Example Griffiths Problem 2.5

Problem 2.5 Find the electric field a distance z above the center of a circular loop of radius r (Fig. 2.9), which carries a uniform line charge λ.

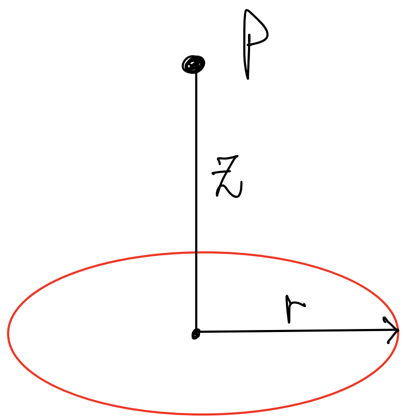
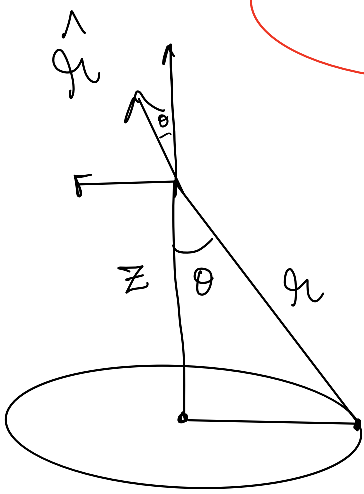


Figure 2.9 of Griffiths.



Again horizontal components cancel leaving :

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left\{\int \frac{\lambda\, dl}{\mathscr{r}^2}\cos\theta\right\}\hat{z}$$

Here, $\mathscr{r}^2 = r^2 + z^2$, $\cos\theta = \frac{z}{\mathscr{r}}$

$\int dl = 2\pi r$, $z, \mathscr{r}$ const.

So,

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, 2\pi r\, z}{(r^2+z^2)^{3/2}}\hat{z}$$

HW Problem 2.6 [Circular disk].

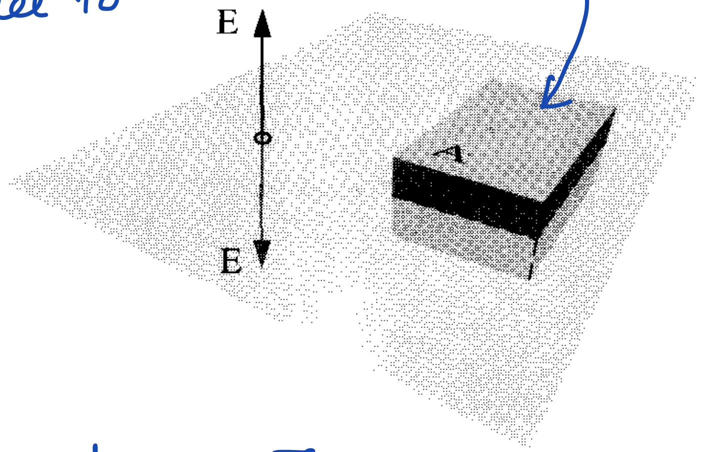
An infinite plane carries a uniform surface charge σ. Find its electric field.

Griffiths Ex – 2.4

$\vec{E}$ points upwards (above), downwards (below)

$A =$ area of pillbox's surface parallel to the plane.

pillbox



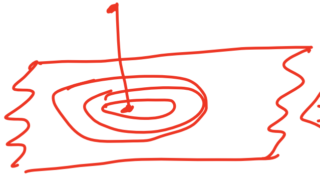
$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}}.$$

$$\oint \mathbf{E} \cdot d\mathbf{a} = 2A|\mathbf{E}|,$$

$$Q_{enc} = \int \sigma \, da = \sigma A$$

$\sigma =$ Constant

$$2A\,|\mathbf{E}| = \frac{1}{\epsilon_0} \sigma A,$$



$\Leftarrow$ Infinite, so $\vec{E}$ is constant at a given z.

$$\boxed{\mathbf{E} = \frac{\sigma}{2\epsilon_0} \hat{\mathbf{n}}}$$

$$E^{\perp}_{above} = \frac{\sigma}{2\varepsilon_0}$$

$$E^{\perp}_{below} = -\frac{\sigma}{2\varepsilon_0}.$$

** What is the discontinuity of the electric field as it crosses the surface charge?

$$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{1}{\epsilon_0} \sigma,$$