Analysis of Structures

- For the equilibrium of structures made up of several connected parts, the internal as well as external forces are considered.
- In the interaction b/w connected parts, Newton's 3rd law states that the reaction b/w the bodies in contact must have the same magnitude, same line of action, and opposite sense.
- Three categories of engineering structures are considered
  - (1) Truss
  - (2) Frame } → designed to support load and are usually stationary and fully constrained
  - (3) Machine → structures containing moving ~~loads~~ parts and designed to transmit & modify forces

(1) **Trusses**: formed from two-force members, i.e., straight members with end point connections (typically considered as pinned joints in structural analysis).

(2) **Frame**: contains at least one multi-force member, i.e., member acted upon by 3 or more forces.

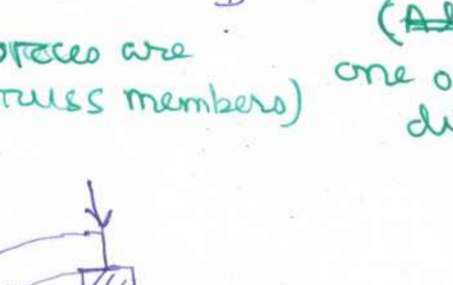
(3) **Machine**: structures containing moving parts and designed to transmit and modify forces.

Example

① Truss

(All the forces are all the truss members)

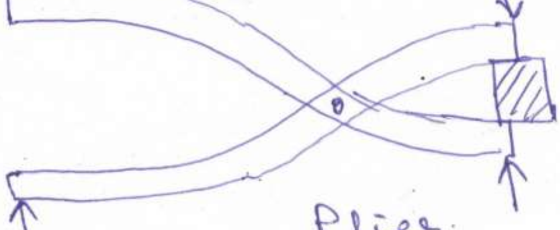
② Frame



→ multi-force member (In this member one of the forces is not along the memb[er])

→ multi-force member. (~~Alon~~ In this member, one of the forces are not directed along the member)

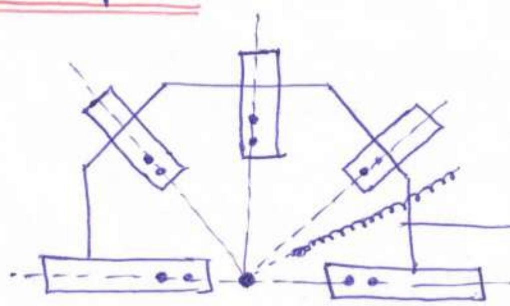
③ Machines



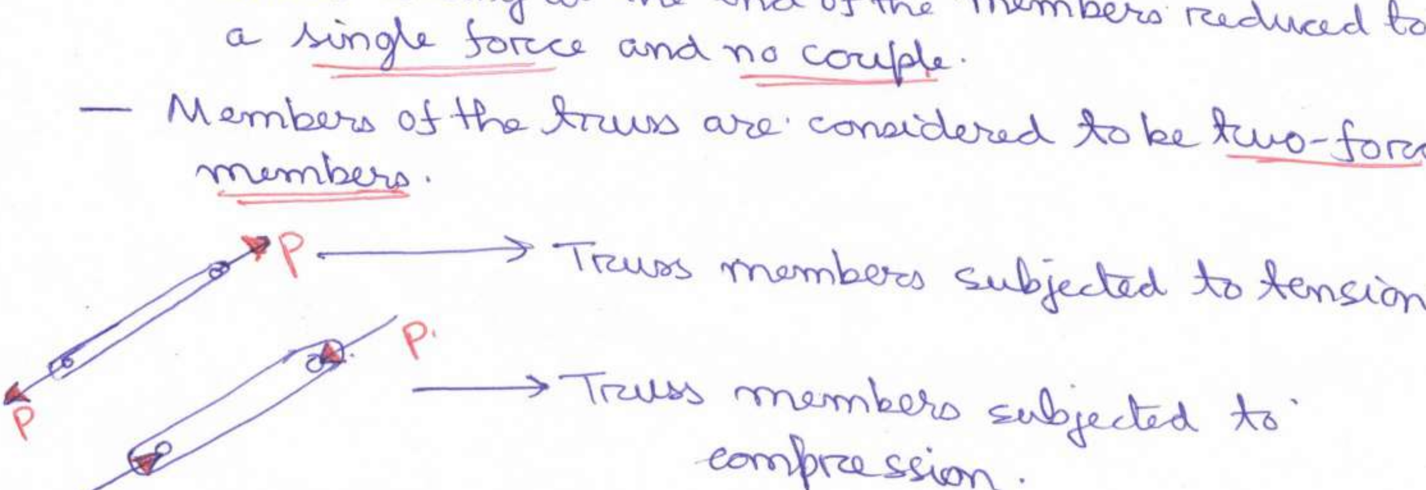
Plier

Trusses.

- Trusses are employed to support transverse load.
- Members forming the truss are subjected to axial loads though the external loading is in the transverse direction.
- Each truss is designed to carry those loads which acts in its plane. Hence, each truss may be treated as a two-dimensional structure and can be considered as a plane truss for the purpose of analysis.
- No member is continuous through a joint. The members are connected to a plate called "Gusset Plate" in such a way that the centroidal axes of these members intersect at a common-point.



- The joints can be made by welding, riveting, bolting, etc. But from the analysis point of view, they are assumed to be pinned joints (frictionless pins).
  ⇒ Forces acting at the end of the members reduced to a single force and no couple.
- Members of the truss are considered to be two-force members.



- The displacement of the truss is assumed to be small under the action of the loads (small deformation analysis).

Determinancy and Stability

If 'm' = no. of truss members
r = no. of reaction supports.
j = no. of joints

The condition of statical determinancy is given by –

$$\boxed{m + r = 2j}$$

⇒ No. of unknown forces in the members + No. of unknown reaction forces = [The no. of equilibrium equations which can be written at a joint i.e., 2 and hence for joints 'j', it becomes 2j]

Statically Indeterminate Internally.

– If in a structure, the number of unknown forces are higher than the no. of available equilibrium equations, then the unknown forces cannot be solved for. In that case, the structure is said to be statically indeterminate internally. Mathematically,

$$\boxed{m + r > 2j}$$ ($r = 3$ can be considered in this case).

– Statically indeterminate (internally) structures are stable structures.

Statically Indeterminate Externally.

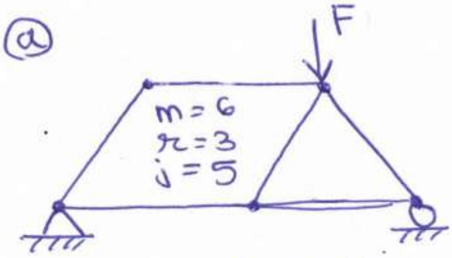
– If in a structure, the no. of support reactions are more (3 in case of plane truss), then the structure is externally indeterminate. Mathematically, statically external indeterminancy = $(r - 3)$.

Statically Unstable

$$\boxed{m + r < 2j}$$

⇒ Some of the members are missing to form triangular sub-elements of the structure.

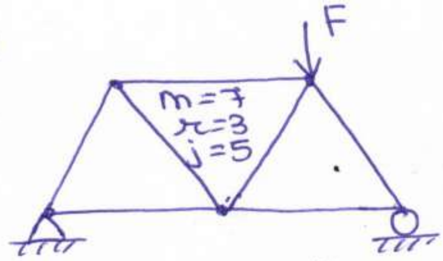
(a)



$m + r < 2j$

$\boxed{\text{Unstable}}$

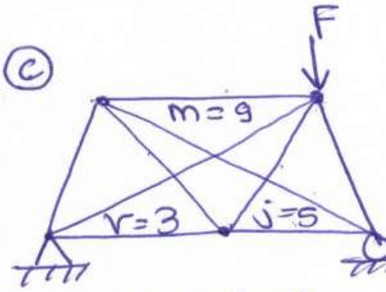
(b)



$m + r = 2j$

$\boxed{\text{Statically determinate and stable.}}$

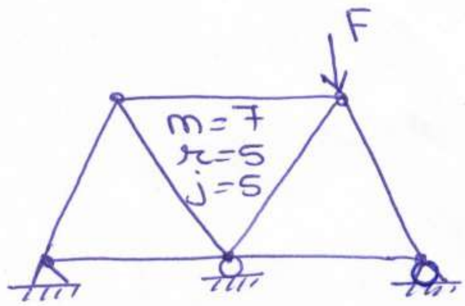
(c)



$m + r > 2j$

$\boxed{\text{Statically indeterminate internally but stable.}}$

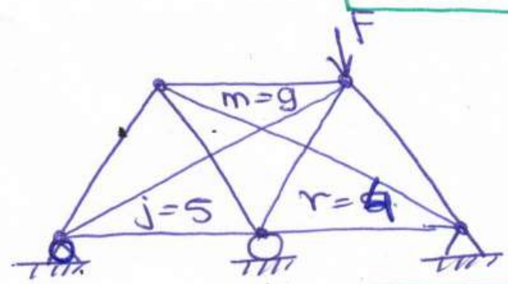
(d)



$r > 3$

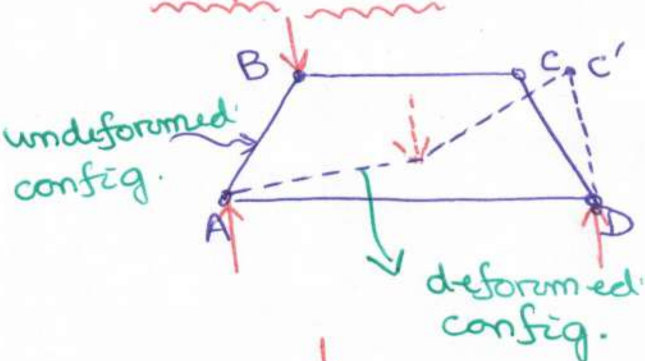
$\boxed{\text{Statically indeterminate externally, but stable.}}$

(e)

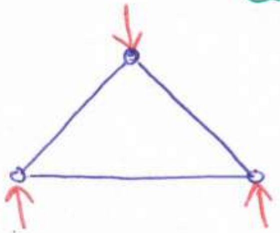


$\boxed{\text{Statically indeterminate both internally and externally, but stable.}}$

Simple Truss.

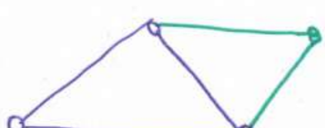


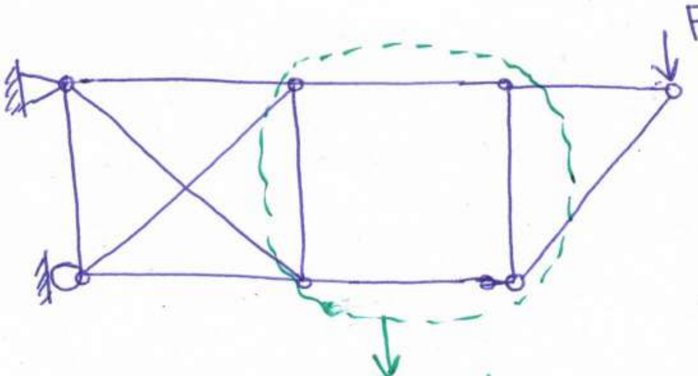
→ A poorly designed truss that cannot support a load.
⇒ unstable structure



— A rigid truss will not collapse under the application of a load.
This is the most basic form of a truss
Also known as Simple Truss

— A simple truss is constructed by successively adding two members and one connection to form the basic triangular truss.

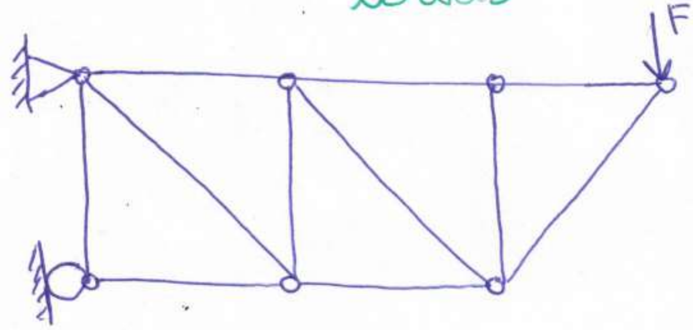




unstable under external loads

$m = 11,\ r = 3,\ j = 7$

$m + r = 2j$ is satisfied only for overall structure. But this truss is collapsible because of the presence of a four-membered sub-structure



$m = 11,\ r = 3,\ j = 7$

$m + r = 2j$ is satisfied for every conceivable sub-system

A Truss can be analyzed by two methods-

(a) Method of joints

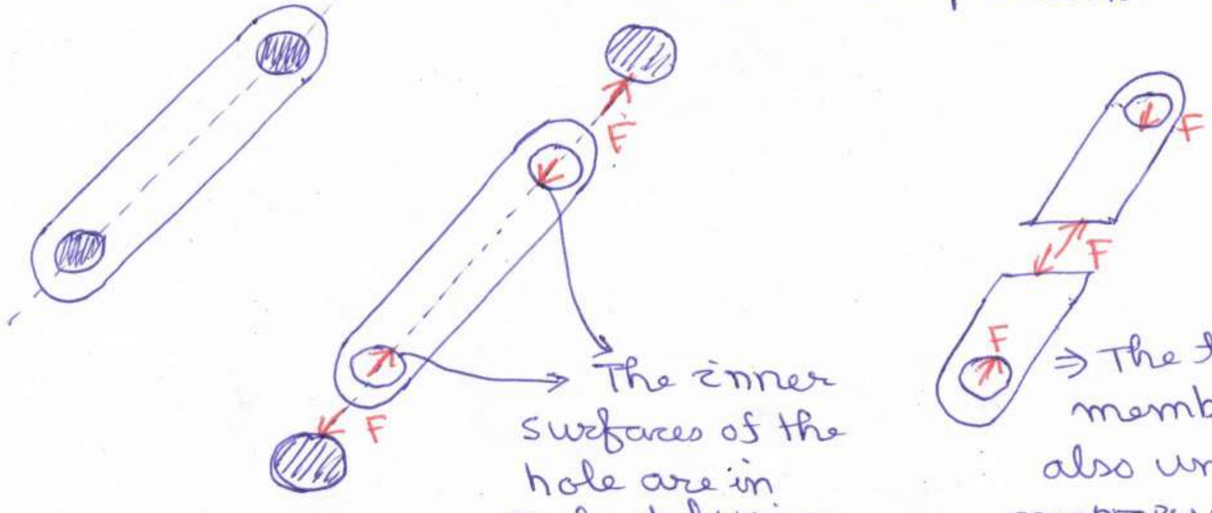
(b) Method of Sections.

Method of Joints

- Equilibrium of each joint is considered separately
- The reaction force at the supports are evaluated by using the force and moment equilibrium equations of the entire truss.
- The force system acting at the joint is concurrent and co-planar
- The solution of the truss problem is started at a joint, where two unknown forces act to satisfy two independent equations of equilibrium.
- An imaginary section is passed to isolate a single joint of the truss.

Association of Tension/Compression in the Truss Members

- Assume that the member is under compression.

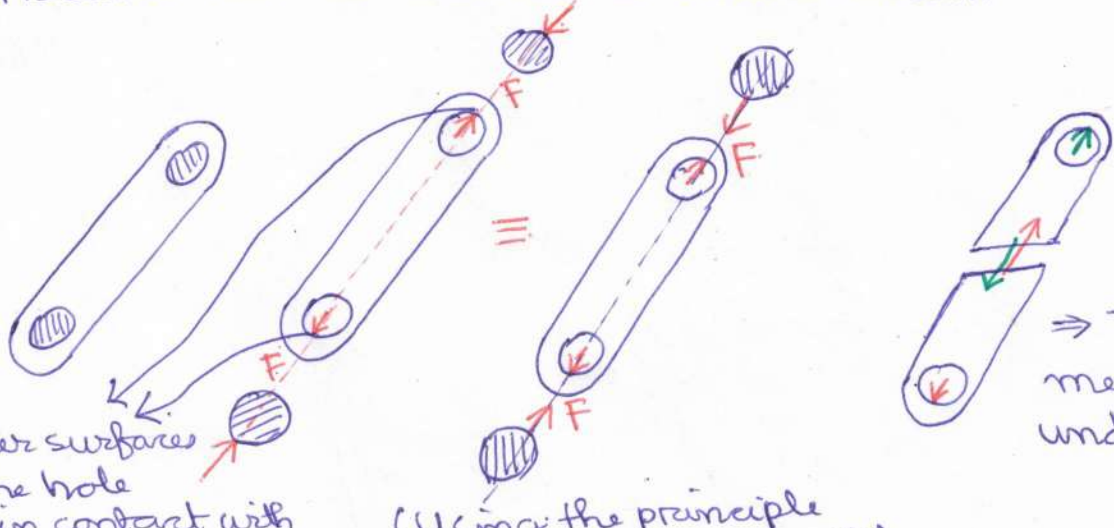


The inner surfaces of the hole are in contact during compression.

⇒ The pinned-joints are also under compression using Newton's 3rd law.

⇒ The truss member is also under compression.

- Assume that the member is under tension.



Outer surfaces of the hole are in contact with the pin during tension.

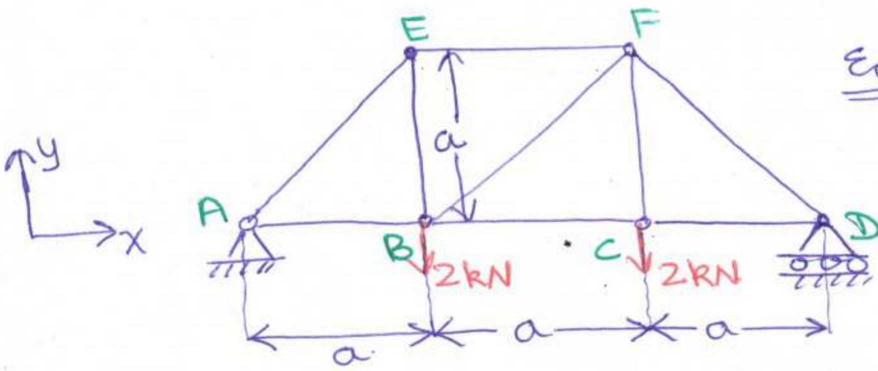
(Using the principle of force transmissibility, the force on the pin is moved to the other side)

⇒ The truss member is also under tension.

Convention is to indicate the force at the pin on the same side of the member.

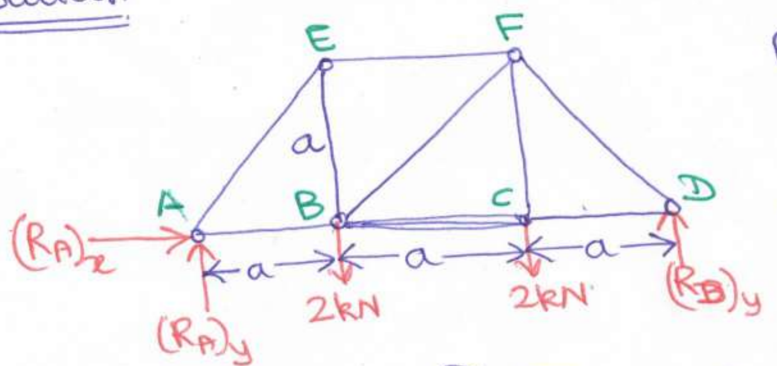
Method of Joints:

- This method is useful in finding forces in all the members of the truss.



Eg. Determine the internal forces acting on the truss members using the method of joints

Solution



(*) $\sum F_x = 0$

$\Rightarrow \boxed{(R_A)_x = 0}$

(*) $\sum F_y = 0$

$\Rightarrow \boxed{(R_A)_y + (R_D)_y = 4\,kN}$

(*) Check for Determinacy or Indeterminacy

$m = 9$

$j = 6$

$r = 3$

$m + r = 2j$ is satisfied

$\Rightarrow$ Determinate structure (Statically).

(*) $\sum M_A = 0$.

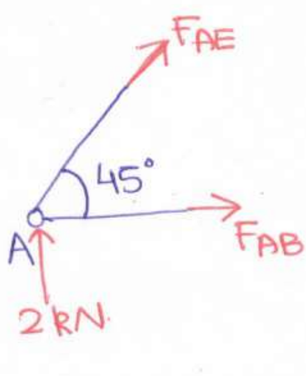
$\Rightarrow (R_D)_y (3a) = (2)(a) + (2)(2a)$

$\Rightarrow \boxed{(R_D)_y = 2\,kN}$

$\therefore \boxed{(R_A)_y = 2\,kN}$

Determination of Member Forces.

① Considering eqlbm. of joint A:-



$\sum F_y = 0$

$\Rightarrow F_{AE} \sin 45° + 2 = 0$

$\Rightarrow \boxed{F_{AE} = -2.83\,kN}$

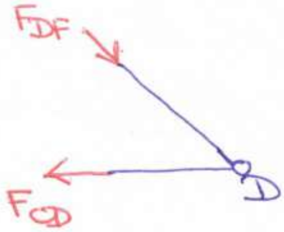
$\sum F_x = 0$

$\Rightarrow F_{AB} + F_{AE} \cos 45°$

$\Rightarrow F_{AB} = -F_{AE} \times \frac{1}{\sqrt{2}}$

$\Rightarrow \boxed{F_{AB} = 2\,kN}$.

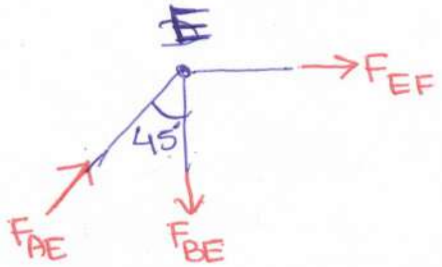
② Considering symmetricity of the problem, member forces at 'D'



$\boxed{F_{DF} = 2.83\,kN}$

$\boxed{F_{CD} = 2\,kN}$

③ Considering eqlb^m. of joint 'E' –



$\Sigma F_x = 0$

$\Rightarrow F_{EF} + F_{AE}\cos 45° = 0$

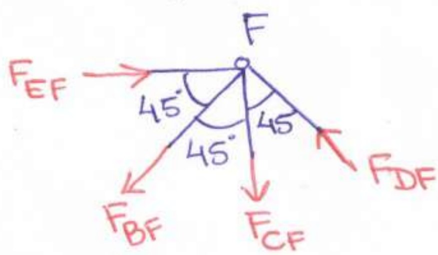
$\Rightarrow F_{EF} = -F_{AE}\cos 45°$

$\Rightarrow \boxed{F_{EF} = -2.83 \times \frac{1}{\sqrt{2}} = -2\,kN}$

$\Sigma F_y = 0 \Rightarrow F_{AE}\sin 45° = F_{BE}$

$\Rightarrow \boxed{F_{BE} = 2.83 \times \frac{1}{\sqrt{2}} = 2\,kN}$

④ Considering eqlb^m. of joint 'F':



$\Sigma F_x = 0$

$\Rightarrow F_{EF} = F_{BF}\cos 45° + F_{DF}\cos 45°$

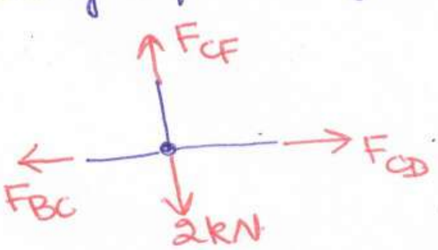
$\Rightarrow 2 = F_{BF}\cos 45° + 2.83 \times \frac{1}{\sqrt{2}}$

$\Rightarrow \boxed{F_{BF} = 0}$

$\Sigma F_y = 0 \Rightarrow F_{CF} = F_{DF}\sin 45°$

$\Rightarrow \boxed{F_{CF} = 2.83 \times \frac{1}{\sqrt{2}} = 2\,kN}$

⑤ Considering eqlb^m. of joint 'C':

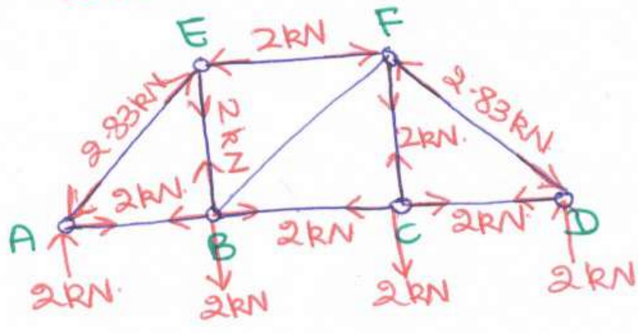


$\Sigma F_x = 0$

$\Rightarrow \boxed{F_{BC} = F_{CD} = 2\,kN}$

Final diagram

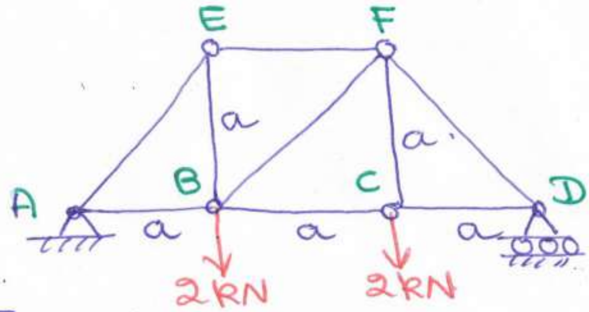
AB: T	BE: T
BC: T	CF: T
CD: T	FD: C
AE: C	BF: –
EF: C	



Method of Sections.

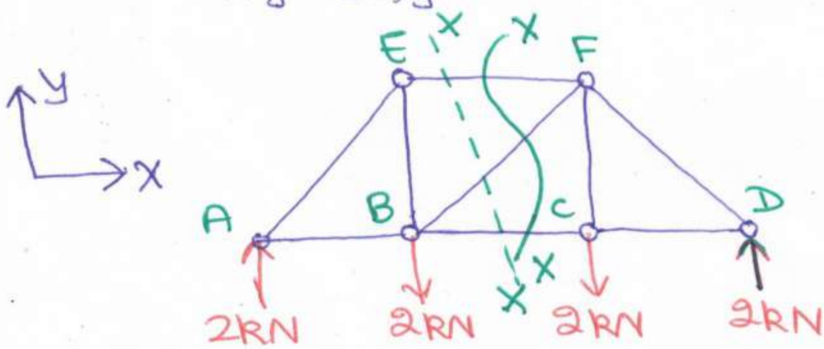
- It is based on the principle that if the entire truss is in equilibrium, then any segment of the truss is also in equilibrium.
- The method of section works very well when the force in only one member or the forces in a very few members are desired.
- In practice, the portion of the truss to be utilized is obtained by passing a section through three members of the truss, one of which is a desirable member.

Example

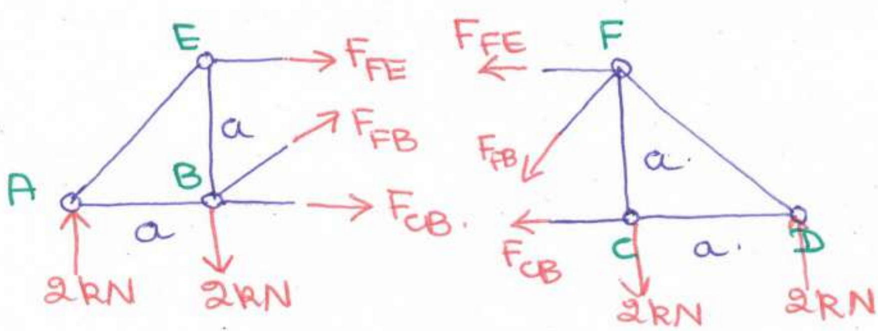


Find the force in the member EF using method of section.

Solution: From the previous analysis of Method of Joints, we know
$(R_A)_y = (R_D)_y = 2\,kN$



(*) A curve cut can also be made across the truss sections!!!



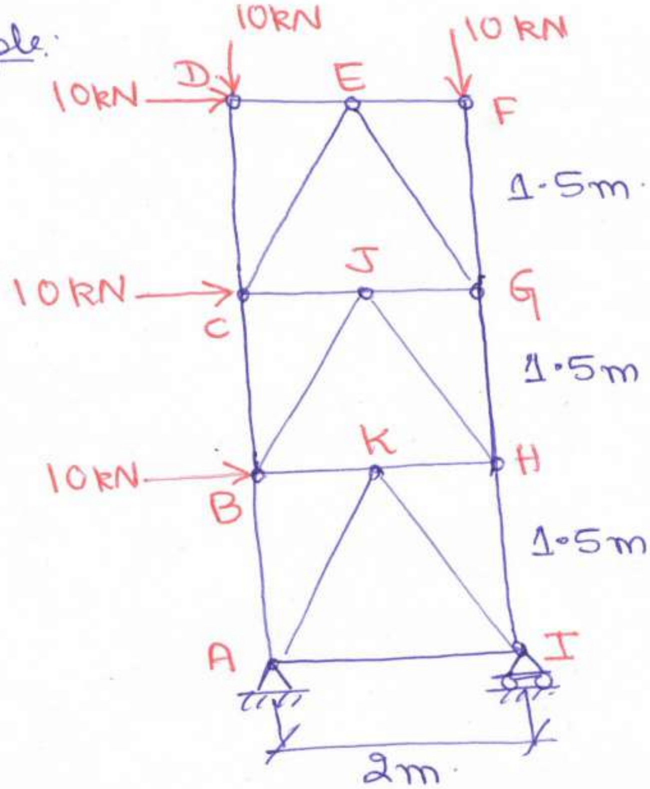
Taking moment about joint B —

$\sum M_B = 0$

$\Rightarrow 2(a) = -F_{FE}(a)$. $\Rightarrow$ The member FE is under a compressive load of 2 kN.

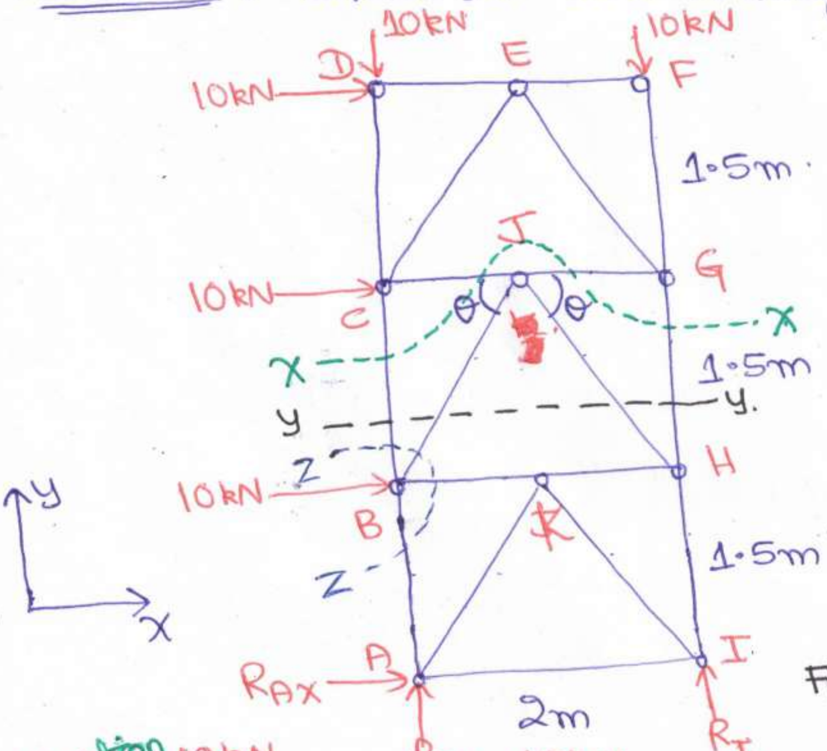
$\Rightarrow \boxed{F_{FE} = -2kN}$

Example:



Determine the forces in the members BC, BJ, BK.

Solution: Check for determinacy/indeterminacy



$m = 19$

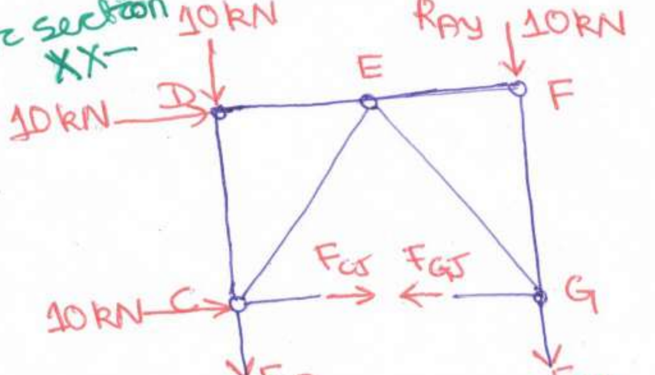
$r = 3$

$j = 11$

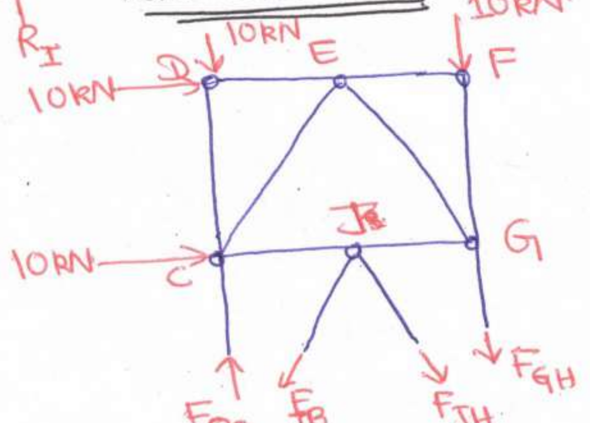
$\Rightarrow m + r = 2j$ is satisfied and each sub member/section is also triangular, hence the truss is stable and statically determinate.

$\cancel{\cos\theta}\ \tan\theta = \frac{1.5}{1} = 56.3^\circ$

For section XX



For section yy.



$F_{JB} = 18.0$...

$\Sigma M_G = 0$

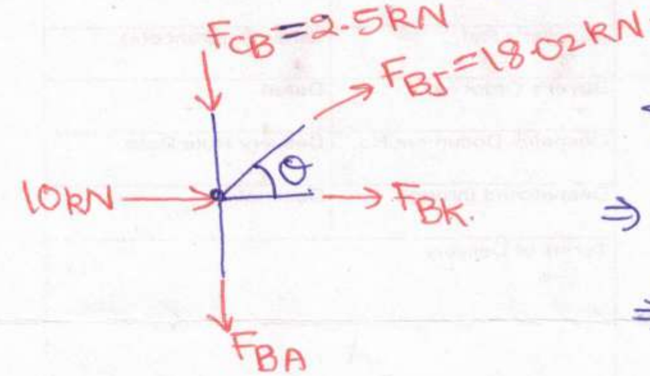
$\Rightarrow F_{CB}(2) = 10(1.5) - 10(2)$

$\Rightarrow \boxed{\cancel{F_{CB} = 17.5\,kN}} \Rightarrow \boxed{F_{CB} = -2.5\,kN}$

$\Sigma M_H = 0$

$\Rightarrow -10(1.5) - 10(3) + 10(2) - F_{BC}(2) + F_{JB}\cos(56.3)(1.5) + F_{JB}\sin 56.3(1) = 0$

Considering joint B (Section ZZ)



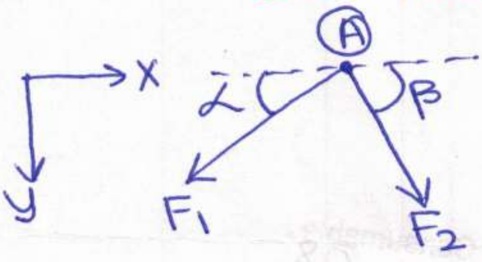
$\Sigma F_x = 0$

$\Rightarrow F_{BK} + 10 + F_{BJ} \cos 56.3 = 0$

$\Rightarrow \boxed{F_{BK} = -25 kN}$

Identification of Zero Force Members in a Truss.

Case-I : When no load is acting on a joint connecting two members



As load is acting on joint A—

$\Sigma F_x = 0$

$\Rightarrow F_1 \cos\alpha = F_2 \cos\beta$

$\Rightarrow F_1 = F_2 \left(\frac{\cos\beta}{\cos\alpha}\right)$

$\Sigma F_y = 0$

$\Rightarrow F_1 \sin\alpha + F_2 \sin\beta = 0$

$\Rightarrow F_2 \left(\frac{\sin\alpha \cos\beta}{\cos\alpha} + \sin\beta\right) = 0$

$\Rightarrow F_2 \sin(\alpha + \beta) = 0$

For any generalized α and β,

$\boxed{F_2 = 0}$

$\Rightarrow \boxed{F_1 = 0}$

Also, $F_2 \sin(\alpha+\beta) = 0$

$\Rightarrow \sin(\alpha+\beta) = 0$

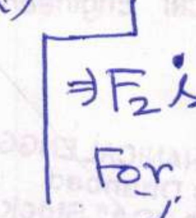
$\Rightarrow \alpha + \beta = n\pi$

① $\alpha + \beta = 0$

$\Rightarrow \boxed{\alpha = -\beta}$

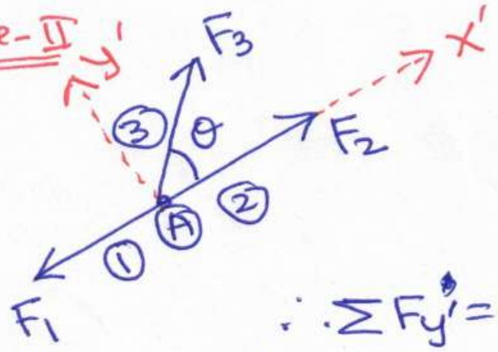
$\therefore \boxed{F_1 = F_2}$

② $\alpha + \beta = \pi$



$\Rightarrow$ The members overlap on each other, which is not a feasible case.

Case-II

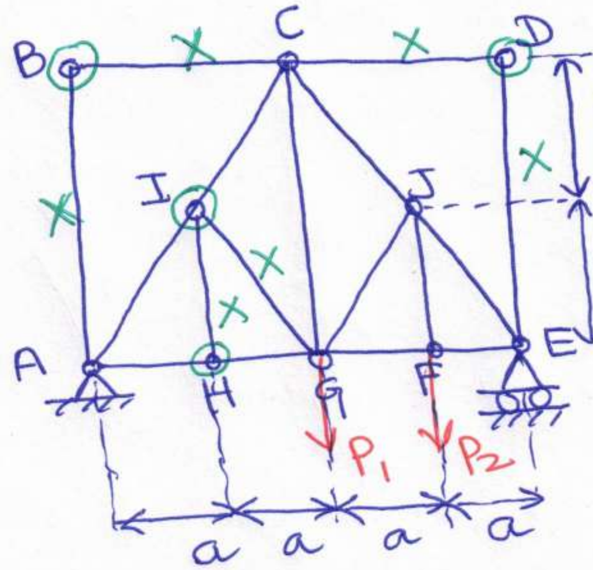


Members 1 and 2 are collinear and member 3 is at any inclined position. No load acts at joint (A)

$$\therefore \Sigma F_{y'} = 0$$
$$\Rightarrow F_3 \sin\theta = 0$$
$$\Rightarrow \boxed{F_3 = 0}$$

$$\Rightarrow \Sigma F_{x'} = 0$$
$$\Rightarrow F_1 = F_2 + F_3 \cos\theta$$
$$\Rightarrow \boxed{F_1 = F_2}$$

Example.



Identify zero force members in the truss by method of inspection.

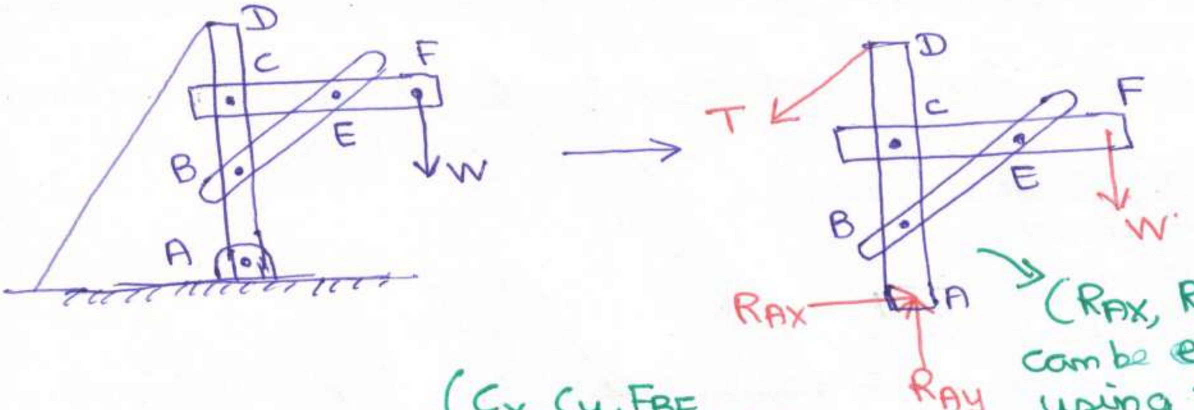
Using Case I and Case-II :—

Zero Force Members in the truss are:

$$\left.\begin{array}{l} AB \\ BC \\ CD \\ DE \\ HI \\ GI \end{array}\right\}$$

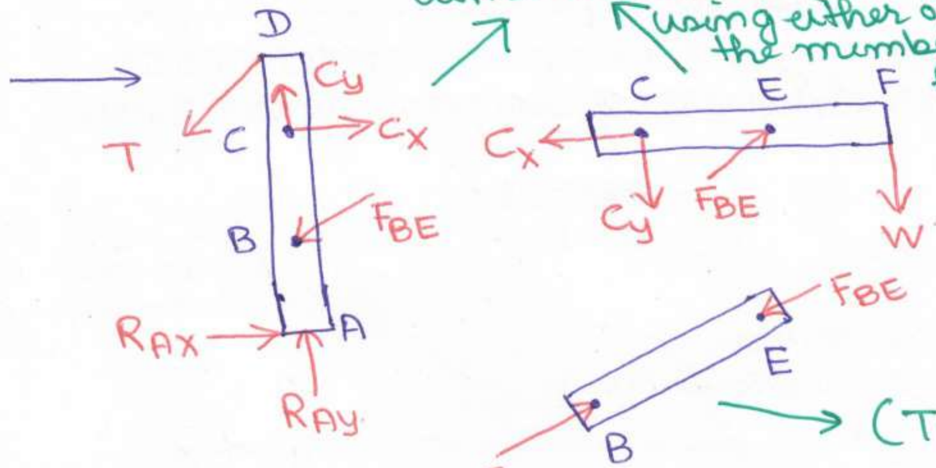
Analysis of Frames

- Frames and machines are structures with atleast one multi-force member, designed to support loads and are usually stationary.
- A FBD of the complete frame is used to determine the external forces acting on the frame.
- Internal forces are determined by dismembering the frame and creating the FBD of each component.



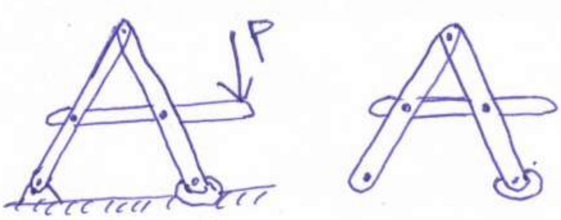
(R_{AX}, R_{AY}, T can be evaluated using $\Sigma F_x = \Sigma F_y = 0$ and $\Sigma M_A = 0$)

(C_x, C_y, F_{BE} can be evaluated using either of the members to be in eqlb^m.)



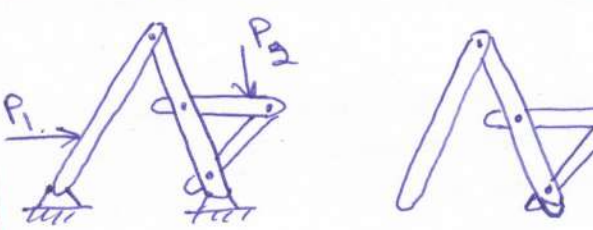
(Two-force member).

Rigid Non-Collapsible Frame



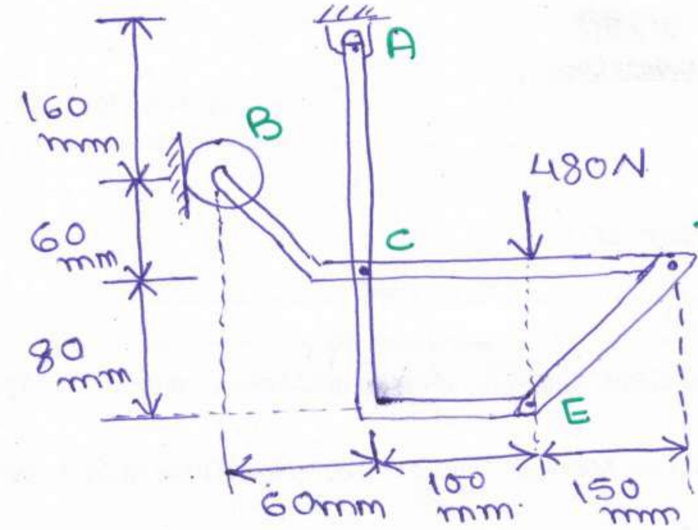
- A frame constitutes a rigid unit by itself, when removed from the supports.

Non-Rigid Collapsible Frame



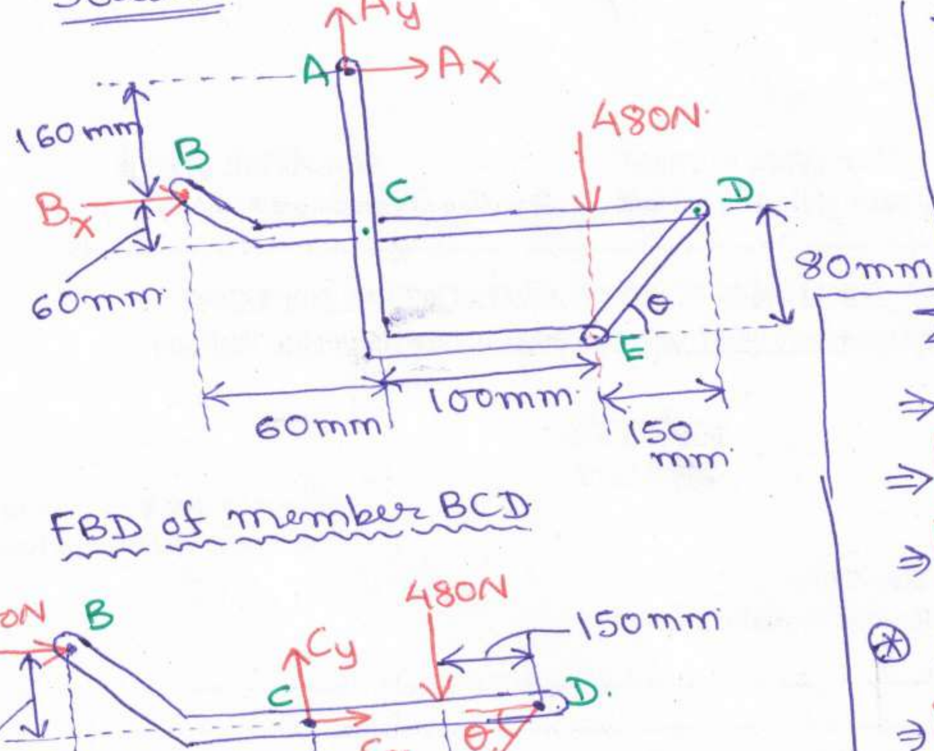
→ A frame is said to non-rigid and cannot sustain by itself when removed from the supports. These units require external supports for rigidity.

Example



Members ACE and BCD are connected by a pin at 'C' and by link DE. For the loading shown, determine the force in the link DE and the components of force exerted at 'C' on member BCD.

Solution



$\sum F_y = 0$

$\Rightarrow \boxed{A_y = 480N}$

$\sum F_x = 0$

$A_x + B_x = 0$

$\sum M_A = 0$

$\Rightarrow B_x(160) = 480(100)$

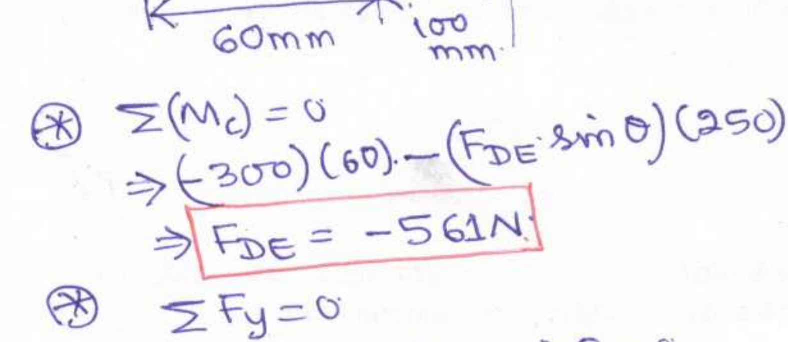
$\Rightarrow \boxed{B_x = 300N}$

$\Rightarrow \boxed{A_x = -300N}$

$\tan\theta = \frac{80}{150}$

$\Rightarrow \boxed{\theta = 28.07^\circ}$

FBD of member BCD



$\sum(M_c) = 0$

$\Rightarrow (-300)(60) - (F_{DE}\sin\theta)(250) - 480(100) = 0$

$\Rightarrow \boxed{F_{DE} = -561N}$

$\sum F_y = 0$

$\Rightarrow C_y - 480 - F_{DE}\sin\theta = 0$

$\Rightarrow C_y = F_{DE}\sin\theta + 480$

$\Rightarrow C_y = (-561)\sin 28.07^\circ + 480$

$\Rightarrow \boxed{C_y = 242.3N}$

$\sum F_x = 0$

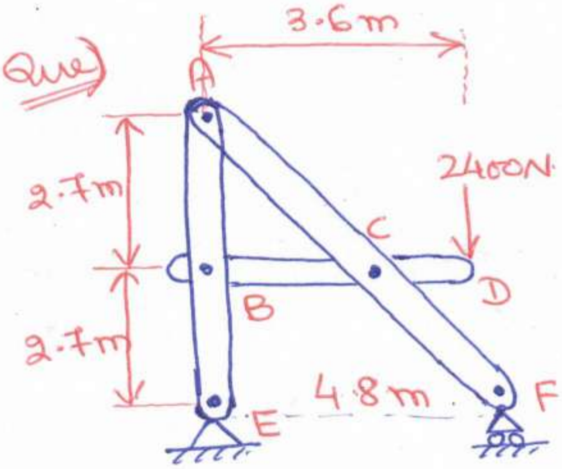
$\Rightarrow 300 + C_x - F_{DE}\cos\theta = 0$

$\Rightarrow C_x = F_{DE}\cos\theta - 300$

$= -561\cos 28.07^\circ - 300$

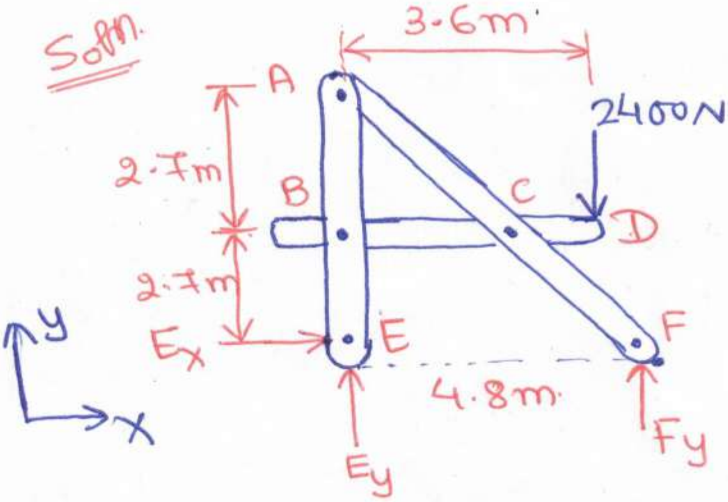
$\Rightarrow \boxed{C_x = -795N}$

Que)



Determine the components of the forces acting on each member of the frame shown.

Soln.



• $\Sigma M_E = 0$

$\Rightarrow (F_y)(4.8) - 2400(3.6) = 0$

$\Rightarrow F_y = 1800\,N$

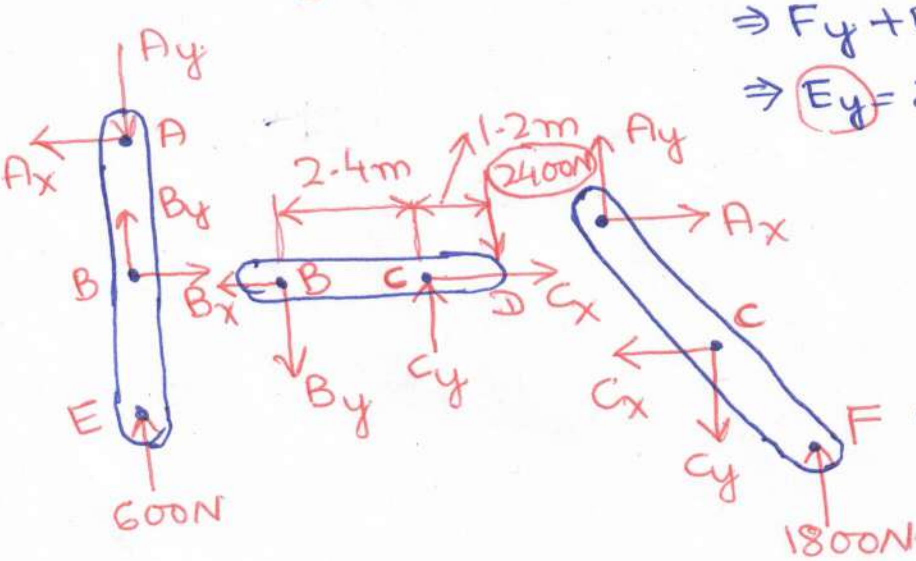
• $\Sigma F_x = 0$

$\Rightarrow E_x = 0$

• $\Sigma F_y = 0$

$\Rightarrow F_y + E_y = 2400$

$\Rightarrow E_y = 2400 - 1800 = 600\,N$



From the ΔABC & ΔAEF

$$\frac{AB}{BC} = \frac{AE}{EF}$$

$$\Rightarrow BC = \frac{AB}{AE} \cdot EF = \frac{2.7}{2(2.7)}(4.8)$$

$\Rightarrow BC = 2.4\,m$.

Considering the member BCD,

• $\Sigma M_B = 0$

$\Rightarrow (C_y)(2.4) = (2400)(3.6)$

$$\Rightarrow \boxed{C_y = \frac{(3.6)(2400)}{2.4} = 3600\,N}$$

• $\Sigma F_x = 0$

$\Rightarrow B_x = -C_x$

• $\Sigma F_y = 0$

$\Rightarrow B_y + 2400 = C_y$

$\Rightarrow B_y = 3600 - 2400$

$\Rightarrow \boxed{B_y = 1200\,N}$

– Considering the member ABE,

- $\sum M_A = 0$

$\Rightarrow (B_x)(2.7) = 0$

$\Rightarrow \boxed{B_x = 0} \Rightarrow \boxed{C_x = 0}$

- $\sum M_E = 0$

$\Rightarrow A_x(5.4) = 0$

$\Rightarrow \boxed{A_x = 0}$

- $\sum F_y = 0$

$\Rightarrow -A_y + B_y + 600 = 0$

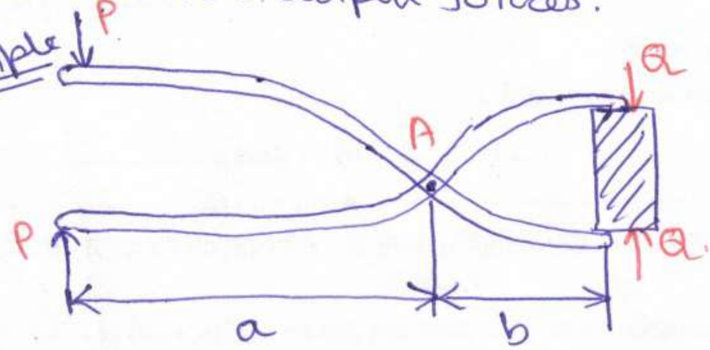
$\Rightarrow A_y = 1200 + 600$

$\Rightarrow \boxed{A_y = 1800N}$

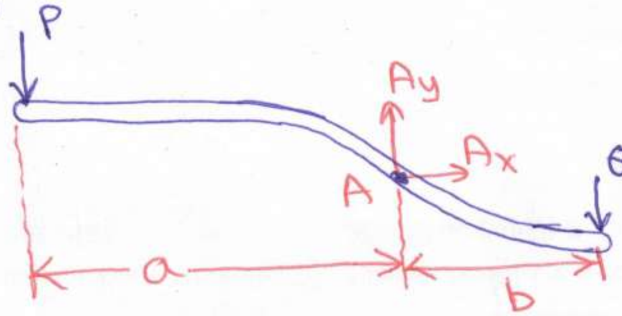
Machines

- Machines are structures designed to transmit and modify forces. Their main purpose is to transform input forces into output forces.

Example



Solution



$$\sum M_A = 0$$

$$\Rightarrow P(a) = Q(b)$$

$$\Rightarrow \boxed{Q = \frac{Pa}{b}}$$

As $b < a \Rightarrow$ The force 'Q' is magnified and is $\left(\frac{a}{b}\right)$ times P