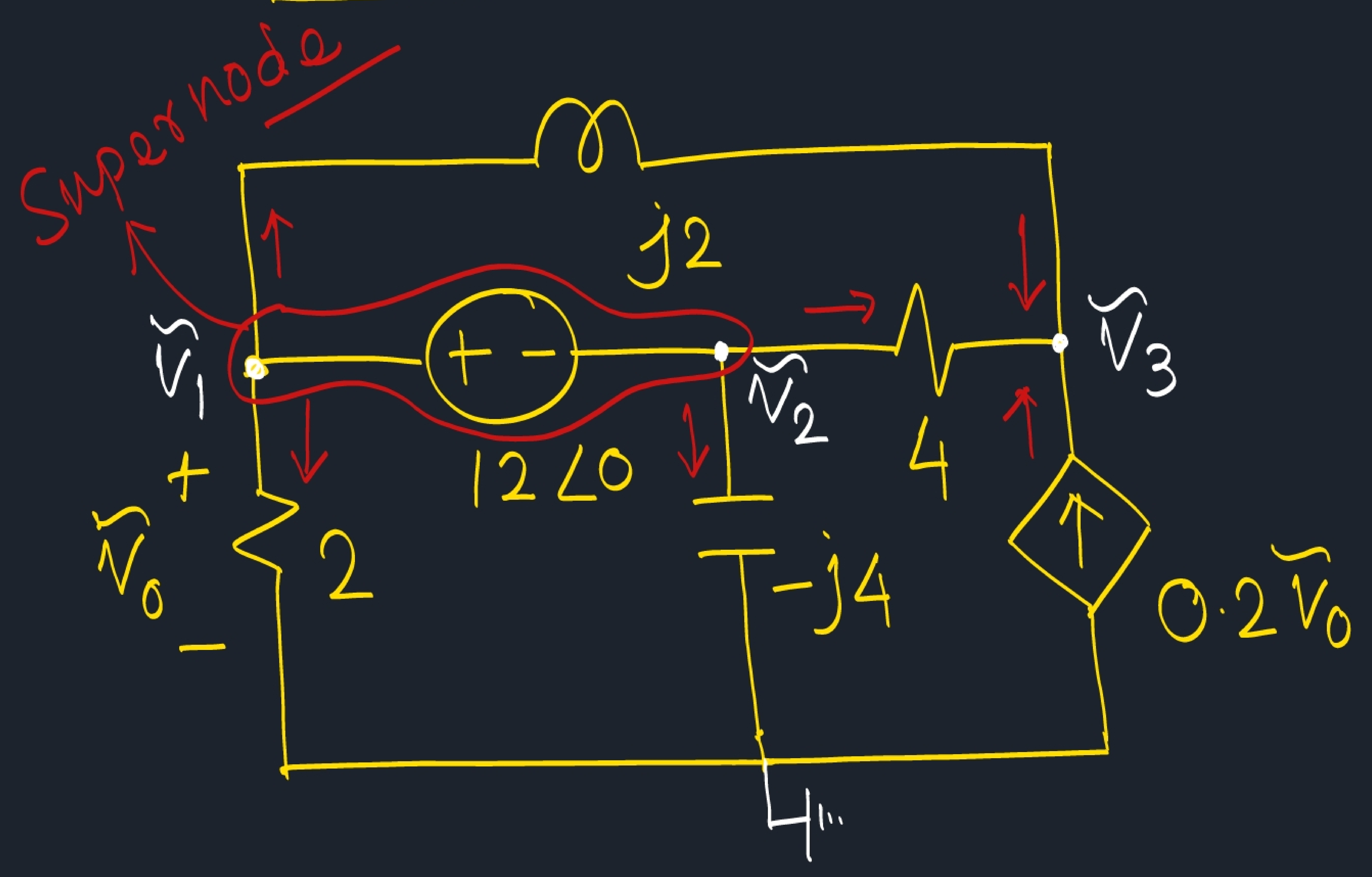
$$v(t) = 16\sin(4t-10)\ \text{Volt}$$

$$v_o(t) = ? = 8.2318\sin(4t-69.0362^\circ)\ \underline{\text{Amp.}}$$

$$\hat{I}_x = 5.4879\angle -69.0362\ A$$

$$\hat{V}_o = 8.2318\angle -69.0362\ A$$

Nodal Analysis:-



$$\tilde{V}_1 - \tilde{V}_2 = 12\angle 0 \quad \text{--- (i)}$$

$$\tilde{V}_0 = ? \quad 7.682\angle 50.19\ \text{V}$$

$$\frac{\tilde{V}_1}{2} + \frac{\tilde{V}_1 - \tilde{V}_3}{j2} + \frac{\tilde{V}_2}{-j4} + \frac{\tilde{V}_2 - \tilde{V}_3}{4} = 0$$

$$\Rightarrow \tilde{V}_1\left(\frac{1}{2} + \frac{1}{j2}\right) + \tilde{V}_2\left(\frac{1}{-j4} + \frac{1}{4}\right) + \tilde{V}_3\left(-\frac{1}{j2} - \frac{1}{4}\right) = 0$$

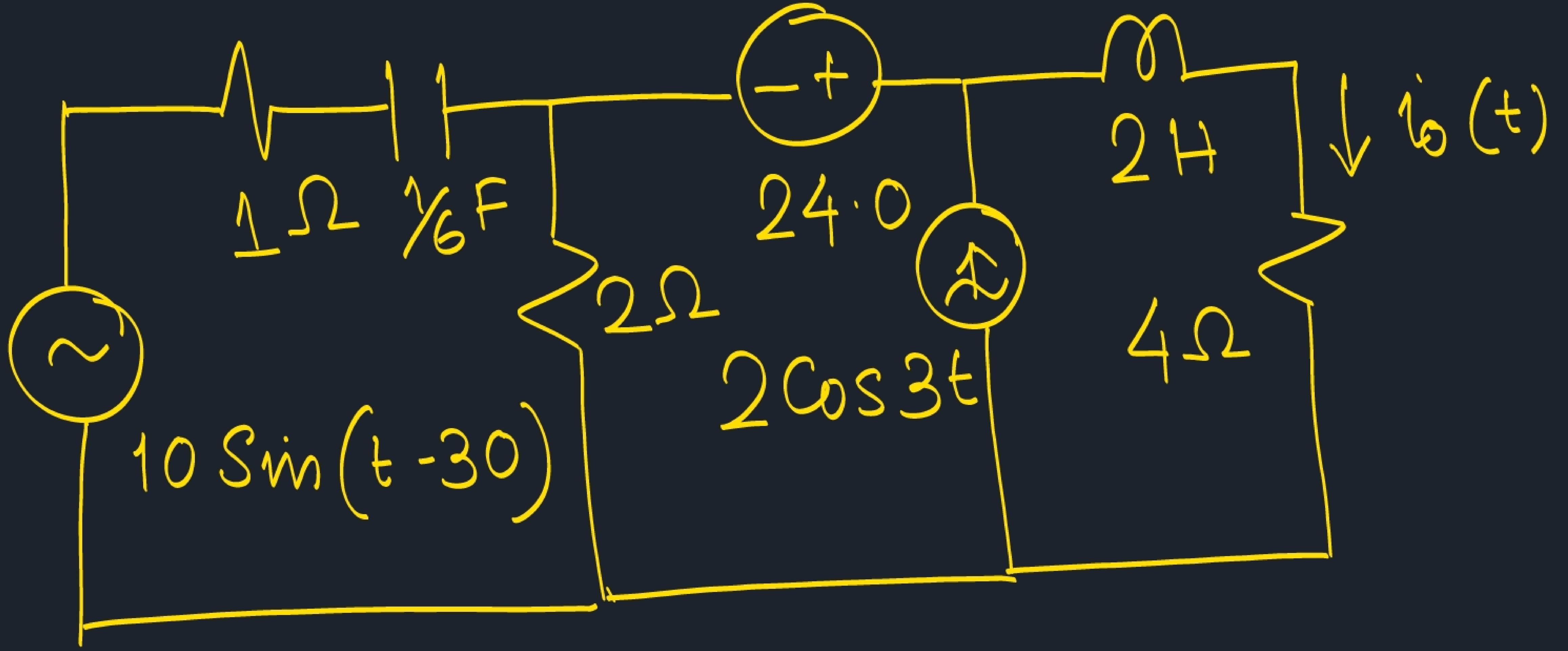
$$\text{----- (ii)}$$

$$\frac{\tilde{V}_2 - \tilde{V}_3}{4} + \frac{\tilde{V}_1 - \tilde{V}_3}{j2} + 0.2\tilde{V}_1 = 0 \qquad \tilde{V}_0 = \tilde{V}_1$$

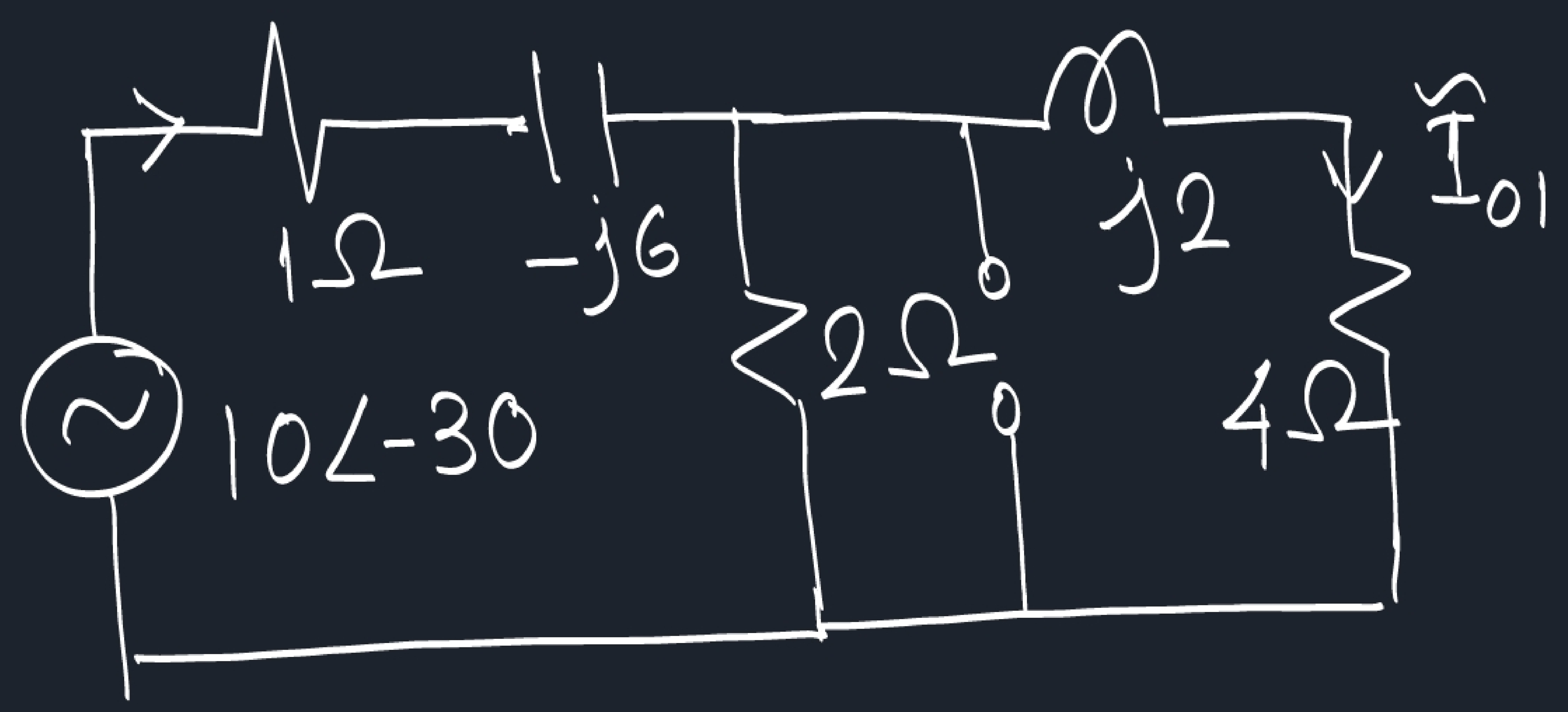
$$\Rightarrow \tilde{V}_1\left(0.2 + \frac{1}{j2.0}\right) + \tilde{V}_2\left(\frac{1}{4}\right) + \tilde{V}_3\left(-\frac{1}{4} - \frac{1}{j2}\right) = 0$$

$$\text{---- (iii)}$$

Problem:- $i_o(t) = ?$

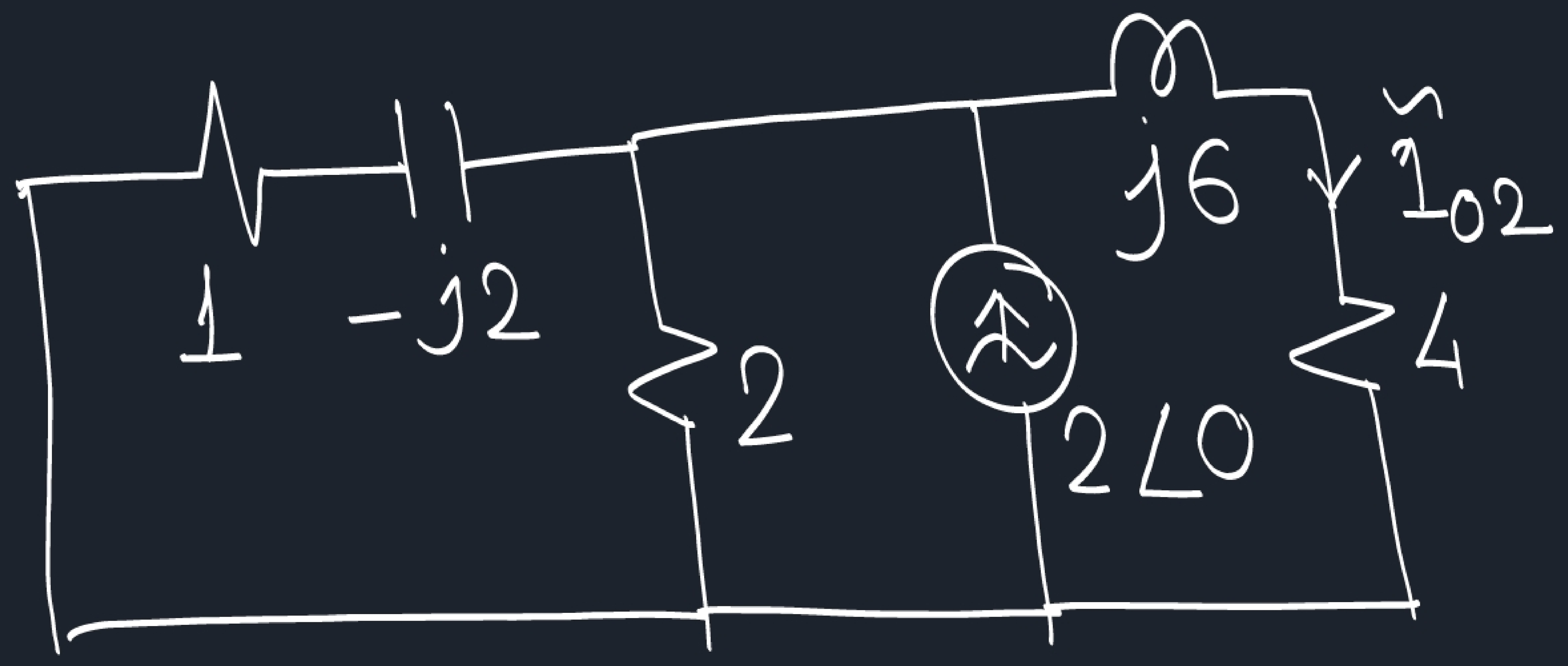


⇒

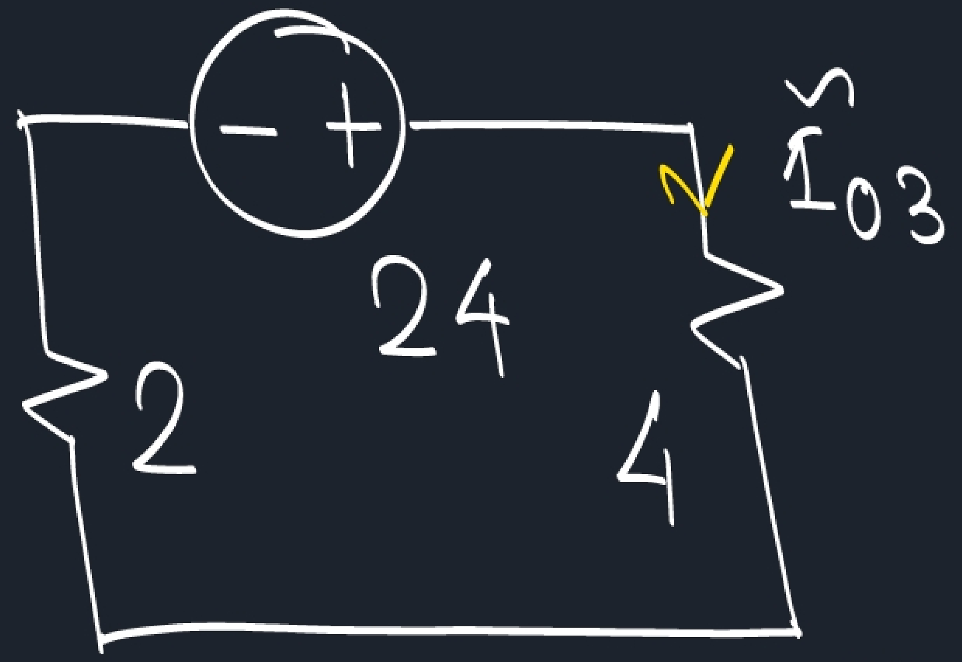


$$\hat{I}_{01} = \frac{10\angle -30}{(1-j6) + (4+j2)\,\|\,(2)} \times \frac{2}{2+4+j2} \text{ Amps.}$$

$$= 0.5038\angle 19.0856^\circ \text{ Amp}$$



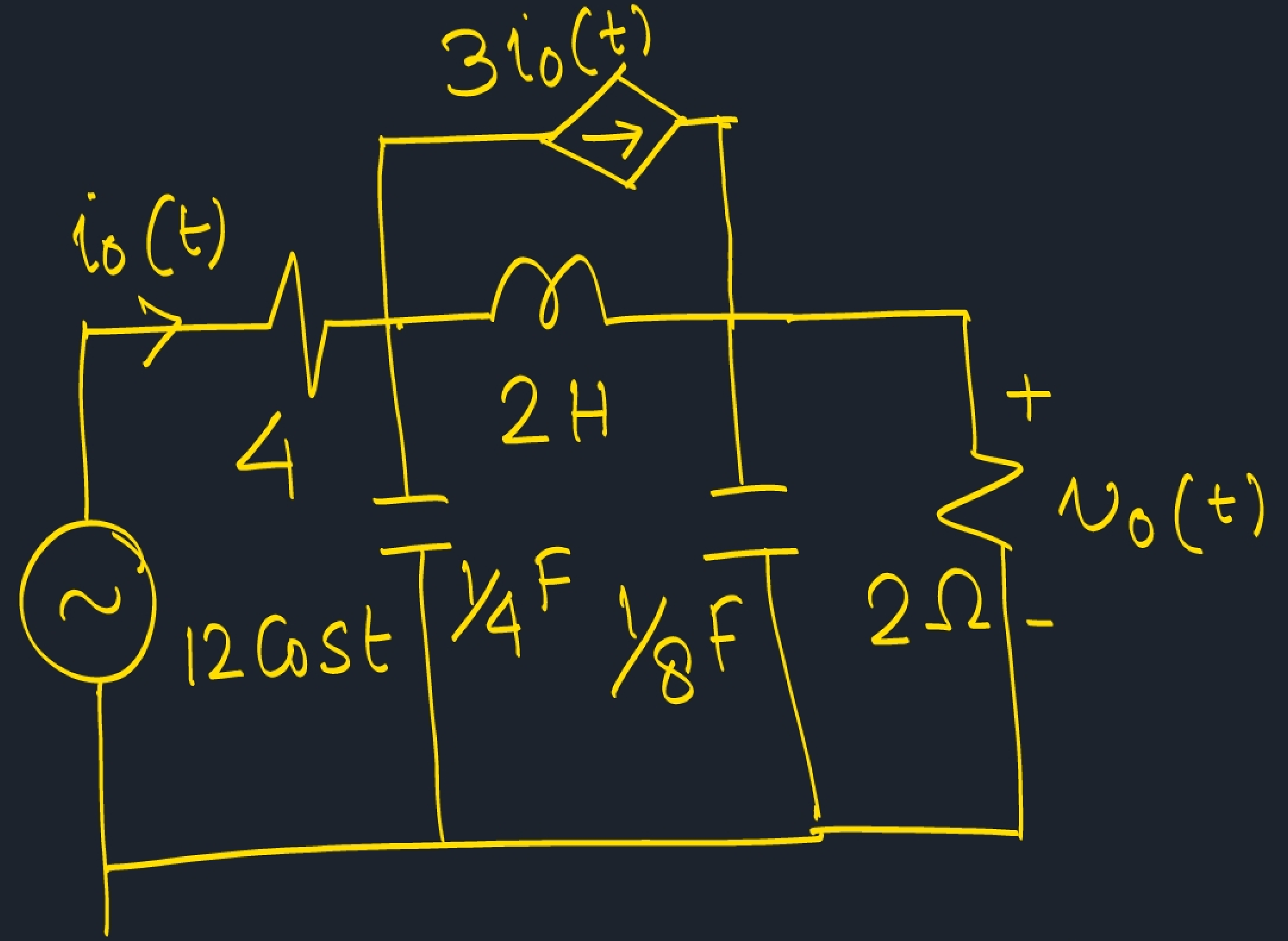
$$\hat{I}_{02} = 2\angle 0 \times \frac{2\,\|\,(1-j2)}{(2\,\|\,(1-j2)) + (4+j6)} = 0.33518\angle -76.4298^\circ \text{ A}$$



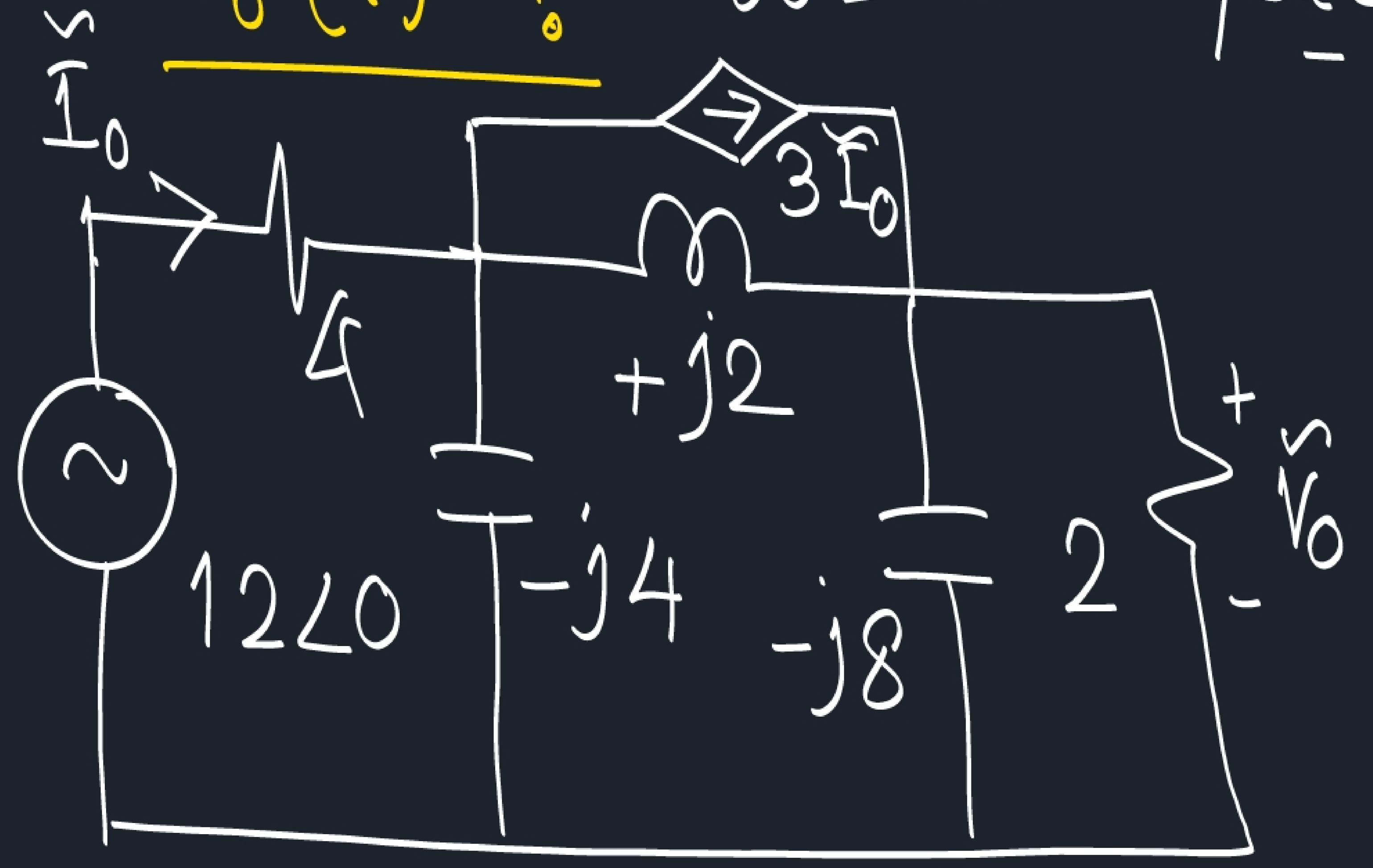
$\hat{I}_{03} = 4\angle 0$

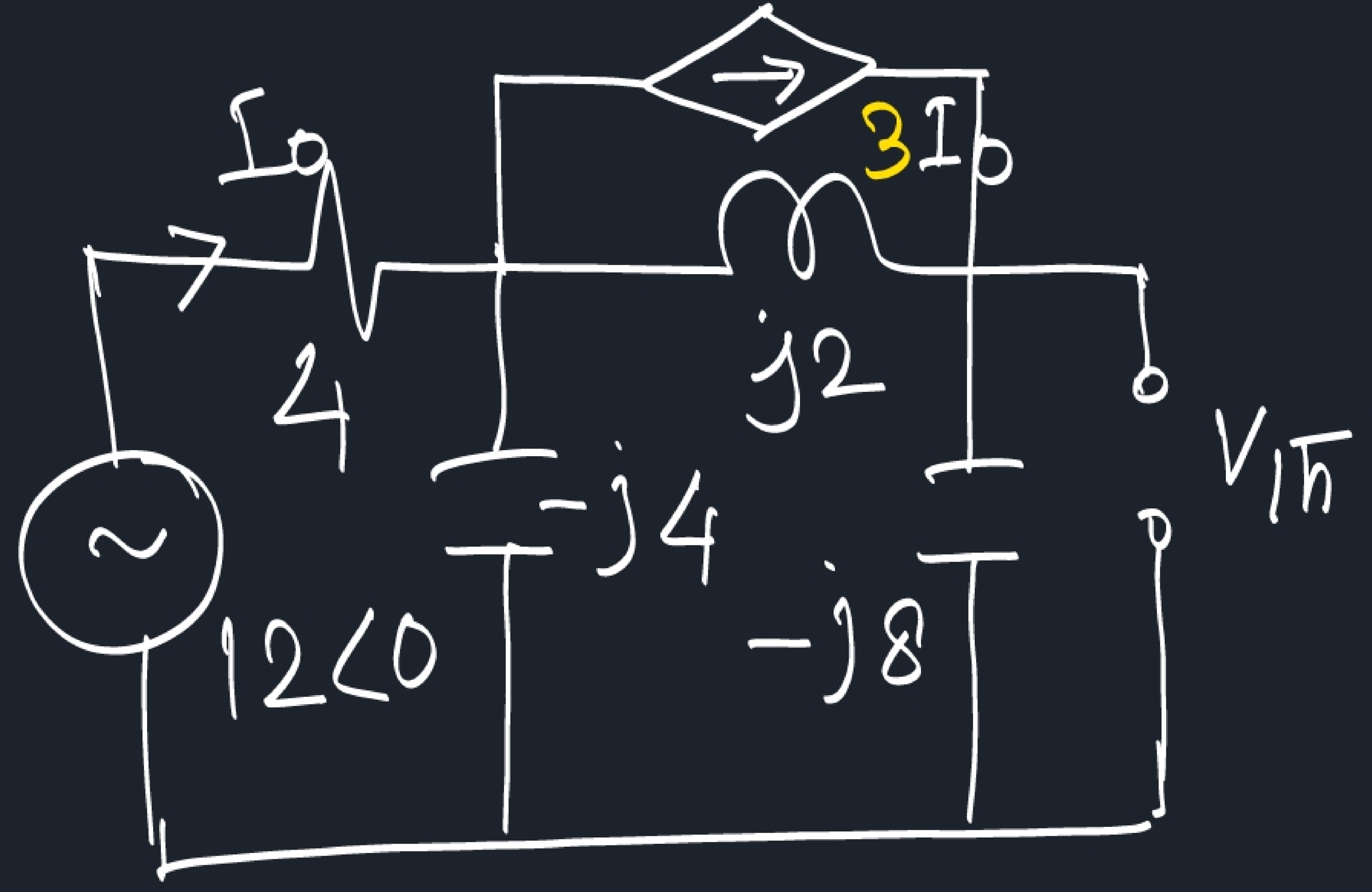
$$i_o(t) = 0.5038\,\mathrm{Sin}(t + 19.0856^\circ) + 0.33518\,\mathrm{Cos}(3t - 76.4298^\circ) + 4$$

Thevenin's Theorem:-

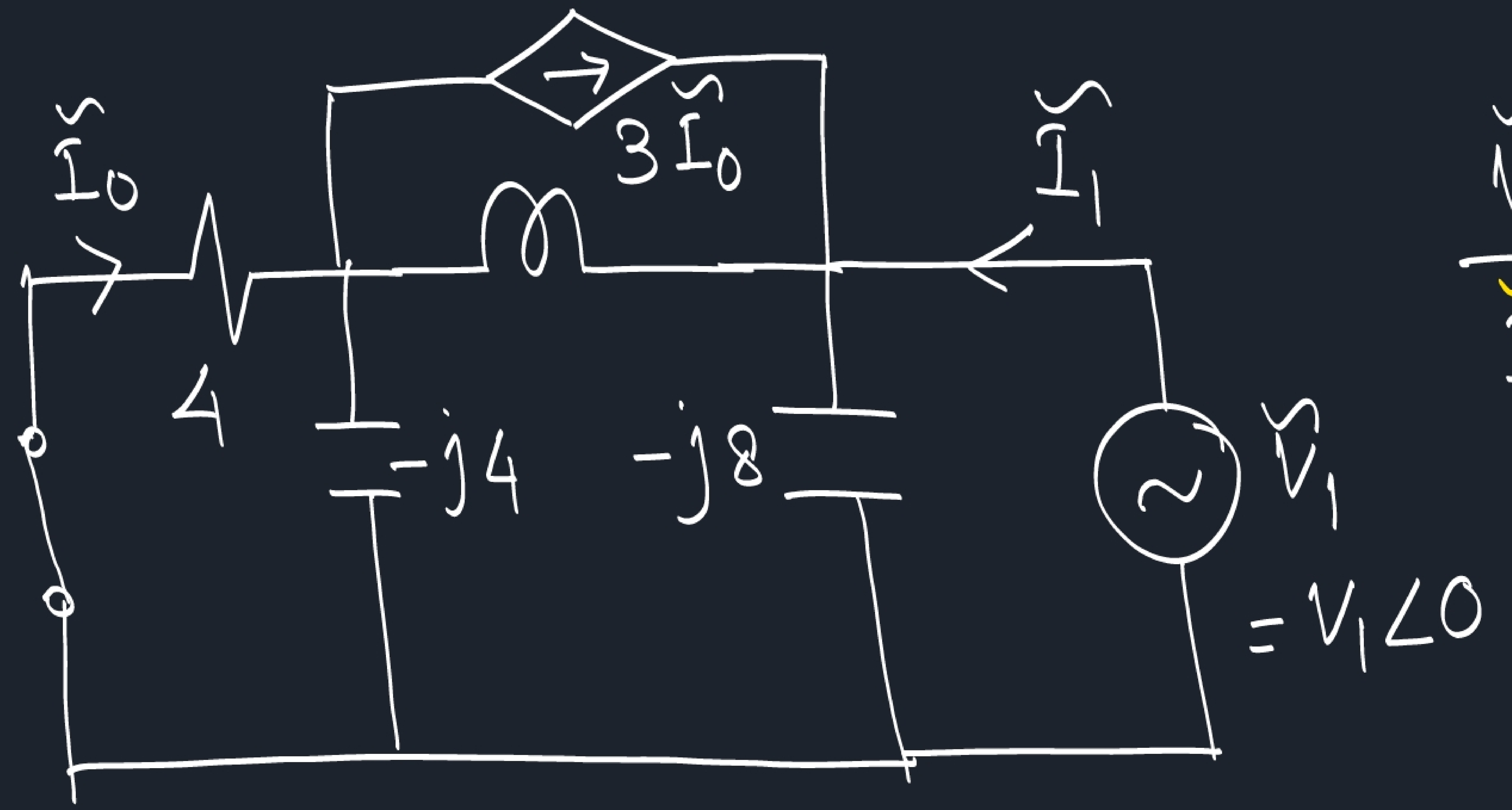


$i_o(t) = ?$ $\quad \omega = 1\,rad/sec.$

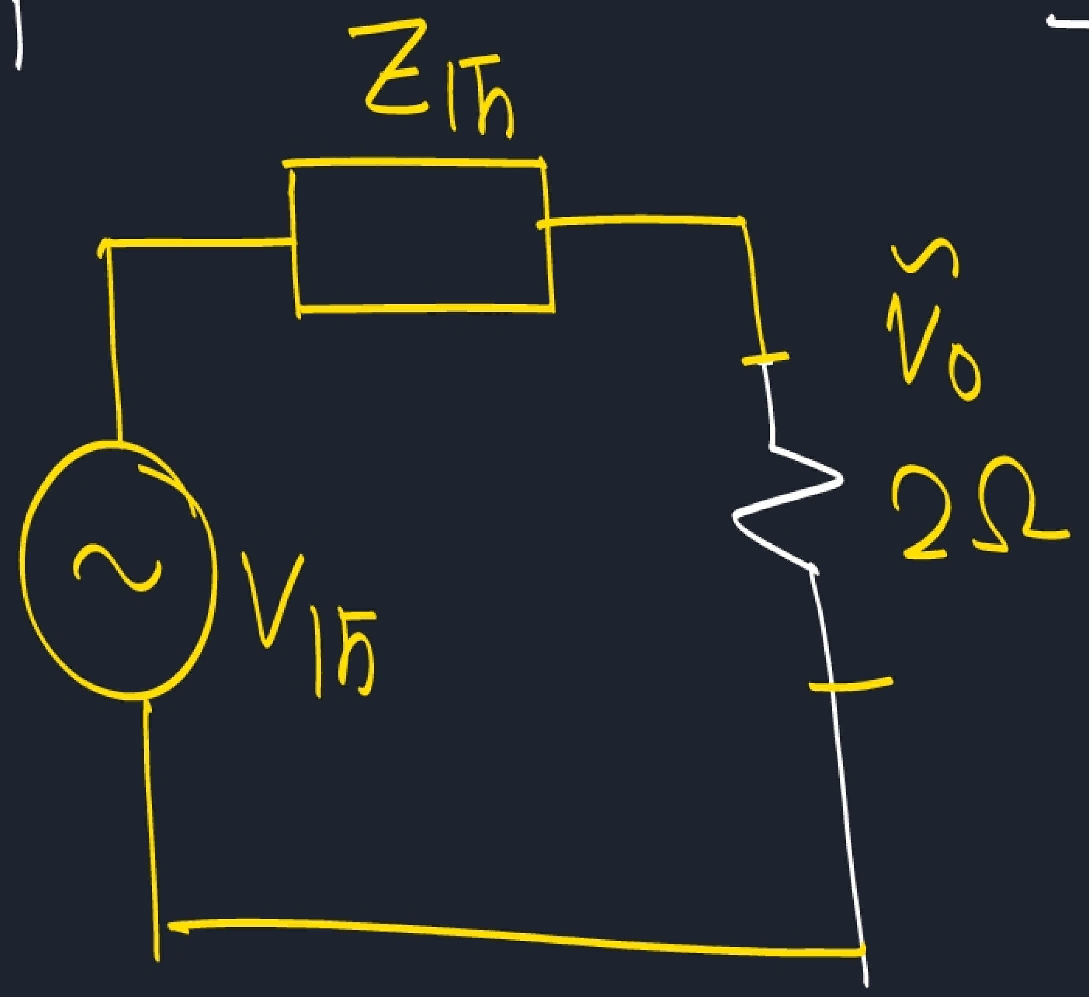




$$\hat{V}_{Th} = 3.0727\angle 140.2165 \text{ Volt.}$$

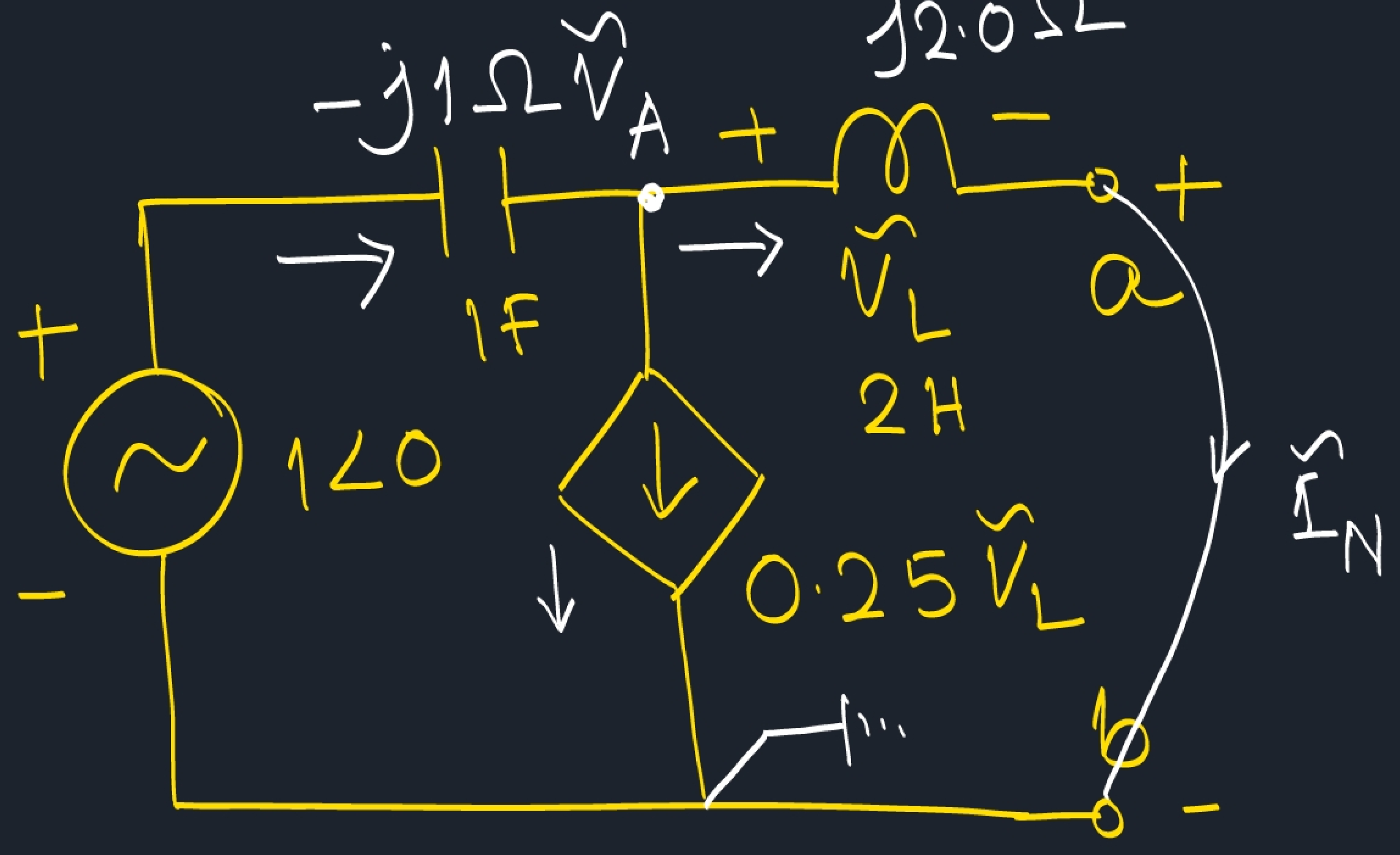


$$\frac{\hat{V}_1}{\hat{I}_1} = 2.2904\angle -103.2405\ \Omega$$



$$\underbrace{v_o(t) = 2.2986\cos\left(t - 163.2786^\circ\right) V}$$

Problem:-



$\omega = 1$ rad/sec.

Norton's Equivalent.

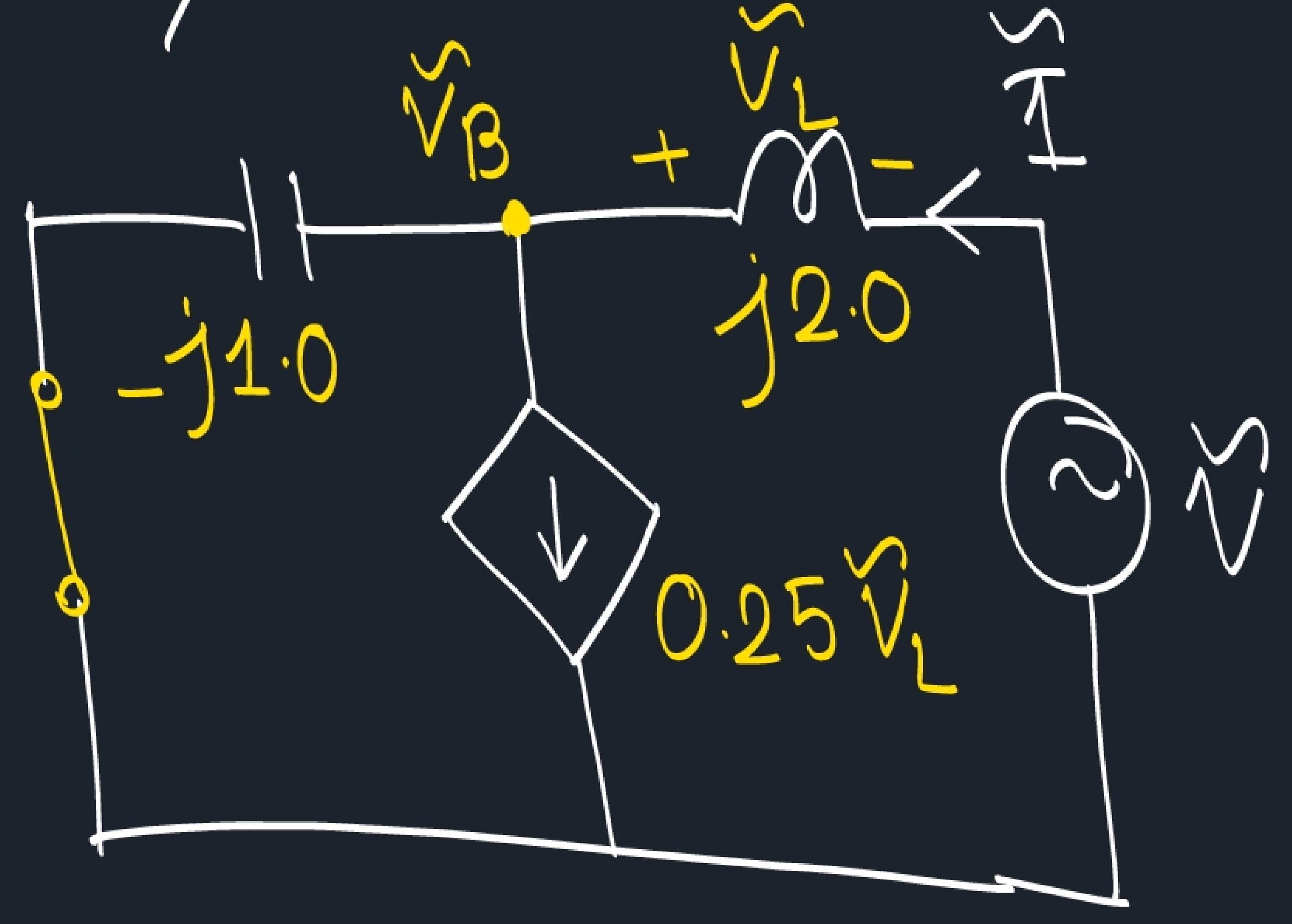
$\hat{V}_A = \hat{V}_L$

$$\frac{1\angle 0 - \hat{V}_A}{-j1.0} = 0.25\hat{V}_A + \frac{\hat{V}_A - 0}{j2.0}$$

$\hat{I}_N = 0.8944\angle -63.435°$ A

$$\Rightarrow V_A\left(\frac{1}{-j1.0} + 0.25 + \frac{1}{j2.0}\right) = \frac{1.0\angle 0}{-j1.0}$$

$$\Rightarrow \hat{V}_A = \hat{V}_L = 1.7888\angle 26.565° \text{ Volt.}$$



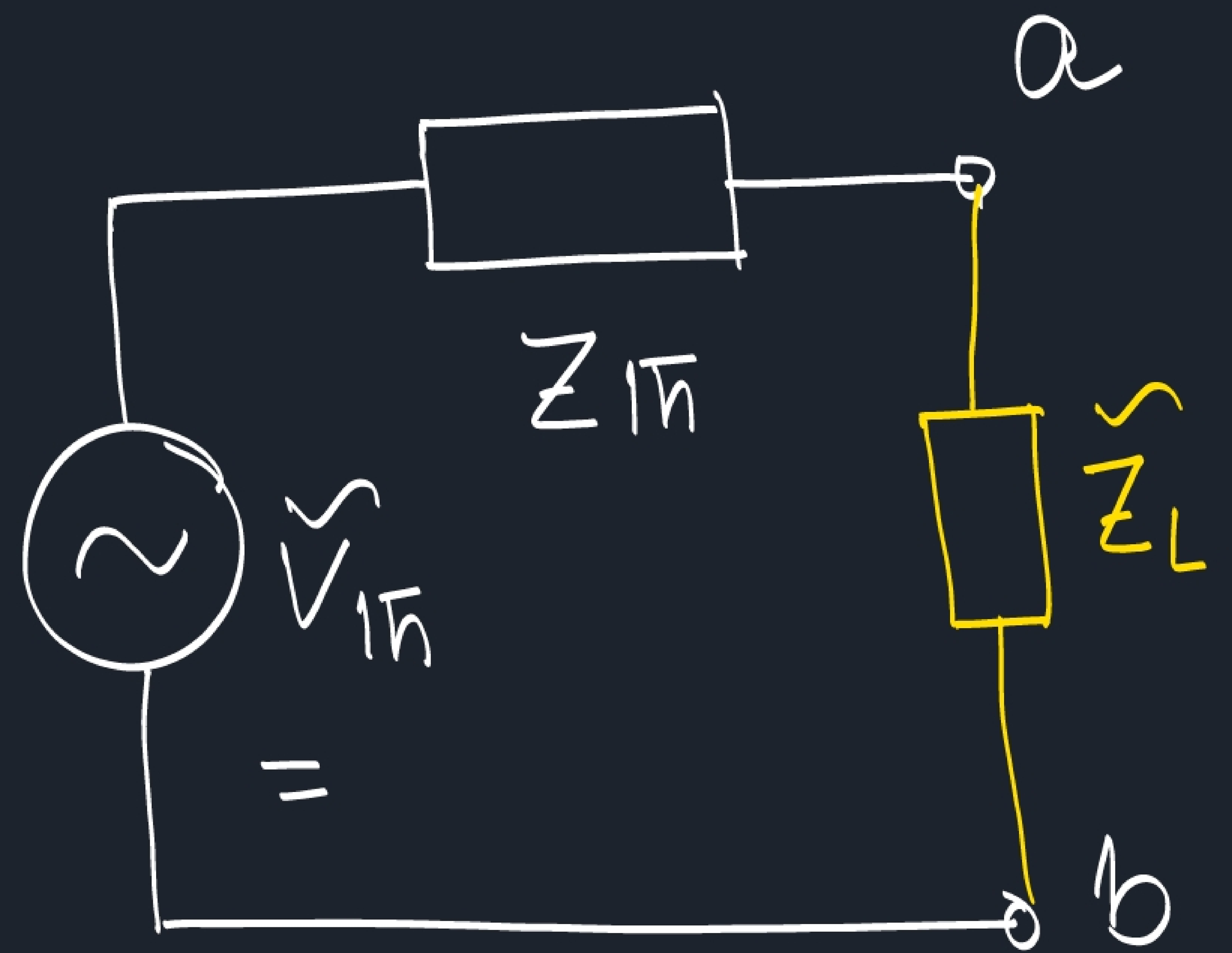
$\hat{V}_L = \hat{V}_B - \hat{V}$

$\hat{V}_B = \hat{V} - \hat{I} \times (j2.0)$

$\hat{I} = 0.25(V_B - V) + \frac{V_B}{-j1}$

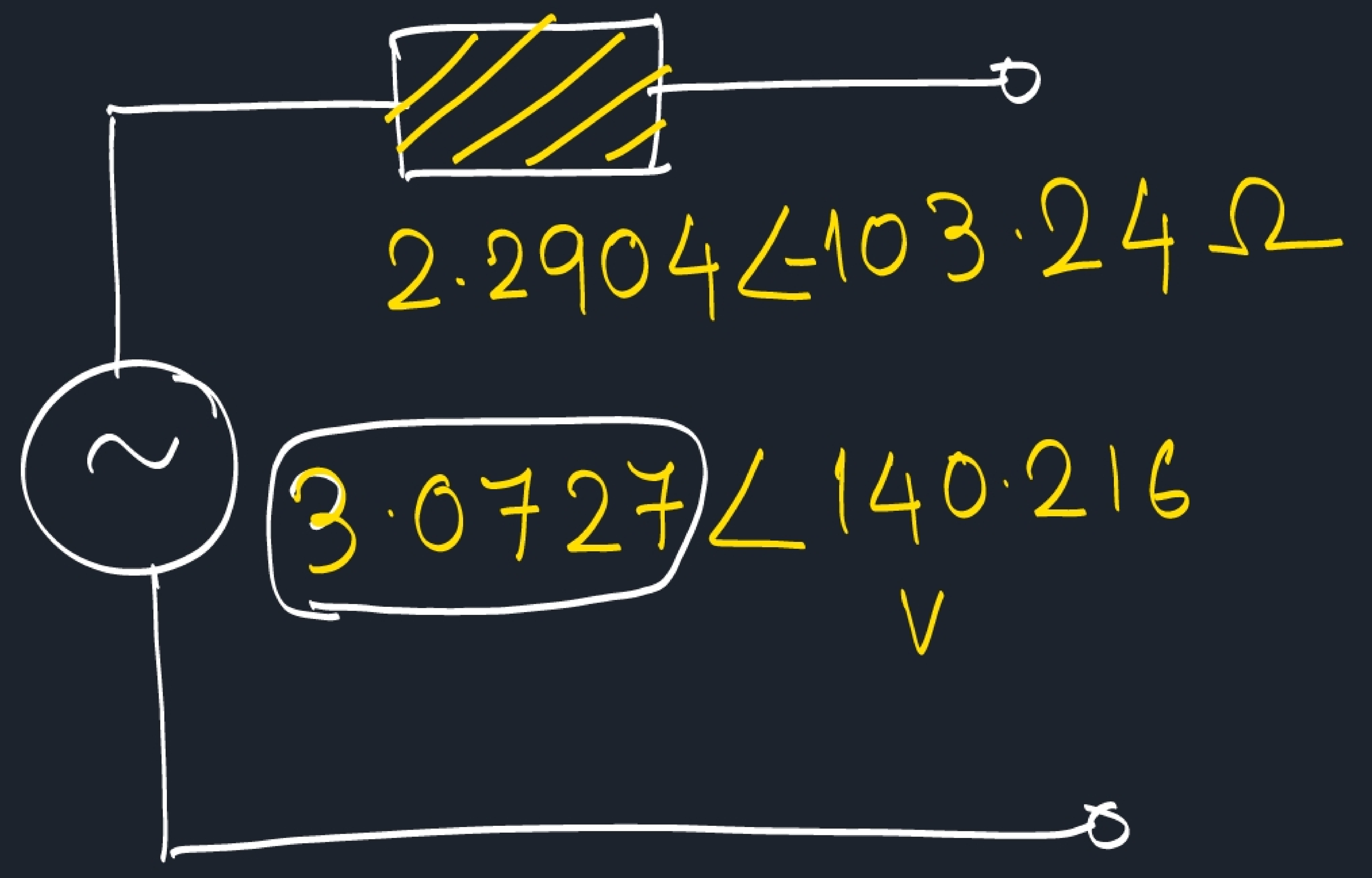
$Z_N = 1.1180\angle 63.4349°\ \Omega$

Max. Power Transfer:-



$Z_{Th} = R_{Th} \pm jX_{Th} = C\angle \pm d^\circ$

$\hat{Z}_L = \check{Z}_{Th}^* = R_{Th} \mp jX_{Th} = C\angle \mp d^\circ$



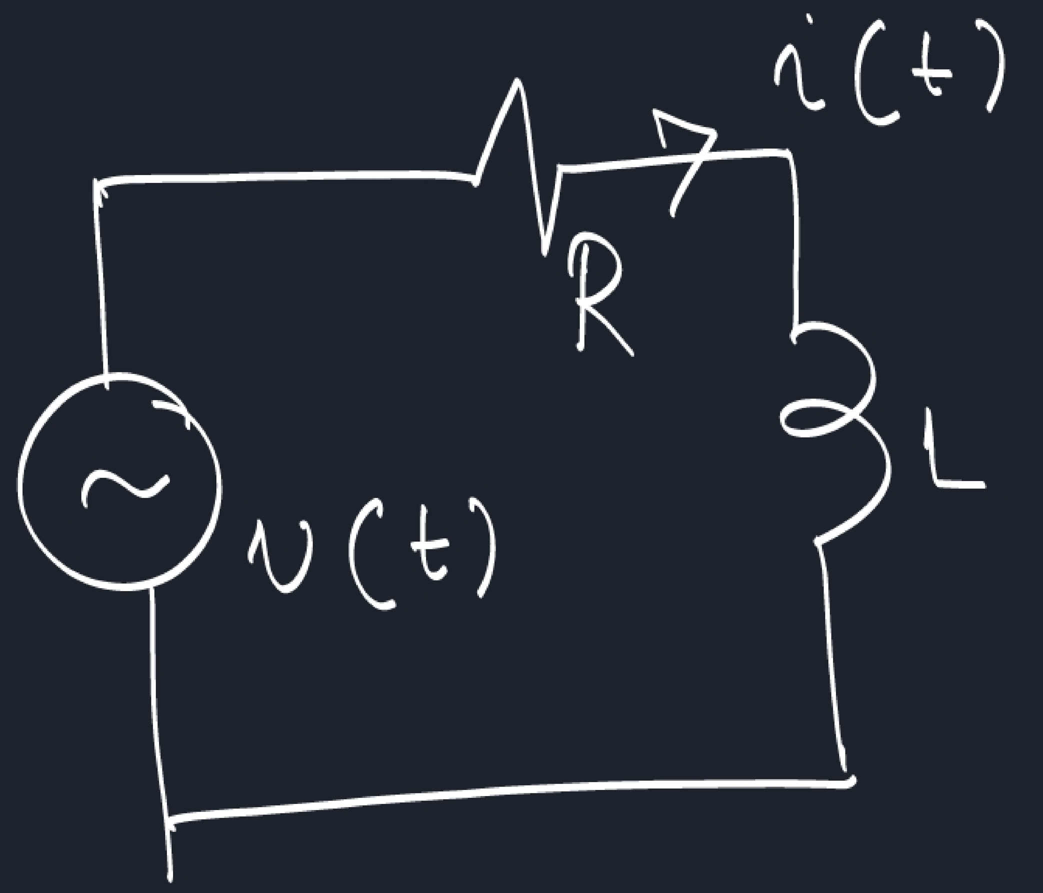
for transferring the max^m power to the load

$Z_L = Z_{Th}^* = 2.2904\angle +103.24$
$= -0.5245 + j2.2295$

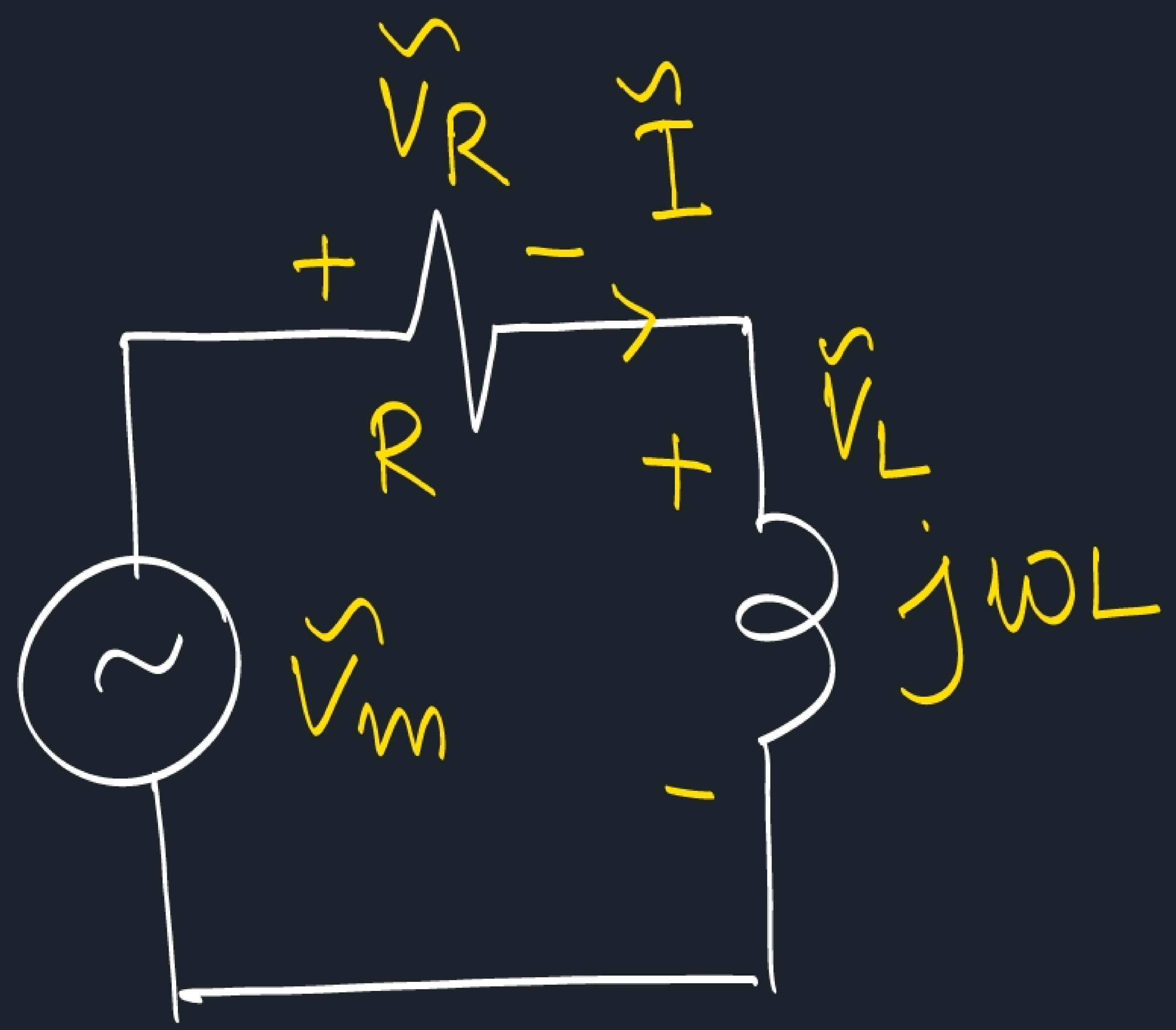
$\frac{|V_{Th}|^2}{4R_{Th}}$

$P_{max} = \frac{(3.0727)^2}{4\times 0.5245} = -?$

Series R-L :-



Phasor Domain

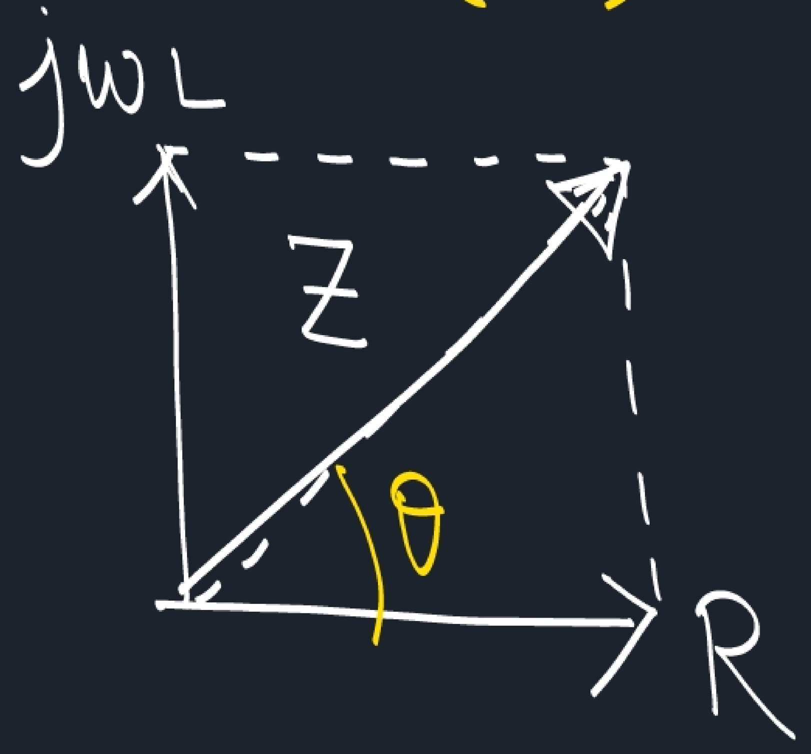


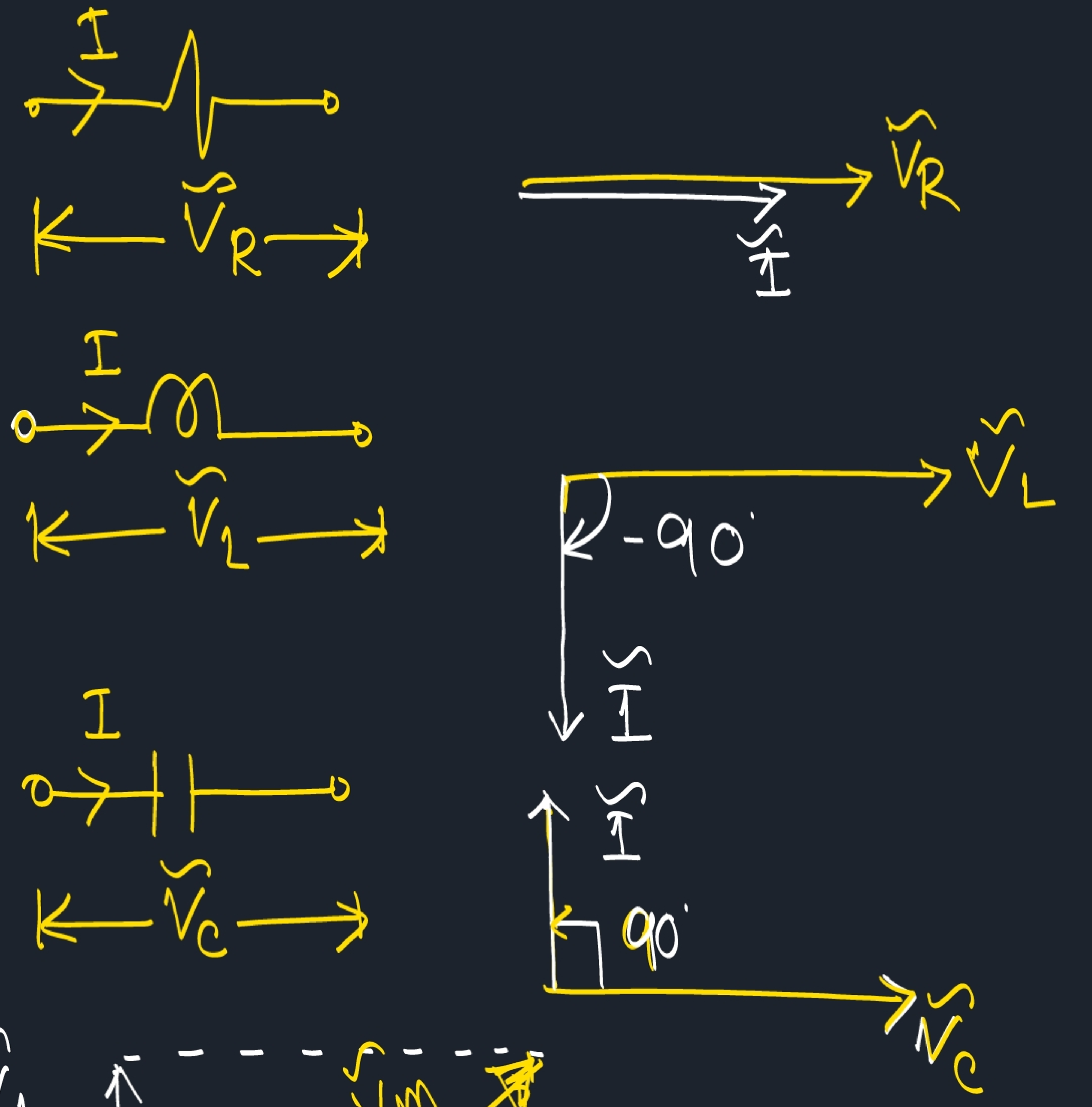
$$v(t) = V_m \cos \omega t$$

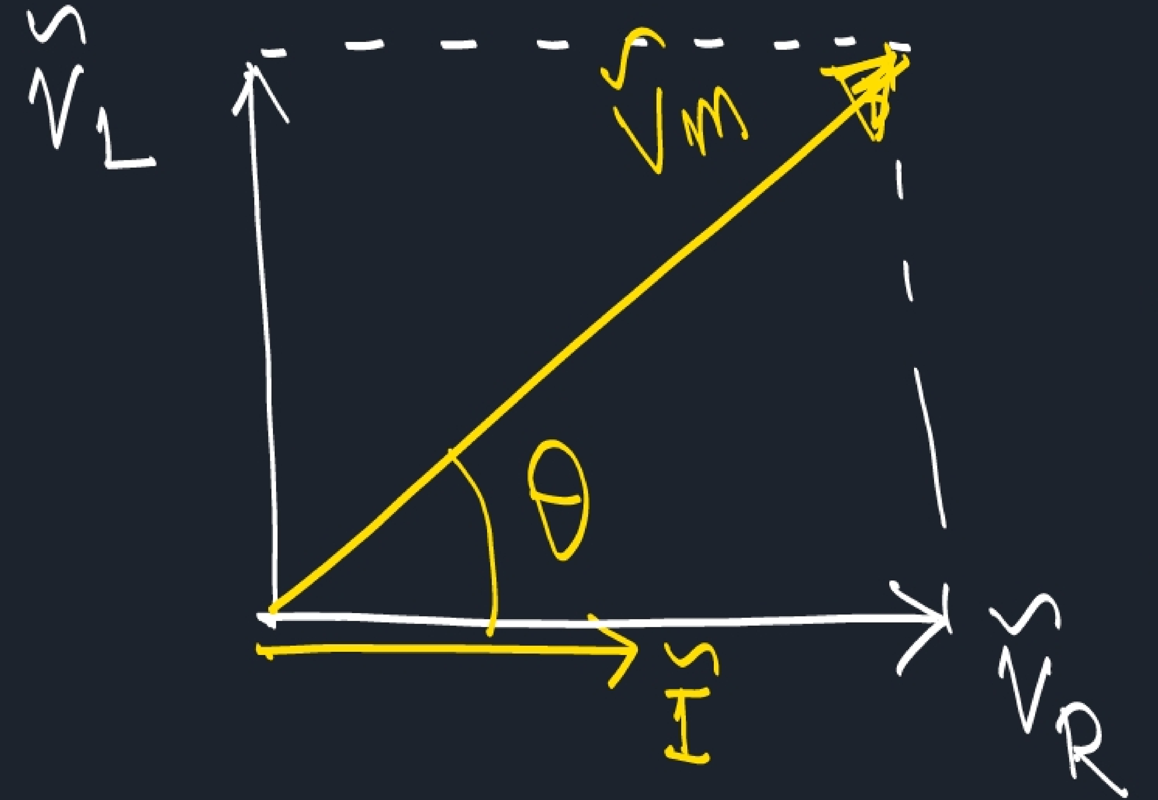
$$\hat{I} = \frac{V_m \angle 0}{R + j\omega L}$$

$$i(t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \angle 0 - \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\hat{V}_m = \hat{V}_R + \hat{V}_L$$







$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$Z = R + j\omega L$ = Impedance.