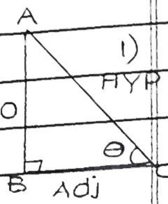
TRIGONOMETRY FORMULA



1) In right triangle ABC

$\sin\theta = \dfrac{\text{side opposite to angle } \theta}{\text{Hypotenuse}}$

$\cos\theta = \dfrac{\text{side adjacent to angle } \theta}{\text{Hypotenuse}}$

$\tan\theta = \dfrac{\text{side opposite to angle } \theta}{\text{side adjacent to angle } \theta}$

2)

sin positive	All positive
tan positive	cos positive

3)

	0	30	45	60	90
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	ND
$\operatorname{cosec}\theta$	ND	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	ND
$\cot\theta$	ND	$\sqrt{3}$	1	$1/\sqrt{3}$	0

4) $\operatorname{cosec}\theta = 1/\sin\theta$

$\sec\theta = 1/\cos\theta$

$\cot\theta = 1/\tan\theta$

$\cos^2\theta + \sin^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

5) Negative angles (Even-odd Identities)

$\cos(-x) = \cos x$	$\sec(-x) = \sec x$
$\sin(-x) = -\sin x$	$\operatorname{cosec}(-x) = -\operatorname{cosec} x$
$\tan(-x) = -\tan x$	$\cot(-x) = -\cot x$

6) shifting angle by $\pi/2, \pi, 3\pi/2$ (Co-Function identities or Periodicity identities)

$\sin(\pi/2 - x) = \cos x$	$\cos(\pi/2 - x) = \sin x$
$\sin(\pi/2 + x) = \cos x$	$\cos(\pi/2 + x) = -\sin x$
$\sin(3\pi/2 - x) = -\cos x$	$\cos(3\pi/2 - x) = -\sin x$
$\sin(3\pi/2 + x) = -\cos x$	$\cos(3\pi/2 + x) = \sin x$
$\sin(\pi - x) = \sin x$	$\cos(\pi - x) = -\cos x$
$\sin(\pi + x) = -\sin x$	$\cos(\pi - x) = -\cos x$
$\sin(2\pi - x) = -\sin x$	$\cos(2\pi - x) = \cos x$
$\sin(2\pi + x) = \sin x$	$\cos(2\pi + x) = \cos x$

7) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

8) $2\cos x \cos y = \cos(x+y) + \cos(x-y)$

$-2\sin x \sin y = \cos(x+y) - \cos(x-y)$

$2\sin x \cos y = \sin(x+y) + \sin(x-y)$

$2\cos x \sin y = \sin(x+y) - \sin(x-y)$

9) $\sin 2x = 2\sin x \cos x = \dfrac{2\tan x}{1+\tan^2 x}$

$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

$= \dfrac{1-\tan^2 x}{1+\tan^2 x}$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$\cos^2 x/2 = 1/2\,(1+\cos x)$

$\sin^2 x/2 = 1/2\,(1-\cos x)$

$\sin 3x = 3\sin x - 4\sin^3 x$

$\cos 3x = 4\cos^3 x - 3\cos x$

$\tan 3x = \dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

10) $\cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

$\cos x - \cos y = -2\sin\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

$\sin x + \sin y = 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

$\sin x - \sin y = 2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

INVERSE TRIGONOMETRY FORMULA

1) $\sin^{-1}(-x) = -\sin^{-1}x$

$\cos^{-1}(-x) = \pi - \sin^{-1}x$

$\tan^{-1}(-x) = -\tan^{-1}x$

$\mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1}x$

$\sec^{-1}(-x) = -\sec^{-1}x$

$\cot^{-1}(-x) = \pi - \cot^{-1}x$

2) $\sin^{-1}(1/x) = \mathrm{cosec}^{-1}x$ $\quad \sin^{-1}x + \cos^{-1}x = \pi/2$

$\cos^{-1}(1/x) = \sec^{-1}x$ $\quad \tan^{-1}x + \cot^{-1}x = \pi/2$

$\tan^{-1}(1/x) = \tan^{-1}x$ $\quad \sec^{-1}x + \mathrm{cosec}^{-1}x = \pi/2$

3) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\dfrac{x+y}{1-xy}$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\dfrac{x-y}{1+xy}$

$2\tan^{-1}x = \tan^{-1}\dfrac{2x}{1-x^2} = \sin^{-1}\dfrac{2x}{1+x^2} = \cos^{-1}\dfrac{2x}{1+x^2}$

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DERIVATIVE

1) $y = f(x)$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

2) $\frac{d}{dx}(x^n) = nx^{n-1}$

3) $\frac{d}{dx}(\log x) = \frac{1}{x}$

4) $\frac{d}{dx}(\log_a x) = \frac{1}{x\log a}$

5) $\frac{d}{dx}(e^x) = e^x$

6) $\frac{d}{dx}(a^x) = a^x \log a$

7) $\frac{d}{dx}(\sin x) = \cos x$

8) $\frac{d}{dx}(\cos x) = -\sin x$

9) $\frac{d}{dx}(\tan x) = \sec^2 x$

10) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

11) $\frac{d}{dx}(\sec x) = \sec x \tan x$

12) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

13) $\frac{d}{dx}(\sin^{-1} x/a) = \frac{1}{\sqrt{a^2-x^2}}$

14) $\frac{d}{dx}(\cos^{-1} x/a) = \frac{-1}{\sqrt{a^2-x^2}}$

15) $\frac{d}{dx}(\tan^{-1} x/a) = \frac{a}{x^2+a^2}$

INTEGRATION

1) $\frac{d}{dx} f(x) = g(x)$

$\int g(x)\,dx = f(x) + C$

2) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1$

3) $\int \frac{1}{x}\,dx = \log x + C$

4) $\int e^x\,dx = e^x + C$

5) $\int a^x\,dx = \frac{a^x}{\log a} + C$

6) $\int \cos x\,dx = \sin x + C$

7) $\int \sin x\,dx = -\cos x + C$

8) $\int \tan x\,dx = \log \sec x + C$

9) $\int \cot x\,dx = \log \sin x + C$

10) $\int \sec x\,dx = \log(\sec x + \tan x)$

$= \log(\tan \pi/4 + x/2) + C$

11) $\int \operatorname{cosec} x\,dx = \log(\operatorname{cosec} x - \cot x) + C$

$= \log(\tan x/2) + C$

12) $\int \sec^2 x\,dx = \tan x + C$

13) $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$

14) $\int \sec x \tan x\,dx = \sec x + C$

15) $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x$

16) $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)$

17) $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

18) $\int \frac{1}{x\sqrt{x^2-a^2}} = \sec^{-1}\left(\frac{x}{a}\right)$

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16) $\frac{d}{dx}\left(\cot^{-1}\frac{x}{a}\right) = \frac{-a}{x^2+a^2}$

17) $\frac{d}{dx}\left(\sec^{-1}\frac{x}{a}\right) = \frac{1}{x\sqrt{x^2-a^2}}$

18) $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-a^2}}$

19) $\frac{d}{dx} f(ax+b) = a\frac{df}{dx}$

20) $\frac{d}{dx}(K) = 0$

19) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)$

20) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)$

21) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

22) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log(x+\sqrt{x^2+a^2})$

23) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log(x+\sqrt{x^2-a^2}) + C$

Rules of Derivatives

1) $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

2) $\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$

3) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

4) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,{du}/{dx} - u\,{dv}/{dx}}{v^2}$

5) $\frac{d}{dx}(Ku) = K\frac{du}{dx}$

6) $\frac{d}{dx}\left(\frac{K}{u}\right) = \frac{-K}{u^2}\frac{du}{dx}$

7) $\frac{d}{dx} f(g(x)) = \frac{df}{dg}\frac{dg}{dx}$

8) $x = f(t), y = g(t)$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

9) when $x^x, x^{\cos x}, x^y \longrightarrow$ take log on both sides

RULES OF INTEGRATION

1) $\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

2) $\int [f(x)-g(x)]\,dx = \int f(x)\,dx - \int g(x)\,dx$

3) $\int f(x)\,g(x)\,dx = \int uv\,dx$ (ILATE)

$= u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)$ Int by part

4) $\int k\,f(x)\,dx = k\int f(x)\,dx$

5) $\int \frac{u}{v}\,dx = \longrightarrow$ 1) substitution method
2) Trigonometric formula
3) Partial Fraction

6) substitution

i) $\int \frac{f'(x)}{f(x)}\,dx = \log f(x) + C$

ii) $\int e^{f(x)}\,f'(x)\,dx = e^{f(x)} + C$

iii) $\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C$

FUNDAMENTAL THEOREM OF INTEGRAL CALCULAS

$$\int_a^b f(x)\,dx = g(b) - g(a)$$

PROPERTY 1: $\int_a^a f(x)\,dx = 0$

PROPERTY 2: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

PROPERTY 3: $\int_a^b f(x)\,dx = \int_a^b f(t)\,dx$

PROPERTY 4: $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

where $a<c<b$ i.e $c \in [a,b]$

PROPERTY 5: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

PROPERTY 6: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

PROPERTY 7: $\int_0^{2a} f(x)\,dx = \int_0^a f(x) + \int_0^a f(2a-x)\,dx$

PROPERTY 8: $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$, if $f(x)$ even function

$= 0$, if $f(x)$ is odd function