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Problem 1.
Let ABC be a triangle with incentre I. A point P inside the triangle satisfies $PBA + PCA = PBC + PCB$. Show that $AP >= AI$, and that equality holds if and only if $P = I$.

Problem 2.
Let P be a regular polygon with 2006 sides. A diagonal of P is called good if its endpoints divide the boundary of P into two parts, each consisting of an odd number of sides. The sides of P are also called good.

Suppose that P is dissected into triangles by 2003 diagonals, no two of which have a common interior point. Find the maximum number of isosceles triangles having two good sides that can occur in such a dissection.

Problem 3.
Determine the least real number M such that the inequality

$$|ab(a^2 - b^2) + bc(b^2 - c^2) + ca(c^2 - a^2)| <= M(a^2 + b^2 + c^2)^2$$

holds for all real numbers a, b, and c.

Problem 4.
Determine all pairs (x, y) of integers such that

$$1 + 2^x + 2^{2x+1} = y^2.$$

Problem 5.
Let $P(x)$ be a polynomial of degree $n > 1$ with integer coefficients, and let k be a positive integer. Define $Q(x) = P(P(P(x)))$ where P occurs k times. Prove that there are at most n integers t such that $Q(t) = t$.

Problem 6.
For each side b of a convex polygon P, assign the maximum area of a triangle that has b as a side and is contained in P. Show that the sum of the assigned areas is at least twice the area of P.