DEPARTMENT OF ENGINEERING DESIGN

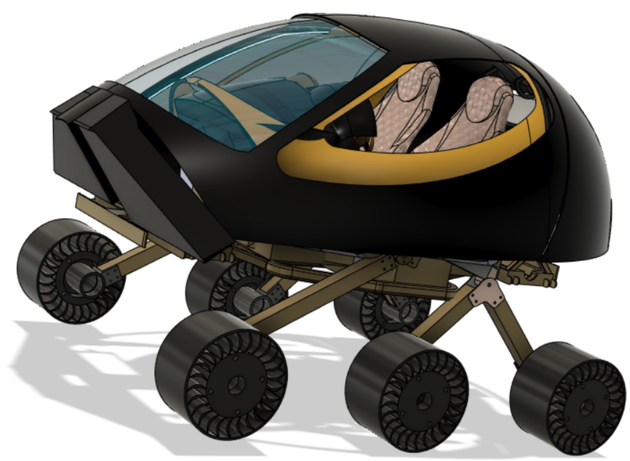
COURSE PROJECT

ED4060

DESIGN OF MECHANICAL SYSTEMS 2

"MULTI TERRAIN VEHICLE"

(Implementing Rocker-Bogie suspension)



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ACKNOWLEDGEMENT

We would like to thank Prof. Jayaganthan for giving us an opportunity to work on this project, with his constant mentoring and guidance. We would also like to thank our TA Sai Prasad Pranav Nadimpalli for his constant guidance and support throughout the project, and for helping us whenever we faced any roadblock. All this would not have been possible without constant encouragement from our professor and TA and also our parents and friends and family whom we would also like to profusely thank.

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"Invention causes things to come into existence from ideas, makes world conform to thought; whereas science, by deriving ideas from observation, makes thought conform to existence."

-Carl Mitchams

Figure 1: MTV we designed

1 ABSTRACT :

The objective of this project is to design an efficient rocker-bogie mechanism for multi-terrain traversal while keeping balance.We are trying to replace the existing spring suspension system found in ATV's with the rocker-bogie suspension system.The mechanism would be capable of traversing in any kind of terrain.

The application of the mechanism could be utilised in several other projects also like
i)Sowing and cultivation in hilly terrains
ii)Autonomous sensing in uneven mines and forests
iii)military and civil purposes
iv)Disaster relief and rescue.

This vehicle can be implemented in healthcare applications such as stair climbing wheelchair for the differently-able people. The applications for the multi terrain vehicle is vast and designing an efficient mechanism would help in several other fields as well.

The critical component of the multi-terrain vehicle is the suspension system and the mechanism that balances the multi-terrain vehicle.Since these are the critical components we are restricting our focus on the suspension system and the balancing mechanism.

We are looking to implement the rocker-bogie suspension system with a differential gear mechanism so as to counter the balance problems that exists with the suspension system in general(spring suspensions).

The objective of the project is to simulate the working of the rocker bogie suspension system with a differential gear mechanism and with a novel idea of incorporating it into a multi-terrain vehicle(MTV).
We are planning to apply for a patent for our idea.

2 PROBLEM-STATEMENT:

In our project we have done a detailed analysis of the rocker-bogie suspension system- the most commonly used suspension systems in rovers, which was our inspiration. We have done parametrisation for the link lengths and have done a kinematic link analysis of the rocker bogie.Our rocker bogie suspension also has a differential involved, which helps maintain balance.

This helps in maintaing contact for the wheels with the ground at all times. We have used a differential gear for the same, coupled with the rocker bogie suspension system.

Finally, we observed that the suspension system in ATVs involved a spring suspension system, which increases the reaction force from the ground to the vehicle and causes the traction of the higher wheel to be more than that of the lower wheel while climbing obstacles.

Our novelty is to try to incorporate the rocker bogie suspension system into the All Terrain Vehicles and to make a new type of vehicle called Multi Terrain Vehicle(MTV), which overcomes the problem of uneven traction.

3 SUSPENSION MECHANISM

Wheeled locomotion's main component is its suspension mechanism which connects the wheels to the main body or platform. This connection can be in several ways like springs, elastic rods or rigid mechanisms. Most of the heavy vehicles like trucks and train wagons use leaf springs.

The role of the suspension system is to support the vehicle weight, to separate the vehicle body from road disturbances, and to maintain the contact between the tire and the road surface also to improve stability and ride comfort of the vehicle.

For comfortable driving, cars use a complex spring, damping and mechanism combination. Generally, multi terrain vehicles are driven on the rough surface which consists of different sized stones and soft sand. For this reason, car suspensions are not applicable for multi terrain vehicles.

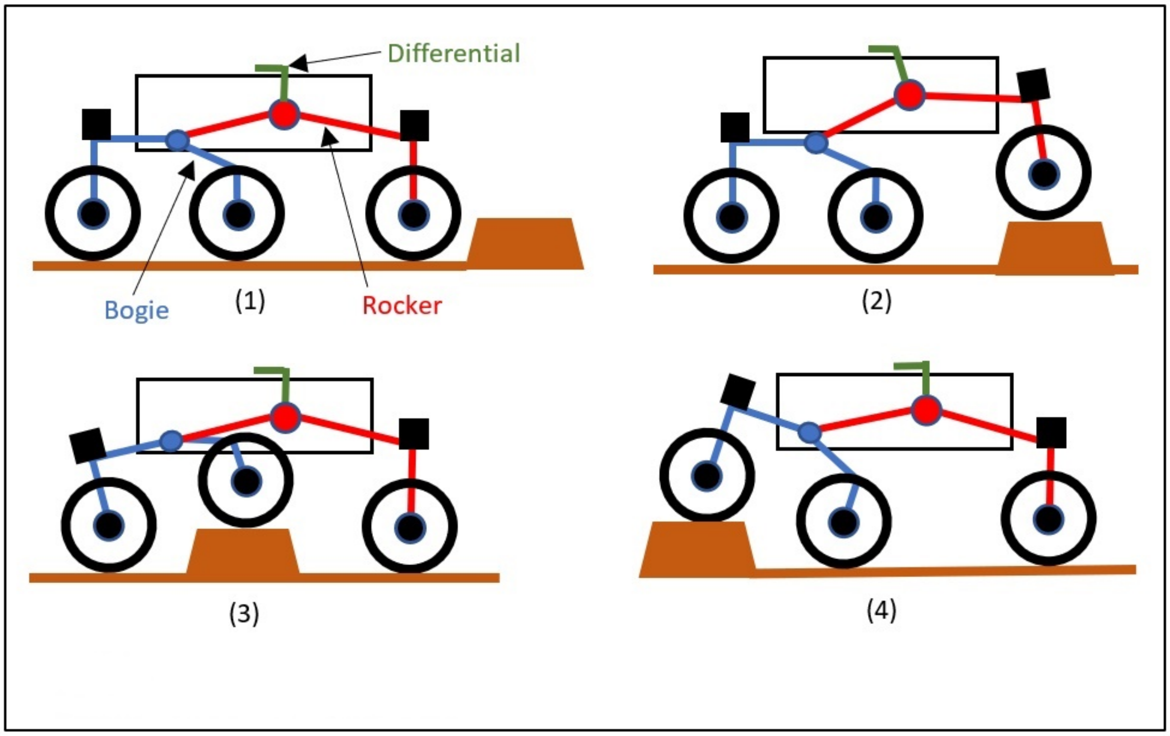

Figure 2: Suspension movement

3.1 The requirements of a MTV's suspension system

- As simple and lightweight as possible.
- Connections should be without spring to maintain equal traction force on wheels.
- Distribute load equally to each wheel for most of the orientation possibilities to prevent from slipping.
- To preserve stability of the vehicle in pitching and rolling while in motion.
- Independent movement of each of the wheels on an axle

3.2 Common Suspension Problems in the current ATV

1. One of the biggest problems in spring suspension system is that the balance of the vehicle may not be maintained in the case of rocky terrains or any other uneven terrains.

This is where the rocker-bogie has a significant advantage. The rocker-bogie helps increase the ability of the vehicle for balance by more than 50 percentage as compared to other suspension systems.

2. Most vehicles cannot move over all domains, as their suspension systems would not be able to change the alignment of the body according to the domain. Since the rocker-bogie differential suspension system mechanism ensures that there is always 6-wheel contact, coupled with the fact that there is weight distribution on 6 wheels rather than 4, the rocker-bogie is a suspension system that can be used for many more domains than other suspension systems.
The only difference that may arise would be that the material of the wheels alone may have to change, i.e the damping coefficient would have to be changed. Soft suspension systems with spring reduce vibrations and effects of impacts between wheel and ground.

Figure 3: Present spring suspension in ATV's

As we can see from figure 2,the current suspension system in ATV's is spring driven. The use of springs will cause the reaction force of pressed spring to increase the force that is transmitted from the wheel to the ground.While climbing over obstacles, the traction of the higher wheel is more than that of the lower wheel which causes slippage.

4 THE ROCKER-BOGIE MECHANISM :

The rocker-bogie mechanism system was initially used for the Mars Rover and is currently NASA's preferred design for rover wheel suspension. The perfectly designed wheel suspension allows the vehicle to travel over very uneven or rough terrain and even proceed over obstacles of several types.
This rocker suspension is a type of mechanism that allows a six wheel vehicle to constantly keep all six wheels in contact with a surface when driving on uneven terrain surfaces.

The rocker-bogie mechanism is one of the most popular suspension mechanisms, which was initially designed for space travel vehicles having its own deep history embedded in its development.By design it is a wheel robot which comprises 6 motorized wheels.

The word "rocker" describes the back part of the larger links present on both sides of the suspension system and these rockers are connected to each other and the vehicle chassis through a selectively modified differential in order to balance the chassis.

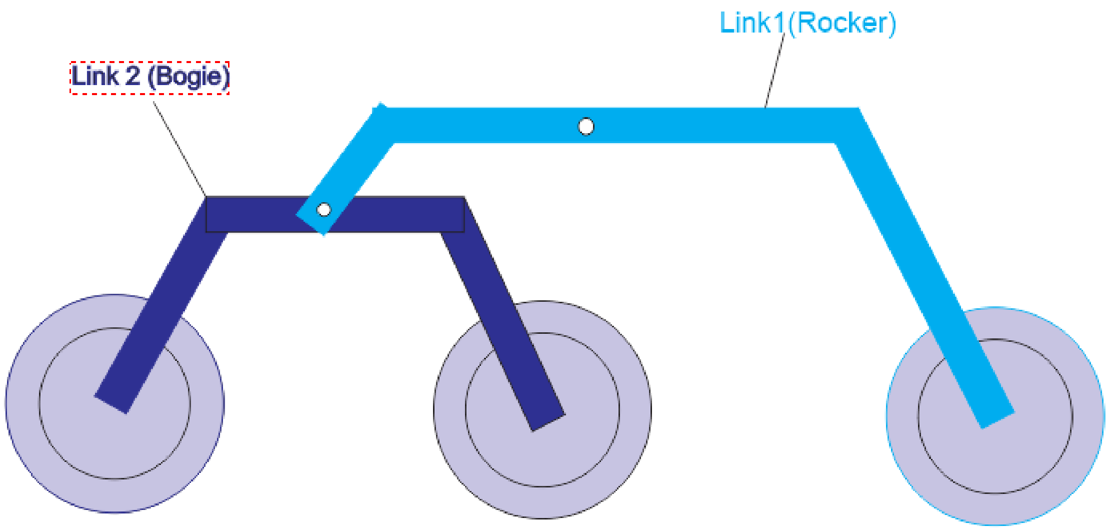

Figure 4: Rocker-Bogie Mechanism

By construction it has a main frame containing two linkages on each side that are called the "rocker" (see Figure 7). One end of the rocker is connected to the back wheel, and the other end is connected to maintain the center of gravity of the entire vehicle. As per the actual design, one end of a rocker is joined with a drive wheel and the other end is pivoted to a bogie which gives required moment and degree of freedom.

4.1 HISTORY OF ROCKER-BOGIE SUSPENSION:

The common suspension mechanism used in all the below rovers is the rocker-bogie mechanism.

4.1.1 Sojourner

Jet Propulsion Laboratory and California was designed to improved rovers with similar structure named Sojourner and Marie-Curie.

Figure 5: JPL Sojourner Rover

4.1.2 Shrimp

Shrimp is also a different six-wheeled rover.It has a one front four-bar to climb over obstacles up to two wheel diameter with no stability problem.
The Middle has four wheels which have parallelogram bogie which balances the wheel reaction forces during climbing. Single back and front wheels connected directly to the main body also driven by motor to increase the climbing capacity.

Figure 6: Shrimp vehicle

4.1.3 Mars Exploration Rovers (MER)

Each Mars Exploration Rover is 1.6 meter long and weighs 174 kilograms.with Rocker-Bogie suspension and 4-wheel steering.

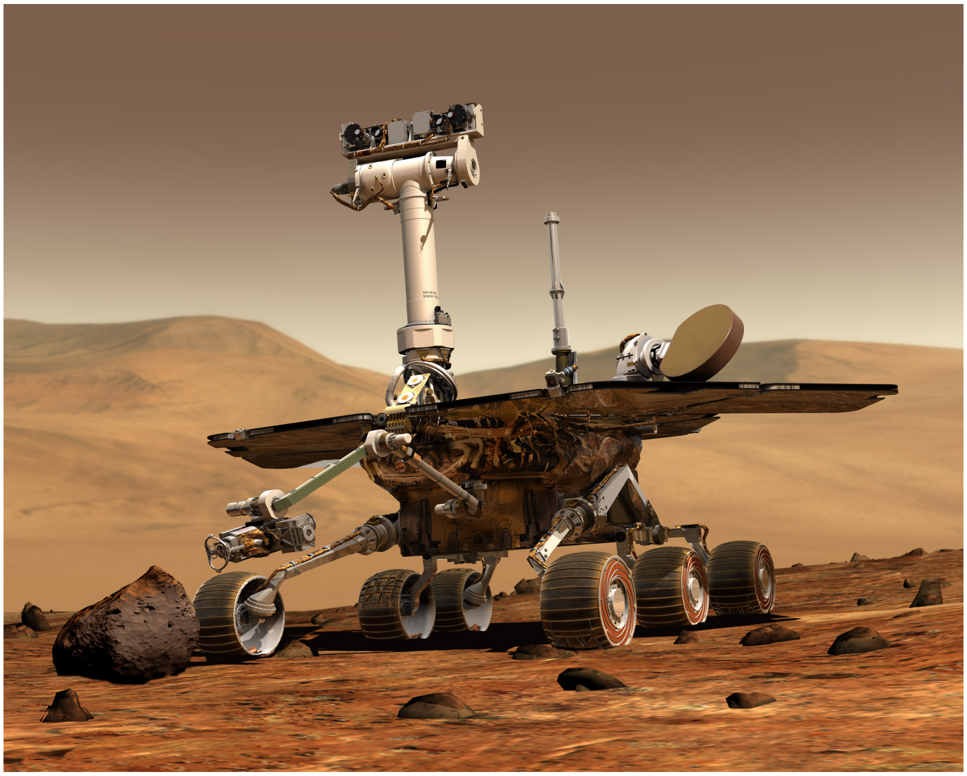

Figure 7: Mars Exploration Rover

4.2 The Mars Exploration Rover and its suspension system

The suspension system in MER is depends on how the wheels are connected to and interact with the rover body.

'Rocker' is used to describe the design of the differential,which enables the body to rock. The most important factor here is how to prevent sudden and dramatic position changes while going over a rocky domain.
In the absence of a differential, which is the suspension system for a rocker-bogie, the entire body would lose balance and may topple. What the differential does is to lower one side of the rocker-bogie when the other side goes up, to balance out the weight load on the 6 wheels. The differential ensures that all 6 wheels are in contact with the surface.

5 DIFFERENTIAL :

5.1 Balancing Mechanism: The Differential

The multi-terrain vehicle that we have designed now has six wheels and uses a rocker-bogie suspension system to drive smoothly over bumpy ground. There is one rocker-bogie assembly on each side of the multi-terrain vehicle.

The rocker bogie mechanism alone can't keep the body level. Without the differential the rover body can tip all the way forward or backward around the rocker pivots.

If you build a Rocker-bogie mechanism and you attach the rockers to the body with an axle or two pivot pins, the body will tip forward or backward until it hits the ground! In order to prevent this tipping and ensure stability of the body, the two rockers on either side are connected through a mechanism called differential which ensures stability. This differential ensures that the body is level. Relative to the body, when one rocker goes up, the other rocker goes down. Relative to the ground, the body angle is halfway between the angles of the two rockers. There are primarily two different types of differential mechanisms: a differential gearbox or a differential bar. Both ensure stability to the body, in this project differential gears are used, hence the discussion here focuses mainly on the differential gear and its working.

5.2 Differential Gear

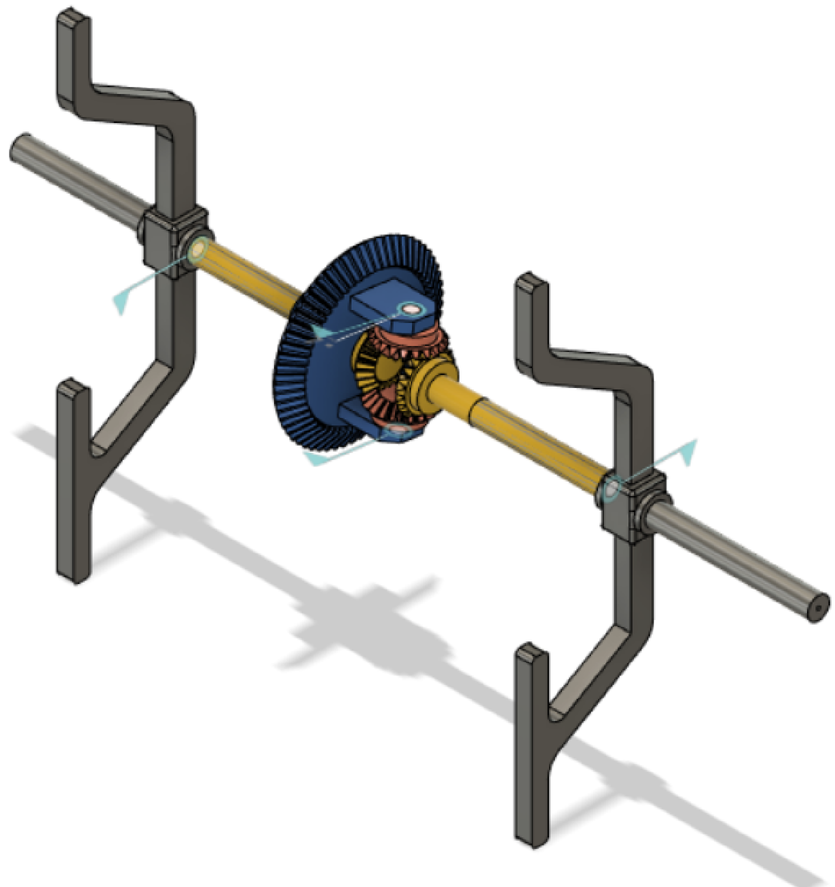

Figure 8: The differential gear (sun planetary system)

There are different types of differential gears. The one that was initially modelled for this project (shown in Fig 10) is a sun-planetary type differential gear. Both of the yellow gears have their shafts connected rigidly to the rockers on their side. The pink gears rotate about their axes and also revolve around the yellow gears hence giving the name of this type of gear system as sun-planetary gears. When one side of the rocker rotates it also rotates the yellow gear, which rotates the pink gear which also revolves around the yellow gears and effectively results in the rotation of the other yellow gear. The two yellow gears and the two pink gears have the same radius. The purpose of the blue gear is to just support the pink gears.

Since the two yellow and the two pink gears are of the same radius, when one yellow gear rotates the other yellow gear also rotates with the same speed in the opposite direction and since the shafts of the yellow gears are rigidly connected to the rockers they also rotate along with the gears. This leads us to the result that we initially wanted ie. when one rocker rotates the other rocker must rotate in the opposite direction which will ensure that at any point all six wheels of the rocker bogie are in contact with the ground thus the normal force is evenly distributed and the chances of tipping over is vastly reduced and stability of the body is ensured.

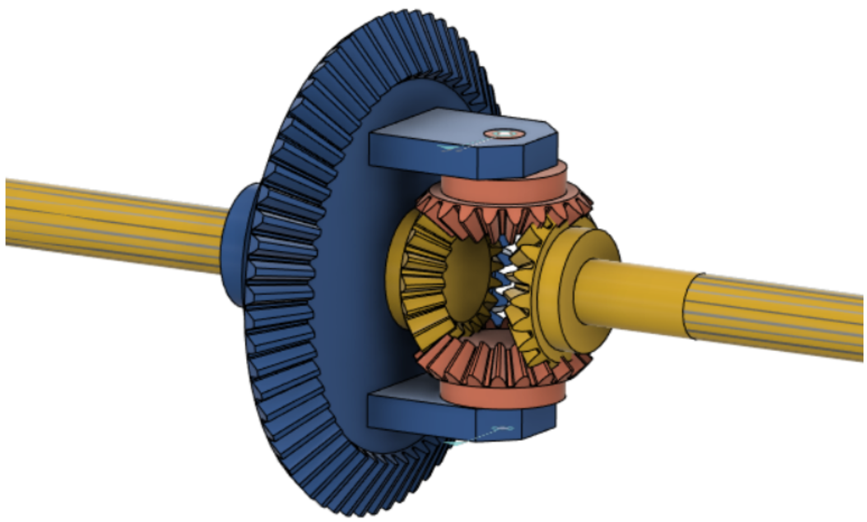

Figure 9: Visual of the sun planetary differential gear system

Even though the above differential is widely used it proved to be challenging to model the gear relations and simulate the gear hence a three gear differential was used. The above differential is functionally equivalent to the simple three gear but ensures longevity. For simulation purposes though three gear differential is sufficient.

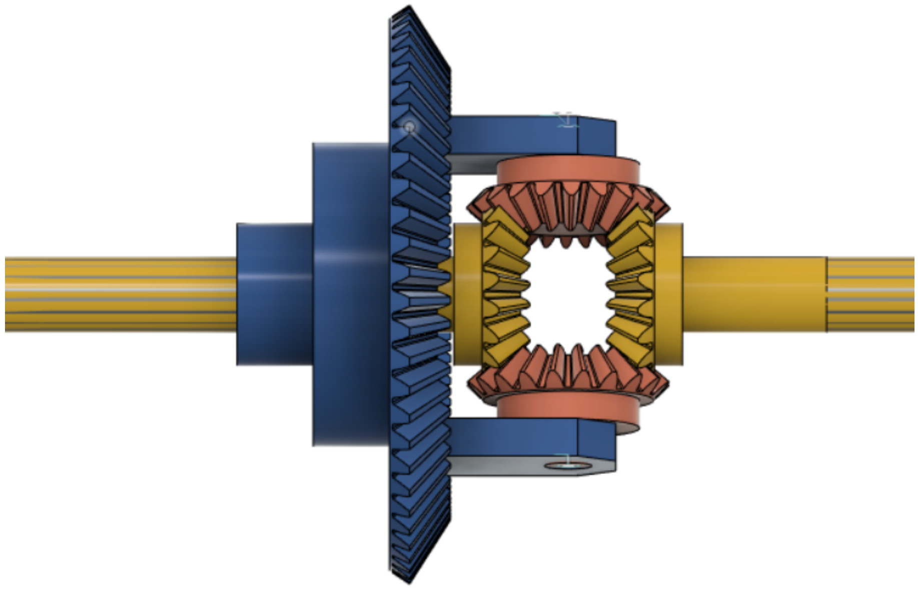

Figure 10: The sun planetary system,Top view

The three gear differential basically has two gears that are collinear and are have shafts that are connected the either sides of the rocker rigidly, these two gears are joined by a perpendicular gear, each pair constitutes a bevel gear, hence the three gear differential is basically a combination of two bevel gears.

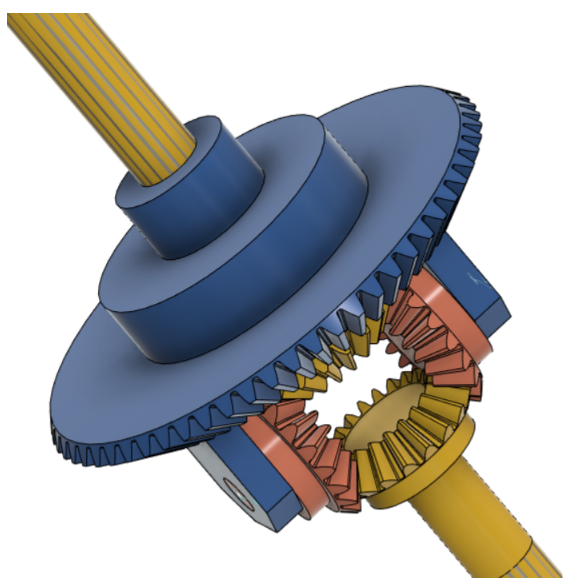

Figure 11: Power transmission through the gears

When one of the gears (say the left gear) rotate in a particular direction (say c.w) the middle gear rotates along its axis with the same angular speed but in opposite direction (c.c.w) this makes the right gear rotate about its axis in the same direction as the left gear (in c.w direction).But when all rotations are considered from a fixed frame one can quickly realize that the axes of the left and right gears are opposite to each other, hence when viewed from a fixed frame the left and right gear rotate in opposite direction, therefore the rockers attached to the left and right gears also rotates in opposite direction.

Figure 12: The 3 Gear differential mechanism

Hence when one rocker goes up the other has to come down which satisfies our initial requirement just like the sun and planetary gear system. Hence either of them can be used for simulation purposes. As mentioned above three gear differential is used in this project. (Simulation of the three gear differential is available in the simulation section of the report).

5.3 Differential Bar

Another type of differential mechanism that is widely used is the differential bar mechanism, the differential bar mechanism utilizes kinematic linkages instead of gear relations to obtain stability in the rocker bogie mechanism. The differential bar mechanism offers extreme longevity compared to the differential gear mechanism as it avoids the wear and tear that occurs when gears interact with each other but on the other hand it is also heavier and more expensive than the differential gears, hence it is primarily used in applications where change of parts is not possible, like in mars rovers.

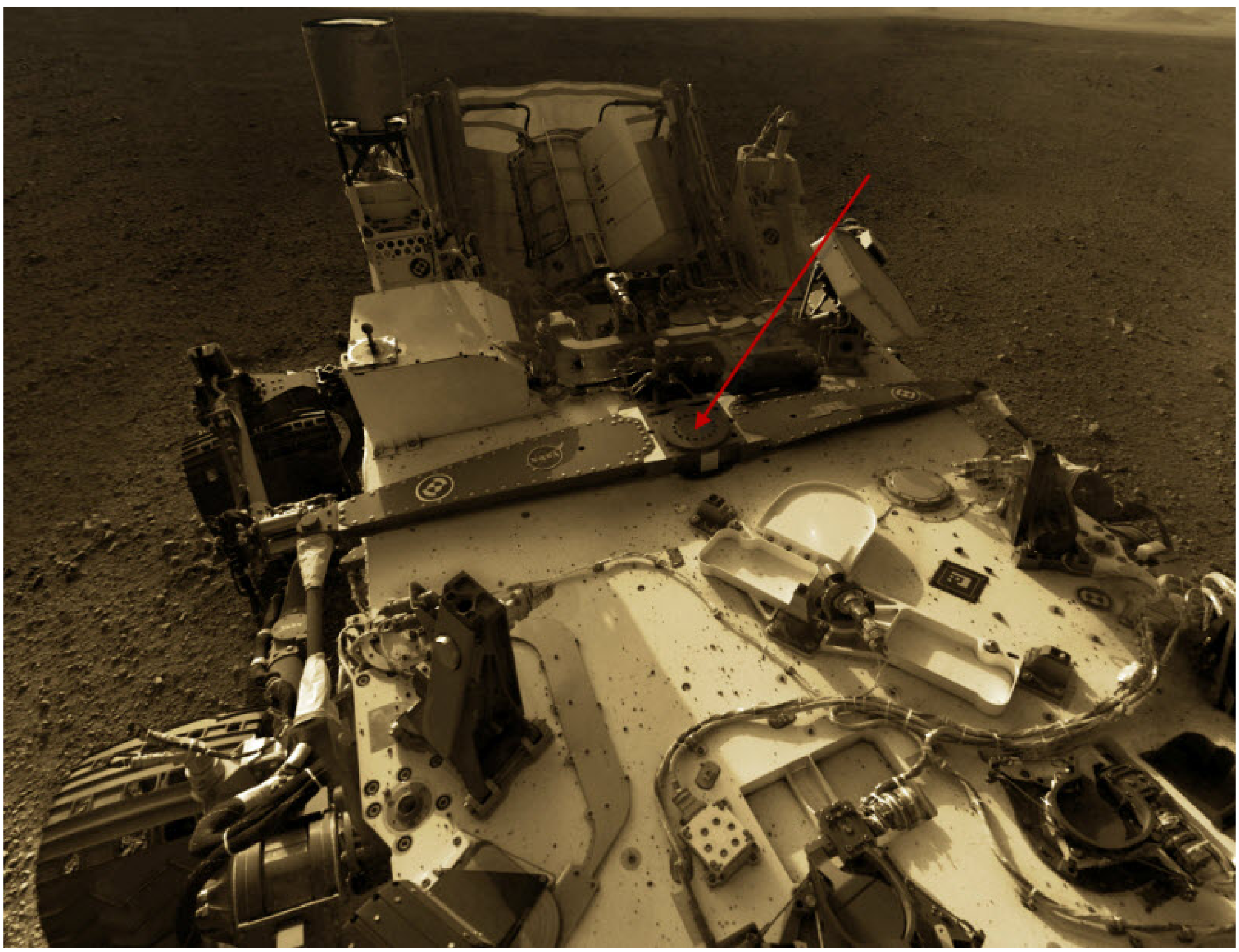

Figure 13: The differential bar mechanism

6 KINEMATIC AND DYNAMIC ANALYSIS:

6.1 Overview of D'Alembert's principle / Euler - Lagrange:

- The equations of motion for a system can be arrived at using D' Alembert's principle. We found this approach to analyze the Rocker bogie suspension to be simpler in comparison to using newton's laws directly.
- D'alembert's principle stems from the notion of virtual work and its extension to systems that are not in equilibrium. It is alternatively known as the Euler - Lagrange equations.
- Its simplicity is due to its use of scalar quantities (T, V) and because it is easy to express it using any type coordinates (generalized coordinates) and does not require the rewriting equations to suite the coordinate system like in Newton's formulation. Its power lies in its generality.
- In other words, if one knows the expressions for T and V along with the constraints and generalized driving forces it is straight forward to derive the EOM(equations of motion).

D'Alembert's principle / Euler - Lagrange equations:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_{potential,\,i} + Q_{driving,\,i}, \quad where\; i = 1, \ldots, n$$

Where,

- n is the no. of DOF
- T represents the kinetic energy of the system
- q_i represents the i-th generalized coordinate
- $\dot{q}_i$ represents the i-th generalized velocity
- t represents time
- $Q_{potential,\,i}$ represents the generalized forces caused by potentials
- $Q_{driving,\,i}$ represents the generalized driving/ actuator forces

We can rewrite $Q_{potential,\,i}$ as $-\frac{\partial V}{\partial q_i}$ The equation can be modified to incorporate constraints as well
Modified version to incorporate constraints:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_{potential,\, i} + Q_{driving,\, i} + \sum_{j=1}^{m} \lambda_j \frac{\partial C_j}{\partial q_i}$$

or

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = -\frac{\partial V}{\partial q_i} + Q_{driving,\, i} + \sum_{j=1}^{m} \lambda_j \frac{\partial C_j}{\partial q_i}$$

Where,

- C_j represents the j - th constraint.
- The total number of constraints are denoted by m, here
- The terms given by are the generalized constraint forces due to constraint C_j
- The variables denoted by λ_j are known as Lagrange multipliers

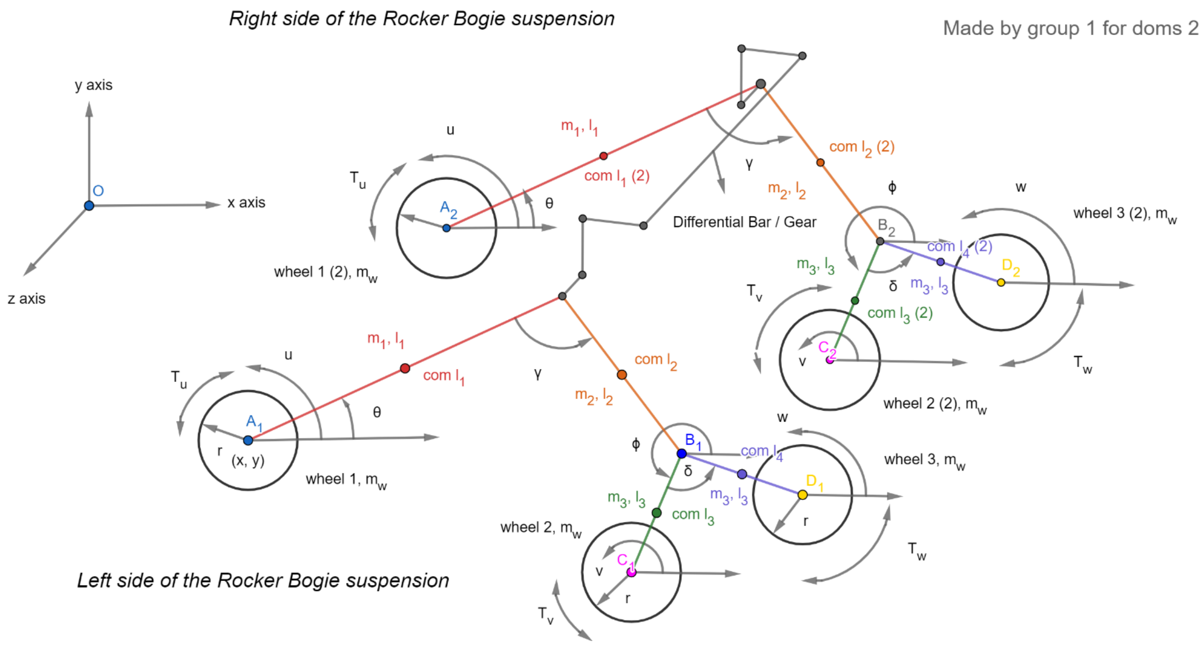

Figure 14: Rocker-bogie with differential constraint

The above diagram shows a rocker bogie suspension with both its left and right side lying in planes parallel to the x-y plane.
The differential is always parallel to the z axis due to assumptions made on the surface (stated below).

Constants / model parameters/ information:

- Gamma – angle between links 1 and 2
- Delta – angle between links 3 and 4
- Alpha – angle representing the orientation of the box with mass M, it is a constant
- Link 3 and 4 have the same lengths and same masses, hence, the link lengths are l_1, l_2, l_3, l_3 for the four links respectively. The masses of the links are given by m_1, m_2, m_3, m_3
- The gravitational field points in the negative y direction
- The wheels have the same radius r and same mass m_w
- Points B_1 and B_2 have revolute joints
- The COM of the box (including the differential) is taken to be at the meeting point of links 1 and 2 for simplicity. The mass of the box unit is denoted by M and its moment of inertia about its COM is denoted by I.

6.2 Degrees of Freedom analysis:

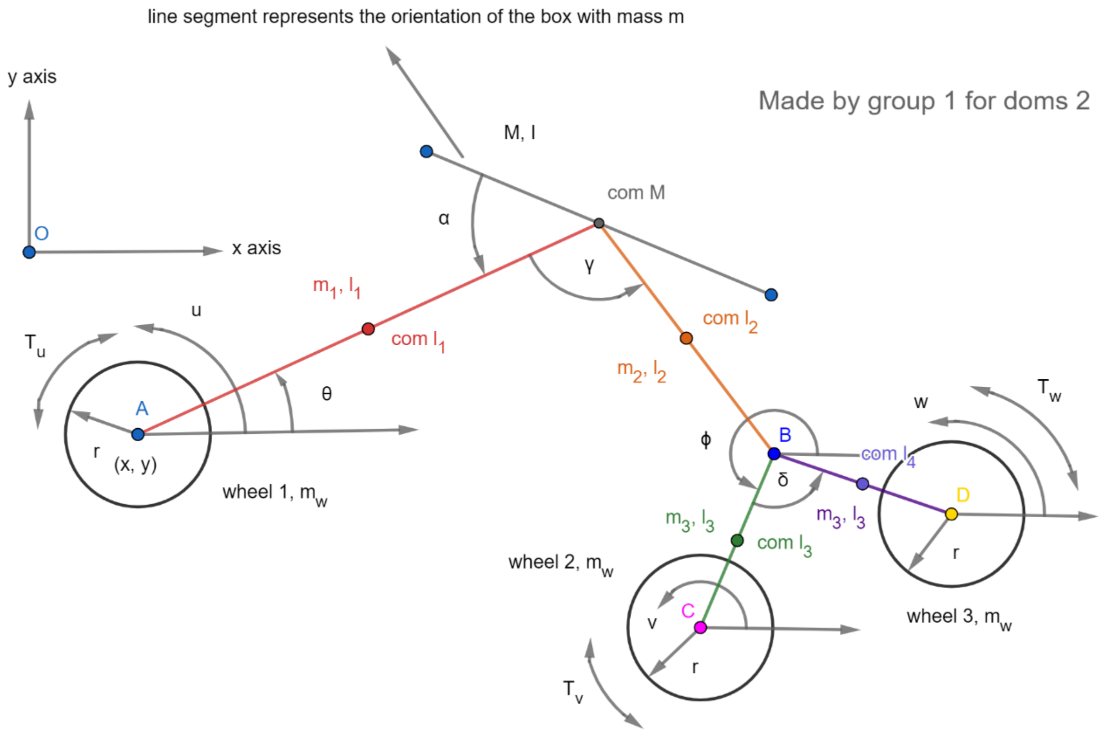

Figure 15: Rocker-bogie side view along the z axis

A simplified version of the first diagram (the rocker bogie in the first diagram is seen in a direction parallel to the z axis and is drawn without the differential as it may be ignored) is shown above.

Assumption on the surface while calculating the Degrees of Freedom:

- The rocker bogie faces the same surface elevation on both sides, hence the differential can be ignored in the analysis. In other words, this causes the problem to become planar

When the first assumption holds true, the rocker bogie has 7 degrees of freedom. The degrees of freedom are the (x, y) coordinates of wheel 1, the angle theta that link 1 makes with the horizontal, the angle phi that link 3 makes with the x axis, the angles rotated by the 3 wheels (u, v, w).

The (x, y) coordinates and theta specify the positions of the center of wheel 1 and completely specify the position of links 1 and 2. Due the presence of a revolute joint at B, the angle phi must be known to completely specify the positions of links 3 and 4 and the centers of wheels 2 and 3. As we have found only the centers of the wheels, 3 angles, u, v, w, one for each wheel must be known to determine their orientation.

The same conclusion can be arrived at using Gruebler's equation.
Gruebler's equation: no. of DOF = 3L – $2J_1$ – J_2 – 3G
Where,
Here,
L = 5 ($l_1$and $l_2$count as a single link as they form a rigid body,l_3 and l_4 count as a single link as they form a rigid body, wheel 1, wheel 2, wheel 3)
$J_1 = 4$ (4 revolute joints exist, they are present at A, B, C, D)
$J_2 = 0$
G = 0

Hence, using Gruebler's equation, no. of DOF = $3 \times 5 – 2 \times 4 – 0 – 0 = 7$.

Assumption on the moments of inertia of some components about their COM to simplify calculations:

- The links can be considered as 1-D sticks for moment of inertia calculations (a 1-D stick of length l and mass m has a moment of inertia of $ml^2/12$ about an axis passing through its center of mass (COM))
- The wheels can be considered as rings/ hollow cylinders for moment of inertia calculations (a ring/ hollow cylinder of mass m has a moment of inertia of $m * r^2$ about an axis passing through its center of mass (COM), that is perpendicular to its plane)

The DOF will be chosen as generalized coordinates.

To use D'Alembert's principle we must first identify the positions and velocities of the COMs all the components in terms of the generalized coordinates (here, DOF).

6.3 Position Analysis :

Position of wheel 1's COM :

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

Position of link 1's COM :

$$\begin{bmatrix} x(t) + \ l_1 \cos(\theta(t)) \\ y(t) + \ l_1 \sin(\theta(t)) \end{bmatrix}$$

Position of COM of M:

$$\begin{bmatrix} x(t) + 2\,l_1 \cos(\theta(t)) \\ y(t) + 2\,l_1 \sin(\theta(t)) \end{bmatrix}$$

Position of link 2's COM :

$$\begin{bmatrix} x(t) + 2\,l_1 \cos(\theta(t)) - l_2 \cos(\gamma + \theta(t)) \\ y(t) + 2\,l_1 \sin(\theta(t)) - l_2 \sin(\gamma + \theta(t)) \end{bmatrix}$$

Position of link 3's COM:

$$\begin{bmatrix} x(t) + l_3 \cos(\phi(t)) + 2\,l_1 \cos(\theta(t)) - 2\,l_2 \cos(\gamma + \theta(t)) \\ y(t) + l_3 \sin(\phi(t)) + 2\,l_1 \sin(\theta(t)) - 2\,l_2 \sin(\gamma + \theta(t)) \end{bmatrix}$$

Position of wheel 2's COM :

$$\begin{bmatrix} x(t) + 2\,l_3 \cos(\phi(t)) + 2\,l_1 \cos(\theta(t)) - 2\,l_2 \cos(\gamma + \theta(t)) \\ y(t) + 2\,l_3 \sin(\phi(t)) + 2\,l_1 \sin(\theta(t)) - 2\,l_2 \sin(\gamma + \theta(t)) \end{bmatrix}$$

Position of link 4's COM :

$$\begin{bmatrix} x(t) + 2\,l_1 \cos(\theta(t)) + l_3 \cos(\delta + \phi(t)) - 2\,l_2 \cos(\gamma + \theta(t)) \\ y(t) + 2\,l_1 \sin(\theta(t)) + l_3 \sin(\delta + \phi(t)) - 2\,l_2 \sin(\gamma + \theta(t)) \end{bmatrix}$$

Position of wheel 3's COM :

$$\begin{bmatrix} x(t) + 2\,l_1 \cos(\theta(t)) + 2\,l_3 \cos(\delta + \phi(t)) - 2\,l_2 \cos(\gamma + \theta(t)) \\ y(t) + 2\,l_1 \sin(\theta(t)) + 2\,l_3 \sin(\delta + \phi(t)) - 2\,l_2 \sin(\gamma + \theta(t)) \end{bmatrix}$$

6.4 Velocity Analysis :

We can differentiate positions to find velocities. The dot notation is used to denote time derivatives, here.

Velocity of wheel 1's COM :

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

Velocity of link 1's COM :

$$\begin{bmatrix} \dot{x} - l_1\,\dot{\theta}\sin\left(\theta\left(t\right)\right) \\ \dot{y} + l_1\,\dot{\theta}\cos\left(\theta\left(t\right)\right) \end{bmatrix}$$

Velocity of COM of M:

$$\begin{bmatrix} \dot{x} - 2\,l_1\,\dot{\theta}\sin\left(\theta\left(t\right)\right) \\ \dot{y} + 2\,l_1\,\dot{\theta}\cos\left(\theta\left(t\right)\right) \end{bmatrix}$$

Velocity of link 2's COM :

$$\begin{bmatrix} x\left(t\right) + 2\,l_1\cos\left(\theta\left(t\right)\right) + 2\,l_3\cos\left(\delta + \phi\left(t\right)\right) - 2\,l_2\cos\left(\gamma + \theta\left(t\right)\right) \\ y\left(t\right) + 2\,l_1\sin\left(\theta\left(t\right)\right) + 2\,l_3\sin\left(\delta + \phi\left(t\right)\right) - 2\,l_2\sin\left(\gamma + \theta\left(t\right)\right) \end{bmatrix}$$

Velocity of link 3's COM :

$$\begin{bmatrix} \dot{x} + l_2\,\dot{\theta}\sin\left(\gamma + \theta\left(t\right)\right) - 2\,l_1\,\dot{\theta}\sin\left(\theta\left(t\right)\right) \\ \dot{y} - l_2\,\dot{\theta}\cos\left(\gamma + \theta\left(t\right)\right) + 2\,l_1\,\dot{\theta}\cos\left(\theta\left(t\right)\right) \end{bmatrix}$$

Velocity of wheel 2's COM :

$$\begin{bmatrix} \dot{x} + 2\,l_2\,\dot{\theta}\sin\left(\gamma + \theta\left(t\right)\right) - 2\,l_3\,\dot{\phi}\sin\left(\phi\left(t\right)\right) - 2\,l_1\,\dot{\theta}\sin\left(\theta\left(t\right)\right) \\ \dot{y} - 2\,l_2\,\dot{\theta}\cos\left(\gamma + \theta\left(t\right)\right) + 2\,l_3\,\dot{\phi}\cos\left(\phi\left(t\right)\right) + 2\,l_1\,\dot{\theta}\cos\left(\theta\left(t\right)\right) \end{bmatrix}$$

Velocity of link 4's COM :

$$\begin{bmatrix} \dot{x} - l_3\,\dot{\phi}\sin\left(\delta + \phi\left(t\right)\right) + 2\,l_2\,\dot{\theta}\sin\left(\gamma + \theta\left(t\right)\right) - 2\,l_1\,\dot{\theta}\sin\left(\theta\left(t\right)\right) \\ \dot{y} + l_3\,\dot{\phi}\cos\left(\delta + \phi\left(t\right)\right) - 2\,l_2\,\dot{\theta}\cos\left(\gamma + \theta\left(t\right)\right) + 2\,l_1\,\dot{\theta}\cos\left(\theta\left(t\right)\right) \end{bmatrix}$$

Velocity of wheel 3's COM :

$$\begin{bmatrix} \dot{x} - 2\,l_3\,\dot{\phi}\sin\left(\delta + \phi\left(t\right)\right) + 2\,l_2\,\dot{\theta}\sin\left(\gamma + \theta\left(t\right)\right) - 2\,l_1\,\dot{\theta}\sin\left(\theta\left(t\right)\right) \\ \dot{y} + 2\,l_3\,\dot{\phi}\cos\left(\delta + \phi\left(t\right)\right) - 2\,l_2\,\dot{\theta}\cos\left(\gamma + \theta\left(t\right)\right) + 2\,l_1\,\dot{\theta}\cos\left(\theta\left(t\right)\right) \end{bmatrix}$$

Angular velocities about axes passing through the COM of each element, that are parallel to the z axis:

- link 1, link 2 and M – $\dot{theta}$ (They have the same angular velocity because they form a rigid body)
- link 3 and link 4 -$\dot{phi}$ (They have the same angular velocity because they form a rigid body)
- wheel 1 - $\dot{u}$
- wheel 2 - $\dot{v}$
- wheel 3 - $\dot{w}$

6.5 Equations of motion:

These equations do not account for constraint forces and constraint forces will be accounted once the constraint are stated for the wall climbing scenario. This is done in the next section using the modified version of $D'Alembert's$ principle.

T is given by:

T =

$$m_2\,(4\,l_1^2\,\dot\theta^2 - 4\cos(\gamma)\,l_1\,l_2\,\dot\theta^2 - 4\sin(\theta(t))\,l_1\,\dot\theta\,\dot x + 4\cos(\theta(t))\,l_1\,\dot\theta\,\dot y + l_2^2\,\dot\theta^2 + 2\sin(\gamma+\theta(t))\,l_2\,\dot\theta\,\dot x - 2\cos(\gamma+\theta(t))\,l_2\,\dot\theta\,\dot y + \dot x^2 + \dot y^2)\,\cdot$$

$$+\frac{I\dot\theta^2}{2} + m_3\,(4\,l_1^2\,\dot\theta^2 - 8\cos(\gamma)\,l_1\,l_2\,\dot\theta^2 + 4\cos(\delta+\phi(t)-\theta(t))\,l_1\,l_3\,\dot\phi\,\dot\theta - 4\sin(\theta(t))\,l_1\,\dot\theta\,\dot x + 4\cos(\theta(t))\,l_1\,\dot\theta\,\dot y + 4\,l_2^2\,\dot\theta^2 - 4\cos(\delta-\gamma+\phi(t)-\theta(t))\,l_2\,l_3\,\dot\phi\,\dot\theta + 4\sin(\gamma+\theta(t))\,l_2\,\dot\theta\,\dot x - 4\cos(\gamma+\theta(t))\,l_2\,\dot\theta\,\dot y + l_3^2\,\dot\phi^2$$

$$^2 - 2\sin(\delta+\phi(t))\,l_3\,\dot\phi\,\dot x + 2\cos(\delta+\phi(t))\,l_3\,\dot\phi\,\dot y + \dot x^2 + \dot y^2) + m_w\,(4\,l_1^2\,\dot\theta^2 - 8\cos(\gamma)\,l_1\,l_2\,\dot\theta^2 + 8\cos(\delta+\phi(t)-\theta(t))\,l_1\,l_3\,\dot\phi\,\dot\theta - 4\sin(\theta(t))\,l_1\,\dot\theta\,\dot x + 4\cos(\theta(t))\,l_1\,\dot\theta\,\dot y + 4\,l_2^2\,\dot\theta^2 - 8\cos(\delta-\gamma+\phi(t)-\theta(t))\,l_2\,l_3\,\dot\phi\,\dot\theta$$

$$\dot\theta + 4\sin(\gamma+\theta(t))\,l_2\,\dot\theta\,\dot x - 4\cos(\gamma+\theta(t))\,l_2\,\dot\theta\,\dot y + 4\,l_3^2\,\dot\phi^2 - 4\sin(\delta+\phi(t))\,l_3\,\dot\phi\,\dot x + 4\cos(\delta+\phi(t))\,l_3\,\dot\phi\,\dot y + \dot x^2 + \dot y^2) + \frac{M\,(4\,l_1^2\,\dot\theta^2 - 4\sin(\theta(t))\,l_1\,\dot\theta\,\dot x + 4\cos(\theta(t))\,l_1\,\dot\theta\,\dot y + \dot x^2 + \dot y^2)}{2} + m_w\,(\dot x^2 + \dot y^2)\,\cdot$$

$$) + m_1\,(l_1^2\,\dot\theta^2 - 2\sin(\theta(t))\,l_1\,\dot\theta\,\dot x + 2\cos(\theta(t))\,l_1\,\dot\theta\,\dot y + \dot x^2 + \dot y^2) + m_3\,(4\,l_1^2\,\dot\theta^2 - 8\cos(\gamma)\,l_1\,l_2\,\dot\theta^2 + 4\cos(\phi(t)-\theta(t))\,l_1\,l_3\,\dot\phi\,\dot\theta - 4\sin(\theta(t))\,l_1\,\dot\theta\,\dot x + 4\cos(\theta(t))\,l_1\,\dot\theta\,\dot y + 4\,l_2^2\,\dot\theta^2 - 4\cos(\gamma-\phi(t)+\theta(t))\,l_2\,l_3\,\dot\phi\,\dot\theta$$

$$+4\sin(\gamma+\theta(t))\,l_2\,\dot\theta\,\dot x - 4\cos(\gamma+\theta(t))\,l_2\,\dot\theta\,\dot y + l_3^2\,\dot\phi^2 - 2\sin(\phi(t))\,l_3\,\dot\phi\,\dot x + 2\cos(\phi(t))\,l_3\,\dot\phi\,\dot y + \dot x^2 + \dot y^2) + m_w\,(4\,l_1^2\,\dot\theta^2 - 8\cos(\gamma)\,l_1\,l_2\,\dot\theta^2 + 8\cos(\phi(t)-\theta(t))\,l_1\,l_3\,\dot\phi\,\dot\theta - 4\sin(\theta(t))\,l_1\,\dot\theta\,\dot x + 4\cos(\theta(t))\,l_1\,\dot\theta\,\dot y + 4\,l_2^2\,\dot\theta^2$$

$$^2 - 8\cos(\gamma-\phi(t)+\theta(t))\,l_2\,l_3\,\dot\phi\,\dot\theta + 4\sin(\gamma+\theta(t))\,l_2\,\dot\theta\,\dot x - 4\cos(\gamma+\theta(t))\,l_2\,\dot\theta\,\dot y + 4\,l_3^2\,\dot\phi^2 - 4\sin(\phi(t))\,l_3\,\dot\phi\,\dot x + 4\cos(\phi(t))\,l_3\,\dot\phi\,\dot y + \dot x^2 + \dot y^2) + \frac{2\,l_3^2\,m_3\,\dot\phi^2}{3} + \frac{l_1^2\,m_1\,\dot\theta^2}{3} + \frac{l_2^2\,m_2\,\dot\theta^2}{3} + m_w\,r^2\,\dot u^2 + m_w\,r^2\,\dot v^2 + m_w\,r^2\,\dot w^2$$

V is given by:

$$(2\,g\,l_3\,m_3 + 4\,g\,l_3\,m_w)\sin(\delta + \phi(t))$$
$$+\,(2\,M\,g\,l_1 + 2\,g\,l_1\,m_1 + 4\,g\,l_1\,m_2 + 8\,g\,l_1\,m_3 + 8\,g\,l_1\,m_w)\sin(\theta(t))$$

$$+ \left(2\,g\,l_3\,m_3 + 4\,g\,l_3\,m_w\right)\sin\left(\phi\left(t\right)\right)$$
$$+ \left(M\,g + 2\,g\,m_1 + 2\,g\,m_2 + 4\,g\,m_3 + 6\,g\,m_w\right)y\left(t\right)$$
$$+ \left(-2\,g\,l_2\,m_2 - 8\,g\,l_2\,m_3 - 8\,g\,l_2\,m_w\right)\sin\left(\gamma + \theta\left(t\right)\right)$$

The generalized forces due to potential are:

$$\begin{bmatrix} 0 \\ -M\,g - 2\,g\,m_1 - 2\,g\,m_2 - 4\,g\,m_3 - 6\,g\,m_w \\ (2\,g\,l_2\,m_2 + 8\,g\,l_2\,m_3 + 8\,g\,l_2\,m_w) \\ \cos(\gamma + \theta(t)) + (-2\,M\,g\,l_1 - 2\,g\,l_1\,m_1 - 4\,g\,l_1\,m_2 - 8\,g\,l_1\,m_3 - 8\,g\,l_1\,m_w)\cos(\theta(t)) \\ (-2\,g\,l_3\,m_3 - 4\,g\,l_3\,m_w)\cos(\delta + \phi(t)) + (-2\,g\,l_3\,m_3 - 4\,g\,l_3\,m_w)\cos(\phi(t)) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The generalized driving forces are:

$$\begin{bmatrix} 0 \\ 0 \\ -T_u \\ -T_v - T_w \\ T_u \\ T_v \\ T_w \end{bmatrix}$$

Equation of motion corresponding to x

$$(M + 2\,m_1 + 2\,m_2 + 4\,m_3 + 6\,m_w)\,\frac{\partial^2}{\partial t^2}\,x(t) - \left(\frac{\partial}{\partial t}\,\phi(t)\right)^2 (2\,l_3\,m_3\cos(\delta + \phi(t)) + 4\,l_3\,m_w\cos(\delta + \phi(t)) + 2\,l_3\,m_3\cos(\phi(t)) + 4\,l_3\,m_w\cos(\phi(t)))$$

$$- \left(\frac{\partial}{\partial t}\,\theta(t)\right)^2 (2\,M\,l_1\cos(\theta(t)) - 8\,l_2\,m_3\cos(\gamma + \theta(t)) - 8\,l_2\,m_w\cos(\gamma + \theta(t)) - 2\,l_2\,m_2\cos(\gamma + \theta(t)) + 2\,l_1\,m_1\cos(\theta(t)) + 4\,l_1\,m_2\cos(\theta(t)) + 8\,l_1\,m_3\cos(\theta(t)) + 8\,l_1\,m_w\cos(\theta(t)))$$

$$- (2\,l_3\,m_3\sin(\delta + \phi(t)) + 4\,l_3\,m_w\sin(\delta + \phi(t)) + 2\,l_3\,m_3\sin(\phi(t)) + 4\,l_3\,m_w\sin(\phi(t)))\,\frac{\partial^2}{\partial t^2}\,\phi(t)$$

$$- (2\,M\,l_1\sin(\theta(t)) - 8\,l_2\,m_3\sin(\gamma + \theta(t)) - 8\,l_2\,m_w\sin(\gamma + \theta(t)) - 2\,l_2\,m_2\sin(\gamma + \theta(t)) + 2\,l_1\,m_1\sin(\theta(t)) + 4\,l_1\,m_2\sin(\theta(t)) + 8\,l_1\,m_3\sin(\theta(t)) + 8\,l_1\,m_w\sin(\theta(t)))\,\frac{\partial^2}{\partial t^2}\,\theta(t) = 0$$

Equation of motion corresponding to y

$$(2\,M\,l_1\cos(\theta(t)) - 8\,l_2\,m_3\cos(\gamma + \theta(t)) - 8\,l_2\,m_w\cos(\gamma + \theta(t)) - 2\,l_2\,m_2\cos(\gamma + \theta(t)) + 2\,l_1\,m_1\cos(\theta(t)) + 4\,l_1\,m_2\cos(\theta(t)) + 8\,l_1\,m_3\cos(\theta(t)) + 8\,l_1\,m_w\cos(\theta(t)))\,\frac{\partial^2}{\partial t^2}\,\theta(t) + (M + 2\,m_1 + 2\,m_2 + 4\,m_3 + 6\,m_w)\,\frac{\partial^2}{\partial t^2}\,y(t)$$

$$+ (2\,l_3\,m_3\cos(\delta + \phi(t)) + 4\,l_3\,m_w\cos(\delta + \phi(t)) + 2\,l_3\,m_3\cos(\phi(t)) + 4\,l_3\,m_w\cos(\phi(t)))\,\frac{\partial^2}{\partial t^2}\,\phi(t) - \left(\frac{\partial}{\partial t}\,\phi(t)\right)^2 (2\,l_3\,m_3\sin(\delta + \phi(t)) + 4\,l_3\,m_w\sin(\delta + \phi(t)) + 2\,l_3\,m_3\sin(\phi(t)) + 4\,l_3\,m_w\sin(\phi(t)))$$

$$- \left(\frac{\partial}{\partial t}\,\theta(t)\right)^2 (2\,M\,l_1\sin(\theta(t)) - 8\,l_2\,m_3\sin(\gamma + \theta(t)) - 8\,l_2\,m_w\sin(\gamma + \theta(t)) - 2\,l_2\,m_2\sin(\gamma + \theta(t)) + 2\,l_1\,m_1\sin(\theta(t)) + 4\,l_1\,m_2\sin(\theta(t)) + 8\,l_1\,m_3\sin(\theta(t)) + 8\,l_1\,m_w\sin(\theta(t)))$$

$$= -M\,g - 2\,g\,m_1 - 2\,g\,m_2 - 4\,g\,m_3 - 6\,g\,m_w$$

Equation of motion corresponding to theta

$$\left(I+4\,M\,l_1^2+\frac{8\,l_1^2\,m_1}{3}+8\,l_1^2\,m_2+16\,l_1^2\,m_3+\frac{8\,l_2^2\,m_2}{3}+16\,l_2^2\,m_3+16\,l_1^2\,m_w+16\,l_2^2\,m_w-8\,l_1\,l_2\,m_2\cos(\gamma)-32\,l_1\,l_2\,m_3\cos(\gamma)-32\,l_1\,l_2\,m_w\cos(\gamma)\right)\frac{\partial^2}{\partial t^2}\,\theta(t)$$

$$+\,(2\,M\,l_1\cos(\theta(t))-8\,l_2\,m_3\cos(\gamma+\theta(t))-8\,l_2\,m_w\cos(\gamma+\theta(t))-2\,l_2\,m_2\cos(\gamma+\theta(t))+2\,l_1\,m_1\cos(\theta(t))+4\,l_1\,m_2\cos(\theta(t))+8\,l_1\,m_3\cos(\theta(t))+8\,l_1\,m_w\cos(\theta(t)))\frac{\partial^2}{\partial t^2}\,y(t)$$

$$+\,(4\,l_1\,l_3\,m_3\cos(\phi(t)-\theta(t))+8\,l_1\,l_3\,m_w\cos(\phi(t)-\theta(t))-4\,l_2\,l_3\,m_3\cos(\delta-\gamma+\phi(t)-\theta(t))-8\,l_2\,l_3\,m_w\cos(\delta-\gamma+\phi(t)-\theta(t))+4\,l_1\,l_3\,m_3\cos(\delta+\phi(t)-\theta(t))+8\,l_1\,l_3\,m_w\cos(\delta+\phi(t)-\theta(t))$$

$$-\,4\,l_2\,l_3\,m_3\cos(\gamma-\phi(t)+\theta(t))-8\,l_2\,l_3\,m_w\cos(\gamma-\phi(t)+\theta(t)))\frac{\partial^2}{\partial t^2}\,\phi(t)-(2\,M\,l_1\sin(\theta(t))-8\,l_2\,m_3\sin(\gamma+\theta(t))-8\,l_2\,m_w\sin(\gamma+\theta(t))-2\,l_2\,m_2\sin(\gamma+\theta(t))+2\,l_1\,m_1\sin(\theta(t))+4\,l_1\,m_2\sin(\theta(t))+8\,l_1\,m_3\sin(\theta(t))$$

$$+\,8\,l_1\,m_w\sin(\theta(t)))\frac{\partial^2}{\partial t^2}\,x(t)-\left(\frac{\partial}{\partial t}\,\phi(t)\right)^2(4\,l_1\,l_3\,m_3\sin(\delta+\phi(t)-\theta(t))+8\,l_1\,l_3\,m_w\sin(\delta+\phi(t)-\theta(t))+4\,l_2\,l_3\,m_3\sin(\gamma-\phi(t)+\theta(t))+8\,l_2\,l_3\,m_w\sin(\gamma-\phi(t)+\theta(t))+4\,l_1\,l_3\,m_3\sin(\phi(t)-\theta(t))+8\,l_1\,l_3\,m_w\sin(\phi(t)-\theta(t))$$

$$-\,4\,l_2\,l_3\,m_3\sin(\delta-\gamma+\phi(t)-\theta(t))-8\,l_2\,l_3\,m_w\sin(\delta-\gamma+\phi(t)-\theta(t)))=-T_u-2\,g\,m_2\,(2\,l_1\cos(\theta(t))-l_2\cos(\gamma+\theta(t)))-4\,g\,m_3\,(2\,l_1\cos(\theta(t))$$

$$-\,2\,l_2\cos(\gamma+\theta(t)))-4\,g\,m_w\,(2\,l_1\cos(\theta(t))-2\,l_2\cos(\gamma+\theta(t)))-2\,M\,g\,l_1\cos(\theta(t))-2\,g\,l_1\,m_1\cos(\theta(t))$$

Equation of motion corresponding to phi

$$\left(\frac{16\,l_3^2\,m_3}{3}+16\,l_3^2\,m_w\right)\frac{\partial^2}{\partial t^2}\,\phi(t)+(2\,l_3\,m_3\cos(\delta+\phi(t))+4\,l_3\,m_w\cos(\delta+\phi(t))+2\,l_3\,m_3\cos(\phi(t))+4\,l_3\,m_w\cos(\phi(t)))\frac{\partial^2}{\partial t^2}\,y(t)+(4\,l_1\,l_3\,m_3\cos(\phi(t)-\theta(t))+8\,l_1\,l_3\,m_w\cos(\phi(t)-\theta(t))-4\,l_2\,l_3\,m_3\cos(\delta-\gamma+\phi(t)-\theta(t))$$

$$-\,8\,l_2\,l_3\,m_w\cos(\delta-\gamma+\phi(t)-\theta(t))+4\,l_1\,l_3\,m_3\cos(\delta+\phi(t)-\theta(t))+8\,l_1\,l_3\,m_w\cos(\delta+\phi(t)-\theta(t))-4\,l_2\,l_3\,m_3\cos(\gamma-\phi(t)+\theta(t))-8\,l_2\,l_3\,m_w\cos(\gamma-\phi(t)+\theta(t)))\frac{\partial^2}{\partial t^2}\,\theta(t)$$

$$-\,(2\,l_3\,m_3\sin(\delta+\phi(t))+4\,l_3\,m_w\sin(\delta+\phi(t))+2\,l_3\,m_3\sin(\phi(t))+4\,l_3\,m_w\sin(\phi(t)))\frac{\partial^2}{\partial t^2}\,x(t)+\left(\frac{\partial}{\partial t}\,\theta(t)\right)^2(4\,l_1\,l_3\,m_3\sin(\delta+\phi(t)-\theta(t))+8\,l_1\,l_3\,m_w\sin(\delta+\phi(t)-\theta(t))+4\,l_2\,l_3\,m_3\sin(\gamma-\phi(t)+\theta(t))$$

$$+\,8\,l_2\,l_3\,m_w\sin(\gamma-\phi(t)+\theta(t))+4\,l_1\,l_3\,m_3\sin(\phi(t)-\theta(t))+8\,l_1\,l_3\,m_w\sin(\phi(t)-\theta(t))-4\,l_2\,l_3\,m_3\sin(\delta-\gamma+\phi(t)-\theta(t))-8\,l_2\,l_3\,m_w\sin(\delta-\gamma+\phi(t)-\theta(t)))$$

$$=-T_v-T_w-2\,g\,l_3\,m_3\cos(\delta+\phi(t))-4\,g\,l_3\,m_w\cos(\delta+\phi(t))-2\,g\,l_3\,m_3\cos(\phi(t))-4\,g\,l_3\,m_w\cos(\phi(t))$$

Equation of motion corresponding to u

$$2\,m_w\,r^2\frac{\partial^2}{\partial t^2}\,u(t)=T_u$$

Equation of motion corresponding to v

$$2\,m_w\,r^2\frac{\partial^2}{\partial t^2}\,v(t)=T_v$$

Equation of motion corresponding to w

$$2\,m_w\,r^2\frac{\partial^2}{\partial t^2}\,w(t)=T_w$$

6.5.1 Analysing the rocker-bogie when climbing a wall

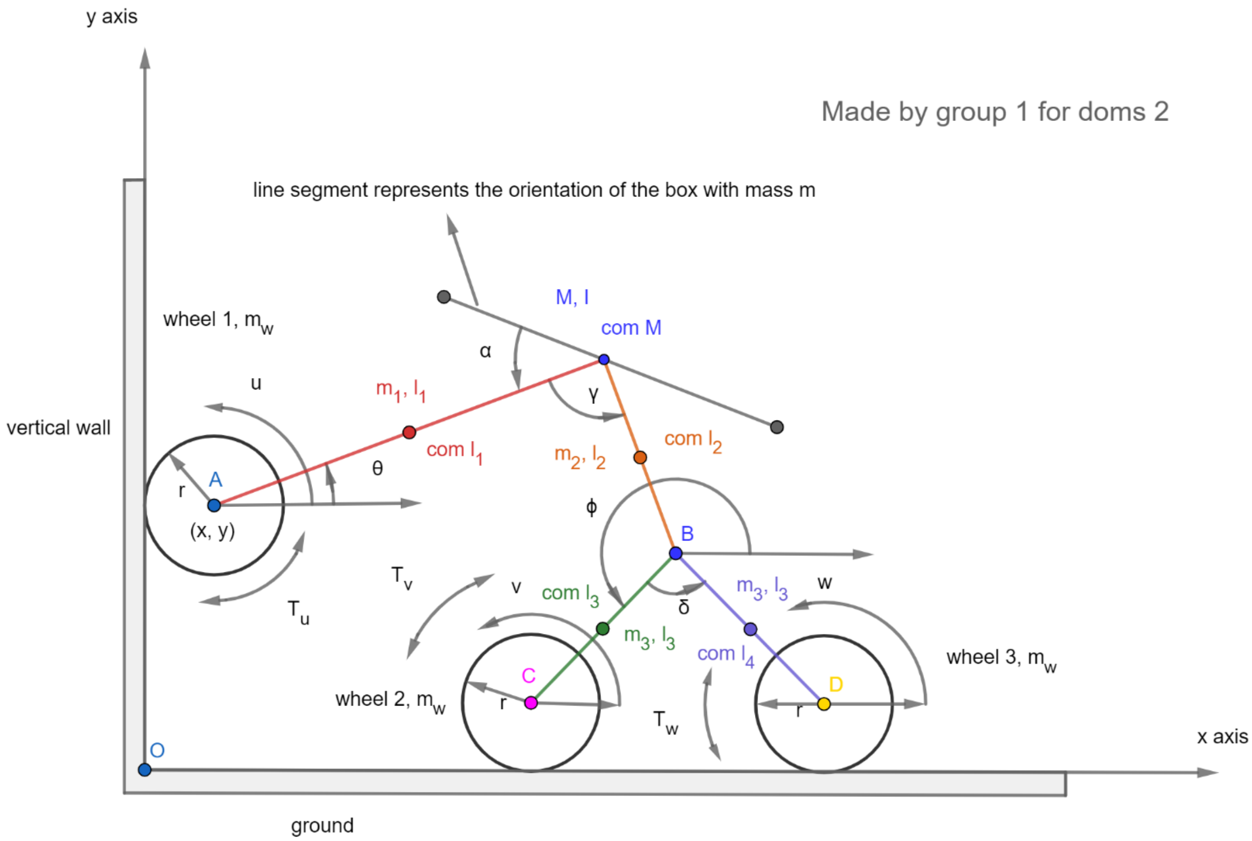

Figure 16: Analyzing it when it is climbing a wall:

Assumptions / simplifications for wall climbing analysis:

- The coefficient of friction is high enough to ensure pure rolling always occurs when the wheels are in contact with the walls and the ground

They are in the order of the generalized coordinates/ DOF which is x, y, theta, phi, u, v, w.
Constraints:

- $x\left(t\right) - r = 0$ - wheel 1's contact constraint with the vertical wall
- $y\left(t\right) - r + 2\,l_3 \sin\left(\phi\left(t\right)\right) + 2\,l_1 \sin\left(\theta\left(t\right)\right) - 2\,l_2 \sin\left(\gamma + \theta\left(t\right)\right) = 0$ - wheel 2's contact constraint with the ground

- $y(t) - r + 2\,l_1 \sin(\theta(t)) + 2\,l_3 \sin(\delta + \phi(t)) - 2\,l_2 \sin(\gamma + \theta(t)) = 0$ - wheel 3's contact constraint with the ground
- $y(t) - r - r\,u(t) = 0$ - wheel 1's pure rolling constraint with the vertical wall
- $x(t) - \text{w2xi} + 2\,l_3 \cos(\phi(t)) + 2\,l_1 \cos(\theta(t)) + r\,v(t) - 2\,l_2 \cos(\gamma + \theta(t)) = 0$ - wheel 2's pure rolling constraint with the ground
- $x(t) - \text{w2xi} - 4\,l_3 \sin\left(\frac{\delta}{2}\right) + 2\,l_1 \cos(\theta(t)) + r\,w(t) + 2\,l_3 \cos(\delta + \phi(t)) - 2\,l_2 \cos(\gamma + \theta(t)) = 0$ – wheel 3's pure rolling constraint with the ground

Where, w2xi is the initial x coordinate of the wheel-2.
Upon solving the 6 constraint equations, taking theta as the independent variable, we obtain the other six dof in terms of theta. Hence the system is solely characterized by theta and thus has a single dof.

The expressions we get are:

x(t) = r

y(t) =

$$r - 2\,l_1 \sin(\theta(t)) + 2\,l_2 \sin(\gamma + \theta(t)) + 2\,l_3 \cos\left(\frac{\delta}{2}\right)$$

phi(t) =

$$\frac{3\,\pi}{2} - \frac{\delta}{2}$$

u(t) =

$$\frac{2\,l_2 \sin(\gamma + \theta(t)) - 2\,l_1 \sin(\theta(t)) + 2\,l_3 \cos\left(\frac{\delta}{2}\right)}{r}$$

v(t) =

$$-\frac{r - \text{w2xi} + 2\,l_3 \cos\left(\frac{\delta}{2} - \frac{3\,\pi}{2}\right) + 2\,l_1 \cos(\theta(t)) - 2\,l_2 \cos(\gamma + \theta(t))}{r}$$

w(t) =

$$-\frac{r - \text{w2xi} + 2\,l_3 \cos\left(\frac{\delta}{2} - \frac{3\,\pi}{2}\right) - 4\,l_3 \sin\left(\frac{\delta}{2}\right) + 2\,l_1 \cos(\theta(t)) - 2\,l_2 \cos(\gamma + \theta(t))}{r}$$

An alternative way to see that these constraints result in a 1 DOF system is to use Gruebler's equation:

Gruebler's equation: no. of DOF = 3L – $2J_1$ – J_2 – 3G Where,

L = total no. of links
J_1 = no. of full joints
J_2 = no. of half joints
G = no. of grounded links
Here,
L = 5
(l_1 and l_2 count as a single link as they form a rigid body, l_3 and l_4 count as a single link as they form a rigid body, wheel 1, wheel 2, wheel 3)
J_1 = 7
(4 revolute joints exist, they are present at A, B, C, D and three pure rolling constraints)
J_2 = 0
G = 0

Hence, using Gruebler's equation, no. of DOF = 3×5–2×7–0–0 = 1.
Substituting these expressions in the EOM and taking the terms containing the Lagrange multipliers to the LHS of each equation and storing the equations in matrix form (each row respresents an equation)
A = $[TermsDtheta^2 + terms_D 2theta - (Q_{driving} + Q_{potential}]$ Where,

- A is a matrix whose columns represent the Lagrange multipliers and their coefficients in each equation are stored row wise. A is of size 7×6
- $TermsDtheta^2$ is the column vector containing the coefficients of the $\left(\frac{d}{dt}\theta\right)^2$ (each row stores coefficients corresponding to the respective equation) . It is of size 7×1
- $TermsD2theta$ is is the column vector containing the coefficients of the$\left(\frac{d^2}{dt^2}\theta\right)$ (each row stores coefficients corresponding to the respective equation) . It is of size 7×1
- $Q_{driving} + Q_{potential}$ is the column vector whose entries (row wise) correspond to the sum of the generalized driving forces and generalized forces due to the potential in the respective equation. It is of size 7×1

Converting both sides into RREF (row reduced echelon form) we get expressions for the Lagrange multipliers and an ODE governing the time evolution of theta in the $7^t h$row.

The six Lagrange multipliers are denoted by
$\lambda_1, \lambda_2, \lambda_3, \mu_1, \mu_2, \mu_3$.

The following expressions have been obtained for the first 3 Lagrange multipliers:
$\lambda_1 =$

$$\left(\frac{\partial}{\partial t}\,\theta(t)\right)^2 \left(\left(2\,l_2\,m_2+8\,l_2\,m_3+16\,l_2\,m_w\right)\cos(\gamma+\theta(t))+\left(-2\,M\,l_1-2\,l_1\,m_1-4\,l_1\,m_2-8\,l_1\,m_3-16\,l_1\,m_w\right)\cos(\theta(t))\right)+\frac{T_v}{r}+\frac{T_w}{r}$$
$$+\left(\left(2\,l_2\,m_2+8\,l_2\,m_3+16\,l_2\,m_w\right)\sin(\gamma+\theta(t))+\left(-2\,M\,l_1-2\,l_1\,m_1-4\,l_1\,m_2-8\,l_1\,m_3-16\,l_1\,m_w\right)\sin(\theta(t))\right)\frac{\partial^2}{\partial t^2}\,\theta(t)$$

$$\lambda_2 =$$

$$-\frac{-\frac{T_u}{4}-\left(\frac{\partial}{\partial t}\theta(t)\right)^2\left(\left(l_2^2 m_3 \cot\left(\frac{\delta}{2}\right)\right)\cos(2\gamma+2\theta(t))+\left(\frac{M l_1^2}{2}+\frac{l_1^2 m_1}{2}+l_1^2 m_2+2 l_1^2 m_3\right)\sin(2\theta(t))+\left(l_1^2 m_3 \cot\left(\frac{\delta}{2}\right)\right)\cos(2\theta(t))+\left(\frac{l_2^2 m_2}{2}+2 l_2^2 m_3\right)\sin(2\gamma+2\theta(t))\right.}{}$$

$$\frac{) + m_3\left(\cot\left(\frac{\delta}{2}\right) l_1^2-2\cot\left(\frac{\delta}{2}\right)\cos(\gamma)\, l_1 l_2+\cot\left(\frac{\delta}{2}\right) l_2^2\right)+\frac{m_w\left(2 l_1^2\cos\left(\frac{\delta}{2}\right)+2 l_2^2\cos\left(\frac{\delta}{2}\right)+l_2^2\cos\left(2\gamma-\frac{\delta}{2}+2\theta(t)\right)2+2 l_1^2\cos\left(\frac{\delta}{2}-2\theta(t)\right)-2 l_1 l_2\cos\left(\frac{\delta}{2}+\gamma\right)-l_1 l_2\cos\left(\gamma-\frac{\delta}{2}+2\theta(t)\right)4-2 l_1 l_2\cos\left(\frac{\delta}{2}-\gamma\right)\right)2}{\sin\left(\frac{\delta}{2}\right)}}{}$$

$$\left.+\left(-2 l_1 l_2 m_3\cot\left(\frac{\delta}{2}\right)\right)\cos(\gamma+2\theta(t))+\left(-\frac{M l_1 l_2}{2}-\frac{l_1 l_2 m_1}{2}-\frac{3 l_1 l_2 m_2}{2}-4 l_1 l_2 m_3\right)\sin(\gamma+2\theta(t))-\frac{M l_1 l_2\sin(\gamma)}{2}-\frac{l_1 l_2 m_1\sin(\gamma)}{2}-\frac{l_1 l_2 m_2\sin(\gamma)}{2}\right)$$

$$\frac{+T_v\left(\frac{l_1\cos(\theta(t))-l_2\cos(\gamma+\theta(t))}{4 l_3\sin\left(\frac{\delta}{2}\right)}+\frac{\frac{l_1\cos\left(\frac{\delta}{2}-\theta(t)\right)}{2}-\frac{l_2\cos\left(\gamma-\frac{\delta}{2}+\theta(t)\right)}{2}}{r\sin\left(\frac{\delta}{2}\right)}\right)+T_w\left(\frac{l_1\cos(\theta(t))-l_2\cos(\gamma+\theta(t))}{4 l_3\sin\left(\frac{\delta}{2}\right)}+\frac{\frac{l_1\cos\left(\frac{\delta}{2}-\theta(t)\right)}{2}-\frac{l_2\cos\left(\gamma-\frac{\delta}{2}+\theta(t)\right)}{2}}{r\sin\left(\frac{\delta}{2}\right)}\right)+\left(-\left(-\frac{l_2^2 m_2}{2}-2 l_2^2 m_3\right)\cos(2\gamma+2\theta(t))-\frac{I}{4}\right.}{l_1\cos(\theta(t))-l_2\cos(\gamma+\theta(t))}$$

$$\frac{-m_2\left(l_1^2-\frac{\cos(\gamma)\, l_1 l_2}{2}+\frac{l_2^2}{6}\right)-M\left(\frac{l_1^2}{2}+\frac{l_2\cos(\gamma)\, l_1}{2}\right)-m_1\left(\frac{l_1^2}{6}+\frac{l_2\cos(\gamma)\, l_1}{2}\right)-m_3\left(2 l_1^2+2 l_2^2-4 l_1 l_2\cos(\gamma)+\frac{2\cos\left(\frac{\delta}{2}\right)\left(l_1\cos(\theta(t))-l_2\cos(\gamma+\theta(t))\right)\left(l_1\sin(\theta(t))-l_2\sin(\gamma+\theta(t))\right)}{\sin\left(\frac{\delta}{2}\right)}\right)-\left(\frac{M l_1 l_2}{2}+\frac{l_1 l_2 m_1}{2}+\frac{3 l_1 l_2 m_2}{2}\right.}{}$$

$$\frac{\left.+4 l_1 l_2 m_3\right)\cos(\gamma+2\theta(t))+\frac{m_w\left(l_1^2\sin\left(\frac{\delta}{2}-2\theta(t)\right)-l_1^2\sin\left(\frac{\delta}{2}\right)-l_2^2\sin\left(\frac{\delta}{2}\right)-l_2^2\sin\left(2\gamma-\frac{\delta}{2}+2\theta(t)\right)+l_1 l_2\sin\left(\frac{\delta}{2}+\gamma\right)+l_1 l_2\sin\left(\gamma-\frac{\delta}{2}+2\theta(t)\right)2+l_1 l_2\sin\left(\frac{\delta}{2}-\gamma\right)\right)4}{\sin\left(\frac{\delta}{2}\right)}}{}$$

$$\frac{\left.-\left(-\frac{M l_1^2}{2}-\frac{l_1^2 m_1}{2}-l_1^2 m_2-2 l_1^2 m_3\right)\cos(2\theta(t))\right)\frac{\partial^2}{\partial t^2}\theta(t)+m_2\left(\frac{g l_2\cos(\gamma+\theta(t))}{2}-g l_1\cos(\theta(t))\right)+m_3\left(2 g l_2\cos(\gamma+\theta(t))\right.}{}$$

$$\frac{\left.)-2 g l_1\cos(\theta(t))\right)+m_w\left(2 g l_2\cos(\gamma+\theta(t))-2 g l_1\cos(\theta(t))\right)-\frac{M g l_1\cos(\theta(t))}{2}-\frac{g l_1 m_1\cos(\theta(t))}{2}}{}$$

$\lambda_3 =$

$$\frac{\frac{T_u}{4}+T_v\left(\frac{\frac{l_1\cos\left(\frac{\delta}{2}+\theta(t)\right)}{2}-\frac{l_2\cos\left(\frac{\delta}{2}+\gamma+\theta(t)\right)}{2}}{r\sin\left(\frac{\delta}{2}\right)}+\frac{l_1\cos(\theta(t))-l_2\cos(\gamma+\theta(t))}{4\,l_3\sin\left(\frac{\delta}{2}\right)}\right)+T_w\left(\frac{\frac{l_1\cos\left(\frac{\delta}{2}+\theta(t)\right)}{2}-\frac{l_2\cos\left(\frac{\delta}{2}+\gamma+\theta(t)\right)}{2}}{r\sin\left(\frac{\delta}{2}\right)}+\frac{l_1\cos(\theta(t))-l_2\cos(\gamma+\theta(t))}{4\,l_3\sin\left(\frac{\delta}{2}\right)}\right)-\left(\frac{\partial}{\partial t}\theta(t)\right)^2\left(-\left(-l_2^2\,m_3\cot\left(\frac{\delta}{2}\right)\right)\cos(2\gamma+2\theta(t))-\left(\right.\right.}{}$$

$$-\left(2\,l_1\,l_2\,m_3\cot\left(\frac{\delta}{2}\right)\right)\cos(\gamma+2\theta(t))-\left(\frac{M\,l_1^2}{2}+\frac{l_1^2\,m_1}{2}+l_1^2\,m_2+2\,l_1^2\,m_3\right)\sin(2\theta(t))-\left(\frac{l_2^2\,m_2}{2}+2\,l_2^2\,m_3\right)\sin(2\gamma+2\theta(t))+m_3\left(\cot\left(\frac{\delta}{2}\right)l_1^2-2\cot\left(\frac{\delta}{2}\right)\cos(\gamma)\,l_1\,l_2+\cot\left(\frac{\delta}{2}\right)l_2^2\right)+\frac{2\,m_w\left(2\,l_1^2\cos\left(\frac{\delta}{2}\right)+2\,l_2^2\cos\left(\frac{\delta}{2}\right)+\right.}{}$$

$$\frac{+2\,l_2^2\cos\left(\frac{\delta}{2}+2\gamma+2\theta(t)\right)+2\,l_1^2\cos\left(\frac{\delta}{2}+2\theta(t)\right)-2\,l_1\,l_2\cos\left(\frac{\delta}{2}+\gamma\right)-4\,l_1\,l_2\cos\left(\frac{\delta}{2}+\gamma+2\theta(t)\right)-2\,l_1\,l_2\cos\left(\frac{\delta}{2}-\gamma\right)\Big)}{\sin\left(\frac{\delta}{2}\right)}-\left(-\frac{M\,l_1\,l_2}{2}-\frac{l_1\,l_2\,m_1}{2}-\frac{3\,l_1\,l_2\,m_2}{2}-4\,l_1\,l_2\,m_3\right)\sin(\gamma+2\theta(t))-\left(-l_1^2\,m_3\cot\left(\frac{\delta}{2}\right)\right)\cos(2\theta(t))$$

$$\frac{+\frac{M\,l_1\,l_2\sin(\gamma)}{2}+\frac{l_1\,l_2\,m_1\sin(\gamma)}{2}+\frac{l_1\,l_2\,m_2\sin(\gamma)}{2}\Big)-\Big(-\left(-\frac{l_2^2\,m_2}{2}-2\,l_2^2\,m_3\right)\cos(2\gamma+2\theta(t))-\frac{I}{4}-m_2\left(l_1^2-\frac{\cos(\gamma)\,l_1\,l_2}{2}+\frac{l_2^2}{6}\right)-M\left(\frac{l_1^2}{2}+\frac{l_2\cos(\gamma)\,l_1}{2}\right)-m_1\left(\frac{l_1^2}{6}+\frac{l_2\cos(\gamma)\,l_1}{2}\right)-m_3\Big(2\,l_1^2+2\,l_2^2-4\,l_1\,l_2\cos(\gamma)}{l_1\cos(\theta(t))-l_2\cos(\gamma+\theta(t))}$$

$$-\frac{2\cos\left(\frac{\delta}{2}\right)\left(l_1\cos(\theta(t))-l_2\cos(\gamma+\theta(t))\right)\left(l_1\sin(\theta(t))-l_2\sin(\gamma+\theta(t))\right)}{\sin\left(\frac{\delta}{2}\right)}\Big)-\left(\frac{M\,l_1\,l_2}{2}+\frac{l_1\,l_2\,m_1}{2}+\frac{3\,l_1\,l_2\,m_2}{2}+4\,l_1\,l_2\,m_3\right)\cos(\gamma+2\theta(t))+\frac{4\,m_w\left(l_1^2\sin\left(\frac{\delta}{2}+2\theta(t)\right)-l_1^2\sin\left(\frac{\delta}{2}\right)-l_2^2\sin\left(\frac{\delta}{2}\right)+l_2^2\sin\left(\frac{\delta}{2}+2\gamma+2\theta(t)\right)\right.}{\sin\left(\frac{\delta}{2}\right)}$$

$$\frac{+l_1\,l_2\sin\left(\frac{\delta}{2}+\gamma\right)-2\,l_1\,l_2\sin\left(\frac{\delta}{2}+\gamma+2\theta(t)\right)+l_1\,l_2\sin\left(\frac{\delta}{2}-\gamma\right)\Big)}{}-\left(-\frac{M\,l_1^2}{2}-\frac{l_1^2\,m_1}{2}-l_1^2\,m_2-2\,l_1^2\,m_3\right)\cos(2\theta(t))\Bigg)\frac{\partial^2}{\partial t^2}\theta(t)-m_2\left(\frac{g\,l_2\cos(\gamma+\theta(t))}{2}-g\,l_1\cos(\theta(t))\right)$$

$$\Big)-m_3\left(2\,g\,l_2\cos(\gamma+\theta(t))-2\,g\,l_1\cos(\theta(t))\right)-m_w\left(2\,g\,l_2\cos(\gamma+\theta(t))-2\,g\,l_1\cos(\theta(t))\right)+\frac{M\,g\,l_1\cos(\theta(t))}{2}+\frac{g\,l_1\,m_1\cos(\theta(t))}{2}$$

Lambda 1 is the normal force on wheel 1 from the vertical wall.
Lambda 2 is the normal force on wheel 2 from the ground.
Lambda 3 is the normal force on wheel 3 from the ground.
The ODE governing the time evolution of theta is :

$$\Big(\left(2\,l_1\,l_2\,m_1+2\,l_1\,l_2\,m_2+8\,l_1\,l_2\,m_3+24\,l_1\,l_2\,m_w\right)\cos(\gamma)-\frac{I}{2}-m_1\left(\frac{4\,l_1^2}{3}+2\,l_2^2\right)-m_2\left(2\,l_1^2+\frac{4\,l_2^2}{3}\right)-m_3\left(4\,l_1^2+4\,l_2^2\right)-m_w\left(12\,l_1^2+12\,l_2^2\right)-M\left(l_1^2+l_2^2\right)+\left(M\,l_1^2+2\,l_1^2\,m_2+4\,l_1^2\,m_3+4\,l_1^2\,m_w\right)\cos(2\theta(t))$$

$$+\left(4\,l_2^2\,m_3-2\,l_2^2\,m_1-M\,l_2^2+4\,l_2^2\,m_w\right)\cos(2\gamma+2\theta(t))+\left(2\,l_1\,l_2\,m_1-2\,l_1\,l_2\,m_2-8\,l_1\,l_2\,m_3-8\,l_1\,l_2\,m_w\right)\cos(\gamma+2\theta(t))\Big)\frac{\partial^2}{\partial t^2}\theta(t)+\Big(\left(2\,l_1\,l_2\,m_2-2\,l_1\,l_2\,m_1+8\,l_1\,l_2\,m_3+8\,l_1\,l_2\,m_w\right)\sin(\gamma+2\theta(t))$$

$$+\left(M\,l_2^2+2\,l_2^2\,m_1-4\,l_2^2\,m_3-4\,l_2^2\,m_w\right)\sin(2\gamma+2\theta(t))+\left(-M\,l_1^2-2\,l_1^2\,m_2-4\,l_1^2\,m_3-4\,l_1^2\,m_w\right)\sin(2\theta(t))\Big)\left(\frac{\partial}{\partial t}\theta(t)\right)^2+\left(\frac{T_u\,l_2}{r}-2\,g\,l_2\,m_1-g\,l_2\,m_2-2\,g\,l_2\,m_w-M\,g\,l_2\right)\cos(\gamma+\theta(t))$$

$$-\frac{T_u}{2}+\left(g\,l_1\,m_1+2\,g\,l_1\,m_w-\frac{T_u\,l_1}{r}\right)\cos(\theta(t))+\left(\frac{T_v\,l_1}{r}+\frac{T_w\,l_1}{r}\right)\sin(\theta(t))+\left(-\frac{T_v\,l_2}{r}-\frac{T_w\,l_2}{r}\right)\sin(\gamma+\theta(t))=0$$

6.5.2 Solving ODE numerically

We now proceed to solve to ODE numerically (in Matlab using ode15s) in the range of motion for which the constraints hold.
The range of motion where the constraints hold true can be determined by finding when the normal forces from the walls switch signs from positive (away from the wall) to negative (toward the wall)

We take the set of sample parameters required to solve the ODE numerically to be:

```
Editor - C:\Users\1\Desktop\kionep1\3) sem-wise\4th sem\ed4060\project do
parameter_values.m  ode_theta_plots.m  normal_forces.m
load('C:\Users\1\Desktop\kionep1\3) sem-wis
% known parameters

g = 10; % gravity
material_density = 2700; % aluminium
T_u = 280;
T_v = 50;
T_w = 50;
M = 40;
I = 1/12 * M * (l_1^2 + l_2^2);

% unknown parameters

l_1 = 0.75;
l_2 = 0.375;
l_3 = 0.375;
r1 = 0.2575;
r2 = 0.2425;
h = 0.5;
area = pi*(0.06^2 - 0.04^2);
delta = pi/2;
gamma = pi/2;

% dependent parameters

zeta = (0.75^2 + 0.375^2)^(1/2);
r = (r1 + r2) / 2;
m_1 = material_density*2*l_1*area;
m_2 = material_density*2*l_2*area;
m_3 = material_density*2*l_3*area;
m_w = material_density*pi*(r1^2 - r2^2)*h;

save('C:\Users\1\Desktop\kionep1\3) sem-wis
```

Using these values in the ODE and solving we get the following plots:
Insert labelled plots

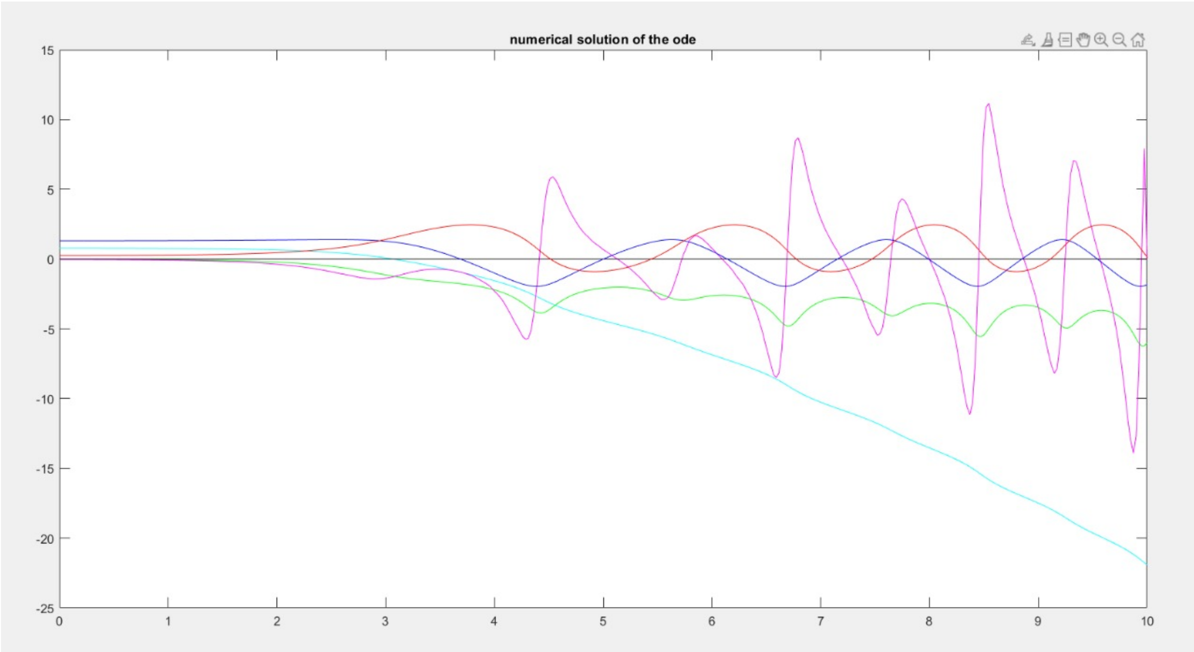

Figure 17: Cyan-$\theta(t)$,Green-first derivative of θ,Magenta- second derivative of θ,Blue-x co-ordinate of wheel 2,Red-y coordinate of wheel 1

This plot(fig.17) is valid for time values less than 2.73 s as the normal force becomes 0.

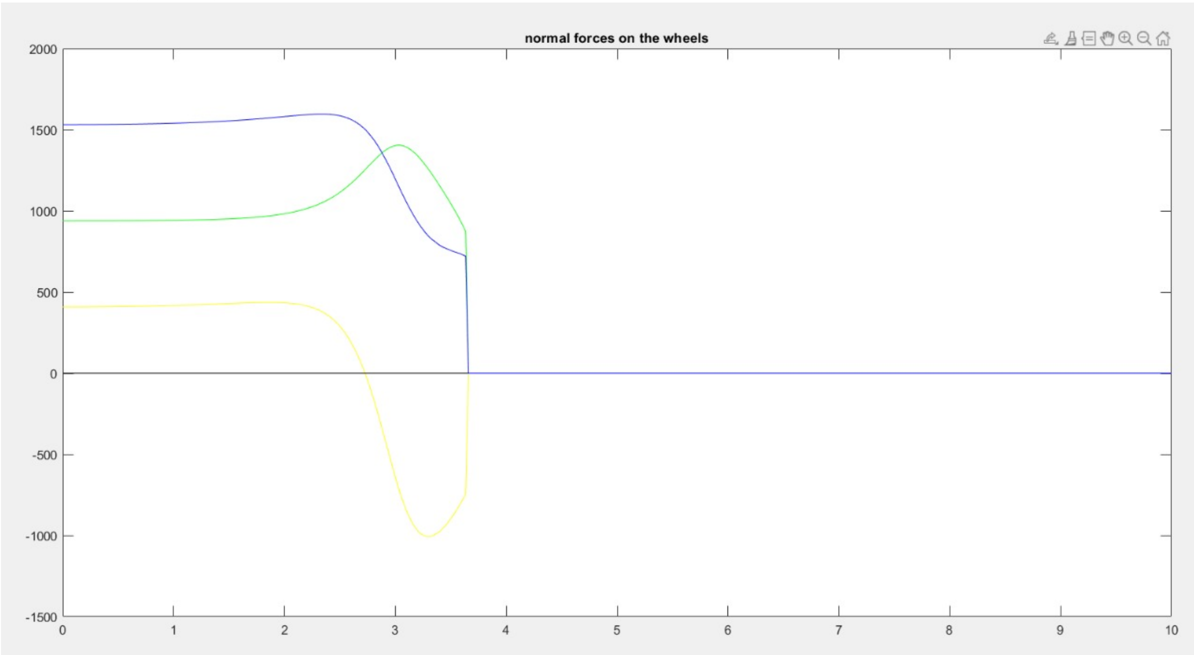

Figure 18: Yellow- normal force on wheel 1,Green-normal force on wheel 2,Blue-normal force on wheel 3

The rocker wheel stays in contact till t = 2.73 s (the normal force on wheel 1 switches signs at this instant of time)
wheel 1 leaves the vertical wall at this instant.
After this instant the constraints change. Only the bogie's wheels stay on the floor. This requires rewriting the EOM and solving the coupled ODE system numerically.

The parameters in the code can be changed to obtain the numerical solutions for θ for different rocker-bogie suspension system.
We used GeoGebra to draw diagrams and Matlab to derive and simplify the eom (equations of motion)

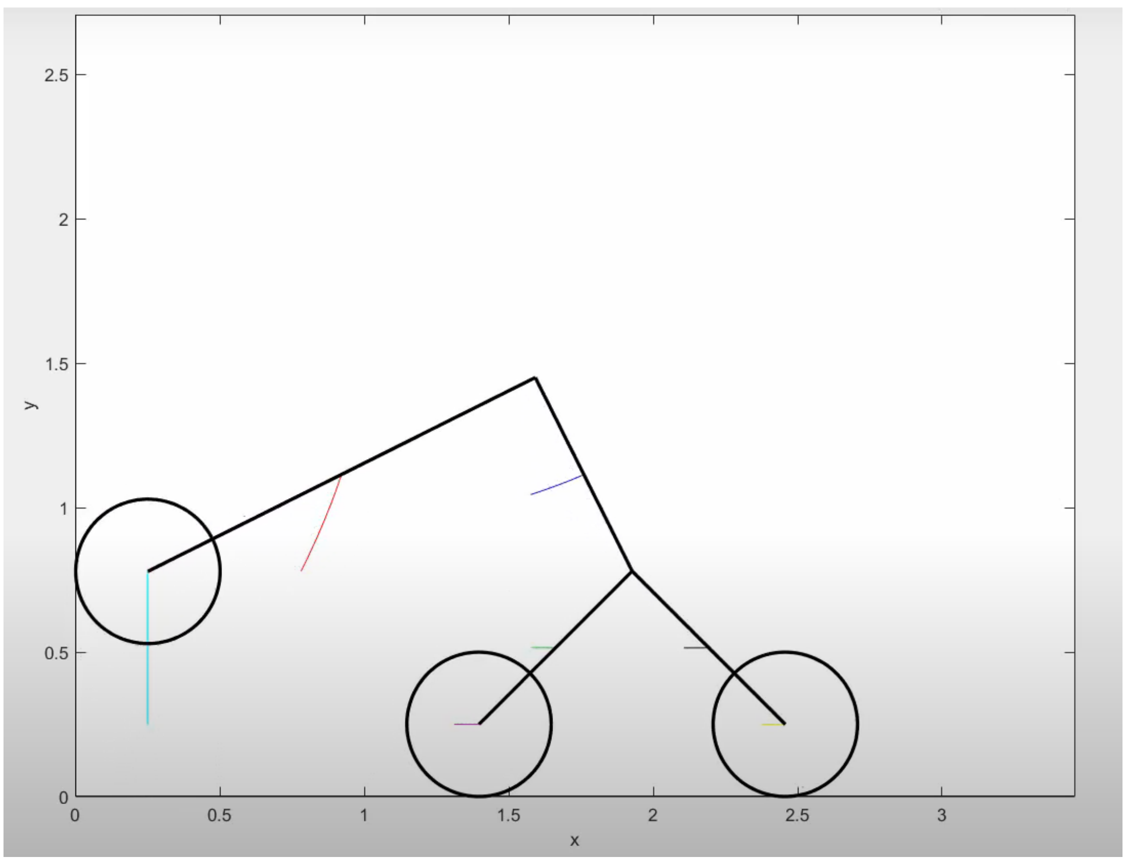

Figure 19: Animation of rocker-bogie with the sample parameters climbing a wall **(click on the image to see the simulation)**

7 CAD MODELLING AND COMPUTER SIMULATION

Computer modelling and simulation are powerful tools for designers.Even though mathematical and geometrical considerations complete an analysis in the theoretical level, they alone don't paint the entire picture in the practical level.Hence without the computer modelling and simulations presented below this analysis on multi-terrain vehicles is not complete.

7.1 CAD Modelling

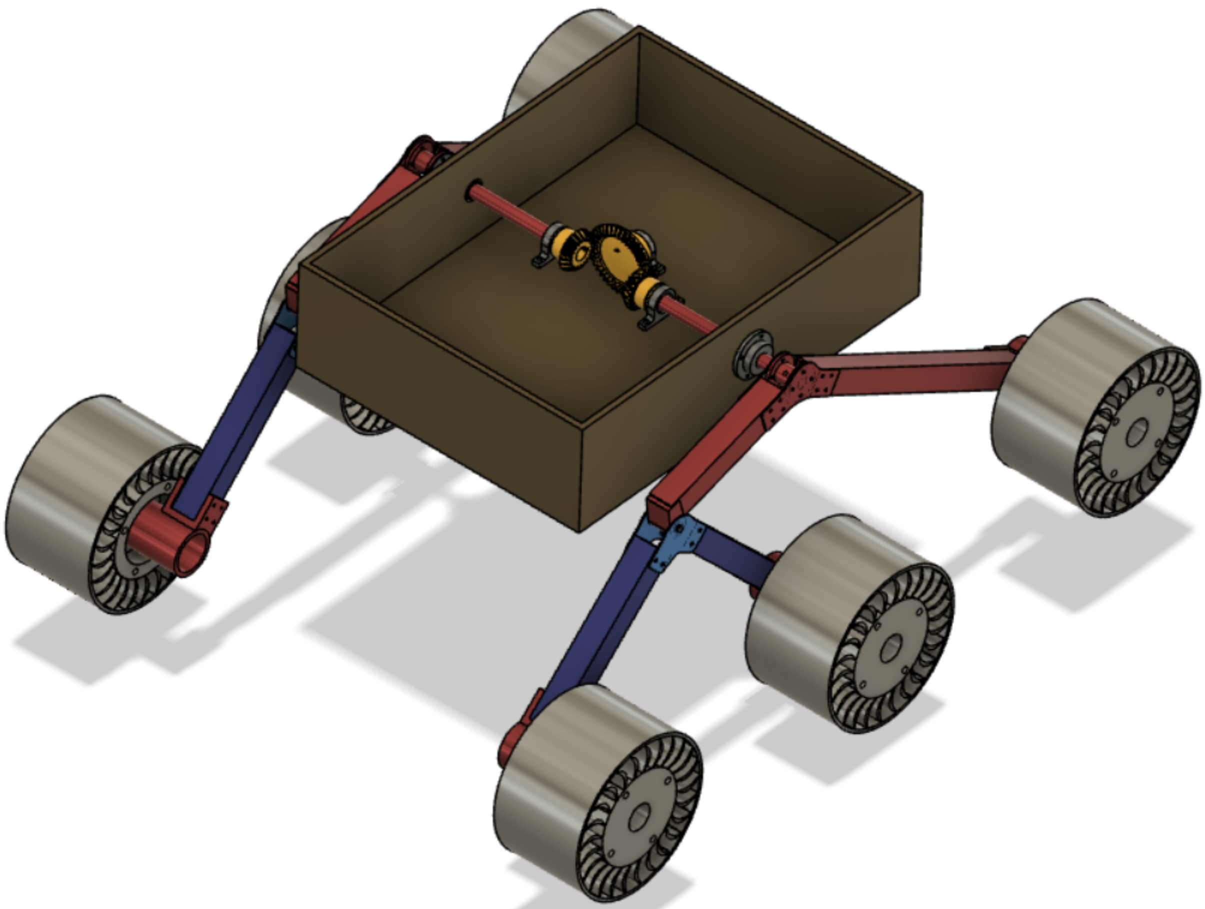

Figure 20

This is the final cad model of the proposed multi terrain vehicle. The CAD modelling is done using the cad modelling software Fusion 360

The part in red is the rocker whose one end is connected to the rear wheel and the other end is connected to the bogie. The angle between the two rocker arms is a constant. There are shafts that connect each rocker arm with its respective gear. The gears are represented by yellow color. There is a third gear in the middle which connects both the gears on the sides.
The part in brown is the chassis of the vehicle whose shape can be modified and optimized later on further analysis such as CFD analysis, but for this project no modifications are made since such analysis are out of the scope of this project.

The part in blue is the bogie. The bogie has three connections, two to the wheels and one with the rocker. The angle between the two bogie arms is a constant.

7.2 Modelling Joints

The joints were modelled using an online CAD modelling software called Onshape, this was because the Onshape software had a direct method through which the model along with the joints can be imported into MATLAB and simulink which was the platform we used for our simulation. This was not possible in Fusion 360. Therefore we imported our basic model from Fusion 360 to Onshape and modelled the joints in Onshape and carried on from there.

7.3 Rocker- Bogie:

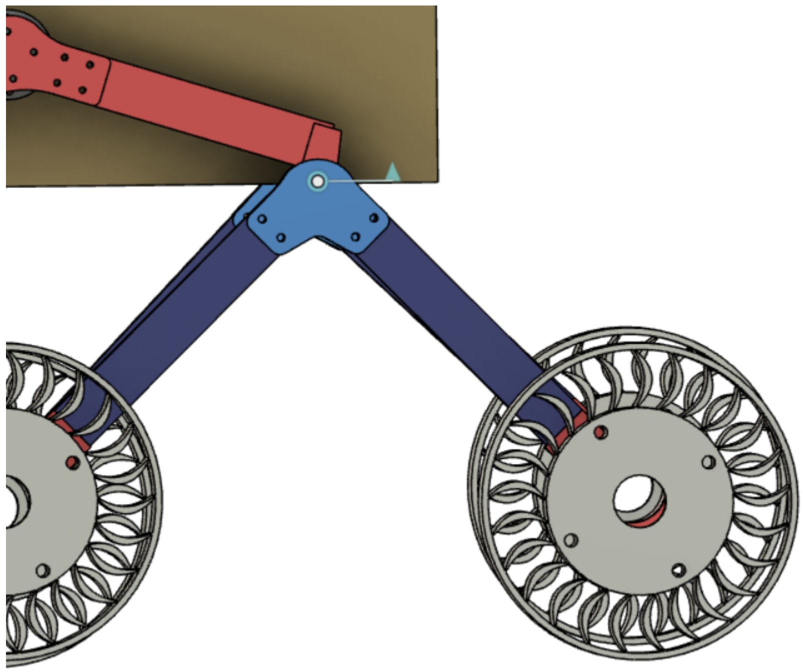

Figure 21

The connection between rocker and bogie is given between one end of the rocker and the middle vertex of the bogie. This joint is a revolute joint ie. There is one relative degree of freedom that exists between the rocker and bogie.
This revolute joint plays a key role in the rocker bogie mechanism and is the point of difference between the rocker bogie mechanisms and other mechanisms. When the bogie encounters an obstacle which it has to climb over, it does so by changing the angle between the bogie and the rocker, therefore the bogie can climb the obstacle without compromising the contact of the rocker which is the key difference between the rocker bogie and other mechanisms. Therefore it wouldn't be a big stretch to say that this joint is the most important in the entire multi-terrain vehicle.

7.3.1 Wheel - Rocker/bogie:

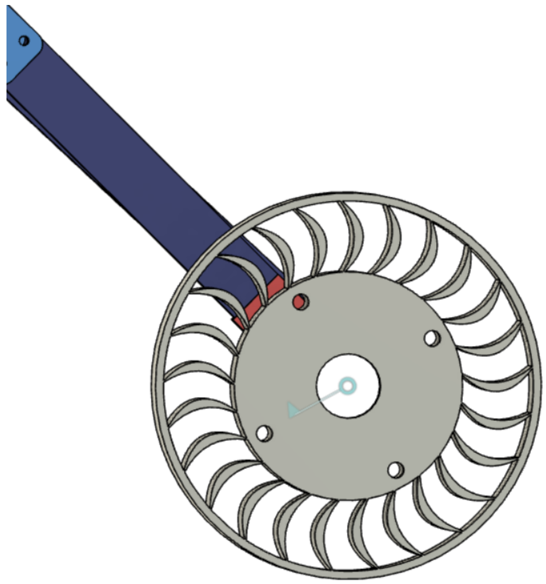

Figure 22

The joint between the wheel and either the rocker or bogie is similar to the joints found in most modern vehicles for the wheels. It has a **revolute** joint with some damping. Without this joint the wheel would not be able to rotate and the vehicle won't be able to move.

7.3.2 Shaft-Flange:

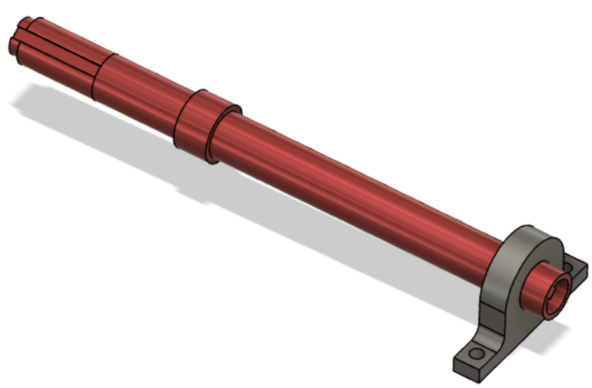

Figure 23

This is the joint that exists between the flange in the chassis and the shafts on either side. This is also a **revolute** joint. This joint is key to the functioning of the differential. The shafts are rigidly connected on both sides to the gear and the rocker respectively. Therefore when the rocker rotates this shaft rotates with respect to the flange which is rigidly connected to the chassis and thus rotates the gears and enables the functioning of the differential. Without this joint there would be no use for the differential.

7.4 Modelling Gear Relations and Simulating the Differential gear

Figure 24

As mentioned in the differential section of the report, the three gear differential was chosen as the differential mechanism for the project. Before modelling the gear relations it is important to realize that the gears in contact must not overlap and the tooth of the gears must mesh exactly for any simulation of the differential gears to actually work. When the meshing was not accurate or when the gears overlapped the solver gave various errors ranging from position violation errors to kinematic singularity solutions. Hence it is very important to ensure that the gears are meshed correctly and that no part of gears overlap with each other.

The gears simulated here (ref fig) each have a pitch radius of 2.8cm.

The above differential was modelled in MATLAB simscape multibody using the following link diagram.

The three blocks in the leftmost are the world frame are the world frame and solver configuration blocks. They are connected to the gear blocks through rigid transform blocks that align the gears in their exact positions and ensure that they don't overlap. Also there are three revolute joint blocks which give the gears exactly one degree of freedom with respect to the ground and lets it rotate

.

After the gears are aligned using appropriate rigid transformations and given revolute joints, the next step is to model the gear relations. Since the three gear differential is modelled using bevel gears we use a bevel constrain block. This block connects the left and the right gears to the middle gear. The connection is made at the point R, which is the center of the pitch circle for each of the gears.

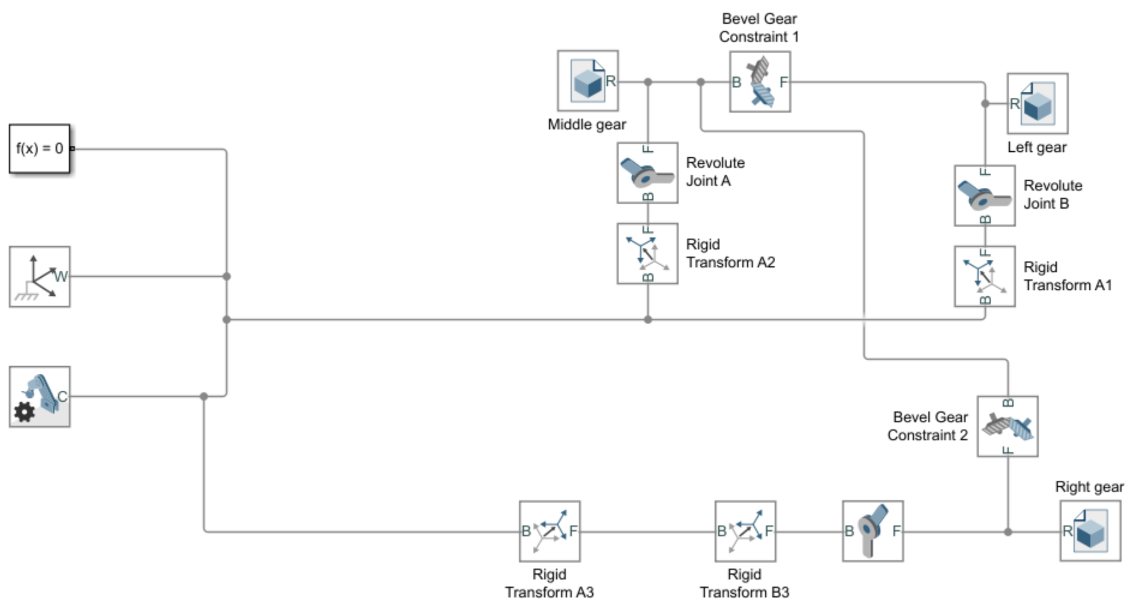

Figure 25: simscape multibody model of the differential gear

Simulation of the above three gear differential system is available below with initial conditions that the left gear is given an initial angular speed of 0.1 rev/s. We can see that the simulation behaves as expected, the left gear and right gear rotates in opposite directions with respect to the ground frame and the middle gear rotates with its axis perpendicular to the axes of the other two gears.

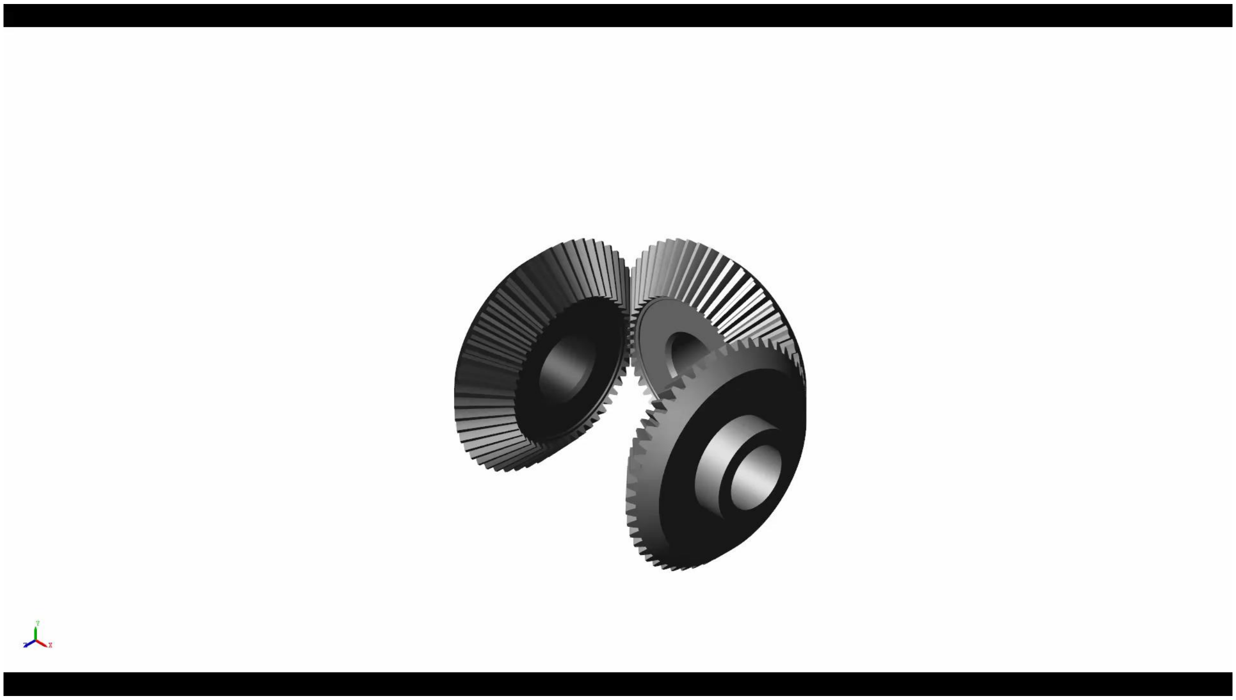

Figure 26: Simulation of differential gears(**click on the image to see the VIDEO**)

The next step after modelling and simulating the differential is incorporating it into the rocker bogie mechanism modelled in the previous section. This can be done by connecting the left and the right gears rigidly to the shafts connected to the rockers at either ends. While connecting the above differential it was realized that the radii of the gears has to be changed to ensure that they are aligned exactly with the shafts, also the radius of the middle gear must be greater than the radii of the other gears by a precise amount so that the middle gear is aligned exactly with the flange connected to the chassis. Hence a new model of the differential was created (shown below) which is incorporated into the rocker bogie and used in further simulations.

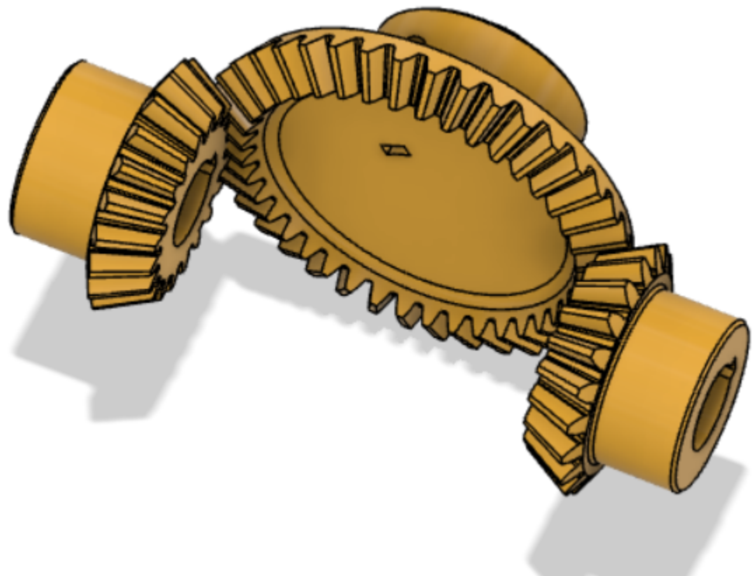

Figure 27

The pitch radius of the left and the right gears is 29.753
The pitch radius of the middle gear 52.351

The modelling of the gear and relations and simulation of the gear relations for this differential gear follows exactly the same process as described in the above differential gear. Once the gear relations are modelled and simulated this differential gear can be used in the multi terrain vehicle that was modelled by attaching the left and right gears rigidly to the left and right shafts respectively and connecting the middle gear to the flange in the chassis through a revolute joint. Once again it is emphasized that the distances between the shafts and the flange in the MTV model must be very accurate such that the gears attached to them have their teeth meshed but don't overlap. Extreme caution must be taken when this process is done.

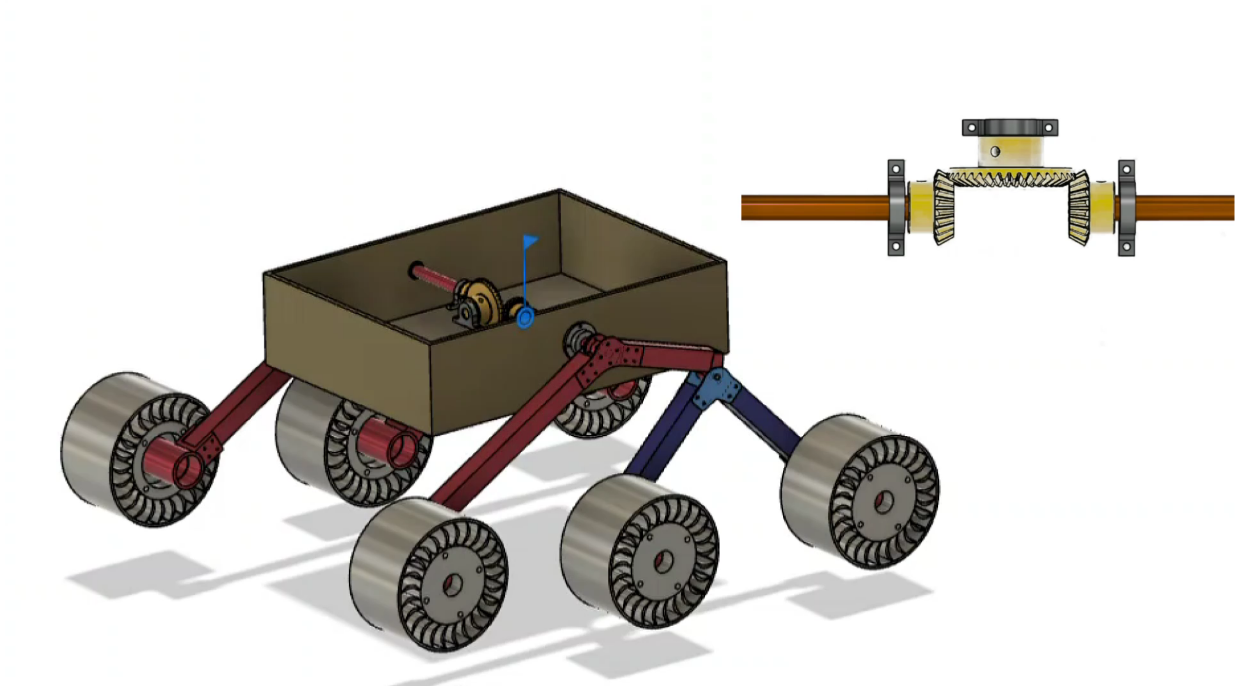

Figure 28: Simulation of differential gears(**click on the image to see the VIDEO**)

7.5 Simulation of the multi-terrain vehicle in different terrains

Simulation of the suspension system is one of the most important parts of any suspension analysis. Only a mathematical analysis is not sufficient as it is impossible to mathematically model different terrains and the intricacies involved in the traversal in those terrains. Hence a computer simulation using mechanical simulation softwares is of utmost importance.

After incorporating the differential into the multi-terrain vehicle the last and the most important thing left is to simulate the entire mechanism. The main performance criteria considered here is the distribution of normal forces on the wheels which is a good measure of the stability of the system. If the normal forces vary largely on different wheels then it causes a moment which may result in the toppling of the mechanism.

7.5.1 One sided obstacle

This test is designed to determine the efficiency of the differential gear. There is an obstacle on only one side of the multi-terrain vehicle, this causes uneven normal forces on either side of the vehicle and may cause the vehicle to topple without the presence of the differential.But the differential vastly reduces the toppling effect in the rocker bogie by balancing it.

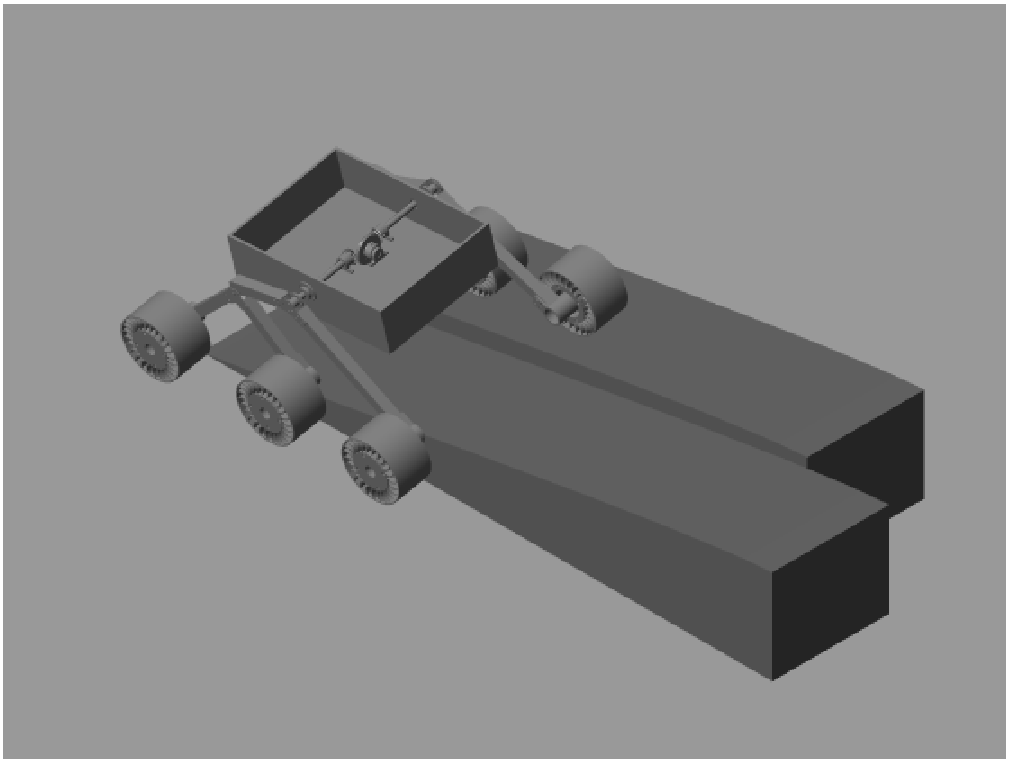

Figure 29: Simulation of MTV on one sided obstacle(**click on the image to see the VIDEO**)

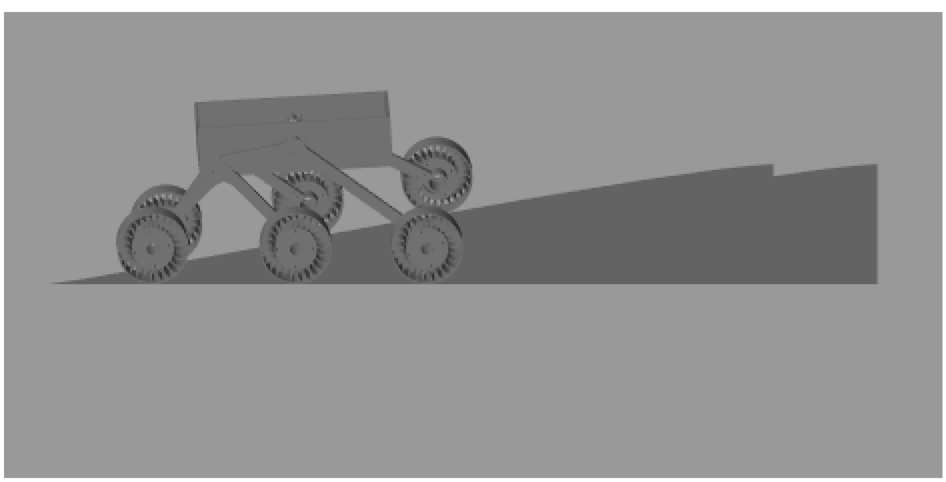

Figure 30: it can be seen in this side view that the chassis tilts only by a small angle due to the presence of the differential

Simulation parameters:

Maximum height of the obstacles in terms of wheel radius: The maximum height of the obstacle is 9.2 times the radius of the wheel.
Coefficient of static friction between the wheels and the obstacles: 0.3
Coefficient of static friction between the wheels and the ground: 0.3
Coefficient of kinetic friction between the wheels and the obstacles: 0.3
Coefficient of kinetic friction between the wheels and the ground: 0.3
Input torque provided to each wheel : 3Nm
Material used for the multi-terrain vehicle: stainless steel (Density 8000 Kg/m^3)
Acceleration due to gravity: 9.8 m/s^2
Damping effects of the joints are not considered as the objective of this simulation is to primarily determine the effect of differential. In the further simulations damping effects are considered.

Simulation run time: 15 secs.
The contact between the vehicle and the ground and the contact between the vehicle and the obstacle is modelled by the solver inherently as a spring damper system with stiffness and damping ratio of the order 10^6 and 10^4 to accurately model the contact between these surfaces.

The following graph is between normal forces on the left side of the vehicle(in N) (ie. the side that climbs over the obstacle) and time(in s). The values of the normal forces were calculated only at discrete times of interval 0.05s as the simulation and computation of normal forces for finer intervals or for continuous time would require way more memory and simulation time. This is why the following graph is discrete and not continuous. All further graphs are similar to this and not continuous for the same reason mentioned above.

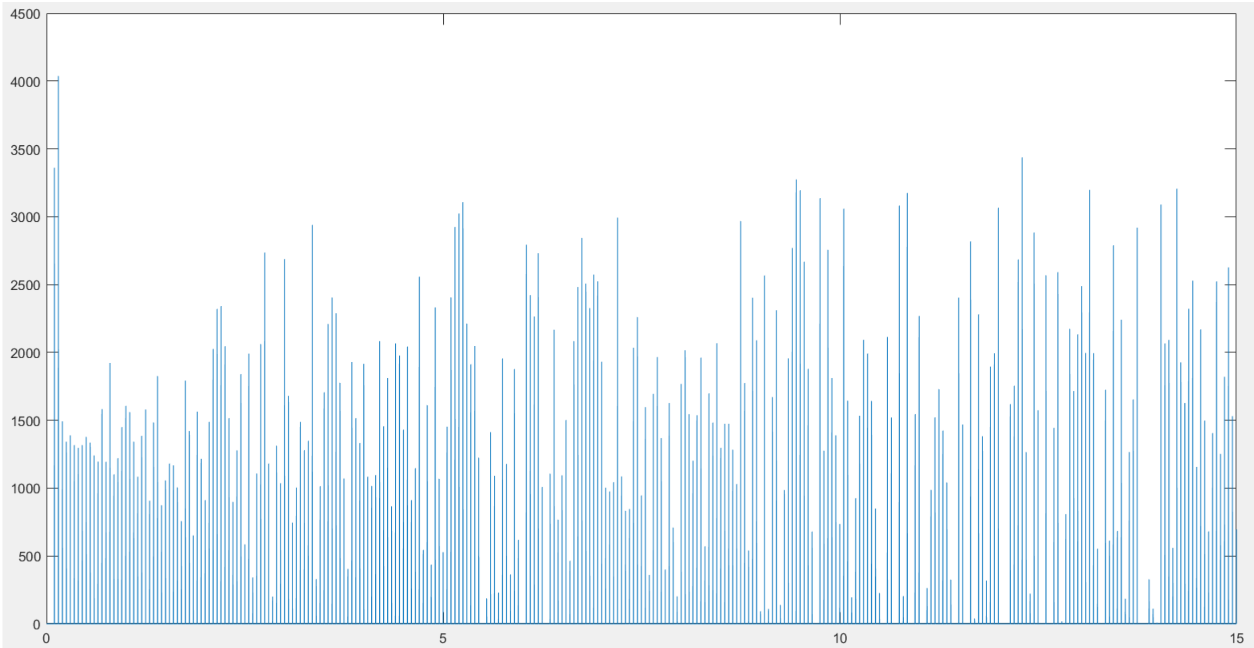

Figure 31: Plot of normal forces on left wheels of the vehicle(in N) vs time (in s)

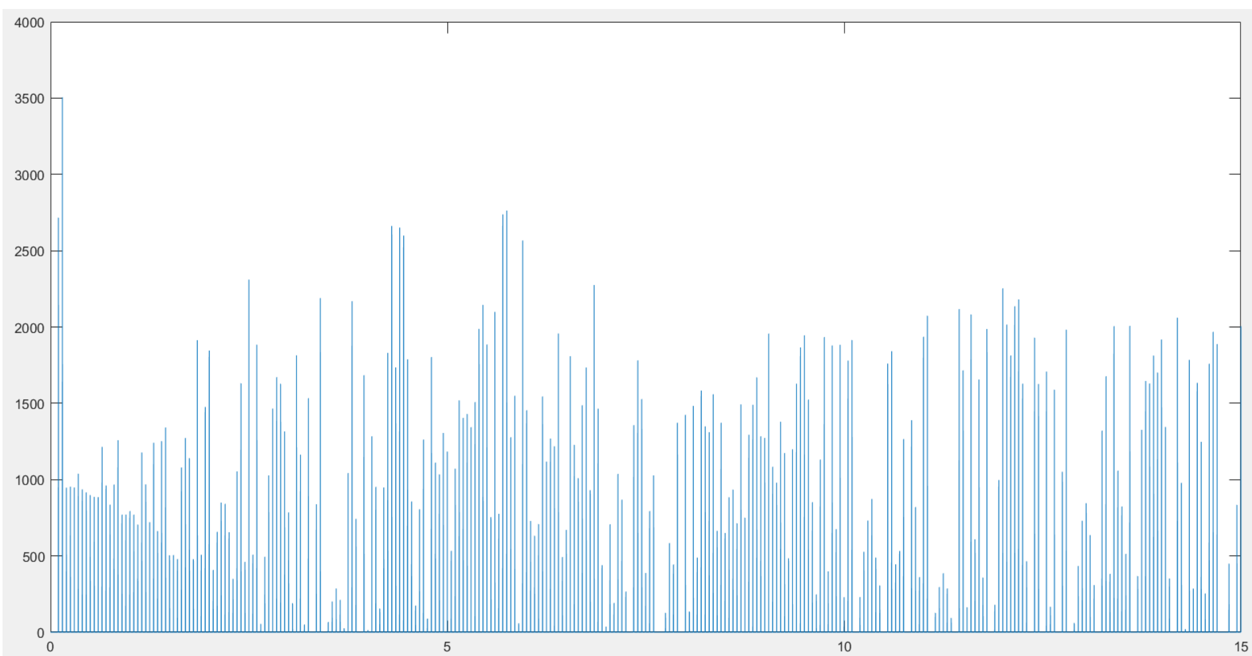

Figure 32: Plot of the normal forces on the right wheels(in N) vs time(s)

One can observe from the graph that the normal force on the left side is higher than on the right side which is expected because the obstacle is on the left side. The difference between the maximum normal forces is 510N.
The average of the normal forces on the right wheels is 1357N.

The average of the normal forces on the left wheels is 1781N.
The average difference of the normal forces on both sides is 424N.
This average difference of normal forces is a performance metric that can be used to analyze the effect and optimize the differential.
Without the differential this difference would increase further to the point that the vehicle wouldn't be able to land safely like it does in this simulation(full simulation available as a video above) and would topple even before traversing the full obstacle unlike the multi-terrain vehicle which seamlessly transitions over this obstacle.

7.5.2 Step Obstacle

Another challenging terrain type for multi terrain vehicles is step climbing. This challenge gives an evaluation of the performance of the rocker-bogie . This vertical motion also needs a high surface friction coefficient between wheel and the step.
The performance criterion in this simulation is the comparison of the normal forces on the front wheels, rear wheels and middle wheels which evaluates the stability of the vehicle against toppling forward.

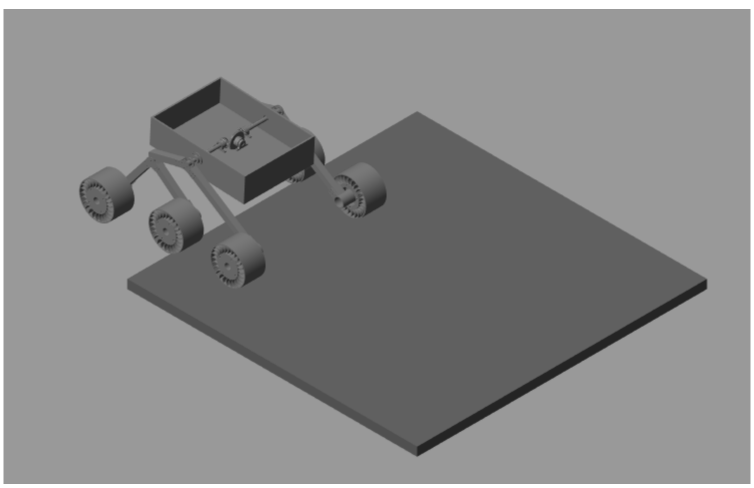

Figure 33: Simulation of MTV on step obstacle(**click on the image to see the VIDEO**)

Simulation parameters:
Height of the step: 7cm
Coefficient of static friction between the wheels and the step: 0.3
Coefficient of static friction between the wheels and the ground: 0.3
Coefficient of kinetic friction between the wheels and the step: 0.3

Coefficient of kinetic friction between the wheels and the ground: 0.3
Input torque provided to each wheel : 3Nm
Material used for the multi-terrain vehicle: stainless steel (Density 8000 Kg/m^3)
Acceleration due to gravity: 9.8 m/s^2
Damping effects of the joints are not considered as the objective of this simulation is to primarily determine the effect of the rocker bogie suspension.

Simulation run time: 15 secs.

The contact between the vehicle and the ground and the contact between the vehicle and the obstacle is modelled by the solver inherently as a spring damper system with stiffness and damping ratio of the order 10^6 and 10^4 to accurately model the contact between these surfaces.

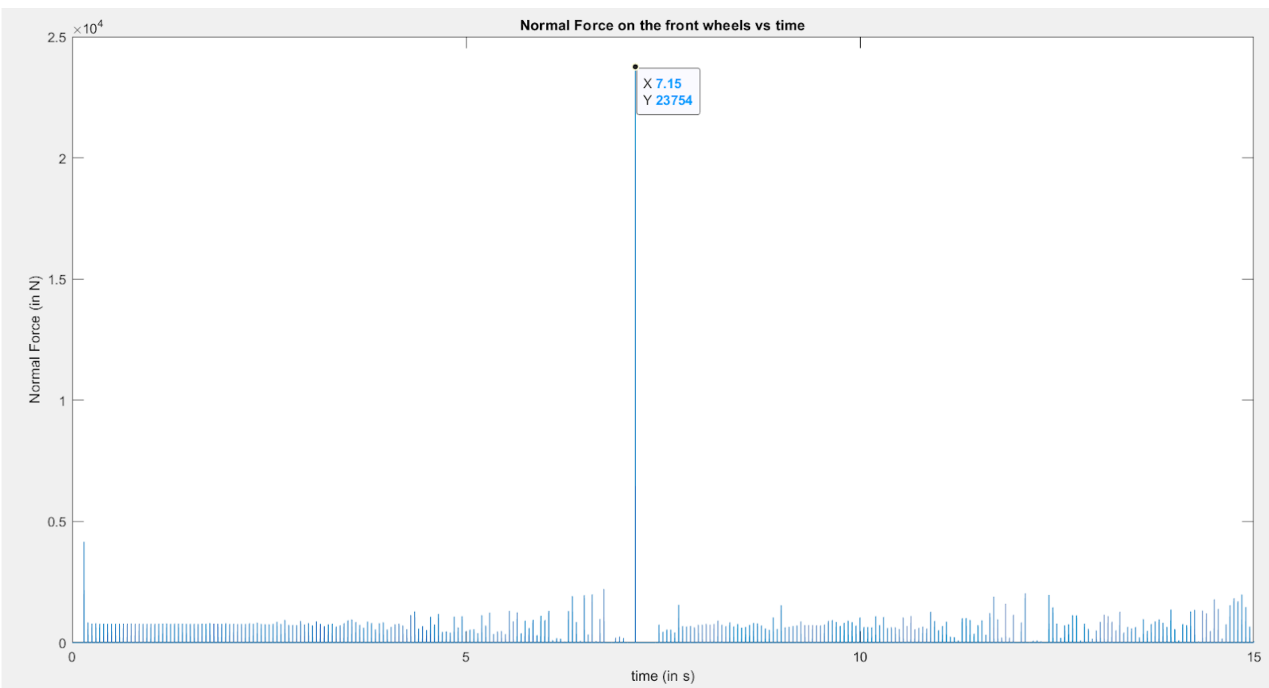

Figure 34: The spike in normal force arises when the wheel hits the step. The spike is maximum for front wheels because when the front wheels hit the step the speed of the vehicle is maximum

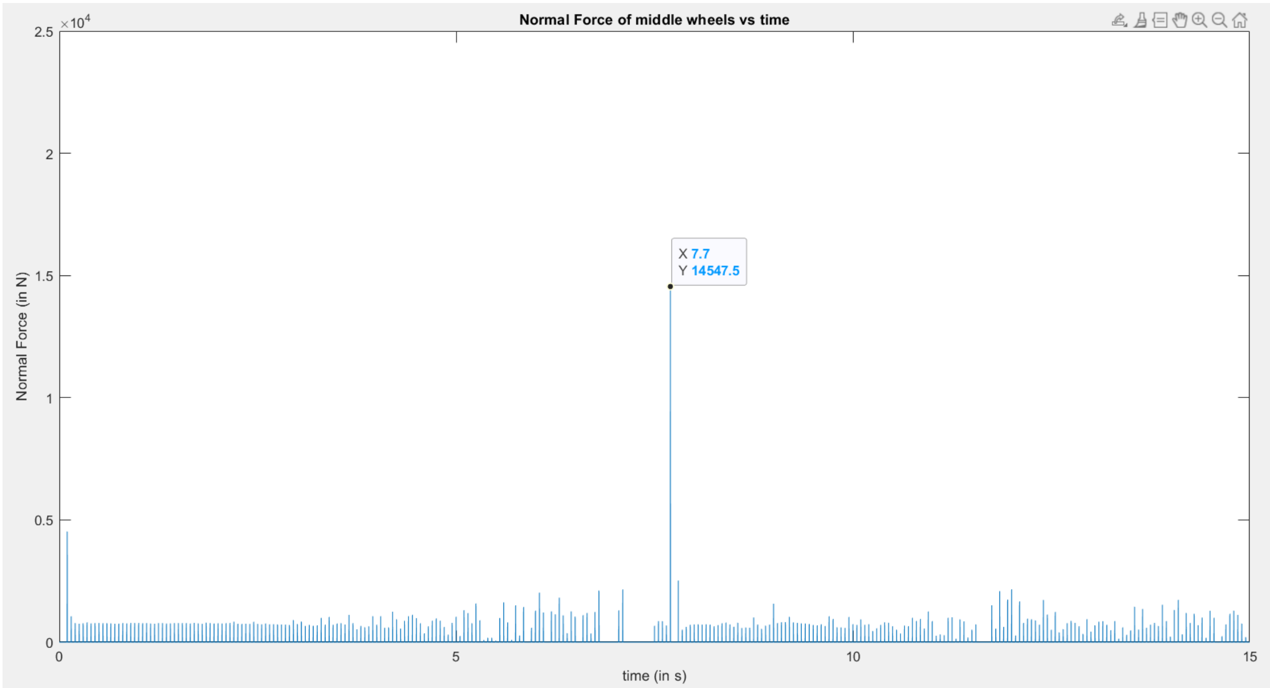

Figure 35

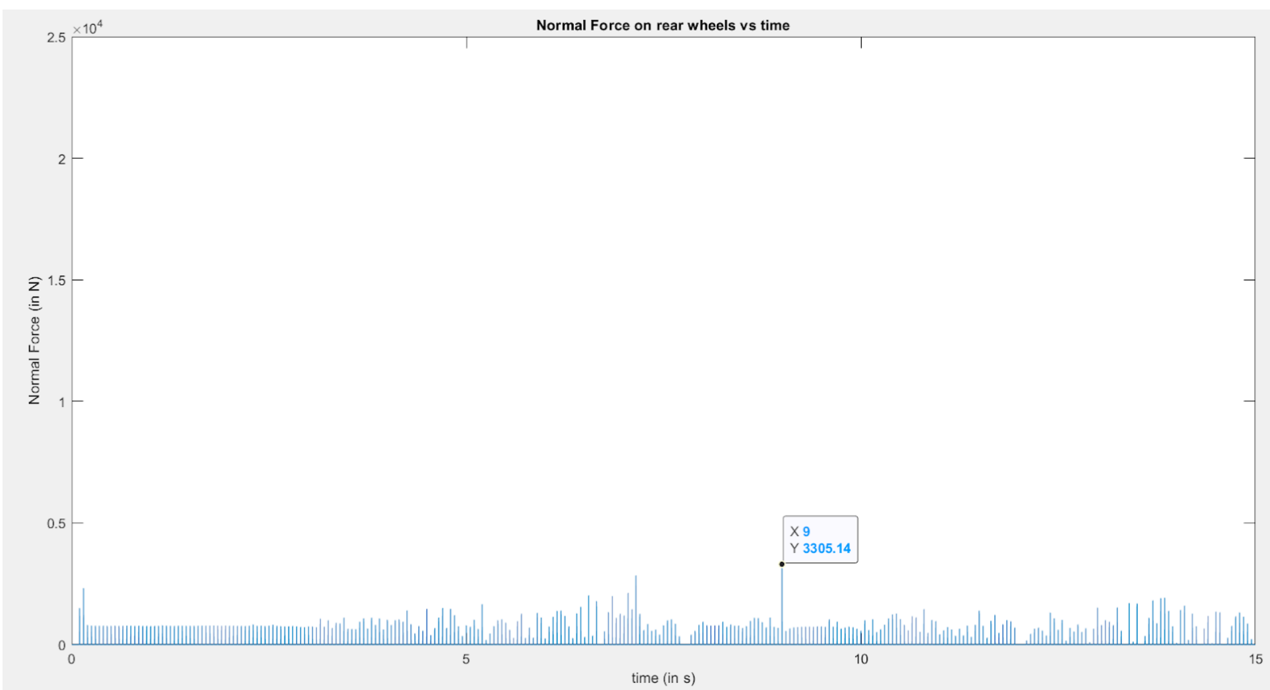

Figure 36: The spike in rear wheels is least because when the rear wheels hit the step the speed of the vehicle is already reduced due to the collisions of the front and the middle wheel

We can clearly see from the above normal force graphs that the normal force on the front, rear and the middle wheels are almost exactly identical in

most parts of the time. There is a spike in all three graphs, this spike can not be avoided as this spike arises when the wheels make contact for the first time with the step at the edge of the step. This spike is maximum for the front wheels because they make the first contact with the step.
The fact that the normal forces is almost identical at all times except the time when the wheels hit the step shows that the entire vehicle is stable and that there are no internal vibrations in the vehicle, if there were vibrations then the normal forces would have varied a lot but that isn't the case which itself is a testimony to the suspension quality of the rocker bogie. In all other spring based suspension systems vibration can't be avoided but since the principle behind rocker bogie is utilizing kinematic linkages for shock absorption we can be sure that there won't be any vibration which is also proven by the simulation results.

The above two simulations prove to be vital in understanding the role and realizing the importance of the differential and the rocker bogie suspension in multi-terrain vehicles. They also give a critical measure that can be used to quantify the performance of the differential and the rocker bogie suspension which is the normal force distribution and by this metric one can easily see that the proposed multi-terrain vehicle with rocker bogie suspension and the differential performs better than any other vehicles that are currently available.

8 RENDERED IMAGES ON DIFFERENT SURFACES

We have designed a MTV(multi-terrain-vehicle)which could traverse in several terrains carrying load without losing its balance.It is clearly a better replacement for the ATV's using the spring based differential as this reduces the wobbling to a much greater extent. We have attached the rendered images of the MTV in different terrains below.

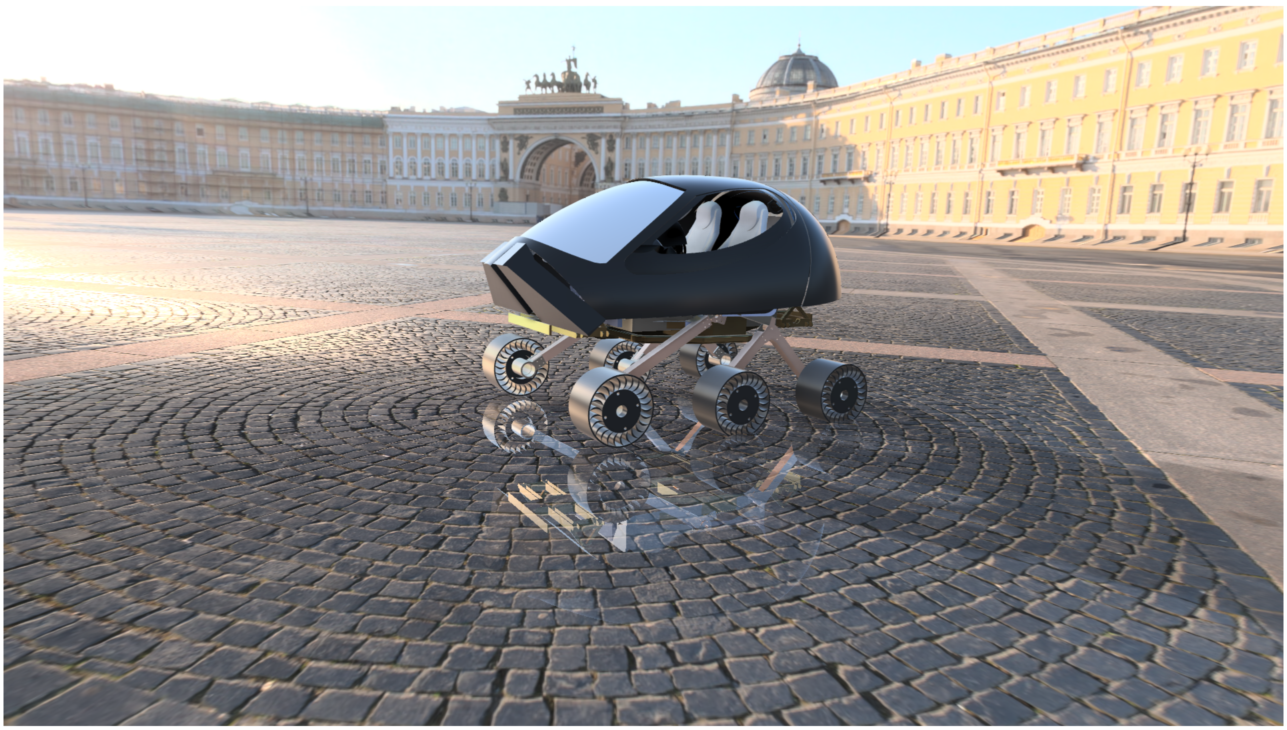

Figure 37: MTV on a irregular stony terrain

Figure 38: MTV on a grassy terrain

Figure 39: MTV on a icy terrain

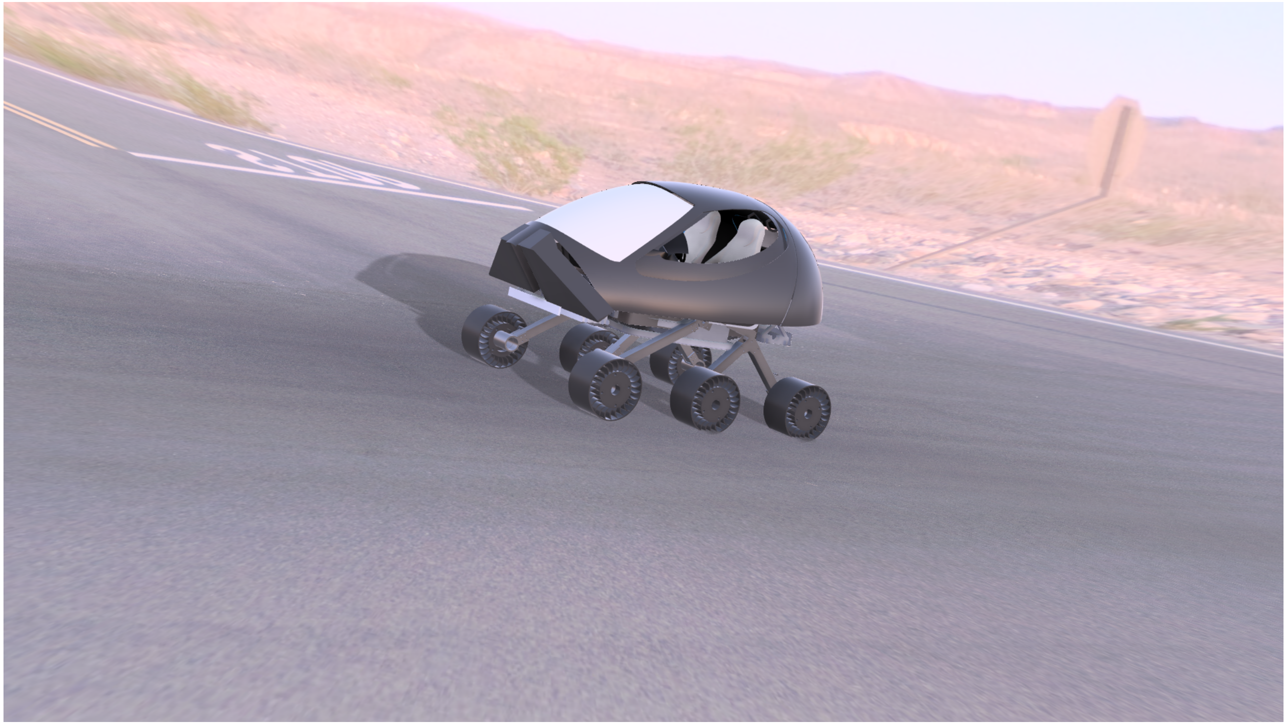

Figure 40: MTV on a hilly tar road terrain

Figure 41: MTV on a dry and sandy terrain (war fields)

9 CONCLUSION :

In this project report, we have done a complete kinematic and dynamic analysis of the rocker bogie mechanism. We were inspired by the Perseverance Rover launched in Jul 2020. We first did a literature search for information about the existing rocker bogie suspension systems. We then found the best softwares for modelling the rocker bogie, for simulating it and for running a dynamic and kinematic analysis of it. After multiple trials, we finally decided on FUSION 360 for modelling, MATLAB -Simulink and Simscape for simulations, and Onshape for importing models from FUSION 360 to MATLAB with joints intact. In the middle of our analysis, we came across a vital component of the rocker bogie called differential. We did our analysis with a 3 gear differential gear mechanism, for which we found that the best way to simulate the gear relations was in MATLAB itself hence it was done in MATLAB. A dynamic and kinematic analysis for the rocker bogie mechanism involving link lengths and obstacle sizes was then done in MATLAB, and we got definite results in terms of graphs of normal forces on the wheels and link analysis.

Then we incorporated the differential into our rocker bogie model to obtain the final prototype of this project with the rocker bogie and the differential. This prototype was then simulated on different terrains, normal force distribution was used as the critical performance metric. Upon this metric the proposed prototype performed better than the existing vehicles.

Somewhere in the middle of our project, we learnt that the current ATVs(All Terrain Vehicles) had a spring suspension system, and the key issue there in addition to balance was that while climbing over obstacles, the higher wheel had a greater traction than the lower wheel, which causes slippage. Seeing this, we decided that having a rocker bogie suspension system in an ATV (we named this new vehicle as MTV-Multi Terrain Vehicle) would be beneficial, especially if there was a need to transport medicines or relief supplies and food to either disaster-stricken areas, or for soldiers on the battlefield. Since there is no existing mechanism like this, we decided to incorporate this proposal as a novelty from our project.

10 REFERENCES:

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12.https://blogs.mathworks.com/student-lounge/2020/08/31/creating-virtual-

11 APPENDIX A

11.1 IDEATION:

We started our project around the time the Perseverance rover was launched by Nasa to Mars. Since this was something that interested all of us, we decided to look for a mechanism involved in the rover. This was how we came across the rocker-bogie mechanism, which is the most commonly used mechanism in rovers and many other vehicles which are used to traverse uneven terrains.

On seeing this rocker bogie, we decided to try to simulate what was seen in the rover as closely as possible, but we soon discovered that it was way more complex than it looked, with a very complex chassis. So we tried to simulate just the 6 wheel mechanism, with the differential to the best of our ability.

On doing some research into the rocker-bogie, we found that it could be used in rescue and search operations on Earth also since this is right now the most popular and widely used mechanism for traversal of uneven and dangerous paths, including but not limited to slippery regions like on mountain slopes and also uneven terrains in terms of obstacles(like on Mars).

We explored multiple softwares-AUTODESK FUSION 360 and Ansys rather extensively for our simulations and decided on MATLAB -SIMULINK and SIMSCAPE because of the variety of the libraries that are available in MATLAB which made simulations across terrains easier. We also managed to find reasonably well-explained tutorials for the same.Another software which we used especially towards the latter part of our project was ONSHAPE,where we could directly import models to MATLAB from FUSION 360 with the joints constraints done in ONSHAPE before importing rather than in MATLAB after importing.

One issue that we faced was that it was rather difficult to make our 3D model in MATLAB. What worked better for us was making the model in Fusion 360 and importing it as an ipt file to MATLAB. We found that we could then simulate the joints and contacts in MATLAB from the imported model. There was a slight nag with respect to the various transformations that we had to give for our imported model, but this was still easier than sketching our model in 3D in MATLAB.

We then decided on using another software Onshape for importing 3D models made in FUSION 360 with the joint constraints applied into MATLAB. We used FUSION 360 to model the differential and since we were not able to give contact constraints in Onshape.
Finally, for the kinematic analysis of the rocker bogie link lengths and the simulation of the rocker bogie, we used MATLAB and SIMULINK and SIMSCAPE once again.

Now, let us look at a more detailed analysis of all the parts of the rocker bogie suspension system with a differential gear along with our timeline of progress.

12 APPENDIX B

12.1 TIME-LINE OF OUR PROJECT:

12.1.1 Literature Search:

We consulted various papers for information and existing studies. There were many papers that we came across, but the main ones we used for our reference were one master thesis, the link for which is there at the ending. We also used various articles from research journals irjnet and iijsrt. From this, along with existing models that we found for the rocker-bogie, we made out a rough model of our rocker bogie. After the initial literature search stages, again

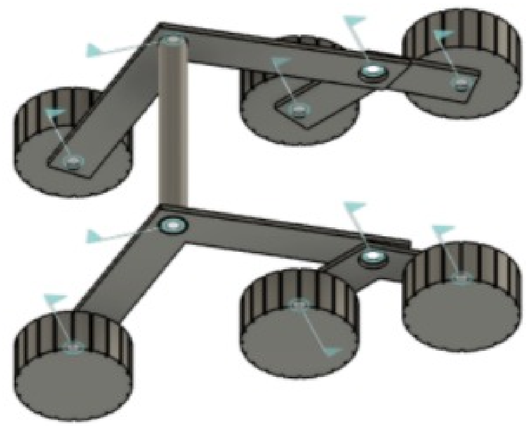


Figure 42: (visual of the mechanism we designed)

before the modelling of the differential, we had another literature search to understand the working the differential, for which we saw various papers and articles. Out of the two commonly used types of differentials, we decide on a differential gear rather than a differential bar mode.

12.1.2 Simulink and Simscape and importing to MatLab:

We modelled our cursory rocker bogie in fusion.
After that, we decided on using MATLAB SIMULINK and SIMSCAPE for modelling the contacts of the surface and wheels.
For this, we first started by modelling a simple wheel first, with 6 DOF, as used conventionally in MATLAB.

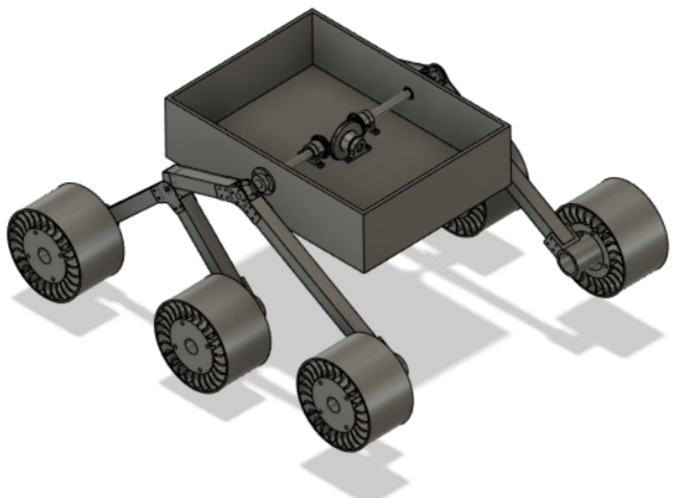


Figure 43: (visual of mechanism with the differential gear)

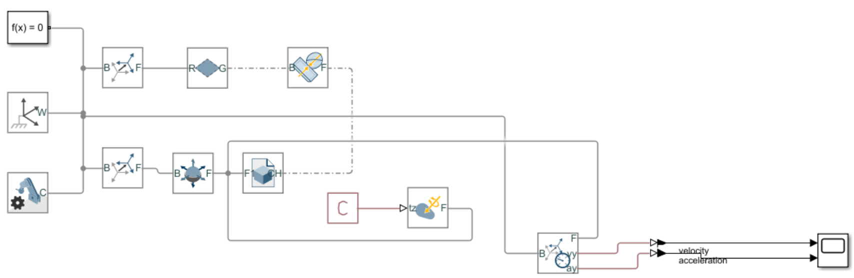


Figure 44: simscape model for a wheel rotating

In the ending, we managed to model a wheel rotating over the surface with all contacts properly specified.

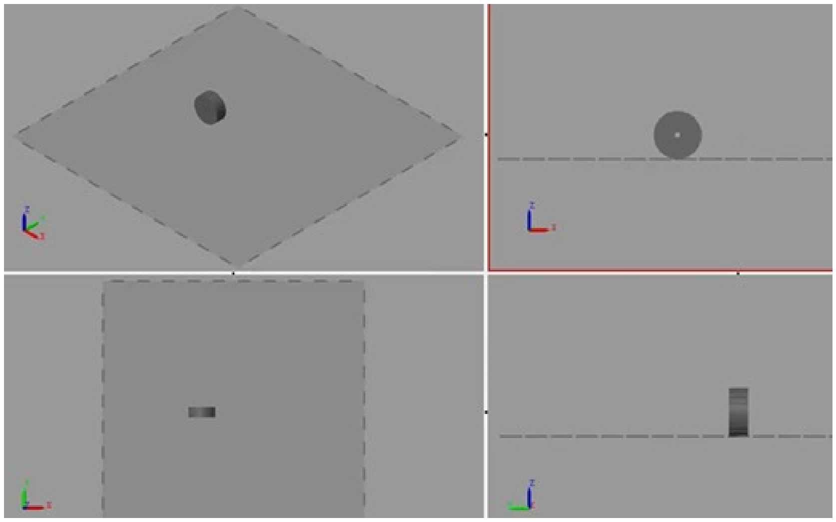


Figure 45: Single wheel simscape model (click on the image to see the simulation)

Subsequent modelling of this was for a simple chassis with 4 wheels. The important things that we had to consider carefully were that different wheels may get oriented in different directions when imported from fusion first, and the transformation that had to be done for these had to be done observing how it was originally imported from FUSION 360 as a .stl file.

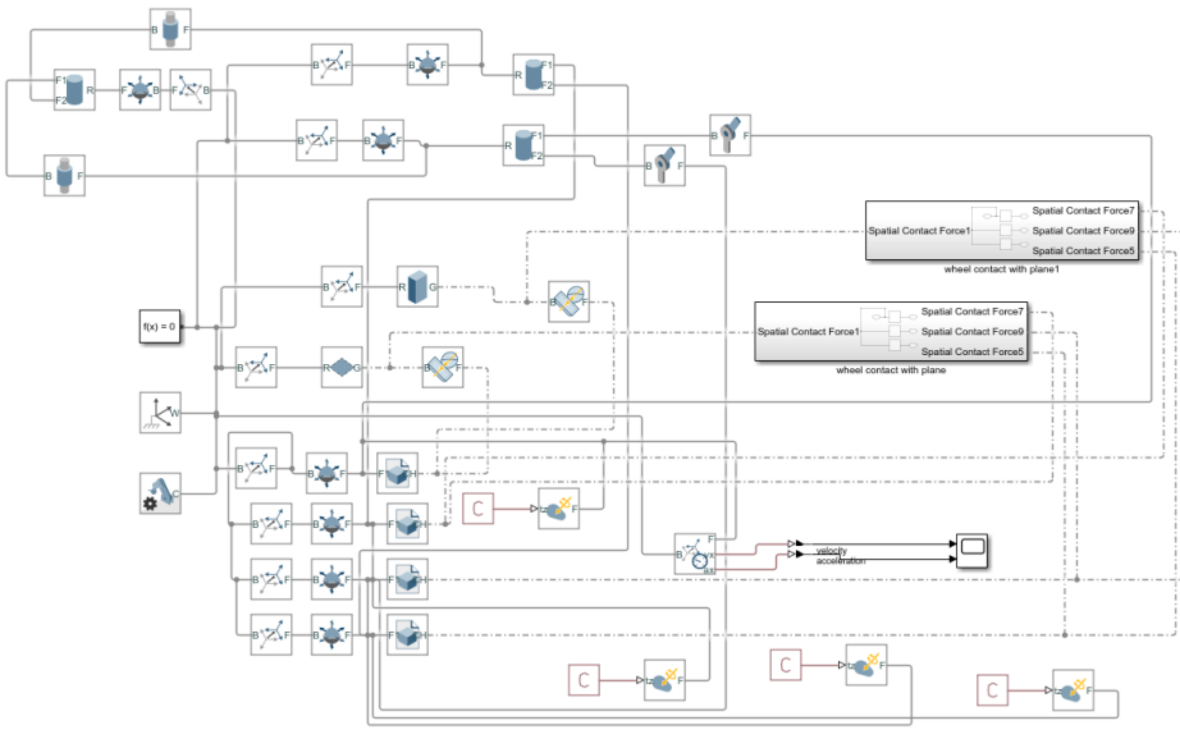
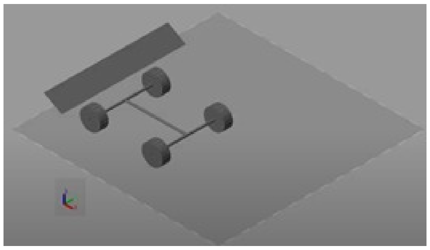
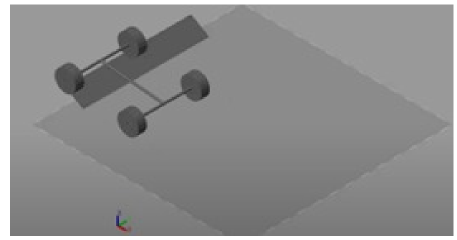


Figure 46: 4 wheeled simscape model (click on the image to see the simulation)

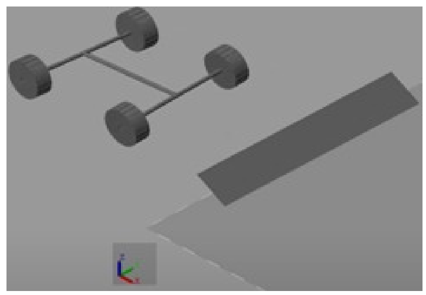
We also decided to have a simple obstacle in the form of an incline. Also, at this stage, we had 2 wheels on 2 axles each, connected by a connecting rod.



(a) Going over smooth surface



(b) Climbing the incline



(c) Going down the incline

Figure 47: First 4 wheeled simscape model simulation

As expected, on crossing the incline the entire set-up had significant wobbling and bouncing, since there was no mechanism like a differential in place to ensure that the chassis was straight.

The next step that we did here was trying to import our initial rocker-bogie model, on which we had not done the link length analysis yet, but were first trying to see how to import the model with all the wheels and links constrained properly. For this, we had applied all the joints and constraints in FUSION 360 first, as it was found that applying the joint constraints was much easier in ONSHAPE rather than in MATLAB.

Here, it was observed that the joints in between the 2 sides of the rocker-bogie had to be a cylindrical joint, and not spherical to keep the orientation of the chassis upright.

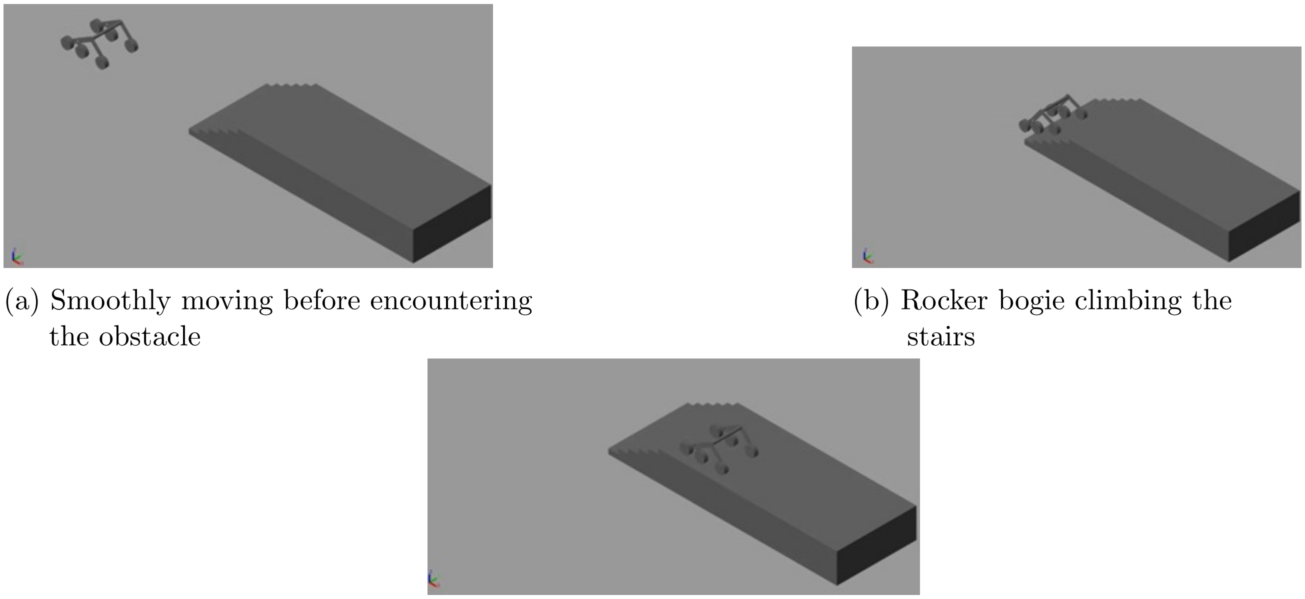
Also, at this stage, in place of a rather complex differential mechanism, we simply used a connecting rod. One thing that we observed was that at this stage the bouncing was comparatively lesser than the previous case, and this was expected since there was a chassis with higher inertia in place and we had also increased the shock-absorbing ability of the wheel by increasing the damping coefficient.

Figure 48: Travelling smoothly before the obstacle

Also, it was found that importing as a .ipt file rather than a .stl file allowed us to interact with the imported model, and change the dimension of the imported model.

Figure 49: We can see the rocker-bogie mechanism climbing the step

In ipt we were able to select reference axes for each wheel individually. The axis in .stl file was found only at the centroid, whereas in .ipt file the reference axis could be chosen individually for each imported part separately, and so we could choose the required reference axis for each of the components separately based on the respective geometric feature.



(a) Smoothly moving before encountering the obstacle

(b) Rocker bogie climbing the stairs

(c) Smooth movement after climbing stairs

Figure 50: Climbing multiple stairs

At the next stage, we now incorporated multiple steps as obstacles for the rocker-bogie imported into MATLAB. This was when the absence of a differential (at this stage) was seen since on climbing the steps there was rather a prominent wobbling.

12.1.3 Modelling of the differential

The differential is one of the key components of a rocker bogie system. As already explained earlier in detail, the differential in the most basic terms helps to maintain balance by maintaining contact between all the wheels and the surface.
We had decided on a differential gear mechanism for our project. When we first tried with differential gear with 4 gears, we were able to get 3 of the 4 gears to rotate properly, with their respectively stationary flanges as they should be. However, we faced issues with the 4th gear, as here instead of the gear rotating, the flange was rotating, and this made the entire chassis rotate too.
Grounding the flange grounded our entire chassis too, and since we need the chassis to be able to move in a rocker bogie, we decided to go with a 3 gear differential mechanism. We were then able to smoothly model our 3 gear

differential mechanism in FUSION 360, which we used in our rocker bogie finally.

12.1.4 ONSHAPE Simulations:

One issue which we faced was that we were not able to import components with the joint constraints applied into MATLAB from FUSION 360. Due to multiple translations and transformations of the axis which were required if we were to model the joints in MATLAB, we felt it better suited to directly import a model with the joints applied elsewhere.
For this we used a software called Onshape. We imported the FUSION 360 model to Onshape and then applied the various joint constraints in Onshape, and from here we were directly able to import models with the joint constraints applied into MATLAB. However, we were unable to use this for specifying the contact constraints in our differential between the various gears involved, as Onshape only helped us to import joint constraints. For this reason, we had to model the various contact constraints for the 3 gears involved in the differential gear mechanism in MATLAB directly.

13 APPENDIX C

13.1 Contributions of team members

SAKETH D(ED19B010)

- Literary search, Report writing and Documentation, Kinematic and Dynamic Analysis using Lagrangian mechanics

KESHAVA KRISHNA S(ED19B015)

- Literary search, Report writing and Documentation, CAD modelling of MTV,Modelling of joints in MTV,Simulation and Modelling of differential gears,Simulation of the MTV in different terrains

SIVAHARI A(ED19B032)

- Literary search, Report writing and Documentation, CAD modelling of MTV, Modelling of joints in MTV, Simulation and Modelling of differential gears

SURYA KUMAR M(ED19B034)

- Literary search, Report writing and Documentation,CAD modelling of MTV, Rendering of the final prototype(Different terrains),Compiling the report in LaTeX.

ANIRUDH R(ED19B043)

- Literary search, Report writing and Documentation, Suspension system and current suspension Problems, CAD modelling of MTV