

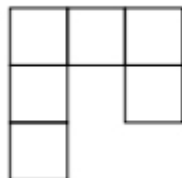


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**Problem 1.** Let  $ABC$  be an acute-angled triangle with  $AB \neq AC$ . The circle with diameter  $BC$  intersects the sides  $AB$  and  $AC$  at  $M$  and  $N$  respectively. Denote by  $O$  the midpoint of the side  $BC$ . The bisectors of the angles  $BAC$  and  $MON$  intersect at  $R$ . Prove that the circumcircles of the triangles  $BMR$  and  $CNR$  have a common point lying on the side  $BC$ .

**Problem 2.** Find all polynomials  $f$  with real coefficients such that for all real numbers  $a, b, c$  such that  $ab + bc + ca = 0$  we have the relation  $f(a - b) + f(b - c) + f(c - a) = 2f(a + b + c)$ .

**Problem 3.** Define a “hook” to be a figure made up of six unit squares as shown below in the picture, or any of the figures obtained by applying rotations and reflections to this figure.



Determine all  $m \times n$  rectangles that can be covered without gaps and without overlaps with hooks such that

- the rectangle is covered without gaps and without overlaps
- no part of a hook covers area outside the rectangle.

**Problem 4.** Let  $n \geq 3$  be an integer. Let  $t_1, t_2, \dots, t_n$  be positive real numbers such that

$$n^2 + 1 > (t_1 + t_2 + \dots + t_n) \left( \frac{1}{t_1} + \frac{1}{t_2} + \dots + \frac{1}{t_n} \right).$$

Show that  $t_i, t_j, t_k$  are side lengths of a triangle for all  $i, j, k$  with  $1 \leq i < j < k \leq n$ .

**Problem 5.** In a convex quadrilateral  $ABCD$  the diagonal  $BD$  does not bisect the angles  $ABC$  and  $CDA$ . The point  $P$  lies inside  $ABCD$  and satisfies  $PBC = DBA$  and  $PDC = BDA$ . Prove that  $ABCD$  is a cyclic quadrilateral if and only if  $AP = CP$ .

**Problem 6.** We call a positive integer alternating if every two consecutive digits in its decimal representation are of different parity. Find all positive integers  $n$  such that  $n$  has a multiple which is alternating.