Graph Paper Grand Prix
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GRAPH PAPER GRAND PRIX

by A. G. Rae, student at the Doncaster College of Education

There are many games available which are an aid to the teaching of the use of grids and co-ordinates, ordered pairs being used to describe points on a board. "Battleships" is one such game. It has even been extended into three dimensions—according to some American children whom I worked with last summer—different levels being used for aircraft, ships and submarines. However, about two years ago I came across a very interesting game which could help specifically in the teaching of vectors.

It was a wet morning during a week's "camp" in central Scotland for Public School boys. Most of the lads were happily engaged in various activities around the house in which we were staying but—as always seems to happen—there was one small group of three or four boys just hanging around with nothing to do. One of my fellow leaders appeared, armed with some pencils and a large pad of graph paper, and challenged the boys to a race over a graph paper race track. Once the rules had been explained and a test run tried, the boys, aged from about $13\frac{1}{2}$ to 16 tackled their first Grand Prix. They found it intriguing, stimulating and great fun. In fact they became so addicted to it that they would have probably ignored lunch to continue playing had we not called a halt. From then on, for the rest of the week one could often see groups of boys (and leaders!) crowded round sheets of graph paper busily working out the best line to take as they completed to be first over the finishing line.

I am led to believe that the idea for graph paper racing originated in the *New Scientist*, or a similar Journal, some time ago. The rules are basically simple and can be reasonably quickly understood by many secondary children. The applications to vectors will become apparent as one reads the rules and studies the examples. The game can also help in the understanding of the concepts of velocity and acceleration. The only equipment required is a writing implement and some graph paper—the bigger the sheets the better! I have never had the opportunity to try this game with girls, but boys certainly seem to get a great deal of fun out of it. With practice, some become very skilful at the game, cornering with a minimum of moves and forcing other competitors to "shunt" or "spin out".

After reading the rules and guidelines, try a test-run on a simple home-made track; then try your hand at Le Mans, using the track printed on page 25.

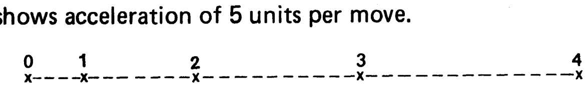
Rules for Graph paper Racing

1. Acceleration Rates

(a) A *maximum* rate must be agreed before each race.

(b) The rates can vary from 1 unit per move to as many as desired, depending on the size of graph paper used. A sensible maximum for normal use would be from 5 to 7 units per move. Figure 1 shows acceleration of 5 units per move.

Figure 1



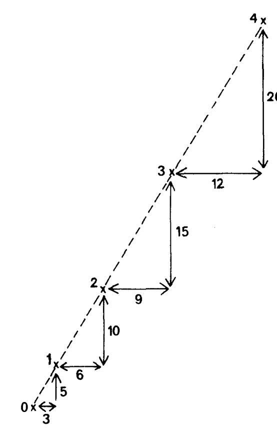
(c) Acceleration can be vertical, horizontal or both. Figure 2 shows an acceleration of 3 units per move horizontally and 5 units per move vertically.

(d) Only *whole number* acceleration rates should be used—this avoids complications.

2. Braking Rates

The same rules as for acceleration apply but there should be a specified *maximum* braking rate for each *maximum* acceleration

Figure 2



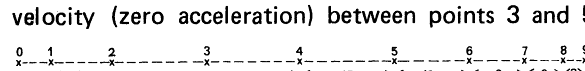
rate according to the formula M.B.R. = $\frac{1}{2}$M.A.R.+1 (for *even* rates) or M.B.R. = $\frac{1}{2}$M.A.R.+$\frac{1}{2}$ (for *odd* rates)—see table 1. Figure

Table 1

A.R. (Max.)	B.R. (Max.)
10	6
9	5
8	5
7	4
6	4
5	3
4	3
3	2

3 shows acceleration of 5 units per move from point 0 to point 3, constant velocity (zero acceleration) between points 3 and 5

Figure 3



and braking of 3 units per move from point 5 to point 9.

3. Cornering

Where a corner is sharp enough to require a 180° change in direction horizontally or vertically, the player must come to a standstill in that direction before accelerating off in the opposite direction. With reference to figure 4, notice that the maximum braking rate is 3 units per move (M.A.R. = 5), but from E to F the only vertical braking rate possible is 1. If any other rate was taken, it would in effect be equivalent to braking and accelerating simultaneously—which is not possible! The same is true for the horizontal motion between K and L.

Table 2 shows each move the player has made from A to P. Vectors, in the form $\binom{x}{y}$, are used to represent acceleration and braking (negative acceleration) and vectors, in the form (x,y) represent velocity at the point reached. Note that the velocity at, for example, B, can be found by adding: $(0,15)+\binom{2}{-2} = (2,13)$. Note also that the negative sign in this case does *not* indicate direction along the x or y axis.

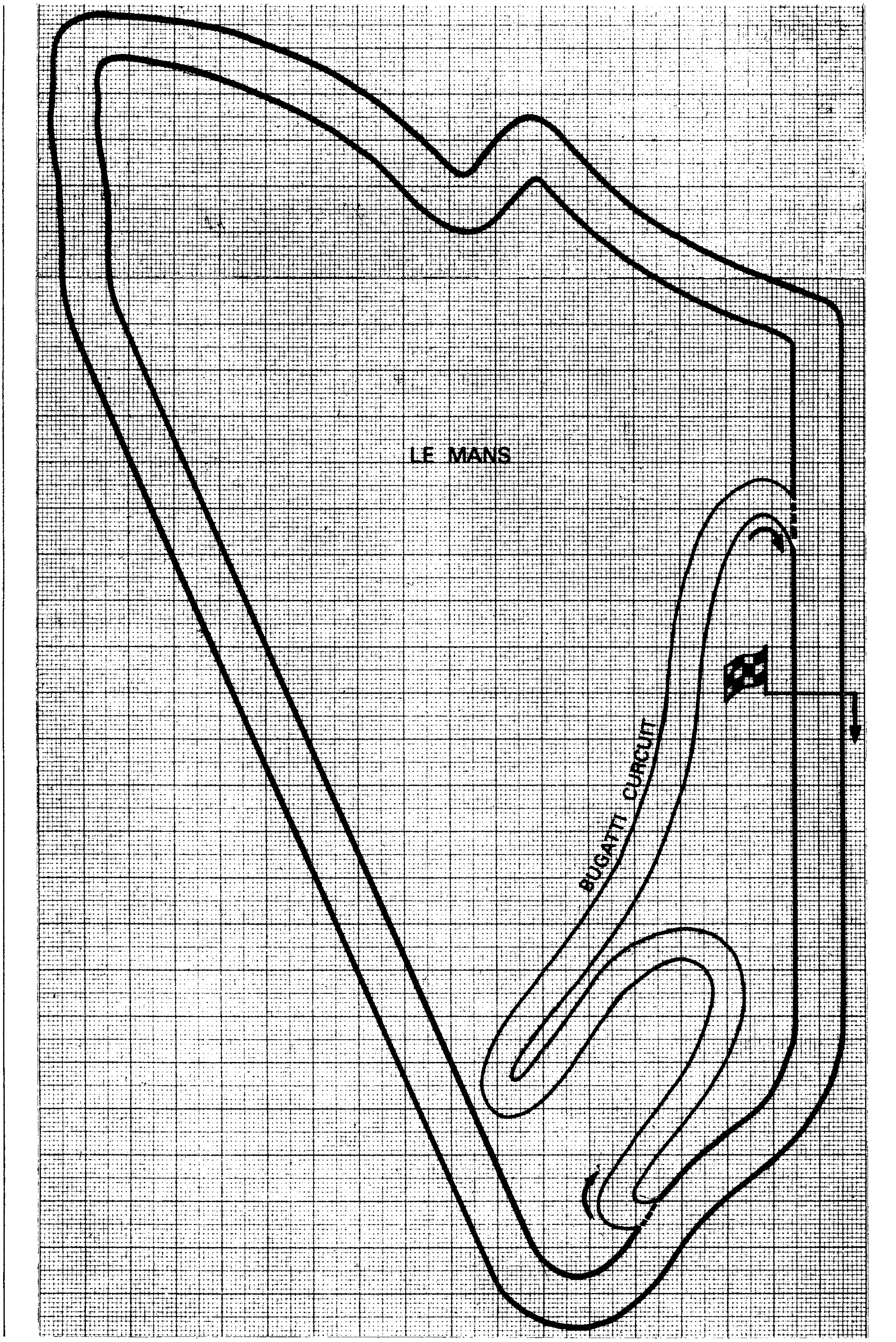

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Table 2

Points	Acceleration	Point	Velocity
		A	(0,15)
A–B	$\binom{2}{-2}$	B	(2,13)
B–C	$\binom{0}{-3}$	C	(2,10)
C–D	$\binom{3}{-3}$	D	(5,7)
D–E	$\binom{1}{-3}$	E	(6,4)
E–F	$\binom{-3}{-3}$	F	(3,1)
F–G	$\binom{-1}{-1}$	G	(2,0)
G–H	$\binom{0}{5}$	H	(2,5)
H–I	$\binom{0}{4}$	I	(2,9)
I–J	$\binom{5}{-3}$	J	(7,6)
J–K	$\binom{-3}{-3}$	K	(4,3)
K–L	$\binom{-3}{-2}$	L	(1,1)
L–M	$\binom{-1}{0}$	M	(0,1)
M–N	$\binom{5}{1}$	N	(5,2)
N–O	$\binom{2}{4}$	O	(7,6)
O–P	$\binom{-3}{5}$	P	(4,11)

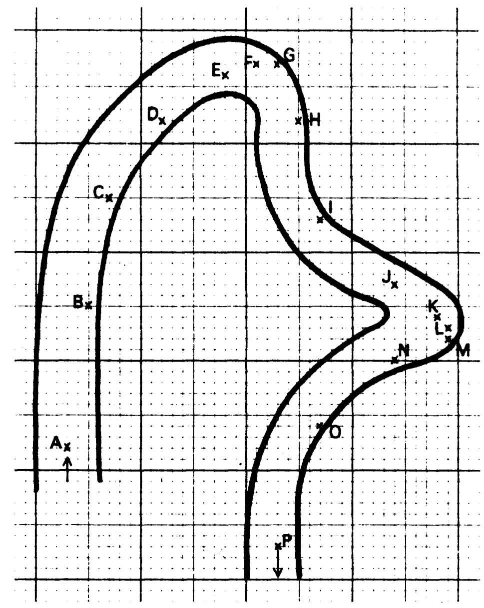


Figure 4

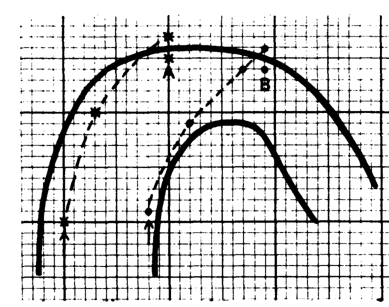


Figure 5

4. Disasters—Shunts and Spin-outs

(a) A "shunt" between cars occurs when a player lands on an intersection already occupied by another player who has already taken his move for that turn.

(b) A "spin-out" occurs when the line a player has chosen forces him to land on an intersection outside the line defining the edge of the track.

(c) The penalties incurred are the same for either type of crash and take the form of missing a certain number of turns. This number is the nearest whole number to $\frac{1}{2}$ the average of the two components of velocity at the time of collision. The person "hit" in a shunt as mentioned in (a) is not penalised.

In figure 5, the player * has misjudged (badly!) his stopping distance. His crash velocity is taken as (7,7); he therefore misses $\frac{1}{2} \times \frac{7+7}{2} = 4$ turns ($3\frac{1}{2}$ is rounded up). The player ● has only made a slight error and by using full braking power (in this case M.A.R. = 5 and M.B.R. = 3) he has cut his crash velocity to (2,2). He will therefore miss only 1 turn. *N.B.* a person whose crash velocity is (1,0) or (0,1) will also miss 1 turn.

(d) A player who has crashed resumes the race after missing the appropriate number of turns by restarting with velocity (0,0) at the intersection perpendicularly nearest to the point reached on crashing. (A and B in Figure 5 for * and ● respectively.) With a shunt, the player who was "hit" continues as though nothing had happened and the one who shunted misses the appropriate number of turns and restarts with velocity (0,0) from the intersection where the shunt took place.

(e) Cutting corners is permissible as long as no rules are infringed in so doing. For example, in Figure 6, the player appears to have cut the corner between C and D. Assume that he is using brake power 3 vertically between A and B, he continues vertical braking and accelerates across 4 from B to C (vertical velocity now 7−3 = 4). He continues braking vertically and accelerating

Puzzles Pastimes Problems Solutions

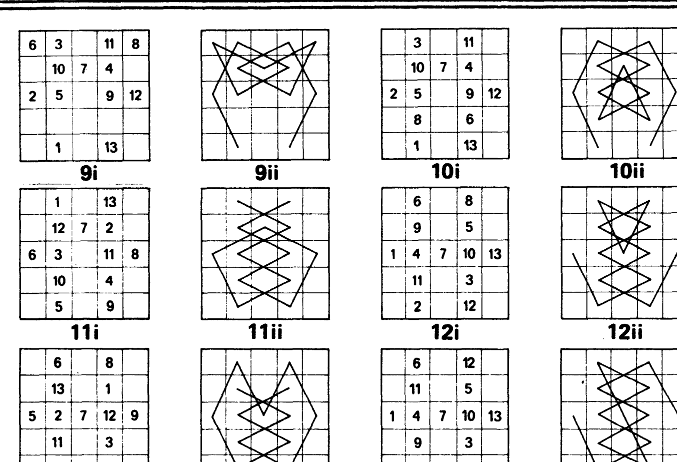
to Puzzles, Pastimes and Problems shown on page 12.

1. A Knight's Tour on a Chess Board

(a) See Figs 9, i and ii.

(b) See Figs 10 i and ii, 11 i and ii.

(c) There are two solutions, Figs 12 i and ii, 13 i and ii, in which the tour diagram has line symmetry and a third solution, Fig. 14 i and ii, in which the tour diagram has rotational symmetry. It is not always easy to be certain that all the solutions have been found. Please send to Canon Eperson any alternatives that you discover, except those that are reversals, reflections or rotations of those shown here.



9i 9ii 10i 10ii 11i 11ii 12i 12ii 13i 13ii 14i 14ii

2. Postage Stamps

(a) 10 at 3½p and 5 at 3p.

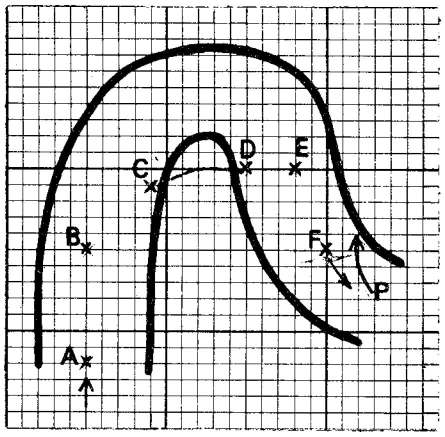
(b) Yes. $3\frac{1}{2}X+3Y = 50$ has two solutions in integers: (4, 12) and (10, 5).

(c) Five ways. $3\frac{1}{2}X+3Y = 100$ has five solutions in integers: (2, 31), (8, 24), (14, 17), (20, 10) and (26, 3).

3. Noah's Vineyard

(a) Noah's widow would have the central equilateral triangle in Fig. 15. (b) Each son could add one-third

Figure 6



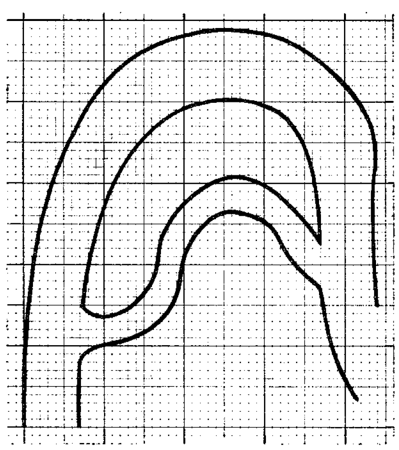
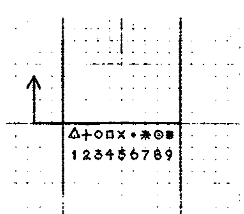


Figure 7

Figure 8



horizontally his velocity at D being 4+2 = 6 across and 4−3 = 1 up. He then brakes 3 horizontally and 1 vertically. This brings his vertical velocity to zero prior to direction change (Rule 3). So at E he has velocity of (3,0). He then brakes at 1 across and accelerates at 5 down giving him a velocity at F of (2,5).

(f) If the line indicating the limits of the track passes through an intersection—for example at P in Figure 6—that point is not available for play; in other words, anyone landing on it has "crashed". It is worthwhile constructing a table of braking distances, like Table 3, for different braking rates and velocities. For example, if velocity in one direction is 15 it will take 21 units to stop if braking power is 4 units per move.

Table 3

Velocity	Brake Power 1	2	3	4	5	6
5	10	4	2	1	–	–
6	15	6	3	2	1	–
7	21	9	5	3	2	1
8	28	12	7	4	3	2
9	36	16	9	6	4	3
10	45	20	12	8	5	4
11	55	25	15	10	7	5
12	66	30	18	12	9	6
13	78	36	22	15	11	8
14	91	42	26	18	13	10
15	105	49	30	21	15	12
16	120	56	35	24	18	14
17	136	64	40	28	21	16
18	153	72	45	32	24	18
19	171	81	51	36	27	21
20	190	90	57	40	30	24
21	210	100	63	45	34	27
22	231	110	70	50	38	30
23	253	121	77	55	42	33
24	276	132	84	60	46	36
25	300	144	92	66	50	40

5. Either/or

An interesting element of choice may be built into a race track in the form of a narrow alternative route which a player can take instead of the main route if he thinks it would be quicker. Figure 7 shows an example.

6. Finish

The finishing line can be crossed at any speed; in other words stopping distance can be ignored. Obviously, the winner is the first player to cross the finishing line.

7. Grand Prix Circuits

Design your own racing circuits, or for increased reality, try drawing plans of actual circuits.

8. How many can take part?

In theory there is no limit, but if there are too many racing on too small a sheet of graph paper the field gets a bit crowded and "car" identification is difficult. One solution is to hold qualifying heats. Starting grid placing may also be a problem unless players draw lots for position and order of playing! Different symbols or colours or both can be used to represent the cars. Figure 8 shows some examples. ■

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of the central plot to his triangular plot, making it into a rhombus (Fig. 16).

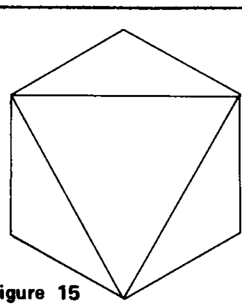


Figure 15

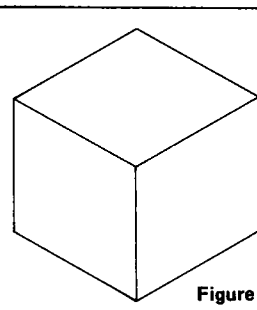
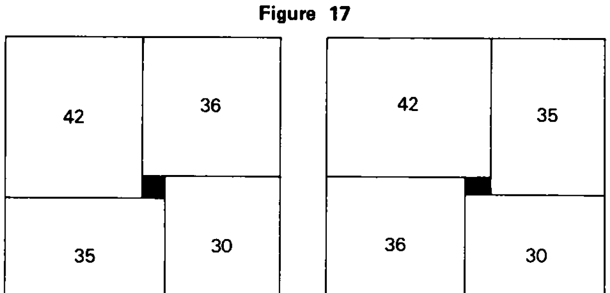


Figure 16

Figure 17



4. Guineas—Now and Then

(a) £1.05, or 105p. (b) N guineas is $(252N)$d and $252 = 3 \times 3 \times 4 \times 7$. (c) N guineas is $(105N)$p and $105 = 3 \times 5 \times 7$. (d) 72. (e) 30.

5. The Shah's Rolls-Royces

It is impossible to carry out the terms of the will, but by adding one loaned car to a fleet of 23, they could be shared out with the elder son receiving 16, the second son 4, the third son 3, and the loaned car could be returned to King Solomon.

If, however, the Shah had had 47 cars, the same procedure would result in the allocation of double the previous number of cars to each son. In fact if $24N-1$ is a prime number (e.g. when $N = 1, 2, 6, 7$, etc.) the procedure will apparently be in accordance with the will!

6. The Damaged Carpet

The first plan showed that the missing square foot had one corner at the centre of the square. A plan that results in four rectangular pieces can be made by producing the four sides of the missing square to meet an outer edge of the whole square, either clockwise or anticlockwise. This leads to the two solutions shown in Fig. 17. ■