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12 July 2006

**Problem 1.**

Let  $ABC$  be a triangle with incentre  $I$ . A point  $P$  inside the triangle satisfies  $PBA + PCA = PBC + PCB$ . Show that  $AP \geq AI$ , and that equality holds if and only if  $P = I$ .

**Problem 2.**

Let  $P$  be a regular polygon with 2006 sides. A diagonal of  $P$  is called good if its endpoints divide the boundary of  $P$  into two parts, each consisting of an odd number of sides. The sides of  $P$  are also called good.

Suppose that  $P$  is dissected into triangles by 2003 diagonals, no two of which have a common interior point. Find the maximum number of isosceles triangles having two good sides that can occur in such a dissection.

**Problem 3.**

Determine the least real number  $M$  such that the inequality

$$|ab(a^2 - b^2) + bc(b^2 - c^2) + ca(c^2 - a^2)| \leq M(a^2 + b^2 + c^2)^2$$

holds for all real numbers  $a$ ,  $b$ , and  $c$ .

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**Problem 4.**

Determine all pairs  $(x, y)$  of integers such that

$$1 + 2^x + 2^{2x+1} = y^2.$$

**Problem 5.**

Let  $P(x)$  be a polynomial of degree  $n > 1$  with integer coefficients, and let  $k$  be a positive integer. Define  $Q(x) = P(P(P(x)))$  where  $P$  occurs  $k$  times. Prove that there are at most  $n$  integers  $t$  such that  $Q(t) = t$ .

**Problem 6.**

For each side  $b$  of a convex polygon  $P$ , assign the maximum area of a triangle that has  $b$  as a side and is contained in  $P$ . Show that the sum of the assigned areas is at least twice the area of  $P$ .