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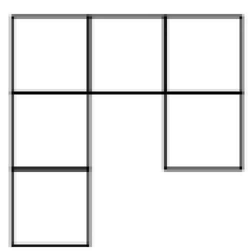
Problem 1.
Let ABC be an acute-angled triangle with AB not equal to AC. The circle with diameter BC intersects the sides AB and AC at M and N respectively. Let O be the midpoint of BC. The bisectors of the angles BAC and MON intersect at R. Prove that the circumcircles of triangles BMR and CNR have a common point on the side BC.

Problem 2.
Find all polynomials f with real coefficients such that for all real numbers a, b, c satisfying $ab + bc + ca = 0$,
$f(a - b) + f(b - c) + f(c - a) = 2f(a + b + c)$.

Problem 3.
Define a hook to be a figure made up of six unit squares as shown below, or any figure obtained by rotations and reflections of this figure.



Determine all rectangles of size m by n that can be covered using hooks such that:

- the rectangle is covered without gaps and without overlaps;
- no part of a hook lies outside the rectangle.

Problem 4.

Let $n >= 3$ be an integer. Let t_1, t_2, t_3, t_4, t_5 up to t_n be positive real numbers such that
$n^2 + 1 > (t_1 + t_2 + t_3 + t_4 + t_5 + t_n)(1/t_1 + 1/t_2 + 1/t_3 + 1/t_4 + 1/t_5 + 1/t_n)$.
Show that t_i, t_j, t_k are side lengths of a triangle for all i, j, k with $1 <= i < j < k <= n$.

Problem 5.

In a convex quadrilateral $ABCD$, the diagonal BD does not bisect the angles ABC and CDA. A point P lies inside $ABCD$ such that $PBC = DBA$ and $PDC = BDA$. Prove that $ABCD$ is cyclic if and only if $AP = CP$.

Problem 6.

A positive integer is called alternating if every two consecutive digits in its decimal representation have different parity. Find all positive integers n such that n has a multiple which is alternating.