UNIT2

The series of the form $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n(\cos nx) + \sum_{n=1}^{\infty} b_n \sin nx$, where a_0, a_n, b_n are constants is called trigonometric series.

Any periodic function $f(x)$ of period 2π which satisfies certain conditions known as Dirichelt's conditions can be expressed in the form of trigonometric series known as fourier series.

A function $f(x)$ defined on the interval $(\alpha, \alpha+2\pi)$ can be expressed as fourier's series if in the interval ~~no~~

(i) $f(x)$ & its integrals are finite & single valued

(ii) $f(x)$ is piecewise continuous and bounded in the interval i.e $f(x)$ has discontinuity finite in no.

(iii) $f(x)$ has finite no. of maxima & minima

FOURIER SERIES OF $f(x)$ in the interval $(\alpha, \alpha+2l)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

where a_0, a_n, b_n are constants known as Fourier coefficient

$$a_0 = \frac{1}{l}\int_{\alpha}^{\alpha+2l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_{\alpha}^{\alpha+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_{\alpha}^{\alpha+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

A $f(x)$ on $[0, 2\pi]$ or $(0, 2\pi)$

Fourier series of $f(x)$ defined on $(0, 2\pi)$ and $f(x) = f(x+2\pi)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

When Fourier coeff $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$, $a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) = \cos nx\,dx$, $b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$

Q Find the Fourier series expansion of $f(x) = e^{ax}$, $[0, 2\pi]$ and $f(x) = f(x+2\pi)$

– Given: $f(x) = e^{ax}$, $[0, 2\pi]$

$f(x) = f(x+2\pi)$

Fourier series of $f(x)$ is

$$f(x) = \frac{a_0}{2} + \sum a_n \cos nx + \sum b_n \sin nx \quad \text{——— (1)}$$

where $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} e^{ax}\,dx = \frac{1}{\pi}\left[\frac{e^{ax}}{a}\right]_0^{2\pi} = \frac{1}{\pi}\left[\frac{e^{2a\pi}-1}{a}\right]$$

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$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} e^{ax}\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{ax}}{a^2+n^2}(a\cos nx + n\sin nx)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{2a\pi}}{a^2+n^2}(a) - \frac{1}{a^2+n^2}(a)\right]$$

$$= \frac{a\left[e^{2a\pi}-1\right]}{\pi(a^2+n^2)}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} e^{ax}\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{ax}}{a^2+n^2}(a\sin nx - n\cos nx)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{2a\pi}}{a^2+n^2}(-n) - \frac{1}{a^2+n^2}(-n)\right]$$

$$= \frac{-n\left[e^{2a\pi}-1\right]}{\pi(a^2+n^2)}$$

$$f(x) = \frac{e^{2a\pi}-1}{2\pi a} + \sum \frac{a(e^{2a\pi}-1)}{\pi(a^2+n^2)}\cos nx + \sum \frac{-n\left[e^{2a\pi}-1\right]}{\pi(a^2+n^2)}\sin nx$$

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Q Obtain FS $f(x) = \left(\frac{\pi - x}{2}\right)^2$ on $0 \le x \le 2\pi$ & $f(x) = f(x+2\pi)$ hence prove that

(i) $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots$

(ii) $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots$

(iii) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

– Given: $f(x) = \left(\frac{\pi - x}{2}\right)^2$, $[0, 2\pi]$

$$f(x) = f(x+2\pi)$$

Fourier series of $f(x)$ is

$$f(x) = \frac{a_0}{2} + \sum a_n\cos nx + \sum b_n \sin nx \quad \text{—(1)}$$

where $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)^2 dx$$

$$= \frac{1}{\pi}\left[\frac{\left(\frac{\pi-x}{2}\right)^3}{3\times -1/2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{(\pi-x)^3}{-\frac{8\times3}{2}}\right]_0^{2\pi}$$

$$= \frac{1}{-12\pi}\left[(-x+\pi)^3\right]_0^{2\pi}$$

$$= \frac{1}{-12\pi}(-2\pi+\pi)^3 - (\pi)^3$$

$$= \frac{1}{-12\pi}(-\pi)^3 - (\pi)^3 = \frac{-1}{12\pi}\times -2\pi^3 = \frac{\pi^2}{6}$$

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$$a_n = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi-x}{2}\right)^2 \cos nx\, dx$$

$$= \frac{1}{4\pi}\int_0^{2\pi}(\pi-x)^2 \cos nx\, dx$$

$$= \frac{1}{4\pi}\left[(\pi-x)^2\left(\frac{\sin nx}{n}\right) - 2(\pi-x)\times -1\left(\frac{-\cos nx}{n^2}\right)\right.$$

$$\left. +(-2)\left(\frac{-\sin nx}{x^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[\frac{-2(-\pi)}{n^2} + \frac{2(\pi)}{n^2}\right] = \frac{1}{4\pi}\left(\frac{4\pi}{n^2}\right) = \frac{1}{n^2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi-x}{2}\right)^2 \sin nx\, dx$$

$$= \frac{1}{4\pi}\int_0^{2\pi}(\pi-x)^2 \sin nx\, dx$$

$$= \frac{1}{4\pi}\left[(\pi-x)^2\left(\frac{-\cos nx}{n}\right) - (-2(\pi-x))\left(\frac{-\sin nx}{n^2}\right) + \right.$$

$$\left. (2)\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[\left(\frac{-\pi^2}{n} + \frac{2}{n^3}\right) - \left(\frac{-\pi^2}{n} + \frac{2}{n^3}\right)\right]$$

$b_n = 0$

$\therefore$ substitute all values in (1);

$$\left(\frac{\pi-x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\frac{1}{n^2}\cos nx \quad \text{—(2)}$$

(i) put $x = \pi$

$$0 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\frac{\cos nx\pi}{n^2}$$

$$-\frac{\pi^2}{12} = \frac{\cos \pi}{1} + \frac{\cos 2\pi}{2^2} + \frac{\cos 3\pi}{3^2} + \frac{\cos 4\pi}{4^2} + \ldots$$

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$$-\frac{\pi^2}{12} = -\frac{1}{1} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} - \ldots$$

$$\frac{\pi^2}{12} = \frac{1}{1} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots \quad \text{—(3)}$$

Put $x = 0$

$$\left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\frac{\pi^2}{4} = \frac{\pi^2}{12} + \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots \quad \text{—(4)}$$

Add (3)+(4)

$$\frac{3\pi^2}{12} = \left[\frac{1}{1} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\right] + \left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\right]$$

$$\frac{\pi^2}{4} = \frac{2}{1^2} + \frac{2}{3^2} + \ldots$$

$$\frac{\pi^2}{4} = 2\left[\frac{1}{1^2} + \frac{1}{3^2} + \ldots\right]$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \ldots$$

12/2022 UNIT 2

1 $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}$, $f(x) = f(x+2\pi)$

– Its Fourier series

$$f(x) = \frac{a_0}{2} + \sum a_n \cos nx + \sum b_n \sin nx \quad \text{--- (1)}$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$a_0 = \frac{1}{\pi}\left[\int_0^{\pi} x\,dx + \int_{\pi}^{2\pi}(2\pi - x)\,dx\right]$$

$$= \frac{1}{\pi}\left\{\left[\frac{x^2}{2}\right]_0^{\pi} + \left[2\pi x - \frac{x^2}{2}\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\left[\frac{x^2}{2}\right]_0^{\pi} + \left[\left(4\pi^2 - \frac{4\pi^2}{2}\right) - \left(2\pi^2 - \frac{\pi^2}{2}\right)\right]\right\}$$

$$= \frac{1}{\pi}\left\{\left(\frac{\pi^2}{2} - 0\right) + \left[\left(4\pi^2 - \frac{4\pi^2}{2}\right) - \left(\frac{2\pi^2 - \pi^2}{2}\right)\right]\right\}$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} + \left(2\pi^2 - \frac{3\pi^2}{2}\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} + \frac{\pi^2}{2}\right] = \frac{1}{\pi} \times \pi^2 = \pi$$

$$a_0 = \pi$$

$$a_n = \frac{1}{\pi}\left[\int_0^{\pi} x\cos nx\,dx + \int_{\pi}^{2\pi}(2\pi - x)\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - 1\left(\frac{-\cos nx}{n^2}\right)\right]_0^{\pi} + \frac{1}{\pi}\left[(2\pi - x)\left(\frac{\sin nx}{n}\right) - (-1)\left(\frac{-\cos nx}{n^2}\right)\right]_{\pi}^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right] + \frac{1}{\pi}\left[\frac{-1}{n^2} + \frac{(-1)^n}{n^2}\right]$$

$$\left(\because \sin 0 = \sin n\pi = \sin 2n\pi = 0,\ \cos 0 = \cos 2n\pi = 1,\ \cos n\pi = (-1)^n\right)$$

$$a_n = \frac{2[(-1)^n - 1]}{\pi n^2}$$

$$b_n = \frac{1}{\pi}\left[\int_0^{\pi} x\sin nx\,dx + \int_{\pi}^{2\pi}(2\pi - x)\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - 1\left(\frac{-\sin nx}{n^2}\right)\right]_0^{\pi} + \frac{1}{\pi}\left[(2\pi - x)\left(\frac{-\cos nx}{n}\right) - (-1)\left(\frac{-\sin nx}{n^2}\right)\right]_{\pi}^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{-\pi(-1)^n - 0}{n}\right] + \frac{1}{\pi}\left[\frac{0 + \pi(-1)^n}{n}\right]$$

$$\left(\because \sin 0 = \sin n\pi = \sin 2n\pi = 0,\ \cos 0 = \cos 2n\pi = 1,\ \cos n\pi = (-1)^n\right)$$

$$b_n = 0$$

Fourier series of $f(x)$ on $[0, 2l]$ with period $2l$

The fourier series of $f(x)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

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Q Obtain the Fourier series of $f(x)=\pi x,\ 0\le x\le 2$, $f(x)=f(x+2)$

- Given: $f(x)=\pi x,\ 0\le x\le 2$

$(0,2)=(0,2l)$

$2=2l \quad \therefore l=1$

$\therefore$ Its Fourier series

$$f(x)=\frac{a_0}{2}+\sum a_n\cos(n\pi x)+\sum b_n\sin(n\pi x)$$

$$a_0=\frac{1}{l}\int_0^{2l}f(x)\,dx=\int_0^2 f(x)\,dx \quad (\text{Put } l=1)$$

$$a_n=\int_0^2 f(x)\cos(n\pi x)\,dx$$

$$b_n=\int_0^2 f(x)\sin(n\pi x)\,dx$$

$$a_0=\int_0^2 \pi x\,dx=\pi\left[\frac{x^2}{2}\right]_0^2=\pi\left[\frac{4}{2}\right]=2\pi$$

$$a_n=\int_0^2 \pi x\cos(\pi n x)\,dx$$

$$=\pi\left[x\left(\frac{\sin(n\pi x)}{n\pi}\right)-\left(\frac{-\cos(n\pi x)}{n^2\pi^2}\right)\right]_0^2$$

$$=\pi\left[\frac{1}{n^2\pi^2}-\frac{1}{n^2\pi^2}\right] \quad \left(\begin{matrix}\cos 0=\cos 2n\pi=1\\ \sin 0=\sin 2n\pi=0\end{matrix}\right)$$

$$a_n=0$$

$$b_n=\int_0^2 \pi x\sin(n\pi x)\,dx$$

$$=\pi\left[x\left(\frac{-\cos(n\pi x)}{n\pi}\right)-1\left(\frac{-\sin(n\pi x)}{n^2\pi^2}\right)\right]_0^2$$

$$=\pi\left[\frac{-2}{n\pi}\right]$$

$$b_n=\frac{-2}{n}$$

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$\therefore$ Fourier series

$$\pi x=\frac{2\pi}{2}+0+\sum\frac{-2}{n}\sin(n\pi x)$$

$$\pi x=\pi-\sum\frac{2}{n}\sin(n\pi x)$$

$f(x)$ defined on $(-\pi,\pi)$ with period 2π

1) $f(x)$ is neither even nor odd

Its Fourier series

$$f(x)=\frac{a_0}{2}+\sum a_n\cos nx+\sum b_n\sin nx$$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\,dx$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx\,dx$$

2) $f(x)$ is even

Its Fourier series

$$f(x)=\frac{a_0}{2}+\sum a_n\cos nx$$

where $a_0=\frac{2}{\pi}\int_0^{\pi}f(x)\,dx$

$$a_n=\frac{2}{\pi}\int_0^{\pi}f(x)\cos nx\,dx$$

$$b_n=0$$

3) $f(x)$ is odd

Its Fourier series

$$f(x)=\sum b_n\sin nx$$

$a_0=0 \qquad b_n=\frac{2}{\pi}\int_0^{\pi}f(x)\sin nx\,dx$

$a_n=0$

Obtained the fourier series of $f(x) = \pi x, -\pi \le x \le \pi$ with period 2π

- Given $f(x) = \pi x \quad -\pi \le x \le \pi$

$f(-x) = -\pi x = -f(x)$

$\because f(x)$ is odd function

$\because$ It's forier series

$f(x) = \sum b_n \sin nx$ —— (1)

where $a_0 = 0$ $\quad b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin x \, dx$

$a_n = 0$

$b_n = \frac{2}{\pi}\int_0^{\pi} \pi x \sin nx \, dx$

$= 2\left[x\left(\frac{-\cos nx}{n}\right) - 1\left(\frac{-\sin nx}{n^2}\right)\right]_0^{\pi}$

$= 2\left[\frac{-x\cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{\pi}$

$= 2\left[\frac{-\pi(-1)^n}{n} - 0\right]$ --- $\left(\begin{matrix}\sin 0 = 0 = \sin\pi \\ \cos 0 = 1 = \cos n\pi = (-1)^n\end{matrix}\right)$

$= \frac{-2\pi(-1)^n}{n}$

$\because$ Req Fourier series

$f(x) = \sum \frac{-2\pi(-1)^n}{n} \sin nx$

Q obtained the fourier series of $f(x) = x^2$ in $(-\pi, \pi)$ hence deduce that

$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$

- Given: $f(x) = x^2 \quad (-\pi, \pi)$

$f(-x) = (-x)^2 = x^2$

$f(x) = f(-x)$

$\because$ Given function is even

$\because$ Its Fourier series

$f(x) = \frac{a_0}{2} + \sum a_n \cos nx$

$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$

$= \frac{2}{\pi}\int_0^{\pi} x^2\,dx$

$= \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi}$

$= \frac{2}{\pi} \times \left(\frac{\pi^3}{3}\right)$

$= \frac{2\pi^2}{3}$

$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$

$= \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx\,dx$

$= \frac{2}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right) - 2x\left(\frac{-\cos nx}{n}\right) + 2\left(\frac{-\sin nx}{n^3}\right)\right]_0^{\pi}$

$= \frac{2}{\pi}\left[-2\pi\left(\frac{-(-1)^n}{n^2}\right) - 0\right]$

$= \frac{2}{\pi} \times \frac{2\pi}{n^2}(-1)^n = \frac{4}{n^2}(-1)^n \cancel{\cos nx}$

$\because$ Req Fourier series

$x^2 = \frac{2\pi^2}{6} + \sum \frac{4(-1)^n}{n^2}\cos nx$

Put $x = \pi$

$\pi^2 = \frac{\pi^{\cancel{3}\,2}}{3} + \sum \frac{4(-1)^n \cos n\pi}{n^2}$

$\pi^2 - \frac{\pi^2}{3} = \sum \frac{4(-1)^n \cos n\pi - (-1)^n}{n^2}$

$$\frac{2\pi^2}{3\times 4} = \sum \frac{((-1)^n)^2}{n^2}$$

$$\frac{2\pi^2}{12} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$$

Homework

(i) $f(x) = x^3, \ -\pi \le x \le \pi$

— $f(x) = x^3 \quad (-\pi, \pi)$

$f(-x) = (-x^3) = -x^3$

$f(x) = -f(-x)$

∴ Given function is odd

∴ Its Forier series

$f(x) = \sum b_n \sin nx$ — (1)

where $a_0 = 0$

$a_n = 0$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} x^3 \sin nx\,dx$$

$$= \frac{2}{\pi}\left[x^3\left(\frac{-\cos nx}{n}\right) - 3x^2\left(\frac{-\sin nx}{n^2}\right) + 6x\left(\frac{\cos nx}{n^3}\right) - 6\left(\frac{\sin nx}{n^4}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[(\pi)^3\left(\frac{-(-1)^n}{n}\right) - 3\pi^2(0) + 6\pi\left(\frac{(-1)^n}{n^3}\right) - 6(0)\right]$$

$$= \frac{2}{\pi}\left[\pi^3\frac{-(-1)^n}{n} + 6\pi\left(\frac{(-1)^n}{n^3}\right)\right]$$

$$= \frac{-2\pi^2(-1)^n}{n} + 12\left(\frac{(-1)^n}{n^3}\right)$$

∴ Req Fourier series

$$f(x) = \sum \frac{-2\pi^2(-1)^n}{n} + \frac{12(-1)^n}{n^3}\sin nx$$

(2) $f(x) = x + x^2 \quad -\pi \le x \le \pi$

Given: $f(x) = x + x^2 \ (-\pi, \pi)$

$f(-x) = (-x) + (-x)^2$

$= -x + x^2$

$f(x) \ne f(-x)$

$f(x) \ne -f(-x)$

∴ Given function is neither even nor odd

∴ Its Fourier series

$$f(x) = \frac{a_0}{2} + \sum a_n \cos nx + \sum b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x + x^2\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2} + \frac{x^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\left(\frac{\pi^2}{2} + \frac{\pi^3}{3}\right) - \left(\frac{\pi^2}{2} - \frac{\pi^3}{3}\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{2\pi^3}{3}\right]$$

$$a_0 = \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x + x^2)\cos nx\,dx$$