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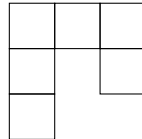
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1. **Problem 1.** Let ABC be an acute-angled triangle with $AB \neq AC$. The circle with diameter BC intersects the sides AB and AC at M and N respectively. Denote by O the midpoint of the side BC . The bisectors of the angles $\angle BAC$ and $\angle MON$ intersect at R . Prove that the circumcircles of the triangles BMR and CNR have a common point lying on the side BC .
2. **Problem 2.** Find all polynomials f with real coefficients such that for all real numbers a, b, c satisfying $ab + bc + ca = 0$ we have

$$f(a - b) + f(b - c) + f(c - a) = 2f(a + b + c).$$

3. **Problem 3.** Define a “hook” to be a figure made up of six unit squares as shown below, or any of the figures obtained by applying rotations and reflections to this figure.



Determine all $m \times n$ rectangles that can be covered without gaps and without overlaps with hooks such that

- the rectangle is covered without gaps and without overlaps;
 - no part of a hook covers area outside the rectangle.
4. **Problem 4.** Let $n \geq 3$ be an integer. Let t_1, t_2, \dots, t_n be positive real numbers such that

$$n^2 + 1 > (t_1 + t_2 + \dots + t_n) \left(\frac{1}{t_1} + \frac{1}{t_2} + \dots + \frac{1}{t_n} \right).$$

Show that t_i, t_j, t_k are side lengths of a triangle for all $1 \leq i < j < k \leq n$.

5. **Problem 5.** In a convex quadrilateral $ABCD$ the diagonal BD does not bisect the angles $\angle ABC$ and $\angle CDA$. The point P lies inside $ABCD$ and satisfies

$$\angle PBC = \angle DBA \quad \text{and} \quad \angle PDC = \angle BDA.$$

Prove that $ABCD$ is a cyclic quadrilateral if and only if $AP = CP$.

6. **Problem 6.** We call a positive integer alternating if every two consecutive digits in its decimal representation are of different parity. Find all positive integers n such that n has a multiple which is alternating.