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HALF RANGE SERIES

1) Half range Fourier cosine series of $f(x)$ on $(0, l)$

$$f(x) = \frac{a_0}{2} + \sum a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx \qquad a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

2) Half range Fourier sine series on $(0, l)$

$$f(x) = \sum b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

3) Half range Fourier cosine series on $(0, \pi)$

$$f(x) = \frac{a_0}{2} + \sum a_n \cos(nx)$$

$$a_0 = \frac{2}{\pi}\int_0^\pi f(x)\,dx$$

$$a_n = \frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx$$

4) Half Range

$$f(x) = \sum b_n \sin nx$$

$$b_n = \frac{2}{\pi}\int_0^\pi f(x)\sin nx\,dx$$

Q Obtain Fourier sine series of $f(x) = x$ on $(0, l)$

– Here $f(x) = x$, $(0, l)$

$\therefore$ Fourier sine series

$$f(x) = \sum b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

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$$b_n = \frac{2}{l}\int_0^l x\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{l}\left[x\left(\frac{-\cos(n\pi x/l)}{n\pi/l}\right) - 1\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right)\right]_0^l$$

$$= \frac{2}{l}\left[\frac{-l \cdot l}{n\pi}(-1)^n - 0\right] \qquad \left(\begin{array}{l}\cos n\pi = (-1)^n \\ \cos 0 = 1 \\ \sin 0 = \sin n\pi = 0\end{array}\right)$$

$$= \frac{-2l(-1)^n}{n\pi}$$

Q Obtain Fourier cosine series of $f(x) = x$ on $(0, 2)$

– Here $f(x) = x$ on $(0, 2)$

FCS is

$$f(x) = \frac{a_0}{2} + \sum a_n \cos\left(\frac{n\pi x}{2}\right)$$

$$a_0 = \frac{2}{2}\int_0^2 f(x)\,dx$$

$$= \int_0^2 x\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^2$$

$$a_0 = 2$$

$$a_n = \int_0^2 x\cos\left(\frac{n\pi x}{2}\right)dx$$

$$= \left[x\left(\frac{\sin\left(\frac{n\pi x}{2}\right)}{\frac{n\pi}{2}}\right) - 1\left(\frac{-\cos\left(\frac{n\pi x}{2}\right)}{\frac{n^2\pi^2}{4}}\right)\right]_0^2$$

$$= \left[\frac{4(-1)^n}{n^2\pi^2} - \frac{4(1)}{n^2\pi^2}\right]$$

$$= \frac{4[(-1)^n - 1]}{n^2\pi^2}$$

Req FCS is

$$f(x) = \frac{2}{2} + \sum \frac{4[(-1)^n - 1]}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right)$$

Q Obtain the half range FCS of $f(x) = x - x^2$, $0 \le x \le 1$

– Here $f(x) = x - x^2$, $0 \le x \le 1$

Fourier series

$$f(x) = \frac{a_0}{2} + \sum a_n \cos\left(\frac{n\pi x}{l}\right)$$

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But $l=1$

$$f(x)=\frac{a_0}{2}+\sum a_n \cos(n\pi x) \quad \text{---(1)}$$

$$a_0=\frac{2}{l}\int_0^l f(x)\,dx = 2\int_0^1 f(x)\,dx = 2\int_0^1 (x-x^2)\,dx$$

$$=2\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1$$

$$=2\left[\frac{1}{2}-\frac{1}{3}\right]=2\left[\frac{1}{6}\right]=\frac{1}{3}$$

$$a_n=\frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$=2\int_0^1 (x-x^2)\cos(n\pi x)\,dx$$

$$=2\left[(x-x^2)\left(\frac{\sin(n\pi x)}{n\pi}\right)-(1-2x)\left(\frac{-\cos n\pi x}{n^2\pi^2}\right)\right.$$

$$\left.+(-2)\left(\frac{-\sin(n\pi x)}{n^3\pi^3}\right)\right]_0^1$$

$$=2\left[\frac{-(-1)^n}{n^2\pi^2}-\frac{1}{n^2\pi^2}\right]$$

$$=\frac{-2\left[(-1)^n+1\right]}{n^2\pi^2}$$

Q Find cosine series for $f(x)=\pi-x$ on $(0,\pi)$

- Here $f(x)=\pi-x$ $(0,\pi)$

Fourier series,

$$f(x)=\frac{a_0}{2}+\sum a_n \text{cosine}(nx)$$

$$a_0=\frac{2}{\pi}\int_0^\pi f(x)\,dx$$

$$=\frac{2}{\pi}\int_0^\pi (\pi-x)\,dx$$

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$$=\frac{2}{\pi}\left[\pi x-\frac{x^2}{2}\right]_0^\pi$$

$$=\frac{2}{\pi}\left[\pi^2-\frac{\pi^2}{2}\right]$$

$$=2\pi-\pi$$

$$a_0=\pi$$

$$a_n=\frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx$$

$$=\frac{2}{\pi}\int_0^\pi (\pi-x)\cos nx\,dx$$

$$=\frac{2}{\pi}\left[(\pi-x)\left(\frac{\sin nx}{n}\right)-(-1)\left(\frac{-\cos nx}{n^2}\right)\right]_0^\pi$$

$$=\frac{2}{\pi}\left[\left\{0-(-1)\left(\frac{-(-1)^n}{n^2}\right)\right\}-\left\{\pi(0)-(-1)\left(\frac{-1}{n^2}\right)\right\}\right]$$

$$=\frac{2}{\pi}\left[\frac{-(-1)^n}{n^2}+\frac{1}{n^2}\right]$$

$$=\frac{2}{\pi}\left[\frac{1-(-1)^n}{n^2}\right]$$

Req FCS is

$$f(x)=\frac{\pi}{2}+\sum\frac{2}{\pi}\left[\frac{1-(-1)^n}{n^2}\right]\cos(nx)$$

Q Find HRFSS $f(x)=x^2$ on $(0,\pi)$

- Here $f(x)=x^2$, $(0,\pi)$

Fourier Series,

$$f(x)=\sum b_n \sin nx$$

$$b_n=\frac{2}{\pi}\int_0^\pi f(x)\sin nx\,dx$$

$$=\frac{2}{\pi}\int_0^\pi x^2\sin nx\,dx$$