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EXERCISE 4.1

Evaluate the determinants in Exercises 1 and 2.

1. $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

2. (i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ (ii) $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

3. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$.

4. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$.

5. Evaluate the determinants

   (i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ (ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

   (iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ (iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

6. If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, find $|A|$.

7. Find values of x, if

   (i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ (ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

8. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

   (A) 6 (B) ± 6 (C) -6 (D) 0

4.3 Properties of Determinants

In the previous section, we have learnt how to expand the determinants. In this section, we will study some properties of determinants which simplify its evaluation by obtaining maximum number of zeros in a row or a column. These properties are true for determinants of any order. However, we shall restrict ourselves upto determinants of order 3 only.

Property 1

The value of the determinant remains unchanged if its rows and columns are interchanged.

Verification Let

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Expanding along first row, we get

$\Delta = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$

By interchanging the rows and columns of Δ, we get $\Delta_1 = a_1(b_2c_3 - c_2b_3) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$

Hence

$\Delta = \Delta_1$.

> **Note** If R_i is ith row and C_i is ith column, then for interchange of row and column, we symbolically write $C_i \leftrightarrow R_i$. Let us verify the above property by example.

Example 6

Verify Property 1 for

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

Solution

Expanding the determinant along first row, we have

$$\Delta = 2\begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix} - (-3)\begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} + 5\begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix}$$

$= 2(0 - 20) + 3(-42 - 4) + 5(30 - 0)$

$= -40 - 138 + 150$

$= -28$

By interchanging rows and columns, we get

$$\Delta_1 = \begin{vmatrix} 2 & 6 & 1 \\ -3 & 0 & 5 \\ 5 & 4 & -7 \end{vmatrix}$$

Expanding along first column, we get

$$\Delta_1 = 2\begin{vmatrix} 0 & 5 \\ 4 & -7 \end{vmatrix} - (-3)\begin{vmatrix} 6 & 1 \\ 4 & -7 \end{vmatrix} + 5\begin{vmatrix} 6 & 1 \\ 0 & 5 \end{vmatrix}$$

$= 2(0 - 20) + 3(-42 - 4) + 5(30 - 0)$

$= -28$

Clearly

$\Delta = \Delta_1$.

Hence, Property 1 is verified.

Property 2

If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.

Verification Let

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Expanding along first row, we get

$\Delta = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$

Interchanging first and third rows, the new determinant obtained is

$$\Delta_1 = \begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

Expanding Δ_1 along third row, we get

$\Delta_1 = a_1(c_2b_3 - b_2c_3) - a_2(c_1b_3 - c_3b_1) + a_3(b_2c_1 - b_1c_2)$

$= -\left[a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)\right]$

Clearly

$\Delta_1 = -\Delta$.

Similarly, we can verify the result by interchanging any two columns.