# LECTURE - 1, 2

SECTION A : } 28/07/2025
SECTION B : } 04/08/2025

(Lecture - 1)

SECTION C : } 31/07/2025
SECTION D : } 31/07/2025

(Lecture - 2)

SECTION C : } 05/08/2025
SECTION D : } 07/08/2025

# LECTURE PLAN PART 1 SECTION A, B, C, D

Classical Mechanics and Electrodynamics → 14 Lecture Hours

Mechanics of Many-body Systems

→ 1 – 2 Lecture Hours

→ Lecture notes will be uploaded periodically on MIS

Lagrangian and Hamiltonian Equations

→ 3 – 6 Lecture Hours

→ Topics covered in Class are very important for exams.

Electrodynamics
→ Maxwell's Equations
→ Wave equation
→ Energy density, Poynting's Theorem
→ 6 – 7 Lecture Hours

→ Also follow the lecture plan provided on MIS for details about Marks and weightage distribution.
[LECTURE PLAN FOLDER]

LECTURES SHALL ALSO INCLUDE PROBLEM SOLVING

→ PROBLEM SOLVING SESSIONS COULD PROVE VERY BENEFICIAL

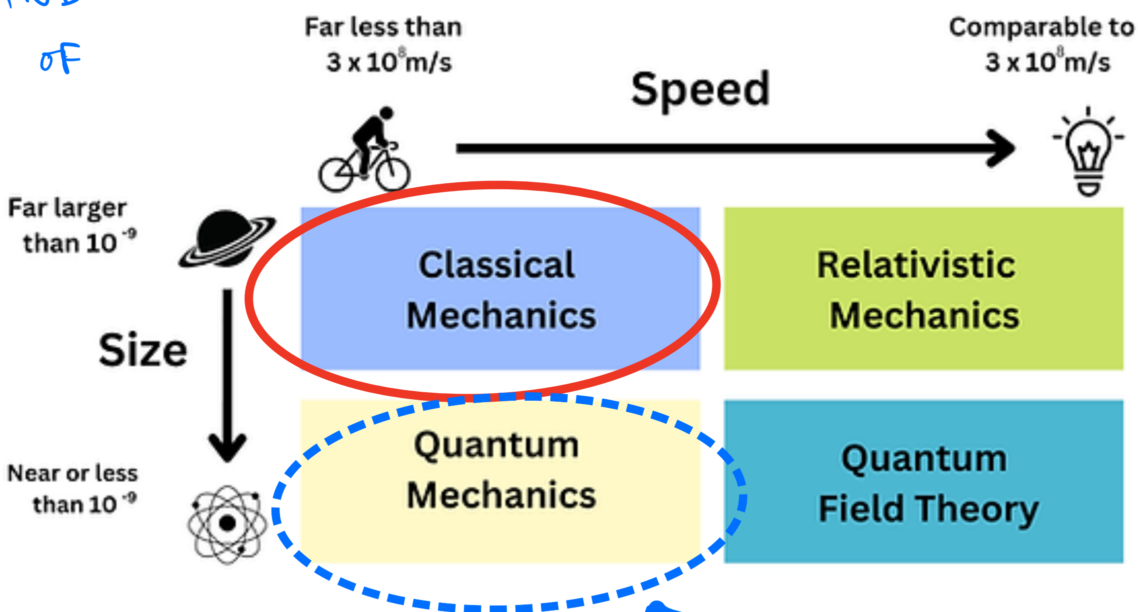
→ TEXTBOOK NAMES SHALL BE MENTIONED DURING LECTURE.

# LECTURE - 1

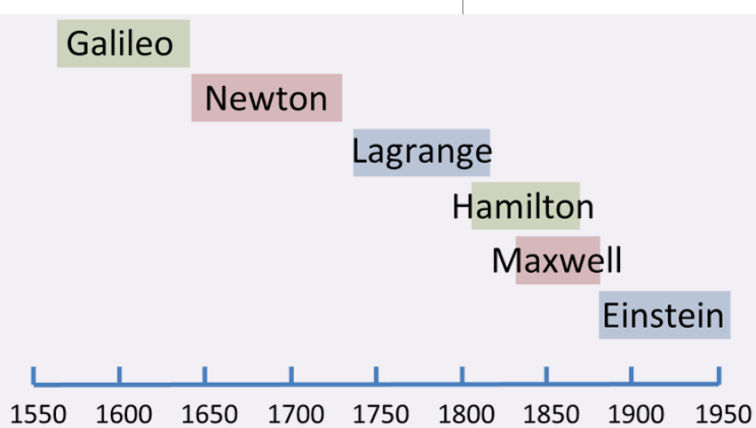
## PLAN :

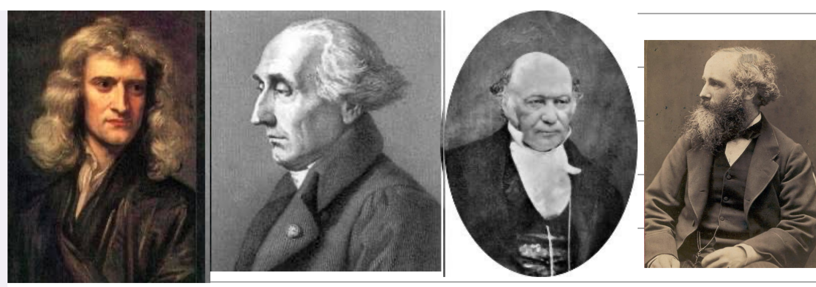
SYNOPSIS AND OVERVIEW OF COURSE

THIS COURSE




IN MODERN PHYSICS PART



NEWTON LAGRANGE HAMILTON MAXWELL

⇓

Electrodynamics

# 1 Why study Classical Mechanics?

Newton's laws date back to 1687, when he published Philosophiae Naturalis Principia Mathematica, laying out his laws of motion and universal gravitation. Now, over 300 years later, we understand the world in terms of relativistic quantum field theory or even fundamental string. Classical mechanics is known to fail in at least three ways. At distances smaller than $\frac{\hbar}{mv}$, we must use quantum mechanics or quantum field theory to describe the motions and interactions of matter. Second, at velocities close to the speed of light we must make relativistic corrections, working in a unified spacetime instead of Newton's abstraction of ideal Euclidan space and universal time. Finally, the law of universal gravitation has been replaced by general relativity. Why then, study classical mechanics at all?

## 1.1 Range of applicability

One answer lies in the wide range of applicability. Planck's constant $\hbar$ is small, so quantum considerations usually only become important for phenomena on the scale of atoms whose size is determined by $\frac{\hbar}{mv}$ for the orbiting electrons. Similarly, special relativity gives significant corrections only when objects move at a substantial fraction of the speed of light. The fastest man-made object ever produced was not the Voyager spacecraft (35,000 mi/hr) as is often claimed, but the solar probes Helios-A and Helios-B which reached a maximum speed of 252,792 km/hr (see http://en.wikipedia.org/wiki/Helios_(spacecraft) ). This is still only $0.000234c$, with $c$ the speed of light, so the relativistic corrections $\left(\sim \frac{v^2}{c^2}\right)$ are only a few parts in $10^8$. Interestingly, second place appears to be held by a nuclear powered manhole cover (nuclear testing, Pascal B, gone wrong) (see http://savvyparanoia.com/the-fastest-man-made-object-ever-a-nuclear-powered-manhole-cover-true/ ) which would have been traveling at about 237,500 mph. This still is only $\frac{v}{c} = 2.2 \times 10^{-4}$. These small corrections are important only for extremely fine measurements, where they are easily measured by modern atomic clocks. Finally, general relativity is important for cosmology, precise orbit predictions in planetary and satellite science, and the GPS system, but errors using Newtonian gravity are of order $\frac{GM}{Rc^2}$, where $R$ is the distane from a mass $M$. This is of order $6.95 \times 10^{-7}$ near the surface of Earth.

Therefore, for sizes larger than atoms and smaller than the solar system, and ordinary velocities, classical mechanics is an excellent approximation.

## 1.2 Mathematical techniques

$\Rightarrow$ Knowledge about integration, differentiation

$\Rightarrow$ Vector Calculus (Gradient, divergence, Curl)

$\Rightarrow$ Solution of differential equations

$\Rightarrow$ FUNCTIONAL CALCULUS (NOT REQUIRED FOR US)

## 1.3 First approximation (NEWTONIAN MECHANICS)

Because the corrections to Newtonian mechanics are so small, the Newtonian solution to problems is close to the exact solution, and therefore makes a good place to start in making a perturbative approximation to the full solution. Alternatively, if we have an exact solution in general relativity or quantum mechanics, we may be able to make sense of it by comparing terms in the classical solution.

## 1.4 Intuition

We have a great deal of direct experience with the world, and the terms of classical mechanics line up well with this experience. We can use this familiarity to guess how a system will behave. With more precise theories, having a similar picture of what is going on becomes difficult.

# 2 Review of Newtonian Mechanics

## Basic definitions

We define several important concepts. We picture the world as a 3-dimensional Euclidean space with points labeled by triples of numbers, often simply the Cartesian $(x, y, z)$, but others as well. Events are parameterized by the passage of time, so a point particle is described by a curve

$$\begin{aligned} \mathbf{r}(t) &= (x(t), y(t), z(t)) \\ &= (r(t), \theta(t), \varphi(t)) \end{aligned}$$

in whatever coordinates we choose, with boldface denoting a vector. Notice that the position vector $\mathbf{r}(t)$ is a *dynamical variable*, not a coordinate. As $t$ varies, $\mathbf{r}(t)$ traces out a curve in space.

The time-rate-of-change of the position vector is called the *velocity,*

$$\begin{aligned} \mathbf{v}(t) &= \frac{d\mathbf{r}(t)}{dt} \\ &= \dot{\mathbf{r}}(t) \end{aligned}$$

and is tangent to this curve. Notice that for simplicity we will sometimes denote the time derivative with a dot over the variable. The *acceleration* is the rate of change of velocity,

$$\begin{aligned} \mathbf{a}(t) &= \frac{d\mathbf{v}(t)}{dt} \\ &= \frac{d^2\mathbf{r}(t)}{dt^2} \\ &= \ddot{\mathbf{r}}(t) \end{aligned}$$

Particles are characterized by a constant called the *mass*, $m$, which reflects their resistance to change of velocity. Defining the *momentum* as the product

$$\mathbf{p} = m\mathbf{v}$$

we write Newton's second law as

$$\mathbf{F} = \frac{d\mathbf{p}(t)}{dt}$$

The force, $\mathbf{F}$, is to be taken intuitivelty and is determined by the particular problem. It is essentially that effort which produces a change of momentum. For example, if we stretch a spring it has the ability to move a mass attached to the end. Since this ability doubles if we double the stretch of the spring, we may write the force of a spring as proportional to the difference between the location of its endpoints,

$$\mathbf{F}_{spring} = -k(\mathbf{r}_2 - \mathbf{r}_1)$$

where the spring constant, $k$, characterizes the strength of the spring. This form of force is Hooke's Law.

There are many forces that have been identified:

$$\begin{array}{cc}
0 & Zero\,force \\
-k\left(\mathbf{r}_2 - \mathbf{r}_1\right) & Hooke's\,law \\
q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) & Lorentz\,force \\
\mathbf{N} & Normal\,force \\
-\mu\mathbf{N} & Friction \\
-\frac{GMm}{r^2}\hat{\mathbf{r}} & Gravitation \\
-\frac{kQq}{r^2}\hat{\mathbf{r}} & Coulomb's\,law
\end{array}$$

Once the forces on a particle have been identified, Newton's second law becomes an ordinary, second order differential equation

$$\sum \mathbf{F} = \frac{d}{dt}\left(m\frac{d\mathbf{r}\left(t\right)}{dt}\right)$$

where the sum is over all forces on the particle. This means that two initial conditions are required to give a unique solution to a problem. If the time starts at $t = t_0$, then the initial conditions may be taken as the position and velocity at $t_0$,

$$\begin{aligned}
\mathbf{r}_0 &= \mathbf{r}\left(t_0\right) \\
\mathbf{v}_0 &= \mathbf{v}\left(t_0\right)
\end{aligned}$$

## Conservation laws

We now develop three conservation laws.

### Conservation of linear momentum

If no force acts, $\mathbf{F} = 0$ and we have

$$\frac{d\mathbf{p}\left(t\right)}{dt} = 0$$

Integrating gives

$$\mathbf{p}\left(t\right) = p_0 = m\mathbf{v}\left(t_0\right)$$

The linear momentum is therefore constant; we say that $\mathbf{p}$ is *conserved.*

### Conservation of angular momentum

We define the *angular momentum* of a particle at position $\mathbf{r}\left(t\right)$, relative to a fixed position $\mathbf{R}$, to be

$$\mathbf{L} = \left(\mathbf{r}\left(t\right) - \mathbf{R}\right) \times \mathbf{p}$$

and the *torque* about the same location to be

$$\mathbf{N} = \left(\mathbf{r}\left(t\right) - \mathbf{R}\right) \times \mathbf{F}$$

Then taking the cross product of the relative position, $\mathbf{r}\left(t\right) - \mathbf{R}$, with Newton's second law, we have

$$\begin{aligned}
\left(\mathbf{r}\left(t\right) - \mathbf{R}\right) \times \mathbf{F} &= \left(\mathbf{r}\left(t\right) - \mathbf{R}\right) \times \frac{d\mathbf{p}\left(t\right)}{dt} \\
\mathbf{N} &= \frac{d}{dt}\left(\left(\mathbf{r}\left(t\right) - \mathbf{R}\right) \times \mathbf{p}\left(t\right)\right) - \left(\frac{d}{dt}\left(\mathbf{r}\left(t\right) - \mathbf{R}\right)\right) \times \mathbf{p}\left(t\right)
\end{aligned}$$

where we use the product rule on the right. Since

$$\begin{aligned}\frac{d}{dt}\left(\mathbf{r}\left(t\right)-\mathbf{R}\right) &= \frac{d\mathbf{r}\left(t\right)}{dt}-\frac{d\mathbf{R}}{dt}\\ &= \mathbf{v}\left(t\right)-0\end{aligned}$$

and with $\mathbf{p}\left(t\right)=m\mathbf{v}\left(t\right)$, the right side becomes

$$\begin{aligned}\frac{d}{dt}\left(\left(\mathbf{r}\left(t\right)-\mathbf{R}\right)\times\mathbf{p}\left(t\right)\right)-\left(\frac{d}{dt}\left(\mathbf{r}\left(t\right)-\mathbf{R}\right)\right)\times\mathbf{p}\left(t\right) &= \frac{d\mathbf{L}}{dt}-\mathbf{v}\left(t\right)\times m\mathbf{v}\left(t\right)\\ &= \frac{d\mathbf{L}}{dt}\end{aligned}$$

$\Rightarrow \vec{N} = \dot{\vec{L}}$ [Similarity with NL II for an angular system]

since $\mathbf{v}\left(t\right)\times\mathbf{v}\left(t\right)=0$. We therefore have

$$\mathbf{N} = \frac{d\mathbf{L}}{dt}$$

$\Rightarrow$ If $\vec{N} = \vec{0}$, then $\dot{\vec{L}} = \vec{0}$ $\Rightarrow \vec{L} =$ const.

It follows immediately that $\mathbf{L}$ is conserved if the torque vanishes.

Angular momentum is conserved if no torque acts! (about an axis)

### Conservation of energy

Suppose the force on a particle is a function of particle position, $\mathbf{F}=\mathbf{F}\left(\mathbf{r}\right)$. Then we may integrate the second law by taking the dot product with the velocity:

$$\begin{aligned}\mathbf{F} &= \frac{d\mathbf{p}\left(t\right)}{dt}\\ \mathbf{F}\cdot\mathbf{v} &= \frac{d\mathbf{p}}{dt}\cdot\mathbf{v}\\ \mathbf{F}\cdot\frac{d\mathbf{r}}{dt} &= m\frac{d\mathbf{v}}{dt}\cdot\mathbf{v}\\ \mathbf{F}\cdot d\mathbf{r} &= m\mathbf{v}\cdot d\mathbf{v}\end{aligned}$$

Integrating from the initial values to the values at a general time, $t$, we have

$$\begin{aligned}\int_{\mathbf{r}_0}^{\mathbf{r}(t)}\mathbf{F}\cdot d\mathbf{r} &= m\int_{\mathbf{v}_0}^{\mathbf{v}(t)}\mathbf{v}\cdot d\mathbf{v}\\ &= \frac{1}{2}m\mathbf{v}^2-\frac{1}{2}m\mathbf{v}_0^2\\ &= T-T_0\end{aligned}$$

where we define the *kinetic energy*, $T\left(t\right)=\frac{1}{2}m\mathbf{v}^2$. In general, the integral on the left side depends on the path of integration. Such a path-dependent integral is *not* a function, but is called instead a *functional.* Along any path we define the *work* as

$$W_{12}=\int_{\mathbf{r}_1}^{\mathbf{r}_2}\mathbf{F}\cdot d\mathbf{r}$$

so that the work is equal to the change in kinetic energy,

$$W_{12}=T_2-T_1$$

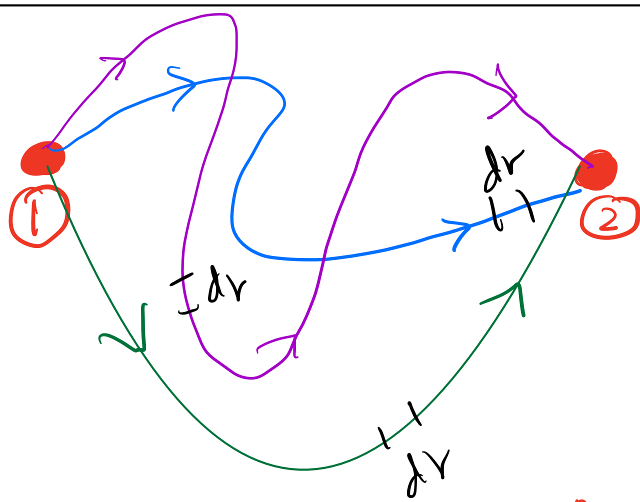
This is the work-energy theorem.

- Forces can be either conservative or Non-conservative

# START OF LECTURE - 2

## Conservation of Energy

Work done is FORCE times DISTANCE moved in the direction of the force.




Work done by a Force $\vec{F}$ along path from ① → ②

$$W_{12} = \int_{①}^{②} \vec{F} \cdot d\vec{r}$$

Line integral (depends on Path!)

infinitesimal displacement vector

[Work]
$= [F]\,[ds]$
$= MLT^{-2}\,L$
$= ML^{2}T^{-2}$
$\equiv$ Joule

$$d\vec{r} = \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{pmatrix} dt = \vec{v}\, dt$$

Constant mass, m : $\vec{F} = m\vec{a}$

$$\vec{F} = m\vec{a}$$

$$W_{12} = m \int_{1}^{2} \vec{a} \cdot d\vec{r}$$

$$= m \int_{1}^{2} \frac{d\vec{v}}{dt} \cdot d\vec{r}$$

$$= m \int_{1}^{2} \frac{d\vec{v}}{dt} \cdot \vec{v}\, dt$$

$$= \frac{1}{2} m \int_{1}^{2} \frac{d}{dt} \left(|\vec{v}|^2\right) dt$$

$$\frac{d}{dt}(\vec{v} \cdot \vec{v}) = \frac{d\vec{v}}{dt} \cdot \vec{v} + \vec{v} \cdot \frac{d\vec{v}}{dt} = 2 \frac{d\vec{v}}{dt} \cdot \vec{v}$$

$$= \frac{1}{2} m \left(v_2^2 - v_1^2\right)$$

$$\boxed{W_{12} = T_2 - T_1}$$

Work done is change in KE

where $T_i$ is the kinetic energy (KE) $\frac{1}{2} m v_i^2$ of the particle at position, 'i'.

Another definition:

⇒ Conservative : NO mechanical energy destroyed or dissipated.

Total Energy can be simply written as, $E = KE + PE$

Work done is independent of path.

e.g. gravitation, electrostatics, Hooke's law for springs

⇒ Non-conservative : mechanical energy <u>is</u> dissipated.

⇒ Work <u>depends</u> on path taken.

e.g. Friction, magnetism (Lorentz force)

BUT: All Fundamental Forces are <u>conservative</u>!

| |
|---|
| Q. WHAT ARE THE CONSEQUENCES OF PATH INDEPENDENT CONSERVATIVE FORCES ? |

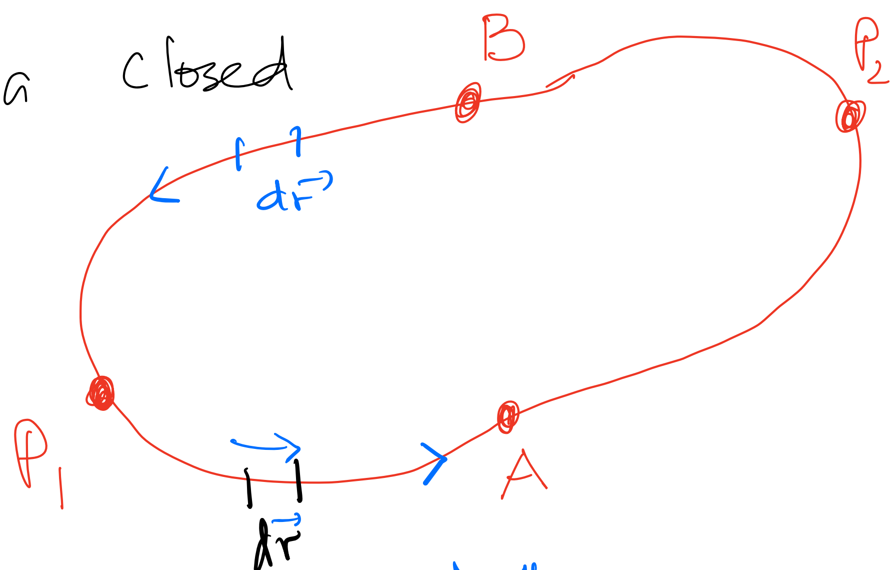
ANSWER: Scalar Potential

Path independence of work done by a conservative force implies the existence of a <u>scalar potential</u>.

Let's prove the following as well:

Prove that if $\int_{P_1}^{P_2} \vec{F}\cdot d\vec{r}$ is independent of the path joining any two points $P_1$ and $P_2$ in a given region, then $\oint \vec{F}\cdot d\vec{r} = 0$ for all closed paths in the region and conversely.

Let $P_1 A P_2 B P_1$ be a closed curve. Then,




$$\oint \vec{F}\cdot d\vec{r}$$

$$= \int_{P_1 A P_2 B P_1} \vec{F}\cdot d\vec{r}$$

$$= \int_{P_1 A P_2} \vec{F}\cdot d\vec{r} + \int_{P_2 B P_1} \vec{F}\cdot d\vec{r}$$

$$= \int_{P_1 A P_2} \vec{F}\cdot d\vec{r} - \int_{P_1 B P_2} \vec{F}\cdot d\vec{r} = 0$$

Hence,

$$\oint \vec{F}\cdot d\vec{F}$$

$$= 0$$

as $\int_{P_1}^{P_2} \vec{F}\cdot d\vec{r}$ is independent of path

$\Rightarrow$ Since the integral from $P_1$ to $P_2$ through 'A' is same as the integral through 'B'.

[Recall hypothesis that $\int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$ is independent of path joining $P_1$, $P_2$.]

Conversely, if $\oint \vec{F} \cdot d\vec{r} = 0$, then

$$\Rightarrow \int_{P_1AP_2BP_1} \vec{F} \cdot d\vec{r} = \int_{P_1AP_2} \vec{F} \cdot d\vec{r} + \int_{P_2BP_1} \vec{F} \cdot d\vec{r} = 0$$

$$= \int_{P_1AP_2} \vec{F} \cdot d\vec{r} - \int_{P_1BP_2} \vec{F} \cdot d\vec{r}$$

$$= 0$$

Implies :

$$\int_{P_1AP_2} \vec{F} \cdot d\vec{r} = \int_{P_1BP_2} \vec{F} \cdot d\vec{r}$$

$\vec{F}$ is Conservative

Say,

$$W_{12} = \int_{\text{Path}} \vec{F} \cdot d\vec{r} \Rightarrow \oint_{\substack{\text{Closed}\\ \text{loop}}} \vec{F} \cdot d\vec{r} = 0$$

$\Downarrow$

Work done is zero for a conservative force along a closed path.

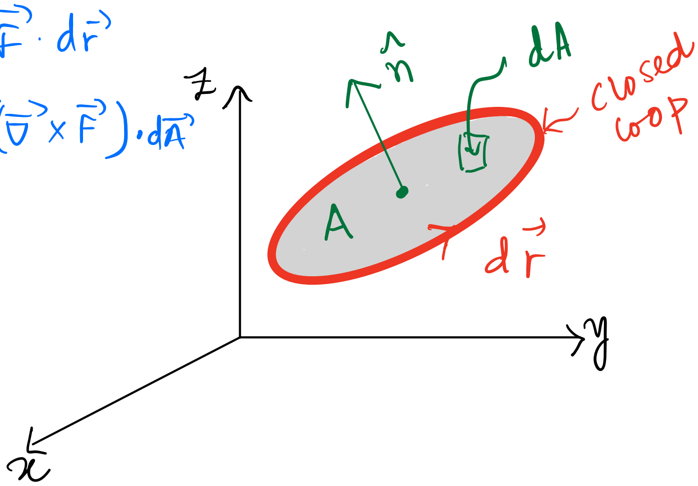
$\Rightarrow$ NECESSARY CONDITION

→ Path independence implies the result depends on the START and END point only.

→ Work done also depends on the difference b/w the kinetic energies (KE), and, if there is no dissipation while a particle traverses from point 1 to point 2, then the KE's do not change.

$$W_{12} = \int_{\text{Path}} \vec{F} \cdot d\vec{r} \Rightarrow \oint_{\substack{\text{Closed}\\ \text{loop}}} \vec{F} \cdot d\vec{r} = 0$$

STOKES' THEOREM: $\oint \vec{F} \cdot d\vec{r} = \int (\vec{\nabla} \times \vec{F}) \cdot d\vec{A}$




$\downarrow$ Stokes' Theorem

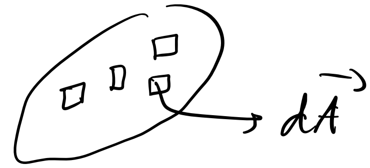
$$\int_{\substack{\text{Area bounded}\\ \text{by original}\\ \text{loop.}}} (\vec{\nabla} \times \vec{F}) \cdot d\vec{A} = 0$$

$\downarrow$

Conservative forces $\Rightarrow \vec{\nabla} \times \vec{F} = \vec{0}$ have no curl!

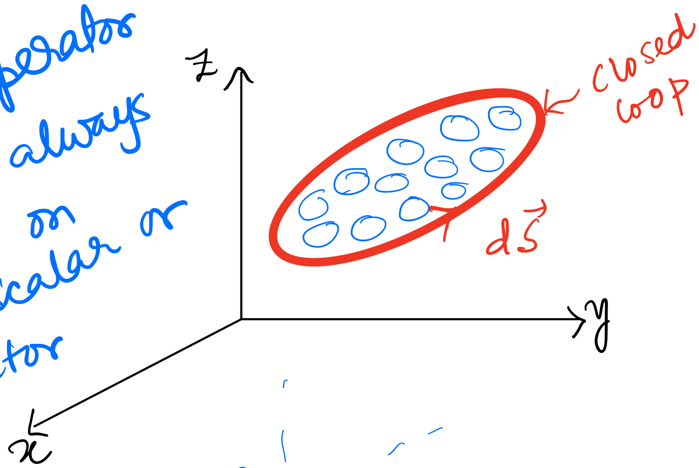
$\Rightarrow$ Existence of 3 constraint equations.

$$\vec{\nabla} \times \vec{A} = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{pmatrix}$$

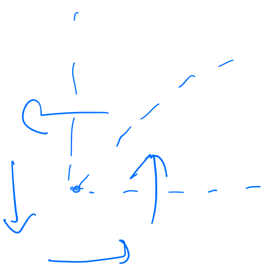


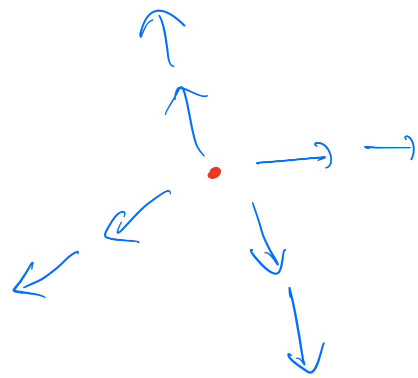

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

Vector operator as it always operates on some scalar or a vector




$$\oint \vec{F} \cdot d\vec{s} = \int (\vec{\nabla} \times \vec{F}) \cdot d\vec{A}$$





$$\frac{\hat{r}}{r^2}$$

Curl of a vector : Suggests the rotation of a vector field at a point in space.

MORE ON THIS PART BEFORE THE START OF ELECTRICITY AND MAGNETISM.

Constraint Equations:

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \Rightarrow \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\Rightarrow \quad \frac{\partial F_y}{\partial z} - \frac{\partial F_z}{\partial y} = 0 \; ; \quad \frac{\partial F_z}{\partial x} - \frac{\partial F_z}{\partial z} = 0 \; ;$$

$$\frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x} = 0$$

Encapsulate the above mathematics into one function $\Rightarrow$ Scalar potential!

Define a potential $V(\vec{r})$ such that,

$$\boxed{\vec{F} = -\vec{\nabla} V}$$

$$\underbrace{\vec{\nabla} \times \vec{F} = 0}_{\text{Conservative}}$$

We Need ! $\quad \vec{\nabla} \times \vec{F} = \vec{0}$ (Use the above)

$$\Rightarrow \quad -\vec{\nabla} \times \vec{\nabla} V = \vec{0} \quad \Leftarrow \text{Mathematical Identity.}$$

Examples :

(i) Gravitation :

$$F_G = G \frac{m_1 m_2}{r^2}$$

$$V_G \propto \frac{1}{r}$$

(ii) Electrostatics :

$$F_{el} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$$V_{el} \propto \frac{1}{r}$$

Non-conservative forces :

(kinetic) $\vec{F}_{viscous} = -k\vec{v}$

$\vec{F}_{mag} = q(\vec{v} \times \vec{B})$

depend on $\vec{v}$

$\therefore$ No scalar potential energies for these forces.

(kinetic) → depends on the relative velocity b/w two surfaces in contact with each other.

# Potential Energy

Scalar potential $V(\vec{r})$ only defined up to a constant.

$$\vec{F} = -\vec{\nabla} V$$

let's define, $\tilde{V}(\vec{r}) = V(\vec{r}) + V_0$

Then, $\vec{F} = -\vec{\nabla}\left(\tilde{V}(\vec{r}) - V_0\right)$

$$= -\vec{\nabla}\tilde{V}(\vec{r})$$

$$\because \quad \vec{\nabla} V_0 = 0$$

→ Same force (and hence, same physics) independent of added constant '$V_0$'.

** FOR INTERESTED STUDENTS

→ This is like "GAUGE SYMMETRY".

→ We will encounter this in 'ELECTRODYNAMICS' part. (Unknown upto a gauge field)

**

→ Absolute SCALAR POTENTIAL is ambiguous.

LET'S SEE HOW THIS WORKS IN PRACTICE!

$$W_{12} = \int_{①}^{②} \vec{F} \cdot d\vec{r} = - \int_{①}^{②} \vec{\nabla} V \cdot d\vec{r} = V_1 - V_2$$

$\uparrow$ potential

$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$ ; $\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}$

Check this out in the 1 dimensional version :

1d version: $-\int_{x_1}^{x_2} \frac{d}{dx} V(x)\, dx = -\left[V(x_2) - V(x_1)\right]$

$$W_{12} = T_2 - T_1 = V_1 - V_2 \Leftarrow$$ Conservative force

$$\Rightarrow \boxed{V_1 + T_1 = V_2 + T_2}$$

Total energy at the start of journey at pt. 1 is equal to the total energy at the end of journey at pt. 2.

$$\boxed{E = T + V = \text{constant}}$$

$\Rightarrow$ CONSERVATION OF ENERGY