12/22 TUE UNIT 2

HARMONIC ANALYSIS

The function $y=f(x)$ defined on the interval $(0, 2\pi)$ in discrete form

x	x_0	x_1	x_2	x_3	...	x_{m-1}	x_m
y	y_0	y_1	y_2	y_3	...	y_{m-1}	y_m

where $(x_0, x_m) = (0, 2\pi)$

The interval (x_0, x_m) is divided into n equal sub-intervals $(x_0, x_1), (x_1, x_2), \dots, (x_{m-1}, x_m)$ with length $h = \frac{x_m - x_0}{m} = \frac{2\pi - 0}{m} = \frac{2\pi}{m}$

then the fourier series of $f(x)$ on $(x_0, x_m) = (0, 2\pi)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \dots$$

$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \dots$$

$$= \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) + \dots$$

where the term $a_1 \cos x + b_1 \sin x$ is called 1st harmonic

$a_2 \cos 2x + b_2 \sin 2x$ is called 2nd harmonic

$a_3 \cos 3x + b_3 \sin 3x$ is called 3rd harmonic

$$a_0 = \frac{2}{m} \sum_{i=0}^{m-1} y_i$$

$$a_n = \frac{2}{m} \sum_{i=0}^{m-1} y_i \cos(n x_i)$$

Don't take last value becze $y_0 = y_n$

$$b_n = \frac{2}{m} \sum_{i=0}^{m-1} y_i \sin(n x_i)$$

$$a_1 = \frac{2}{m} \sum y_i \cos x_i, \quad a_2 = \frac{2}{m} \sum y_i \cos 2x_i$$

$$b_1 = \frac{2}{m} \sum y_i \sin x_i, \quad b_2 = \frac{2}{m} \sum y_i \sin 2x_i$$

(II) on $(0, 2l)$

m = no. of intervals

The Fourier series of $f(x)$

$$f(x) = \frac{a_0}{2} + \left(a_1 \cos \frac{\pi x}{l} + b_1 \sin \frac{\pi x}{l}\right) + \left(a_2 \cos \frac{2\pi x}{l} + b_2 \sin \frac{2\pi x}{l}\right) + \dots$$

$$a_0 = \frac{2}{m} \sum y_i$$

$$a_1 = \frac{2}{m} \sum y_i \cos\left(\frac{n x_i}{l}\right) \quad a_2 = \frac{2}{m} \sum y_i \cos\left(\frac{2\pi x_i}{l}\right)$$

$$b_1 = \frac{2}{m} \sum y_i \sin\left(\frac{\pi x_i}{l}\right) \quad b_2 = \frac{2}{m} \sum y_i \sin\left(\frac{2\pi x_i}{l}\right)$$

(III) on $(0, l)$ Find Fourier sine series

The FS of $f(x)$

$$f(x) = b_1 \sin \frac{\pi x}{l} + b_2 \sin \frac{2\pi x}{l} + \dots$$

(IV) on $(0, l)$ Find FCS

$$f(x) = \frac{a_0}{2} + a_1 \cos \frac{\pi x}{l} + a_2 \cos \frac{2\pi x}{l} + \dots$$

(V) FSS $(0, \pi)$

$$f(x) = b_1 \sin x + b_2 \sin 2x + \dots$$

$$b_1 = \frac{2}{m} \sum y_i \sin x_i \quad b_1 = \frac{2}{m} \sum y_i \sin\left(\frac{\pi x}{l}\right)$$