Date

NUM Using Taylor's thm. Find the expansion of $\tan x$ in ascending powers of $(x-\pi/4)$

– Let $f(x) = \tan x$, $a = \pi/4$

By Taylor's thm.

$$f(x) = f(\pi/4) + \frac{(x-\pi/4)}{1!} f'(\pi/4) + \frac{(x-\pi/4)^2}{2!} f''(\pi/4) + \dots \quad (1)$$

$f(x) = \tan x$ $\qquad f(\pi/4) = 1$

$f'(x) = \sec^2 x$ $\qquad f'(\pi/4) = 2$

$f''(x) = 2\sec^2 x \tan x$ $\qquad f''(\pi/4) =$

$f'''(x) = 2\sec^4 x + 4\sec^2 x \tan^2 x$ $\qquad f'''(\pi/4) = 12$

Substitute all values in eq (1).

$$\tan x = 1 + \frac{(x-\pi/4)}{1!}(2) + \frac{(x-\pi/4)^2}{2!}(4) + \dots$$

$$= 1 + 2(x-\pi/4) + 2(x-\pi/4)^2 + \dots$$

NUM Expand $3x^3 - 2x^2 + x - 6$ in powers of $x - 3$

– Let $f(x) = 3x^3 - 2x^2 + x - 6$ $\quad a = 3$

By Taylor's thm.

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \dots$$

$$f(x) = f(3) + (x-3) f'(3) + \frac{(x-3)^2}{2!} f''(3) + \frac{(x-3)^3}{3!} f'''(3) + \dots$$

$f(x) = 3x^3 - 2x^2 + x - 6$ $\qquad f(3) = 60$

$f'(x) = 9x^2 - 4x + 1$ $\qquad f'(3) = 70$

$f''(x) = 18x - 4$ $\qquad f'''(3) = 50$

$f'''(x) = 18$ $\qquad f'''(3) = 18$

Substitude in eq (1);

$$f(x) = 60 + (x-3)70 + \frac{(x-3)^2}{2!} 50 + \frac{(x-3)^3}{3!} 18 + \dots$$

$$= 60 + 70(x-3) + 25(x-3)^2 + 3(x-3)^3 + \dots$$

Num Expand $3x^3-2x^2+x-6$ in powers of $x-2$

Num Expand $3x^3-2x^2+x-6$, $a=2$

- Let $f(x)=3x^3-2x^2+x-6$

By Taylor's thm

$f(x)=f(a)+\frac{(x-a)}{1!}f'(a)+\frac{(x-a)^2}{2!}f''(a)+\frac{(x-a)^3}{3!}f'''(a)+\dots$

$=f(2)+\frac{(x-2)}{1!}f'(2)+\frac{(x-2)^2}{2!}f''(2)+\frac{(x-2)^3}{3!}f'''(2)+\dots$

$f(x)=3x^3-2x^2+x-6$ $\quad f(2)=12$

$f'(x)=9x^2-4x+1$ $\quad f'(2)=29$

$f''(x)=18x-4$ $\quad f''(2)=32$

$f'''(x)=18$ $\quad f'''(2)=18$

Substitute in eq (1)

$f(x)=12+\frac{(x-2)}{1!}29+(x-2)^2 16+(x-2)^3\times 3$

$=12+29(x-2)+16(x-2)^2+3(x-2)^3$

Num Expand x^3+7x^2+x-6 in powers of $x-3$

- Let $f(x)=x^3+7x^2+x-6$ $\quad f(3)=87$

$f'(x)=3x^2+14x+1$ $\quad f'(3)=70$

$f''(x)=6x+14$ $\quad f''(3)=32$

$f'''(x)=6$ $\quad f'''(4)=6$

$f^{IV}(x)=0$ $\quad f^{IV}(3)=0$

Taylor's series about $x=a$

$f(x)=f(a)+\frac{(x-a)}{1!}f'(a)+\frac{(x-a)^2}{2!}f''(a)+\dots$

Put $a=3$

$f(x)=f(3)+\frac{(x-3)}{1!}f'(3)+\frac{(x-3)^2}{2!}f''(3)+\dots$

$=87+70(x-3)+\frac{32}{2}(x-3)^2+\frac{6(x-3)^2}{3!}+0$

$=87+70(x-3)+16(x-3)^2+(x-3)^2$

Num Using Taylor's theorem expand

$2x^3+3x^2-8x+7$

2/12/2022 UNIT 1

Expand $40+53(x-2)+19(x-2)^2+2(x-3)^3$ in ascending powers of x

- Let $f(x+h)=40+53(x-2)+19(x-2)^2+2(x-3)^3=f(x-2)$

$f(x)=40+53(x)+19(x)^2+2(x)^3$

By Taylor's theorem,

$f(x+h)=f(h)+\frac{x}{1!}f'(h)+\frac{x^2}{2!}f'''(h)+\frac{x^3}{3!}f'''(h)+\dots$

$f(x-2)=f(-2)+\frac{x}{1!}f'(-2)+\frac{x^2}{2!}f''(-2)+\frac{x^3}{3!}f'''(-2)+\dots$ —(1)

$f(x)=40+53x+19x^2+2x^3$ $\quad f(-2)=-6$

$f'(x)=53+19\times 2x+6x^2$ $\quad f'(-2)=1$

$f''(x)=38+12x$ $\quad f''(-2)=14$

$f'''(x)=12$ $\quad f'''(-2)=12$

$f^{IV}(x)=0$ $\quad f^{IV}(-2)=0$

$f(x-2)=-6+\frac{x}{1!}+14\frac{x^2}{2!}+12\frac{x^3}{3!}+\dots$

$=-6+x+7x^2+2x^3$

Prove that $\tan(x+h)=\tan x+h\sec^2 x+h^2\sec^2 x\tan x+$

- Let $f(x+h)=\tan(x+h)$

$f(x)=\tan x$

By Taylor's thm,

$\therefore f(x+h)=f(x)+\frac{h}{1!}f'(x)+\frac{h^2}{2!}f''(x)+\dots$ —(1)

$f(x) = \cos x$

$f(x+h) = \cos(x+h) = \cos(45+1)$

$x = 45° = \pi/4$

$h = 1° = 1 \times \pi/180°$

Page No

Date

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$f(x) = \tan x$

$f'(x) = \sec^2 x$

$f''(x) = 2\sec^2 x \tan x$

$\vdots$

Substitute all values in (1)

$\therefore \tan(x+h) = \tan x + h\sec^2 x + \frac{h^2}{2} \times 2\sec^2 x \tan x + \ldots$

Expand $f(x) = \log(\cos x)$ about $x = \pi/3$

- Let Given:

$f(x) = \log(\cos x)$, $a = \pi/3$

By Taylor's thm.

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \ldots$$

$$= f(\pi/3) + \frac{(x-\pi/3)}{1!} f'(\pi/3) + \frac{(x-\pi/3)^2}{2!} f''(\pi/3) + \ldots \quad (1)$$

$\therefore f(x) = \log(\cos x)$ $\qquad f(\pi/3) = \log \cos(\pi/3)$

$= \log 1/2$

$= \log 2^{-1}$

$= -\log 2$

$f'(x) = \frac{1}{\cos x} \times -\sin x$ $\qquad f'(\pi/3) = -\tan(\pi/3)$

$= -\sqrt{3}$

$= -\tan x$

$f''(x) = -\sec^2 x$ $\qquad f''(\pi/3) = -\sec^2(\pi/3)$

$= -4$

$\vdots$

Substitute all values in (1).

$$f(x) = -\log 2 + (x - \pi/3) \times -\sqrt{3} + \frac{(-4)(x-\pi/3)^2}{2!} + \ldots$$

$$= -\log 2 - \sqrt{3}(x - \pi/3) - 2(x - \pi/3)^2 + \ldots$$

Expand Find the value of $\sqrt{10}$ by Taylor's thm

- Let $f(x) = \sqrt{x}$

$f(x+h) = \sqrt{x+h} = \sqrt{9+1}$

$x = 9$, $h = 1$

By Taylor's thm

$$f(x+h) = f(x) + \frac{h}{1!} f'(x) + \frac{h^2}{2!} f''(x) + \ldots$$

$$f(10) = \sqrt{10} = f(9) + \frac{1}{1!} f'(9) + \frac{1}{2!} f''(9) + \ldots$$

$f(x) = \sqrt{x}$ $\qquad f(9) = \sqrt{9} = 3$

$f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2}(x)^{-1/2}$ $\qquad f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{2 \times 3} = 0.16$

$f''(x) = \frac{1}{2}\left(\frac{-1}{2}\right) x^{-3/2}$ $\qquad f''(9) = \frac{-1}{4 \times 9 \times 3} = -9.25 \times 10^{-3}$

$= \frac{-1}{4x^{3/2}} = \frac{-1}{4x\sqrt{x}}$

$\sqrt{10} = 3 + 0.16 - 0.00925 = 3.15$

Show that $e^{x\cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{3} - \frac{11}{24} x^4 + \ldots$

- We know Maclaurin's Expansion,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$$

Consider,

$$e^{x\cos x} = 1 + \frac{x\cos x}{1!} + \frac{x^2\cos^2 x}{2!} + \frac{x^3\cos^3 x}{3!} + \frac{x^4\cos^4 x}{4!} + \ldots$$

$$= 1 + x\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\right] + \frac{x^2}{2}\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\right]^2 +$$

$$\frac{x^3}{3!}\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\right]^3 + \frac{x^4}{4!}\left[1 - \frac{x^2}{2} + \frac{x^4}{4} - \ldots\right] + \ldots$$

Find maclaurin's expansion of $\sqrt{1+\sin x}$ upto x^6

Page No.
Date

$$= 1 + x\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots\right] + \frac{x^2}{2}\left[1 - x^2 + \dots\right] + \frac{x^3}{6}\left[1 + \dots\right] +$$

$$\frac{x^4}{24}\left[1 + \dots\right] + \dots$$

$$= 1 + x + \frac{x^2}{2} - \frac{x^3}{3} + \frac{11}{24}x^4 - \dots$$

Find maclaurin's expansion of $\sqrt{1+\sin x}$ upto x^6

consider

$$\sqrt{1+\sin x} = \sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$$

$$= \sin\frac{x}{2} + \cos\frac{x}{2} \quad \text{---(1)}$$

$$\left[\sin y = y - \frac{y^3}{3!} + \frac{y^5}{5!} + \dots\right] \left[\cos y = 1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \frac{y^6}{6!} + \dots\right]$$

From (1)

$$= \left[y/2 - \frac{1}{6}\left(\frac{y}{2}\right)^3 + \frac{1}{120}\left(\frac{y}{2}\right)^5 + \dots\right] +$$

$$\left[1 - \frac{1}{2}\left(\frac{y}{2}\right)^2 + \frac{1}{24}\left(\frac{y}{2}\right)^4 - \frac{1}{720}\left(\frac{y}{2}\right)^6 + \dots\right]$$

$$= 1 + \frac{y}{2} - \frac{y^2}{8} - \frac{y^3}{48} + \frac{y^4}{384} + \frac{y^5}{3840} - \frac{y^6}{46080} + \dots$$

Page No.
Date 5 | 12 | 22

UNIT 1

Num Prove that $x\operatorname{cosec}x = 1 + \frac{x^2}{6} + \frac{7}{360}x^4 + \dots$

$$x\operatorname{cosec}x = \frac{x}{\sin x}$$

$$= \frac{x}{x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots}$$

$$= \frac{x}{x\left[1 - \frac{x^2}{6} + \frac{x^4}{5!} + \dots\right]}$$

$$= \frac{1}{\left[1 - \frac{x^2}{6} + \frac{x^4}{120} - \dots\right]}$$

$$= \left[1 - \left(\frac{x^2}{6} \mp \frac{x^4}{120} + \dots\right)\right]^{-1}$$

$$= 1 + \left(\frac{x^2}{6} - \frac{x^4}{120} + \dots\right) + \left(\frac{x^2}{6} - \frac{x^4}{120} + \dots\right)^2 + \dots$$

$$= 1 + \frac{x^2}{6} - \frac{x^4}{120} + \frac{x^4}{36} + \dots$$

$$= 1 + \frac{x^2}{6} + \frac{7x^4}{360} + \dots$$

Num Expand $(1+x)^x$ in ascending powers of x upto 5^{th} powers of x

Let $y = (1+x)^x$

$$\log y = x\log(1+x) \quad (\because \log(1+x) = x - x^2/2 + x^3/3 - x^4/4 + \dots$$

$$= x\left[x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots\right]$$

$$= \left[x^2 - \frac{x^3}{2} + \frac{x^4}{3} - \frac{x^5}{4} + \dots\right]$$

$$y = e^{\left[x^2 - x^3/2 + x^4/3 - x^5/4 + \dots\right]}$$

$$\left[x^2 - \frac{x^3}{2} + \frac{x^4}{3} + \dots\right] + \frac{1}{2!}\left[x^2 - \frac{x^3}{2} + \frac{x^4}{3} + \dots\right]^2$$

$$= 1 + \frac{1}{1!}\left[x^2 - \frac{x^3}{2} + \frac{x^4}{3}\right] + \frac{1}{2!}\left[x^4 - 2(x^2)\left(-\frac{x^3}{2}\right) + \dots\right] + \dots$$

$$= 1 + x^2 - \frac{x^3}{2} + \frac{x^4}{3} + \dots + \frac{1}{2}\left[x^4 - 2(x^2)\left(-\frac{x^3}{2}\right) + \dots\right] + \dots$$

$$= 1 + x^2 - \frac{x^3}{2} + \frac{x^4}{3} + \frac{x^4}{2} + \frac{x^5}{2} + \dots$$

$$= 1 + x^2 - \frac{x^3}{2} + \frac{5x^4}{6} + \frac{x^5}{2} + \dots$$

Expansion by using Differentiation & Integration

Num Prove that $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

Let $y = \tan^{-1}x$

Diff w.r.t x

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$= (1+x^2)^{-1}$$

$$= 1 - x^2 + (x^2)^2 - (x^2)^3 + \dots \quad \left[\because (1+z)^{-1} = 1 - z + z^2 - z^3 + \dots\right]$$

$$= 1 - x^2 + x^4 - x^6 + \dots$$

$$\int_0^x dy = \int_0^x (1 - x^2 + x^4 - x^6 + \dots)\,dx$$

$$[y]_0^x = \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots\right]_0^x$$

$$[\tan^{-1}x]_0^x = \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots\right]_0^x$$

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

Num Expand $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ in ascending powers of x

Let $x = \tan\theta \quad \therefore \theta = \tan^{-1}x$

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$= 2\tan^{-1}x$$

$$= 2\left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots\right]$$

Num Expand $\cos^{-1}\left(\frac{x^{-1}-x}{x^{-1}+x}\right) = \cos^{-1}\left(\frac{1/x - x}{1/x + x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put $x = \tan\theta \quad \therefore \theta = \tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

$$\cos^{-1}\left(\frac{x^{-1}-x}{x^{-1}+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$= \cos^{-1}(\cos 2\theta)$$

$$= 2\theta$$

$$= 2\tan^{-1}x$$

$$= 2\left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots\right]$$

Num $\tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right) = \frac{1}{2}\left(x - \frac{x^3}{3} + \frac{x^5}{5} + \dots\right)$ show

Put $x = \tan\theta \quad \therefore \theta = \tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$

$$\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta} - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$= \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$

$= \tan^{-1}\left[\frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2}\right]$

$= \tan^{-1}\left[\tan\theta/2\right]$

$= \frac{\theta}{2} = \frac{\tan^{-1}x}{2} = \frac{1}{2}\left[x - \frac{x^3}{3} + \frac{x^5}{5} - \dots\right]$

1/12/22 UNIT 1

1) $\lim_{x\to 0} \frac{\sin x}{x} = 1$

2) $\lim_{x\to 0} \frac{\tan x}{x} = 1$

3) $\lim_{x\to 0} \frac{\sin^2 x}{x} = 1$

4) $\lim_{x\to 0} \frac{\tan^{-1}x}{x} = 1$

5) $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$

6) $\lim_{x\to 0} \frac{a^x - 1}{x} = \log a$

7) $\lim_{x\to 0} (1+x)^{1/x} = e$

Q Evaluate $\lim_{x\to 0} \frac{e^{ax} - e^{-bx}}{\log(1+bx)}$

— Let $L = \lim_{x\to 0} \frac{e^{ax} - e^{-ax}}{\log(1+bx)}$ —— $\left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{ae^{ax} + ae^{-ax}}{b/1+bx}$ —— (By LHOSP Rule)

$L = \frac{2a}{b}$

Q Evaluate $\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}$

— Let $L = \lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}$ —— $\left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{xe^x + e^x - 1/1+x}{2x}$ —— $\left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{xe^x + e^x + e^x + 1/(1+x)^2}{2}$ — By LHOSP Rule

$= 3/2$

Q Evaluate $\lim_{x\to 0} \frac{e^{2x} - (1+x)^2}{x\log(1+x)}$

— Let $L = \lim_{x\to 0} \frac{e^{2x} - (1+x)^2}{x\log(1+x)}$ —— $(0/0)$

$= \lim_{x\to 0} \frac{2e^{2x} - 2(1+x)}{x \times \frac{1}{1+x} + \log(1+x)}$ —— $\left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{4e^{2x} - 2}{\frac{(1+x) - x}{(1+x)^2} + \frac{1}{1+x}}$ — (By 2 LHOSP Rule) $\frac{1}{1} + 1$

$= 1$

7/12 UNIT 1

Q $\lim_{x\to 0} \frac{e^{2x} - (1+x)^2}{x\log(1+x)}$

$\lim_{x\to 0} \frac{e^{2x} - (1+x)^2}{\frac{x^2}{x}\log(1+x)}$

$\lim_{x\to 0} \frac{e^{2x} - (1+x)^2}{x^2 \log(1+x)^{1/x}}$

$\lim_{x\to 0} \frac{e^{2x} - (1+x)^2}{x^2} \lim_{x\to 0} \frac{1}{\log(1+x)^{1/x}}$

$$\lim_{x\to 0} \frac{e^{2x}-(1+x)^2}{x^2 \log e}$$

By LHOSP Rule

$$\lim_{x\to 0} \frac{2e^{2x}-2(1+x)}{2x}$$

$$\lim_{x\to 0} \frac{4e^{2x}-2}{2}$$

$$\lim_{x\to 0} \frac{2}{2} = 1$$

Q $L = \lim_{x\to \pi/2} \frac{\tan 3x}{\tan x}$ —— $\left(\frac{\infty}{\infty}\right)$ (By LHOSP Rule)

$$= \lim_{x\to \pi/2} \frac{3\sec^2 3x}{\sec^2 x}$$ —— (By LHOSP Rule)

$$= \lim_{x\to \pi/2} \frac{3\ {}^1/\cos^2 3x}{{}^1/\cos^2 x}$$

$$= \lim_{x\to \pi/2} \frac{3\cos^2 x}{\cos^2 3x}$$

$$= \lim_{x\to \pi/2} \frac{-3\times 2\sin x \cos x}{-2\cos 3x \sin 3x \times 3}$$

$$= \lim_{x\to \pi/2} \frac{-\cancel{6}\sin x\cos x}{-\cancel{6}\sin 3x\cos 3x}$$

$$= \lim_{x\to \pi/2} \frac{\cos x}{\cos 3x} \lim_{x\to \pi/2} \frac{\sin x}{\sin 3x}$$

$$= \lim_{x\to \pi/2} \frac{\cos x}{\cos 3x} \quad x-1$$

$$= -1 \times \lim_{x\to \pi/2} \frac{\cos x}{\cos 3x}$$

$$= -\lim_{x\to \pi/2} \frac{-\sin x}{-3\sin 3x}$$

$$= -\left(\frac{-1}{-3\times(-1)}\right) = \frac{1}{3}$$

Q $\lim_{x\to \pi/2} \frac{\tan 3x}{\tan x}$

$$\lim_{x\to \pi/2} \frac{\frac{\sin 3x}{\cos 3x}}{\frac{\sin x}{\cos x}}$$

$$\lim_{x\to \pi/2} \frac{\sin 3x \cos x}{\cos 3x \sin x}$$

$$\lim_{x\to \pi/2} \frac{\sin 3x}{\sin x} \lim_{x\to \pi/2} \frac{\cos x}{\cos 3x}$$

$$-1 \lim_{x\to \pi/2} \frac{\cos x}{\cos 3x}$$

$$-\lim_{x\to \pi/2} \frac{-\sin x}{-3\sin 3x}$$

$$-\left(\frac{-1}{-3\times -1}\right) = \frac{1}{3}$$

Q $L = \lim_{x\to 0} \log_x \tan x$

$$= \lim_{x\to 0} \frac{\log \tan x}{\log x} —— \left(\frac{\infty}{\infty}\right)$$

By LHOSP Rule.

$$= \lim_{x\to 0} \frac{{}^1/\tan x \times \sec^2 x}{{}^1/x}$$

$$= \lim_{x\to 0} \frac{\sec^2 x}{\frac{\tan x}{x}}$$

$$= \lim_{x\to 0} \sec^2 x$$

$$= 1$$

Q $\lim_{x\to 1} \frac{x}{x-1} - \frac{1}{\log x}$ —— $(\infty - \infty)$

$$\lim_{x\to 1} \frac{x\log x - x + 1}{(x-1)\log x} —— \left(\frac{0}{0}\right)$$

By Hosp Rule.

$$\lim_{x\to1} \frac{x \times 1/x + \log x - 1}{\frac{(x-1)}{x} + \log x}$$

$$\lim_{x\to1} \frac{\log x}{\frac{(x-1)}{x} + \log x}$$

$$\lim_{x\to1} \frac{\log x}{\log x + 1 - 1/x}$$

$$\lim_{x\to1} \frac{1/x}{1/x + 1/x^2}$$

$$\lim_{x\to1} \frac{1}{1+1} = \frac{1}{2}$$

Q $\lim_{x\to0} \frac{1+\sin x - \cos x + \log(1-x)}{x\tan^2 x}$ — $\left(\frac{0}{0}\right)$

$$\lim_{x\to0} \frac{1+\sin x - \cos x + \log(1-x)}{\frac{x \tan^2 x \times x^2}{x^2}}$$

$$\lim_{x\to0} \frac{1+\sin x - \cos x + \log(1-x)}{x^3}$$

$$\lim_{x\to0} \frac{\cos x + \sin x + 1/1-x \times -1}{3x^2}$$

$$\lim_{x\to0} \frac{\cos x + \sin x - 1/1-x}{3x^2} \text{ — } \left(\frac{0}{0}\right)$$

$$\lim_{x\to0} \frac{-\sin x + \cos x + 1/(1-x)^2 \times -1}{6x}$$

$$\lim_{x\to0} \frac{-\sin x + \cos x - 1/(1-x)^2}{6x} \text{ — } \left(\frac{0}{0}\right)$$

$$\lim_{x\to0} \frac{-\cos x - \sin x - (-2) \times 1/(1-x)^3 \times -1}{6}$$

$$\lim_{x\to0} \frac{-\cos x - \sin x - 2/(1-x)^3}{6}$$

$$\frac{-1-0-2}{6} = \frac{-3}{6} = -\frac{1}{2}$$

Q Evaluate $\lim_{x\to0} (\cot x)^{\sin x}$

— Let $L = \lim_{x\to0} (\cot x)^{\sin x}$ — (∞^0)

$$\log L = \lim_{x\to0} \sin x \log(\cot x)$$

$$\cancel{\log L = \lim_{x\to0} \sin x \times \frac{-\operatorname{cosec}^2 x}{\cot x} + \log(\cot x)}$$

$$\log L = \lim_{x\to0} \frac{\log(\cot x)}{\operatorname{cosec} x} \text{ — } \left(\frac{\infty}{\infty}\right)$$

$$\log L = \lim_{x\to0} \frac{1/\cot x \times -\operatorname{cosec}^{\cancel{2}} x}{-\cancel{\operatorname{cosec} x} \cot x}$$

$$\log L = \lim_{x\to0} \frac{\operatorname{cosec} x}{\cot^2 x} \text{ — } \left(\frac{\infty}{\infty}\right)$$

$$\log L = \lim_{x\to0} \frac{1/\cancel{\sin x}}{\frac{\cos^2 x}{\sin^{\cancel{2}} x}}$$

$$\log L = \lim_{x\to0} \frac{\sin x}{\cos^2 x}$$

$$\log L = 0$$

$$L = e^0 = 1$$

Q $\lim_{x\to e} (\log x)^{1/1-\log x}$ — (1^∞)

$$\log L = \lim_{x\to e} \frac{1}{1-\log x} \log(\log x)$$

$$\log L = \lim_{x\to e} \frac{\log(\log x)}{1-\log x} \text{ — } \left(\frac{0}{0}\right)$$

By L Hosp Rule.

$$\log L = \lim_{x\to e} \frac{1/\log x \times \cancel{1/x}}{-\cancel{1/x}}$$

$$= -\lim_{x\to e} \frac{1}{\log x}$$

$\log L = -1$

$L = e^{-1}$

Q Evaluate $\lim_{x \to 0}\left(\frac{2^x+3^x+5^x}{2}\right)^{1/x}$

$- \quad L = \lim_{x \to 0}\left(\frac{2^x+3^x+5^x}{3}\right)^{1/x} \quad - (1^\infty)$

$\log L = \lim_{x \to 0} \frac{1}{x} \log\left(\frac{2^x+3^x+5^x}{3}\right)$

$\log L = \lim_{x \to 0} \frac{1}{x} \log(2^x+3^x+5^x) - \log 3$

$\log L = \lim_{x \to 0} \frac{\log(2^x+3^x+5^x) - \log 3}{x} \quad - \left(\frac{0}{0}\right)$

$\log L = \lim_{x \to 0} \frac{1}{2^x+3^x+5^x} \times 2^x \log 2 + 3^x \log 3 + 5^x \log 5$

$\log L = \lim_{x \to 0} \frac{\log 2 + \log 3 + \log 5}{3}$

$\log L = \frac{\log 30}{3}$

$L = e^{1/3 \log 30}$

$L = \sqrt[3]{30}$

Q Evaluate unknowns

If $\lim_{x \to 0} \frac{\sin 2x + p\sin x}{x^3}$ is finite then find value of p. Hence find value of limit.

$- \quad L = \lim_{x \to 0} \frac{\sin 2x + p\sin x}{x^3} \quad - \left(\frac{0}{0}\right)$

By LHosp Rule

$= \lim_{x \to 0} \frac{2\cos 2x + p\cos x}{3x^2} \quad - \left(\frac{2+p}{0}\right)$

Denominator $= 0$ and the value of limit is finite

$\therefore 2 + p = 0$

$p = -2$

$L = \lim_{x \to 0} \frac{\sin 2x - 2\sin x}{x^3} \quad - \left(\frac{0}{0}\right)$

Q Find value of a & b if $\lim_{x \to 0} (x^{-3}\sin x + a x^{-2} + b) = 0$

$- \quad L = \lim_{x \to 0} (x^{-3}\sin x + a x^{-2} + b) = 0$

$= \lim_{x \to 0} \frac{\sin x}{x^3} + \frac{a}{x^2} + b = 0$

$= \lim_{x \to 0} \frac{\sin x + a x + b x^3}{x^3} = 0 \quad - \left(\frac{0}{0}\right)$

By LHosp Rule,

$= \lim_{x \to 0} \frac{\cos x + a + 3b x^2}{3x^2} = 0 \quad - \left(\frac{1+a}{0}\right)$

Since limit is finite

Denominator $= 0$

Num $= 0$

$1 + a = 0$

$a = -1$

$= \lim_{x \to 0} \frac{-\sin x + 6bx}{6x} = 0 \quad - \left(\frac{0}{0}\right)$

By LHosp Rule,

$= \lim_{x \to 0} \frac{-\cos x + 6b}{6} = 0 \quad - \left(\frac{-1+b6}{6}\right)$

$\frac{-1+6b}{6} = 0$

$b = 1/6$