



Keshava Reddy V  
ID: COMETFWC054  
47th International Mathematical Olympiad 2006

12 July 2006

1. Let  $ABC$  be a triangle with incentre  $I$ . A point  $P$  in the interior of the triangle satisfies  $\angle PBA + \angle PCA = \angle PBC + \angle PCB$ .

Show that  $AP \geq AI$ , and that equality holds if and only if  $P = I$ .

2. Let  $P$  be a regular 2006-gon. A diagonal of  $P$  is called good if its endpoints divide the boundary of  $P$  into two parts, each composed of an odd number of sides of  $P$ . The sides of  $P$  are also called good.

Suppose  $P$  has been dissected into triangles by 2003 diagonals, no two of which have a common point in the interior of  $P$ . Find the maximum number of isosceles triangles having two good sides that could appear in such a configuration.

3. Determine the least real number  $M$  such that the inequality  $|ab(a^2 - b^2) + bc(b^2 - c^2) + ca(c^2 - a^2)| \leq M(a^2 + b^2 + c^2)^2$  holds for all real numbers  $a, b$  and  $c$ .

13 July 2006

4. Determine all pairs  $(x, y)$  of integers such that  $1 + 2^x + 2^{2x+1} = y^2$ .
5. Let  $P(x)$  be a polynomial of degree  $n > 1$  with integer coefficients and let  $k$  be a positive integer. Consider the polynomial  $Q(x) = P(P(P(x)))$ , where  $P$  occurs  $k$  times. Prove that there are at most  $n$  integers  $t$  such that  $Q(t) = t$ .
6. Assign to each side  $b$  of a convex polygon  $P$  the maximum area of a triangle that has  $b$  as a side and is contained in  $P$ . Show that the sum of the areas assigned to the sides of  $P$  is at least twice the area of  $P$ .