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Problem 1. Let ABC be a triangle with incenter I. A point P in the interior of the triangle satisfies $PBA + PCA = PBC + PCB$.

Show that $AP >= AI$, and that equality holds if and only if $P = I$.

Problem 2. Let P be a regular 2006-gon. A diagonal of P is called good if its endpoints divide the boundary of P into two parts, each composed of an odd number of sides of P. The sides of P are also called good.

Suppose P has been dissected into triangles by 2003 diagonals, no two of which have a common point in the interior of P. Find the maximum number of isosceles triangles having two good sides that could appear in such a configuration.

Problem 3. Determine the least real number M such that

$|ab(a^2 - b^2) + bc(b^2 - c^2) + ca(c^2 - a^2)| <= M(a^2 + b^2 + c^2)^2$

holds for all real numbers a, b and c.

Time allowed: 4 hours 30 minutes Each problem is worth 7 points

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Problem 4. Determine all pairs (x, y) of integers such that

$1 + 2^x + 2^{2x+1} = y^2$.

Problem 5. Let $P(x)$ be a polynomial of degree $n > 1$ with integer coefficients and let k be a positive integer.

Define $Q(x) = P(P(P(x)))$ where P is applied k times.

Prove that there are at most n integers t such that $Q(t) = t$.

Problem 6. Assign to each side b of a convex polygon P the maximum area of a triangle that has b as a side and is contained in P.

Show that the sum of the areas assigned to the sides of P is at least twice the area of P.

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