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**Problem 1.** Six points are chosen on the sides of an equilateral triangle  $ABC$ :  $A_1, A_2$  on  $BC$ ,  $B_1, B_2$  on  $CA$  and  $C_1, C_2$  on  $AB$ , such that they are the vertices of a convex hexagon  $A_1A_2B_1B_2C_1C_2$  with equal side lengths. Prove that the lines  $A_1B_2$ ,  $B_1C_2$  and  $C_1A_2$  are concurrent.

**Problem 2.** Let  $a_1, a_2, \dots$  be a sequence of integers with infinitely many positive and negative terms. Suppose that for every positive integer  $n$  the numbers  $a_1, a_2, \dots, a_n$  leave  $n$  different remainders upon division by  $n$ . Prove that every integer occurs exactly once in the sequence  $a_1, a_2, \dots$

**Problem 3.** Let  $x, y, z$  be three positive real numbers such that  $xyz \geq 1$ . Prove that

$$\frac{x^5 - x^2}{x^5 + y^2 + z^2} + \frac{y^5 - y^2}{x^2 + y^5 + z^2} + \frac{z^5 - z^2}{x^2 + y^2 + z^5} \geq 0.$$

**Problem 4.** Determine all positive integers relatively prime to all the terms of the infinite sequence

$$a_n = 2^n + 3^n + 6^n - 1, \quad n \geq 1.$$

**Problem 5.** Let  $ABCD$  be a fixed convex quadrilateral with  $BC = DA$  and  $BC$  not parallel to  $DA$ . Let two variable points  $E$  and  $F$  lie on the sides  $BC$  and  $DA$ , respectively, and satisfy  $BE = DF$ . The lines  $AC$  and  $BD$  meet at  $P$ , the lines  $BD$  and  $EF$  meet at  $Q$ , the lines  $EF$  and  $AC$  meet at  $R$ . Prove that the circumcircles of the triangles  $PQR$ , as  $E$  and  $F$  vary, have a common point other than  $P$ .

**Problem 6.** In a mathematical competition, in which 6 problems were posed to the participants, every two of these problems were solved by more than  $\frac{2}{5}$  of the contestants. Moreover, no contestant solved all the 6 problems. Show that there are at least 2 contestants who solved exactly 5 problems each.